SEC. 7.1 MATRICES AND SYSTEMS OF EQUATIONS

1. MATRIX: A RECTANGULAR ARRAY OF NUMBERS ORGANIZED INTO ROWS AND COLUMNS.

EX.

$$\underset{\text{ROWS}}{M} \times \underset{\text{COLUMNS}}{N} = \underbrace{\left.\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}\right\}}_{n \text{ COLUMNS}} \begin{matrix} m \\ \text{ROWS} \end{matrix}$$

2. AUGMENTED MATRIX

$$\begin{cases} 3x - 2y + z = 5 \\ x + 3y - z = 0 \\ -x \qquad\quad + 4z = 11 \end{cases} \qquad \left[\begin{array}{ccc:c} 3 & -2 & 1 & 5 \\ 1 & 3 & -1 & 0 \\ -1 & 0 & 4 & 11 \end{array}\right]$$

3. ELEMENTARY ROW OPERATIONS
   1) INTERCHANGE ROWS
   2) ADD TWO ROWS TOGETHER
   3) MULTIPLY A ROW BY A NON-ZERO NUMBER

4. NOTATION FOR ROW OPERATIONS

1) $R_1 \leftrightarrow R_2$ " INTERCHANGE ROW_1 WITH ROW_2

2) $R_1 + R_3 \rightarrow R_3$ " ADDING R_1 AND R_3 THEN REPLACE R_3.

3) $-2R_2 \rightarrow R_2$ " MULTIPLY R_2 BY -2 THEN REPLACE R_2.

5. ROW-ECHELON FORM & <u>REDUCED</u> ROW-ECHELON FORM

A) ROW-ECHELON FORM (THINK TRIANGULAR FORM)

1) THE FIRST NON-ZERO NUMBER IN EACH ROW (READING LEFT TO RIGHT) IS A 1. THIS IS CALLED THE "LEADING ENTRY."

2) THE LEADING ENTRY IN EACH ROW IS TO THE RIGHT OF THE LEADING ENTRY IN THE ROW JUST ABOVE IT.

3) ALL ROWS CONSISTING ENTIRELY OF ZEROS MUST BE THE BOTTOM ROW.

B) <u>REDUCED</u> ROW-ECHELON FORM

4) EVERY NUMBER ABOVE & BELOW A LEADING ENTRY MUST BE A ZERO.

$$\begin{bmatrix} 1 & 3 & -6 & 10 & 0 \\ 0 & 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$ ROW - ECHELON FORM

$$\begin{bmatrix} 0 & 1 & 2 & 0 & 6 \\ 1 & 0 & 3 & 4 & -5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 & 2 \end{bmatrix}$$ NEITHER

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$ REDUCED ROW ECHELON

EXAMPLE:

$$\begin{cases} x - y + 3z = 4 \\ x + 2y - 2z = 10 \\ 3x - y + 5z = 14 \end{cases}$$

WRITE AUGMENTED MATRIX:

$$\begin{array}{c} \begin{array}{cccc} -3 & -3 & -3 & -3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & -1 & 3 & 4 \\ 1 & 2 & -2 & 10 \\ 3 & -1 & 5 & 14 \end{array}\right] \end{array}$$

$-1R_1 + R_2 \rightarrow R_2$

$-3R_1 + R_3 \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 4 \\ 0 & 3 & -5 & 6 \\ 0 & 2 & -4 & 2 \end{array}\right]$$

$\frac{1}{2}R_3 \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 4 \\ 0 & 3 & -5 & 6 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

$R_3 \leftrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 4 \\ 0^{-3} & 1^{-3} & -2^{-3} & 1^{-3} \\ 0 & 3 & -5 & 6 \end{array}\right]$$

$-3R_2 + R_3 \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 4 \\ 0^{-3} & 1^{-3} & -2^{-3} & 1^{-3} \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$x - y + 3z = 4$
$x - 7 + 3(3) = 4$
$x + 2 = 4$

