9/20 The product rule

If $f(x) = u(x)\, v(x)$

then

$$f'(x) = u'(x)\, v(x) + u(x) \cdot v'(x)$$

Example: $u(0) = 3$ $\quad u'(0) = 2$ $\quad (0,3)$ $m=2$

$v(0) = 7$ $\quad v'(0) = 5$ $\quad (0,7)$ $m=5$

$$f'(0) = 2 \cdot 7 + 3 \cdot 5 = 29$$

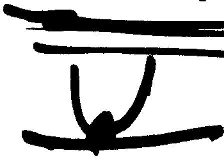
particular

$u(x) = 2x + 3$

$v(x) = 5x + 7$

$f(0) = 21$

$$f(x) = u(x)\, v(x) = (2x+3)(5x+7) = 10x^2 + 29x + 21$$



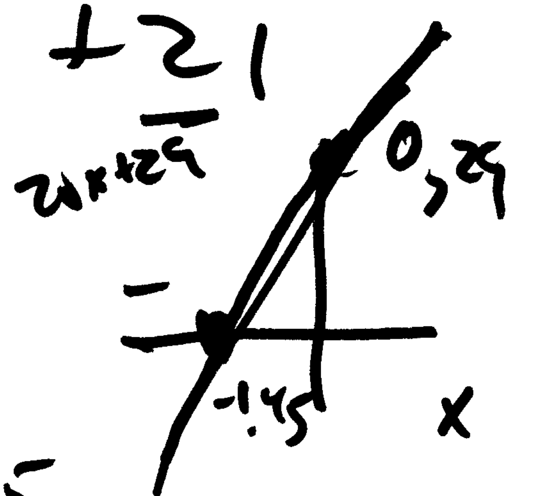
$$f'(x) = 20x + 29 = 5 \cdot 3 + 2 \cdot 7$$

