第4章 刚体力学-1

在线直播课

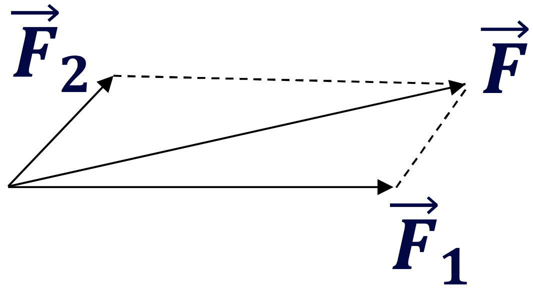
2022-4-1

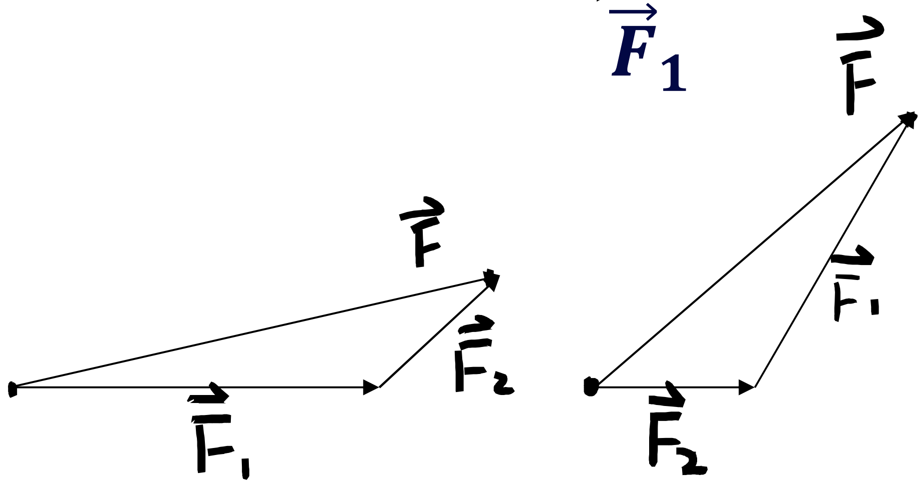
矢量：大小、方向

运算规则：平行四边形或三角形法则、加法交换律

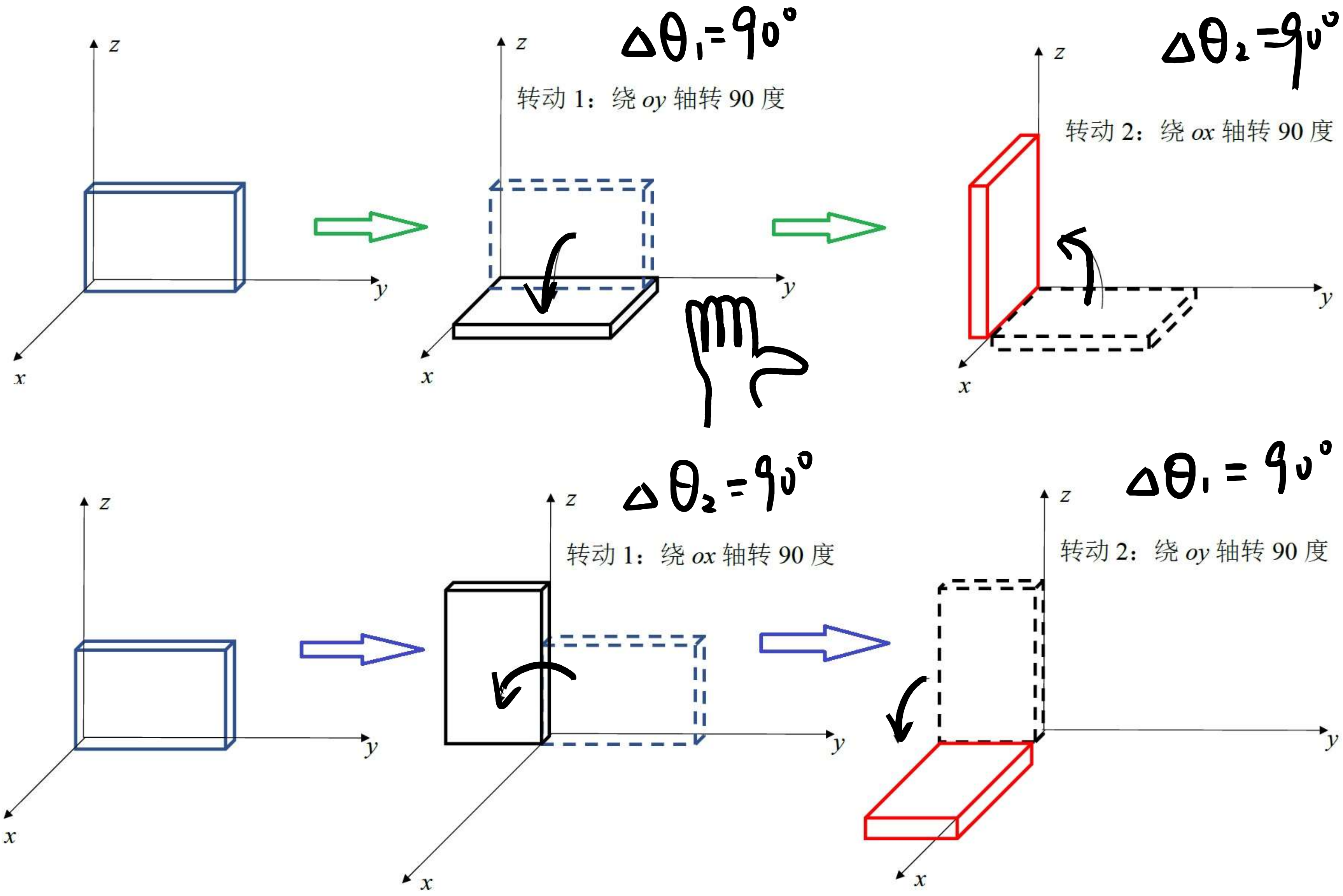
$$\vec{F} = \vec{F}_1 + \vec{F}_2$$

$$= \vec{F}_2 + \vec{F}_1$$



