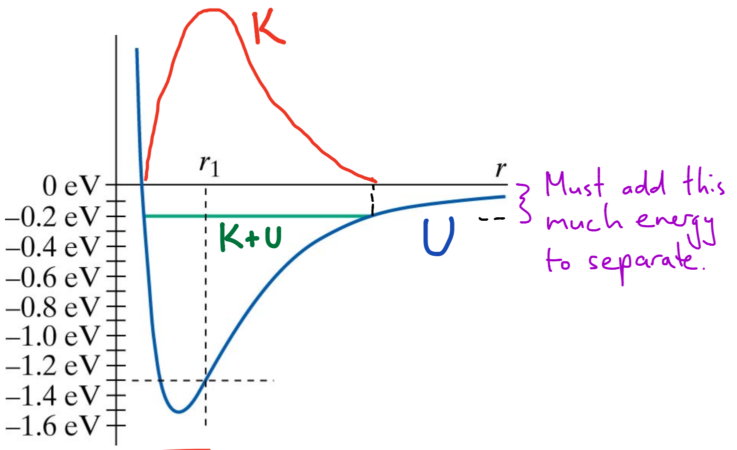
University Physics A(2) 2014

Worksheet #8: Internal Energy

Name (名字): Student number (学号):

Problems Show all working.

(1) [M&I.7.P.32] The figure below is a potential energy curve for the interaction of two neutral atoms. The two-atom system is in a vibrational state indicated by the horizontal green line.



(a) At $r = r_1$, what are the approximate values of the kinetic energy K, the potential energy U, and quantity $K + U$?

$U = \boxed{-1.3\ eV}$

$K + U = \boxed{-0.2\ eV}$

$\therefore\ K = (K+U) - U = -0.2 + 1.3 = \boxed{1.1\ eV}$

(b) What minimum energy must be supplied to cause these two atoms to separate?

To separate to $r \to \infty$, need $K + U \geq 0$.

$\therefore$ We must add at least $\boxed{0.2\ eV}$, to reach $K+U=0$.

(c) In some cases, when r is large, the interatomic potential energy can be expressed approximately as $U = -a/r^6$. For large r, what is the formula for the magnitude of the force the two atoms exert on each other in this case?

$$F_r = -\frac{dU}{dr} = -\frac{d}{dr}\left(-ar^{-6}\right) = -\left[-(-6)ar^{-7}\right] = \boxed{\frac{-6a}{r^7}}$$

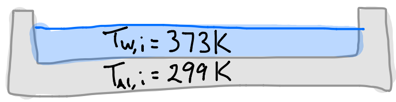
(This is called the "Lennard-Jones" potential)

(2) [M&I.7.P.46] 180 grams of boiling water (temperature 100°C, specific heat capacity 4.2 J/K/gram) are poured into an aluminum pan with mass 1050 grams and initial temperature 26°C (the heat capacity of aluminum is 0.9 J/K/gram).

(a) After a short time, what is the temperature of the water?

We assume that this "short time" is long enough for the pan and water to reach equilibrium, but not long enough to exchange energy with the surroundings.

Initial state:



SYSTEM: pan, water

SURROUNDINGS: negligible

ENERGY PRINCIPLE:

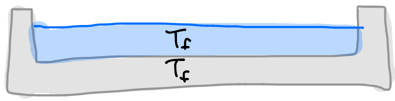
$$\Delta E = \cancel{W}^{\,0} + \cancel{Q}^{\,0}$$

$$\Delta E_w + \Delta E_{Al} = 0$$

$$C_w m_w \Delta T_w + C_{Al} m_{Al} \Delta T_{Al} = 0$$

$$C_w m_w (T_f - T_{w,i}) + C_{Al} m_{Al} (T_f - T_{Al,i}) = 0$$

Final state: $T_{w,f} = T_{A,f} = T_f$



$$T_f = \frac{C_w m_w T_{w,i} + C_{Al} m_{Al} T_{Al,i}}{C_w m_w + C_{Al} m_{Al}}$$

$$= \boxed{332\,K \text{ or } 58.9°C}$$

(b) What simplifying assumptions did you have to make in part (a)? (You may choose more than one.)

(A) The thermal energy of the water doesn't change.

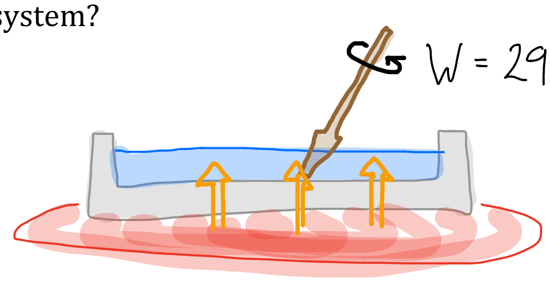
(B) The thermal energy of the aluminum doesn't change.

(C) Energy transfer between the system (water plus pan) and the surroundings was negligible during this time.

(D) The heat capacities for both water and aluminum do not change with temperature in this temperature range.