$$f'(0) = 20 \cdot 0 + 29 = 29$$

29·29 - 4·10·21

$f(x) = 10x^2 + 29x + 21$

$f'(x) = 20x + 29$ =



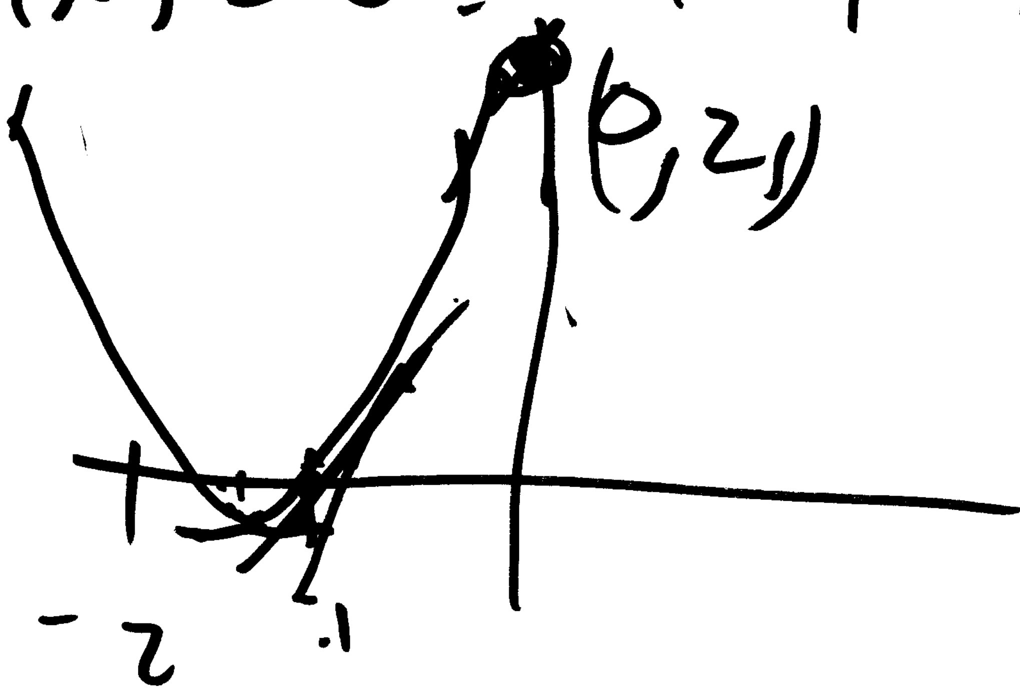
$f'(x) > 0 \quad x > -1.45$

$10(1.45)^2 + 29(1.45)$
$+ 21$

$f'(x) < 0 \quad x < -1.45$

$f'(x) = 0 = 20x + 29 \qquad x = \frac{-29}{20}$

-1.45



$x^3+x^2 \quad (x+h)^3+(x+h)^2 - (x^3+x^2)$

$$(u+v)'(x) = \lim_{h \to 0} \frac{(u+v)(x+h) - (u+v)(x)}{h}$$

$$= \lim_{h \to 0} \frac{u(x+h)+v(x+h) - (u(x)+v(x))}{h}$$

$$= \lim_{h \to 0} \frac{u(x+h)-u(x)}{h} + \frac{v(x+h)-v(x)}{h}$$

$$u'(x) + v'(x)$$

$$\frac{(x+h)^3-x^3}{h} + \frac{(x+h)^2-x^2}{h}$$

$$\downarrow 3x^2 \qquad \downarrow 2x$$

$$3x^2+2x$$

$u(x) = x^3$

$$u'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}} =$$

$$\lim_{h \to 0} 3x^2 + 3\cancel{x}h + \cancel{h^2}$$

$$= 3x^2$$

$$V(x) = x^2$$

$$V'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$= \lim_{h \to 0} 2x + h$$

$$= 2x$$

$$u+v(x) = u(x) + v(x) = x^3 + x^2$$

$$\lim_{h \to 0} \frac{(x+h)^3 + (x+h)^2 - (x^3 + x^2)}{h}$$

$$3x^2 + 2x$$

If $f(x) = 10x^4 - 3x^3 + 5x^2 - 7x + 11x^0$

$f'(x) = 40x^3 - 9x^2 + 10x - 7$

$u(x) = 7x + 2$ $\quad u'(x) = 7$

$v(x) = -3x + 1$ $\quad v'(x) = -3$

$f(x) = u(x) + v(x) = 4x + 3$

$f'(x) = 4$

$f(x) = 5u(x) = 35x + 10$ $\quad f'(x) = 35$

$g(x) = cx^3$ $\quad x^4$

$g'(x) = c(3x^2)$ $\quad nx^{n-1}$

$$f(x) = 7x^3$$

$$f'(x) = \lim_{h \to 0} \frac{7(x+h)^3 - 7x^3}{h}$$

$$= \lim_{h \to 0} \frac{7\left[(x+h)^3 - x^3\right]}{h}$$

$$\vdots$$

$$7 \cdot 3x^2$$

If $f(x) = \overset{7}{C} \; \overset{x^3}{u(x)}$

$$f'(x) = \lim_{h \to 0} \frac{c\,u(x+h) - c\,u(x)}{h}$$

$$= \lim_{h \to 0} c \left(\frac{u(x+h) - u(x)}{h} \right)$$

$$= \cancel{\lim_{h \to 0}}\, C \lim_{h \to 0} \frac{u(x+h) - u(x)}{h}$$

$$C \quad u'(x)$$

$f(x) = 2^x \qquad f'(x) \neq x 2^{x-1}$

$$f'(x) = \lim_{h\to 0} \frac{2^{x+h} - 2^x}{h}$$

$2^{2+3} = 2^5$

$2^2 \cdot 2^3$

$32 = 4 \cdot 8$

$$= \lim_{h\to 0} \frac{2^x 2^h - 2^x}{h}$$

$$= 2^x \left(\lim \frac{2^{0+h} - 1}{h} \right) \qquad 2^0 = 1$$

$f'(0)$

$f(x) = a^x \qquad f'(x) = C a^x$

$C \to (\ln a)$