SEC 4.3 REAL ZEROS OF POLYNOMIALS

1. RATIONAL ZERO THEOREM

$$P(x) = \overset{Q}{a_n} x^n + a_{n-1} x^{n-1} + \ldots \, ax + \overset{P}{a_0}$$

LIST POSSIBLE ZEROS: $\dfrac{P}{Q}$ $\dfrac{\text{(FACTORS)}}{\text{(FACTORS}}$

EXAMPLE: $\overset{Q}{1}x^3 - x^2 - 14x + \overset{P}{24} = P(x)$

$$\frac{P}{Q} = \frac{\pm 1 \ \pm 2 \ \pm 3 \ \pm 4 \ \pm 6 \ \pm 8 \ \pm 12 \ \pm 24}{\pm 1}$$

2. DESCARTES RULE OF SIGNS

A) THE NUMBER OF POSITIVE ZEROS OF P(x) IS EITHER EQUAL TO THE NUMBER OF SIGN CHANGES OR LESS BY AN EVEN (2)

EXAMPLES 4 SIGN CHANGES

4 OR 2 OR 0

$$P(x) = x^3 - x^2 - 14x + 24$$

1 1

2 OR 0 POSITIVE ZEROS

B) THE NUMBER OF NEGATIVE REAL ZEROS OF P(x) IS EITHER EQUAL TO THE NUMBER OF SIGN CHANGES WHEN (-x) IS SUBSTITUTED IN FOR EACH x OR LESS BY A EVEN NUMBER (2).

$$P(x) = x^3 - x^2 - 14x + 24$$

$$(-x)^3 - (-x)^2 - 14(-x) + 24$$

$$\downarrow$$

$$-x^3 - x^2 + 14x + 24$$

1

SO 1 NEGATIVE ZERO EXISTS

3. THE UPPER AND LOWER BOUNDS THEOREM

A) IF A VALUE (c) IS USED BY SYNTHETIC DIVISION AND THE QUOTIENT ARE <u>ALL</u> POSITIVE COEFFICIENTS, YOU HAVE FOUND AN UPPER BOUND

B) IF A VALUE (c) IS USED BY SYNTHETIC DIVISION AND THE QUOTIENT ARE ALTERNATING SIGNS (+ - + -) THEN YOU HAVE FOUND A LOWER BOUND.

EXAMPLE: $P(x) = x^3 \underbrace{-}_{1} x^2 - 14x \underbrace{+}_{1} 24$

$$\frac{P}{Q} = \frac{\pm 1 \quad \pm 2 \quad \pm 3 \quad \pm 4 \quad \pm 6 \quad \pm 8 \quad \pm 12 \quad \pm 24}{\pm 1}$$

2 OR 0 POSSIBLE POSITIVE ZEROS

$(-x)^3 - (-x)^2 - 14(-x) + 24$

$-x^3 \underbrace{- x^2 + }_{1} 14x + 24$

1 POSSIBLE NEGATIVE ZERO

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -14 & 24 \\ & \downarrow & 1 & 1 & -15 \\ \hline & 1 & 0 & -15 & \boxed{9} \end{array}$$

$$\begin{array}{r|rrrr} 6 & 1 & -1 & -14 & 24 \\ & \downarrow & 6 & 30 & 96 \\ \hline & 1 & 5 & 16 & + \end{array}$$

6 IS AN UPPER BOUND

$$\begin{array}{r|rrrr} 4 & 1 & -1 & -14 & 24 \\ & \downarrow & 4 & 12 & -8 \\ \hline & 1 & 3 & -2 & \boxed{16} \end{array}$$

$$\begin{array}{r|rrrr} 3 & 1 & -1 & -14 & 24 \\ & \downarrow & 3 & 6 & -24 \\ \hline & 1x^2 & +2x & -8 & \boxed{0} \end{array}$$

$x^2 + 2x - 8$

$(x+4)(x-2)$

$\{-4, 2, 3\}$

#25. $P(x) = 4x^4 - 25x^2 + 36$

144: $1 \cdot 144$
$2 \cdot 72$
$3 \cdot 48$
$4 \cdot 36$
$6 \cdot 24$
$8 \cdot 18$
$-9 \cdot -16$

$$4x^4 - 9x^2 - 16x^2 + 36$$
$$x^2(4x^2 - 9) - 4(4x^2 - 9)$$
$$(4x^2 - 9)(x^2 - 4)$$
$$(2x+3)(2x-3)(x+2)(x-2)$$

$$\left\{ -\frac{3}{2}, \frac{3}{2}, -2, 2 \right\}$$

#35 $P(x) = x^5 + 3x^4 - 9x^3 - 31x^2 + 36$

$$\frac{P}{Q} = \frac{\pm 1 \pm 2 \pm 3 \pm 4 \pm 6 \pm 9 \pm 12 \pm 18 \pm 36}{\pm 1}$$

2 OR 0 POSSIBLE POSITIVE ZEROS

CHECK FOR POSSIBLE ZEROS (DESCARTES' RULE OF SIGNS)

$$(-x)^5 + 3(-x)^4 - 9(-x)^3 - 31(-x)^2 + 36$$
$$-x^5 + 3x^4 + 9x^3 - 31x^2 + 36$$

3 OR 1 POSSIBLE NEGATIVE ZEROS

1	$1x^5$	3	-9	-31	0	36	
	1	1	4	-5	-36	-36	
	$1x^4$	4	-5	-36	-36		0

3	$1x^4$	4	-5	-36	-36	
		3	21	48	36	
	$1x^3$	7	16	12		0

-2	$1x^3$	7	16	12	
		-2	-10	-12	
	$1x^2$	$+5x$	$+6$		0

$(x+2)(x+3)$

$\{-2, -3, -2, 3, 1\}$

$\{-2^{\text{MULT OF 2}}, -3, 3, 1\}$