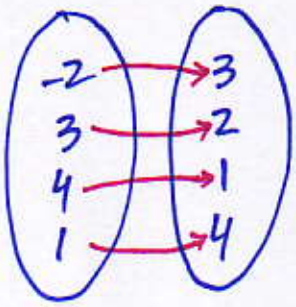
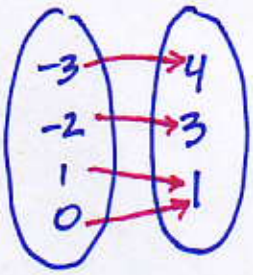
SEC 3.7 ONE-TO-ONE FUNCTIONS AND THEIR INVERSES.

1. ONE-TO-ONE FUNCTION: FOR EVERY x THERE IS ONLY ONE y AND FOR EVERY y THERE IS ONLY ONE x.

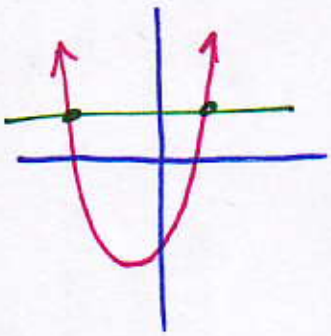


1-1.

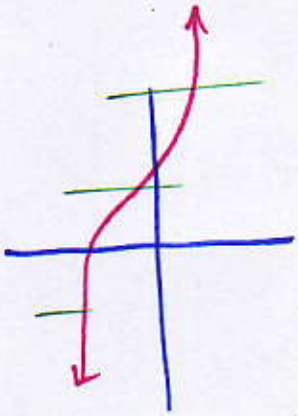


NOT 1-1

2. HORIZONTAL LINE TEST: A FUNCTION IS ONE-TO-ONE IF AND ONLY IF NO HORIZONTAL LINE INTERSECTS THE GRAPH MORE THAN ONCE.



NOT ONE-TO-ONE



ONE-TO-ONE

GRAPHS MUST PASS BOTH THE VERTICAL & HORIZONTAL LINE TESTS TO BE 1-1.

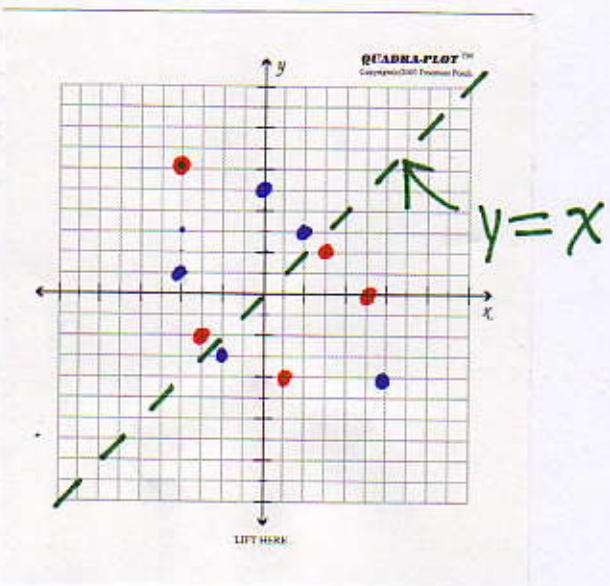
? WHY DO WE NEED TO KNOW IF SOMETHING IS 1-1 ?

ANSWER: THEN IT HAS AN INVERSE.

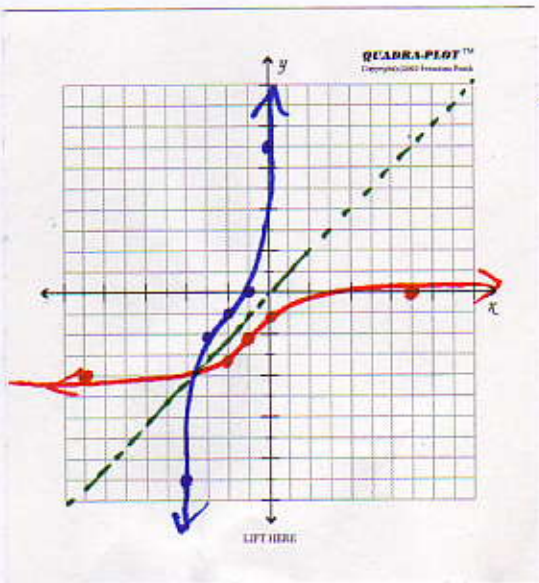
3. INVERSE : SWITCH THE x & y COORDINATES

EX. $\{(2,3)\ (-4,1)\ (0,5)\ (-2,-3)\ (6,-4)\}$

INVERSE $f^{-1}(x) = \{(3,2)\ (1,-4)\ (5,0)\ (-3,-2)\ (-4,6)\}$



$y = x$



$f(x) = (x+2)^3 - 1$

4. HOW TO FIND THE INVERSE OF A FUNCTION.

STEPS

1. CHANGE $f(x)$ TO y.
2. SWITCH THE x & y COORDINATES
3. SOLVE FOR y
4. CHANGE y TO $f^{-1}(x)$.

EXAMPLE

$\cancel{f(x)} = (x+2)^3 - 1$

$y = (x+2)^3 - 1$

SWITCH VARIABLES

$x = (y+2)^3 - 1$

$+1 \qquad +1$

$\sqrt[3]{x+1} = \sqrt[3]{(y+2)^3}$

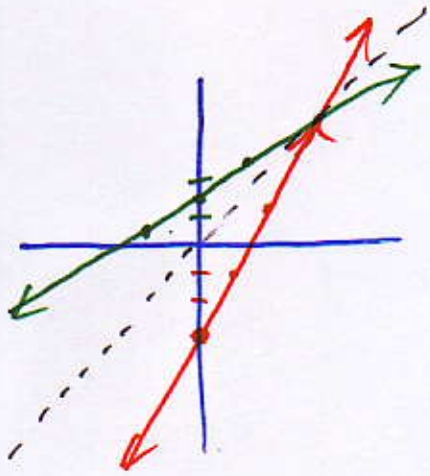
$\sqrt[3]{x+1} = y + 2$

$-2 \qquad -2$

$\sqrt[3]{x+1} - 2 = y$

$f^{-1}(x) = \sqrt[3]{x+1} - 2$

EX. $f(x) = 2x - 3$ *



$y = 2x - 3$

$+3 \quad x = 2y - 3 \quad +3$

$\frac{x+3}{2} = \frac{2y}{2}$

$f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$ *

TEST EX. $f(x) = \frac{x+3}{x-4}$

$y = \frac{x+3}{x-4}$

$(y-4)x = \frac{(y+3)(y-4)}{y-4}$

$-3 \quad xy - 4x = y + 3 \quad -3$

$xy - 3 - 4x = y - xy$

$-xy \quad -xy$

$-3 - 4x = y - xy$

$\frac{-3-4x}{1-x} = \frac{y(1-x)}{1-x}$

$f^{-1}(x) = \frac{-3-4x}{1-x}$

5. VERIFYING INVERSES: BY COMPOSITION OF FUNCTIONS.

$f(x) = 2x - 5$ $\quad$ $g(x) = \frac{x+5}{2}$

$(f \circ g)(x) = x$ $\quad$ $(g \circ f)(x) = x$

$2\left(\frac{x+5}{2}\right) - 5$

$x + 5 - 5$

x

$\frac{(2x-5)+5}{2}$

$\frac{2x}{2}$

x