SEC 4.5 RATIONAL FUNCTIONS & ASYMTOTES

1. ASYMPTOTE: THE LINE TO WHICH THE GRAPH APPROACHES.
(HORIZONTAL, VERTICAL OR SLANT)

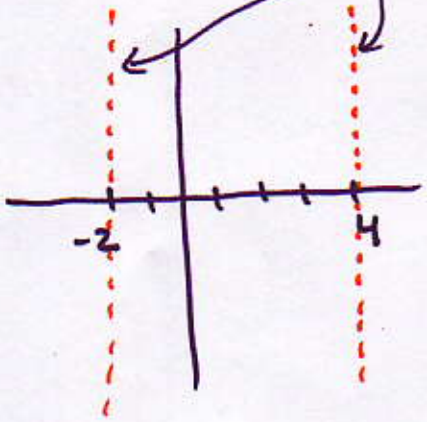
2. VERTICAL ASYMPTOTE: TO FIND THEM, SET THE DENOMINATOR EQUAL TO ZERO AND SOLVE.

EXAMPLE: $r(x) = \dfrac{x+1}{x^2-2x-8}$

$x^2-2x-8=0$

$(x-4)(x+2)=0$

$\{4, -2\}$



SO $x=4$, $x=-2$ } TWO VERTICAL ASYMPTOTES EQUATIONS

DOMAIN: $(-\infty, -2) \cup (-2, 4) \cup (4, \infty)$

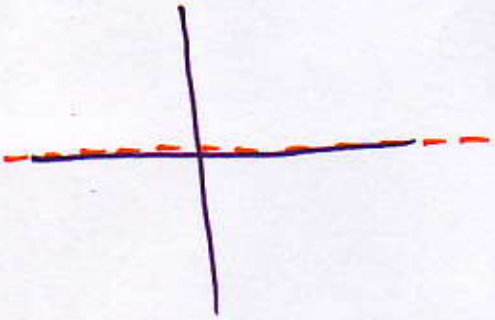
$\{x \mid x \neq 4 \text{ OR } -2\}$ (SET-BUILDER NOTATION)

3. HORIZONTAL ASYMPTOTES: (DON'T PLAY BY THE RULES) THE GRAPH CAN TOUCH AN HORIZONTAL ASYMPTOTE. IT CAN EVEN CROSS IT.

3 WAYS OF FINDING A HORIZONTAL ASYMPTOTE

1) H.A. WILL BE $y=0$ WHEN THE DEGREE OF THE NUMERATOR IS LESS THAN THE DEGREE OF DENOMINATOR.

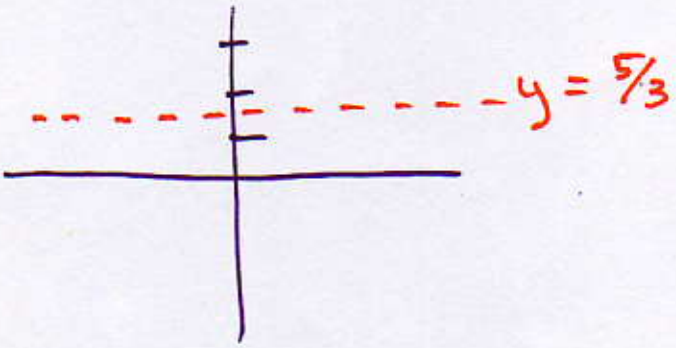
EX. $r(x) = \dfrac{x+3}{x^2-2x-8}$ THEREFORE $y=0$ IS H.A.



2) H.A. WILL BE $y = \dfrac{\text{LEADING COEFFICIENT (NUM)}}{\text{LEADING COEFFICIENT (DEN)}}$

WHEN THE DEGREE OF NUMERATOR IS THE SAME AS THE DEGREE OF DENOMINATOR.

EX. $r(x) = \dfrac{5x^2+4}{3x^2-x-6}$ so $y = \dfrac{5}{3}$



3) NO HORIZONTAL ASYMPTOTE EXISTS WHEN THE DEGREE OF THE NUMERATOR IS GREATER THAN THE DEGREE OF THE DENOMINATOR.

Ex. $r(x) = \dfrac{x^3+2x^2-x+1}{x^2+4}$

4. SLANT ASYMPTOTES: MUST USE LONG DIVISION WHEN THE DEGREE OF THE NUMERATOR IS EXACTLY 1 DEGREE MORE THAN THE DEGREE OF THE DENOMINATOR

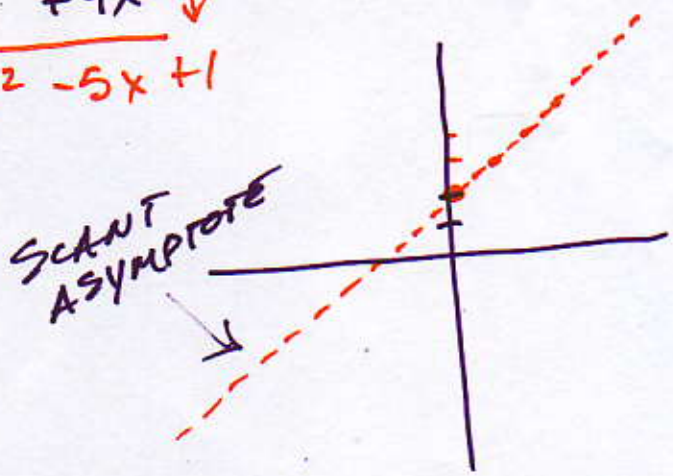
Ex. $r(x) = \dfrac{x^3+2x^2-x+1}{x^2+4}$

$$\begin{array}{r} x+2 \\ x^2+4 \;\overline{)\; x^3+2x^2-x+1} \\ \underline{-x^3 \quad\quad -4x} \\ 2x^2-5x+1 \end{array}$$

