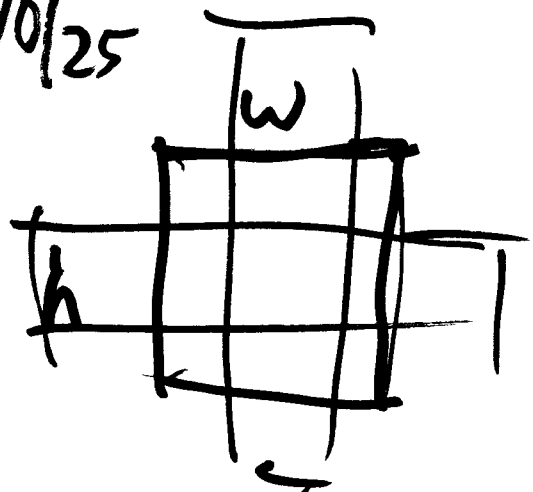
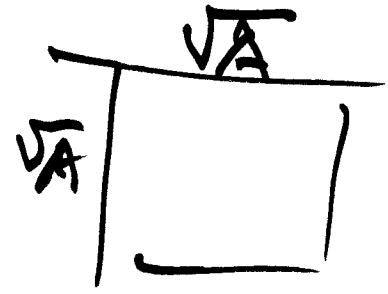
10/25





$A = wh$

$L = 2w + 2h$

Fix A Minimize L

$w = \frac{A}{h} = Ah^{-1}$

$L(h) = 2h + 2Ah^{-1}$

$L'(h) = 2 - 2Ah^{-2} = 0 \quad 1 = \frac{A}{h^2}$

$2 = \frac{2A}{h^2}$

$A = h^2 \quad h = \sqrt{A} \quad w = \frac{A}{h} = \frac{A}{\sqrt{A}} = \sqrt{A}$

Fix L

Maximize $A = h \cdot w = h\left(\frac{L-2h}{2}\right)$

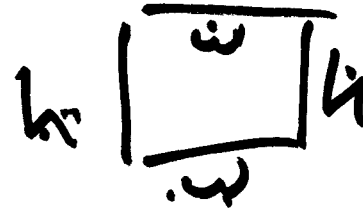
$L = 2w + 2h \quad w = \frac{L-2h}{2}$

$A(h) = h \cdot \frac{1}{2}(L-2h) = -h^2 + \frac{1}{2}Lh$

$= -h\left(h - L/2\right)$

$h_{max} = L/4$

$A'(h) = -2h + \frac{1}{2}L = 0$



$L = 2w + 2h$

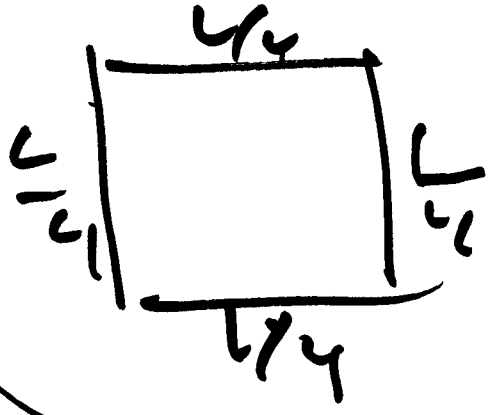
Minimum L for fixed A

was when $h = w = \sqrt{A}$

$L = 4\sqrt{A}$ $\Rightarrow \frac{L}{4} = \sqrt{A}$ $A = \frac{L^2}{16}$

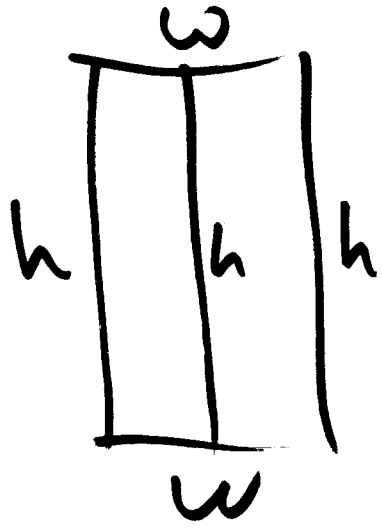
Maximum A for fixed L

was when $h = \frac{L}{4}$ $w = \frac{L}{4}$



$A = \left(\frac{L}{4}\right)^2 = \frac{L^2}{16}$

$A = \frac{L^2}{16}$ $16A = L^2$ $L = 4\sqrt{A}$



$$L = 3h + 2w$$
$$A = w \cdot h$$

Fix A min L:

$$L(h) = 3h + 2Ah^{-1}$$
$$L'(w) = 3 - \frac{2A}{h^2} = 0 \therefore h^2 = \frac{2A}{3}$$

$$h = \sqrt{\frac{2A}{3}} \qquad w = \frac{A}{h} = \frac{A}{\sqrt{\frac{2A}{3}}} = \frac{\sqrt{A}\sqrt{3}}{\sqrt{A}\sqrt{2}} \cdot \frac{\sqrt{A}\cdot\sqrt{3}}{\sqrt{2}}$$

$$L = 3 \cdot \frac{\sqrt{2A}\sqrt{3}}{\sqrt{3}} + 2\frac{\sqrt{3A}}{\sqrt{2}} = 2\sqrt{6A}$$

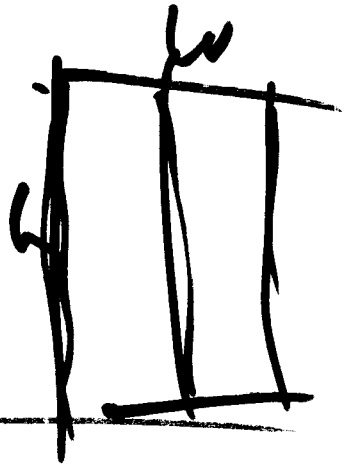
$$\frac{x}{\sqrt{x}} = \sqrt{x} \qquad \sqrt{x}\sqrt{y} = \sqrt{xy} \qquad \frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}$$

$$L = \sqrt{24A}$$

Fix L Maximize A

$L = 3h + 2w$

$A = wh$



$w = \frac{1}{2}(L - 3h)$

$A = h \cdot \frac{1}{2}(L - 3h)$

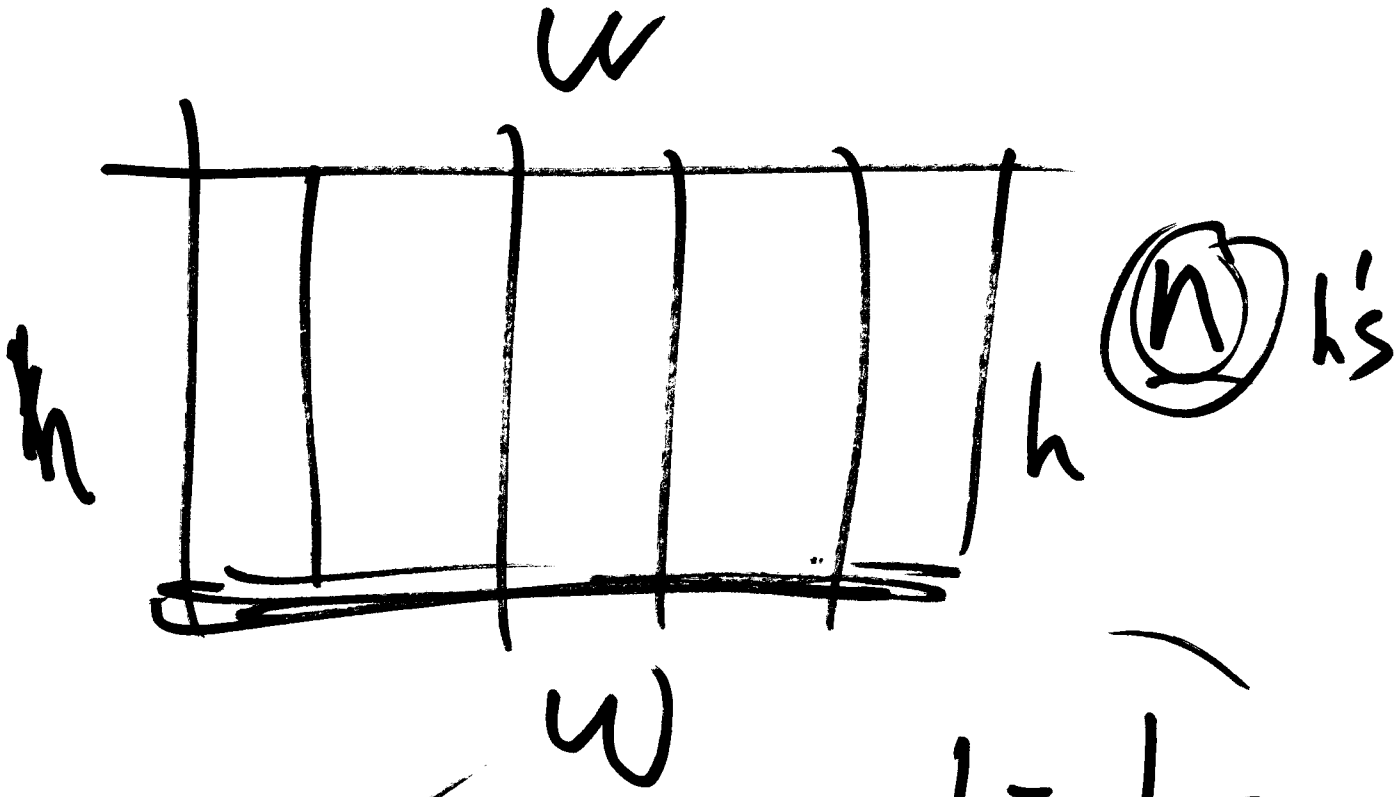
$A(w) = -\frac{3}{2}h^2 + \frac{1}{2}Lh$ ∩

