# SEC. 4.1 POLYNOMIAL FUNCTIONS & THEIR GRAPHS

## 1. POLYNOMIAL FUNCTION:

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_1 x^1 + a_0$$

- $n$: DEGREE
- $a_n$: LEADING COEFFICIENT
- $a_n x^n$: LEADING TERM
- $a_0$: CONSTANT

EX.

$P(x) = -2x^3 + 4x^2 - 5x + 3$

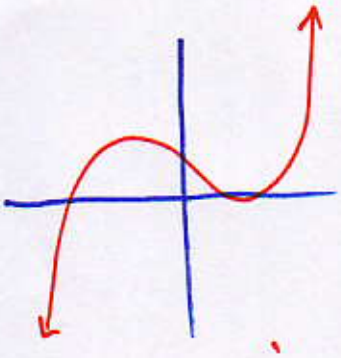
$Q(x) = x^2 + 2x - 1$

$R(x) = 4x - 7$

$S(x) = -5$

| | DEGREE | L.C. |
|---|---|---|
| $P(x)$ | 3 | -2 |
| $Q(x)$ | 2 | 1 |
| $R(x)$ | 1 | 4 |
| $S(x)$ | 0 | -5 |

## 2. GRAPH OF A POLYNOMIAL: SMOOTH AND CONTINUOUS

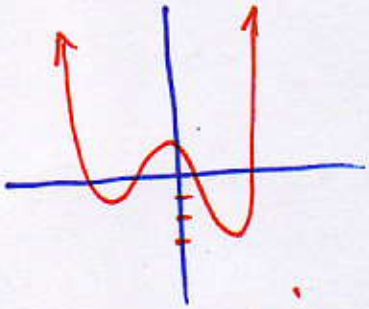


SMOOTH & CONTINUOUS

POLYNOMIAL FUNCTION

DOMAIN: $(-\infty, \infty)$

RANGE: $(-\infty, \infty)$

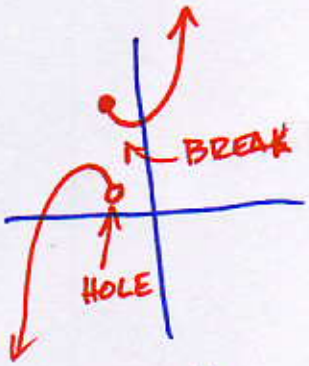


SMOOTH & CONTINUOUS

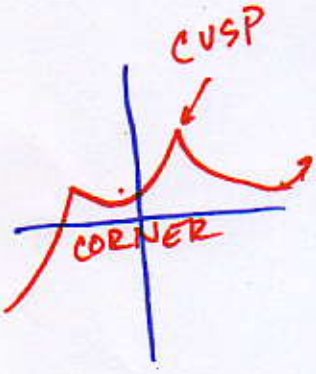
POLYNOMIAL FUNCTION

DOMAIN: $(-\infty, \infty)$

RANGE: $[-3, \infty)$




NOT A POLYNOMIAL FUNCTION



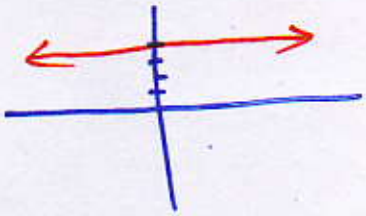

