7.4 DETERMINANTS & CRAMER'S RULE

1. DETERMINANT: AN ASSIGNED NUMBER TO A NXN (SQUARE) MATRIX.

2. FINDING THE DETERMINANT OF A 2X2 MATRIX.

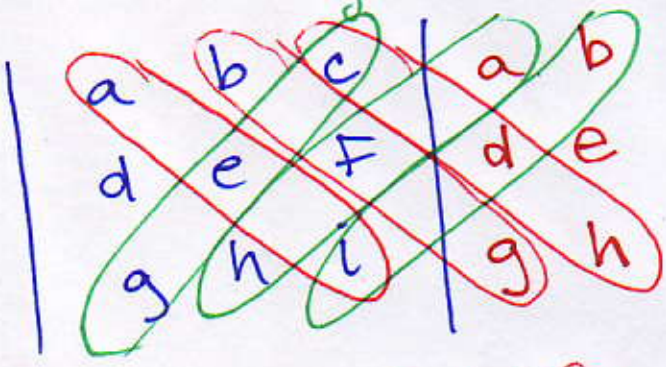
$$\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$ad - bc = \det$

EX. $\begin{vmatrix} 6 & -3 \\ 2 & 3 \end{vmatrix}$

$18 + (+6) = \boxed{24}$

3. FINDING THE DETERMINANT OF A 3X3 MATRIX.



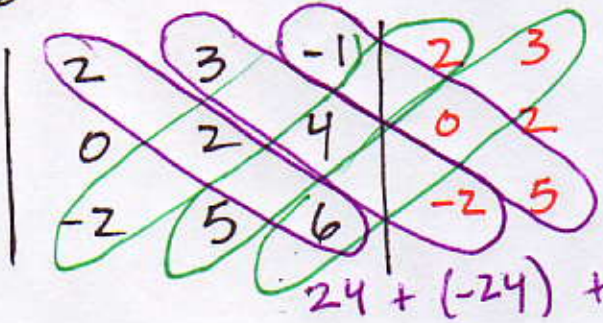
gec + hfa + idb = TOTAL (UP)

DOWN - UP = detA

aei + bfg + cdh = TOTAL (DOWN)

4 + 40 + 0 = 44 (UP TOTAL)

EXAMPLE



$0 - 44 = \boxed{-44}$

24 + (-24) + 0 = 0 (DOWN TOTAL)

4. ANOTHER WAY ... COFACTOR & MINOR

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$a \cdot (-1)^{(1+1)} \begin{vmatrix} e & f \\ h & i \end{vmatrix} + b(-1)^{(1+2)} \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c(-1)^{(1+3)} \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

Ex.

$$\begin{vmatrix} 2 & 3 & -1 \\ 0 & 2 & 4 \\ -2 & 5 & 6 \end{vmatrix}$$

$$2 \begin{vmatrix} 2 & 4 \\ 5 & 6 \end{vmatrix} - 0 \begin{vmatrix} \cdot & \cdot \\ \cdot & \cdot \end{vmatrix} + (-2) \begin{vmatrix} 3 & -1 \\ 2 & 4 \end{vmatrix}$$

$12-20$ $\qquad$ $12-(-2)$

$$2 \cdot -8$$

$$-16 + 0 + (-28) = \boxed{-44}$$

5. CRAMER'S RULE : ANOTHER WAY TO SOLVE A SYSTEM

$$\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$$

CONSTANTS

COEFICIENT MATRIX $\begin{vmatrix} a & b \\ d & e \end{vmatrix} = D$

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D}$$

$$x = \frac{\begin{vmatrix} c & b \\ f & e \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a & c \\ d & f \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}$$

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$$\begin{cases} 2x - y = -9 \\ x + 2y = 8 \end{cases}$$

$$X = \frac{Dx}{D} = \frac{\begin{vmatrix} -9 & -1 \\ 8 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix}} \quad -18 + (+8) \quad \frac{-10}{5}$$

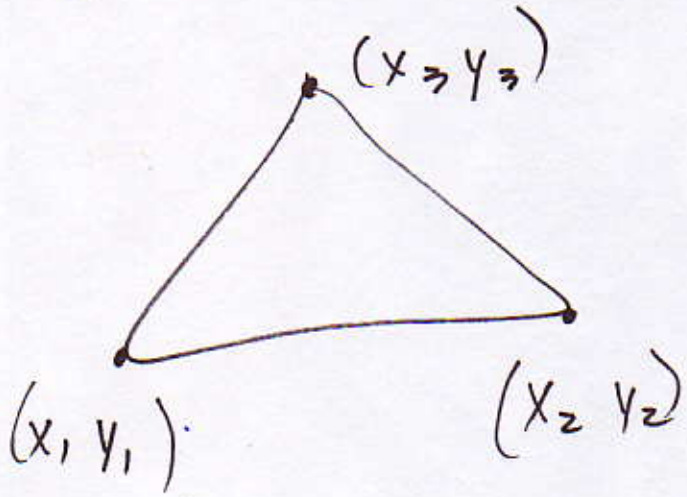
$X = -2$

$$y = \frac{Dy}{D} = \frac{\begin{vmatrix} 2 & -9 \\ 1 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix}} \quad 16 - (-9) = \frac{25}{5}$$

$4 - (-1) = 5$

$y = 5$

6. Area of a Triangle



$$A = \pm \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Ex. $(1,2)$ $(3,6)$ $(-1,4)$

$-6 + 4 + 6 = 4$

$$A = \pm \frac{1}{2} \begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ -1 & 4 & 1 \end{vmatrix}$$

$16 - 4 = 12$

$6 + (-2) + 12 = 16$

$12\left(\frac{1}{2}\right) =$ **6 sq units**