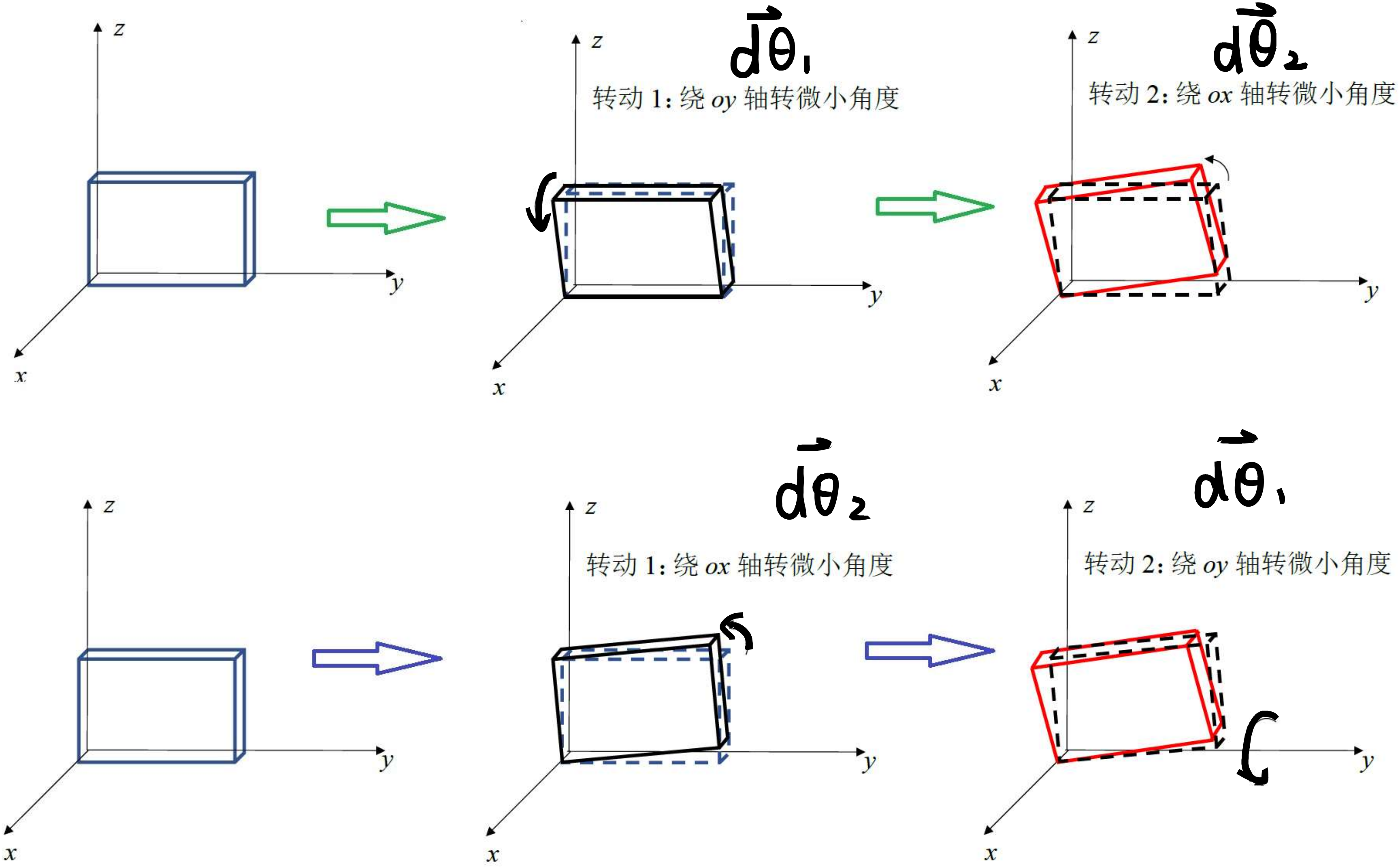


有限大小角位移——标量



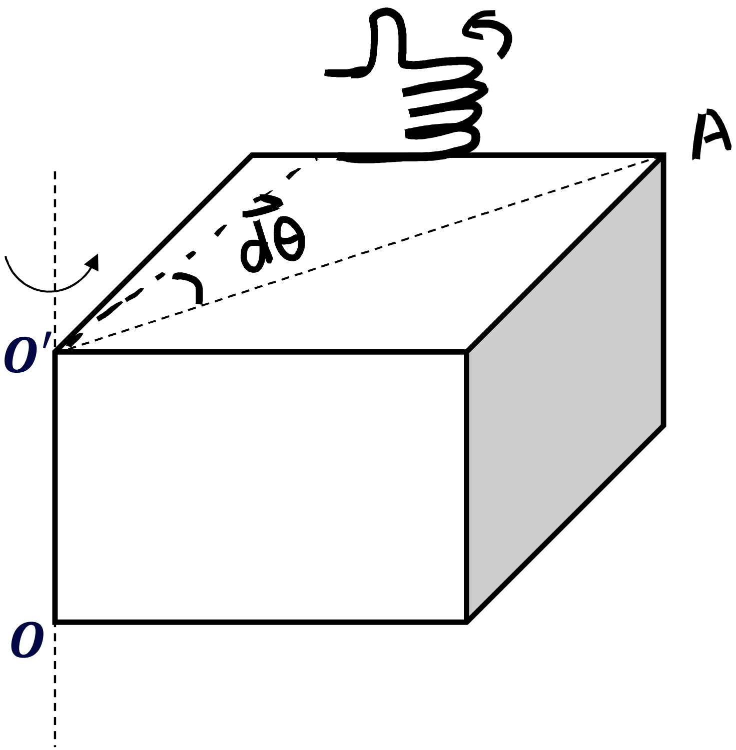
z
y
x
转动 1：绕 oy 轴转 90 度
$\Delta\theta_1 = 90^\circ$
转动 2：绕 ox 轴转 90 度
$\Delta\theta_2 = 90^\circ$
转动 1：绕 ox 轴转 90 度
$\Delta\theta_2 = 90^\circ$
