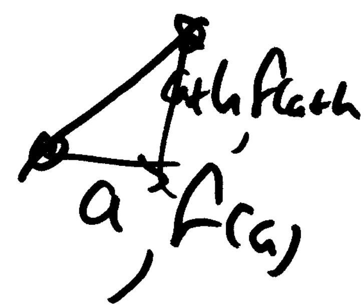
10/1 Derivatives what they mean
How to find them efficiently

$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$



Slope, Instantaneous Change

> 0 Increase < 0 Decrease

$f'' > 0$ Concave up $f'' < 0$ Concave Down

slope increasing Slope decreasing

Find $f'(x)$ if $f(x)$ is a polynomial

$x^2 \to 2x \quad x^3 \to 3x^2 \quad 5x^3 - 2x^2 \to$

$\frac{x^5}{5!} \to \frac{x^4}{4!} = \frac{1}{5!}(5x^4) \quad 15x^2 - 4x$

$\frac{p(x)}{q(x)}$: Rational $\to \frac{qp' - pq'}{q^2}$ $\quad x^{1/2}$ $\quad x^{-1/3}$

$$\left(\overset{x}{a},\ \overset{y}{f(a)} \right)$$

$y = f(x)$

$x = a$

$y = 3$

Slope $\underset{m}{f'(a)}$

Tangent Line

$f'(x) = 2x$

$y = x^2$

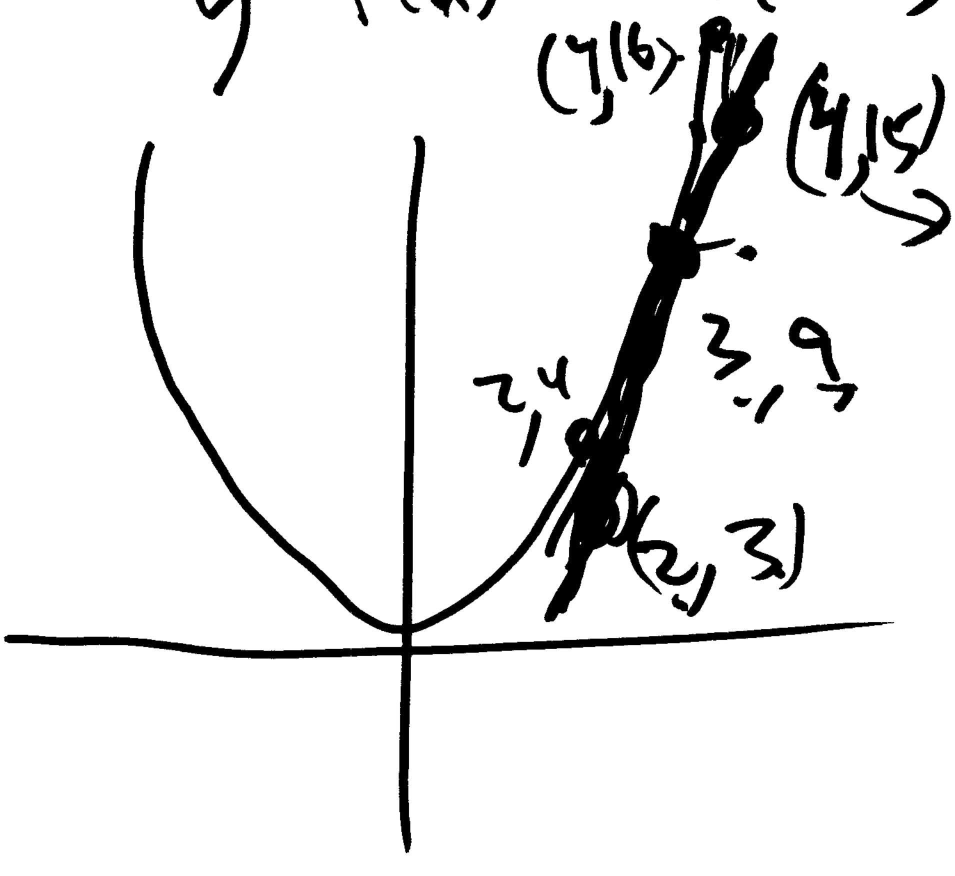
$(3, 9)$

6

$$y - f(a) = f'(a)(x - a)$$

$$y - y_1 = m(x - x_1)$$

$$y - 9 = (6)(x - 3)$$



$$f(x) = \frac{(1)}{\sqrt[3]{\dfrac{x^2+4}{x-2}}} \quad \frac{1}{(\quad)^{1/3}} = (\quad)^{-1/3}$$

