10/16

4.1 x exceeds its square by

$f(x) = x - x^2$ if $f(x) > 0$

$\frac{1}{10} > \frac{1}{100}$ by $\frac{9}{100}$ 2 doesn't exceed $2^2 = 4$

-3 doesn't exceed $-3^2 = 9$

$x < 0 \Rightarrow \underline{x} < 0 < \underline{x}^2$

$x > 1 \Rightarrow x \cdot x > x \cdot 1$ $x^2 > x$
>0

So only for $0 < x < 1$

$x^2 = x \cdot x < x \cdot 1 = x$

$f'(x) = 1 - 2x$ exists everywhere

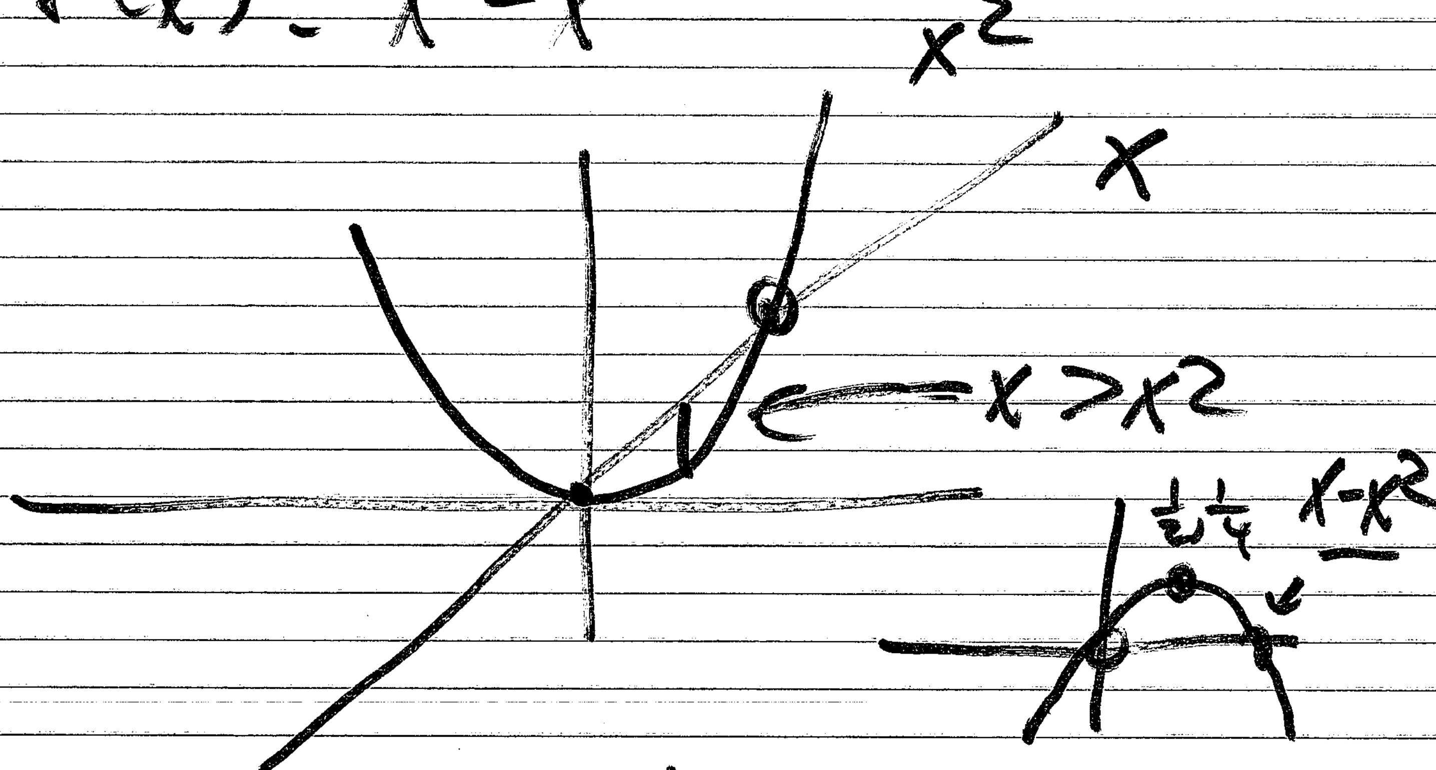
$f'(x) = 0$: $2x = 1$ $x = \frac{1}{2}$ $f(\frac{1}{2}) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

endpoints $f(0) = 0 = f(1)$.

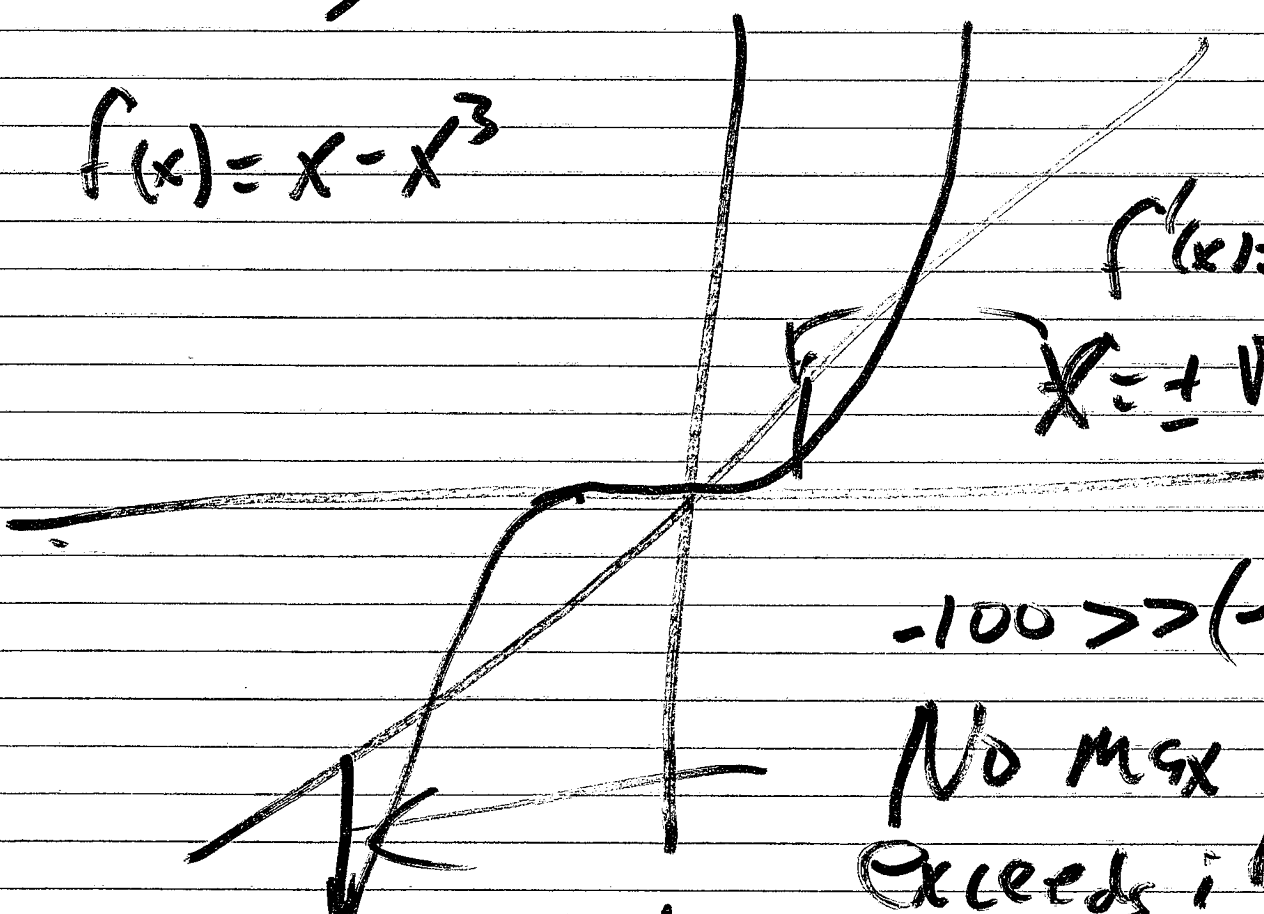
$x = \frac{1}{2}$ exceeds its square by max of $1/4$

