Introduction to Calculus, Math 1100, Fall 2012, Bob Palais

Practice Exam 1, Chapters 1 and 2

Limits and Derivatives

Show all your work on the exam for full credit.

1. (20 points) Find the equation of the line l that is perpendicular to the segment joining the points $A = (3, 0)$ and $B = (0, 4)$ and that contains the midpoint M of the segment. (In other words, the line l is the perpendicular bisector of the segment $\overline{AB}$.)

Bonus) (5 points) Find the distance from the point A to the line l. (Hint: There is an easier way and a harder way to find that distance.)

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2. (20 points) Find the derivative of each of the following functions:

a) (10 points) Use the form provided by filling in the blanks.

$$f(x) = 9 + 8x - 7\frac{x^2}{2} + 5\frac{x^3}{6}$$

$$f'(x) = \_\_ + \_\_\, x + \_\_\, \frac{x^2}{2} + \_\_\, \frac{x^3}{6}$$

b) (10 points)

$$g(t) = (1-t)(1+t+t^2)$$

Bonus. (5 points) Find the general form of an antiderivative of $f(x)$ and $g(x)$ from a) and b):

$$\int f(x)\, dx = \_\_ + \_\_\, x + \_\_\, \frac{x^2}{2} + \_\_\, \frac{x^3}{6} + \_\_\, \frac{x^4}{24}$$

$$\int g(x)\, dt =$$

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3. Derivatives and Tangent lines. Let

$$f(x) = x^2 + 3x.$$

a) (10 points) Find $f'(1)$ using the definition of the derivative.

b) (10 points) Find $f'(1)$ using the power rule. Find an equation for the line tangent to the curve $y = f(x) = x^2 + 3x$ at $x = 1$. Sketch the curve and the tangent line.

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4. (20 pts) If a demand function $P(x)$ gives the price per unit at which x units will be sold, then the revenue function is $R(x) = x\ P(x)$.

a) (10 points) Solve $R(x) = x\ P(x)$ for the demand function:

$$P(x) =$$

Then find the 'marginal demand' function $P'(x)$ in terms of $R(x)$ and $R'(x)$.

b) (10 points) If $R(x) = \sqrt{x^2 + 1}$ find $P'(x)$.

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5. A football player attempts a field goal, kicking the ball with an initial vertical velocity of +48 feet per second. It experiences a constant downward acceleration deceleration due to gravity of -32 feet per second per second. Neglect air resistance in your calculations.

a) (5 points) What is the vertical velocity of the ball t seconds after it is hit?

b) (5 points) What is its maximum height above the field?

c) (5 points) How long after being kicked does it land again?

d) (5 points) If it is kicked directly toward the goalposts and reaches them $t = 5/2$ seconds after leaving the ground, does it score a field goal? (Field goal posts are 10 feet above the ground.)

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