$$= \left(\frac{x^2+4}{x-2}\right)^{-1/3}$$

$$u^{-1/3}$$

$$-\frac{1}{3}u^{-4/3} \cdot u'$$

$$f'(x) = -\frac{1}{3}\left(\frac{x^2+4}{x-2}\right)^{-4/3} \cdot \left[\frac{(x-2)(2x) - (x^2+4)1}{(x-2)^2}\right]$$

$$f'(x) = \frac{(\quad)(0) - (1)\frac{1}{3}(\quad)^{-2/3}}{(\quad)}$$

$$f(x) = \frac{1}{\sqrt{\dfrac{x^2+4}{x-2}}} = \sqrt[3]{\frac{x-2}{x^2+4}}$$

$$\frac{1}{\sqrt{a}} = \frac{\sqrt{1}}{\sqrt{a}} = \sqrt{1/a}$$

$$= \frac{(x-2)^{1/3}}{(x^2+4)^{1/3}}$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$f(x) = \frac{x+2}{x+7} \qquad f'(x) = \frac{(x+7)(1) - (x+2)1}{(x+7)^2}$$

$$= \frac{5}{(x+7)^2} = 5(x+7)^{-2}$$

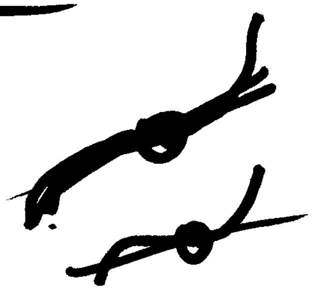
