

Suggested Types of Problems

College Algebra

Functions and Graphs

- ✓1. (Easy) Determine whether the relation $x^2 + y^2 = 16$ is a function.
- ✓2. (Medium) For the function $f(x) = \sqrt{2-x}$, evaluate $f(-3)$, $f(c)$, and $f(x+h)$.
- ✓3. (Hard) For the function $f(x) = x^2 + 2x + 4$, evaluate the difference quotient: $\frac{f(x+h)-f(x)}{h}$.
- ✓4. (Medium) Find the domain of the function: $f(x) = \frac{\sqrt{2x-1}}{x-2}$. Give your solution in interval notation.
- ✓5. (Medium) Determine whether the function $f(x) = \frac{1}{x} + 2x$ is even, odd, both or neither.
- ✓6. (Medium) Find the average rate of change for the function $f(x) = 16 - x^2$ from $x = 1$ to $x = 3$.
- ✓7. (Medium) Sketch the graph of the piecewise defined function
$$f(x) = \begin{cases} x & \text{for } x > 1 \\ 1 & \text{for } -1 \leq x \leq 1 \\ x^2 & \text{for } x < 1 \end{cases} \quad (1)$$
- ✓8. (Hard) A t-shirt design company charges \$50 for a set-up fee, \$10 per shirt for the first 100 shirts ordered and \$8 for every additional shirt. Write the cost function $C(x)$ as a function of the number of shirts x ordered, where x is a nonnegative integer.
- ✓9. (Hard) Write the function whose graph is the graph of $f(x) = \sqrt[3]{x}$, but is shifted to the left 3 units, up 2 units and reflected across the x -axis.
- ✓10. (Medium) Suppose the graph of f is given. Describe how the graph of $y = 2f(x+3) - 1$ can be obtained from the graph of f .
- ✓11. (Medium) Sketch the graph of the function $g(x) = -3|x+1| - 2$ using transformations. Show all intermediate steps.
- ✓12. (Medium) Sketch the graph of the function $f(x) = \frac{1}{2}\sqrt{x+4} - 3$ using transformations. Show all intermediate steps.

- ✓13. (Easy) Sketch the graph of the function $f(x) = \frac{1}{x-3} + 2$ using transformations. Show all intermediate steps.
- ✓14. (Medium) Complete the square to transform the equation $f(x) = 2x^2 + 4x - 3$ into the form $f(x) = c(x - h)^2 + k$ and then sketch the graph of $f(x)$.
- ✓15. (Hard) Find the maximum or minimum value of the quadratic function: $f(x) = -4x^2 - 4x + 3$
16. (Easy) For the functions $f(x) = x^2 - 4$ and $g(x) = \frac{1}{x+3}$, answer the following:
- Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$ and $(f/g)(x)$ and state the domain of each.
 - Find $(g \circ f)(2)$ and $(f \circ g)(2)$.
 - Find a formula and give the domain for the functions $(f \circ g)(x)$ and $(g \circ f)(x)$.
- ✓17. (Easy) Write the function $h(x) = \sqrt[3]{x^2 + x}$ as a composite of two functions f and g where $h(x) = f(g(x))$.
- ✓18. (Medium) For the functions $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x - 1}$, show that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$.
- ✓19. (Easy) Find the inverse of the function: $f(x) = (x - 1)^3$
- ✓20. (Medium/Hard) Find the inverse of the function: $f(x) = \frac{x-1}{x-2}$
- ✓21. (Medium) Sketch the graph of the function $f(x) = \sqrt{x - 3}$, $x \geq 3$ and its inverse. State the domain and range of both f and f^{-1} .

Polynomial and Rational Functions

- Find the standard form for the quadratic function, identify the vertex, state the range of the function, and sketch the graph.
 - (Easy) $f(x) = x^2 - 4x + 4$
 - (Easy) $f(x) = x^2 + 2x - 1$
 - (Medium) $f(x) = 2x^2 + 8x + 3$
 - (Medium) $f(x) = -4x^2 + 16x - 7$

✓(e) (Medium) $f(x) = -2x^2 + 6x - 5$

✓(f) (Medium) $f(x) = \frac{1}{4}x^2 - \frac{1}{2}x - \frac{3}{4}$

- ✓2. (Hard) A rancher has 600 feet of fencing and wants to build a rectangular enclosure, subdivided into two equal parts. What dimensions would enclose the maximum amount of area? What is the maximum area?

- ✓3. (Medium) A person standing near the edge of a cliff 100 feet above a lake throws a rock upward with an initial speed of 32 feet per second. The height of the rock above the lake at the bottom of the cliff is a function of time, $h(t) = -16t^2 + 32t + 100$, where t is measured in seconds. How many seconds will it take for the rock to reach its maximum height? What is the maximum height?

