Multi-Class Logistic Regression and Perceptron

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Some slides adapted from Dan Jurfasky, Brendan O'Connor and Marine Carpuat

MultiClass Classification

- Q: what if we have more than 2 categories?
  - Sentiment: Positive, Negative, Neutral
  - Document topics: Sports, Politics, Business, Entertainment, …

Q: How to easily do Multi-label classification?

Two Types of MultiClass Classification

- Multi-label Classification
  - each instance can be assigned more than one labels

- Multinominal Classification
  - each instance appears in exactly one class (classes are exclusive)

Multinominal Classification

- Pretty straightforward with Naive Bayes.

$$P(\mathrm{spam}|D) \propto P(\mathrm{spam}) \prod_{w \in D} P(w|\mathrm{spam})$$

Log-Linear Models

$$P(y|x) \propto e^{w \cdot f(x,y)}$$

$$P(y|x) = \frac{1}{Z(w)} e^{w \cdot f(x,y)}$$

Multinominal Logistic Regression

$$P(y|x) \propto e^{w \cdot f(x,y)}$$

$$P(y|x) = \frac{1}{Z(w)} e^{w \cdot f(x,y)}$$

$$\boxed{P(y|x) = \frac{e^{w \cdot f(x,y)}}{\sum_{y' \in Y} e^{w \cdot f(x,y')}}}$$

normalization term (Z) so that probabilities sum to 1

(a.k.a) Softmax Regression

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Softmax function

From Wikipedia, the free encyclopedia

In mathematics, the **softmax function**, or **normalized exponential function**,[1]:198 is a generalization of the logistic function that "squashes" a K-dimensional vector $\mathbf{z}$ of arbitrary real values to a K-dimensional vector $\sigma(\mathbf{z})$ of real values in the range (0, 1) that add up to 1. The function is given by

$$\sigma(\mathbf{z})_j = \frac{e^{z_j}}{\sum_{k=1}^{K} e^{z_k}} \quad \text{for } j = 1, \ldots, K.$$

Q: what if there are only 2 categories?

$$P(y = j|x_i) = \frac{e^{w_j \cdot x_i}}{\sum_k e^{w_k \cdot x_i}}$$

Q: what if there are only 2 categories?

$$P(y=1|x) = \frac{e^{w_1 \cdot x}}{e^{w_0 \cdot x + w_1 \cdot x - w_1 \cdot x} + e^{w_1 \cdot x}}$$

Q: what if there are only 2 categories?

$$P(y=1|x) = \frac{e^{w_1 \cdot x}}{e^{w_0 \cdot x - w_1 \cdot x} e^{w_1 \cdot x} + e^{w_1 \cdot x}}$$

Q: what if there are only 2 categories?

$$P(y=1|x) = \frac{e^{w_1 \cdot x}}{e^{w_1 \cdot x}(e^{w_0 \cdot x - w_1 \cdot x} + 1)}$$

Q: what if there are only 2 categories?

$$P(y = 1|x) = \frac{1}{e^{w_0 \cdot x - w_1 \cdot x} + 1}$$

Q: what if there are only 2 categories?

$$P(y = 1|x) = \frac{1}{e^{-w' \cdot x} + 1}$$

Sigmoid (logistic) function

Multinominal Logistic Regression

- Binary (two classes):
  - We have one feature vector that matches the size of the vocabulary
- Multi-class in practice:
  - one weight vector for each category

$$w_{\text{pos}} \qquad w_{\text{neg}} \qquad w_{\text{neut}}$$

In practice, can represent this with one giant weight vector and repeated features for each category.

Maximum Likelihood Estimation

$$w_{\mathrm{MLE}} = \mathrm{argmax}_w \log P(y_1, \ldots, y_n | x_1, \ldots, x_n; w)$$

$$= \mathrm{argmax}_w \sum_i \log P(y_i | x_i; w)$$

$$= \mathrm{argmax}_w \sum_i \log \frac{e^{w \cdot f(x_i, y_i)}}{\sum_{y' \in Y} e^{w \cdot f(x_i, y')}}$$