4.1

$f(x) = x - x^2$



$f(x) = x - x^3$



$f'(x) = 1 - 3x^2 \stackrel{?}{=} 0$

$x = \pm\sqrt{1/3}$

$-100 >> (-100)^3$

No max x

Exceeds its cube

unless also $x > 0$!

10/18

$$f(x) = \frac{x'}{1+x^2} = \frac{(x-0)'}{x^2+1}$$

$$\frac{\sqrt{3}}{1+3} = \frac{\sqrt{3}}{4}$$

$$\frac{1}{1+1^2}$$

$$\left(1, \frac{1}{2}\right)$$

$$\frac{-1}{1+(-1)^2} = \frac{-1}{2}$$

$$f'(x) = \frac{(1+x^2)\cdot 1 - \overset{-2x^2}{x\cdot 2x}}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$$

$$= \frac{(1-x)(1+x)}{(1+x^2)^2} = 0 \text{ when } x = 1, -1$$

$$= -\frac{(x-1)(x-(-1))}{(x^2+1)^2}$$

$$f'(x) = \frac{1-x^2}{(1+x^2)^2} = (1-x^2)(1+x^2)^{-2}$$

$$f''(x) = -2x(1+x^2)^{-2} + (1-x^2)\cdot -2(1+x^2)^{-3}\cdot 2x$$

$$= \frac{-2x}{(1+x^2)^3}\left[(1+x^2) + 2(1-x^2)\right]$$

$+2-2x^2$

$3-x^2$

$$= \frac{+2x}{(1+x^2)^3}(x^2-3) = 0$$

$= 0$

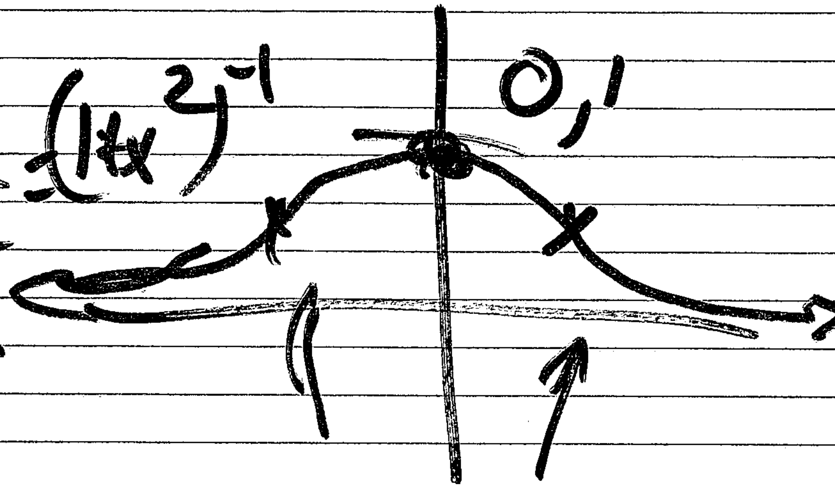
at $x = 0, \pm\sqrt{3}$

$$f'(x) = \frac{1-x^2}{(1+x^2)^2}$$

$$f''(x) = \frac{(1+x^2)^{\cancel{2}\,1} \cdot -2x - (1-x^2) \cdot 2 \cdot \cancel{(1+x^2)} \cdot 2x}{(1+x^2)^{\cancel{4}\,3}}$$