4. (Easy) Find all the zeros of the polynomial.

✓
(a) $p(x) = x^2 - x - 6$
(b) $p(x) = x^3 - 9x$
(c) $p(x) = (x + 1)(x - 4)(x + 6)$
(d) $p(x) = 2(x + 3)(x - \frac{1}{2})$

5. (Easy) Find the multiplicity of the given zero in the polynomial.

✓
(a) $p(x) = (x + 1)(x - 3), c = -1$
(b) $p(x) = (x - 2)^3(x - 5), c = 2$
(c) $p(x) = x^2 - 2x - 8, c = 4$
(d) $p(x) = x^2 - 4x + 4, c = 2$

6. (Easy) Find a polynomial with the given conditions. (Answers may vary)

✓
(a) degree 3, zeros -2, 0, 3
(b) degree 3, zeros 0 (multiplicity 2) and 1

7. (Easy) Find the unique polynomial with the given conditions.

✓
(a) degree 2, zeros -3 and -1, leading coefficient $a_n = -3$
(b) degree 3, zeros 2 (multiplicity 2) and -4, $p(3) = 21$

8. (Easy) Divide $p(x)$ by $d(x)$ using long division.

(a) $p(x) = 2x^3 + x^2 + 5x + 15$; $d(x) = 2x + 3$

(b) $p(x) = 6x^4 - 4x^3 + 3x^2 - 14x + 2$; $d(x) = 3x - 2$

(c) $p(x) = 2x^3 - 5x^2 + 1$; $d(x) = x^2 - 3$

(d) $p(x) = 10x^3 + x^2 + 18x + 2$; $d(x) = 2x^2 - x + 4$

9. (Medium) Use synthetic division to do the following for $p(x)$ and $d(x) = x - c$:

i) Divide $p(x)$ by $d(x) = x - c$.

ii) Rewrite $p(x)$ in $d(x)q(x) + r(x)$ form.

iii) Evaluate $p(x)$ at $x = c$.

(a) $p(x) = 2x^4 - 3x^3 - 5x^2 + x - 6$; $d(x) = x - 1$

(b) $p(x) = 3x^4 - 2x^3 - 4x + 8$; $d(x) = x + 2$

(c) $p(x) = x^3 - 7x^2 + 17x - 20$; $d(x) = x - 4$

10. (Medium) Use synthetic division to do the following for $p(x)$ and the given value of c :

i) Divide $p(x)$ by $d(x) = x - c$.

ii) Rewrite $p(x)$ in $d(x)q(x) + r(x)$ form.

iii) Evaluate $p(x)$ at $x = c$.

(a) $p(x) = x^5 + 4x^4 - 2x^3 - 7x^2 + 3x - 5$; $c = -3$

(b) $p(x) = 2x^6 - 4x^4 - 5x^3 + x - 5$; $c = 2$

(c) $p(x) = 2x^4 + 3x^3 + 2x^2 - 1$; $c = -1$

11. Do the following for each polynomial:

i) Use the Rational Zero Theorem to determine all possible rational zeros

ii) Use Descartes's rule of signs to determine possible combinations of positive and negative zeros

iii) Factor the polynomial

iv) Sketch the graph

(a) (Medium) $p(x) = x^3 - 7x + 6$

(b) (Medium) $p(x) = -x^3 + 4x^2 - x - 6$

(c) (Medium) $p(x) = 6x^3 + 17x^2 + x - 10$

- (d) (Medium) $p(x) = 6x^3 + x^2 - 5x - 2$
 (e) (Hard) $p(x) = x^4 + 4x^3 - 3x^2 - 10x + 8$
 (f) (Hard) $p(x) = -2x^4 + 5x^3 + 6x^2 - 20x + 8$
 (g) (Hard) $p(x) = x^5 - 3x^4 - x^3 + 7x^2 - 4$
 (h) (Hard) $p(x) = 2x^6 - x^5 - 13x^4 + 13x^3 + 19x^2 - 32x + 12$

12. (Medium) Find the least integer upper bound and the greatest integer lower bound for the real zeros of $p(x)$, obtained by using the Upper and Lower Bounds Theorem.

- (a) $p(x) = x^4 - 3x^2 - 2x + 5$
 (b) $p(x) = -2x^5 + 3x^3 - 4x^2 - 4$

13. (Medium) Use the Intermediate Value Theorem to show that the polynomial has a zero in the given interval.

