SEC 4.4 COMPLEX ZEROS & THE FUNDAMENTAL THEOREM OF ALGEBRA

REVIEW

$$x^2 - 9 = 0$$
$$(x+3)(x-3) = 0$$
$$\{-3, 3\}$$

BUT $x^2 - \cancel{9} = 0$
$\quad\;\; +9 \quad +9$

$$\sqrt{x^2} = \sqrt{9}$$
$$x = \pm 3$$

$$x^2 + 16 = 0$$
$$(x + 4i)(x - 4i) = 0$$
$$x^2 - \cancel{4ix} + \cancel{4ix} - 16i^2$$
$$(-1)$$
$$x^2 + 16$$

$x^2 + \cancel{16} = 0$
$\quad\;\; -16 \quad -16$

$$\sqrt{x^2} = \sqrt{-16} = \sqrt{-1} \cdot \sqrt{16}$$
$$x = \pm 4i$$

SINCE $\sqrt{-1}^2 = i^2$

$$-1 = i^2$$

$$x^2 + 49 = 0$$
$$(x+7i)(x-7i) = 0$$
$$\{\pm 7i\}$$

OR $\sqrt{x^2} = \sqrt{-49}$

$$x = \pm 7i$$

TRY $x^2 + 1x + 4 = 0$ $\qquad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 4}}{2}$$
$$= \frac{-1 \pm \sqrt{1 - 16}}{2}$$
$$= \frac{-1 \pm i\sqrt{-15}}{2}$$
$$= \frac{-1 \pm i\sqrt{15}}{2}$$
$$= -\tfrac{1}{2} + i\sqrt{15} \;\;\&\;\; -\tfrac{1}{2} - i\sqrt{15}$$

$a = 1$
$b = 1$
$c = 4$

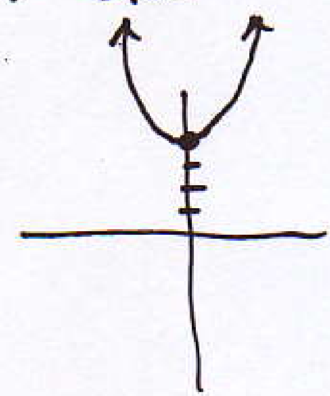
1. FUNDAMENTAL THEOREM OF ALGEBRA.

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a x + a_0$$

$n \geq 1$

$a_n \neq 0$

IF THE POLYNOMIAL HAS A COMPLEX COEFFICIENT THEN AT LEAST ONE COMPLEX ZERO EXISTS.



EX. $x^2 + 4 = 0$

$$(x+2i)(x-2i) = 0$$

$$x^2 - 2ix + 2ix - 4i^2$$

COMPLEX COEFFICENT

$$\{2i, -2i\}$$

2. COMPLETE FACTORIZATION THEOREM

$$P(x) = a_n x^n + \dots + a_0$$

DEGREE = # OF REAL & COMPLEX ZERO

$$P(x) = (x-c_1)(x-c_2)(x-c_3)\cdots(x-c_n)$$

EXAMPLE

$P(x) = 3x^4 - 2x^3 - x^2 - 12x - 4$

$\frac{\pm 1 \pm 2 \pm 4}{\pm 1 \pm 3}$

1

1 POS REAL ZERO

3 OR 1 NEG. REAL ZERO $3x^4 + 2x^3 - x^2 + 12x - 4$

1 1 1

2	$3x^4$	-2	-1	-12	-4
	↓	6	8	14	4
$-\frac{1}{3}$	$3x^3$	4	7	2	0
	↓	-1	-1	-2	
	$3x^2$	$+3x$	$+6$	0	

$3(x^2 + x + 2)$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1$

$b = 1$

$c = 2$

$\frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1}$

$\frac{-1 \pm \sqrt{1-8}}{2}$

$\frac{-1 \pm i\sqrt{-7}}{2} = \frac{-1 + i\sqrt{7}}{2} \quad \frac{-1 - i\sqrt{7}}{2}$