(c) Next you place the pan on a hot electric stove. While the stove is heating the pan, you use a beater to stir the water, doing 29,541 J of work, and the temperature of the water and pan increases to 86.9°C. How much energy transfer due to a temperature difference was there from the stove into the (water+pan) system?

$W = 29{,}541\ \mathrm{J}$



$\Delta T = 86.9°C - 58.9°C$
$= 28\ \mathrm{K}$

$Q = ?$

SYSTEM: pan, water

SURROUNDINGS: stove, beater

ENERGY PRINCIPLE:

$$\Delta E = W + Q$$

$$\Delta E_w + \Delta E_{Al} = W + Q$$

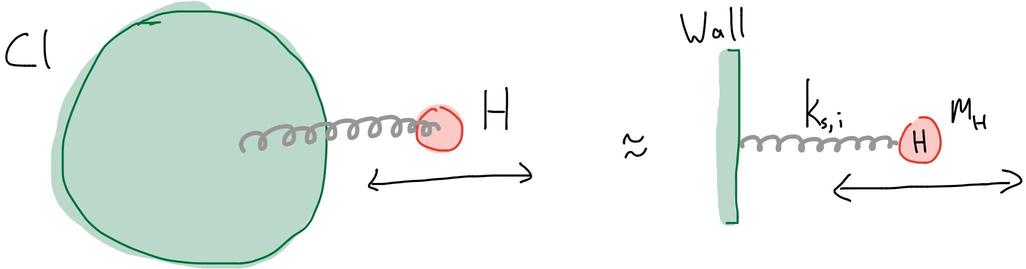
$$\rightarrow \quad Q = \Delta E_w + \Delta E_{Al} - W$$

$$= C_w m_w \Delta T + C_{Al} m_{Al} \Delta T - W$$

$$= \left(4.2 \tfrac{\mathrm{J}}{\mathrm{g \cdot K}} \cdot 180\,\mathrm{g} + 0.9 \tfrac{\mathrm{J}}{\mathrm{g \cdot K}} \cdot 1050\,\mathrm{g}\right)(28\ \mathrm{K}) - 29{,}541\ \mathrm{J}$$

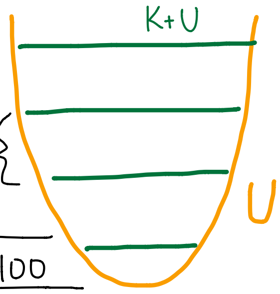
$$= \boxed{18{,}087\ \mathrm{J}}$$

(3) [M&I.P.35] (a) A HCl molecule can be considered to be a quantized harmonic oscillator, with quantized vibrational energy levels that are evenly spaced. Estimate this uniform energy spacing, in eV (where 1 eV = 1.6×10^{-19} J). You can assume the "spring stiffness" for the chemical bond between the two atoms is about 100 N/m. The mass of a proton is 1.7×10^{-27} kg.



Because $m_{Cl} >> m_H$, we can assume the Cl atom is stationary.

One "quantum" of energy:

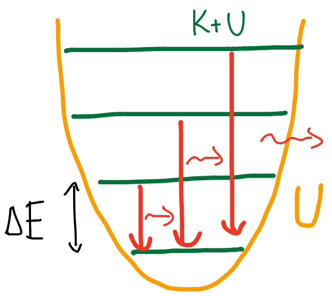


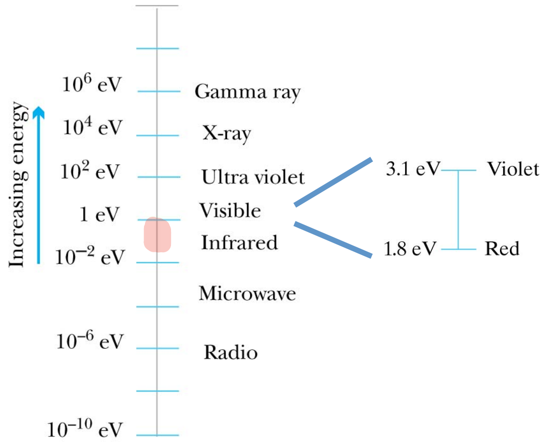
$$\Delta E = \hbar\sqrt{\frac{k_{s,i}}{m_H}}$$

$$\simeq (1\times10^{-34})\sqrt{\frac{100}{1.7\times10^{-27}}}$$

$$= 2.4\times10^{-20}\text{ J} \text{ or } \boxed{0.15\text{ eV}}$$

(b) List several photon energies that would be emitted if a number of these vibrational energy levels were occupied due to electron excitation. To what region of the spectrum (x-ray, visible, microwave, etc.) do these photons belong? (Refer to the figure below.)





Emitted photons would have energies

$1\times\Delta E,\ 2\times\Delta E,\ 3\times\Delta E,\ \dots$

$= \boxed{0.15\text{ eV},\ 0.3\text{ eV},\ 0.45\text{ eV},\dots}$ Infrared