- (a) $p(x) = x^3 - 3x^2 + 5x - 5$; $(1, 2)$
 (b) $p(x) = x^4 + 4x^3 + 2x^2 + 4x + 1$; $(-4, -3)$

14. Find a polynomial with real coefficients that satisfies the given conditions.

- (a) (Easy) degree 4; zeros 1, -2, $3i$
 (b) (Easy/Medium) degree 5; zeros 2 (mult. 2), -1, $-1+2i$
 (c) degree 3; zeros 0, i ; $p(1) = 4$
 (d) degree 4; zeros -1 (mult. 2), $1-i$; $p(2) = -18$

15. Given a zero of the polynomial, factor the polynomial into linear factors.

- (a) (Medium) $p(x) = x^3 - 3x^2 + 4x - 2$; 1
 (b) (Medium) $p(x) = x^3 + 5x^2 + 17x + 13$; -1
 (c) (Hard) $p(x) = x^4 - 6x^3 + 14x^2 - 24x + 40$; $2i$
 (d) (Hard) $p(x) = x^4 + 2x^3 + 10x^2 + 24x + 80$; $1 + 3i$

16. (Hard) Do the following for each polynomial:

- i) Factor into linear and irreducible quadratic factors.
 ii) Completely factor into linear factors.

- (a) $p(x) = x^4 - 3x^3 + 26x^2 - 22x - 52$
 (b) $p(x) = x^5 - x^4 - 4x^3 - 4x^2 - 5x - 3$

17. For each of the following, give an equation for each asymptote. Sketch the graph using the asymptotes and x-intercept(s), if any.

✓ (a) (Easy) $f(x) = \frac{-1}{x^2-2x-8}$

(b) (Medium) $f(x) = \frac{x-2}{x^2+x-2}$

(c) (Medium) $f(x) = \frac{2x^2-x-3}{x^2-3x}$

(d) (Medium) $f(x) = \frac{-x^2+x}{x^2-2x-3}$

✓ (e) (Medium) $f(x) = \frac{x^2+x-12}{x+1}$

(f) (Medium) $f(x) = \frac{x^3-3x+2}{x^2+3x-10}$

(g) (Hard) $f(x) = \frac{x^2+x-2}{x^2-4x+3}$

(h) (Hard) $f(x) = \frac{x^2+x}{x^3+2x^2-11x-12}$

Exponential and Logarithmic Functions

1. (Easy) Evaluate $(\frac{1}{9})^{-3/2}$ without using a calculator.
2. (Easy) Graph $f(x) = 3^x - 1$ using transformations. State the domain and the range and plot several points. Give an equation of the horizontal asymptote.
3. (Easy) Graph $f(x) = 2 - e^x$ using transformations. State the domain and the range and plot several points. Give an equation of the horizontal asymptote.
4. (Easy) Graph $f(x) = e^{-x} + 1$ using transformations. State the domain and the range and plot several points. Include the horizontal asymptote.
5. (Easy) Graph $f(x) = (\frac{1}{2})^{x-2} + 1$ using transformations. State the domain and the range and plot several points. Give an equation of the horizontal asymptote.
6. (Easy) Use $A = p(1 + \frac{r}{n})^{nt}$ or $A = Pe^{rt}$ whichever is appropriate: Determine how much money should be put in a savings account that earns 4% a year in order to have \$32000 in 18 years if
 - (a) the account is compounded quarterly.
 - (b) the account is compounded continuously.
7. (Easy) If you put \$3200 in a savings account that pays 2% a year compounded continuously, how much will you have in the account in 15 years? Use $A = p(1 + \frac{r}{n})^{nt}$ or $A = Pe^{rt}$ whichever is appropriate.
8. (Easy) Write $\log_{81} 3 = \frac{1}{4}$ in its equivalent exponential form.
9. (Easy) Write $e^x = 6$ in its equivalent logarithmic form.

