

SEC 3.3

DOUBLE ANGLES / HALF - ANGLES

1. DOUBLE ANGLES

$$\sin(\alpha + \beta)$$

$$\sin \alpha \cos \beta + \cos \alpha \sin \beta \quad \leftarrow$$

$$\text{BUT } \sin(\alpha + \alpha) = \sin(2\alpha)$$

$$= 2 \sin \alpha \cos \alpha$$

$$\cos(\alpha + \alpha) = \cos(2\alpha)$$

$$\cos \alpha \cos \alpha - \sin \alpha \sin \alpha$$

$$\cos^2 \alpha - \sin^2 \alpha$$

$$\tan(2\alpha) = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

DOUBLE ANGLE IDENTITIES

$$1) \sin 2x = 2 \sin x \cos x$$

$$2) \cos 2x = \cos^2 x - \sin^2 x$$

$$\left. \begin{array}{l} 1 - 2\sin^2 x \\ 2\cos^2 x - 1 \end{array} \right\} \leftarrow \text{ALT. FORMS}$$

$$\begin{array}{l} 1 - \sin^2 x - \sin^2 x \\ \cos^2 x + (-1 + \cos^2 x) \end{array}$$

$$3) \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

2. HALF-ANGLES

$$1) \sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$2) \cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$3) \tan \frac{x}{2} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$$