转动 2：绕 oy 轴转 90 度
$\Delta\theta_1 = 90^\circ$

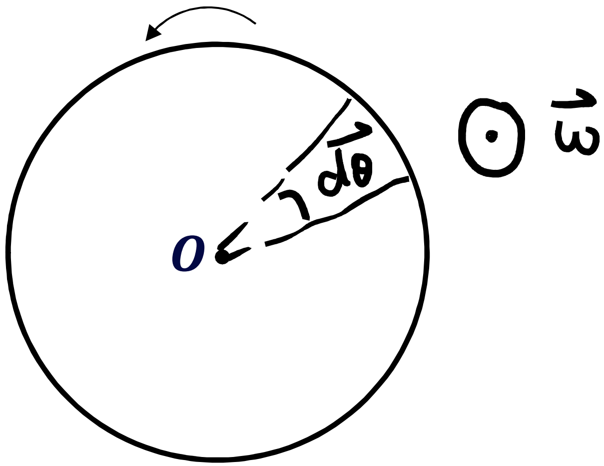
无限小角位移——矢量


$d\vec{\theta}_1$
转动 1：绕 *oy* 轴转微小角度
$d\vec{\theta}_2$
转动 2：绕 *ox* 轴转微小角度
$d\vec{\theta}_2$
转动 1：绕 *ox* 轴转微小角度
$d\vec{\theta}_1$
转动 2：绕 *oy* 轴转微小角度
z
y
x

