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How to calculate $f'(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

more easily than directly from definition?

$$f(x) = \frac{\sqrt{x} - 7}{\sqrt{x} + 7} = \left(\frac{u}{v}\right) = \frac{hi}{ho} \quad u \cdot v^{-1}$$

$$\lim_{h \to 0} \frac{\left(\frac{\sqrt{x+h} - 7}{\sqrt{x+h} - 7}\right) - \left(\frac{\sqrt{x} - 7}{\sqrt{x} + 7}\right)}{h} = ?$$

$$f'(x) = \frac{vu' - uv'}{v^2} = \frac{(\sqrt{x}+7)(\frac{1}{2}x^{-\frac{1}{2}}) - (\sqrt{x}-7)(\frac{1}{2}x^{-\frac{1}{2}})}{(\sqrt{x}+7)^2}$$

$$= 7x^{-\frac{1}{2}} / (\sqrt{x}+7)^2$$

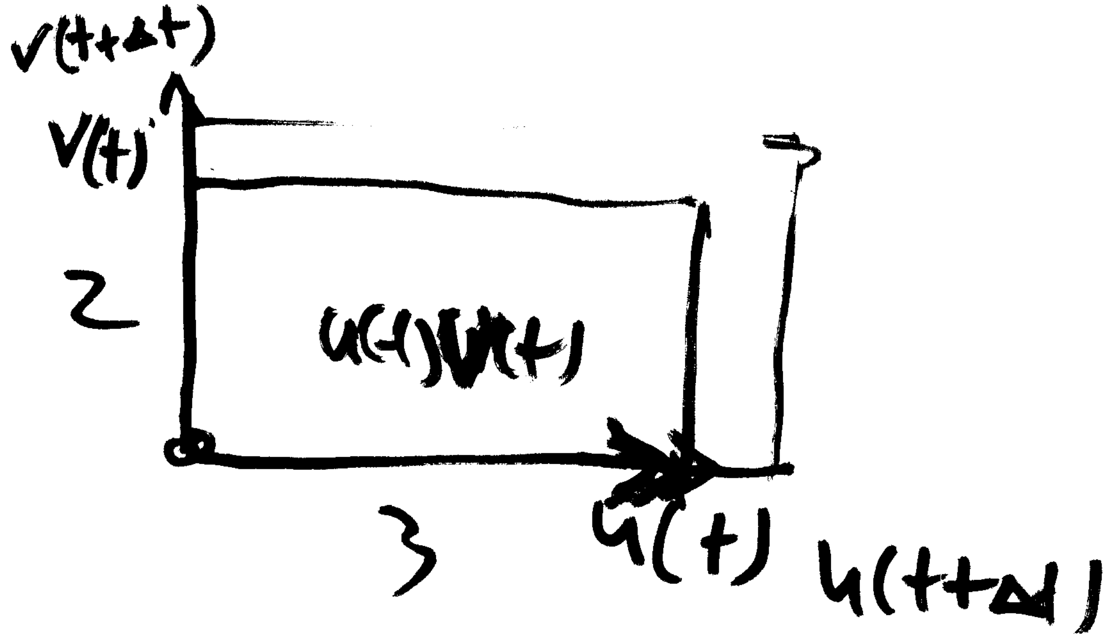
$$f(x) = u(x) \cdot v(x)$$

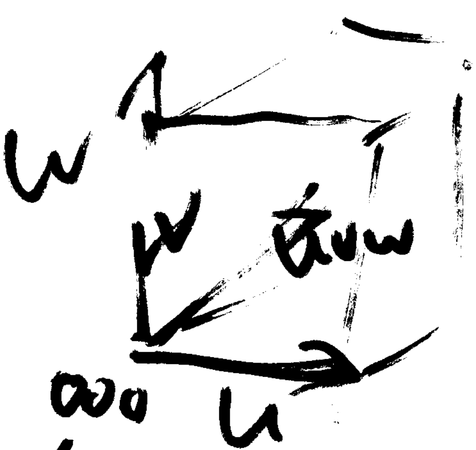
$u \cdot (v)^{-1}$

$u' v^{-1}$

$\frac{u'}{v} + -1 u \cdot v^{-2}$

$$f'(x) = u'(x) \cdot v(x) + u(x) v'(x)$$





$$f(x) = u(x)v(x)w(x)$$

$$f'(x) = u'vw + uv'w + uvw'$$

$$f(x) = \frac{\sin(x)}{\cos(x)} = \tan(x) \qquad \frac{u}{v} \qquad a^2+b^2=1$$

$$\langle \cos, \sin \rangle' = \langle -\sin, \cos \rangle$$

$$f'(x) = \frac{\overbrace{\cos(x)\cos(x) - \sin(x)(-\sin x)}^{\cos^2+\sin^2}}{\cos(x)^2} \qquad \frac{vu' - uv'}{v^2}$$