Multiclass LR Gradient

$$\frac{\partial \mathcal{L}}{\partial w_j} = \sum_{i=1}^{D} f_j(y_i, d_i) - \sum_{i=1}^{D} \sum_{y \in Y} f_j(y, d_i) P(y|d_i)$$

empirical feature count

expected feature count

(a.k.a) Maximum Entropy Classifier

- or MaxEnt

- Math proof of “LR=MaxEnt”:
  - [Klein and Manning 2003]
  - [Mount 2011]

http://www.win-vector.com/dfiles/LogisticRegressionMaxEnt.pdf

Perceptron Algorithm

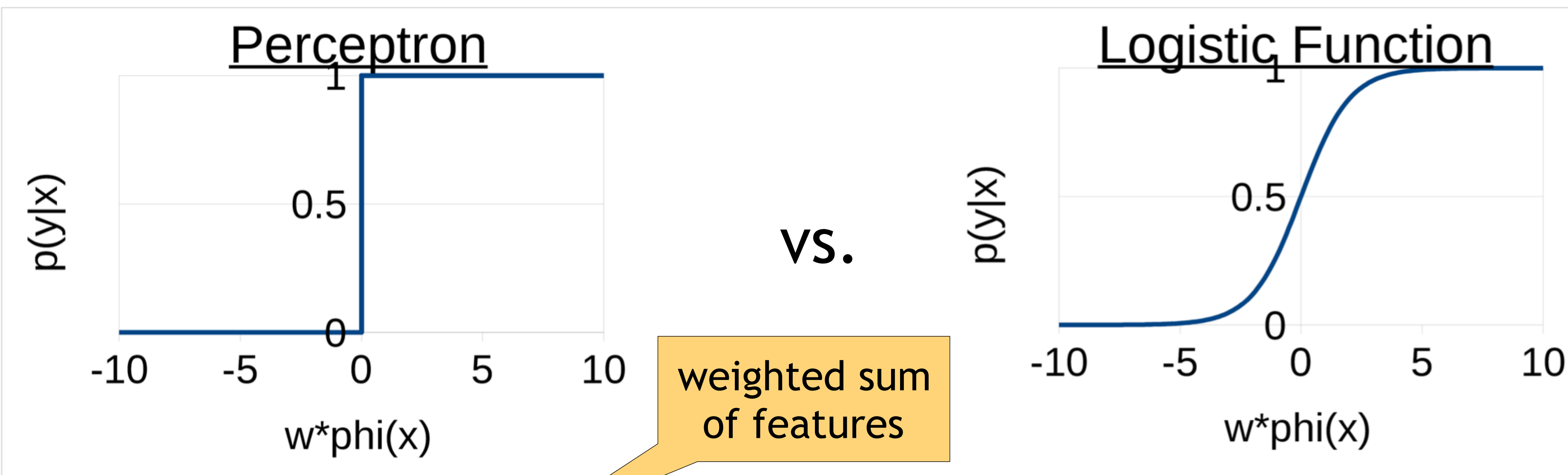
- Very similar to logistic regression
- Not exactly computing gradient

[Rosenblatt 1957]

http://www.peterasaro.org/writing/neural_networks.html

Perceptron Algorithm

- Very similar to logistic regression
- Not exactly computing gradient (simpler)



$$P(y=1|x)=1 \text{ if } w\cdot\varphi(x)\geq 0$$
$$P(y=1|x)=0 \text{ if } w\cdot\varphi(x)<0$$

$$P(y=1|x)=\frac{e^{w\cdot\varphi(x)}}{1+e^{w\cdot\varphi(x)}}$$

Perceptron vs. LR

- The Perceptron is an online learning algorithm.
- Standard Logistic Regression is not

Online Learning

- The Perceptron is an online learning algorithm.
- Logistic Regression is not:

this update is effectively the same as “w += y_i * x_i”

$$
\begin{aligned}
w_{\text{MLE}} &= \operatorname{argmax}_w \log P(y_1, \ldots, y_d | x_1, \ldots, x_d; w) \\
&= \operatorname{argmax}_w \sum_i y_i \log p_i + (1 - y_i) \log(1 - p_i)
\end{aligned}
$$

(Full) Batch Learning

- update parameters after each pass of training set

```
Initialize weight vector w = 0
Create features
Loop for K iterations
   Loop for all training examples x_i, y_i
        …
   update_weights(w)
```

Online Learning

- update parameters for each training example

```
Initialize weight vector w = 0
Create features
Loop for K iterations
    Loop for all training examples x_i, y_i
        …
        update_weights(w, x_i, y_i)
```

Perceptron Algorithm

- Very similar to logistic regression
- Not exactly computing gradient

features of a training example x

$$w \leftarrow w + y\,\varphi(x)$$

weights

label

If y = 1, increase the weights for features in $\varphi(x)$

If y = -1, decrease the weights for features in $\varphi(x)$

Perceptron Algorithm

- Very similar to logistic regression
- Not exactly computing gradient

```
Initialize weight vector w = 0
Loop for K iterations
    Loop For all training examples x_i
        if sign(w * x_i) != y_i        Error-driven!
            w += y_i * x_i
```

The Intuition

- For a given example, makes a prediction, then checks to see if this prediction is correct.

