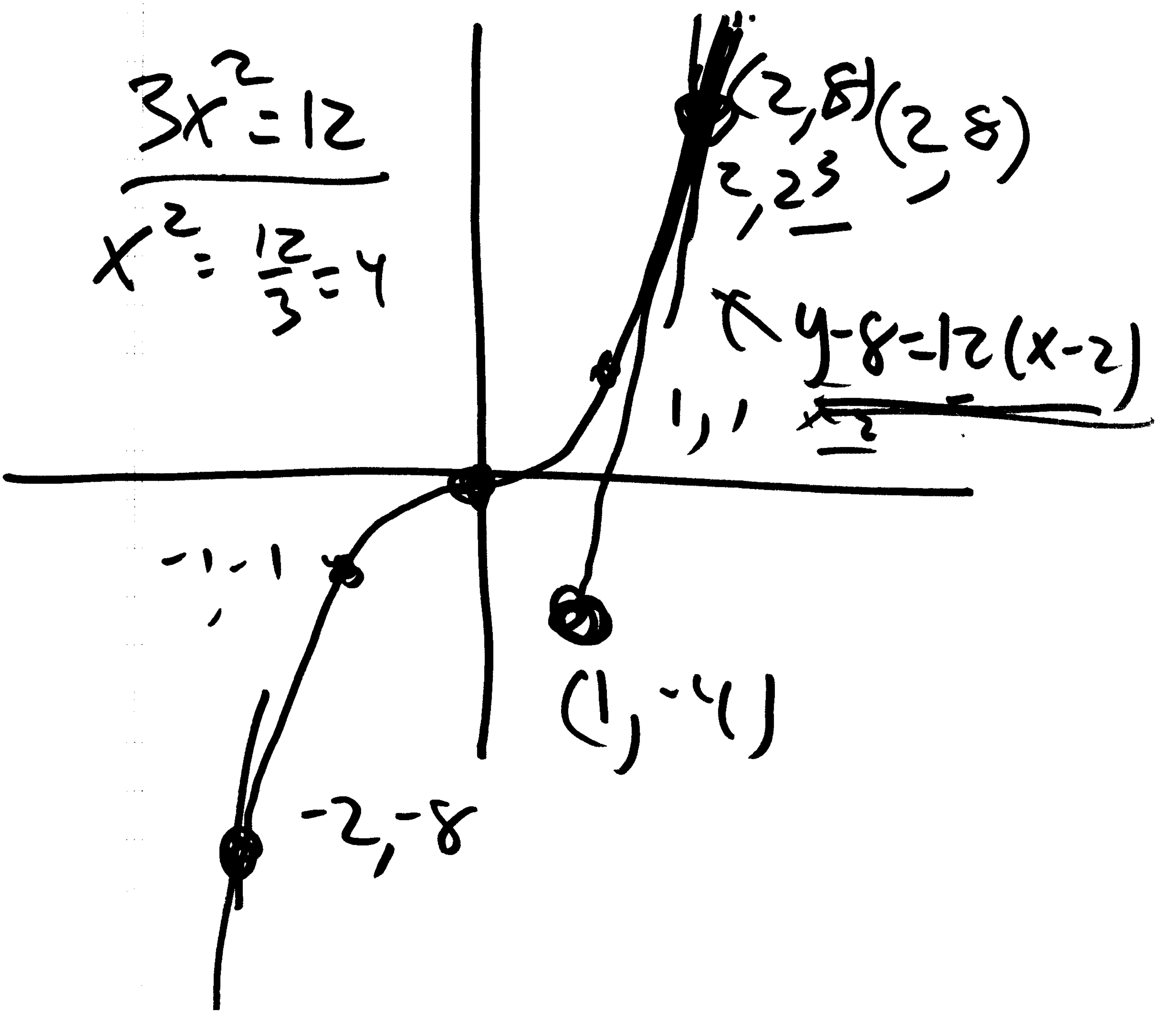
9/24

Where is the slope of

$y = x^3$ equal to 12?

