CS 111: Sample Problems for Midterm 1

See Exam e01 on the course GitHub page for midterm rules, syllabus, and more sample problems.

1. Let x, p, and A be defined by the following numpy statements:

```
x = np.array([31, 41, 59])
p = np.array([2, 0, 1])
A = np.array([[1, 2, 3], [4, 5, 6], [7, 8, 9]])
```

What is the value of y after each of the following?

1a. `y = x[p]`

1b. `y[p] = x`

1c. `y = A[[2, 0], 1:]`

1d. `y = A[:2, p]`

2. Suppose that:

```
A = np.array([[1, 1, 1], [1, 2, 3], [1, 3, 6]])
x = npla.solve(A, [3, 8, 15])
y = A @ [3, -1, 0]
L, U, p = LUfactor(A)
B = linalg.cholesky(A, lower = True)
C = linalg.inv(A)
```

Match each of x, y, L, U, p, B, C with one of the following:

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & .5 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -3 & 1 \\ -3 & 5 & -2 \\ 1 & -2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 5 \\ 0 & 0 & -.5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$$

$$(2, 1, 0)^T$$

$$(0, 1, 2)^T$$

$$(0, 2, 1)^T$$

3. Suppose A is an n by n symmetric, positive definite matrix and b is a column vector of dimension n. Recall that the (lower triangular) Choleksy factor L of A satisfies $LL^T = A$. Fill in the blanks below to compute the solution x to $Ax = b$. You may use L but not A.

```
L = linalg.cholesky(A, lower=True)


y = ______________________________


x = ______________________________
```

4. True or false:

- LU factorization without pivoting works on every nonsingular square matrix.
- LU factorization without pivoting works on every symmetric positive definite matrix.
- LU factorization with partial pivoting works on every nonsingular square matrix.
- Cholesky factorization requires the matrix to be square.
- Cholesky factorization works on every symmetric square matrix.
- Cholesky factorization works on every symmetric positive definite matrix.
- Conjugate gradient requires the matrix to be symmetric positive definite.
- QR factorization requires the matrix to be square.

5a. Let A be a 12-by-5 matrix. What are the dimensions of Q and R after `Q, R = linalg.qr(A)`?

5b. What are the dimensions of Q and R after `Q, R = linalg.qr(A, mode = 'economic')`?

6. (This is a variation on an example I used in class.) Two hikers climbed to the tops of Montecito Peak and La Cumbre Peak. The Montecito Peak hiker's altimeter showed her altitude as 3218 feet, and she triangulated sightings that indicated La Cumbre Peak was 792 feet above her and Cathedral Peak was 115 feet above her. The La Cumbre hiker's altimeter was broken, but his triangulation indicated that Montecito Peak was 771 feet below him and Cathedral Peak was 661 feet below him.

Let x_0, x_1, and x_2 be the altitudes of Montecito Peak, La Cumbre Peak, and Cathedral Peak, respectively. Set up the least squares problem that combines the two hikers' five measurements to find the best approximation to the altitudes of the three peaks as the solution to $Ax \approx b$. What is the matrix A? What is the right-hand side vector b? What numpy routine(s) would you use to solve the problem for x?

(In solving this sample problem, you should go ahead and compute the answer x; but on the exam you won't need to actually run the computation.)