- If the prediction is correct, do nothing.
- If the prediction is incorrect, change its parameters so that it would do better on this example next time around.

Perceptron (vs. LR)

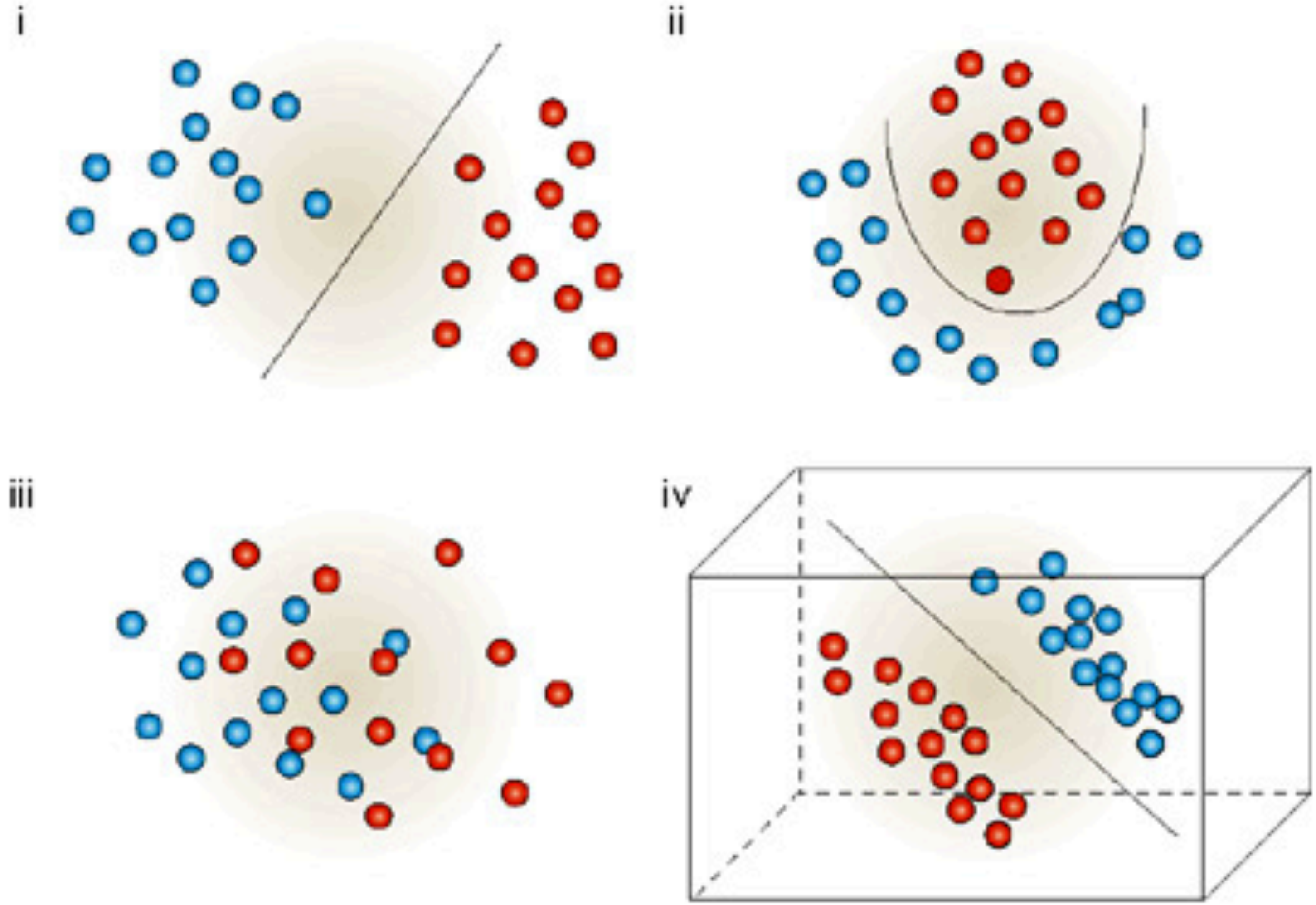
- Only hyperparameter is maximum number of iterations (LR also needs learning rate)

Perceptron (vs. LR)

- Only hyperparameter is maximum number of iterations (LR also needs learning rate)

- Guaranteed to converge if the data is linearly separable (LR always converge)

Linear Separability



What does “converge” mean?

- It means that it can make an entire pass through the training data without making any more updates.
- In other words, it has correctly classified every training example.
- Geometrically, this means that it was found some hyperplane that correctly segregates the data into positive and negative examples

What if non linearly separable?

- In real-world problem, this is nearly always the case.
- The perceptron will not be able to converge.

Q: Then, when to stop?