Research Institute for Future Media Computing
未来媒体技术与研究所

Institute of Computer Vision
计算机视觉研究所

多媒体系统导论

Fundamentals of Multimedia System

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2022年春季课程

Outline of Lecture 08

- Introduction-简介
- Distortion Measures-失真度量
- The Rate-Distortion Theory-比率失真理论
- Quantization-量化
  - Uniform Scalar Quantization-均匀标量量化
  - Nonuniform Scalar Quantization-非均匀标量量化
  - Vector Quantization-向量量化
- Transform Coding-变换编码
- Wavelet-Based Coding-小波编码
- Experiments-实验

Introduction-简介

- What is lossy compression-什么是有损编码

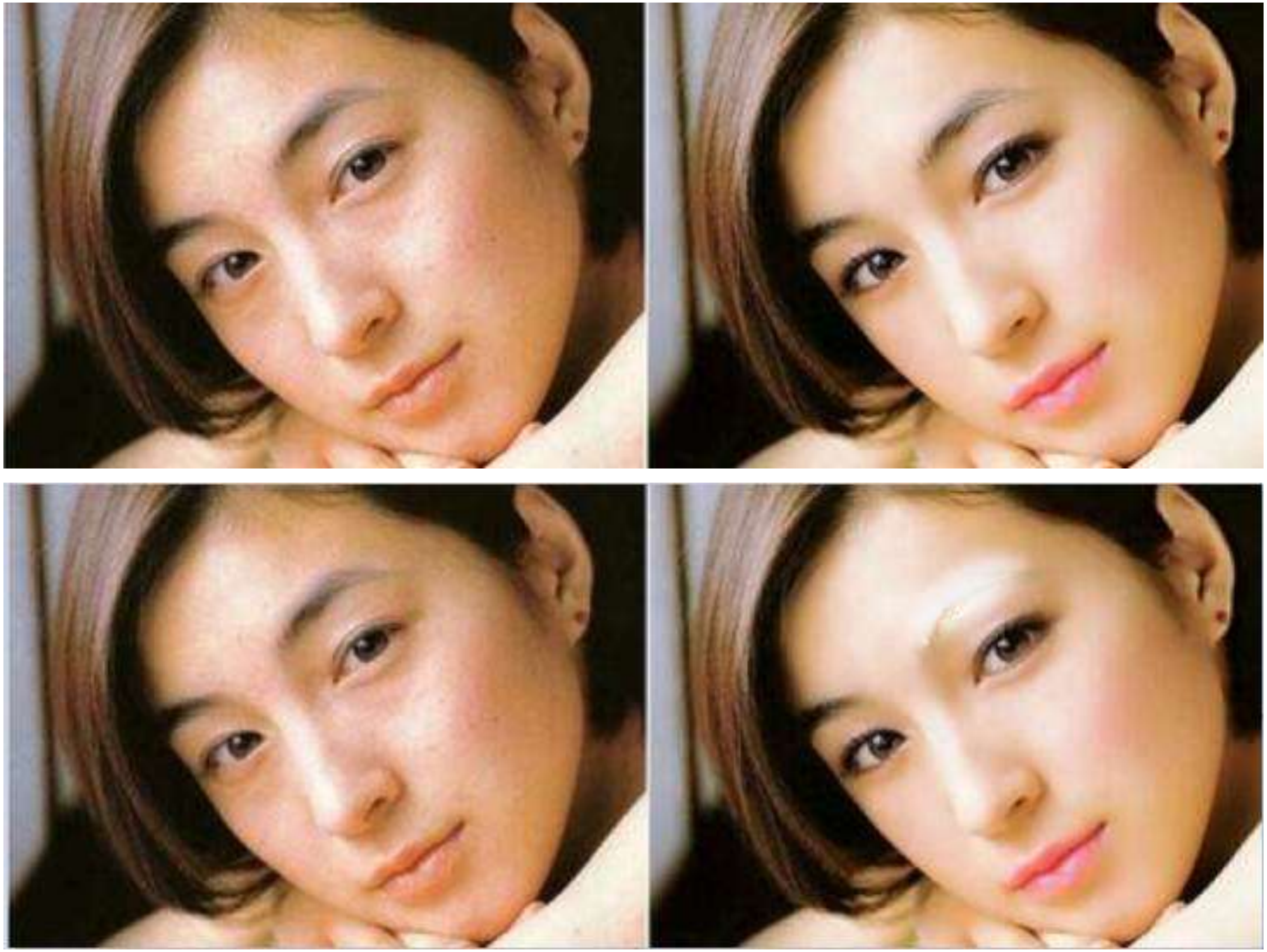
Compression program	Compression ratio			
	Lena	Football	F-18	Flowers
Lossless JPEG	1.45	1.54	2.29	1.26
Optimal lossless JPEG	1.49	1.67	2.71	1.33
compress (LZW)	0.86	1.24	2.21	0.87
gzip (LZ77)	1.08	1.36	3.10	1.05
gzip-9 (optimal LZ77)	1.08	1.36	3.13	1.05
pack (Huffman coding)	1.02	1.12	1.19	1.00

Introduction-简介

- What is lossy compression-什么是有损编码
  - Lossless compression algorithms do not deliver compression ratios that are high enough. Hence, most multimedia compression algorithms are *lossy*-无损编码压缩率不高，多媒体应用需较高压缩率.
  - **Lossy Compression**: The compressed data is not the same as the original data, but a *close approximation* of it-不完全相同，但感知上近似.
  - Yields a much *higher compression ratio* than that of lossless compression-较高压缩率.

Introduction-简介

- What is lossy compression-什么是有损编码



Distortion Measures-失真度量

- Distortion measures in image
  - A *distortion measure* is a mathematical quantity that specifies *how close* an approximation is to its original, using some distortion criteria-近似程度的数学量化.
  - **Mean square error (MSE) –**均方差(平均像素差异):

原始值

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - y_i)^2$$

解压值

  - **Signal to Noise Rate (SNR)-**信噪比:

原数据均方

$$SNR = 10\log_{10}\frac{\sigma_x^2}{\sigma_d^2}$$

均方差

  - **Peak signal to noise ratio (PSNR)-**峰值信噪比:

$$PSNR = 10\log_{10}\frac{x_{peak}^2}{\sigma_d^2}$$

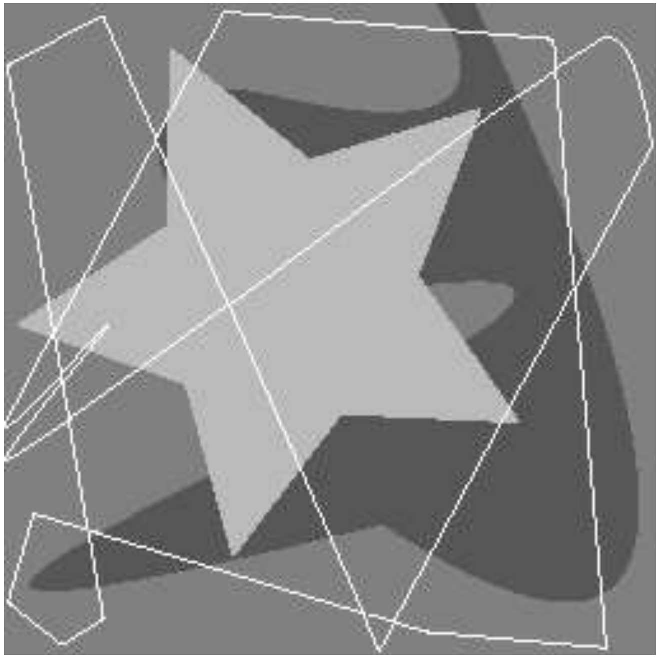
原始数据最大值

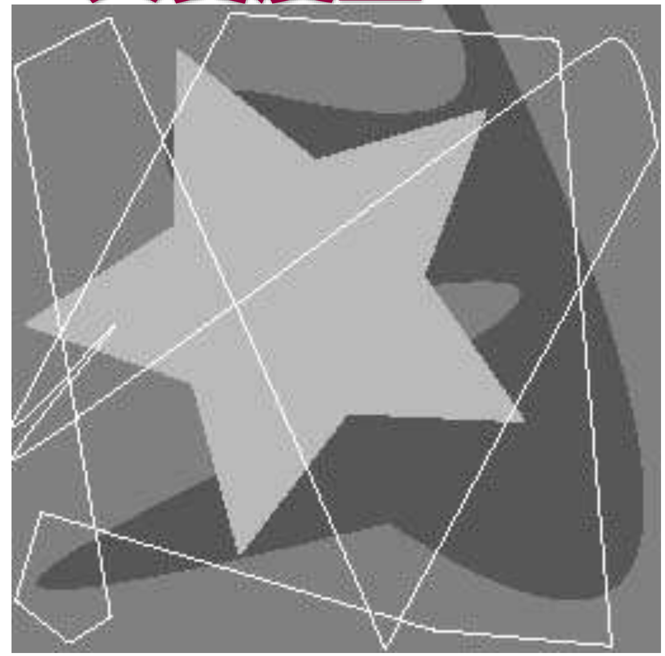
Distortion Measures-失真度量

- Distortion measures in image
  - 例：已知原数据和经过压缩与解压缩之后得到的数据为
  - 原始数据：{12 12 12 12 12 8 8 12 }
  - 处理数据：{12 12 12 8 12 8 12 12}。
  - 请分别计算MSE、SNR和PSNR的值 。
  - $\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - y_i)^2$, 1/8*[(12-8)²+(8-12)²]=4
  - $SNR = 10\log_{10}\frac{\sigma_x^2}{\sigma_d^2}$, 10*$\log_{10}$[(6*12²+2*8²)/8/4]≈ 14.91
  - $PSNR = 10\log_{10}\frac{x_{peak}^2}{\sigma_d^2}$, 10*$\log_{10}$[12²/4]≈ 15.563

