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Precalculus

exponentials/logarithms

$a > 0,\ x, y \in \mathbb{R}$

$a^{x+y} = a^x a^y \quad : \quad 2^{2+3} = 2^2 2^3$

$32 \qquad 4 \cdot 8$

$\log_a(xy) = \log_a(x) + \log_a(y)$

$\log_2(4 \cdot 8) = \log_2(4) + \log_2(8)$

$5 = 2 + 3$

$\log_a(x \cdot x \cdot x \ldots x) = \log_a(x^n) = \log_a x + \log_a(x) + \ldots + \log_a(x)$

$= \underline{n} \log_a(x)$

$a^{x+x+x\ldots+x} = a^{\underline{n} \cdot x} = a^x a^x a^x \ldots a^x$

$= (a^x)^n$

$\underline{a^{\log_a x} = x} \qquad \underline{\log_a a^x = x}$

Natural exponentials & logs : Base e

I $\ln(x)$ is the antiderivative of $\frac{1}{x}$, $\log_a x$ is antiderivative of $\frac{c}{x}$ that passes through $(1,0)$

$\ln x = \int_1^x \frac{1}{t}\,dt$

The missing "n" of power rule

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad n \neq -1$$

II The function $y(x)$ whose derivative is itself and passes through $(0,1)$

$a^1 = a$

$a^0 = \frac{a^1}{a} = 1$

$$y = e^x \qquad y' = y$$

$$y = a^x \qquad y' = c\,y$$

Speed $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \quad \frac{1}{1000000} \to 0$

1hr 2hr 3hr 4hr

Distance $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \cdots$

$\frac{1}{9} + \frac{1}{10} + \cdots \frac{1}{16} \quad \frac{1}{32}$

Speed $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \to 0$

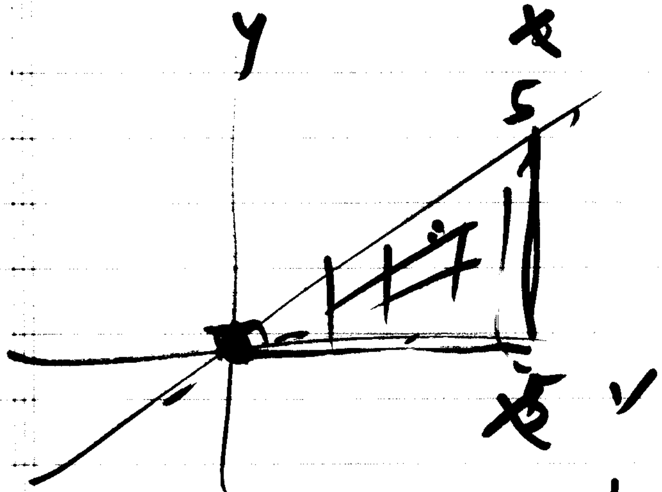
Total Distance $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = 2$