10. (Easy) Evaluate exactly if possible: $\log_5 3125$.
11. (Easy) State the domain of $f(x) = \log_2(4x - 1)$ in interval notation.
12. (Easy) Graph the logarithmic function $\log_3(x - 2) + 1$ using transformations. State the domain and range. Give an equation of the vertical asymptote.
13. (Easy) Graph the logarithmic function $\ln(x + 4)$ using transformations. State the domain and range. Give an equation of the vertical asymptote.
14. (Easy) Calculate the decibels associated with normal conversation given $D = 10 \log\left(\frac{I}{I_t}\right)$ if the intensity $I = 1 \times 10^{-6} \text{ W/m}^2$ and $I_t = 1 \times 10^{-12} \text{ W/m}^2$.
15. (Medium) In 2003, an earthquake centered near Hokkaido, Japan registered 7.4 in magnitude on the Richter scale. Calculate the energy, E , released in joules. Use the Richter scale model $M = \frac{2}{3} \log\left(\frac{E}{E_0}\right)$, where $E_0 = 10^{4.4}$ joules.
16. (Easy) Normal rainwater is slightly acidic and has an approximate hydrogen ion concentration of $10^{-5.6}$. Calculate its pH value. Use the model $\text{pH} = -\log[H^+]$, where $[H^+]$ is the concentration of hydrogen ions. Acid rain and tomato juice have similar concentrations of hydrogen ions. Calculate the concentration of hydrogen ions of acidic rain if its $\text{pH} = 3.8$.
17. (Easy) Simplify each expression using properties of logarithms:
 - (a) $\ln e^3$
 - (b) $7^{-2 \log_7 3}$
 - (c) $\log_2 \sqrt{8}$
18. (Easy) Write each expression as a sum or difference of logarithms (expand):
 - (a) $\log_b(x^3 y^5)$
 - (b) $\log_b\left(\frac{x}{yz}\right)$
 - (c) $\log_b\left[\frac{x^3(x-2)^2}{\sqrt{x^2+5}}\right]$

19. Write each expression as a single logarithm (contract):

(a) (Easy) $3\log_b x + 5\log_b y$

(b) (Medium) $\frac{1}{2}\ln(x+3) - \frac{1}{3}\ln(x+2) - \ln x$

20. (Easy) Use the change of base formula to change $\log_4 19$ to an expression involving only natural logs.

21. Solve the exponential equations exactly for x :

(a) (Easy) $2^{x^2+12} = 2^{7x}$

(b) (Medium) $\left(\frac{2}{3}\right)^{x+1} = \frac{27}{8}$

(c) (Medium) $27 = 2^{3x-1}$

(d) (Medium) $9 - 2e^{0.1x} = 1$

(e) (Medium) $e^{2x} - 4e^x - 5 = 0$

(f) (Hard) $\frac{4}{10^{2x-7}} = 2$

22. Solve the logarithmic equations exactly for x :

(a) (Easy) $\log_3(2x-1) = 4$

(b) (Medium) $\log(x-3) + \log(x+2) = \log(4x)$

(c) (Medium) $\log(2x-5) - \log(x-3) = 1$

(d) (Hard) $\ln(x) + \ln(x-2) = 4$

23. If \$7500 is invested in a savings account earning 5% interest compounded quarterly how many years will pass until there is \$20000? Use formula $a = P\left(1 + \frac{r}{m}\right)^{mt}$.

24. (Medium) Radium-226 has a half life of 1600 years. How long will it take 5 grams of radium-226 to be reduced to 2 grams? Use the model $m = m_0 e^{-kt}$, where m is the amount of radium-226 after t years, m_0 is the initial amount of radium-226, and k is the decay rate.

25. (Medium) An apple pie is taken out of the oven with an internal temperature of 325°F . It is placed on a rack in a room with a temperature of 72°F . After 10 minutes the temperature of the pie is 200°F . What will be the temperature of the pie 30 minutes after coming out of the oven? Use Newton's law of cooling: $T(t) = T_s + D_0e^{-kt}$, where T_s is the temperature of the surroundings, D_0 is the initial temperature difference between the surroundings and the object, and t is time.
26. (Medium/Hard) The number of trout in a particular lake is given by $N = \frac{10000}{1+19e^{-1.56t}}$, where t is time in years. How long will it take for the population to reach 5000?

Systems of linear Equations and Inequalities

1. Solve the following systems of linear equations by substitution:

(a) (Easy)

$$\begin{aligned} 2x - y &= 3 \\ x - 3y &= 4 \end{aligned}$$

(b) (Easy)

$$\begin{aligned} 4x - 5y &= -7 \\ 3x + 8y &= 30 \end{aligned}$$

(c) (Medium)

$$\begin{aligned} \frac{1}{3}x - \frac{1}{4}y &= 0 \\ -\frac{2}{3}x - \frac{3}{4}y &= 2 \end{aligned}$$

2. Solve the following systems of linear equations by using elimination:

(a) (Easy)

$$\begin{aligned} 2x + 5y &= 5 \\ -4x - 10y &= -10 \end{aligned}$$

(b) (Easy)

$$\begin{aligned} 3x - 2y &= 12 \\ 4x + 3y &= 16 \end{aligned}$$

(c) (Medium)

$$\begin{aligned} -0.5x + 0.3y &= 0.8 \\ -1.5x + 0.9y &= 2.4 \end{aligned}$$

3. (Easy) Graph the system of equations to solve:

(a)

$$2x + y = 3$$

$$2x + y = 7$$

(b)

$$x - 2y = 1$$

$$2x - 4y = 2$$

4. (Medium) **Health club Management.** A fitness club has a budget of \$915 to purchase two types of dumbbell sets. One set costs \$30 each and the other deluxe set costs \$45 each. The club wants to purchase 24 new sets of dumbbells. How many of each set should the club purchase?