Distortion Measures-失真度量

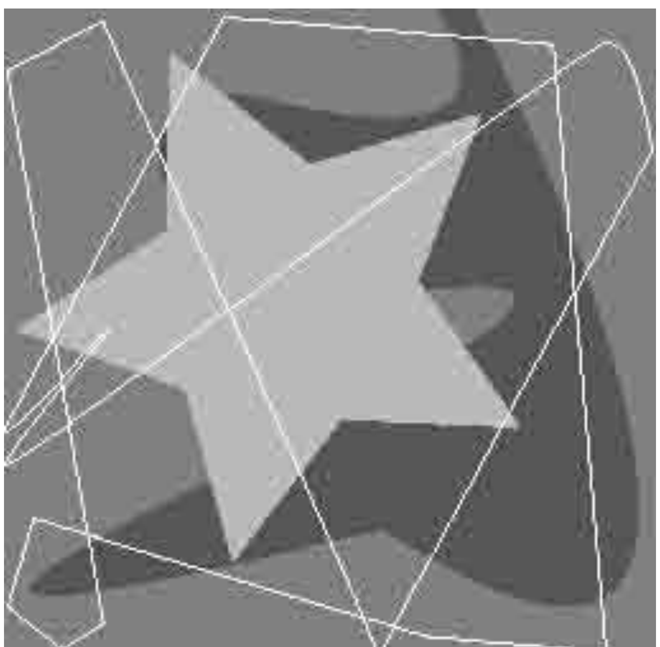
原图像

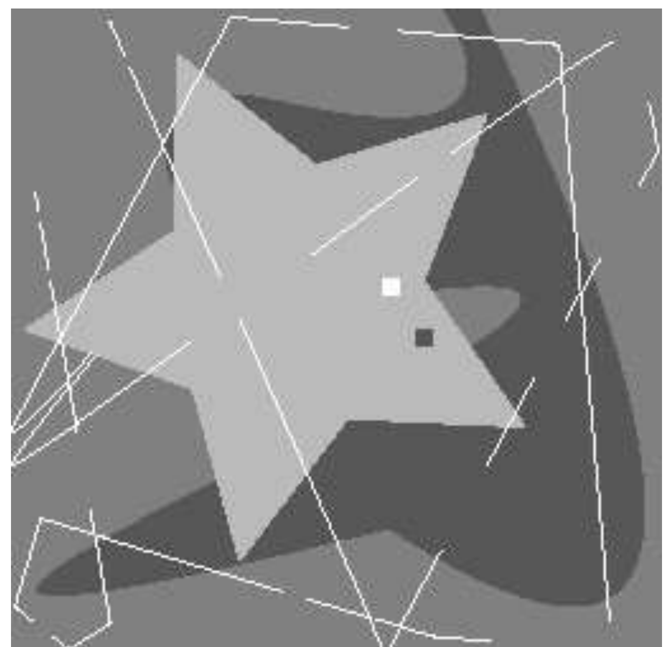




均方根误差
5.17

均方根误差
15.67

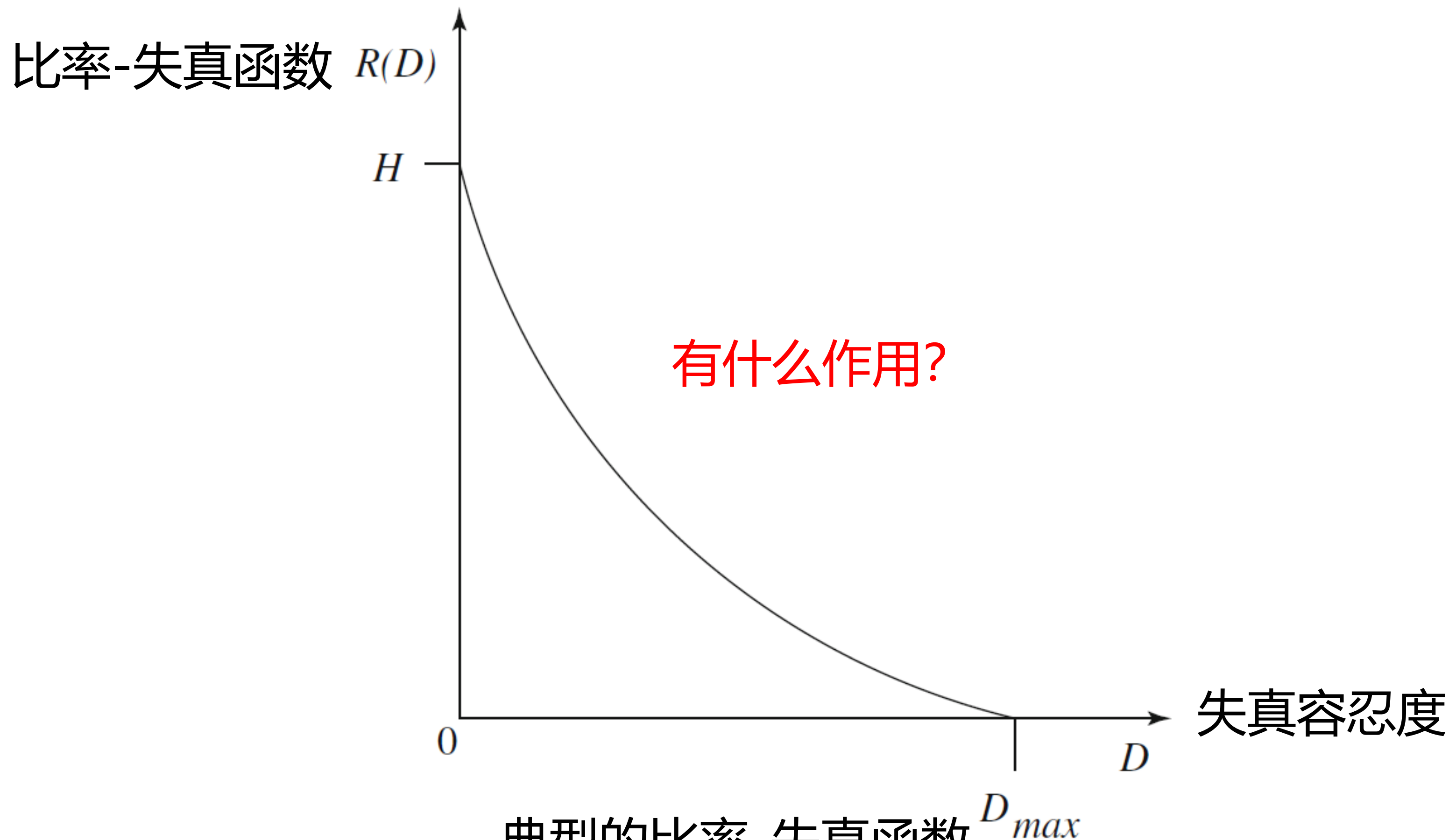




均方根误差
14.17

The Rate-Distortion Theory-比率失真理论

- Tradeoffs between Rate and Distortion-平衡
  - Rate is the average number of bits required to represent each source symbol-位数*vs*失真.



典型的比率-失真函数

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Quantization-量化

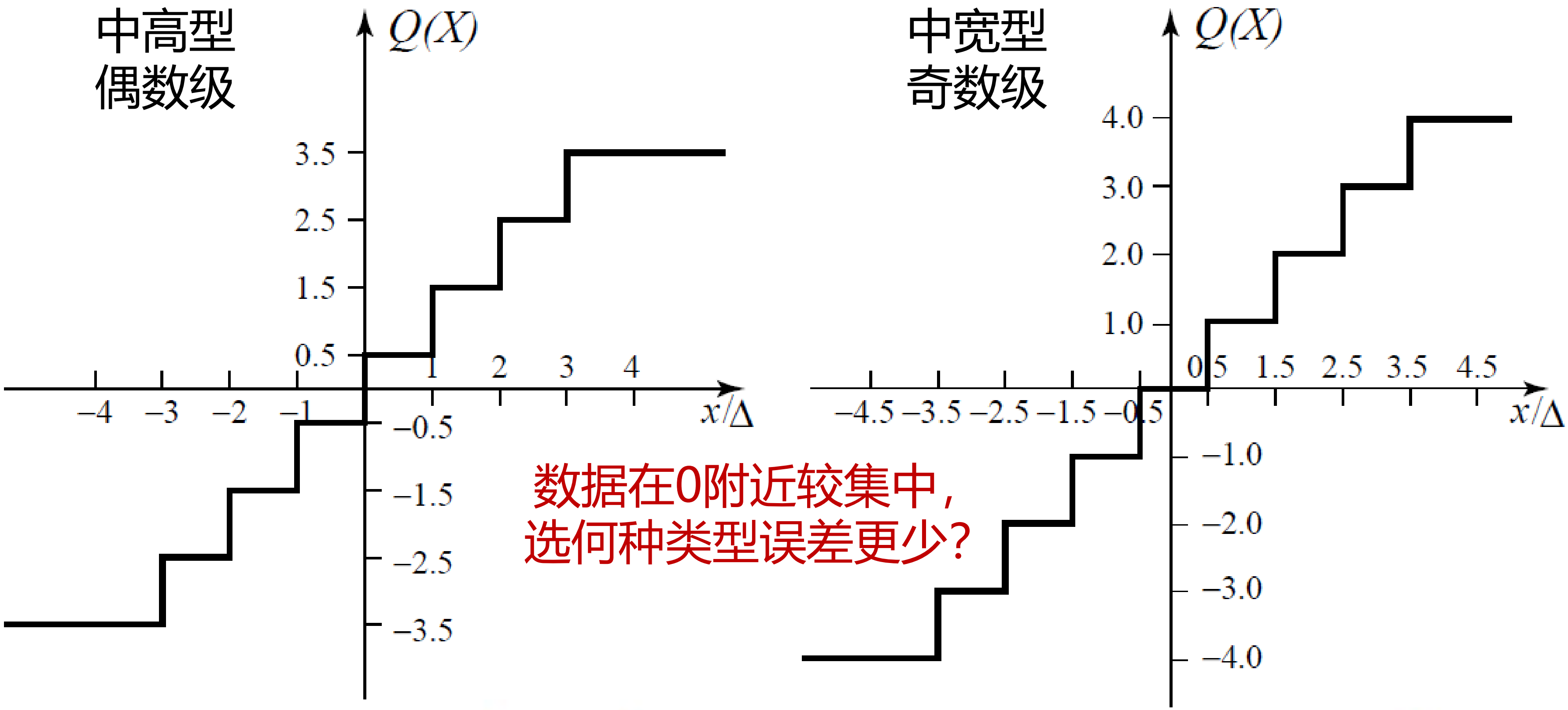
- Further understanding quantization
  - Quantization is the heart of any lossy scheme-量化是任何有损方案的核心.
  - Reduce the number of distinct output values to a much smaller set-不同输出值的数量减少到一个更小的集合.
  - Main source of the “loss” in lossy compression-信息损失主要来源.
  - f: {0, 1, 2, ...32, ..., 64, ..., 94, ..., 128, ..., 192, ..., 255}
  - f' : {0, 64, 128, 192, 255}
  - Uniform/nonuniform scalar quantization-均匀/非均匀
  - Vector quantization-向量量化(LZW编码)

Quantization-量化

- Uniform Scalar Quantization-均匀标量量化
  - A uniform scalar quantizer partitions the domain of input values into equally spaced intervals, except possibly at the two outer intervals-输入值的域划分为等间隔的区间.
  - The endpoints of partition intervals are called the quantizer's decision boundaries-区间端点称决策边界.
  - The length of each interval is referred to as the *step size*, denoted by the symbol Δ-区间长短称为步长.
  - Two types of uniform scalar quantizers: *Midrise* and *Midtread* Quantizers-中高型和中宽型均匀量化器
  - The goal for the design of a successful uniform quantizer is to **minimize the distortion** for a given source input with a desired number of output values by adjusting the *step size* Δ to match the input -调整步长，匹配输入，最小化失真.

Quantization-量化

- Uniform Scalar Quantization-均匀标量量化



数据在0附近较集中，
选何种类型误差更少？

$$Q_{\text{midrise}}(x) = \lceil x \rceil - 0.5$$

(a)

$$Q_{\text{midtread}}(x) = \lfloor x + 0.5 \rfloor$$

(b)

Uniform Scalar Quantizers: (a) Midrise, (b) Midtread

Quantization-量化

◆ Uniform Scalar Quantization-均匀标量量化

- Performance of an M level quantizer. Let $B = \{b_0, b_1, \dots, b_M\}$ be the set of *decision boundaries* and $Y = \{y_1, y_2, \dots, y_M\}$ be the set of *output values*.
- Suppose the input is uniformly distributed in the interval $[-X_{max}, X_{max}]$. The rate of the quantizer is:

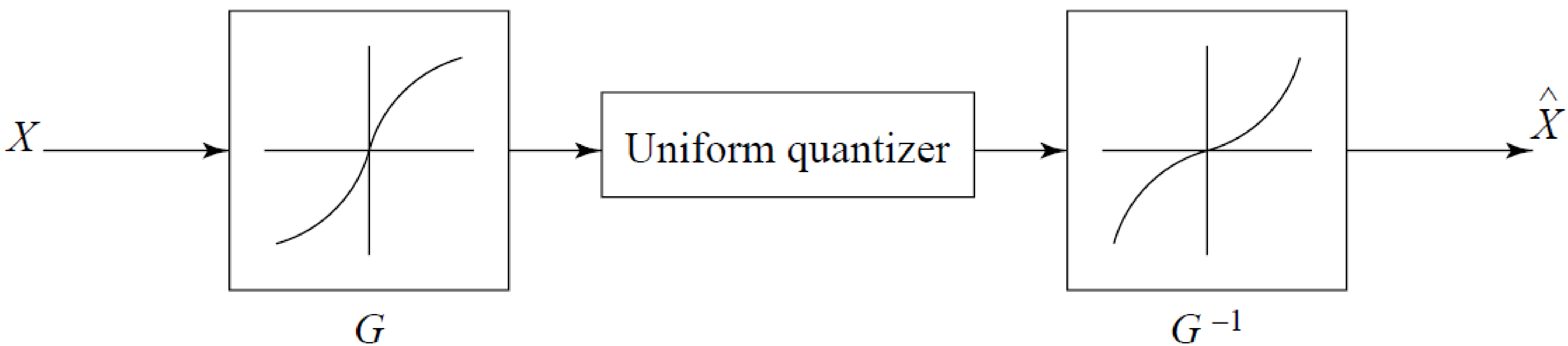
$$R = \lceil logM \rceil$$

- If the quantizer is n bits, $M = 2^n$, then:

$$\begin{aligned} SQNR &= 10\log_{10}\left(\frac{\sigma_x^2}{\sigma_d^2}\right) \\ &= 10\log_{10}\left(\frac{(2X_{\max})^2}{12}\cdot\frac{12}{\Delta^2}\right) \\ &= 10\log_{10} M^2 = 20n\ \log_{10} 2 \\ &= 6.02n\ \text{(dB)}. \end{aligned}$$

Quantization-量化

- Nonuniform Scalar Quantization-非均匀标量量化
  - *Nonuniform quantization* is **nonlinear**-非线性
  - *Companded quantization*: a *compander* consists of a compressor function G, a uniform quantizer, and an expander function G^{-1}-压缩扩展量化器



