

6.5 PARTIAL FRACTIONS

SIMPLY THE FRACTION

$$\frac{1}{(x-1)(2x+1)} + \frac{1}{(2x+1)(x-1)}$$

$$\frac{2x+1 + x-1}{(x-1)(2x+1)}$$

$$\frac{3x}{(x-1)(2x+1)}$$

PARTIAL FRACTION DECOMPOSITION IS GOING BACKWARDS FROM THE ANSWER TO THE SUM OF FRACTIONS

$$\frac{3x}{(x-1)(2x+1)} = \frac{A}{x-1} + \frac{B}{2x+1}$$

$$2Ax + A + Bx - B$$

$$2A + B = 3$$

$$\begin{cases} A - B = 0 \\ 2A + B = 3 \end{cases}$$

$$1 - B = 0$$

$$\boxed{B=1}$$

$$3A = 3$$

$$\boxed{A=1}$$

$$\boxed{\frac{1}{x-1} + \frac{1}{2x+1}}$$

2. SPECIAL CASES

1) REPEATED FACTORS

EX. $\frac{x^2 + 1}{x(x-1)^3} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} + \frac{D}{(x-1)^3}$

2) IRREDUCIBLE QUADRATIC (QUADRATIC THAT DOES NOT FACTOR)

EX. $\frac{2x^2 - x + 4}{x^3 + 4x} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4}$

$x(x^2 + 4)$

↑
IRREDUCIBLE QUADRATIC

3. STEPS

- 1) FACTOR THE DENOMINATOR COMPLETELY
- 2) SET UP FRACTIONS: $\frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$
- 3) MULTIPLY BY THE MISSING FACTORS FROM THE LCD
- 4) DISTRIBUTE, FOLL ... TO SIMPLIFY THE NUMERATORS.
- 5) COLLECT LIKE TERMS AND SET EQUAL TO THE CORRESPONDING TERM FROM

THE ORIGINAL NUMERATOR.

- 6) DIVIDE OUT THE "X" FROM EACH TERM.
- 7) SET UP THE SYSTEM OF EQUATIONS AND SOLVE.
- 8) REWRITE AS PARTIAL FRACTIONS.

EXAMPLE

$$\frac{x+4}{x^2-2x-8}$$

$(x-4)(x+2)$

$$= \frac{A(x+2)}{(x-4)(x+2)} + \frac{B(x-4)}{x+2(x-4)}$$

$$\underline{Ax} + \underline{2A} + \underline{Bx} - \underline{4B} =$$

$$\begin{cases} \cancel{A}x + \cancel{B}x = \cancel{1}x \\ 2A - 4B = 4 \\ 4A + 4B = 4 \end{cases}$$

$$\frac{4}{6}A = \frac{8}{6}$$

$$A = \frac{4}{3}$$

$$\frac{4}{6} + B = \frac{3}{3}$$
$$-\frac{4}{6}$$

$$B = -\frac{1}{3}$$

$$\frac{\frac{4}{3}}{3(x-4)} - \frac{\frac{1}{3}}{3(x+2)}$$

$$\boxed{\frac{4}{3(x-4)} - \frac{1}{3(x+2)}}$$

$$39. \frac{x^4 + x^3 + x^2 - x + 1}{x(x^2 + 1)^2}$$

$$\frac{A}{x} + \frac{Bx + C}{x^2 + 1} + \frac{(Dx + E)x}{(x^2 + 1)^2}$$

$$(x^2 + 1)(x^2 + 1)$$

$$x^4 + x^2 + x^2 + 1$$

$$x^4 + 2x^2 + 1$$

$$\underline{Ax^4} + \underline{2Ax^2} + \underline{A} + \underline{Bx^4} + \underline{Bx^2} + \underline{Cx^3} + \underline{Cx} + \underline{Dx^2} + \underline{Ex}$$

$$\boxed{C=1}$$

$$A + B = 1$$

$$C = 1$$

$$2A + B + D = 1$$

$$1 + B = 1$$

$$\boxed{B=0}$$

$$C + E = -1$$

$$2(1) + 0 + D = 1$$

$$\boxed{A=1}$$

$$\begin{matrix} 2 + D = 1 \\ -2 \end{matrix}$$

$$\boxed{D=-1}$$

$$\begin{matrix} 1 + E = -1 \\ -1 \end{matrix}$$

$$\boxed{E=-2}$$

$$\frac{1}{x} + \frac{1}{x^2 + 1} - \frac{x + 2}{(x^2 + 1)^2}$$