SEC. 5.3 DE MOIVRE'S THEOREM

RECALL $z_1 z_2 = r_1 r_2 (\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$

$(3+5i)^2 = r_1 \cdot r_1 (\cos \theta_1+\theta_1) + i\sin(\theta_1+\theta_1)$

$(a+bi)^2 \quad z^2 = r^2(\cos 2\theta + i\sin 2\theta)$

$(a+bi)^3 \quad z^3 = r^3(\cos 3\theta + i\sin 3\theta)$

DE MOIVRE'S THEOREM $\boxed{z^n = r^n(\cos n\theta + i\sin n\theta)}$

EX. $\left(\underset{\substack{\downarrow \\ r}}{2}(\cos \underset{\substack{\downarrow \\ \theta}}{30} + i\sin 30)\right)^{5 \leftarrow n}$

$z^5 = 2^5(\cos 5\cdot 30 + i\sin 5\cdot 30)$

$\downarrow$

$32(\cos \underset{-\frac{\sqrt{3}}{2}}{\cancel{150}} + i\sin \underset{\frac{1}{2}}{\cancel{150}})$ TRIG FORM

STANDARD FORM

$32\left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)$

$\boxed{-16\sqrt{3} + 16i}$

DE MOIVRE'S THEOREM FOR FINDING ROOTS

LET $z = r(\cos\theta + i\sin\theta)$

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\frac{\theta + 360k}{n} + i\sin\frac{\theta+360k}{n}\right)$$

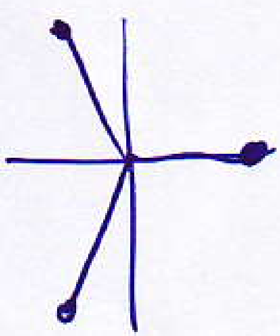
$k = 0, 1, 2, 3, 4 \ldots$

EX. $\underset{\substack{\downarrow \\ r}}{27}\left(\cos \overset{\theta}{0^\circ} + i\sin 0^\circ\right)$ FIND $\boxed{3 = n}$ CUBES ROOTS OF z.

$k=0$ $\quad z_{(1)} = 27^{\frac{1}{3}}\left(\cos\frac{0+360(0)}{3} + i\sin\frac{0+360(0)}{3}\right)$

$k=1$ $\quad z_{(2)} = 27^{\frac{1}{3}}\left(\cos\left(\frac{0+360(1)}{3}\right) + i\sin\left(\frac{0+360(1)}{3}\right)\right)$

$k=2$ $\quad z_{(3)} = 27^{\frac{1}{3}}\left(\cos\left(\frac{0+360(2)}{3}\right) + i\sin\left(\frac{0+360(2)}{3}\right)\right)$



	$a+bi$
$3(\overset{1}{\cos}\, 0 + i\,\overset{0}{\sin}\, 0)$	3
$3(\overset{-\frac{1}{2}}{\cos}\, 120 + i\,\overset{\frac{\sqrt{3}}{2}}{\sin}\, 120)$	$-\frac{3}{2} + \frac{3\sqrt{3}}{2}i$
$3(\overset{-\frac{1}{2}}{\cos}\, 240 + i\,\overset{-\frac{\sqrt{3}}{2}}{\sin}\, 240)$	$-\frac{3}{2} - \frac{3\sqrt{3}}{2}i$