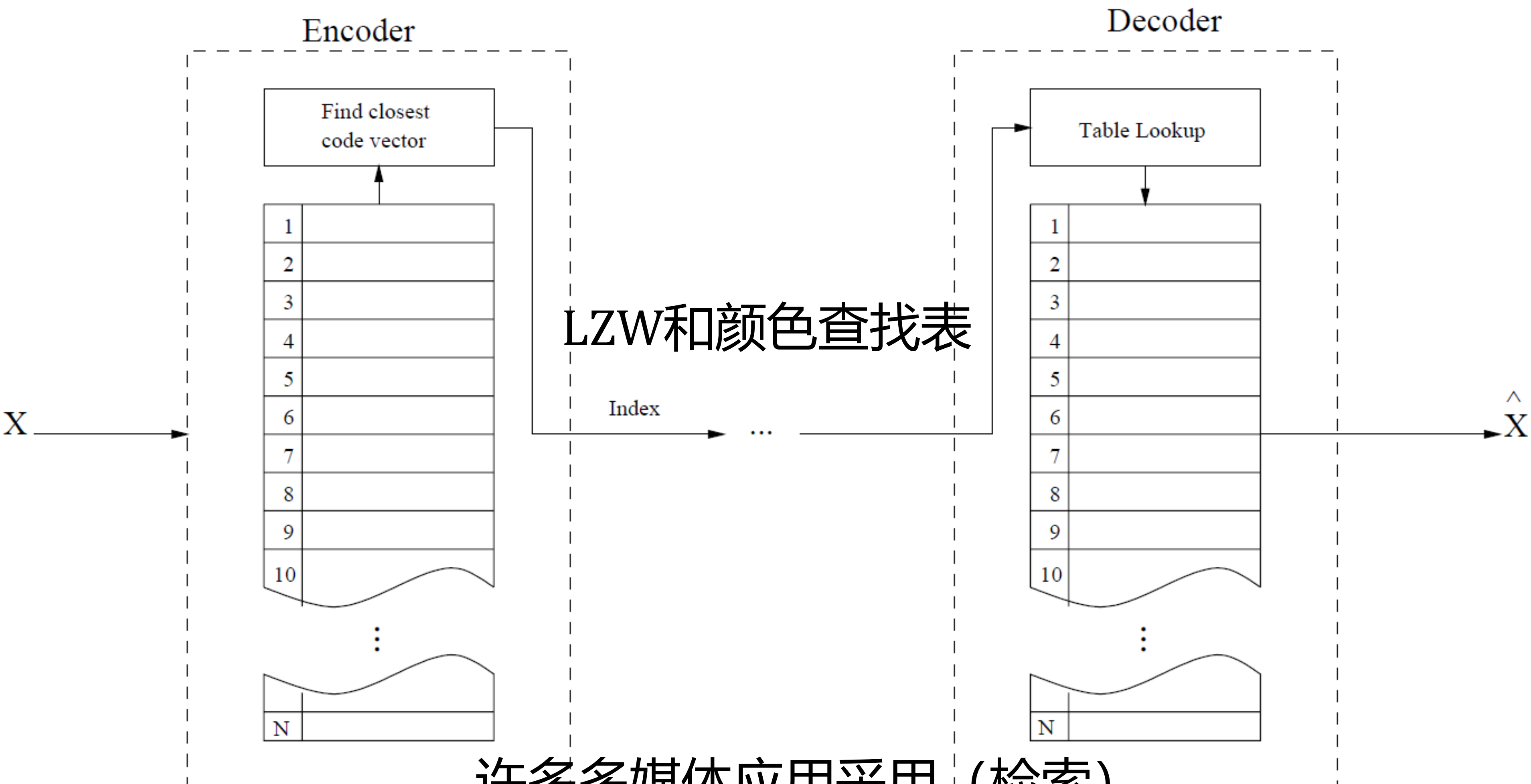
μ律和A律压缩扩展器

Quantization-量化

- Vector Quantization-向量量化
  - According to Shannon's original work on information theory, any compression system performs better if it operates on *vectors* or *groups of samples* rather than individual symbols or samples-非单个样本更好.
  - Form vectors of input samples by simply concatenating a number of consecutive samples into a single vector-一系列样本形成向量.
  - Instead of single reconstruction values as in scalar quantization, in VQ code vectors with n components are used. A collection of these code vectors form the codebook-向量量化码有n个分量，向量码的集合形成码本.

Quantization-量化

◆ Vector Quantization-向量量化



LZW和颜色查找表

许多多媒体应用采用（检索）

Basic vector quantization procedure

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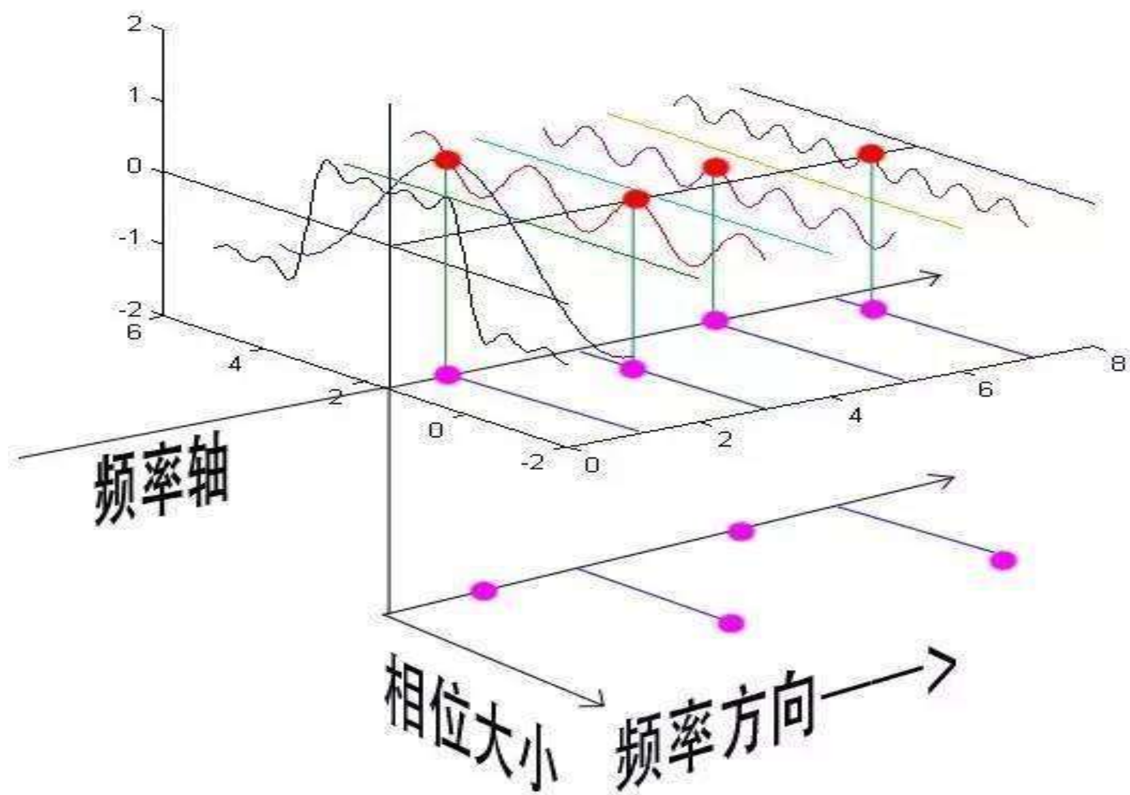
Transform Coding-变换编码

- The rationale behind transform coding-动机
  - Coding vectors is more efficient than coding scalar-向量编码更有效.
  - If Y is the result of a linear transform T of the input vector X, i.e. $Y=T(X)$ in such a way that the components of Y are much *less correlated*, then Y can be coded more efficiently than X-对线性变换T后的Y进行编码比原数据X更有效——变换编码原理.
  - If most information is accurately described by **the first few components** of a transformed vector, then the remaining components can be coarsely quantized, or even **set to zero**, with little signal distortion-大部分信息集中在前几个分量，其它分量可能更粗精度量化，甚至置0.

Transform Coding-变换编码

- The rationale behind transform coding-动机
  - An example: DFT-离散Fourier变换.



https://www.bilibili.com/video/BV1pW411J7s8?from=search&seid=15953141356053172266

Transform Coding-变换编码

- The rationale behind transform coding-动机
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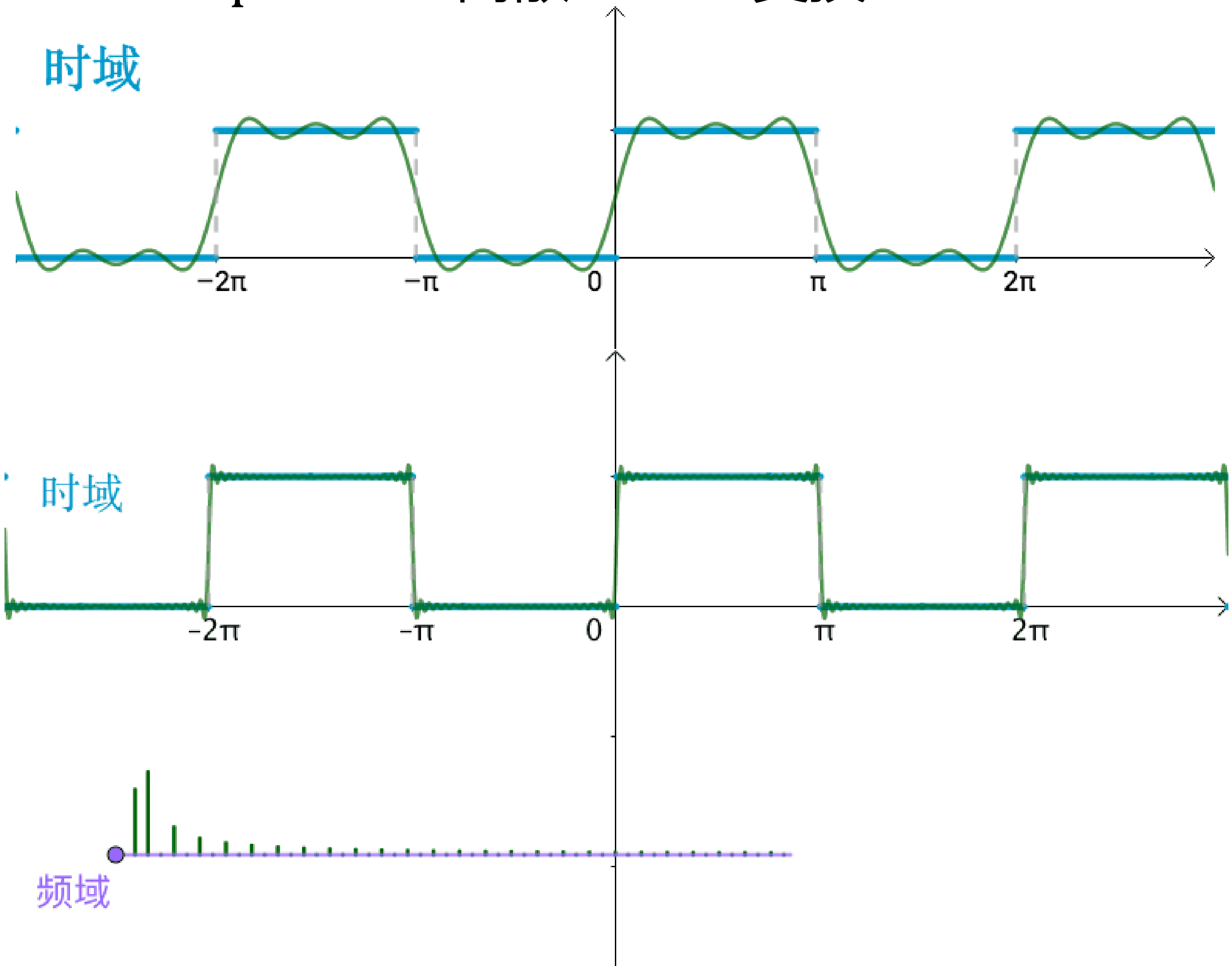


Transform Coding-变换编码

- Discrete Cosine Transform (DCT)-离散余弦变换
  - **Spatial frequency** indicates how many times pixel values change across an image block-图像块像素值变化通过空间频率反映.
  - The DCT formalizes this notion with a measure of how much the *image contents change* in correspondence to **the number of cycles of a cosine wave** per block-余弦函数反映图像内容变化.
  - The role of the DCT is to decompose the original signal into its **DC** and **AC** components; the role of the IDCT is to reconstruct (re-compose) the signal- DCT分解原始信号为直流DC分量和交流AC分量， IDCT为逆变换.