$z = 3$

$x = 2$

$y - 2z = 1$
$y - 2(3) = 1$
$y - 6 = 1$
$y = 7$

$\boxed{(2, 7, 3)}$

6. GAUSS - JORDAN METHOD

(THINK REDUCED ROW-ECHELON FORM)

EXAMPLE:

$$\begin{cases} 4x + 8y - 4z = 4 \\ 3x + 8y + 5z = -11 \\ -2x + y + 12z = -17 \end{cases} \qquad \begin{bmatrix} 4 & 8 & -4 & 4 \\ 3 & 8 & 5 & -11 \\ -2 & 1 & 12 & -17 \end{bmatrix}$$

$$\tfrac{1}{4}R_1 \to R_1 \qquad \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 3 & 8 & 5 & -11 \\ -2 & 1 & 12 & -17 \end{array}\right]$$

$$\begin{array}{l} -3R_1 + R_2 \to R_2 \\ 2R_1 + R_3 \to R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 2 & 8 & -14 \\ 0 & 5 & 10 & -15 \end{array}\right]$$

$$\begin{array}{l} \tfrac{1}{2}R_2 \to R_2 \\ \tfrac{1}{5}R_3 \to R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 1 & 2 & -3 \end{array}\right]$$

$$-1R_2 + R_3 \to R_3 \qquad \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & -2 & 4 \end{array}\right]$$

$$-\tfrac{1}{2}R_3 \to R_3 \qquad \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 1 & -2 \end{array}\right] \qquad z = -2$$

$$-4R_3+R_2 \to R_2 \quad R_3+R_1 \to R_1 \quad \left[\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ \underset{-2}{0} & \underset{-2}{1} & \underset{-2}{0} & \underset{-2}{1} \\ \underset{-4}{0} & \underset{-4}{0} & \underset{-4}{1} & \underset{-4}{-2} \end{array}\right] \qquad y=1$$

$$-2R_2+R_1 \to R_1 \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right] \qquad x=-3$$

$$\boxed{(-3,\ 1,\ -2)}$$

#31 INFINITELY MANY PROBLEM

$$\begin{cases} x + 4y - 2z = -3 \\ 2x - y + 5z = 12 \\ 8x + 5y + 11z = 30 \end{cases} \qquad \begin{array}{cccc} -8 & -8 & -8 & -8 \\ \left[\begin{array}{ccc|c} 1 & 4 & -2 & -3 \\ 2 & -1 & 5 & 12 \\ 8 & 5 & 11 & 30 \end{array}\right] \end{array}$$

$$-2R_1+R_2 \to R_2 \quad -8R_1+R_3 \to R_3 \quad \left[\begin{array}{ccc|c} 1 & 4 & -2 & -3 \\ 0 & -9 & 9 & 18 \\ 0 & -27 & 27 & 54 \end{array}\right]$$

$$-\tfrac{1}{9}R_2 \to R_2 \quad -\tfrac{1}{27}R_3 \to R_3 \quad \left[\begin{array}{ccc|c} 1 & 4 & -2 & -3 \\ 0 & 1 & -1 & -2 \\ 0 & 1 & -1 & -2 \end{array}\right]$$

$$-1R_2+R_3 \to R_3 \quad \left[\begin{array}{ccc|c} 1 & 4 & -2 & -3 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

DEPENDENT SYSTEM

LET $z=t$

$$y - z = -2$$

$$y - \cancel{t} = -2 + t$$
$$\cancel{+t}$$

$$y = t - 2$$

$$x + 4y - 2z = -3$$

$$x + 4(t-2) - 2t = -3$$

$$x + 4t - 8 - 2t = -3$$

$$x + \cancel{2t} - 8 = -3 \quad -2t$$
$$\cancel{-2t} \quad +8 \quad +8$$

$$x = 5 - 2t$$

$$\boxed{(5-2t,\ t-2,\ t)}$$