$$= \frac{-2x}{(1+x^2)^3}\left[(1+x^2) - 2(1-x^2)\right]$$

$$f(x) = \frac{1}{1+x^2} = (1+x^2)^{-1}$$



$$x = \pm\sqrt{\frac{1}{3}} = \frac{\pm 1}{\sqrt{3}} = \frac{\pm\sqrt{3}}{3}$$

$$f'(x) = -(1+x^2)^{-2} \cdot 2x$$

$$= -2\left[\frac{x}{(1+x^2)^2}\right]$$

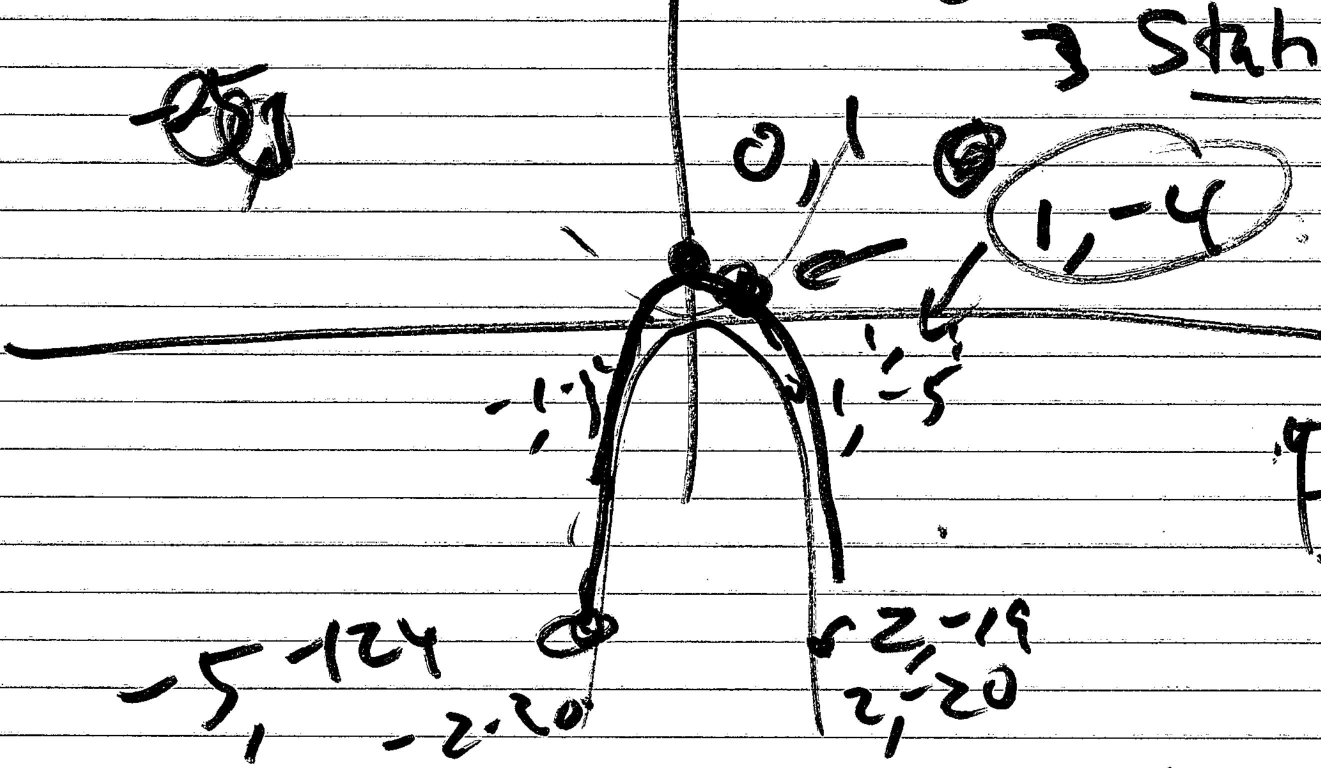
$$f''(x) = -2\left[\frac{(1+x^2)^{\cancel{2}\,1}\cdot 1 - x\cdot 2\cancel{(1+x^2)}\cdot 2x}{(1+x^2)^{\cancel{4}\,3}}\right]$$

$$= -2\left[\frac{1+x^2-4x^2 = 1-3x^2}{(1+x^2)^3}\right]$$

4.2 $f(x) = 1 - 5x^2 = -5x^2 + 1$.

$[-5, 1]$

. Critical points $\begin{cases} 1 & \text{boundary endpoints} \\ 2 & \text{singular pts} \\ 3 & \text{stationary} \end{cases}$



