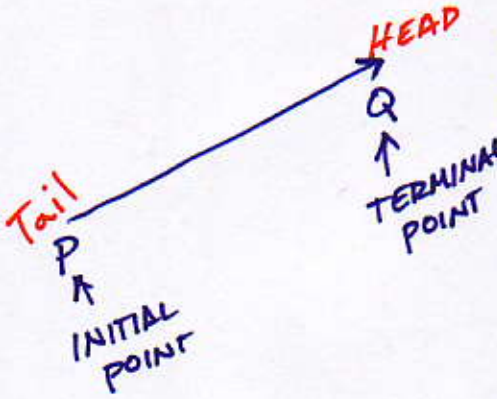
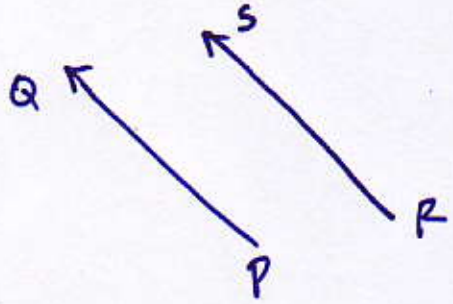
SEC 4.3 VECTORS

1. DIRECTED LINE SEGMENT: A LINE $\overrightarrow{PQ}$ (DLS) WITH MAGNITUDE AND DIRECTION, WHICH STARTS AT AN INITIAL POINT P (TAIL) AND STOPS AT A TERMINAL POINT Q (HEAD).



2. EQUIVALENT DLS's: TWO DLS's WHOSE MAGNITUDE (LENGTH) AND DIRECTION ARE THE SAME.



3. NOTATION: $\overrightarrow{PQ}$, $\vec{v}$, $\mathbf{AB}$

4. MAGNITUDE: LENGTH OF A VECTOR

$\|\overrightarrow{PQ}\|$ $\|\vec{v}\|$ $\|\mathbf{AB}\|$

$\|\vec{v}\| = \langle v_1, v_2 \rangle$

FIND: $\|\vec{v}\| = \sqrt{v_1^2 + v_2^2}$

5. COMPONENT FORM: $P = (p_1, p_2)$ $Q = (q_1, q_2)$

$\overrightarrow{PQ} = \langle q_1 - p_1, q_2 - p_2 \rangle$

H.W. #1

EX. $P_1 = (-3, 0)$ $P_2 = (4, -1)$

$$\overrightarrow{P_1P_2} = \langle 4-(-3), -1-0 \rangle$$

$$v = \langle 7, -1 \rangle$$

MAGNITUDE: $\sqrt{7^2 + (-1)^2}$

$\sqrt{49+1}$

$\sqrt{50} = \boxed{5\sqrt{2} = \|v\|}$

6. STANDARD POSITION: WHEN THE INITIAL POINT IS AT THE ORIGIN.

7. ZERO VECTOR: BOTH INITIAL & TERMINAL POINT IS THE ORIGIN.

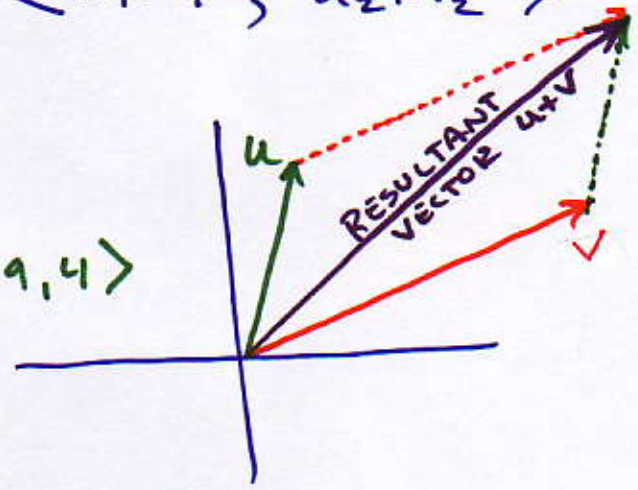
$$\mathbf{0} = \langle 0, 0 \rangle$$

8. VECTOR ADDITION / SUBTRACTION

LET $\mathbf{u} = \langle u_1, u_2 \rangle$ $\mathbf{v} = \langle v_1, v_2 \rangle$

$$\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_2 \rangle$$

GEOMETRICALLY:



