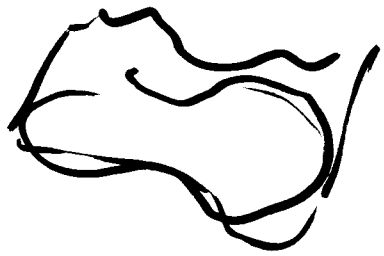
10/26 4.9 Cross section

$$V = A_{\text{base}} \cdot h = \pi r^2 h$$



$$SA = A_{\text{base}} + A_{\text{lateral}}$$

$$SA = \pi r^2 + 2\pi r \cdot h$$

Any cylindrical (constant cross section object): Volume is cross section area times height

Surface area is cross section perimeter times height

Min SA for fixed volume $\Leftrightarrow$ Max volume for fixed surface area

$$SA = \pi r^2 + 2\pi r h$$

$$V = 1 = \pi r^2 h$$

$$r = \sqrt{\frac{1}{\pi h}}$$

$$h = \frac{1}{\pi r^2}$$

$$SA(r) = \pi r^2 + 2\pi r \cdot \frac{1}{\pi r^2}$$

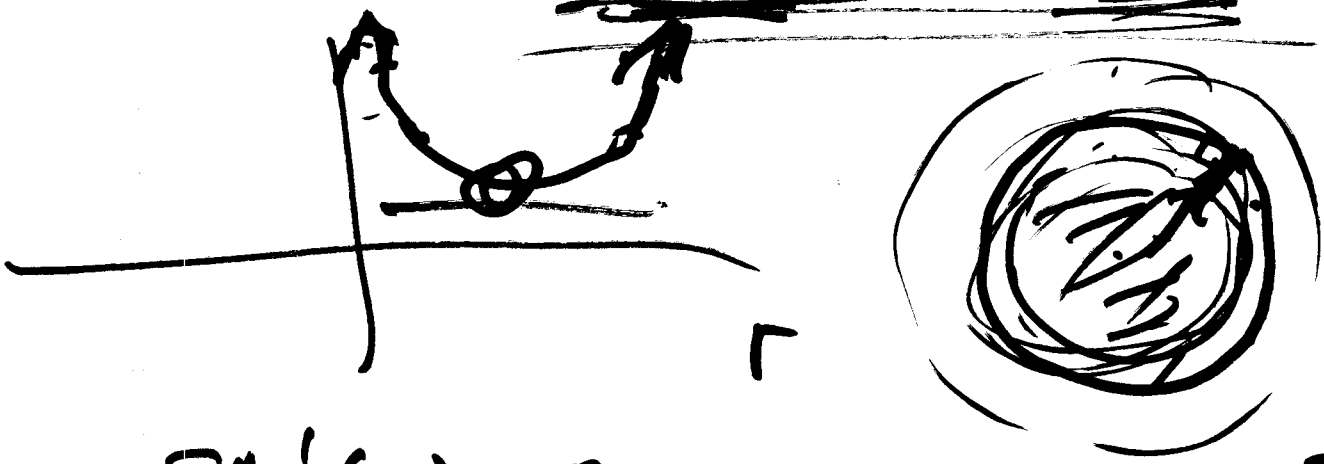
$$= \pi r^2 + \frac{2}{r} = \pi r^2 + 2r^{-1}$$

