University Physics A(2) 2014

Worksheet #10: Entropy (2)

Name (名字): Student number (学号):

Problems Show all working.

$N = 1 \cdot N_A = 6.02 \times 10^{23}$ molecules

(1) [M&I.12.P.62] (a) You have a bottle containing a mole of a monatomic gas such as helium (He) or neon (Ne). You warm up this monatomic gas with an electrical heater, which inputs Q = 580 J of energy. How much does the temperature of the gas increase?

Degrees of freedom: $\underbrace{\rightarrow_x + \uparrow_y + \swarrow_z}_{\text{translation} \times 3} \Rightarrow \bar{E} = \frac{3}{2}k_BT \quad \therefore C_v = \frac{3}{2}k_B$

$$\Delta E = NC_v\Delta T \rightarrow \Delta T = \frac{\Delta E}{NC_v} = \frac{Q}{\frac{3}{2}N_Ak_B} = \frac{2}{3}\frac{Q}{R} = \frac{2}{3}\left(\frac{580}{8.3}\right) = \boxed{47\text{ K}}$$

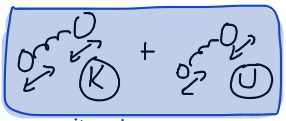
(b) You have a bottle containing a mole of a diatomic gas such as nitrogen (N_2) or oxygen (O_2). The initial temperature is in the range where many rotational levels are excited, but no vibrational levels are excited. You warm up this diatomic gas with an electrical heater, which inputs Q = 580 J of energy. How much does the temperature of the gas increase?

Degrees of freedom: $\underbrace{\rightarrow_x + \uparrow_y + \swarrow_z}_{\text{translation} \times 3} + \underbrace{\circlearrowright + \circlearrowleft}_{\text{rotation} \times 2} \Rightarrow \bar{E} = \frac{5}{2}k_BT \quad \therefore C_v = \frac{5}{2}k_B$

$$\Delta T = \frac{\Delta E}{NC_v} = \frac{Q}{\frac{5}{2}N_Ak_B} = \frac{2}{5}\frac{Q}{R} = \frac{2}{5}\left(\frac{580}{8.3}\right) = \boxed{28\text{ K}}$$

(c) You have a bottle containing a mole of a diatomic gas such as nitrogen (N_2) or oxygen (O_2). The initial temperature is in the range where many rotational levels *and* vibrational levels are excited. You warm up this diatomic gas with an electrical heater, which inputs Q = 580 J of energy. How much does the temperature of the gas increase?

Degrees of freedom: $\underbrace{\rightarrow_x + \uparrow_y + \swarrow_z}_{\text{translation} \times 3} + \underbrace{\circlearrowright + \circlearrowleft}_{\text{rotation} \times 2}$ +



vibration ×2

$\Rightarrow \bar{E} = \frac{7}{2}k_BT$

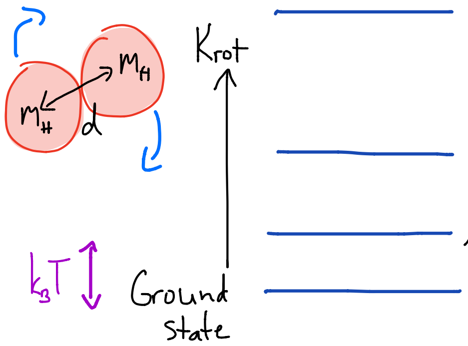
$\therefore C_v = \frac{7}{2}k_B$

$$\Delta T = \frac{\Delta E}{NC_v} = \frac{Q}{\frac{7}{2}N_Ak_B} = \frac{2}{7}\frac{Q}{R} = \frac{2}{7}\left(\frac{580}{8.3}\right) = \boxed{20\text{ K}}$$

(2) [M&I.7.P.66] It is possible to estimate some properties of a diatomic molecule from the temperature dependence of the specific heat capacity.

(a) Below about 80 K, the specific heat capacity at constant volume for hydrogen gas (H_2) is $(3/2)k_B$ per molecule, but at higher temperatures the specific heat capacity increases to $(5/2)k_B$ per molecule due to contributions from rotational energy states. Use these observations to estimate the distance between the hydrogen nuclei in an H_2 molecule.

