SEC 7.2 THE ALGEBRA OF MATRICES

1. EQUALITY OF MATRICES: TWO MATRICES ARE EQUAL IF AND ONLY IF, THEY HAVE THE SAME DIMENSIONS AND EACH CORRESPONDING ENTRY IS EQUAL.

EXAMPLE

$$\underset{2\times 2}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} = \underset{2\times 2}{\begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}}$$

THEREFORE $a=1 \quad b=3 \quad c=5 \quad d=2$

2. SUM, DIFFERENCE AND SCALAR MULTIPLICATION

A) SUM: $[A]+[B]$ THEY MUST HAVE THE SAME DIMENSIONS, THEN ADD THEIR CORRESPONDING ENTRIES.

$$A = \begin{bmatrix} 2 & -3 \\ 0 & 5 \\ 7 & -\frac{1}{2} \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ -3 & 1 \\ 2 & 2 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 2 & -3 \\ 0 & 5 \\ 7 & -\frac{1}{2} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -3 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ -3 & 6 \\ 9 & \frac{3}{2} \end{bmatrix}$$

B) DIFFERENCE: [A] − [B] THEY MUST HAVE THE SAME DIMENSIONS, THEN SUBTRACT THEIR CORRESPONDING ENTRIES

EXAMPLE

$$A - B = \begin{bmatrix} 2 & -3 \\ 0 & 5 \\ 7 & -\frac{1}{2} \end{bmatrix} + \begin{bmatrix} -1 & -0 \\ +3 & -1 \\ -2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 3 & 4 \\ 5 & -\frac{5}{2} \end{bmatrix}$$

C) SCALAR MULTIPLICATION:

A SCALAR IS A REAL NUMBER c. IF c[A], THEN c IS MULTIPLIED TO EVERY ENTRY IN [A].

EXAMPLE 2A

$$2\begin{bmatrix} 2 & -3 \\ 0 & 5 \\ 7 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 4 & -6 \\ 0 & 10 \\ 14 & -1 \end{bmatrix}$$

3. PROPERTIES OF ADDITION & SCALAR MULTIPLICATION. A, B, & C MATRICES
c, d SCALARS

A) $A+B = B+A$ COMMUTATIVE PROP.

B) $(A+B)+C = A+(B+C)$ ASSOCIATIVE PROP.

C) $c(dA) = (cd)A$ ASSOCIATIVE PROP OF SCALAR MULTIPLICATION

D) $(c+d)A = cA + dA$ DISTRIBUTIVE PROP. OF SCALAR MULT.

E) $c(A+B) = cA + cB$

4. MATRIX MULTIPLICATION:

IF MATRIX A IS AN M x N MATRIX
" B " " N x K "

SAME

THEN $A \cdot B = M \times K$

$$\begin{bmatrix} \cancel{a} & \cancel{b} \\ c & d \\ e & f \end{bmatrix} \cdot \begin{bmatrix} g & h \\ i & j \end{bmatrix} = \begin{bmatrix} ag+bi & ah+bj \\ cg+di & ch+dj \\ eg+fi & eh+fj \end{bmatrix}$$

$3 \times 2 \quad 2 \times 2 = 3 \times 2$

SAME

CAN YOU MULTIPLE:

$3 \times 3 \quad 3 \times 4 = 3 \times 4$

$4 \times 1 \quad 1 \times 3 = 4 \times 3$

$2 \times 3 \quad 2 \times 3$ NOT POSSIBLE

EXAMPLE :

$$\begin{bmatrix} 2 & -3 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 2\cdot 1 + (-3)(-1) & 2\cdot 2 + (-3)4 \\ 0\cdot 1 + 1(-1) & 0\cdot 2 + 1\cdot 4 \\ 1\cdot 1 + 2(-1) & 1\cdot 2 + 2\cdot 4 \end{bmatrix}$$

$$\underbrace{3\times 2 \quad \cdot \quad 2\times 2}_{\text{SAME}} \qquad \begin{bmatrix} 5 & -8 \\ -1 & 4 \\ -1 & 10 \end{bmatrix}$$

3×2

Ex

$$\begin{bmatrix} 5 & -3 & 10 \\ 6 & 1 & 0 \\ -5 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 8 \\ -1 \end{bmatrix}$$

$3\times 3 \qquad 3\times 1 \qquad 3\times 1$

5. PROPERTIES OF MATRIX MULTIPLICATION

A) <u>NO</u> COMMUTATIVE PROPERTY

B) $A(BC) = (AB)C$ ASSOCIATIVE PROPERTY

C) $A(B+C) = AB+AC$

$(A+B)C = AC+BC$

DISTRIBUTIVE PROPERTY

6. WRITING A SYSTEM AS A MATRIX EQUATION.

$$\begin{cases} \boxed{\begin{matrix} 2x-5y \\ 3x+2y \end{matrix}} \begin{matrix} =7 \\ =4 \end{matrix} \end{cases}$$

COEFFICIENT MATRIX

$$\begin{bmatrix} 2 & -5 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$$

COEFFICIENT MATRIX

VARIABLE MATRIX

CONSTANT MATRIX