$$f''(x) = 5 \cdot -2(x+7)^{-3}(x+7)'$$

$$= -10(x+7)^{-3}$$

$$g(x) = f(x+a) \qquad g'(x) = f'(x+a)$$

$f(x) = (x+5)(x^2-4)$

$= x^3 + 5x^2 + \ldots$



$f''(x) = 6x + 10 \quad = 0$

$3!$

$x = -10/6$

$f'(x) = 1(x^2-4) + (x+5)(2x)$

$3x^2 + 10x - 4$

$x^3 + x$

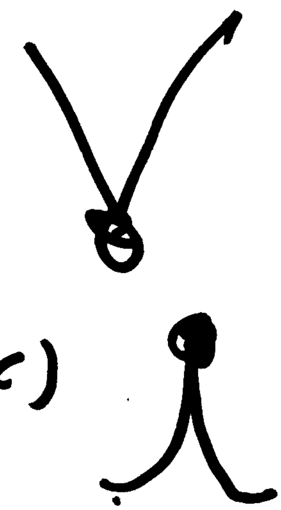
$x^3 - x$

x^3

$$f(x) = (x^3 - 3x)^2$$

$$f'(x) = 2(x^3 - 3x)^1 \cdot (3x^2 - 3)$$

$$f(x) = y(x)^3$$



$$f'(x) = 3\,y(x)^2 \cdot y'(x)$$

$$(x^2 + y_{(x)}^2 = 1)'$$

$$y^2 = 1 - x^2$$

$$y_+ = +\sqrt{1 - x^2}$$

$$y_- = -\sqrt{1 - x^2}$$

$$x(y) = \pm\sqrt{1 - y^2}$$

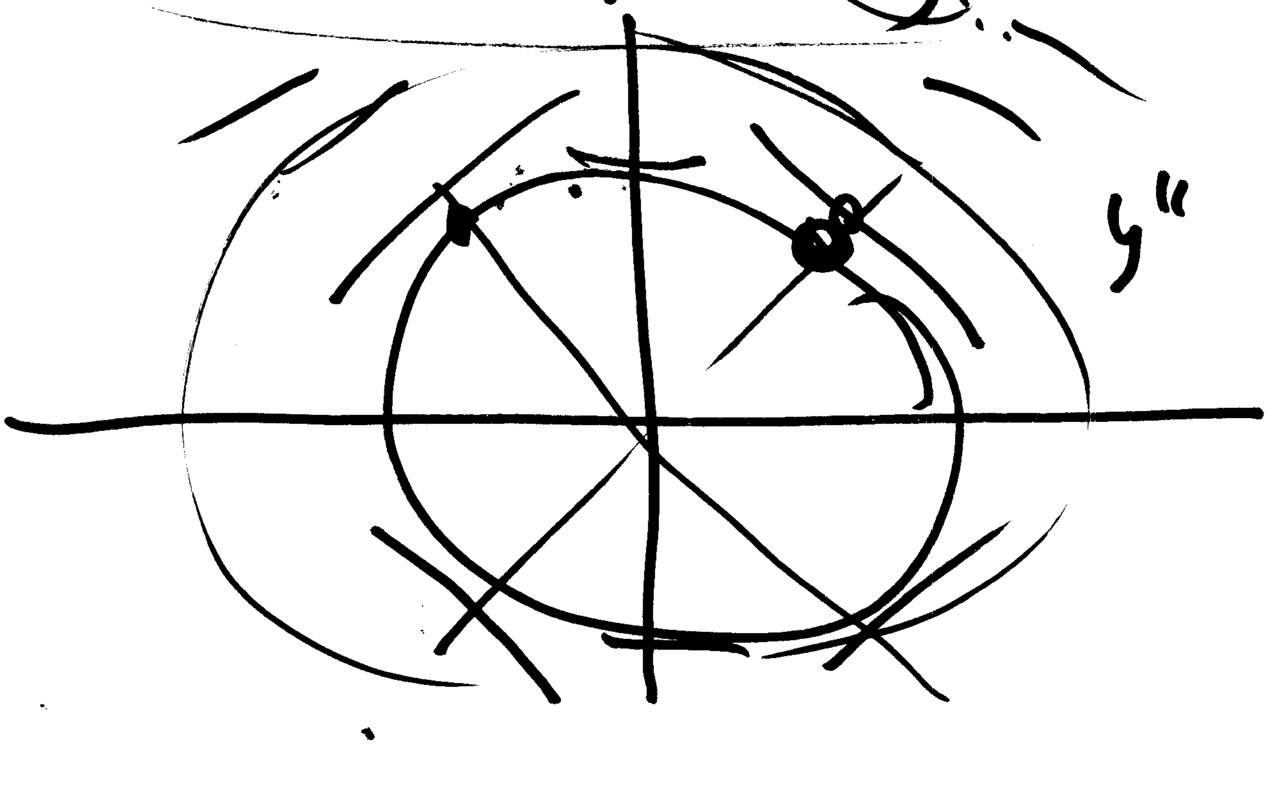
$$2x + 2y(x)^1 \cdot y'(x) = 0 : y' = \frac{-2x}{2y} = -\frac{x}{y}$$