角速度的方向

$$\vec{\omega} = \frac{d\vec{\theta}}{dt}, \qquad \vec{\alpha} = \frac{d\vec{\omega}}{dt}$$





一飞轮绕定轴转动，其角坐标与时间的关系为

$$\theta = a + bt + ct^3$$

式中，a、b、c 均为常量.试求(1)飞轮的角速度和角加速度；(2)距转轴 r 处的质点的切向加速度和法向加速度.

$$\theta \rightleftarrows \omega \rightleftarrows \alpha$$

$$\theta(t) = a + bt + ct^3$$

$$\omega = \frac{d\theta}{dt} = b + 3ct^2$$

$$\alpha = \frac{d\omega}{dt} = 6ct$$

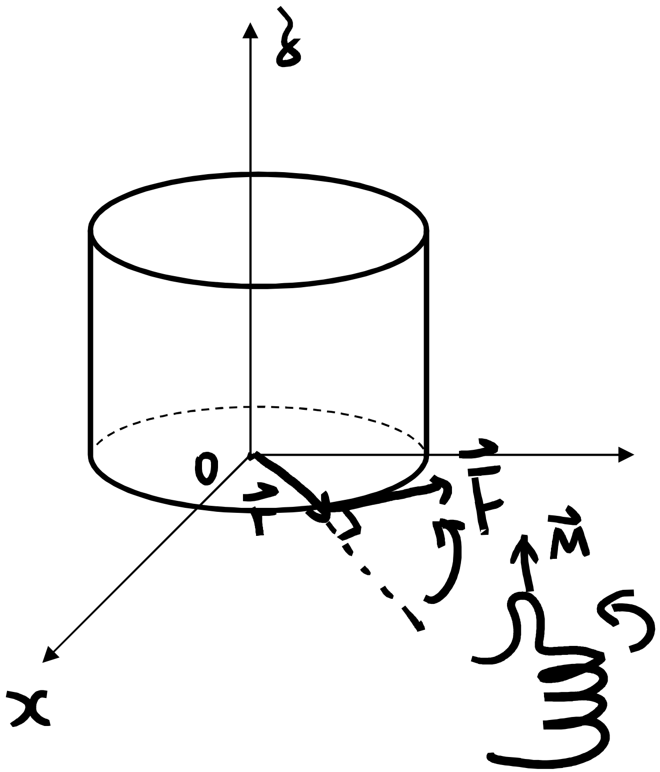
$$a_t = r\alpha$$

$$a_n = r\omega^2$$

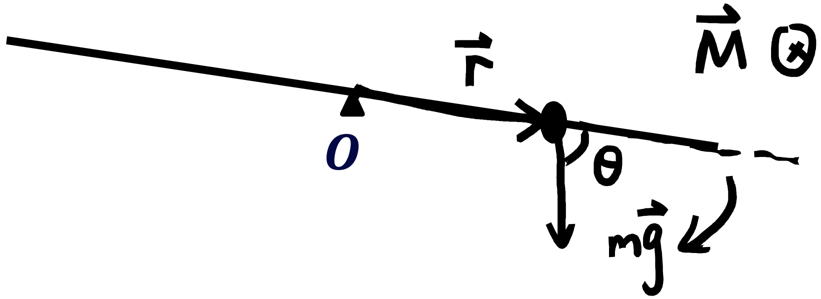
力矩

$M = r F \sin\theta$

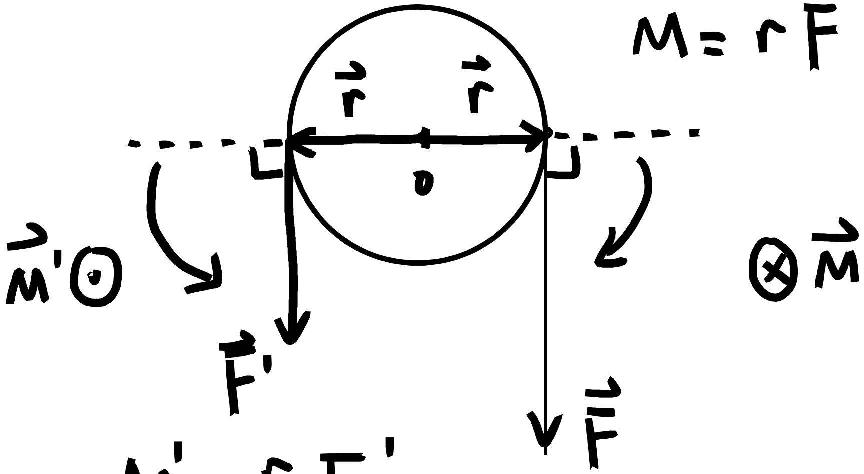
$$\vec{M} = \vec{r} \times \vec{F}$$



$M = r\, mg \sin\theta$



$M = rF$



$M' = rF'$

转动定律

