# SEC 9.6 BINOMIAL EXPANSION THEOREM

BINOMIAL EXPANSION:

$$(a+b)^1 = a+b$$

$$(a+b)^2 = (a+b)(a+b) = a^2+2ab+b^2$$

$$(a+b)^3 = (a+b)(a+b)(a+b) = a^3+3a^2b+3ab^2+b^3$$

$$(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$$

$$(a+b)^5 = a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$$

NOTE $a^3b^2 \Rightarrow 3+2=5$

THE SUM OF THE POWERS ON a & b EQUALS THE DEGREE OF THE POWER.

## PASCAL'S TRIANGLE

| | |
|---|---|
| 1 | $(a+b)^0$ |
| 1  1 | $(a+b)^1$ |
| 1 2 1 | $(a+b)^2$ |
| 1 3 3 1 | $(a+b)^3$ |
| 1 4 6 4 1 | $(a+b)^4$ |
| 1 5 10 10 5 1 | $(a+b)^5$ |
| 1 6 15 20 15 6 1 | $(a+b)^6$ |
| 1 7 21 35 35 21 7 1 | $(a+b)^7$ |

FACTORIAL: $n!$ $\quad n\cdot(n-1)\cdot(n-2)\cdot(n-3)\ldots\cdot 2\cdot 1$

EX. $5! = 5\cdot 4\cdot 3\cdot 2\cdot 1 = 120$

BINOMIAL COEFFICIENTS

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

EX. $$\binom{5}{3} = \frac{5!}{3!\,(5-3)!} = \frac{5!}{3!\cdot 2!}$$

$$= \frac{5\cdot \overset{2}{\cancel{4}}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{3}\cdot\cancel{2}\cdot\cancel{1}\cdot\cancel{2}\cdot 1}$$

$$= 10$$

CONSIDER $(a+b)^{5=n}$ ← ROW BEGINNING 1 5 ...

PASCAL'S TRIANGLE = 1 5 10 10 5 1

$10a^{3} b^{2}$ ($r=3$)

$$\binom{5=n}{3=r} = \frac{5!}{3!\cdot(5-3)!} = \frac{5\cdot \overset{2}{\cancel{4}}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{3}\cdot\cancel{2}\cdot\cancel{1}\cdot\cancel{2}\cdot 1} = 10$$

#9 (TEXTBOOK)

$(2x-3y)^3$

PASCAL'S TRIANGLE:

$(a+b)^3 = 1a^3b^0 \quad 3a^2b^1 \quad 3ab^2 \quad 1a^0b^3$

$(2x-3y)^3 = 1(2x)^3\cancel{(-3y)^0}^{1} \quad 3(2x)^2(-3y)^1 \quad 3(2x)^1(-3y)^2 \quad 1\cancel{(2x)^0}^{1}(-3y$

$3 \cdot 4 \cdot (-3) \qquad 3 \cdot 2 \cdot 9$

$8x^3 - 36x^2y + 5xy^2 - 27y^3$

#35 (TEXTBOOK)

FINDING THE $x^{4=r}$ TERM IN THE EXPANSION OF $(x+2y)^{10=n}$

$\binom{10}{4} x^4 (2y)^6$

$$\frac{10!}{4!\,6!} = \frac{10 \cdot \overset{3}{\cancel{9}} \cdot \cancel{8} \cdot 7 \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 210$$

$210 \cdot x^4 \cdot 2^6 y^6$

$210 \cdot 64\, x^4 y^6$

$13,440\, x^4 y^6$