$$SA(h) = \pi \cdot \frac{1}{\pi h} + 2\pi h \sqrt{\frac{1}{\pi h}}$$

$$= \frac{1}{h} + 2\sqrt{\pi h}$$

$$= h^{-1} + 2\sqrt{\pi}\, h^{1/2}$$

$$SA(r) = \pi r^2 + 2r^{-1}$$



$$0 = SA'(r) = 2\pi r - 2r^{-2}$$

$$2\pi r = \frac{2}{r^2} \qquad r^3 = \frac{1}{\pi}$$

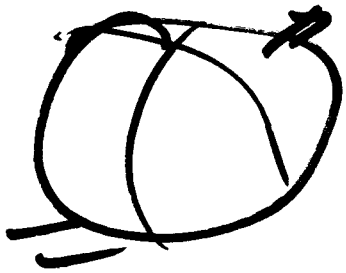
$$r = (1/\pi)^{1/3}$$

Then use $h = 1/\pi r^2 = \pi[(1/\pi)^{1/3}]^3 = \frac{1}{\pi^{1/3}}$

$$\int 2\pi r = 2\pi \frac{r^2}{2} + C = \pi r^2$$

SA of Ball $= 4\pi r^2$

Volume $\int 4\pi r^2 \, dr =$



$$SA(h) = h^{-1} + 2\sqrt{\pi}\, h^{\frac{1}{2}}$$

$$0 = SA'(h) = -h^{-2} + 2\sqrt{\pi} \cdot \frac{1}{2} h^{-\frac{1}{2}}$$

$$0 = -\frac{1}{h^2} + \frac{\sqrt{\pi}}{\sqrt{h}}$$

$$\sqrt{h}\,\frac{1}{h^2} = \frac{\sqrt{\pi}}{\sqrt{h}}\,\sqrt{h}$$

$$\frac{h^{1/2}}{h^{4/2}} = h^{-3/2} = \pi^{\frac{1}{2}}$$

Both sides to 2

$$h^{-3} = \pi \qquad h = \pi^{-1/3} = \frac{1}{\pi^{1/3}}$$