$x^2 - 1 = (x-1)(x+1)$

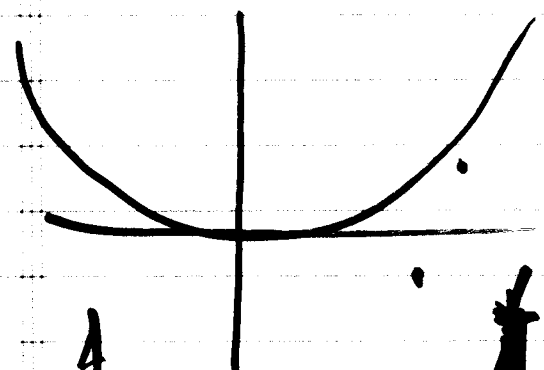
$x^3 - 1 = (x-1)(x^2 + x + 1)$

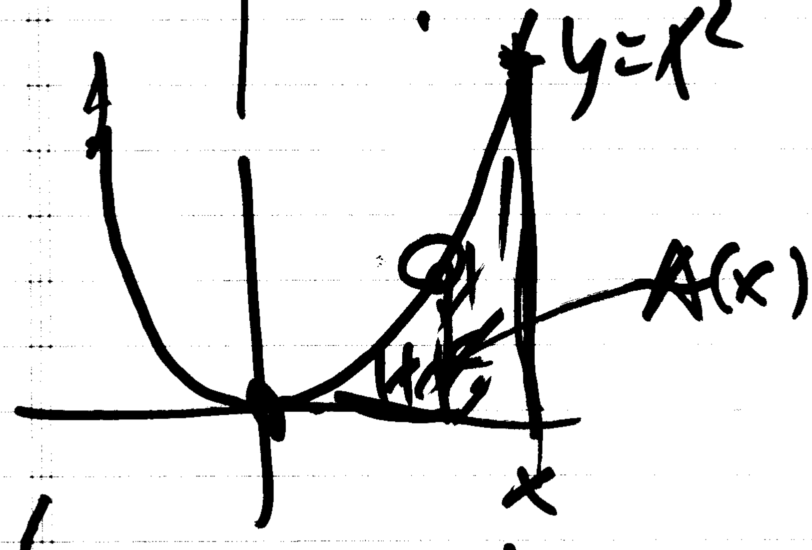
$x^4 - 1 = (x-1)(x^3 + x^2 + x + 1)$

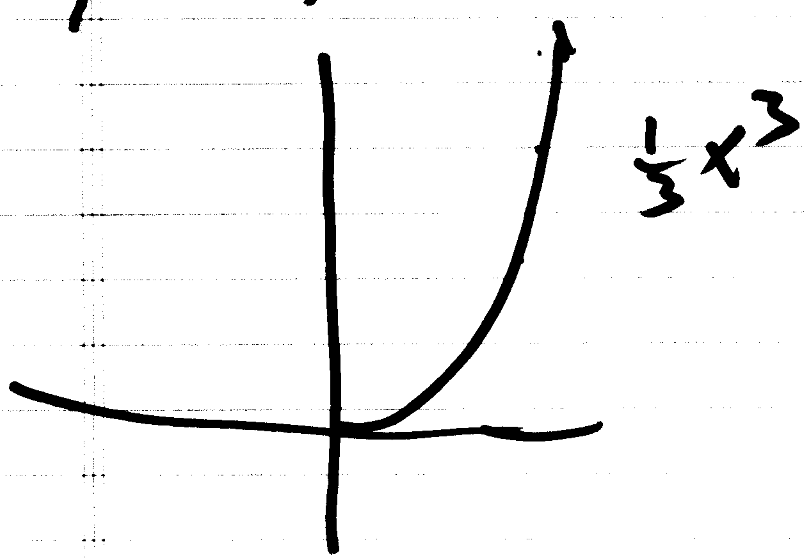
Fundamental Theorem: Area under graph of f from a to x is antiderivative of f(x)



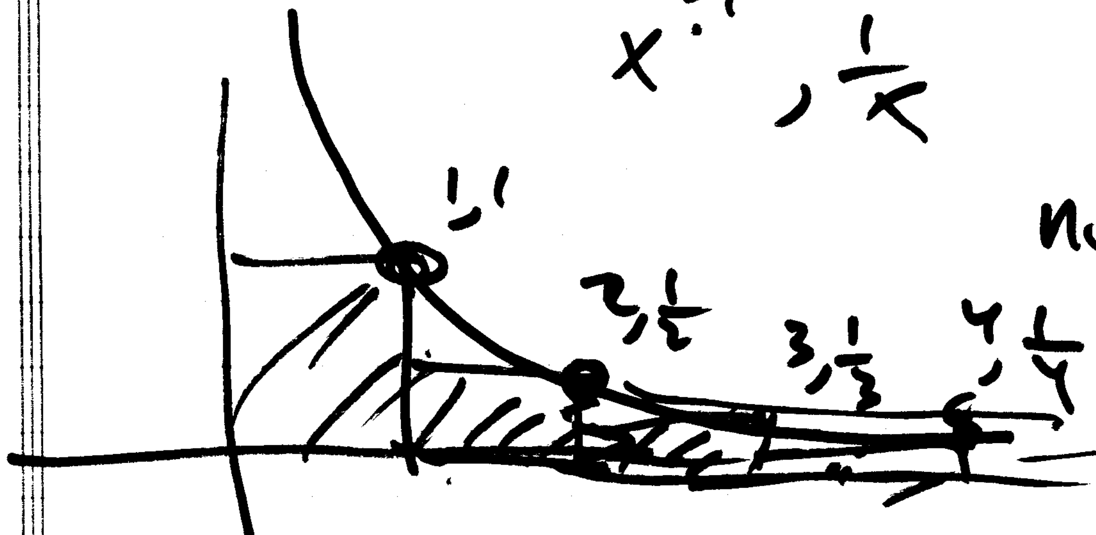
$$\frac{1}{2}x^2$$







x^{-1}, $\frac{1}{x}$



no $\frac{x^{n+1}}{n+1}$

$-1+1=0$

Anti derivative

III. Invest 1 at 100% compounding d divisions/year. How much after t years $t=1$, $d=1$

J $1, \quad 1+1\cdot\frac{100}{100}\cdot 1 = \underline{\underline{2}}$ D

$d=2$ $\quad 1, \quad \underline{1+1\cdot\frac{100}{100}\cdot\frac{1}{2}} = 1.5, \quad \underline{1.5+1.5\cdot\frac{100}{100}\cdot\frac{1}{2}} = 2.25$

J June

$d=365$ $\quad P_0\left(1+\frac{r}{100}\frac{1}{d}\right)^{dt} \quad e = \lim_{d\to\infty}\left(1+\frac{1}{d}\right)^d$