Transform Coding-变换编码

- Definition of DCT-离散余弦变换定义
  - Given an input function $f(i,j)$ over two integer variables i and j (a piece of an image), the 2D DCT transforms it into a new function $F(u,v)$, with integer u and v running over the same range as i and j. The general definition of the transform is-图像像素$f(i,j)$:

$$F(u,v)=\frac{2C(u)C(v)}{\sqrt{MN}}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\cos\frac{(2i+1)\cdot u\pi}{2M}\cdot\cos\frac{(2j+1)\cdot v\pi}{2N}\cdot f(i,j)$$

$$i,u=0,1,\ldots M-1, j,v=0,1,\ldots N-1,$$

$$C(\xi)=\begin{cases}\frac{\sqrt{2}}{2}, & \xi=0\\ 1, & others\end{cases}$$

Transform Coding-变换编码

- 2D DCT and IDCT-二维离散余弦变换及逆变换
  - In the JPEG image compression standard (see Chap. 9), an image block is defined to have dimension $M = N = 8$. Therefore, the definitions for the 2D DCT and its inverse (IDCT) are as follows-JPEG图像压缩二维DCT和IDCT:

$$F(u,v) = \frac{C(u)C(v)}{4} \sum_{i=0}^{7} \sum_{j=0}^{7} \cos\frac{(2i+1)\cdot u\pi}{16} \cdot \cos\frac{(2j+1)\cdot v\pi}{16} \cdot f(i,j)$$

$$\tilde{f}(i,j) = \sum_{u=0}^{7} \sum_{v=0}^{7} \frac{C(u)C(v)}{4} \cdot \cos\frac{(2i+1)\cdot u\pi}{16} \cdot \cos\frac{(2j+1)\cdot v\pi}{16} \cdot F(u,v)$$

$$i,u = 0,1,\ldots 7, j,v = 0,1,\ldots 7, C(\xi) = \begin{cases} \frac{\sqrt{2}}{2}, & \xi = 0 \\ 1, & others \end{cases}$$

Transform Coding-变换编码

- 1D DCT and IDCT-一维离散余弦变换及逆变换
  - **1D Discrete Cosine Transform** (1D DCT)-一维DCT

$$F(u)=\frac{C(u)}{2}\sum_{i=0}^{7}\cos\frac{(2i+1)\cdot u\pi}{16}\cdot f(i),\ i,u=0,1,\ldots 7$$

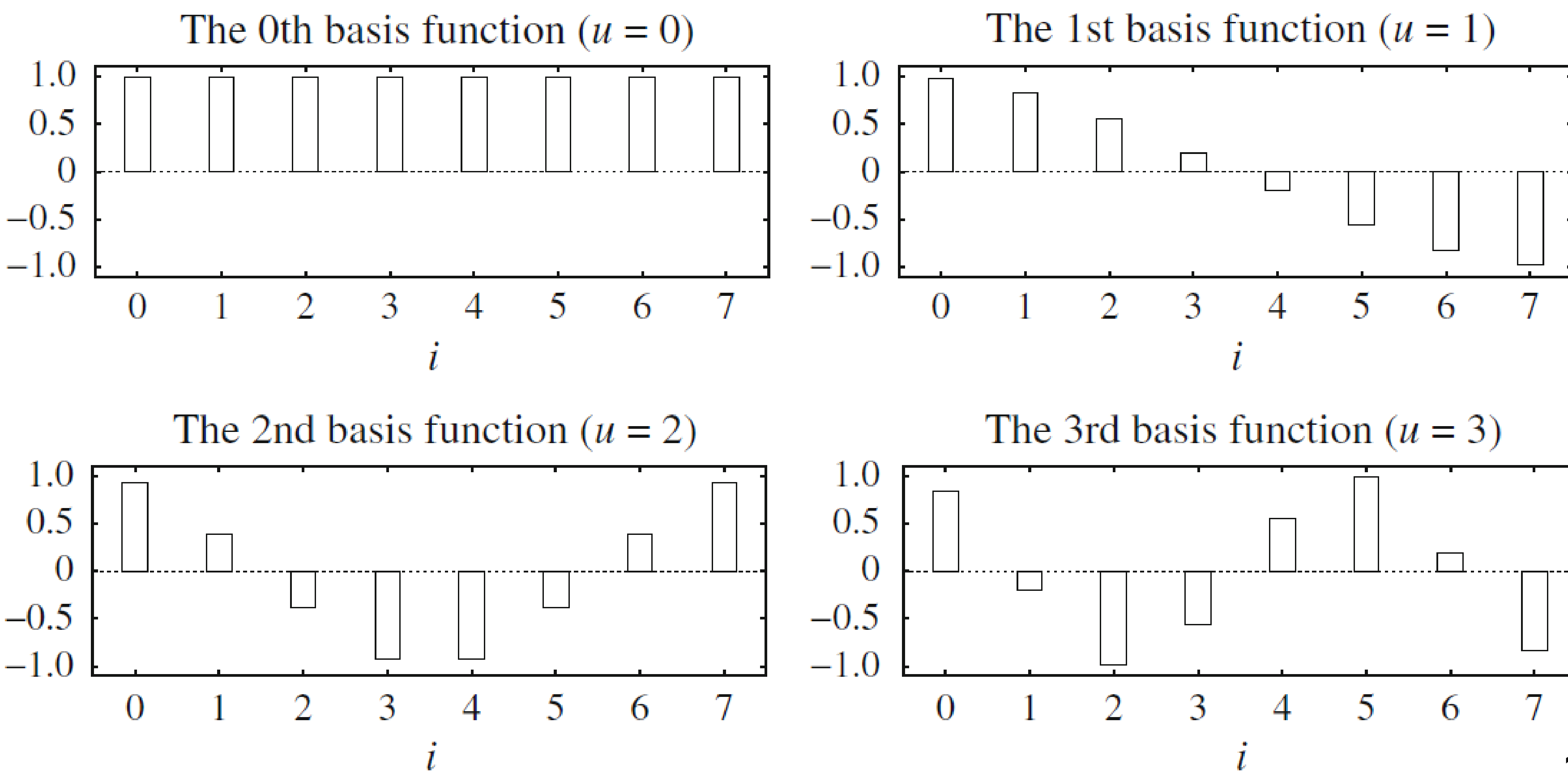
  - 1D Inverse Discrete Cosine Transform (1D IDCT)-一维逆DCT

$$\tilde{f}(i)=\sum_{u=0}^{7}\frac{C(u)}{2}\cdot\cos\frac{(2i+1)\cdot u\pi}{16}\cdot F(u),\ i,u=0,1,\ldots 7$$

Transform Coding-变换编码

- 1D DCT basis functions-一维DCT基函数

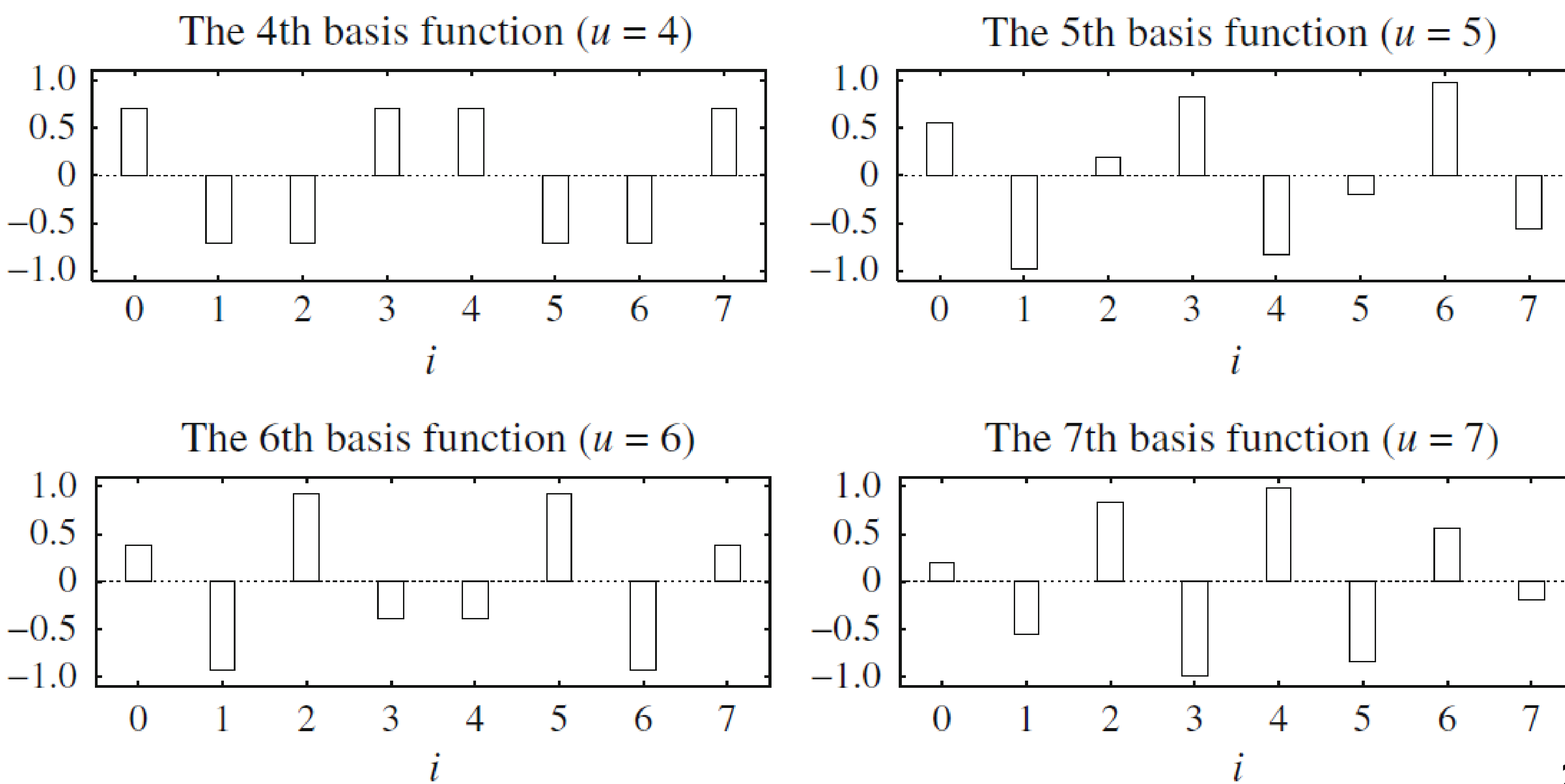
$$F(u) = \frac{C(u)}{2} \sum_{i=0}^{7} \cos \frac{(2i+1) \cdot u\pi}{16} \cdot f(i),\ i, u = 0,1, \ldots 7$$



Transform Coding-变换编码

- 1D DCT basis functions-一维DCT基函数

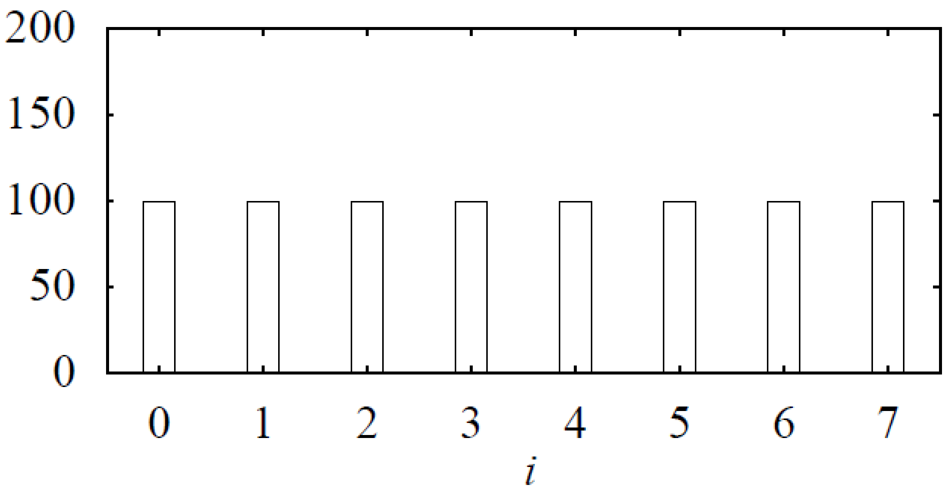
$$F(u) = \frac{C(u)}{2}\sum_{i=0}^{7}\cos\frac{(2i+1)\cdot u\pi}{16}\cdot f(i),\ i,u = 0,1,\dots 7$$

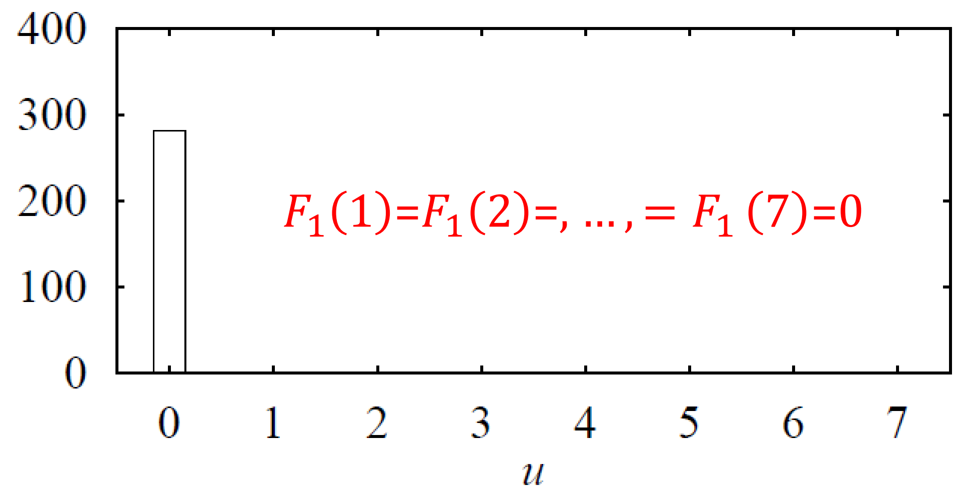


Transform Coding-变换编码

◆ 1D DCT -一维离散余弦变换

- Example: f_1={100, 100, 100, 100, 100, 100, 100, 100}





$$F(u) = \frac{C(u)}{2}\sum_{i=0}^{7}\cos\frac{(2i+1)\cdot u\pi}{16}\cdot f(i)$$

$$F_1(0) = \frac{\sqrt{2}}{2}\cdot\frac{1}{2}(1\cdot 100 + 1\cdot 100 + 1\cdot 100 + 1\cdot 100 + 1\cdot 100 + 1\cdot 100 + 1\cdot 100 + 1\cdot 100)$$
$$\approx 283$$