Left over — not end/boundary
go left and right
f' exists and is >0
or <0

$f'(x) = -10x = 0$ when $x = 0$

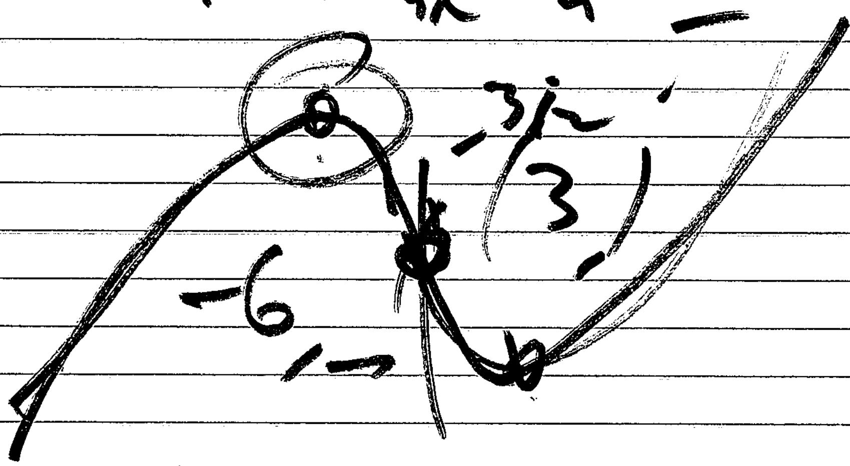
$f(-5) = -124$ $f(0) = 1$ $f(1) = -4$

3 $f(x) = 2x^3 + 9x^2 - 108x - 6$

decreasing on $[-6, 3]$

Increasing on $(-\infty, -6)$ and $(3, \infty)$

Local max at -6



$f''(x) = 12x + 18$

$= 0$ $\quad x = \frac{-18}{12} = \frac{-3}{2}$

$$f'(x) = 6x^2 + 18x - 108$$
$$= 6(x^2 + 3x - 18)$$
$$= 6(x + 6)(x - 3)$$

10/9

$f(x) = x^4 (x-1)^7$ on $[-12, 10]$

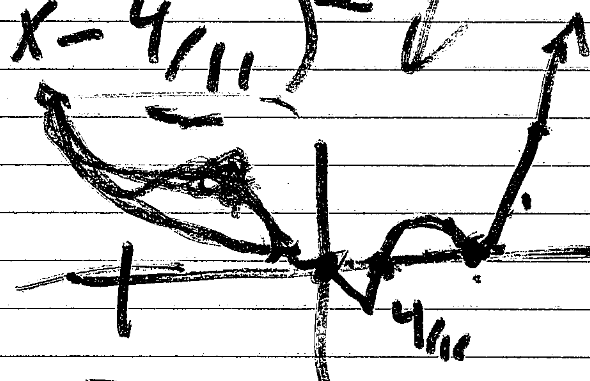
$f'(x) = 4x^3(x-1)^7 + x^4[7(x-1)^6]$

$= x^3(x-1)^6[4(x-1)+7x]$

$= x^3(x-1)^6[11x-4]$ $f'(x)$ $11x^{10}$

$= 11(x-0)^3(x-1)^6(x-4/11)^1$

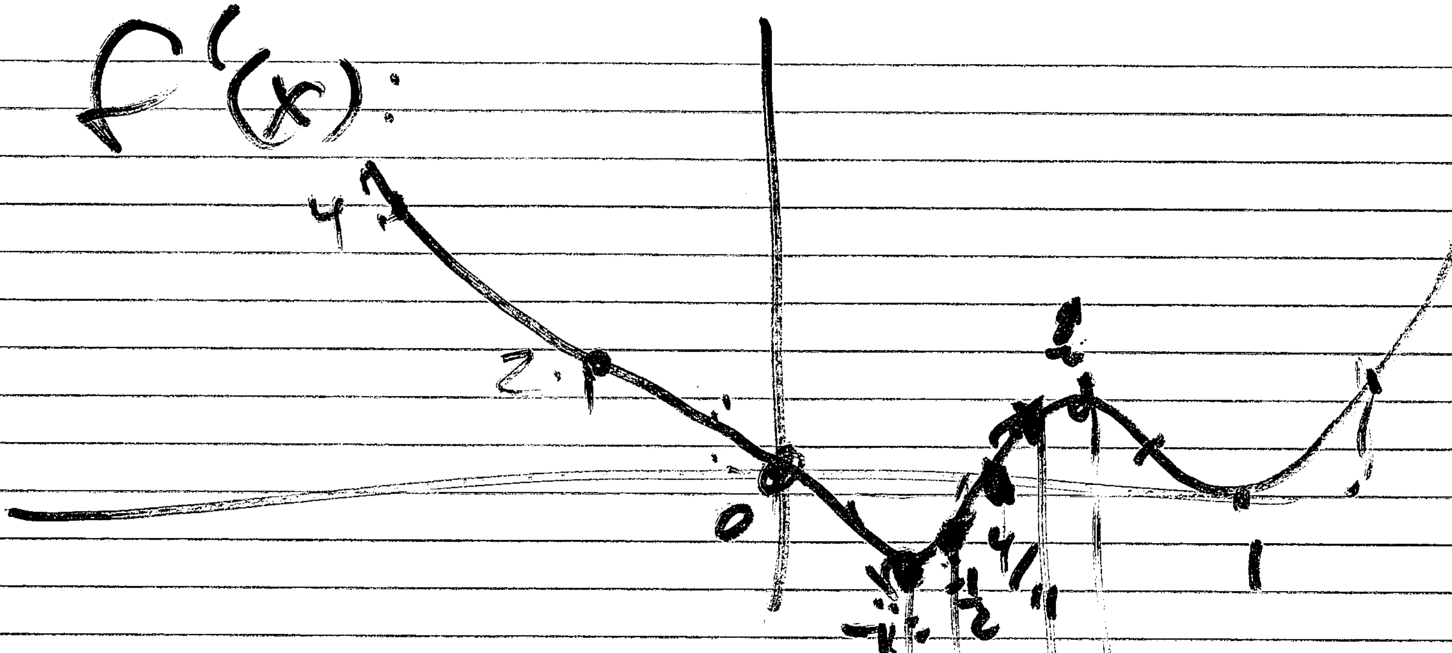
$0, \ 4/11, \ 1$



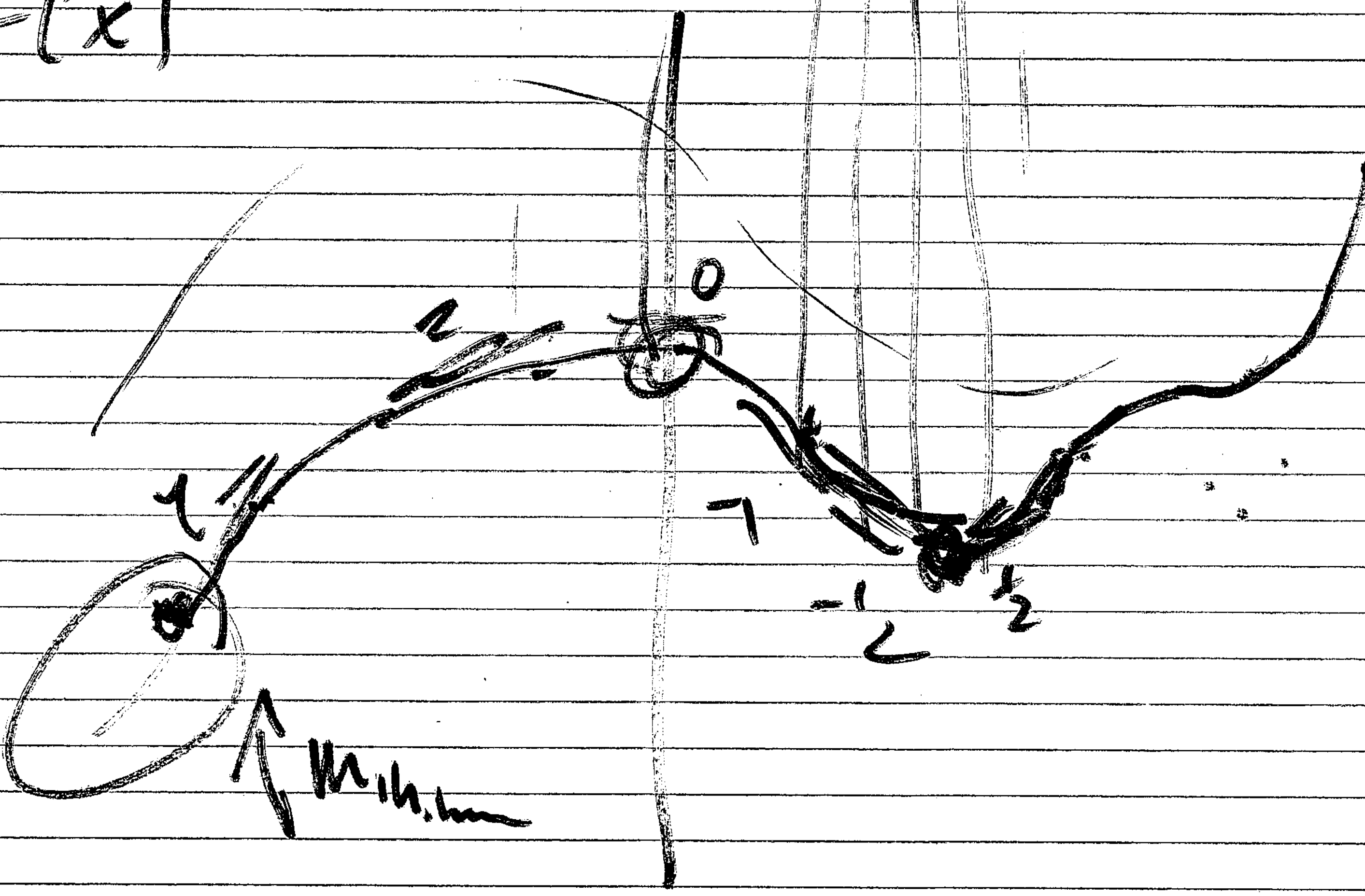
Inc $[-12, 0) \cup [4/11, 9$

Dec $(0, 4/11)$

$f'(x)$:



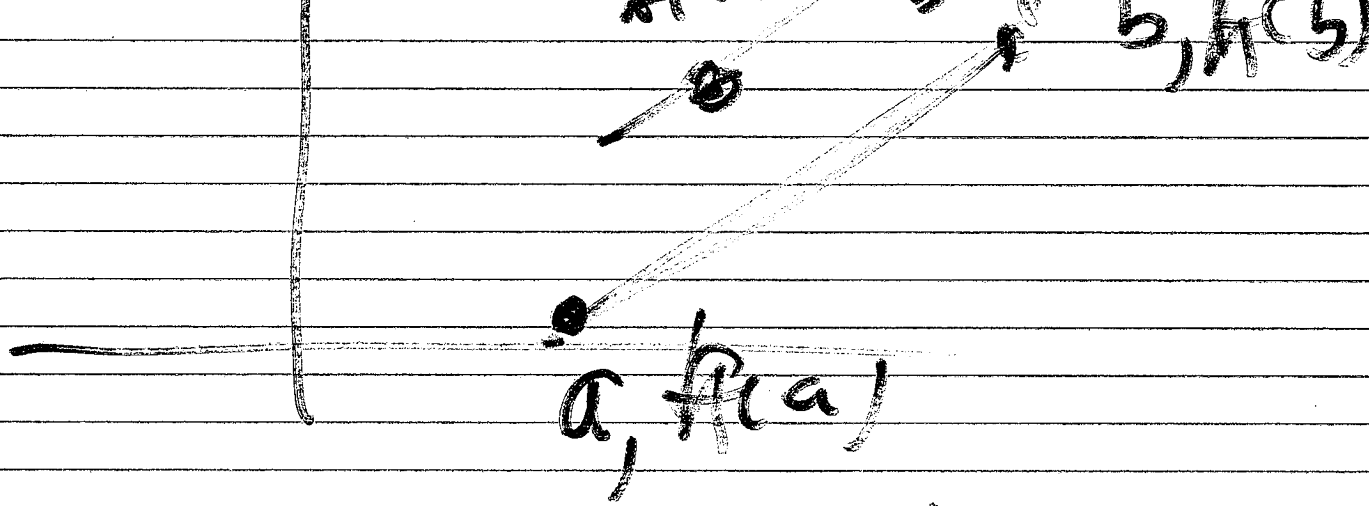
$f(x)$



If $f'(x) = g'(x)$

$h' \quad (f-g)' = f'(x) - g'(x) = 0$

$h'(c) = \frac{h(b)-h(a)}{b-a}$



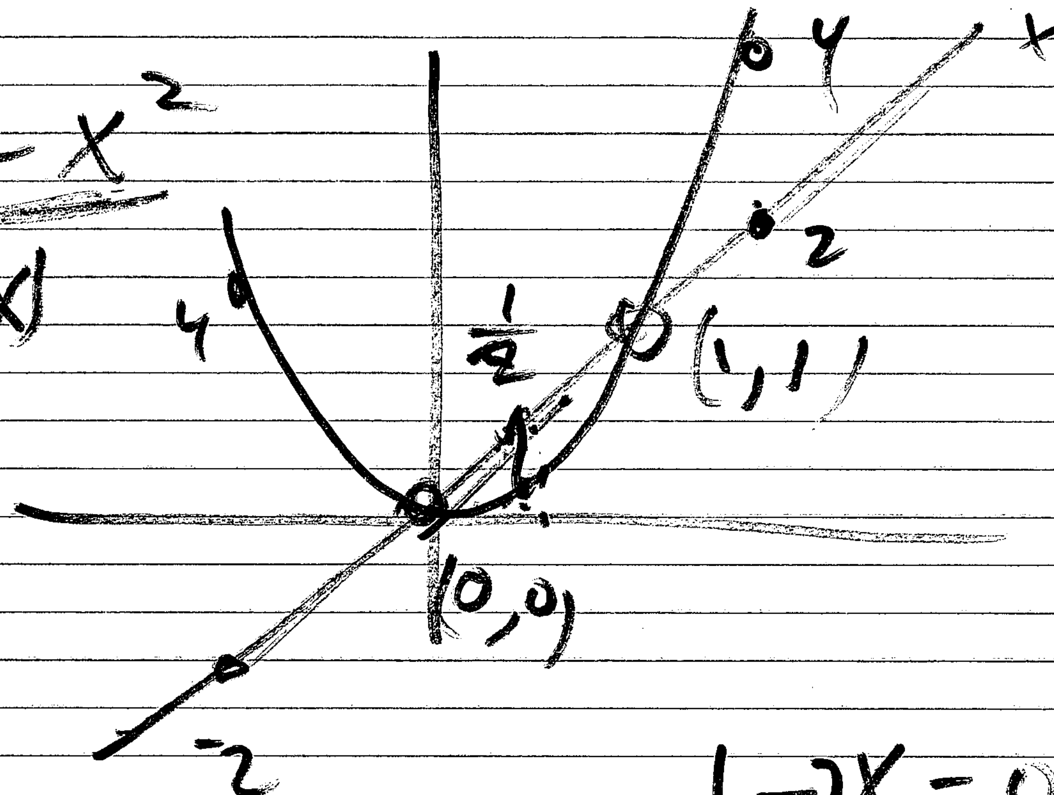
If $h'(c) = 0 \Rightarrow \frac{h(b)-h(a)}{b-a} = 0$