$$3x^2 = 12$$

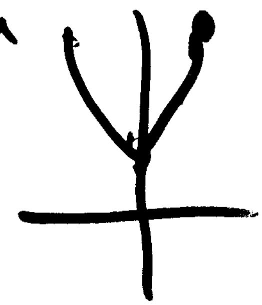
$$x^2 = \frac{12}{3} = 4$$

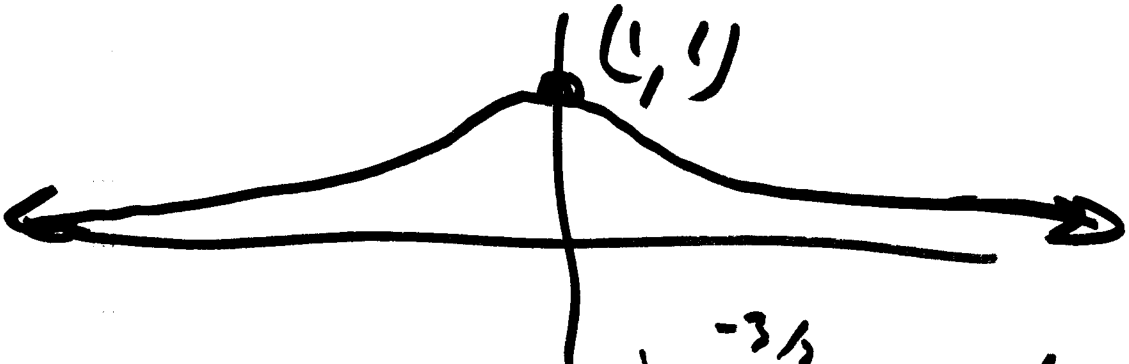


Chain rule example

$$f(x) = \frac{1}{\sqrt{x^2+1}} = \underbrace{(x^2+1)}_{u}{}^{-1/2}$$

$x^2+1 \geq 1$





$-\frac{1}{2}u^{-3/2} \cdot \frac{du}{dx}$

$u^{-1/2} = -\frac{1}{2}u^{-3/2} \cdot \frac{du}{dx}$

$$f'(x) = -\frac{1}{2}(x^2+1)^{-3/2} \cdot (2x)$$

$$= -\frac{x}{(x^2+1)^{3/2}} =$$

$$= -\frac{x}{\left(\sqrt{x^2+1}\right)^3}$$

Chain rule - powerrule motivation

$$f(x) = (x^2+1)^3$$

$$u^3 \downarrow 3u^2 \cdot u'$$

$$f'(x) = 3(x^2+1)^2 \cdot 2x$$

$$f(x) = \overset{u}{(x^2+1)}\overset{u}{(x^2+1)}\overset{u}{(x^2+1)}$$