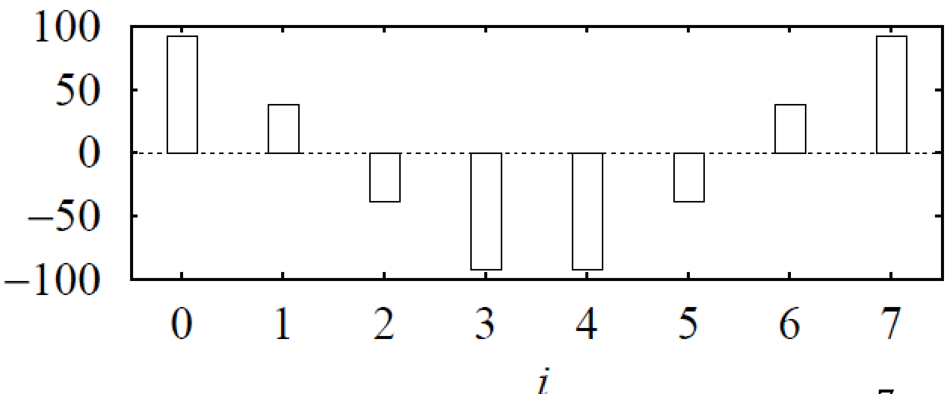
$$F_1(1) = \frac{1}{2}\ (\cos\frac{\pi}{16}\cdot 100 + \cos\frac{3\pi}{16}\cdot 100 + \cos\frac{5\pi}{16}\cdot 100 + \cos\frac{7\pi}{16}\cdot 100 + \cos\frac{9\pi}{16}\cdot 100 + \cos\frac{11\pi}{16}\cdot 100 + \cos\frac{13\pi}{16}\cdot 100 + \cos\frac{15\pi}{16}\cdot 100) = 0$$

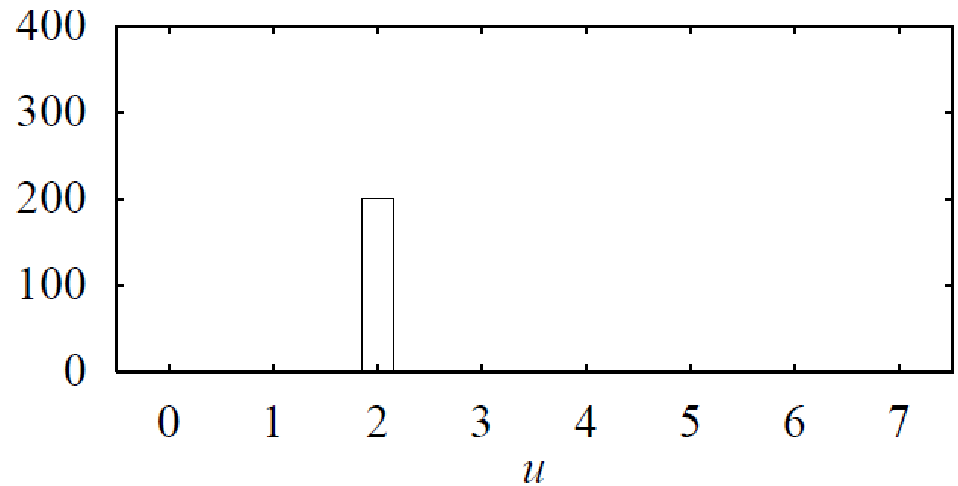
Transform Coding-变换编码

◆ 1D DCT-一维离散余弦变换

- Example: f_2

包含AC分量的变化信号





$$F(u) = \frac{C(u)}{2} \sum_{i=0}^{7} \cos \frac{(2i+1) \cdot u\pi}{16} \cdot f(i)$$

$F_2(0) = 0$

$F_2(2) = 200$

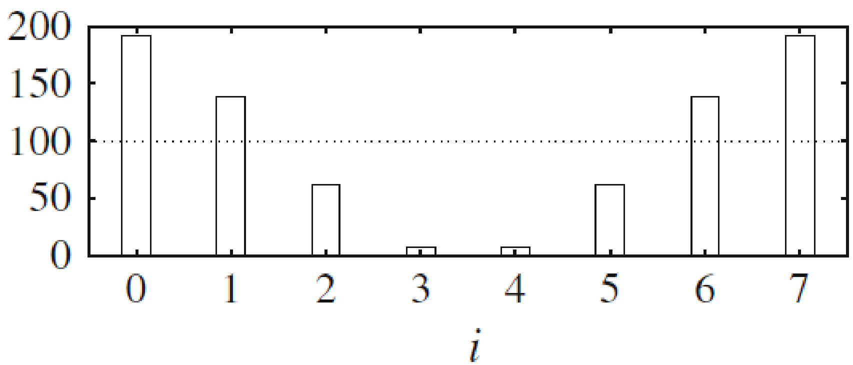
$F_2(1) = F_3(3) = \ldots = F_3(7) = 0$

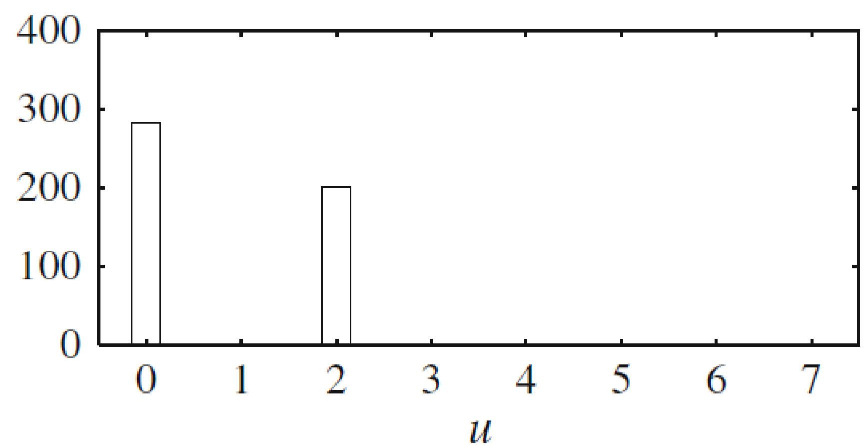
Transform Coding-变换编码

◆ 1D DCT-一维离散余弦变换

- Example: $f_3 = f_1 + f_2$

包含DC+AC分量的变化信号





$$F(u) = \frac{C(u)}{2} \sum_{i=0}^{7} \cos \frac{(2i+1) \cdot u\pi}{16} \cdot f(i)$$

$F_3(0) \approx 283$

$F_3(2) = 200$

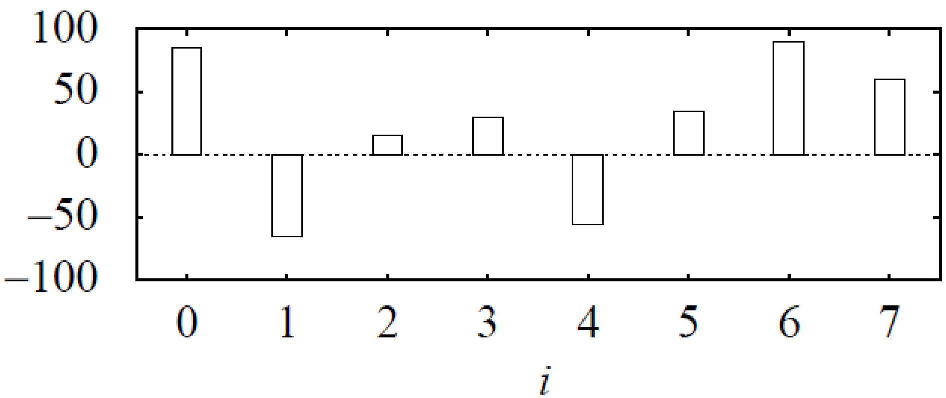
$F_3(1) = F_3(3) = \ldots = F_3(7) = 0$

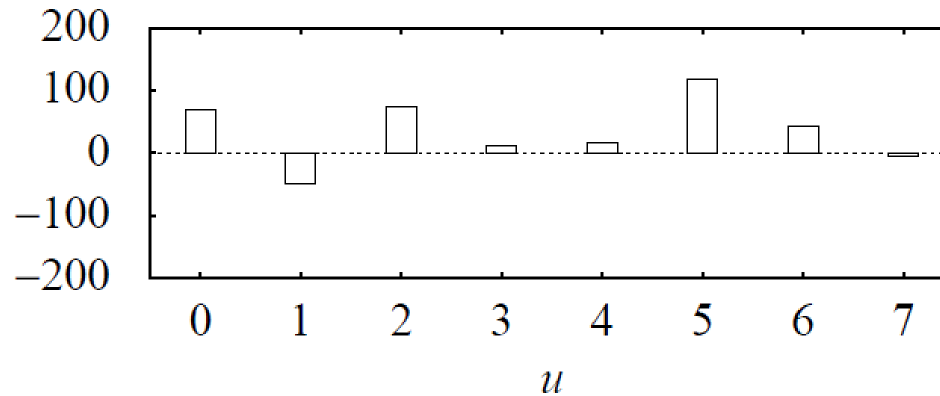
Transform Coding-变换编码

◆ 1D DCT-一维离散余弦变换

- Example: $f_4 = \{85, -65, 15, 30, -56, 35, 90, 60\}$

任意变化信号





$F_4(u)$: $\{69, -49, 74, 11, 16, 117, 44, -5\}$

Transform Coding-变换编码

- 1D DCT-一维离散余弦变换
  - The characteristics of the DCT can be summarized as follows-离散余弦变换特性总结:
  - The DCT produces the spatial frequency spectrum $F(u)$ corresponding to the spatial signal $f(i)$-空间信号$f(i)$的频谱$F(u)$.
  - The 0th DCT coefficient $F(0)$ is the DC component of the signal $f(i)$-$F(0)$是直流分量，与信号的平均值对应$F(0)= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \cdot 8 \cdot f_{avg} = 2\sqrt{2} \cdot f_{avg}$
  - Other seven DCT coefficients reflect the various changing (i.e., AC) components of the signal $f(i)$ at different frequencies-其它系数反映信号不同频率的AC分量.
  - The DCT is a linear transform-离散余弦变换是线性变换.

Transform Coding-变换编码

- 1D IDCT-一维离散余弦逆变换
  - Example: f_4= {85, −65, 15, 30, −56, 35, 90, 60}
  - $F_4(u)$: {69, −49, 74, 11, 16, 117, 44, −5}

$$\tilde{f}(i) = \sum_{u=0}^{7} \frac{C(u)}{2} \cdot \cos \frac{(2i+1) \cdot u\pi}{16} \cdot F(u)$$

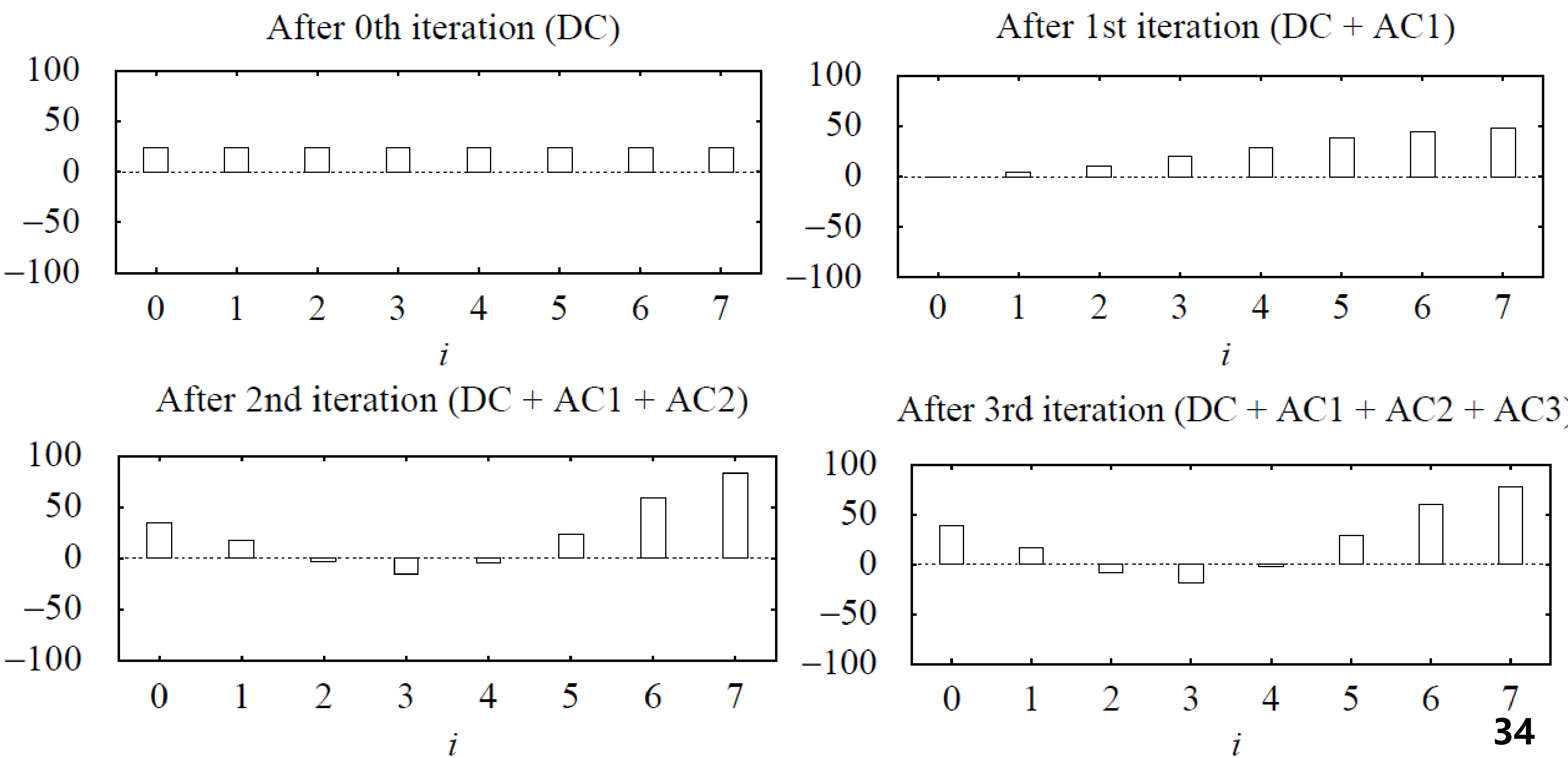
$$\text{Iteration 0:} \quad \tilde{f}(i) = \frac{C(0)}{2} \cdot \cos 0 \cdot F(0) = \frac{\sqrt{2}}{2 \cdot 2} \cdot 1 \cdot 69 \approx 24.3.$$