$\left\{ 2, -\frac{1}{3}, -\frac{1}{2} + \frac{i\sqrt{7}}{2}, -\frac{1}{2} - \frac{i\sqrt{7}}{2} \right\}$

$P(x) = 3(x-2)(x+\frac{1}{3})(x^2+x+2)$

$P(x) = 3(x-2)(x+\frac{1}{3})(x^2+x+2)$

IRREDUCIBLE QUADRATIC

$\{2, -\frac{1}{3}, -\frac{1}{2}+\frac{\sqrt{7}}{2}i, -\frac{1}{2}-\frac{\sqrt{7}}{2}i\}$

3. CONJUGATE PAIRS OF COMPLEX NUMBERS

$(a+bi) \quad (a-bi)$

CONJUGATE PAIRS

#31 IN TEXTBOOK

P HAS A DEGREE OF 2

ZEROS $(1+i)$ & $(1-i)$

CONJUGATE PAIRS

WRITE AS LINEAR FACTORS

$(x-(1+i))(x-(1-i))$

$((x-1)-i)((x-1)+i)$

$(x-1)^2 - i^2$

$x^2-2x+1 \; +1$

$\boxed{x^2-2x+2}$

#35 P HAS A DEGREE OF 3

ZEROS 2 and i so $-i$

CONJUGATE PAIRS

$$(x-2)[(x-i)(x+i)]$$

$$x^2 - i^2_{(-1)}$$

$$(x-2)(x^2+1)$$

$$x^3 + x - 2x^2 - 2$$

$$\boxed{P(x) = x^3 - 2x^2 + x - 2}$$

#49 $P(x) = x^4 + x^3 + 7x^2 + 9x - 18$ $\qquad \dfrac{\pm 1\ \pm 2\ \pm 3\ \pm 6\ \pm 9\ \pm 18}{\pm 1}$

1 POS. REAL ZERO

$$(-x)^4 + (-x)^3 + 7(-x)^2 + 9(-x) - 18$$

$$x^4 - x^3 + 7x^2 - 9x - 18$$

1 1 1   3 OR 1 NEG. REAL ZERO

$$\begin{array}{r|rrrrr} -2 & 1x^4 & 1 & 7 & 9 & -18 \\ & \downarrow & -2 & 2 & -18 & 18 \\ \hline & 1x^3 & -1x^2 & +9x & -9 & \boxed{0} \end{array}$$

$x^2(x-1) + 9(x-1)$

$(x-1)(x^2+9)$

$P(x) = (x+2)(x-1)(x+3i)(x-3i)$

$\{-2, 1, \pm 3i\}$

REVIEW

10. $P(x) = x^4 - 2x^3 + 6x^2 - 18x - 27$

$\dfrac{\pm 1 \pm 3 \pm 9 \pm 27}{\pm 1}$

1 1 1 3 OR 1 POS. REAL ZERO

$(-x)^4 - 2(-x)^3 + 6(-x)^2 - 18(-x) - 27$

$x^4 + 2x^3 + 6x^2 + 18x - 27$

1 NEG REAL ZERO

$$
\begin{array}{r|rrrrr}
1 & 1 & -2 & 6 & -18 & -27 \\
 & \downarrow & 1 & -1 & 5 & -13 \\
\hline
 & 1 & -1 & 5 & -13 & \boxed{-40}
\end{array}
$$

$$
\begin{array}{r|rrrrr}
3 & 1x^4 & -2 & 6 & -18 & -27 \\
 & \downarrow & 3 & 3 & 27 & 27 \\
\hline
 & 1x^3 & +1x^2 & +9x & +9 & \boxed{0}
\end{array}
$$

$x^2(x+1) + 9(x+1)$

$(x+1)(x^2+9)$

$P(x) = (x-3)(x+1)(x+3i)(x-3i)$

$\{3, -1, \pm 3i\}$