Introduction to Calculus, Math 1100, Fall 2012, Bob Palais

Practice Exam 2, Chapters 3 and 4

Differentiation and its Applications, Calculus of Exponential and Logarithmic Functions

1. (20 points) Graphing and Optimization. Let $p(x) = 10 - 6x + x^2$ the price that must be charged to sell $x \in [0, 4]$ units of your product and $c(x) = x + 2$ be the cost of producing those x units. The revenue $R(x) = xp(x) = 10x - 6x^2 + x^3$ is price per unit times the number of units.

The profit from producing and selling those x units is the revenue minus the cost, $P(x) = R(x) - C(x)$:

$$P(x) = x^3 - 6x^2 + 9x - 2.$$

a) Sketch the graph of $P(x)$ on the interval $[0, 4]$ Identify any symmetries, and show clearly any x and y intercepts, intervals of increase and decrease, maximum and minimum points, horizontal and vertical asymptotes, the concavity and inflection points.

b) What price(s) could be charged to yield the maximum profit, and how many units ($x \in [0, 4]$) will be sold at those price(s).

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2. (20 points) Let C be a square centered at the origin with its four vertices at

$$(+h, +h), (-h, +h), (-h, -h), (+h, -h),$$

and whose edge length $2h$ is varying in time. Draw a sketch, then find the instantaneous rate of change of the area A of this square when the edge length $2h$ is equal to $10cm$ and when h is increasing at 2 cm/sec.

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3. (20 points) An initial principal investment of \$150 grows at a fixed exponential interest rate for 30 years to \$450.

a) (10 points) Find an expression for $P(t)$, the amount in the investment t years after the initial investment.

b) (10 points) What is the doubling time for $P(t)$?

Bonus) (5 points) Use differentials to approximate by how much the amount in the investment grows in the next 3.65 days (.01 years) after the 30 year mark. In other words:

What is the change of $P(t)$ between $t = 30$ and $t = 30.01$? (Leave the answer as a formula.)

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4. (20 points) Let

$$y(x) = \frac{(x-2)^{-3}\sqrt{x^2+1}}{e^{x^3}}.$$

a) (10 points) Find a simplified expression for $\ln(y(x))$.

b) (10 points) Use part a) to find the derivative of $\ln(y(x))$.

Bonus) (5 points) Use parts a) and b) to find an expression for y'.
Hint: The logarithmic differentiation formula is obtained by rearranging the formula

$$(\ln(y))' = \frac{1}{y}(y').$$

that comes from the chain rule.

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5. Let

$$y = f(x) = xe^{-x}.$$

a) (10 points) Find $f'(x)$ and $f''(x)$.

b) (10 points) On what intervals is $f(x)$ increasing and decreasing? On what intervals is $f(x)$ concave up and concave down?
Hint: The value of any exponential function is always positive, so factor e^{-x} from $f'(x)$ and $f''(x)$ and look at the rest.

Bonus) (5 points) What is the maximum value of $f(x)$ and for what x is the maximum value taken?

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