$$\begin{aligned} \text{Iteration 1:} \quad \tilde{f}(i) &= \frac{C(0)}{2} \cdot \cos 0 \cdot F(0) + \frac{C(1)}{2} \cdot \cos \frac{(2i+1)\pi}{16} \cdot F(1) \\ &\approx 24.3 + \frac{1}{2} \cdot (-49) \cdot \cos \frac{(2i+1)\pi}{16} \approx 24.3 - 24.5 \cdot \cos \frac{(2i+1)\pi}{16}. \end{aligned}$$

$$\begin{aligned} \text{Iteration 2:} \quad \tilde{f}(i) &= \frac{C(0)}{2} \cdot \cos 0 \cdot F(0) + \frac{C(1)}{2} \cdot \cos \frac{(2i+1)\pi}{16} \cdot F(1) + \frac{C(2)}{2} \cdot \cos \frac{(2i+1)\pi}{8} \cdot \\ &\quad F(2) \\ &\approx 24.3 - 24.5 \cdot \cos \frac{(2i+1)\pi}{16} + 37 \cdot \cos \frac{(2i+1)\pi}{8}. \end{aligned}$$

Transform Coding-变换编码

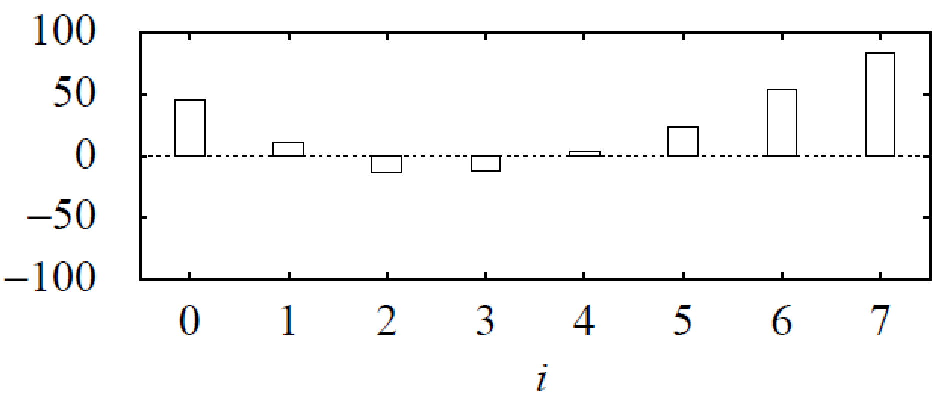
- 1D IDCT-一维离散余弦逆变换
  - Example: f_4= {85, −65, 15, 30, −56, 35, 90, 60}
  - $F_4(u)$: {69, −49, 74, 11, 16, 117, 44, −5}



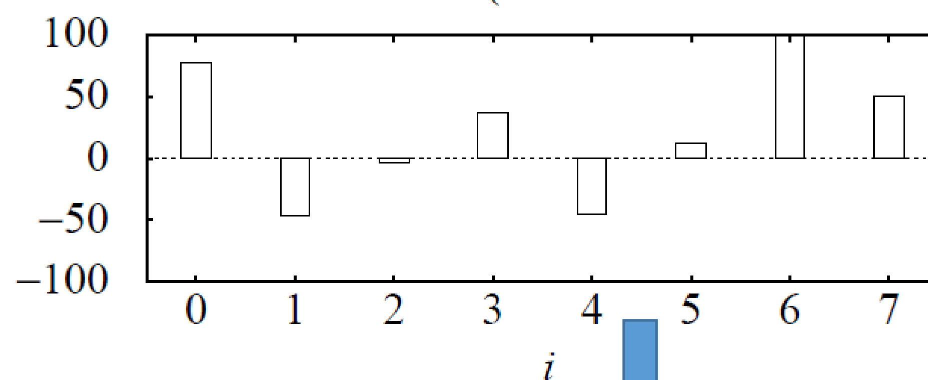
Transform Coding-变换编码

- 1D IDCT-一维离散余弦逆变换
  - Example: f_4= {85, −65, 15, 30, −56, 35, 90, 60}
  - $F_4(u)$: {69, −49, 74, 11, 16, 117, 44, −5}

After 4th iteration (DC + AC1 + . . . + AC4)

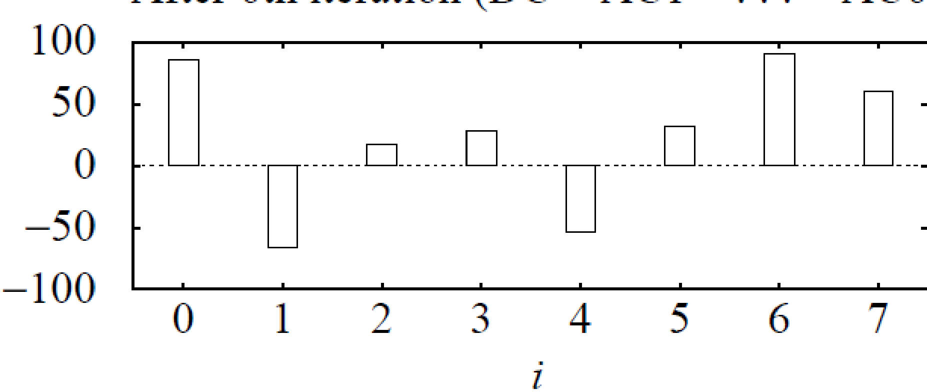


After 5th iteration (DC + AC1 + . . . + AC5)

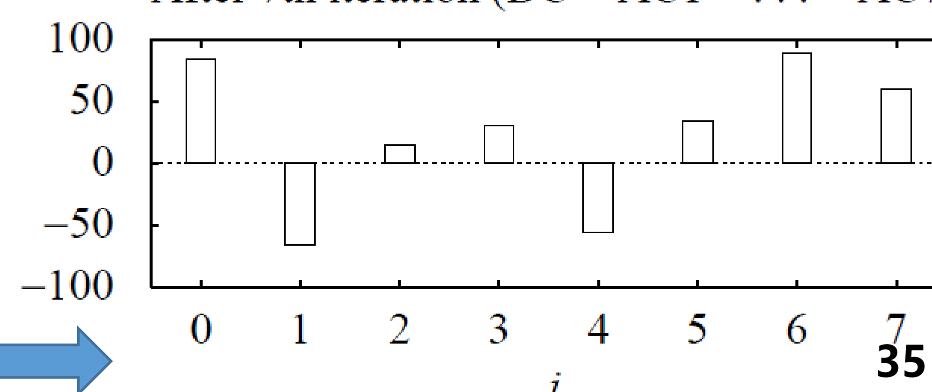


$\tilde{f}(i)$: {85, −65, 15, 30, −56, 35, 90, 60}.

After 6th iteration (DC + AC1 + . . . + AC6)



After 7th iteration (DC + AC1 + . . . + AC7)



Transform Coding-变换编码

- The Cosine Basis Functions-余弦基函数
  - Function $B_p(i)$ and $B_q(i)$ are **orthogonal**, if-正交

$$\sum_i [B_p(i) \cdot B_q(i)] = 0, if\ p \neq q$$

  - Function $B_p(i)$ and $B_q(i)$ are **orthonormal,** if-标准正交

$$\sum_i [B_p(i) \cdot B_q(i)] = 1, if\ p = q$$

  - 欧氏空间坐标系:(1,0,0)，(0,1,0)，(0,0,1)是一个标准正交基，

Transform Coding-变换编码

- The Cosine Basis Functions-余弦基函数
  - With **orthogonal** property, the signal is not amplified during the transform. When the same basis function is used in both the transformation and its inverse, we will get (approximately) the same signal back-正交特性保持变换过程信号不放大，基函数相同，逆变换后信号近似相同.

$$\sum_{i=0}^{7}\left[\cos\frac{(2i+1)\cdot p\pi}{16}\cdot\cos\frac{(2i+1)\cdot q\pi}{16}\right]=0\,,p\neq q$$

$$\sum_{i=0}^{7}\left[\frac{C(p)}{2}\cos\frac{(2i+1)\cdot p\pi}{16}\cdot\frac{C(q)}{2}\cos\frac{(2i+1)\cdot q\pi}{16}\right]=1,p=q$$

标准化参数

Transform Coding-变换编码

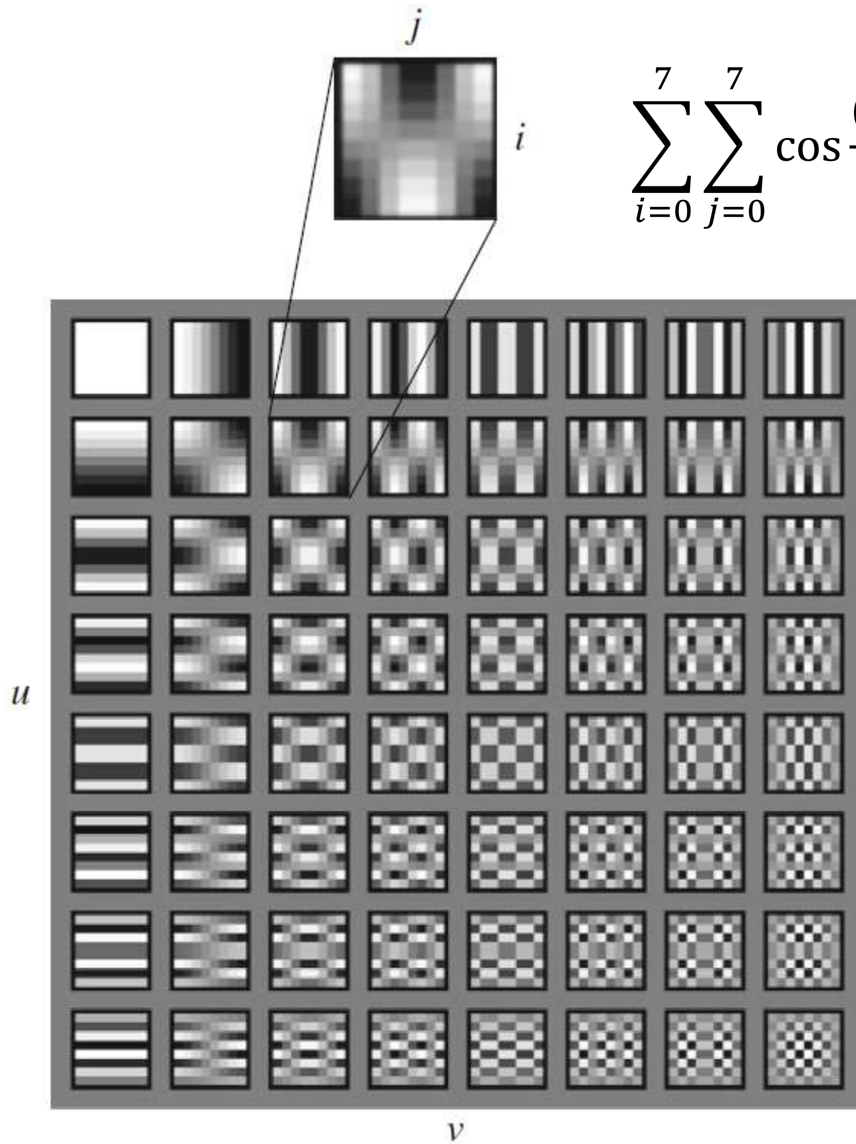
◆ 2D Basis Functions-2D基函数

- For two-dimensional DCT functions, we use the basis depicted as 8×8 image. For a particular pair of u and v, the respective basis function is-具体的对(u, v) 相应基函数:

$$F(u,v) = \frac{C(u)C(v)}{4} \sum_{i=0}^{7} \sum_{j=0}^{7} \cos\frac{(2i+1)\cdot u\pi}{16} \cdot \cos\frac{(2j+1)\cdot v\pi}{16} \cdot f(i,j)$$