$h(b) = h(a) \cdot h(x) = C$

most
X exceeds it square

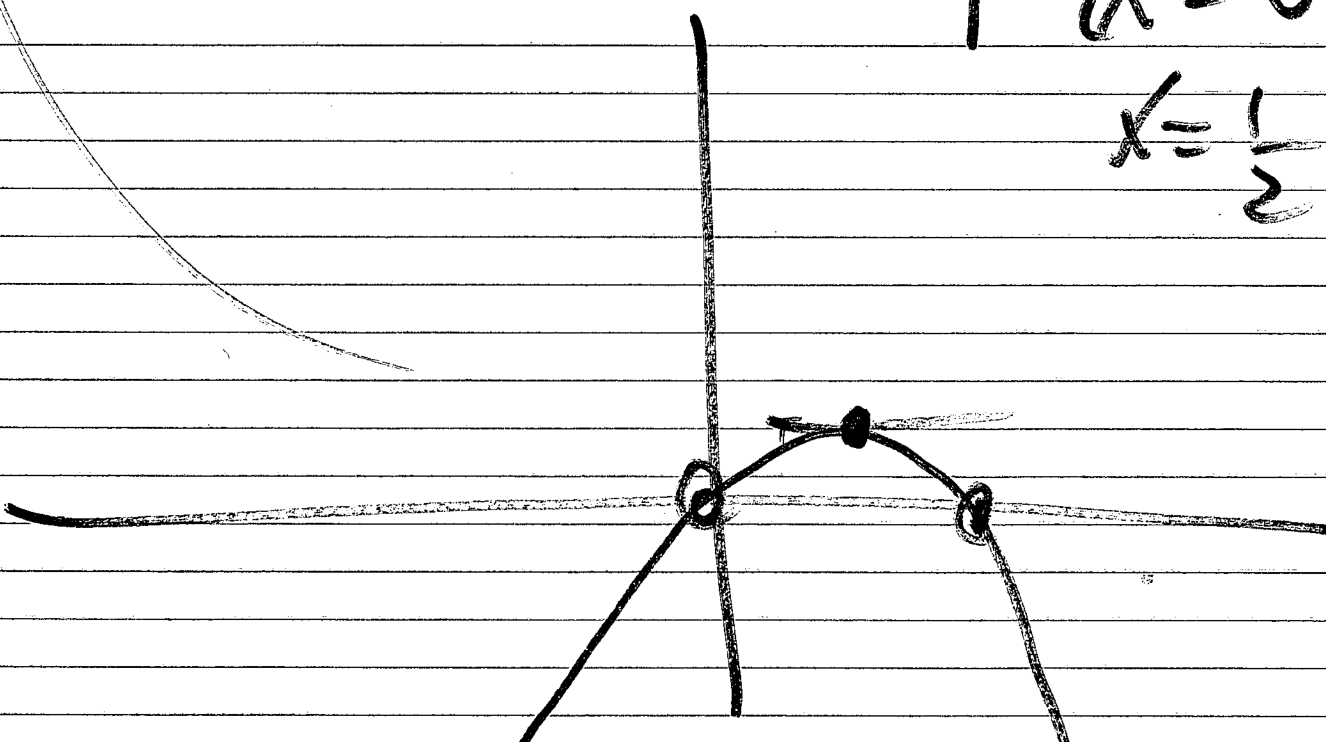
$f(x) = x - x^2$

$x(1-x)$



$1 - 2x = 0$

$x = \frac{1}{2}$

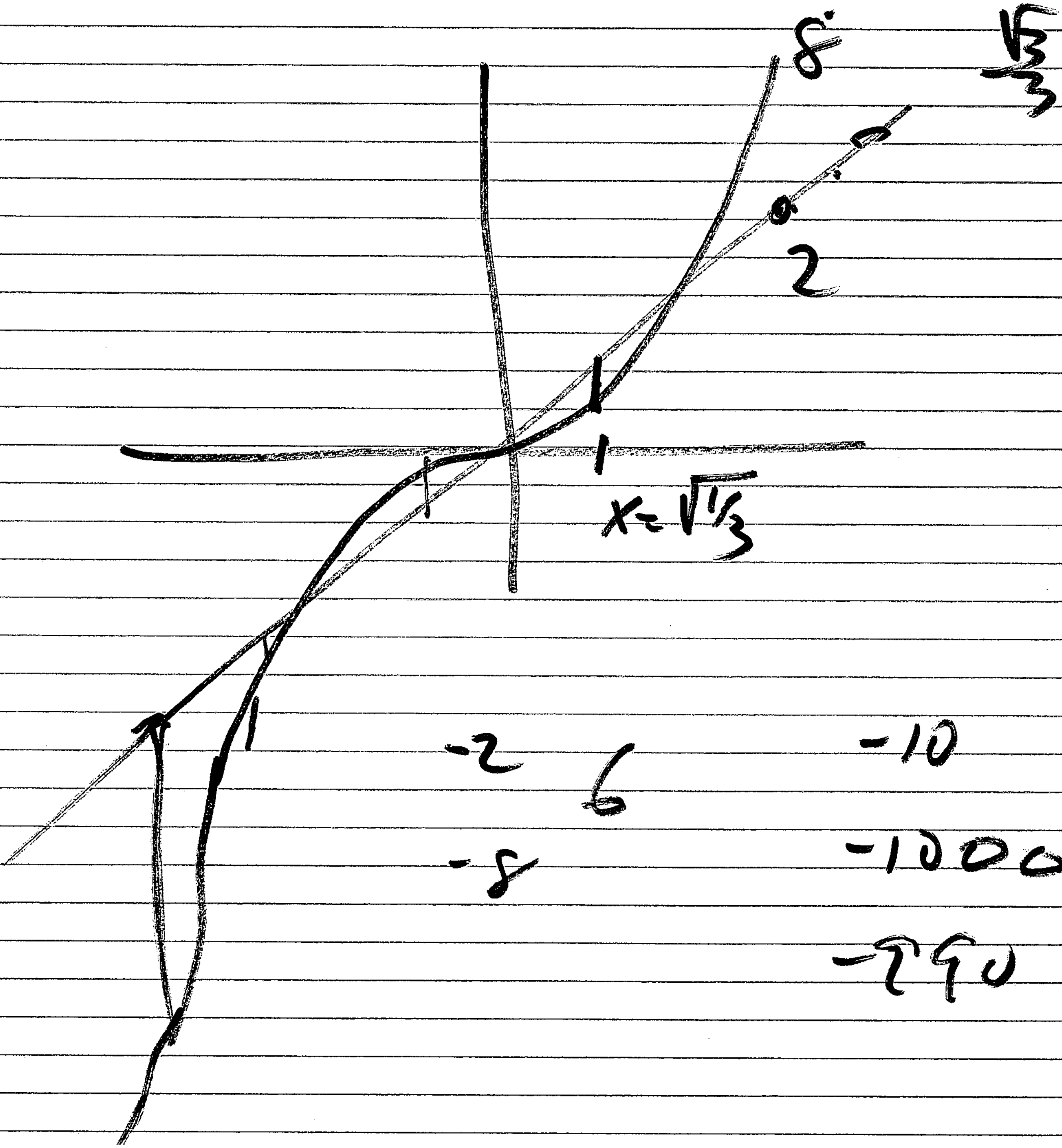


x exceeds its cube by most

$f(x) = x - x^3$

