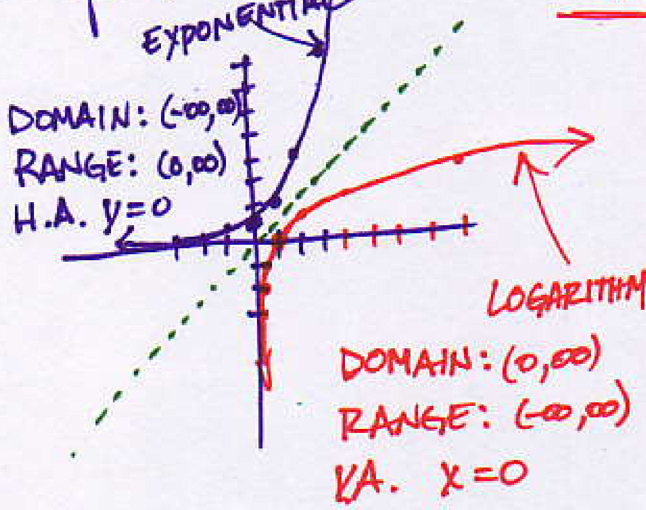
SEC 5.2 LOGARITHMIC FUNCTIONS

EXAMPLE: $f(x) = 2^x$

x	y
0	1
1	2
2	4
3	8
-1	$\frac{1}{2}$
-2	$1/4$
-3	$1/8$



INVERSE

x	y
1	0
2	1
4	2
8	3
$1/2$	-1
$1/4$	-2
$1/8$	-3

1. LOGARITHMIC FUNCTION: IS THE INVERSE OF IT'S EXPONENTIAL FUNCTION.

$$\log_a x = y \Rightarrow a^y = x$$

LOGARITHMIC FORM		EXPONENTIAL FORM
$\log_2 x = y$	$\Longleftrightarrow$	$2^y = x$
$\log_{1/3} x = y$		$\frac{1}{3}^y = x$

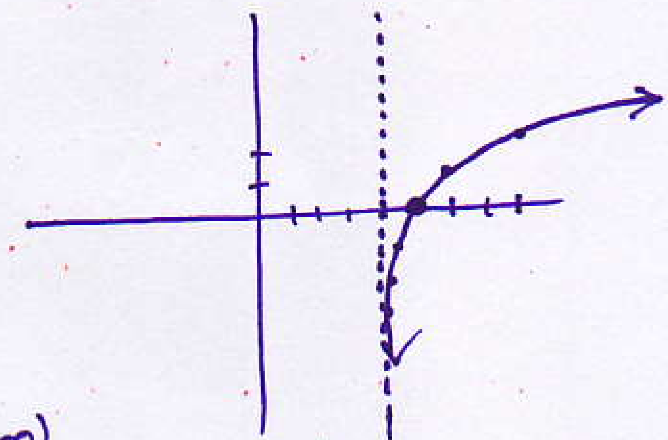
2. GRAPH LOGARITHMIC FUNCTIONS

$\overset{y}{\cancel{f(x)}} = \log_2(x-4) \Rightarrow 2^y = x - \cancel{4}$

$\quad +4 \qquad \cancel{+4}$

$2^y + 4 = x$

x	y
5	0
6	1
8	2
4.5	-1
4.25	-2
4.125	-3



DOMAIN: $(4, \infty)$

RANGE $(-\infty, \infty)$

V.A. $x = 4$

3. FINDING DOMAIN OF A LOGARITHMIC FUNCTION.

$f(x) = \log_4(x-3)$ SET EQUAL TO ZERO

$x - 3 = 0$

$+3 \quad +3$

$x = 3$ SO...

DOMAIN: $(3, \infty)$

$f(x) = \log_4(x+3)$

$x + 3 = 0$

$-3 \quad -3$

$x = -3$

DOMAIN: $(-3, \infty)$

4. SAME BASE PROPERTY:

IF $a^x = a^y$, THEN $x = y$

EXAMPLE: $2^4 = 2^x$ THEN $\boxed{x = 4}$

$3^{x+1} = 3^5$ THEN $x+1 = 5$

$\boxed{x = 4}$

$9^x = 3^3$

$(3^2)^x = 3^3$

$3^{2x} = 3^3$ THEN $\frac{2x}{2} = \frac{3}{2}$ $\boxed{x = 3/2}$

MUST BE THE SAME BASE

5. EVALUATE LOGARITHMS

$\log_2 32 = y \Rightarrow 2^y = 32$

$2^y = 2^5$

$\boxed{y = 5}$

$\log_{16} 4 = y \Rightarrow 16^y = 4$

$(4^2)^y = 4$

$4^{2y} = 4$

$\frac{2y}{2} = \frac{1}{2}$ $\boxed{y = 1/2}$

6. PROPERTIES OF LOGARITHMS

1) $\log_a 1 = 0$ BECAUSE $a^0 = 1$

2) $\log_a a = 1$ BECAUSE $a^1 = a$

3) $\log_a a^x = x$ BECAUSE $a^x = a^x$

4) $a^{\log_a x} = x$ BECAUSE $\log_a x = \log_a x$

$a^y = x \Rightarrow \log_a x = y$

EX. $\log_5 1 = 0$ (PROP #1)

$\log_6 6 = 1$ (PROP #2)

$\log_8 8^3 = 3$ (PROP #3)

$10^{\log_{10} 7} = 7$ (PROP #4)

7. SAME LOGARITHM PROP.

IF $\log_a x = \log_a y$, THEN $x = y$

EX. $\log_5 (x+1) = \log_5 3$

$x+1 = 3$

$x = 2$

8. COMMON LOGARITHM : $\log x = \log_{10} x$

$$\log 5 = \log_{10} 5 \Rightarrow 10^y = 5$$

$$10^{.69897} \approx 5$$

9. CHANGE OF BASE FORMULA

$$\log_2 7 = \frac{\log_{10} 7}{\log_{10} 2} \approx \boxed{2.807}$$

$$\log_a x = \frac{\log x}{\log a}$$

10. THE NATURAL LOGARITHM

$$\log_e x = \ln x$$

EX. $\ln 5 = \approx 1.609$

$$\log_e 5 = \frac{\log 5}{\log e}$$

II. PROPERTIES OF NATURAL LOGS

1) $\ln 1 = 0$ $\qquad \log_e 1 = 0 \quad e^0 = 1$

2) $\ln e = 1$ $\qquad \log_e e = 1 \quad e^1 = e$

3) $\ln e^x = x$ $\qquad \log_e e^x = x \quad e^x = e^x$

4) $e^{\ln x} = x$ $\qquad e^{\log_e x} = x \quad \log_e x = \log_e x$

EX. $\ln e^8 = 8$ (PROP #3)

$e^{\ln 5} = 5$ (PROP #4)

$\ln 1 = 0$ (PROP #1)

$\ln e = 1$ (PROP #2)