SEC. 9.3 GEOMETRIC SEQUENCES

1. GEOMETRIC SEQUENCE: A SEQUENCE GENERATED WHEN WE START WITH A NUMBER (a_1) AND REPEATEDLY MULTIPLY BY A CONSTANT (r).

2. COMMON RATIO: REPEATED MULTIPLICATION

$$r = \frac{a_n}{a_{n-1}}$$

EXAMPLE $\frac{a_7}{a_6} = r$

OR $\frac{a_3}{a_2} = r$

3. DEFINITION:

$a_1, \; a_1 \cdot r, \; a_1 \cdot r \cdot r, \; a_1 \cdot r \cdot r \cdot r, \ldots$

n^{th} TERM $a_n = a_1 \cdot r^{n-1}$ MEMORIZE

EXAMPLE: $a_1 = 3$ $r = 2$ FIND THE FIRST 5 TERMS.

$3, \; 3 \cdot 2, \; 3 \cdot 2^2, \; 3 \cdot 2^3, \; 3 \cdot 2^4$

$3, \; 6, \; 12, \; 24, \; 48$

EXAMPLE: FIND THE FUNCTION OF A GEOMETRIC SEQUENCE WHOSE 1ST TERM IS 5, AND WHOSE COMMON RATIO IS 3.

$a_1 = 5$ $r = 3$

$$a_n = 5 \cdot 3^{n-1}$$

4. PARTIAL SUM (FINITE SUM) OF A GEOMETRIC SEQUENCE.

$$S_n = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \ldots a_1 \cdot r^{n-1}$$

OR

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right) \quad r \neq 1$$

MEMORIZE

#41 (~~TEXTBOOK~~)

$a_3 = 28 \qquad a_6 = 224$

$n = 6$

$a_1 = 7$

$a_3 = a_1 \cdot r^{3-1}$

$\frac{28}{r^2} = a_1 \cdot \frac{r^2}{r^2}$

$a_1 = \frac{28}{r^2}$

$a_1 = \frac{28}{4}$

$a_1 = 7$

$a_6 = a_1 \cdot r^{6-1}$

$224 = a_1 \cdot r^5$

$224 = \frac{28}{r^2} \cdot r^5$

$\frac{224}{28} = \frac{28}{28} r^3$

$\sqrt[3]{8} = \sqrt[3]{r^3}$

$2 = r$

$$S_n = 7 \cdot \left(\frac{1-2^6}{1-2}\right)$$

$$7 \cdot \left(\frac{1-64}{-1}\right)$$

$$7 \cdot \left(\frac{-63}{-1}\right)$$

$$7 \cdot 63 = \boxed{441}$$

5. INFINITE SEQUENCE FORMULA:

$$\sum_{n=1}^{\infty} = S_\infty = \frac{a_1}{1-r}$$ MEMORIZE

EX. $2 + \frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \frac{2}{625} + \dots$

$a_1 = 2$

$r = \frac{\frac{2}{5}}{\frac{2}{1}} \cdot \frac{1}{2} = \boxed{\frac{1}{5} = r}$

$S_\infty = \frac{2}{\frac{5}{5} - \frac{1}{5}} = \frac{2}{\frac{4}{5}} \cdot \frac{5}{4} = \boxed{\frac{5}{2}}$ OR 2.5