5. (Hard) **Mixture.** In chemistry lab, Stephanie has to make a 37 milliliter solution that is 12% HCl. All that is in the lab is 8% and 15% HCl solutions. How much of each should she mix to get the desired solution?

6. Solve the following systems of equations:

(a) (Easy)

$$-x + y - z = -1$$

$$x - y - z = 3$$

$$x + y - z = 9$$

(b) (Medium)

$$3x + 2y + z = 4$$

$$-4x - 3y - z = -15$$

$$x - 2y + 3z = 12$$

(c) (Hard)

$$x - z + y = 10$$

$$2x - 3y + z = -11$$

$$y - x + z = -10$$

(d) (Medium)

$$2x_1 - x_2 + x_3 = 3$$

$$x_1 - x_2 + x_3 = 2$$

$$-2x_1 + 2x_2 - 2x_3 = -4$$

general solutions?

7. (Hard) Suppose you are going to eat only sandwiches for a week (seven days) for lunch and dinner (total of 14 meals). If your goal is a total of 4840 calories and 190 grams of fat, how many of each sandwich would you eat this week to obtain your goal? Consider the following table:

Sandwich	calories	Fat (Grams)
Mediterranean Chicken	350	18
Tuna	430	19
Roast Beef	290	5

8. (Hard) Bob and Betty decide to place \$20,000 of their savings into investments. They put some in a money market account earning 3% interest, some in a mutual fund that has averaged 7% a year, and some in stock that rose 10% last year. If they put \$6,000 more in the money market than in the mutual fund, and the stocks and mutual fund have the same growth in the next year as in the previous year, they will earn \$1,180 in the year. How much money did they put in each of the investments?

9. Find the form of the partial fraction decomposition. Do not solve for the constants:

(a) (Easy) $\frac{3x+2}{x(x^2-25)}$

(b) (Medium) $\frac{3x+2}{x^2(x^2+25)^2}$

(c) (Medium) $\frac{x^2+2x-1}{x^4-9x^2}$

10. Find the partial fraction decomposition for each rational function:

(a) (Easy) $\frac{1}{x(x+1)}$

(b) (Easy) $\frac{9x-11}{(x-3)(x+5)}$

(c) (Medium) $\frac{4x^2-7x-3}{(x+2)(x-1)^2}$

(d) (Medium) $\frac{-2x^2-17x+11}{(x-7)(3x^2-7x+5)}$

(e) (Hard) $\frac{5x+2}{x^3-8}$

11. (Easy) Graph the linear inequalities:

(a) $y < 2x + 3$

(b) $5x + 3y < 15$

(c) $6x - 3y \geq 9$

12. Graph the system of inequalities or indicate that the system has no solution:

(a) (Easy)

$$y > 2x + 1$$

$$y < 2x - 1$$

(b) (Easy)

$$x + 2y > 4$$

$$y < 1$$

$$x \geq 0$$

(c) (Easy)

$$y < x + 2$$

$$y > x - 2$$

$$y < -x + 2$$

$$y > -x - 2$$

(d) (Medium)

$$y + x < 2$$

$$y + x \geq 4$$

$$y \geq -2$$

$$y \leq 1$$

13. (Medium) Maximize $z = 4x + 3y$ subject to: $x \geq 0$, $y \leq -x + 4$, $y \geq x$.

14. (Hard) Minimize $z = \frac{1}{3}x - \frac{2}{5}y$ subject to: $x + y \geq 6$, $-x + y \geq 4$, $-x + y \leq 6$, $x + y \leq 8$.

15. (Hard)**Computer Business** A computer science major and a business major decide to start a small business that builds and sells a desktop and a laptop computer. They buy the parts, assemble them, load the operating system, and sell the computers to other students. The costs for parts, time to assemble the computer, and profit are summarized in the following table:

	Desktop	Laptop
Cost of Parts	\$700	\$400
Assemble time(hours)	5	3
Profit	\$500	\$300

They were able to get a small business loan in the amount of \$10,000 to cover costs. They plan on making the computers over the summer and selling them at the beginning of the fall semester. They can dedicate at most 90 hours in assembling the computers. they estimate the demand for laptops will be at least three times the demand for desktops. How many of each type shall they make to maximize profit?