Transform Coding-变换编码

◆ 2D Basis Functions-2D基函数



$$\sum_{i=0}^{7}\sum_{j=0}^{7}\cos\frac{(2i+1)\cdot 1\cdot \pi}{16}\cdot \cos\frac{(2j+1)\cdot 2\pi}{16}$$

Transform Coding-变换编码

- 2D Separable Basis-2D可分离函数
  - For speed, 2D DCT coefficients—*factorization* into two 1D DCT transforms-二维转换成两个一维DCT加快速度-先列后行.

$$G(u,j)=\frac{C(u)}{2}\sum_{i=0}^{7}\cos\frac{(2i+1)\cdot u\pi}{16}\cdot f(i,j)$$

$$F(u,v)=\frac{C(v)}{2}\sum_{j=0}^{7}\cos\frac{(2j+1)\cdot v\pi}{16}\cdot G(u,j)$$

Transform Coding-变换编码

- 2D DCT-Matrix Implementation-矩阵实现
  - The above factorization of a 2D DCT into two 1D DCTs can be implemented by two consecutive matrix multiplications-二维转换矩阵乘法实现.

$$F(u,v) = \mathbf{T} \cdot f(i,j) \cdot \mathbf{T}^T$$

DCT矩阵

$$\mathbf{T}_8 = \left[\begin{matrix} \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \cdots & \frac{1}{2\sqrt{2}} \\ \frac{1}{2}\cdot\cos\frac{\pi}{16} & \frac{1}{2}\cdot\cos\frac{3\pi}{16} & \frac{1}{2}\cdot\cos\frac{5\pi}{16} & \cdots & \frac{1}{2}\cdot\cos\frac{15\pi}{16} \\ \frac{1}{2}\cdot\cos\frac{\pi}{8} & \frac{1}{2}\cdot\cos\frac{3\pi}{8} & \frac{1}{2}\cdot\cos\frac{5\pi}{8} & \cdots & \frac{1}{2}\cdot\cos\frac{15\pi}{8} \\ \frac{1}{2}\cdot\cos\frac{3\pi}{16} & \frac{1}{2}\cdot\cos\frac{9\pi}{16} & \frac{1}{2}\cdot\cos\frac{15\pi}{16} & \cdots & \frac{1}{2}\cdot\cos\frac{45\pi}{16} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{1}{2}\cdot\cos\frac{7\pi}{16} & \frac{1}{2}\cdot\cos\frac{21\pi}{16} & \frac{1}{2}\cdot\cos\frac{35\pi}{16} & \cdots & \frac{1}{2}\cdot\cos\frac{105\pi}{16} \end{matrix}\right] \begin{matrix} \text{DC} \\ \text{AC1} \\ \text{AC2} \\ \text{AC3} \\ \\ \text{AC7} \end{matrix}$$

Transform Coding-变换编码

- 2D IDCT-Matrix Implementation-逆变换实现
  - Reconstruct $f(i, j)$ from $F(u, v)$ losslessly by matrix multiplications -二维逆变换矩阵乘法实现.

$$f(i,j) = \mathbf{T}^T \cdot F(u,v) \cdot \mathbf{T}$$

  - The DCT-matrix $\mathbf{T}$ is orthogonal, therefore, $\mathbf{T}^T = \mathbf{T}^{-1}$
  - How to drive this formula-如何推导?

$$f(i,j) = \mathbf{T}^{-1} \cdot \mathbf{T} \cdot f(i,j) \cdot \mathbf{T}^T \cdot (\mathbf{T}^T)^{-1}$$

$$F(u,v) = \mathbf{T} \cdot f(i,j) \cdot \mathbf{T}^T$$

$$f(i,j) = \mathbf{T}^{-1} \cdot F(u,v) \cdot (\mathbf{T}^T)^{-1}$$

$$f(i,j) = \mathbf{T}^T \cdot F(u,v) \cdot \mathbf{T}$$

Transform Coding-变换编码

- Comparison of DCT and DFT-与Fourier比较
  - DFT-离散傅里叶变换

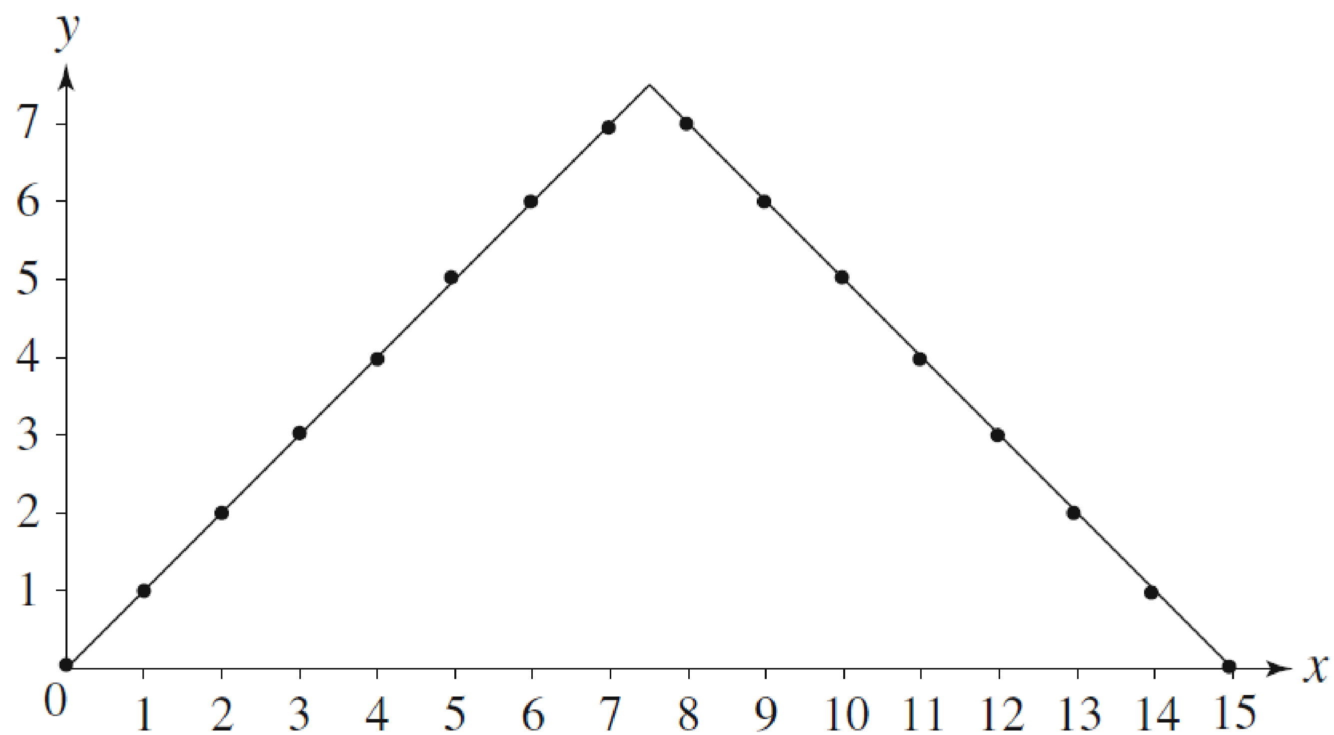
$$F_{\omega} = \sum_{x=0}^{7} f_x \cdot e^{-\frac{2\pi i\omega x}{8}}$$

$$F_{\omega} = \sum_{x=0}^{7} f_x \cdot \cos\left(\frac{2\pi\omega x}{8}\right) - i\sum_{x=0}^{7} f_x \cdot \sin\left(\frac{2\pi\omega x}{8}\right)$$

$$e^{ix} = \cos(x) + i\sin(x)$$

Transform Coding-变换编码

- Comparison of DCT and DFT-与Fourier比较



Symmetric extension of the ramp function-对称斜坡函数

Transform Coding-变换编码

◆ Comparison of DCT and DFT-与Fourier比较

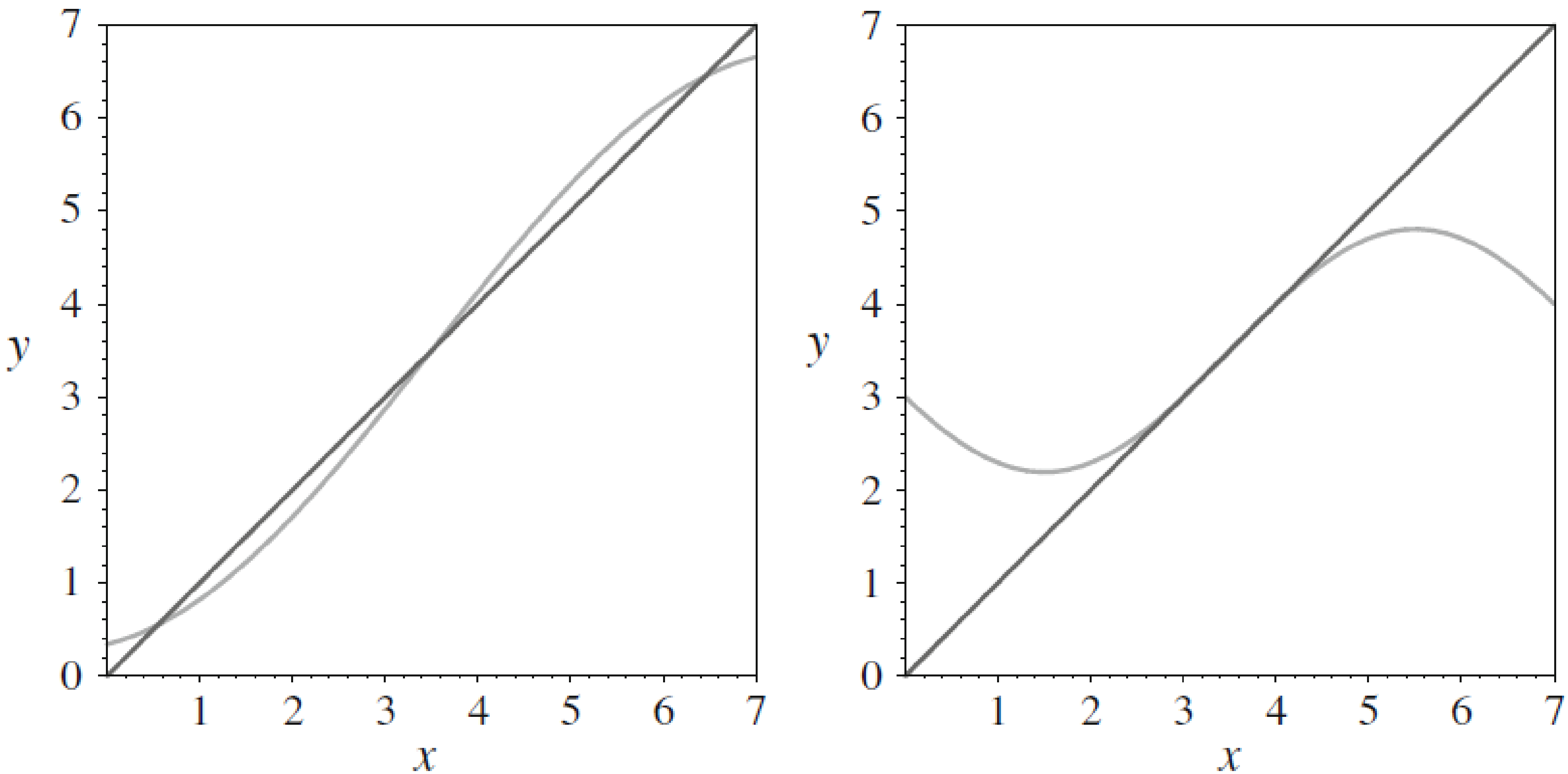
Ramp	DCT	DFT
0	9.90	28.00
1	−6.44	−4.00
2	0.00	9.66
3	−0.67	−4.00
4	0.00	4.00
5	−0.20	−4.00
6	0.00	1.66
7	−0.51	−4.00

DCT and DFT coefficients of the ramp function

DCT前几个系数包含更多的信息

Transform Coding-变换编码

- Comparison of DCT and DFT-与Fourier比较



Approximation of the ramp function: (a) 3 Term DCT Approximation, (b) 3 Term DFT Approximation.