$$I = 2 \times m_H \left(\frac{d}{2}\right)^2 = \frac{1}{2} m_H d^2$$

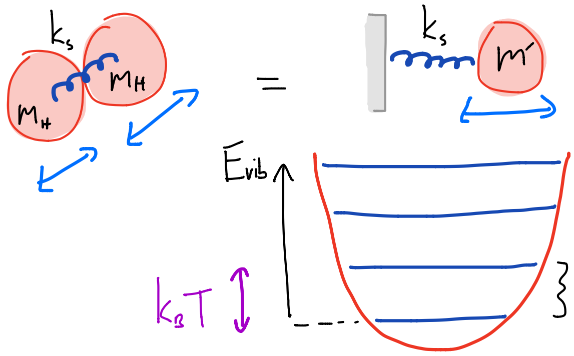


Angular momentum is quantized: $L_n = n\hbar \quad (n = 1, 2, ..)$

$$\Delta K_{rot,1} = \frac{L_1^2}{2I} = \frac{\hbar^2}{2I} = \frac{\hbar^2}{m_H d^2}$$

At 80K, $\boxed{k_B T \simeq K_{rot,1}} \rightarrow \frac{\hbar^2}{m_H d^2} \simeq k_B T \quad \therefore \; d \simeq \hbar \sqrt{\frac{1}{m_H k_B T}} = \boxed{7 \times 10^{-11}\,m}$ $(0.7\,Å)$

(b) At about 2000 K, the specific heat capacity at constant volume for hydrogen gas (H_2) increases to $(7/2)k_B$ per molecule due to contributions from vibrational energy states. Use these observations to estimate the stiffness of the "spring" that approximately represents the interatomic force.



$$m' = \frac{m_1 m_2}{m_1 + m_2} = \frac{m_H^2}{2 m_H} = \frac{1}{2} m_H$$

$$\Delta E_{vib} = \hbar\omega = \hbar\sqrt{\frac{k_s}{m'}} = \hbar\sqrt{\frac{2k_s}{m_H}}$$

At 2000 K, $\boxed{k_B T \simeq \Delta E_{vib}} \rightarrow \hbar\sqrt{\frac{2k_s}{m_H}} \simeq k_B T$

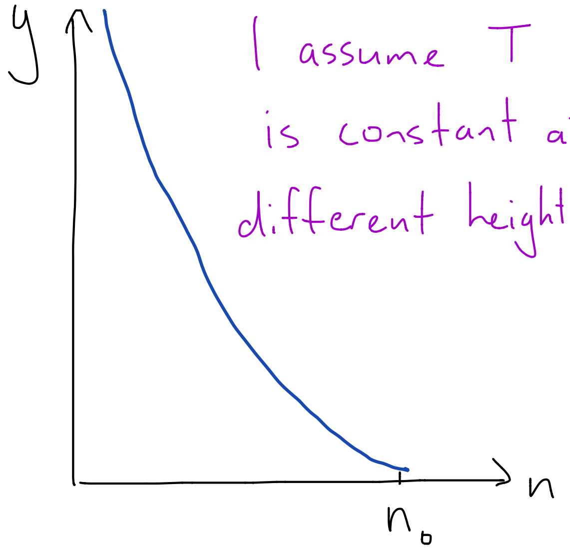
$$\therefore \; k_s \simeq \frac{m_H}{2}\left(\frac{k_B T}{\hbar}\right)^2 = \boxed{65\,N/m}$$

(3) [M&I.P.67] In 1988, telescopes viewed Pluto (冥王星) as it crossed in front of a distant star. As the star came out from behind Pluto, light from the star was slightly dimmed as it went through Pluto's atmosphere (大气层). The observations indicated that the atmospheric density at a height of 50 km above the surface of Pluto is about one-third the density at the surface.

The mass of Pluto about 1.5×10^{22} kg and its radius is about 1200 km. Spectroscopic data (光谱数据) indicate that the atmosphere is mostly nitrogen (N_2 – one mole of N_2 is 28 grams). Estimate the temperature of Pluto's atmosphere.

From the Boltzmann distribution, we know

$$\boxed{n(y) = n_0 e^{-\frac{mgy}{k_B T}}}$$



I assume T is constant at different heights.

$$e^{-mgy/k_B T} = \frac{n}{n_0}$$

$$\frac{-mgy}{k_B T} = \ln \frac{n}{n_0}$$

$$T = \frac{-mgy}{k_B \ln \frac{n}{n_0}} = \frac{mgy}{k_B \ln \frac{n_0}{n}}$$

On Pluto, $g = \frac{GM_p}{R_p^2} = \frac{(6.7\times10^{-11})(1.5\times10^{22})}{(1.2\times10^{6})^2} = 0.7 \text{ m/s}^2.$

and m = mass of one N_2 molecule $= \frac{0.028 \text{ kg/mol}}{6\times10^{23} \text{ molecules/mol}} = 4.7\times10^{-26}$ kg

At $y = 50 \text{ km} = 5\times10^4 \text{ m}$, $\frac{n_0}{n} = 3$:

$$\therefore T \simeq \frac{(4.7\times10^{-26})(0.7)(5\times10^4)}{(1.4\times10^{-23})\ln 3} = \boxed{106 \text{ K}}$$