$$f'(x) = 2x(x^2+1)(x^2+1) + (x^2+1)(2x)(x^2+1) + (x^2+1)(x^2+1)(2x)$$

$$\underbrace{3(x^2+1)^2}_{3u^2} \circ \underbrace{2x}_{u'}$$

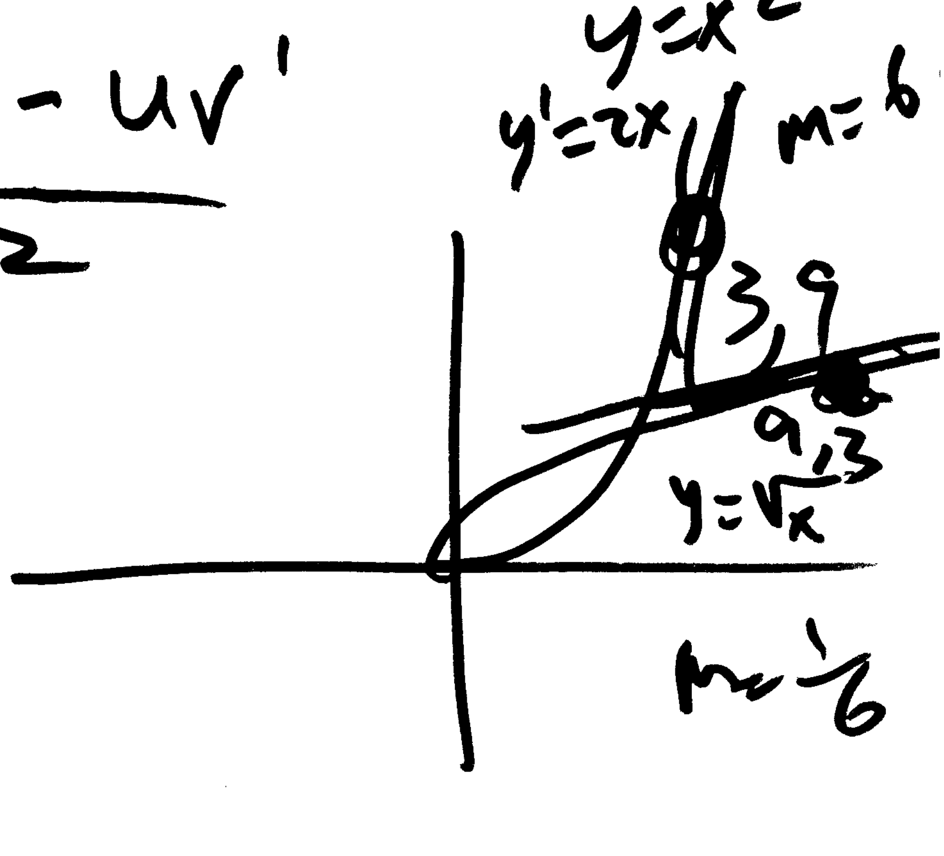
Product + -1 power + Chain = Quotient

$$f(x) = \frac{u(x)}{v(x)} = u(x) \cdot \left(v(x)\right)^{-1}$$

$$f'(x) = u'(x) \cdot v(x)^{-1} + u(x) \cdot -1\, v(x)^{-2} \cdot v'(x)$$

$$= \frac{u'}{v} \cdot \frac{v}{v} - \frac{uv'}{v^2}$$

$$= \frac{vu' - uv'}{v^2}$$



-1 power from definition of derivative in webwork

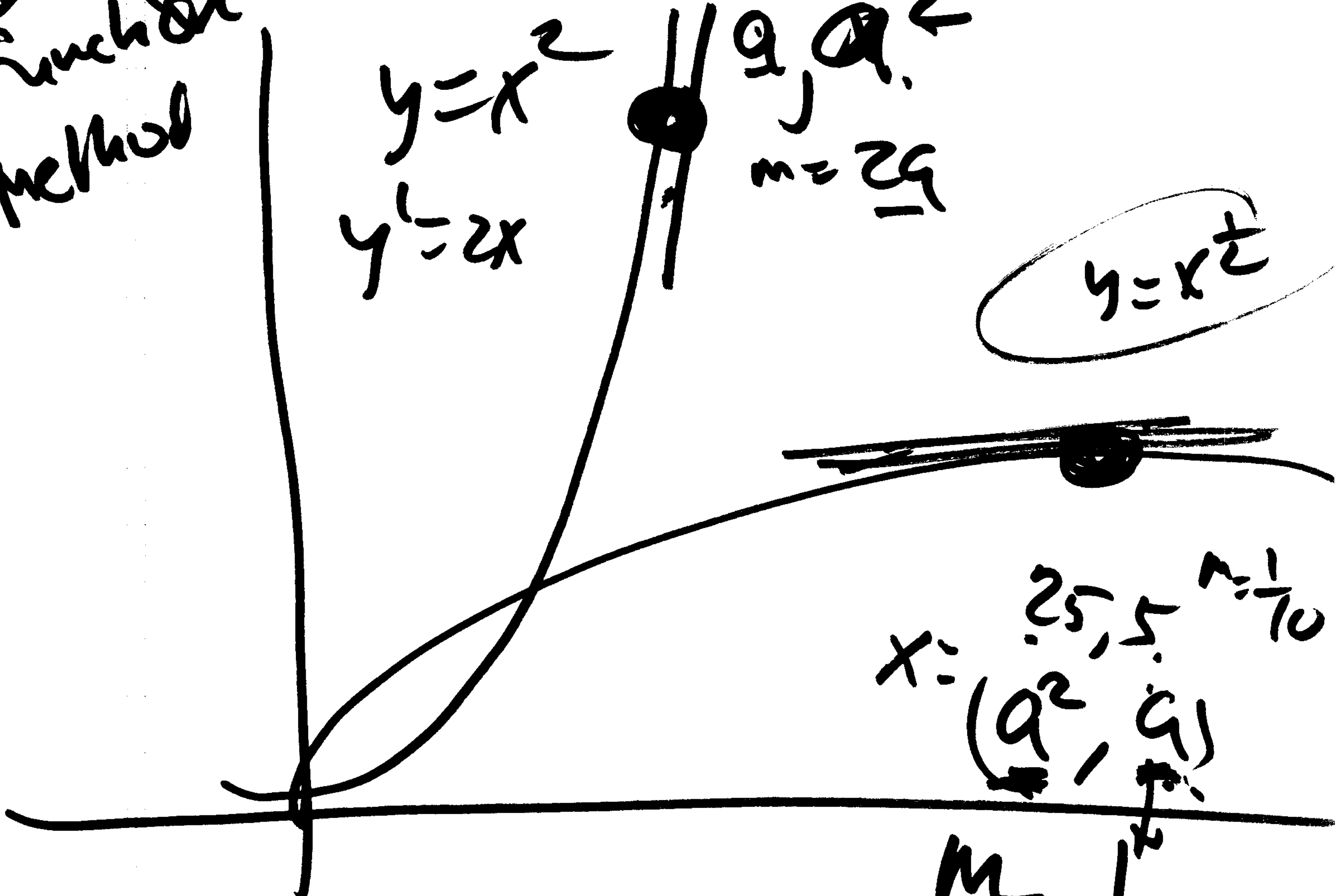
$$b \to 8 \quad \frac{\frac{1}{b} - \frac{1}{8}}{b-8} = \frac{\frac{8-b}{8b}}{b-8}$$