$$x^2 + y^2 = 1 =$$

Explicit $x^2 + y(\underline{x})^2 = 1$

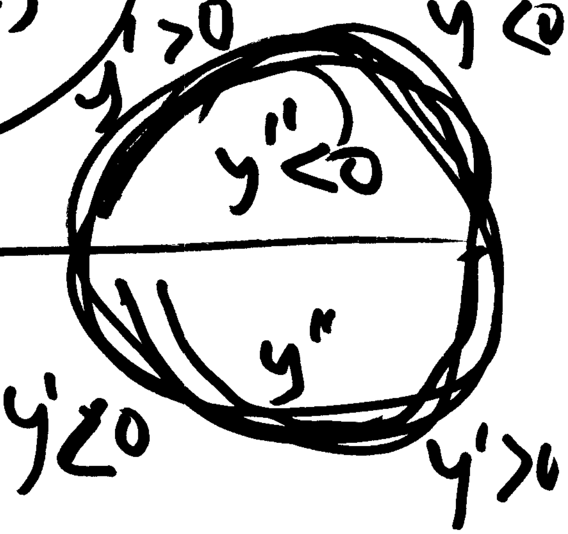
$\frac{d}{dx}$ $2x + 2y(x)y'(x) = 0$

$$y' = -\frac{2x}{2y} = -\frac{x}{y}$$



$$y' = -\frac{x}{y_{(x)}}$$

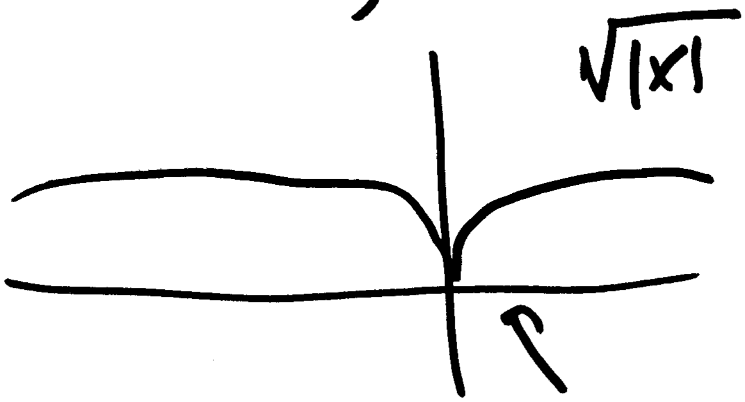
$$y'' = -\left(\frac{y_{(x)}\cdot 1 - x\cdot y'_{(x)}}{y^2}\right)$$

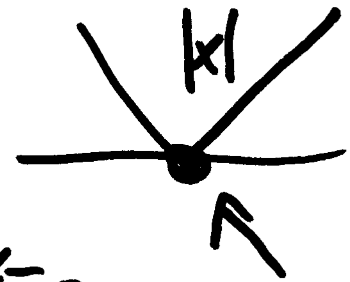


$$= -\left(\frac{y - x\left(\frac{-x}{y}\right)}{y^2}\right)$$

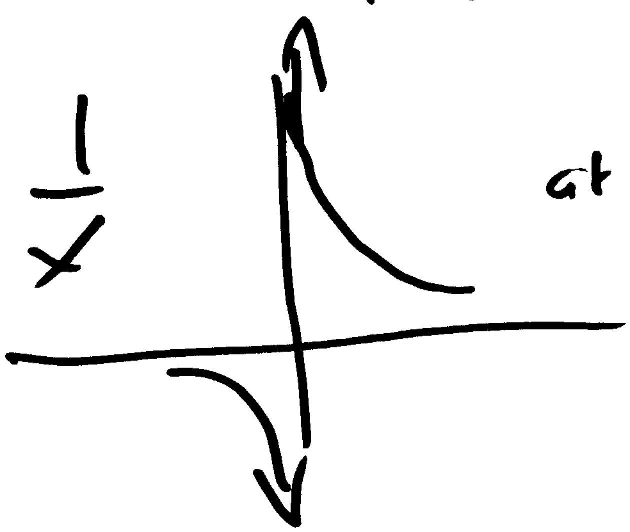
$$= -\left(\frac{y^2 + x^2}{y^3}\right) = -\frac{1}{y^3}$$

Not all functions have derivatives everywhere

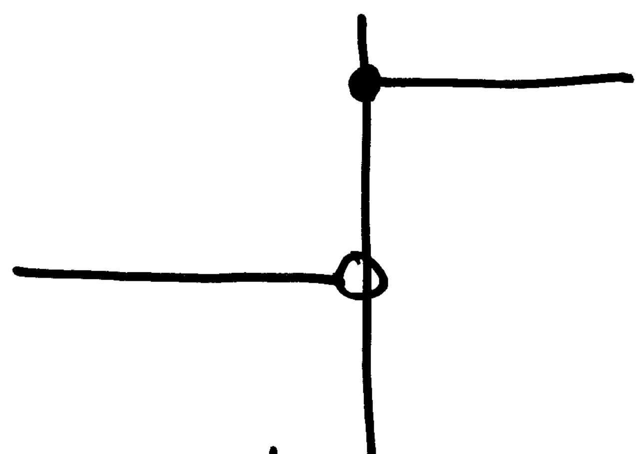




at $x=0$.



at $x=0$



Some don't have a derivative anywhere but are still continuous!