$$\int 4\pi x^2 \, dx =$$

$$\frac{4}{3}\pi r^3$$

$$4\pi \frac{x^3}{3}$$

10. $f(x) = 8x + \frac{8}{x} = \underline{8x^1 + 8x^{-1}}$

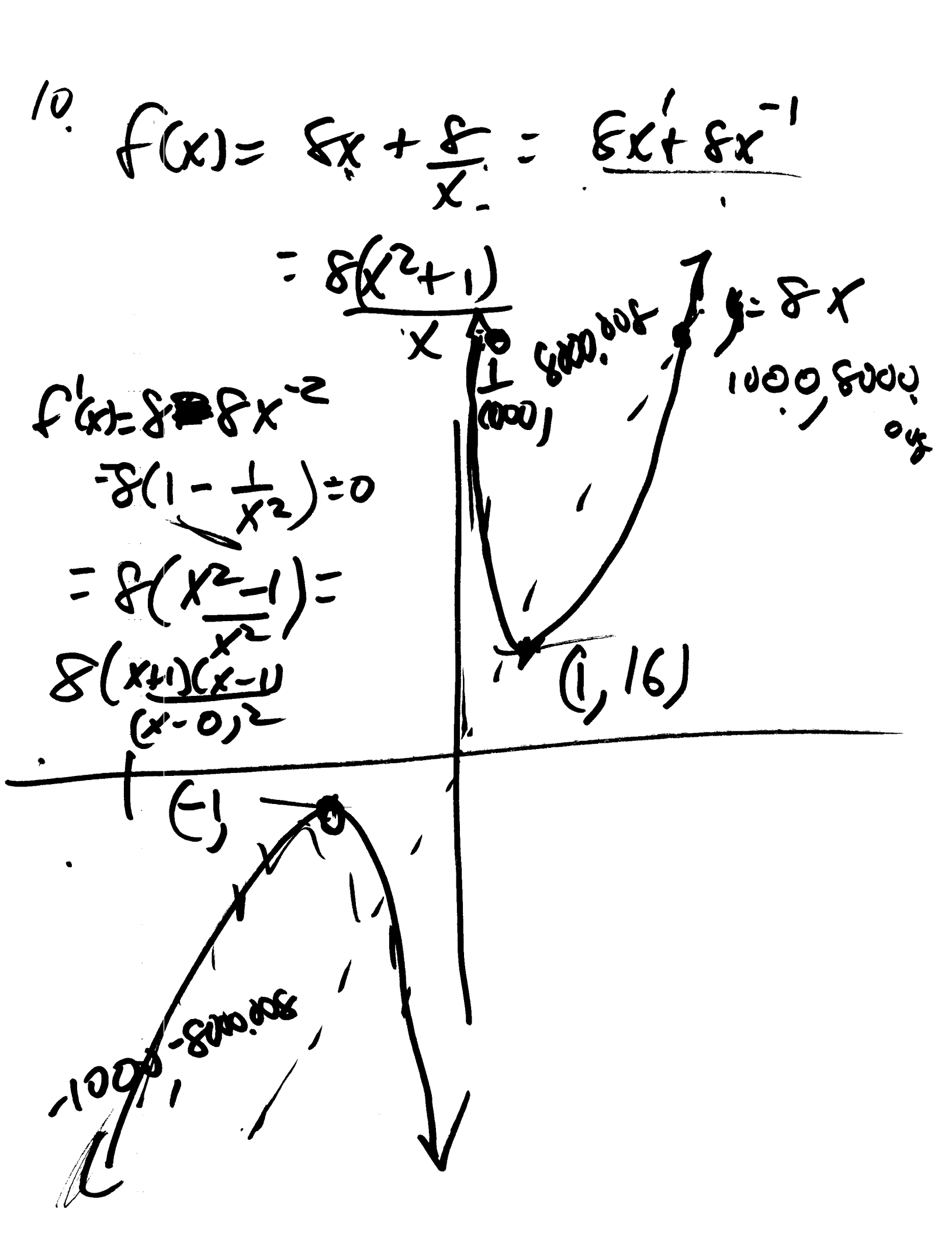
$= \frac{8(x^2+1)}{x}$

$f'(x) = 8 - 8x^{-2}$

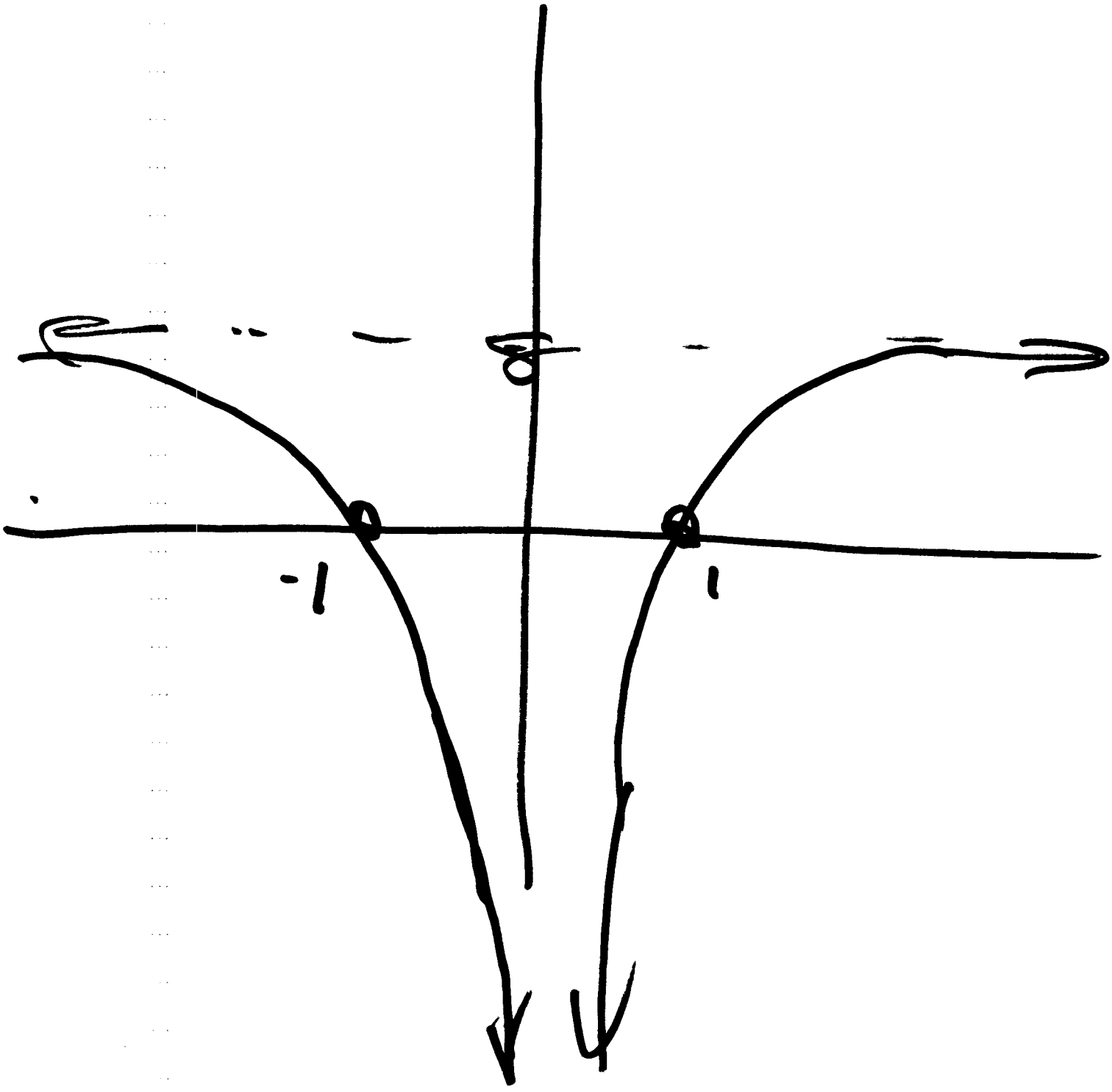
$-8(1 - \frac{1}{x^2}) = 0$

$= 8(\frac{x^2-1}{x^2}) =$

$8\frac{(x+1)(x-1)}{(x-0)^2}$



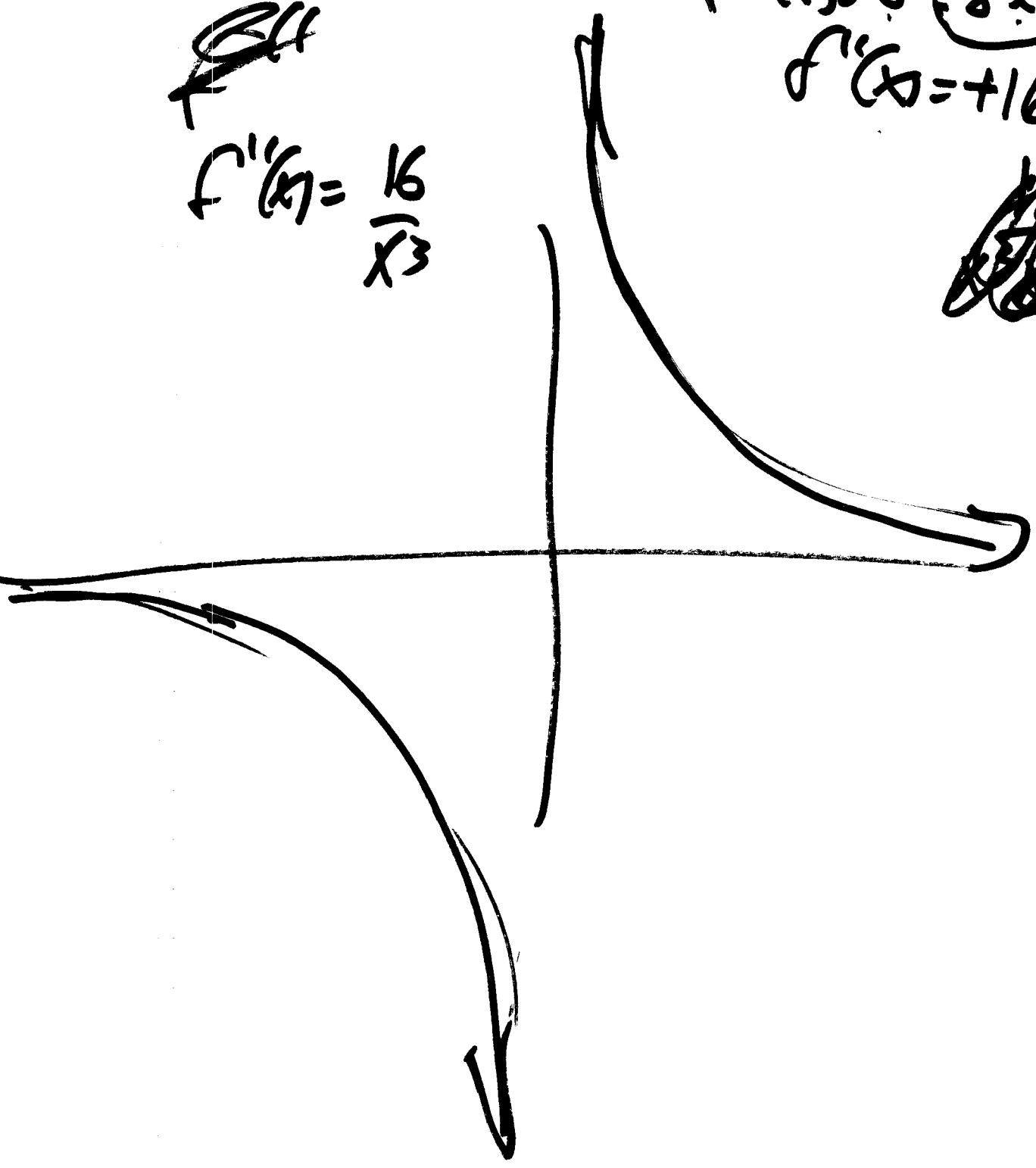
$$f'(x) = 8\frac{(x--1)'(x-1)'}{(x-0)^2} = 8 - \frac{8}{x^2}$$