16. (Hard)**Production** A manufacturer of skis produces two models, a regular ski and a slalom ski. A set of regular skis give \$25 profit and a set of slalom skis give a profit of \$50. The manufacturer expects a customer demand of at least 200 pairs of regular skis and at least 80 pairs of slalom skis. The maximum number of pairs of skis that the manufacturer can produce is 400. How many of each model should be produced to maximize profits?

Matrices

1. (Easy) Determine the order of each matrix.

(a) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$

(b) $B = \begin{bmatrix} -1 & 3 & 6 & 8 \\ 2 & 9 & 7 & 3 \\ 5 & 4 & -2 & -10 \\ 6 & 3 & 1 & 5 \end{bmatrix}$

2. (Easy) Write the augmented matrix for each system of linear equations:

(a)

$$x - y = -4$$

$$y + z = 3$$

(b)

$$2x - 3y + 4z = -3$$

$$-x + y - 2z = 1$$

$$5x - 2y - 3z = 7$$

3. (Easy) Write the system of linear equations represented by the augmented matrix

$$\left[\begin{array}{ccc|c} 3 & 0 & 5 & 1 \\ 0 & -4 & 7 & -3 \\ 2 & -1 & 0 & 8 \end{array} \right]$$

4. (Medium) Perform the indicated row operations on the augmented matrix

$$\left[\begin{array}{cccc|c} 1 & 0 & 5 & -10 & -5 \\ 0 & 1 & 2 & -3 & -2 \\ 0 & 2 & -3 & 0 & -1 \\ 0 & -3 & 2 & -1 & -3 \end{array} \right]$$

$$R_3 - 2R_2 \longrightarrow R_3$$

$$R_4 + 3R_2 \longrightarrow R_4$$

5. (Hard) Use row operations to transform the following matrix to reduced row-echelon form.

$$\left[\begin{array}{ccc|c} -1 & 2 & 1 & -2 \\ 3 & -2 & 1 & 4 \\ 2 & -4 & -2 & 4 \end{array} \right]$$

6. (Medium) Solve the system of linear equations using Gaussian elimination with back-substitution.

$$3x_1 + x_2 - x_3 = 1$$

$$x_1 - x_2 + x_3 = -3$$

$$2x_1 + x_2 + x_3 = 0$$

7. (Hard) Solve the system of linear equations using Gauss-Jordan elimination.

$$x + 2y - z = 6$$

$$2x - y + 3z = -13$$

$$3x - 2y + 3z = -16$$

8. Solve for the indicated variables.

$$(a) \text{ (Easy)} \begin{bmatrix} 3 & 4 \\ 0 & 12 \end{bmatrix} = \begin{bmatrix} x - y & 4 \\ 0 & 2y + x \end{bmatrix}$$

$$(b) \text{ (Medium)} \begin{bmatrix} 9 & 2b + 1 \\ -5 & 16 \end{bmatrix} = \begin{bmatrix} a^2 & 9 \\ 2a + 1 & b^2 \end{bmatrix}$$

9. (Easy) Given the matrices below perform the indicated operations for each expression, if possible.

$$A = \begin{bmatrix} -1 & 3 & 0 \\ 2 & 4 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 2 & 1 \\ 3 & -2 & 4 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \\ 2 & -1 \\ 3 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 2 & -3 \\ 0 & 1 \\ 4 & -2 \end{bmatrix}$$

$$(a) D - B$$

(b) $2\mathbf{B} - 3\mathbf{A}$

(c) $-\frac{1}{5}\mathbf{C}$

(d) $\mathbf{C} - \mathbf{A}$

10. Given the following matrices, perform the indicated operations for each expression, if possible.

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 3 & 1 \\ 5 & 0 & -2 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 & 2 & 1 \\ 3 & -2 & 4 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 2 & 0 & -3 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} -1 & 7 & 2 \\ 3 & 0 & 1 \end{bmatrix} \quad \mathbf{E} = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & 4 \\ -3 & 1 & 5 \end{bmatrix} \quad \mathbf{F} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

(a) (Easy) \mathbf{GB}

(b) (Medium) $\mathbf{B}(\mathbf{A} + \mathbf{E})$

(c) (Easy) $\mathbf{CD} + \mathbf{G}$

(d) (Easy) $\mathbf{FE} - 2\mathbf{A}$

11. (Easy) Write the system of linear equations as a matrix equation.