$A'(h) = -3h + \frac{1}{2}L = 0$

$h = L/6$ $\quad w = L/4$

(Half always to height, half to width)

$A = L^2/24$



$\frac{L}{4}$

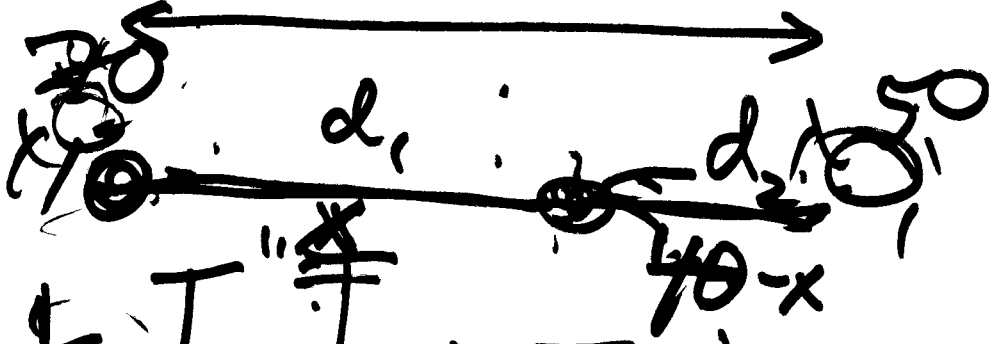
$h = \frac{L}{2 \cdot n}$

$n = 2 \quad \frac{L^2}{16}$

$n = 3 \quad \frac{L^2}{24}$

max given A: $A = \frac{L^2}{8n}$

min given A: $L = \sqrt{8A \cdot n}$

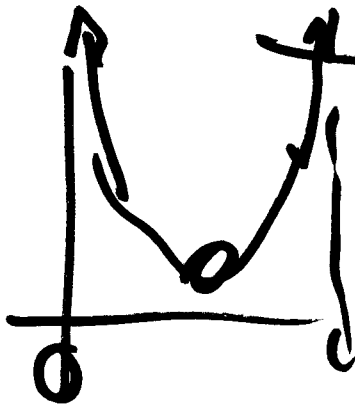


$$L = \frac{k I_1}{d_1^2} + \frac{k I_2}{d_2^2}$$

$x + y = 40$
$y = 40 - x$

$A = 4\pi r^2$

$$L(x) = k\left[\frac{70}{x^2} + \frac{50}{(40-x)^2}\right] = k\left[70x^{-2} + 50(40-x)^{-2}\right]$$



$$0 = L'(x) = k\left(70 \cdot (-2x^{-3}) - 50 \cdot 2(40-x)^{-3} \cdot (-1)\right)$$

$$\frac{a^3}{b^3} = \left(\frac{a}{b}\right)^3 \qquad \frac{2 \cdot 70}{x^3} = \frac{50 \cdot 2}{(40-x)^3} \qquad \left(\frac{40-x}{x}\right)^3 = \frac{50}{70}$$

$$\frac{40-x}{x} = \left(\frac{50}{70}\right)^{1/3} \qquad 40 - x = \left(\frac{50}{70}\right)^{1/3} x$$

$$40 - x = \left(\frac{50}{70}\right)^{1/3} x$$

$$40 = 1x + \left(\frac{50}{70}\right)^{1/3} x =$$

$$= \left(1 + \left(\frac{50}{70}\right)^{1/3}\right) x \qquad b = ax$$

$$x = \frac{40}{\left(1 + \left(\frac{50}{70}\right)^{1/3}\right)} \qquad \frac{b}{a} = x$$