NOT POLYNOMIAL FUNCTION

# 3. GRAPHS & THEIR DEGREES

A) CONSTANT GRAPH

$P(x) = b$

$P(x) = 4$

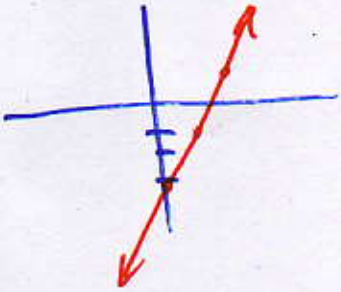


DEGREE: 0

B) LINEAR GRAPH

$P(x) = mx+b$

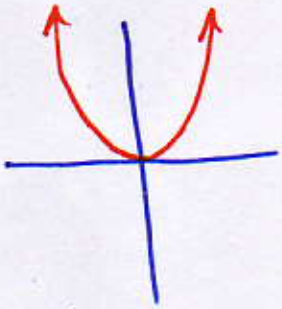
$P(x) = 2x-3$



DEGREE: 1

c) QUADRATIC GRAPH

$P(x) = x^2$

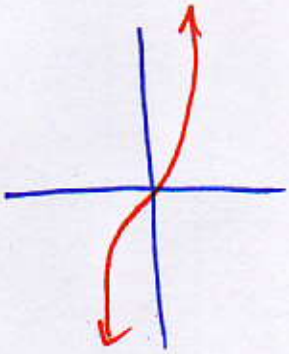


DEGREE: 2

EVEN POWER PARABOLA-LIKE

D) CUBIC GRAPH

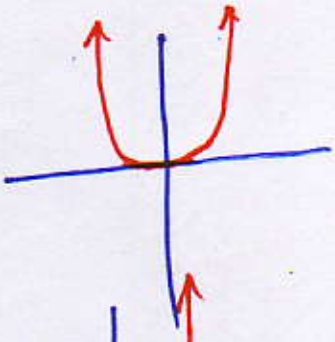
$P(x) = x^3$



DEGREE: 3

E) QUARTIC GRAPH

$P(x) = x^4$

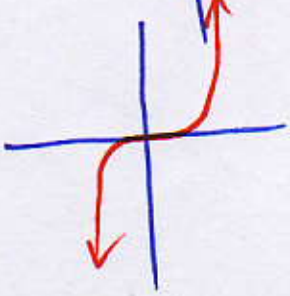


DEGREE: 4

ODD POWERS DISCO-LIKE

F) DEGREE OF 5

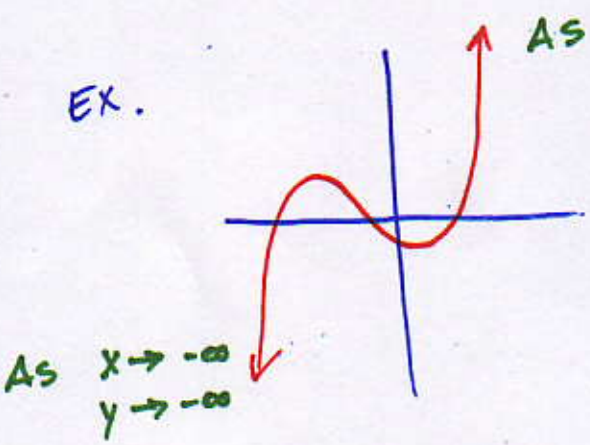
$P(x) = x^5$



DEGREE: 5

# 4. END BEHAVIOR (DEPENDS ON LEADING TERM)

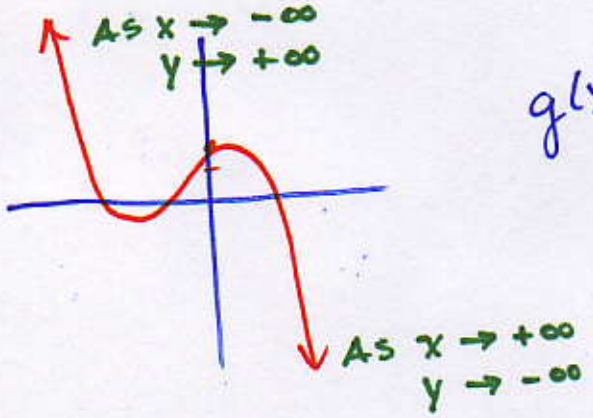
EX.




$f(x) = 3x^3 + 2x - 1$ (ODD)

POSITIVE

EX

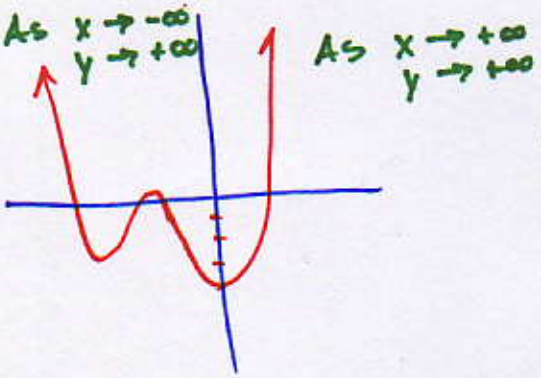



$g(x) = -2x^3 + 5x^2 + x + 2$ (ODD)

NEGATIVE

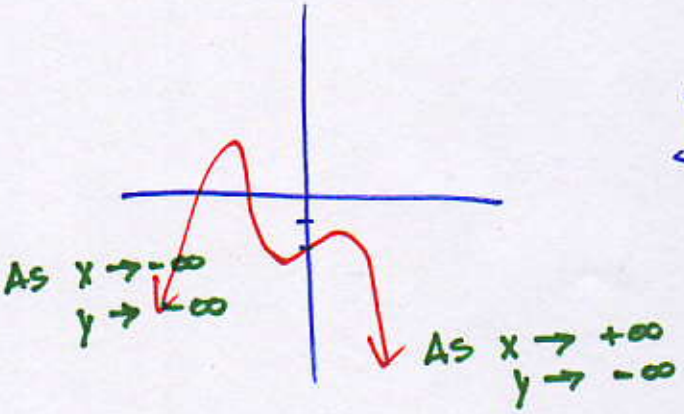
(FLIPS OF x-AXIS)

EX.