$x(x^2+4)$

$y = mx+b$

$y = x+2$

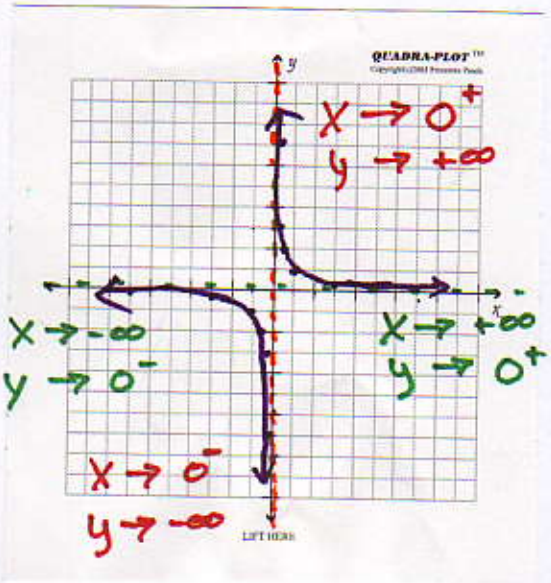


5. RATIONAL FUNCTIONS:

EX. $f(x) = \frac{1}{x}$

x	y
0	UNDEFINED
$\frac{1}{2}$	2
$\frac{1}{3}$	3
3	$\frac{1}{3}$
7	$\frac{1}{7}$
$\frac{1}{7}$	7
1	1

x	y
$-\frac{1}{2}$	-2
$-\frac{1}{3}$	-3
-3	$-\frac{1}{3}$
-7	$-\frac{1}{7}$
$-\frac{1}{7}$	-7
-1	-1



DOMAIN: $(-\infty, 0) \cup (0, \infty)$

RANGE: $(-\infty, 0) \cup (0, \infty)$

V.A $x = 0$

H.A. $y = 0$

#12 (REVIEW)

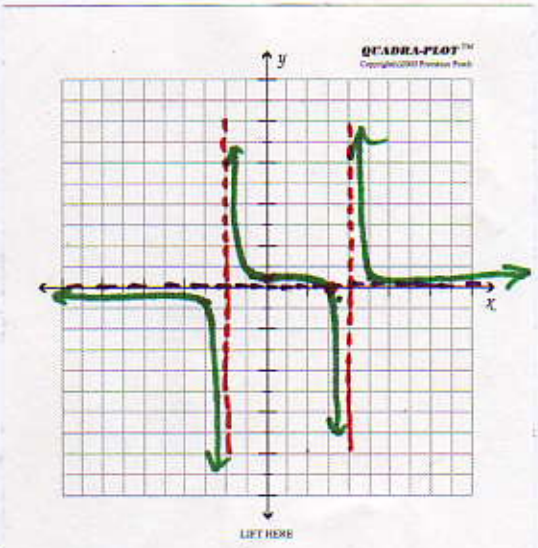
$$r(x) = \frac{x-3}{x^2-2x-8}$$

x-INTERCEPT (3, 0)

y-INTECEPT $(0, \frac{3}{8})$

VERTICAL ASYM. $x = 4, x = -2$

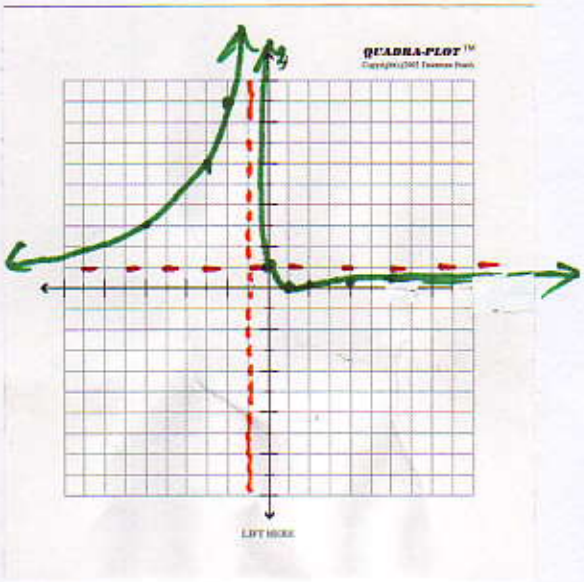
HORIZONTAL ASYM $y = 0$



#43 (BOOK)

$0 = (x-1)(x-1)$

$r(x) = \dfrac{1x^2 - 2x + 1}{1x^2 + 2x + 1 = 0}$



X-INTERCEPT (1, 0)

y-INTERCEPT (0, 1)

V.A. $x = -1$

H.A. $y = 1$

x	
2	$\frac{1}{9}$
-2	9
3	$\frac{4}{16} = \frac{1}{4}$
-3	4
6	$25/49$
-6	$49/25$

$\dfrac{36 - 12 + 1}{36 + 12 + 1}$ $\quad \dfrac{25}{49}$

6. GUIDELINES

1) FIND V.A. BY SETTING THE DENOMINATOR EQUAL TO ZERO AND SOLVING. (VERTICAL DOTTED LINES)

2) FIND HORIZONTAL ASYMPTOTE, BY LOOKING AT THE DEGREES OF NUMERATOR & DENOMINATOR (3 POSSIBILITIES)

3) FIND Y-INTERCEPT

4) FIND X-INTERCEPT BY SETTING NUMERATOR EQUAL TO ZERO.

5) TEST POINTS TO SEE HOW GRAPH APPROACHES THE ASYMPTOTES.

6) SKETCH THE GRAPH.