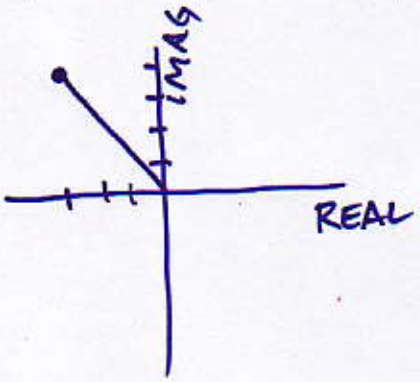
SEC 5.2 TRIGONOMETRIC FORM OF COMPLEX NUMBERS.

1. GRAPH A COMPLEX NUMBER: $a+bi$

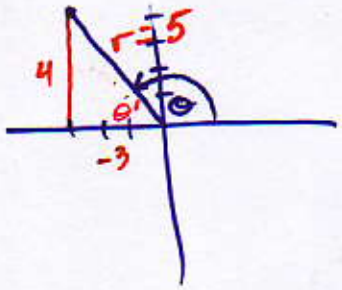


$-3+4i$

↑ REAL ↑ IMAG

2. ABSOLUTE VALUE OF A COMPLEX NUMBER

$$|z| = |a+bi| = \sqrt{a^2+b^2}$$



$$|z| = |-3+4i| = \sqrt{(-3)^2+4^2}$$

$$\sqrt{9+16}$$

$$\sqrt{25}$$

$$|z| = 5$$

3. FIND $\theta = \tan^{-1}\frac{b}{a}$

$$\theta = \tan^{-1}\left(\frac{4}{-3}\right) = -53.13$$

$$180 - 53.13 = 126.87^\circ$$

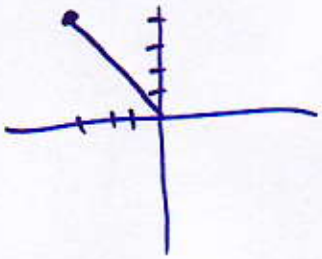
4. TRIGONOMETRIC FORM OF A COMPLEX NUMBER.

$a+bi \Rightarrow$ COMPLEX FORM

$$z = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$$

ABBREVIATED FORM

$-3+4i$



$r = 5$

$\theta = 126.87°$

$$z = 5(\cos 126.87° + i\sin 126.87°)$$

OR $\quad 5\,\text{cis}\,126.87°$

5. MULTIPLYING TWO COMPLEX NUMBERS IN TRIG FORM.

$$z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$$

$$z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$$

$$z_1 \cdot z_2 = r_1 \cdot r_2(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$$

EX. $(2+3i)(4-i) = 8 - 2i + 12i + 3i^2 = 11 + 10i$

TRIG FORM

$r = \sqrt{2^2+3^2}$

$r = \sqrt{13}$

$\theta = \tan^{-1}\frac{3}{2} \approx 56.3°$

$r = \sqrt{4^2+(-1)^2}$

$\sqrt{16+1}$

$\sqrt{17}$

$\theta = \tan^{-1}\frac{-1}{4}$

$\theta = 346$

CONT. $\sqrt{13}(\cos 56.3^\circ + i \sin 56.3^\circ)$ $\sqrt{17}(\cos 346 + i \sin 346)$

MULT. $\sqrt{13}\cdot\sqrt{17}\left(\cos(56.3^\circ + 346^\circ) + i \sin(56.3^\circ + 346^\circ)\right)$

$\sqrt{221}\ \text{cis}\ 402.3^\circ$

6. DIVIDE TWO COMPLEX NUMBERS IN TRIG FORM

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left(\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)\right)$$

EX. #49 H.W.

$$\frac{25(\cos 3.5 + i \sin 3.5)}{5(\cos 1.5 + i \sin 1.5)}$$

$$5\left(\cos(3.5 - 1.5) + i \sin(3.5 - 1.5)\right)$$

$5\ \text{cis}\ 2$ → TRIG FORM

$-2 + 4.5i$ → $a + bi$