$h(x) = 2x^4 - 5x^3 + 2x - 4$ (EVEN)

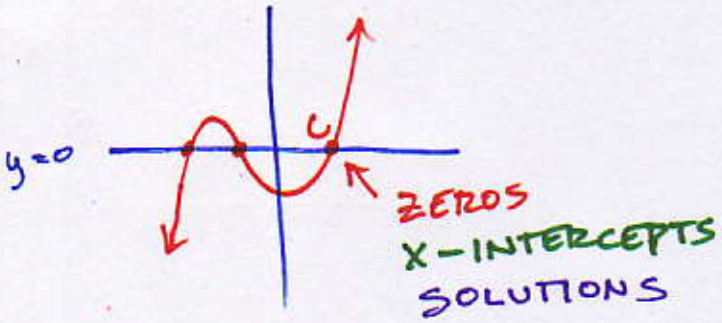
POSITIVE




$j(x) = -\frac{1}{2}x^4 - 2x^3 + x^2 - x - 2$ (EVEN)

NEGATIVE

# 5. REAL ZEROS OF A POLYNOMIAL




IF P IS A POLYNOMIAL AND c IS A REAL NUMBER, THEN THE FOLLOWING ARE EQUIVALENT.

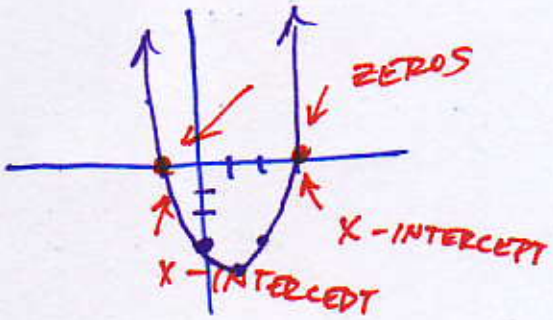
1) c IS A ZERO OF P
2) $x=c$ IS A SOLUTION WHEN $P(x)=0$
3) $x-c$ IS A FACTOR OF $P(x)$.
4) $x=c$ IS A X-INTERCEPT OF THE GRAPH P

EX. $P(x) = x^2 - 2x - 3$

$0 = x^2 - 2x - 3$

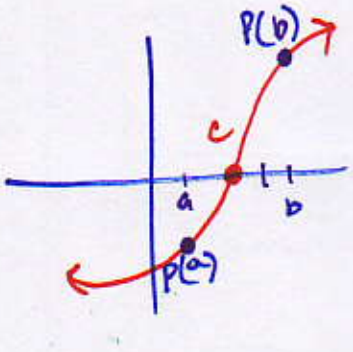
$(x-3)(x+1)$ FACTORS ($x-c$)

$\{3, -1\}$ SOLUTIONS




# 6. INTERMEDIATE VALUE THEOREM

| x | y |
|---|---|
| a | − |
| b | + |




IF P IS A POLYNOMIAL FUNCTION AND P(a) & P(b) ARE OPPOSITE SIGN, THEN AT LEAST ONE ZERO c IS BETWEEN a & b $P(c)=0$

# 7. GUIDELINES FOR GRAPHING POLYNOMIAL FUNCTIONS

1) FIND THE ZEROS. FACTOR THE POLYNOMIAL
2) FIND y-INTERCEPT (IT'S THE CONSTANT)
3) MAKE A TABLE OF VALUES (TEST SOME OTHER POINTS)
4) LOOK AT THE END BEHAVIOR
5) GRAPH IT.