$$= \frac{-1}{8b} \qquad -\frac{1}{8 \cdot 8}$$

$b \to 8$ $\qquad -1x^{-2}$

$f(x) = x^{-1}$ at $x=8$

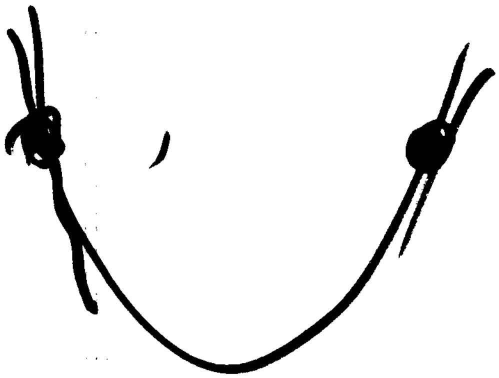
Inverse function method



$m = \frac{1}{2 \cdot 9}$

$\frac{1}{2\sqrt{x}}$

$\frac{1}{2}x^{-\frac{1}{2}}$



$f'(x)$

$y = 9x^2 + 2x + 4$ $\perp$ to

$x + 3y = 1$ slope

$y = \frac{-x + 1}{3}$

$m_{\perp} = 3$ $m = -1/3$

negative reciprocal

$y' = 3$

$18x + 2 = 3$

$x = 1/18$

Initial velocity

$V(0) = 41$

constant deceleration

$V'(t) = a(t) = -2$

$V(t) = 41 - 2t$

$d'(t) =$

4

$d(t) = 41t - 2\left(\frac{t^2}{2}\right) + C_0$

$d(0) = 0$

$t^{99} \to \frac{t^{100}}{100} \to \frac{100}{100} t^{99}$