Solution for Lab 2

> CS410: Artificial Intelligence
> Shanghai Jiao Tong University, Fall 2021

Assignment

Exercise 1

A. Number of 'Lakes'

The idea here is to use a counter to count the number of lakes and we increase the counter each time we enter a lake. One key point is to use graph search to avoid entering one lake more than once.

B. DFGS with Iterative Deepening

First define the functions controling the growth of the depth and then modify the solution for Lab 1, Exercise 2. Then in each run of the DFGS with depth limit, expand one child node if its depth does not exceed the limit of the depth or discard this child node otherwise. Finally keep invoking the DFGS with depth limit until the goal state is found.

Reasonable answers for the discussions are acceptable.

Exercise 2: Least-Cost Path (Uniform-Cost Graph Search)

The exercise could be solved by implementing uniform-cost graph search (UCGS) in Lecture 3, Slide 50.

Reasonable answers for the discussions are acceptable.

Exercise 3: Least-Cost Path with Heuristics

The exercise could be solved by implementing A^* graph search (A^*GS) in Lecture 3, Slide 51. Manhattan distance could be used as a heuristic function which is better than the Euclidean distance.

Reasonable answers for the discussions are acceptable.

For the heuristic function which uses $\mathrm{dis}(P,G) = |x_P - x_G| + |y_P - y_G| - \mathbb{I}\{|x_P - x_G| \neq |y_P - y_G|\}$ as distance, the first thing we should notice about this distance is that it is still admissible (since it is equal or less than the Manhattan distance and Manhattan distance is admissible) but not consistent. To see this, consider 2 states $s_1 = (5,5)$ and $s_2 = (6,5)$ in the maze where the goal state is $G = (10,10)$, it is clear that $dis(s_1, G) = 10 - 5 + 10 - 5 = 10$ but $dis(s_2, G) = 10 - 6 + 10 - 5 - 1 = 8$ and $dis(s_1, G) - dis(s_2, G) = 2 > \mathrm{cost}(s1 \to s_2) = 1$, which violates the triangle inequality.

However, A^* GS could still work under this heuristic function.

Proof:

Follow the proof on Lecture 3, Slide 58. For some n on path to G^* , some worse n' for the same state, and some p which is the ancestor of n when n' is in the priority queue, it could be shown that p will be expanded before n' .

- $f(p) \leq f(n) + 1$:

$$\begin{aligned} f(p) &= g(p) + h(p) \\ &= g(p) + M(p,g) - \mathbb{I}\{|x_p - x_g| \neq |y_p - y_g|\} \\ &\leq g(p) + M(p,g) \\ &= g(p) + M(p,n) + M(n,g) \\ &\leq g(p) + M(p,n) + h(n) + 1 \\ &\leq g(n) + h(n) + 1 \\ &= f(n) + 1\,, \end{aligned}$$

where $M(s_1, s_2)$ denotes the Manhattan distance between states s_1 and s_2.

- $g(n) + 2 \leq g(n')$:

  Denote by P and P' the two paths from the start state to the goal state. Let P_i and P'_i be the direction taken at the i-th step in the path P and P' respectively. Since the maze is a grid, it holds that

$$\sum_{i=1}^{\mathrm{len}(P)} \mathbb{I}\{P_i = w\} - \sum_{i=1}^{\mathrm{len}(P)} \mathbb{I}\{P_i = e\} = \sum_{i=1}^{\mathrm{len}(P')} \mathbb{I}\left\{P'_i = w\right\} - \sum_{i=1}^{\mathrm{len}(P')} \mathbb{I}\left\{P'_i = e\right\} \text{ (the horizontal line distance)} \Rightarrow$$

$$\sum_{i=1}^{\mathrm{len}(P)} \mathbb{I}\{P_i = w\} - \sum_{i=1}^{\mathrm{len}(P')} \mathbb{I}\left\{P'_i = w\right\} = \sum_{i=1}^{\mathrm{len}(P)} \mathbb{I}\{P_i = e\} - \sum_{i=1}^{\mathrm{len}(P')} \mathbb{I}\left\{P'_i = e\right\} = d_1$$

$$\sum_{i=1}^{\mathrm{len}(P)} \mathbb{I}\{P_i = n\} - \sum_{i=1}^{\mathrm{len}(P)} \mathbb{I}\{P_i = s\} = \sum_{i=1}^{\mathrm{len}(P')} \mathbb{I}\left\{P'_i = n\right\} - \sum_{i=1}^{\mathrm{len}(P')} \mathbb{I}\left\{P'_i = s\right\} \text{ (the vertical line distance)} \Rightarrow$$

$$\sum_{i=1}^{\mathrm{len}(P)} \mathbb{I}\{P_i = n\} - \sum_{i=1}^{\mathrm{len}(P')} \mathbb{I}\left\{P'_i = n\right\} = \sum_{i=1}^{\mathrm{len}(P)} \mathbb{I}\{P_i = s\} - \sum_{i=1}^{\mathrm{len}(P')} \mathbb{I}\left\{P'_i = s\right\} = d_2$$

  Based on the above display,

$$\begin{aligned} & \mathrm{len}(P) - \mathrm{len}(P') \\ = \; & \sum_{i=1}^{\mathrm{len}(P)} \left(\mathbb{I}\{P_i = w\} + \mathbb{I}\{P_i = e\} + \mathbb{I}\{P_i = n\} + \mathbb{I}\{P_i = s\}\right) \\ & - \sum_{i=1}^{\mathrm{len}(P')} \left(\mathbb{I}\{P'_i = w\} + \mathbb{I}\{P'_i = e\} + \mathbb{I}\{P'_i = n\} + \mathbb{I}\{P'_i = s\}\right) \\ = \; & (\sum_{i=1}^{\mathrm{len}(P)} \mathbb{I}\{P_i = w\} - \sum_{i=1}^{\mathrm{len}(P')} \mathbb{I}\left\{P'_i = w\right\}) + (\sum_{i=1}^{\mathrm{len}(P)} \mathbb{I}\{P_i = e\} - \sum_{i=1}^{\mathrm{len}(P')} \mathbb{I}\left\{P'_i = e\right\}) \\ & + (\sum_{i=1}^{\mathrm{len}(P)} \mathbb{I}\{P_i = n\} - \sum_{i=1}^{\mathrm{len}(P')} \mathbb{I}\left\{P'_i = n\right\}) + (\sum_{i=1}^{\mathrm{len}(P)} \mathbb{I}\{P_i = s\} - \sum_{i=1}^{\mathrm{len}(P')} \mathbb{I}\left\{P'_i = s\right\}) \\ = \; & 2d_1 + 2d_2 \,, \end{aligned}$$

  which shows that difference between the lengths of two paths in the maze must be even number. Hence $g(n) + 2 \leq g(n')$.

- $f(p) \leq f(n) + 1 < f(n) + 2 = g(n) + h(n) + 2 \leq g(n') + h(n') = f(n')$ and hence p will be expanded before n' and A^* GS could still work under this heuristic function.