$$= \frac{1}{\cos(x)^2} = \sec(x)^2$$

$$\sec^2 x$$

$f_1(x) = {}_1x^1 \qquad f'(x) = 1$

$f_2(x) = x \cdot x \qquad f'(x) = 1 \cdot x + x \cdot 1 = \underline{2x}$

$f_3(x) = x^2 \cdot x \; : \; f'(x) = 2x \cdot x + x^2 \cdot 1 = 3x^2$

$f_3(x) = x \cdot x \cdot x \qquad f'(x) = 1 \cdot x \cdot x + x \cdot 1 \cdot x + x\,x \cdot 1$

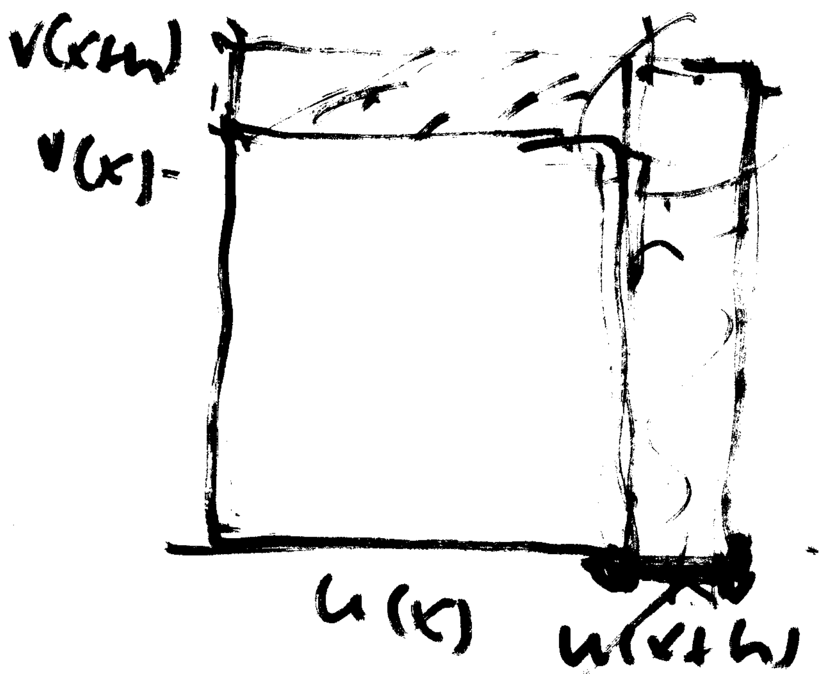
$\langle c, s \rangle' = \langle -s, c \rangle$

$f(x) = \sin x \cdot \sin x + \cos x \cdot \cos x \qquad = 1$

$f'(x) \qquad c \cdot s + s \cdot c + (-s) \cdot c + c \cdot -s = 0$

$f(x) = u(x)\,v(x) \qquad f'(x) =$

$$\lim_{h \to 0} \frac{u(x+h)\,v(x+h) - u(x)\,v(x)}{h}$$



$$= \lim_{h \to 0} \frac{(u(x+h) - u(x))\,v(x+h) + u(x)\,(v(x+h) - v(x))}{h}$$

Sum rule of limits

$$= \lim_{h \to 0} v(x+h)\,\frac{u(x+h) - u(x)}{h} + \lim_{h \to 0} u(x)\,\frac{v(x+h) - v(x)}{h}$$

Product of limits

$$= \lim_{h \to 0} v(x+h) \lim_{h \to 0} \frac{u(x+h) - u(x)}{h}$$

$$+ \cancel{\lim} \; u(x) \lim_{h \to 0} \frac{v(x+h) - v(x)}{h}$$

$$= v(x)\, u'(x) + u(x)\, v'(x)$$

if v' exists -

denom $h \to 0$ numerator $\to 0$

$\Rightarrow v(x+h) \to v(x)$