$\mathbf{u} = \langle 1, 5 \rangle$ $\mathbf{v} = \langle 9, 4 \rangle$

$\mathbf{u}+\mathbf{v} = \langle 1+9, 5+4 \rangle$

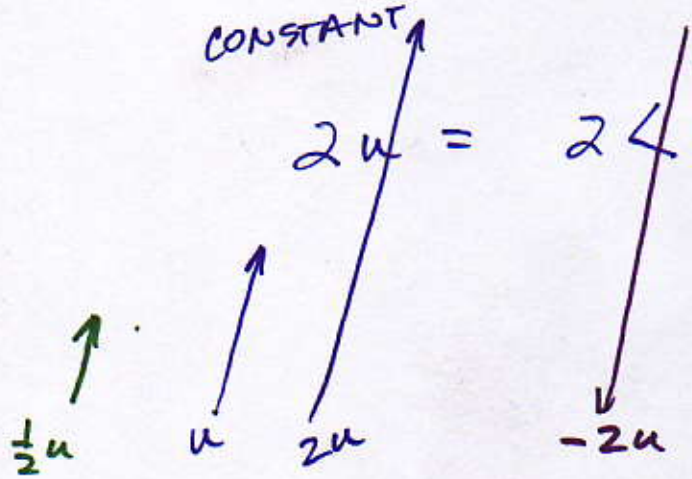
$\langle 10, 9 \rangle$

9. SCALAR MULTIPLICATION:

$$k\mathbf{u} = k\langle u_1, u_2 \rangle = \langle ku_1, ku_2 \rangle$$

(k: CONSTANT; $\mathbf{u}$: VECTOR)

$$2\mathbf{u} = 2\langle 1, 5 \rangle = \langle 2 \cdot 1, 2 \cdot 5 \rangle = \langle 2, 10 \rangle$$



10. VECTOR PROPERTIES

1. $u+v = v+u$ COMMUTATIVE
2. $(u+v)+w = u+(v+w)$ ASSOCIATIVE
3. $u+0 = u$ IDENTITY
4. $u+(-u) = 0$ INVERSE
5. $c(du) = (cd)u$
6. $(c+d)u = cu+du$ DISTRIBUTIVE
7. $c(u+v) = cu+cv$
8. $1 \cdot u = u$
9. $0 \cdot u = 0$
10. $\|cv\| = |c| \cdot \|v\|$

11. UNIT VECTOR : $\|v\| = 1$

FIND A UNIT VECTOR : $v \cdot \frac{1}{\|v\|}$

$v = \langle -2, 5 \rangle$ $\|v\| = \sqrt{(-2)^2 + 5^2}$

$= \sqrt{4+25}$

$\|v\| = \sqrt{29}$

UNIT VECTOR : $\left\langle \frac{-2}{\sqrt{29}}, \frac{5}{\sqrt{29}} \right\rangle$

DEFINITION: $i = \langle 1, 0 \rangle$ $j = \langle 0, 1 \rangle$

$v = \langle 3, 4 \rangle = 3i + 4j$

12. HORIZONTAL & VERTICAL COMPONENTS

$$V = \langle v_1, v_2 \rangle$$

$$\cos\theta = \frac{v_1}{\|v\|} \qquad \sin\theta = \frac{v_2}{\|v\|}$$

$$v_1 = \|v\| \cos\theta \qquad v_2 = \|v\| \sin\theta$$

13. DOT PRODUCT

$$u = \langle u_1, u_2 \rangle \qquad v = \langle v_1, v_2 \rangle$$

$$u \cdot v = u_1 v_1 + u_2 v_2$$

Ex. $v = \langle 6, -2 \rangle \qquad u = \langle -2, 4 \rangle$

$$v \cdot u = 6 \cdot (-2) + (-2) \cdot 4$$

$$-12 + -8$$

$$\boxed{v \cdot u = -20}$$

14. FIND THE ANGLE BETWEEN TWO VECTORS

$$\theta = \cos^{-1}\left(\frac{v \cdot w}{\|v\| \cdot \|w\|}\right)$$

15. PERPENDICULAR VECTORS: $V \cdot W = 0$

(SAID TO BE ORTHOGONAL)