(a)

$$3x + 5y - z = 2$$

$$x + 2z = 17$$

$$-x + y - z = 4$$

(b)

$$x + y - 2z + w = 11$$

$$2x - y + 3z = 17$$

$$-x + 2y - 3z + 4w = 12$$

$$y + 4z + 6w = 19$$

12. Determine whether \mathbf{B} is the inverse of \mathbf{A} using $\mathbf{AA}^{-1} = \mathbf{I}$.

(a) (Easy) $\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} \frac{1}{5} & \frac{3}{5} \\ \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$

(b) (Medium)

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

13. Find the inverse \mathbf{A}^{-1} .

(a) (Medium) $\mathbf{A} = \begin{bmatrix} -2.3 & 1.1 \\ 4.6 & -3.2 \end{bmatrix}$

$$(b) \text{ (Hard)} A = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

14. Apply matrix algebra (use inverses) to solve the system of linear equations.

(a) (Medium)

$$\begin{aligned} \frac{2}{5}x + \frac{3}{7}y &= 1 \\ -\frac{1}{2}x - \frac{1}{3}y &= \frac{1}{6} \end{aligned}$$

(b) (Hard)

$$\begin{aligned} x - y - z &= 0 \\ x + y - 3z &= 2 \\ 3x - 5y + z &= 4 \end{aligned}$$

15. Calculate the determinant of each matrix.

$$(a) \text{ (Easy)} \begin{bmatrix} 1 & -2 \\ -3 & -4 \end{bmatrix}$$

$$(b) \text{ (Easy)} \begin{bmatrix} -1.0 & 1.4 \\ 1.5 & -2.8 \end{bmatrix}$$

$$(c) \text{ (Medium)} \begin{bmatrix} 2 & 1 & -5 \\ 3 & -7 & 0 \\ 4 & -6 & 0 \end{bmatrix}$$

$$(d) \text{ (Hard)} \begin{bmatrix} 5 & -2 & -1 \\ 4 & -9 & -3 \\ 2 & 8 & -6 \end{bmatrix}$$

$$(e) \text{ (Medium)} \begin{bmatrix} \frac{3}{4} & -1 & 0 \\ 0 & \frac{1}{5} & -12 \\ 8 & 0 & -2 \end{bmatrix}$$

16. Use Cramer's rule to solve each system of linear equations in two variables, if possible.

(a) (Easy)

$$\begin{aligned} 3x - 2y &= -1 \\ 5x + 4y &= -31 \end{aligned}$$

(b) (Medium)

$$\begin{aligned} \frac{2}{3}x + \frac{9}{4}y &= \frac{9}{8} \\ \frac{1}{6}x + \frac{1}{4}y &= \frac{1}{12} \end{aligned}$$

17. Use Cramer's rule to solve each system of linear equations in three variables.

(a) (Medium)

$$-x + y + z = -4$$

$$x + y - z = 0$$

$$x + y + z = 2$$

(b) (Hard)

$$\frac{1}{2}x - 2y + 7z = 25$$

$$x + \frac{1}{4}y - 4z = -2$$

$$-4x + 5y = -56$$

Conic sections and nonlinear systems of equations and inequalities

1. Sketch the graph of the parabola. Label the focus and directrix:

(a) (Easy) $x^2 = -4y$

(b) (Medium) $(y - 2)^2 = 8(x + 3)$

2. (Easy) A parabola has a vertex at the origin and directrix $y = -3$. Find an equation of the parabola.

3. (Medium) Rewrite the parabola $x^2 - 4x - 8y - 20 = 0$ in standard form and sketch the graph.

4. (Hard) Rewrite the parabola $y^2 - 4x + 3y + 7 = 0$ in standard form. State the coordinates of the vertex and the focus.

5. Sketch the graph of the ellipse. Label the foci:

(a) (Easy) $\frac{x^2}{4} + \frac{y^2}{25} = 1$

(b) (Medium) $(x + 3)^2 + 4(y - 2)^2 = 16$

6. (Hard) Find an equation of the ellipse having foci $(4, 2)$ and $(8, 2)$ that passes through the point $(7, 3)$.

7. (Easy) Sketch the graph of the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$.

8. Sketch the graph of the hyperbola. Label the foci.

(a) (Medium) $(y + 2)^2 - 9(x - 1)^2 = 36$

(b) (Hard) $16x^2 - y^2 = -4$

(c) (Hard) $y^2 - 9x^2 + 4y + 18x - 41 = 0$

9. (Medium) Find an equation of the hyperbola having vertices (3,2) and (-1,2), and having foci (5,2) and (-3,2).