EX. $P(x) = x^3 - 2x^2 - 3x$ ← DEGREE 3 (ODD)

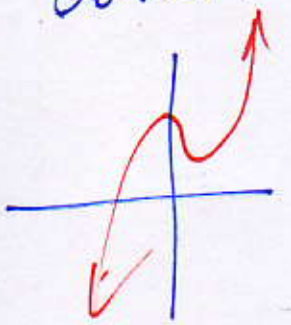
L.C. POSITIVE

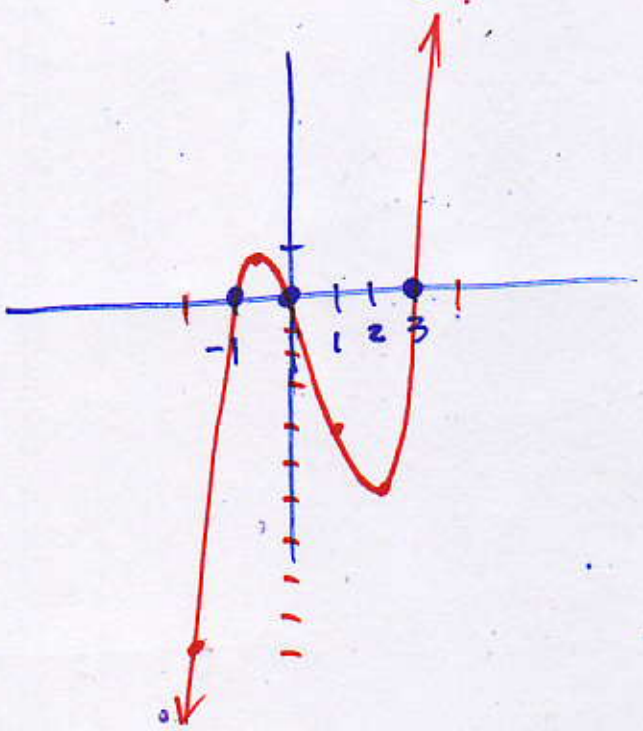
$x(x^2 - 2x - 3)$

$x(x-3)(x+1)$

$\{0, 3, -1\}$

END BEHAVIOR






| X | Y |
|---|---|
| 1 | -4 |
| 2 | -6 |
| 4 | 20 |
| -1/2 | 7/8 |
| -2 | -10 |

$1^3 - 2\cdot 1^2 - 3\cdot 1$
$1 - 2 - 3$

$2^3 - 2\cdot 2^2 - 3\cdot 2$
$8 - 8 - 6$

$4\cdot 1\cdot 5$

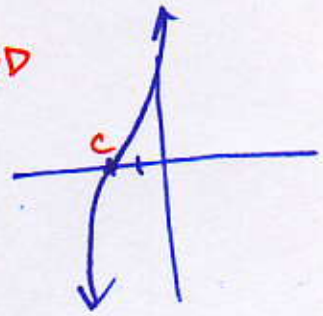
$-2\cdot -5 - 1$

$\left(-\frac{1}{2}\right)^3 - 2\left(-\frac{1}{2}\right)^2 - 3\left(-\frac{1}{2}\right)$
$-\frac{1}{8} - \frac{1}{2} + \frac{3}{2}$
$1 - \frac{1}{8}$

## 8. MULTIPLICITY OF A GRAPH

$f(x) = (x+2)^3$ ← ODD

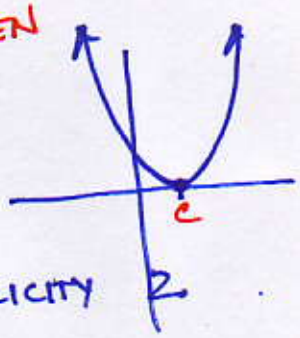
$\{-2, -2, -2\}$




GRAPH GOES THROUGH THE ZERO

A ZERO OF -2 MULTIPLICITY 3

$f(x) = (x-1)^2$ ← EVEN



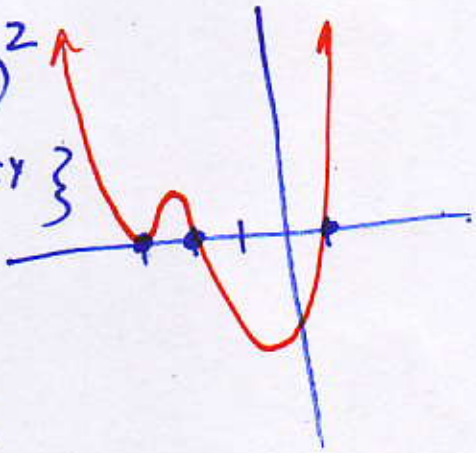

GRAPH TOUCHES

A ZERO OF 1 MULTIPLICITY 2

EX.

$P(x) = (x+2)(x-1)(x+3)^2$

ZERO: $\{-2, 1, -3$ MULTIPLICITY OF 2 $\}$



## 9. LOCAL MAX AND MIN.

IF $P(x)$ IS A POLYNOMIAL WITH DEGREE $n$

THEN THERE ARE AT MOST $n-1$ MAX OR MINS.