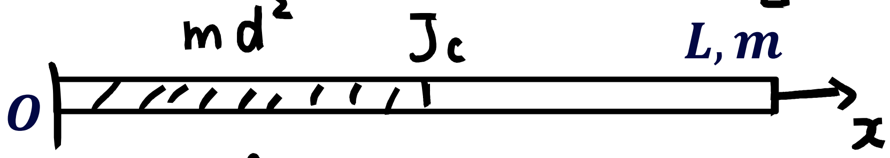
$$\vec{M} = J\vec{\alpha} \qquad \vec{F} = m\vec{a}$$

$\vec{M} \downarrow$ 力矩　　$J \downarrow$ 转动惯量

表4-2 a, b, c, f

$$J = \sum_{i=1}^{n} \underline{\underline{m_i r_i^2}} = \int r^2 dm$$

$r^2 \uparrow x^2$ 　 $dm \uparrow \frac{m}{L}dx$



$dm = \frac{m}{L}dx$

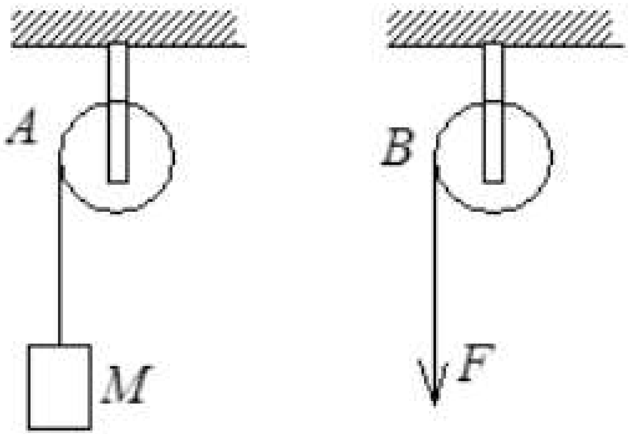
① $J_o = \int_0^L \frac{m}{L}x^2 dx = \frac{m}{L}\frac{L^3}{3} = \frac{1}{3}mL^2$

② $J_o = J_c + m\left(\frac{L}{2}\right)^2, \quad J_c = \frac{1}{12}mL^2$

③ $J = J_1 + J_2$

$J_1 \uparrow \frac{1}{3}mL^2$ 　 $J_2 \uparrow m'L^2$ 　 质点

如图所示,A、B为两个相同的绕着轻绳的定滑轮,它们都可看作是质量均匀分布的圆盘。A滑轮挂一质量为M的物体,B滑轮受拉力F,且 $F = Mg$ 。设A、B两滑轮的角加速度分别为α_A, α_B,不计滑轮轴的摩擦,则有[ ]