Outline of Lecture 08

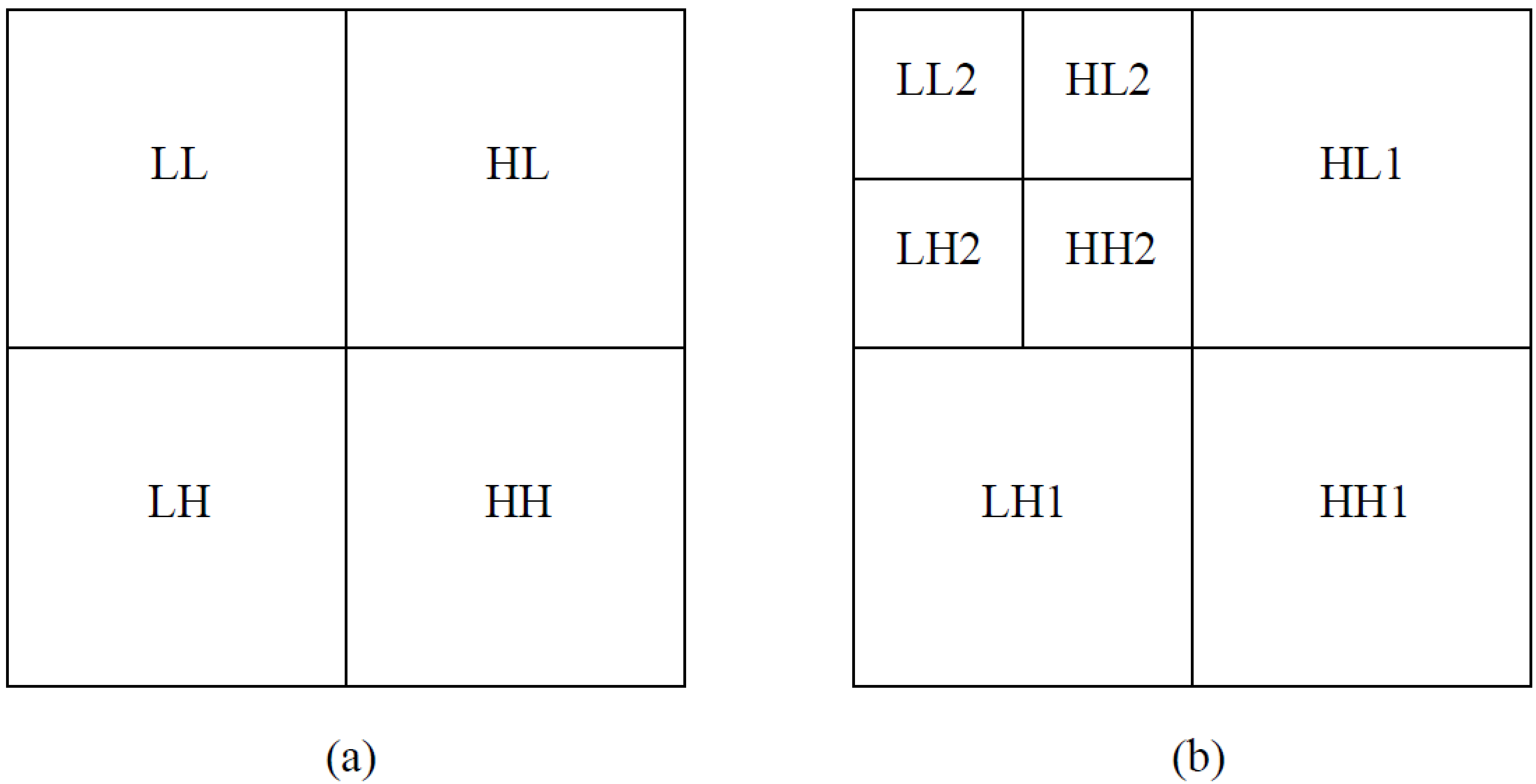
- Introduction-简介
- Distortion Measures-失真度量
- The Rate-Distortion Theory-比率失真理论
- Quantization-量化
  - Uniform Scalar Quantization-均匀标量量化
  - Nonuniform Scalar Quantization-非均匀标量量化
  - Vector Quantization-向量量化
- Transform Coding-变换编码
- Wavelet-Based Coding-小波编码
- Experiments-实验

Transform Coding-变换编码

- Wavelet-Based Coding-小波编码
  - The objective of the wavelet transform is to decompose the input signal into components that are easier to deal with, have *special interpretations*, or have some components that can be thresholded away, for compression purposes-小波变换.
  - We want to be able to at least approximately reconstruct the original signal given these components.
  - The basis functions of the wavelet transform are localized in both time and frequency.
  - There are two types of wavelet transforms: *the continuous wavelet transform* (CWT) and *the discrete wavelet transform* (DWT)-连续和离散小波.

Transform Coding-变换编码

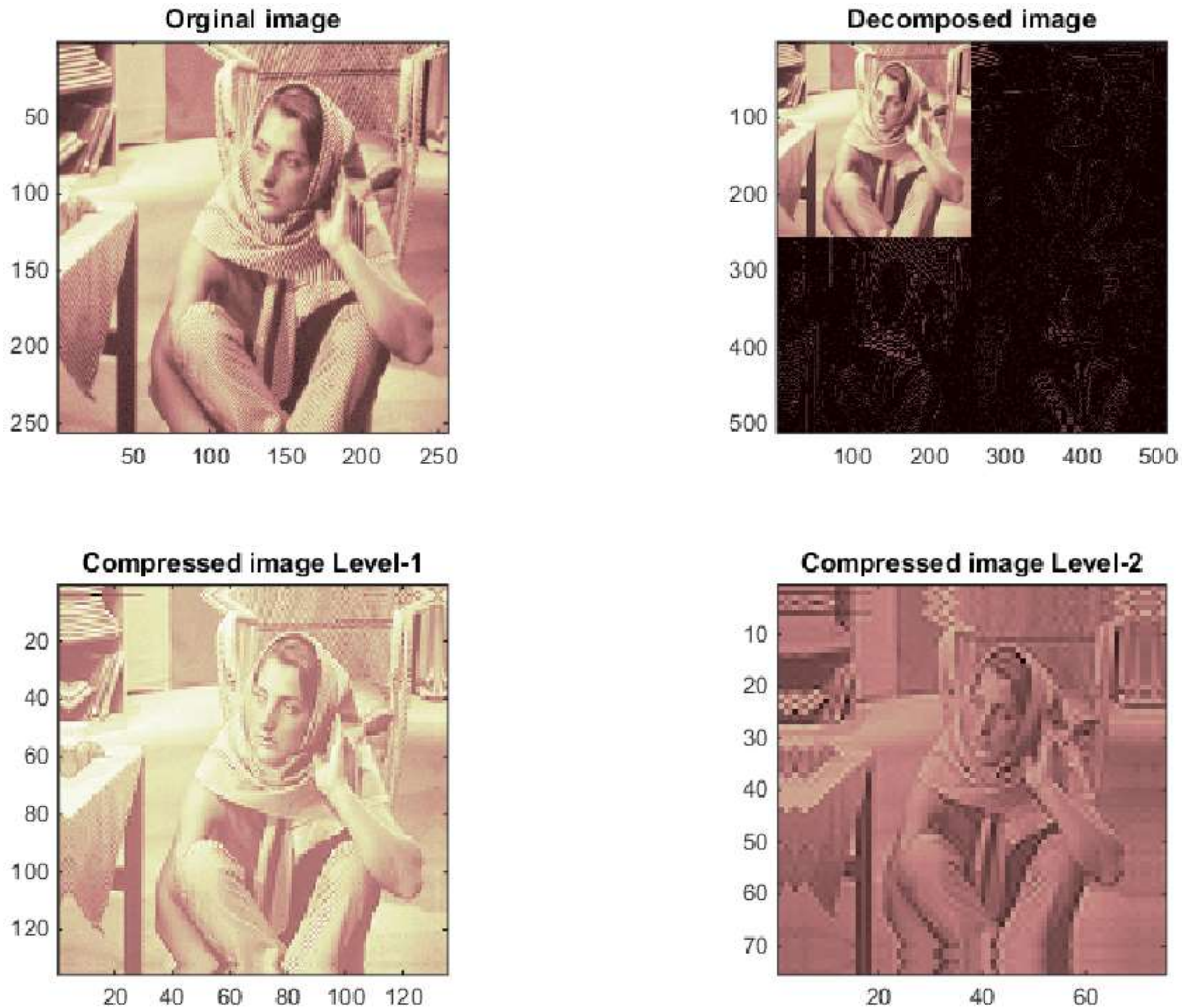
- Wavelet-Based Coding-小波编码
  - *The discrete wavelet transform* (DWT).



The two-dimensional discrete wavelet transform (a) One level transform, (b) two level transform.

Transform Coding-变换编码

- Wavelet-Based Coding-小波编码
  - *The discrete wavelet transform* (DWT).

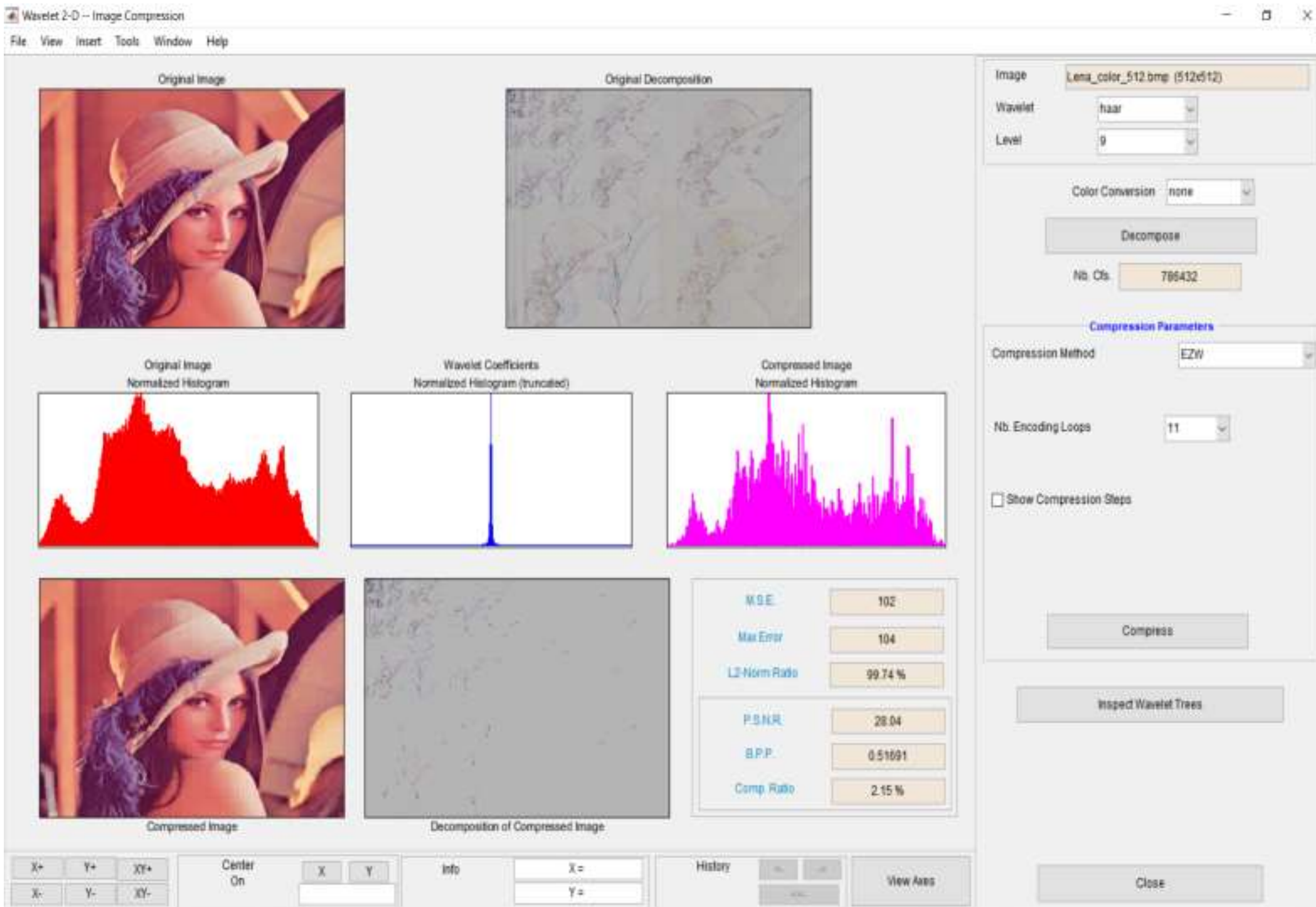


Experiments & Class Assignments

- Experiments-Matlab命令行输入:”wavemenu”

Experiments & Class Assignments

- Experiments-Matlab命令行输入:”wavemenu”



Experiments & Class Assignments

- Experiments
  - DCT Coding--*ch08_dct_demo.m*
  - Wavelet Coding--*ch08_dwt_demo.m*
- Class Assignments

1、已知原来的数据为{12 16 16 12 12 8 8 12 }、经过压缩与解压缩之后得到的数据为{12 12 12 8 12 8 12 12}. 请分别计算MSE、SNR和PSNR的值，请写出计算过程， 其中$\log_{10}(32) \approx 1.505$, $\log_{10}(19) \approx 1.279$.

2、输入信号为$f(i)$=[0 10 20 30 40 50 60 70]，请计算该信号的一维离散余弦变换的$F(0)$.

3、变换编码的基本原理是什么?