10. (Hard) Identify the conic section and sketch the graph: $x^2 + 4y^2 + 6x - 16y + 9 = 0$

11. Solve the system of nonlinear equations:

(a) (Easy)

$$\begin{aligned}x^2 + y^2 &= 1 \\x^2 - y &= -1\end{aligned}$$

(b) (Medium)

$$\begin{aligned}x - y &= -2 \\x^2 + y^2 &= 2\end{aligned}$$

(c) (Medium)

$$\begin{aligned}x^2 + xy - y^2 &= 5 \\x - y &= -1\end{aligned}$$

(d) (Hard)

$$\begin{aligned}x^2 + 2y^2 &= 18 \\xy &= 4\end{aligned}$$

12. (Easy) Graph the nonlinear inequality $y \geq \ln x$

13. Graph the solution set of the system of nonlinear inequalities:

(a) (Medium)

$$\begin{aligned}x^2 + y^2 &> 16 \\ y &\geq 6 + x\end{aligned}$$

(b) (Hard)

$$\begin{aligned}x &> y^2 \\ x^2 - y^2 &< 1\end{aligned}$$

(c) (Hard)

$$\begin{aligned}y &\leq 2^x \\ x - y &= -4\end{aligned}$$

Sequences and Series

1. (Easy) Find the first four terms and the one hundredth term of the sequence given by $a_n = \frac{(-1)^n}{(n+1)^2}$
2. (Hard) Write an expression for the n^{th} term of the sequence whose first few terms are $-\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \frac{16}{81}, \dots$
3. (Medium) Find the first four partial sums and the n^{th} partial sum of the sequence given by $a_n = \frac{1}{n+1} - \frac{1}{n+2}$.
4. (Easy) Evaluate $\sum_{n=0}^4 n^2$
5. (Medium) Write the sum $\frac{2 \times 1}{1} + \frac{3 \times 2 \times 1}{1} + \frac{4 \times 3 \times 2 \times 1}{2 \times 1} + \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} + \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1}$ using sigma notation.
6. (Medium) Write the first five terms of the recursively defined sequence defined by $a_n = a_{n-1}a_{n-2}$, $a_1 = 2$, $a_2 = -3$.
7. (Medium) Don takes a job out of college with a starting salary of \$30,000. He expects to get a 3% raise each year. Write the recursive formula for a sequence that represents his salary n years on the job. Assume $n = 0$ represents his first year making \$30,000.
8. (Easy) Find the first four terms of the sequence $a_n = -3n + 5$. Determine if the sequence is arithmetic, and if so find the common difference d .

9. (Easy) Find the n^{th} term of the arithmetic sequence given the first term $a_1 = 5$ and the common difference $d = -\frac{3}{4}$.
10. (Medium) Find the first term, a_1 , and the common difference, d , of the arithmetic sequence whose 5^{th} term is 44, and whose 17^{th} term is 152.
11. (Easy/Medium) Find the 100^{th} term of the arithmetic sequence $\{9, 2, -5, -12, \dots\}$.
12. (Medium) Find the sum $\sum_{n=1}^{30} (-2n + 5)$
13. (Medium) Find S_{43} , the 43^{rd} partial sum of the arithmetic sequence $\{1, \frac{1}{2}, 0, -\frac{1}{2}, \dots\}$.
14. (Medium) An amphitheater has 40 rows of seating with 30 seats in the first row, 32 in the second row, 34 in the third row, and so on. Find the total number of seats in the amphitheater.
15. (Medium) How many terms of the arithmetic sequence $\{5, 7, 9, \dots\}$ must be added to get 572?
16. (Easy) Determine if the sequence $\{2, -10, 50, -250, 1250, \dots\}$ could be geometric, and if so find the common ratio r .
17. (Easy) Find the eighth term of the geometric sequence $\{5, 15, 45, \dots\}$.
18. (Hard) Find the fifth term of the geometric sequence given that the third term is $\frac{63}{4}$ and the sixth term is $\frac{1701}{32}$.
19. (Medium) Find S_5 , the fifth partial sum of the geometric sequence $\{1, 0.7, 0.49, 0.343, \dots\}$
20. (Easy) Evaluate $\sum_{k=1}^5 (-\frac{2}{3})^k$.
21. (Easy) Find the sum of the infinite geometric series $2 - \frac{2}{5} + \frac{2}{25} - \frac{2}{125} \dots$
22. (Medium) Write $0.\overline{321}$ in reduced fraction form.
23. (Medium) Expand $(2 - 3x)^5$ using Pascal's triangle.

24. (Easy) Calculate the binomial coefficient $\binom{20}{3}$.
25. (Medium) Find the term that contains x^3 in the expansion of $(y - 3x)^{10}$.
26. (Hard) Find the middle term of the expansion $(x^2 + 1)^{18}$.
27. (Medium) Find the coefficient of the simplified third term in the expansion of $(\sqrt{2} + y)^{12}$.