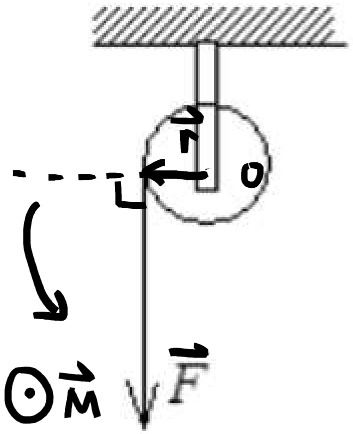
A、 $\alpha_A = \alpha_B$

B、 $\alpha_A > \alpha_B$

C、 $\alpha_A < \alpha_B$

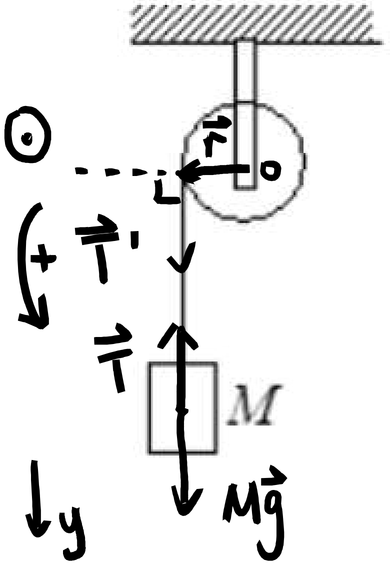
D、 开始时 $\alpha_A = \alpha_B$,以后$\alpha_A < \alpha_B$

$F = Mg$, $J = \frac{1}{2}Mr^2$



$\vec{M} = \vec{r} \times \vec{F}$, $M = rF$, ⊙

$M = rF = J\alpha_1$, $\alpha_1 = \frac{Fr}{J} = \frac{Mgr}{J}$



质点: $M\vec{g} + \vec{T} = M\vec{a}$

盘: $\vec{M} = J\vec{\alpha}_2$

$\vec{M} = \vec{r} \times \vec{T}'$

$M = rT'$

$$\begin{cases} Mg - T = Ma = Mr\alpha_2 \\ rT' = J\alpha_2 \end{cases}$$

$T = T'$

$a = \alpha_2 r$

$T = Mg - Mr\alpha_2$

$r(Mg - Mr\alpha_2) = J\alpha_2$

$\alpha_2 = \frac{Mgr}{J + Mr^2}$

飞轮对自身轴的转动惯量为 J_0,初角速度为 ω_0,作用在飞轮上的阻力矩为 M(常量).试求飞轮的角速度减到 $\omega_0/2$ 时所需的时间 t,以及在这一段时间内飞轮转过的圈数 N.若 $M=\alpha\omega$(α 为常量),再求解以上问题.

k k

$$\theta \rightleftarrows \vec{\omega} \rightleftarrows \vec{\alpha} \rightleftarrows \vec{M}$$

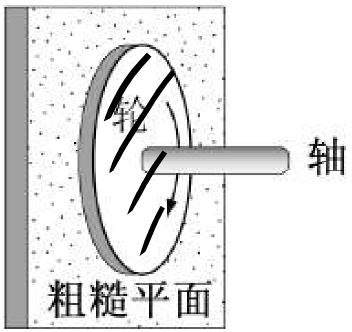
$$\vec{M} = J_0\vec{\alpha}, \quad -M = J_0\alpha, \quad \alpha = -\frac{M}{J_0}$$

$$\omega_t = \omega_0 + \alpha t, \quad \frac{\omega_0}{2} = \omega_0 - \frac{M}{J_0}t, \quad t = ?$$

$$\omega_t^2 - \omega_0^2 = 2\alpha\Delta\theta, \quad \Delta\theta = ?$$

$$M = -k\omega = J_0\alpha = J_0\frac{d\omega}{dt} \qquad \omega = ?$$

以力 F 将一块粗糙平面紧压在轮上，平面与轮之间的滑动摩擦因数为 μ，轮的初角速度为 ω_0，问转过多少角度时轮即停止转动？已知轮的半径为 R，质量为 m，可看作均质圆盘；轴的质量忽略不计；该压力 F 均匀分布在轮面上．



$F/(\pi R^2) = F_0$ 单位面积

盘 → 环 → 弧

面元 (弧) $df = \mu\, dN = \mu F_0\, r\, dr\, d\theta$

$dM_f = r\, df = \mu F_0 r^2 dr\, d\theta$ 方向 ⊙

环 → $dM_f' = \int_0^{2\pi} (\mu F_0 r^2 dr)\, d\theta$

$dM_f' = \mu F_0 2\pi r^2 dr$

盘 $M_f = \int_0^R \mu F_0 2\pi r^2 dr$

$M_f = \frac{2}{3}\mu F_0 \pi R^3 = \frac{2}{3}\mu F R$

$M_f = J\alpha = \frac{2}{3}\mu F R$， $\alpha = ?$