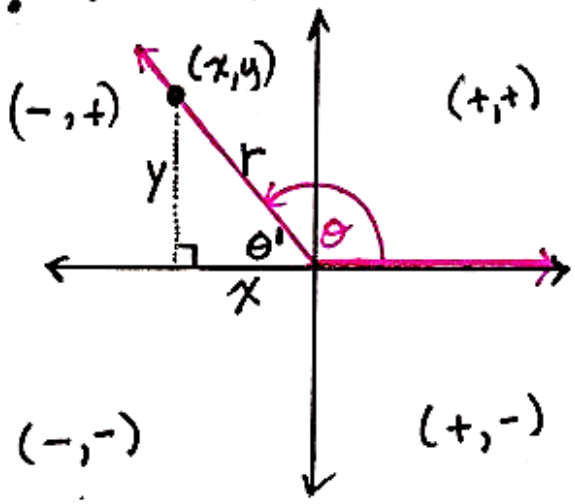
SEC 2.3 TRIGONOMETRIC FUNCTIONS OF ANY ANGLE

1. 6 TRIG FUNCTIONS OF ANY ANGLE:



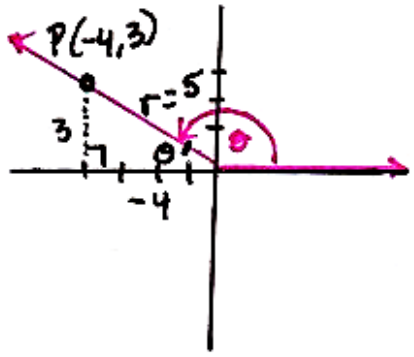
$\sqrt{x^2+y^2} = r$

$\sin\theta' = \frac{y}{r}$ $\quad \csc\theta' = \frac{r}{y}$

$\cos\theta' = \frac{x}{r}$ $\quad \sec\theta' = \frac{r}{x}$

$\tan\theta' = \frac{y}{x}$ $\quad \cot\theta' = \frac{x}{y}$

Ex. $P(\overset{x}{-4}, \overset{y}{3})$



$\sqrt{(-4)^2+3^2} = r$

$\sqrt{25} = r$

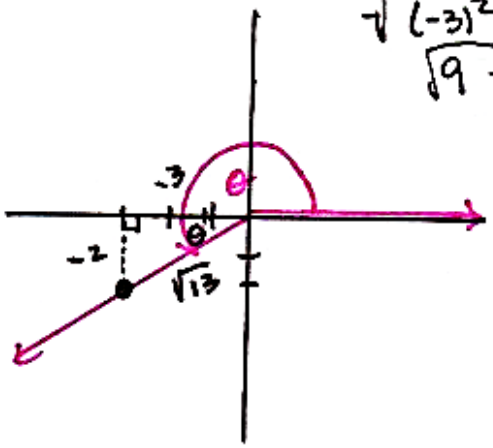
$5 = r$

$\sin\theta' = \frac{3}{5}$ $\quad \csc\theta' = \frac{5}{3}$

$\cos\theta' = \frac{-4}{5}$ $\quad \sec\theta' = \frac{-5}{4}$

$\tan\theta' = \frac{3}{-4}$ $\quad \cot\theta' = \frac{-4}{3}$

Ex. $(-3,-2)$



$\sqrt{(-3)^2+(-2)^2} = r$

$\sqrt{9+4} = r$

$\sqrt{13} = r$

$\sin\theta' = \frac{-2}{\sqrt{13}} = \frac{-2\sqrt{13}}{13}$

$\cos\theta' = \frac{-3}{\sqrt{13}} = \frac{-3\sqrt{13}}{13}$

$\tan\theta' = \frac{-2}{-3} = \frac{2}{3}$

$\csc\theta' = \frac{\sqrt{13}}{-2}$

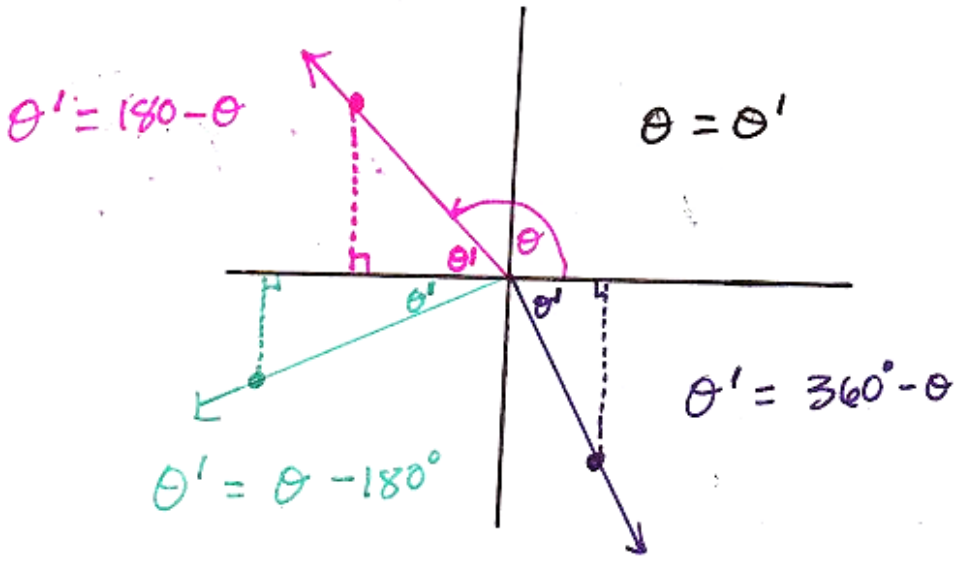
$\sec\theta' = \frac{\sqrt{13}}{-3}$

$\cot\theta' = 3/2$

2. QUADRANT ANGLES: $(0^\circ, 90^\circ, 180^\circ, 270^\circ)$

θ	SIN θ	COS θ	TAN θ	CSC θ	SEC θ	COT θ
0°	0	1	0	UND	1	UND
90°	1	0	UND	1	UND	0
180°	0	-1	0	UND	-1	UND
270°	-1	0	UND	-1	UND	0

3. REFERENCE ANGLE: GIVEN ANY ANGLE θ IN STANDARD POSITION, ITS REFERENCE ANGLE θ' IS THE SMALLEST POSITIVE ANGLE FORMED BY THE TERMINAL SIDE AND THE x-AXIS.



$\theta = 120^\circ$ $\quad \theta' = 180 - 120 = 60^\circ$

$\theta = 345^\circ$ $\quad \theta' = 360 - 345 = 15^\circ$

$\theta = 924^\circ$ $\quad \theta' = 204 - 180 = 24^\circ$

-720

204

$\theta = \frac{9\pi}{5}$

$2\pi - \frac{9\pi}{5}$

$\frac{10\pi}{5} - \frac{9\pi}{5}$

$\theta' = \boxed{\frac{\pi}{5}}$