$f'(x) = 8 - 8x^{-2}$

$f''(x) = +16x^{-3}$

$f''(x) = \frac{16}{x^3}$



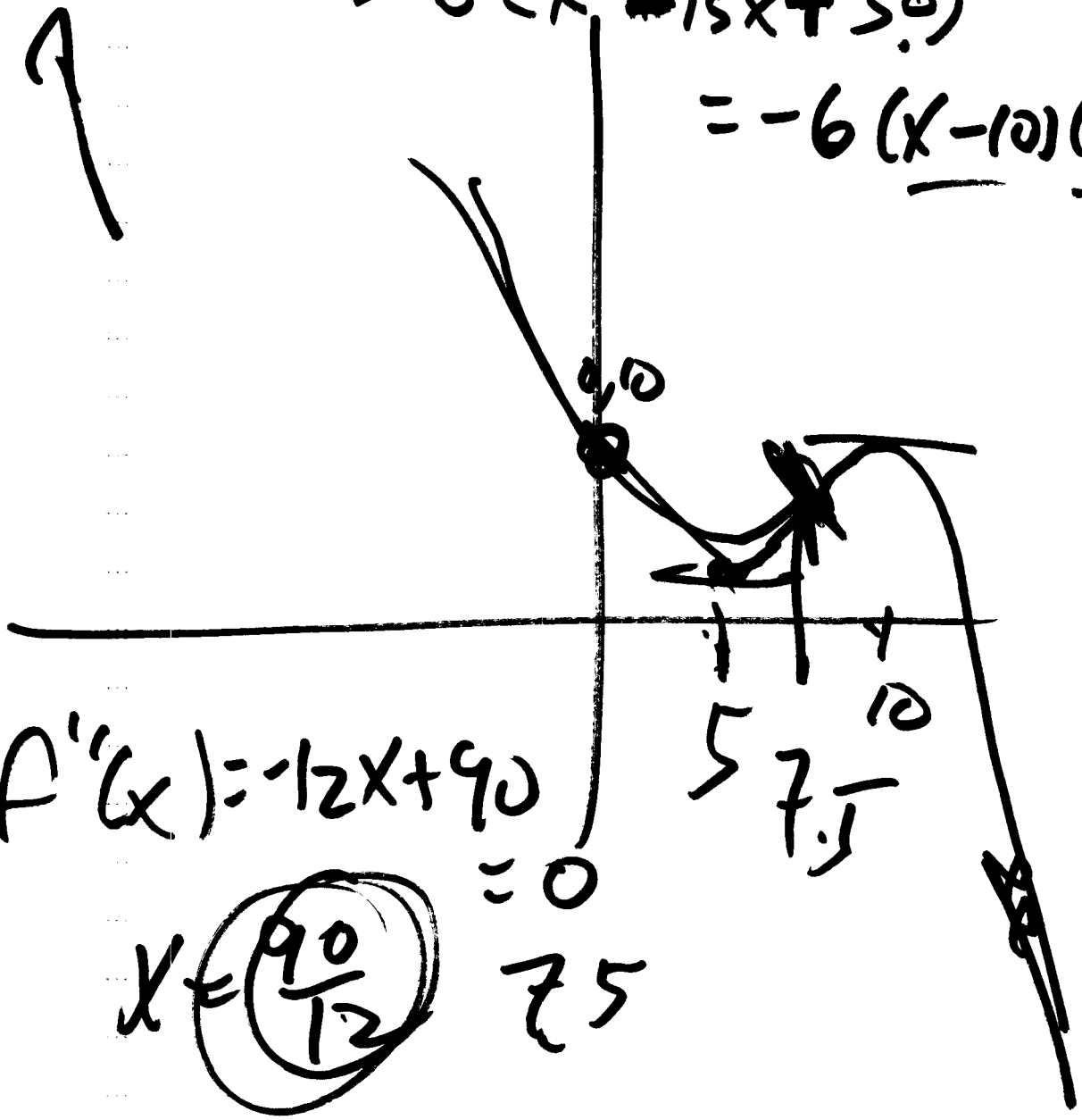
12.

$C(x) = -2x^3 + 45x^2 - 300x + 10$

$C'(x) = -6x^2 + 90x - 300$

$= -6(x^2 - 15x + 50)$

$= -6(x-10)(x-5)$



$C''(x) = -12x + 90$

$= 0$

$x = \frac{90}{12}$

$= 7.5$