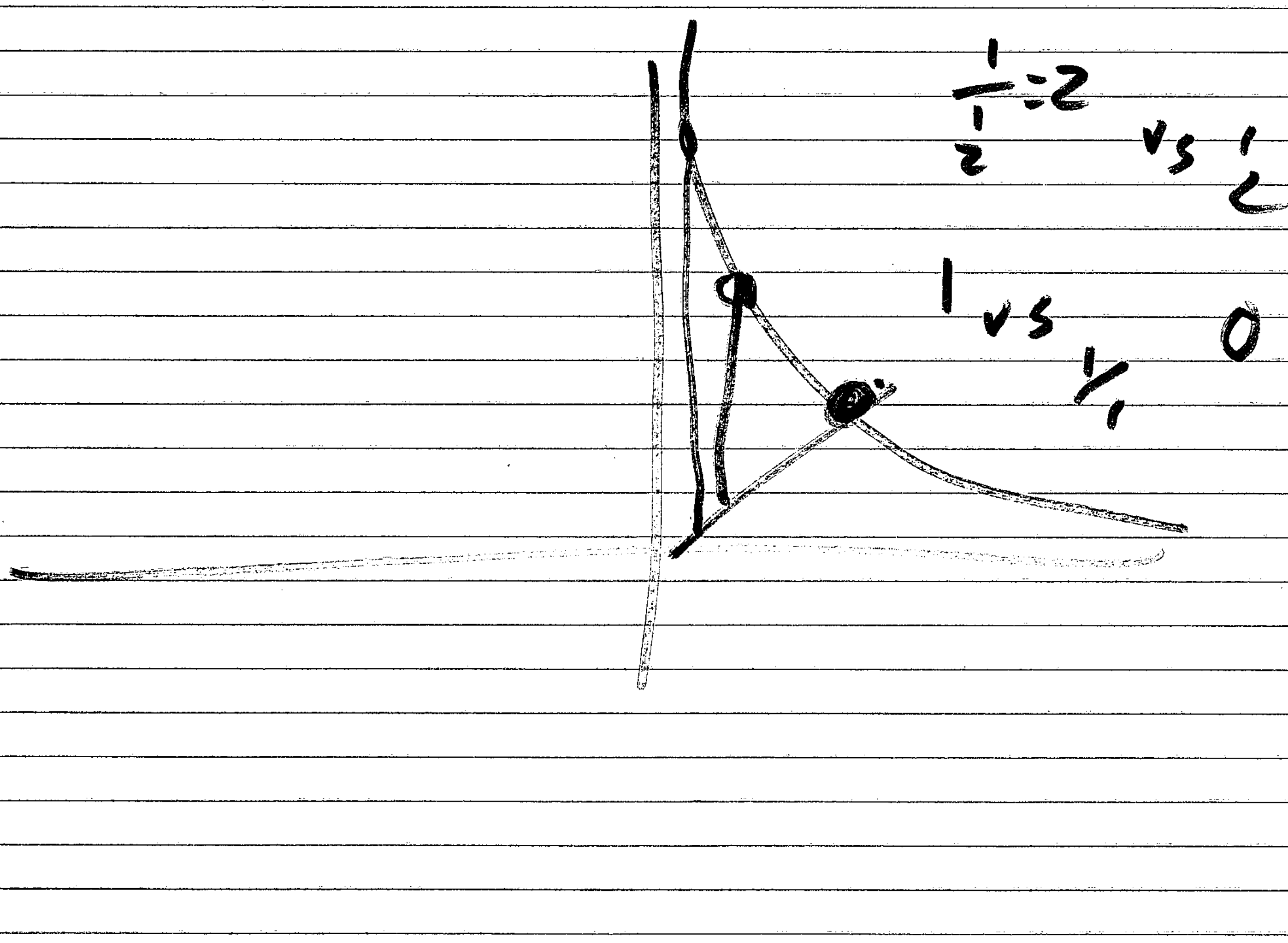
$\boxed{\text{There is no max}}$

$f'(x) = 1 - 3x^2 = 0 \quad x = \pm\sqrt{\frac{1}{3}}$



$0<x<1$

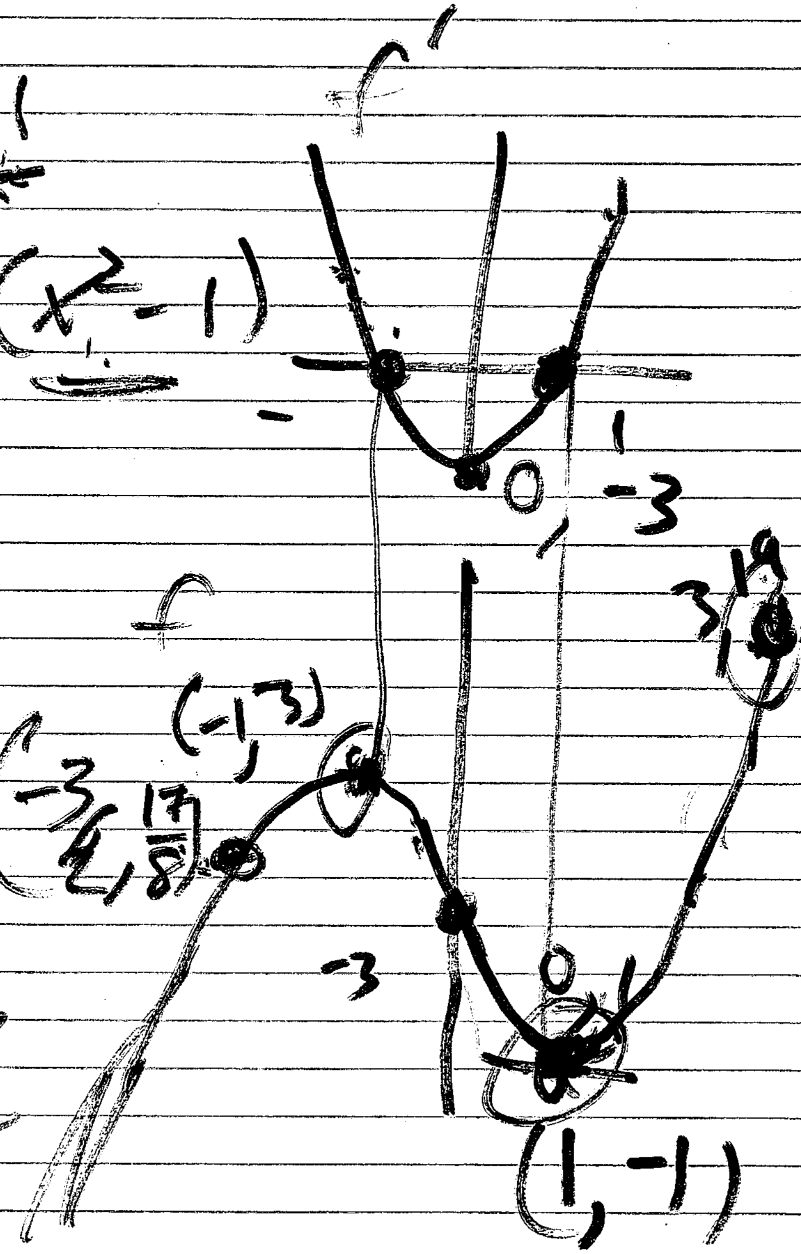
Find $x>0$ for which $\frac{1}{x}$ exceed x by maximum amount



$\left[-3/2 \, , 3\right]$

$f(x) = x^3 - 3x + 1$

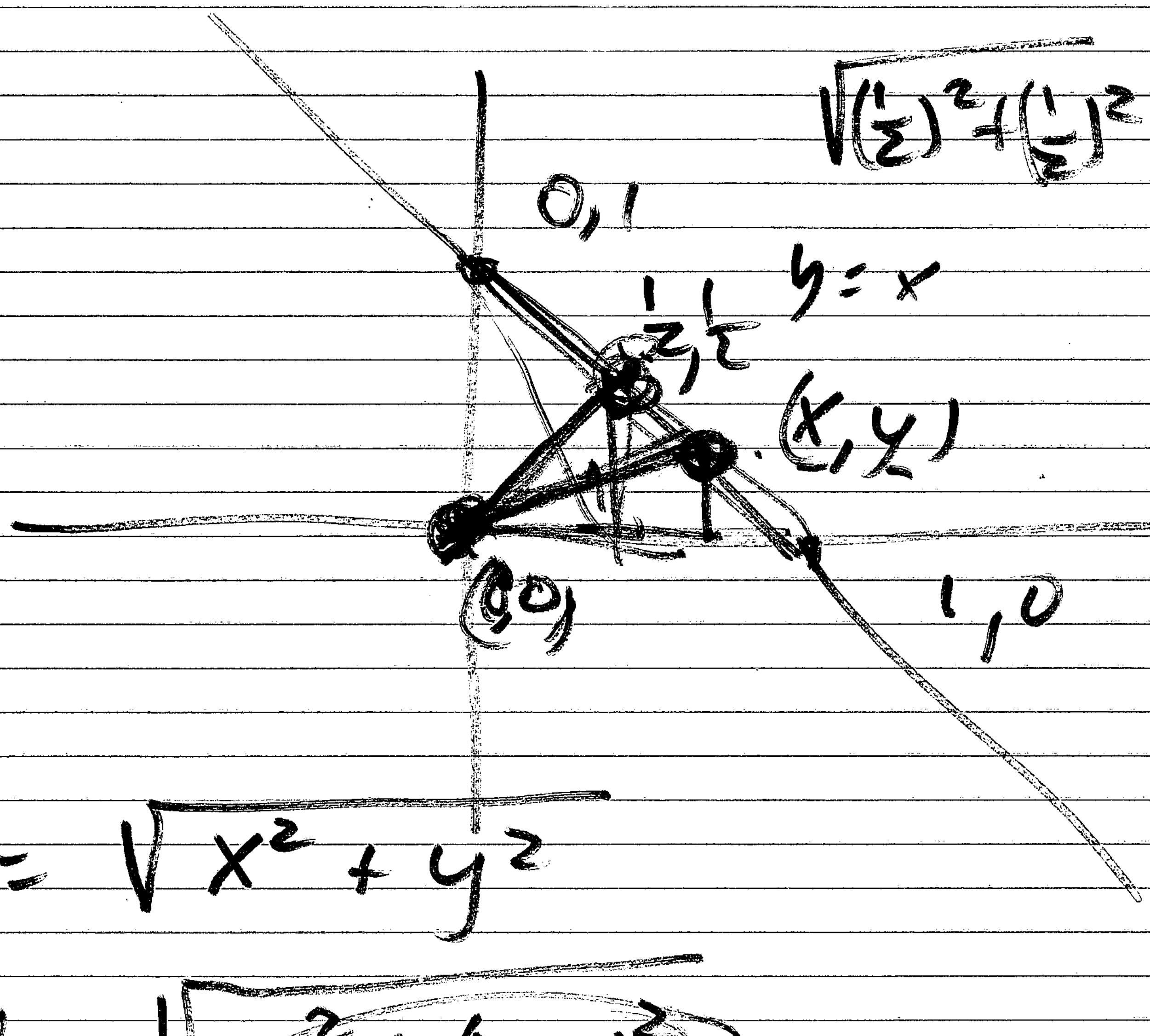
$f'(x) = 3x^2 - 3 = 3(x^2 - 1)$



$f(0) = 1$

$-\frac{27}{8} + \frac{\overset{36}{9}}{8} + \frac{8}{8}$

6. $y = 1 - 1x$



$d = \sqrt{x^2 + y^2}$

$d = \sqrt{x^2 + (1-(x))^2}$

$D(x) = x^2 + (1-x)^2$

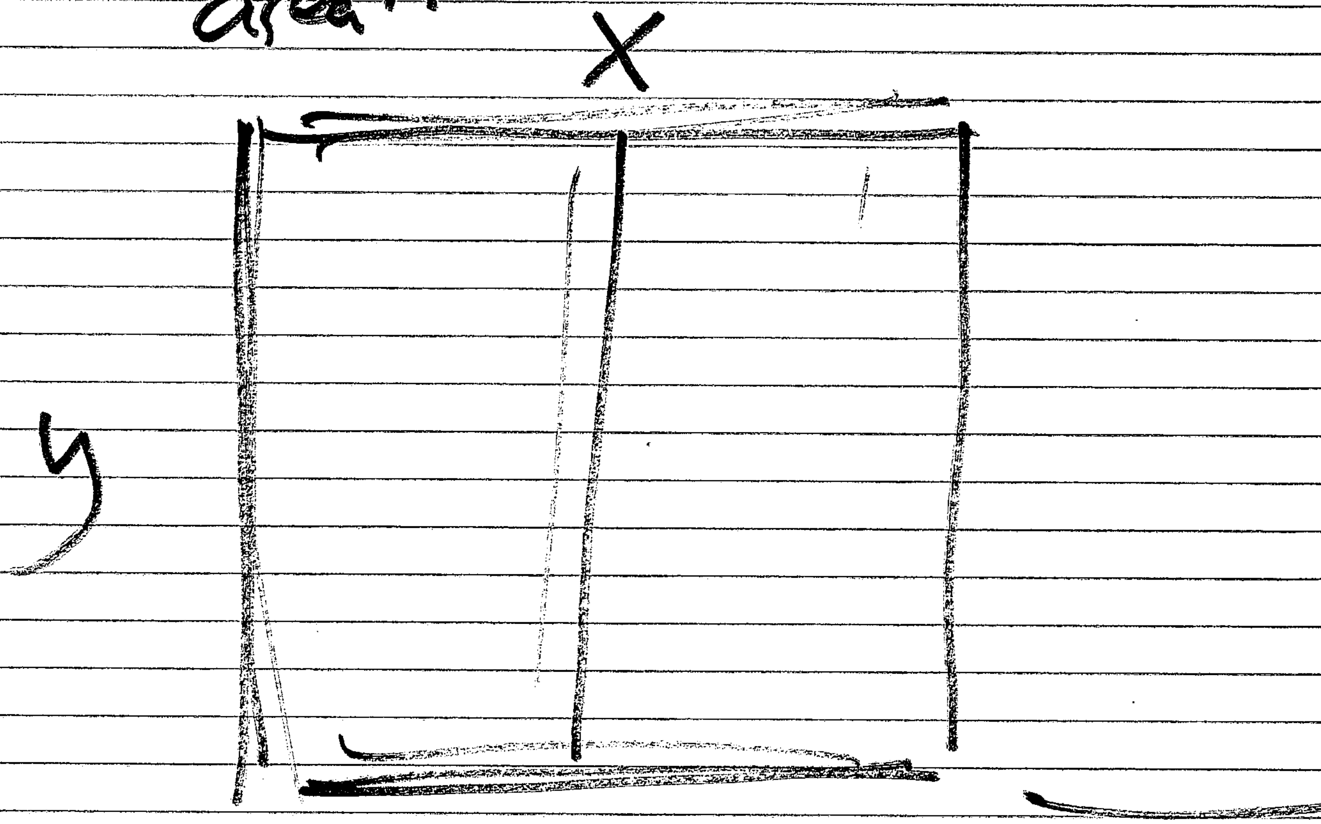
$2x + 2(1-x)' \cdot (-1) = 2x - 2(1-x)$

$4x - 2 = 0$

$x = \frac{1}{2}$

L of fence

area A



$A = xy$ $\quad L = 2x + 3y$

$y = \frac{A}{x}$ $\quad L = 2x + 3\frac{A}{x}$ $\quad x^{-1}$

$L'(x) = 0$

Find $L(x_{min})$