Neural Networks

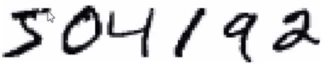
Steve Cygu

January 23, 2019

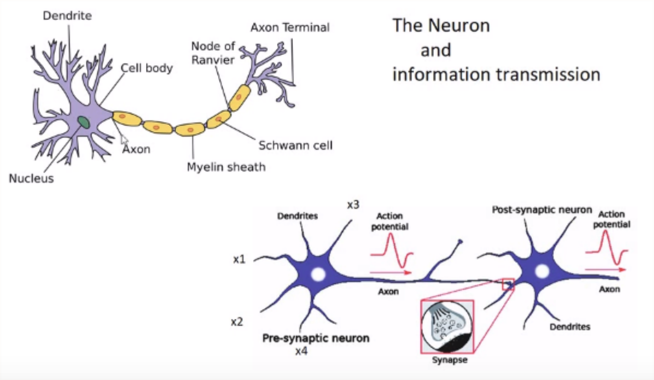
OUTLINE

Why Machine Learning?

Can we write algorithm to correctly identify each of the objects?



Neural Networks and Human Brain



Components

Layers

- **Input nodes**
  No computation
- **Hidden nodes (Neurons)**
  Intermediate processing and computation and transfers (another hidden layer or output).
- **Output nodes**
  Uses a function (not necessarily activation function) to map the input from other layers to desired output format.
  - Sigmoid
  - Softmax

Synapse/Connections

- Transfers the output of neuron i to the input of neuron j.
- Each connection is assigned weight, W_{ij}

Activation function

Introduces nonlinearity into the neuron output.

- Sigmoid (Logistic Activation Function)

$$a(z) = \frac{1}{1 + exp(-z)}$$

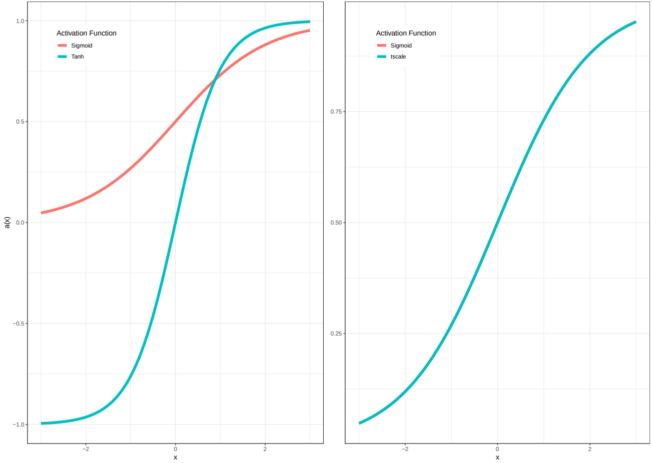
- Tanh (hyperbolic tangent Activation Function)

$$a(z) = tanh(z) = \frac{2}{1 + exp(-2z)} - 1 = 2sigmoid(2z) - 1$$

- ReLU (Rectified Linear Unit Activation Function)

$$a(z) = max(0, z)$$

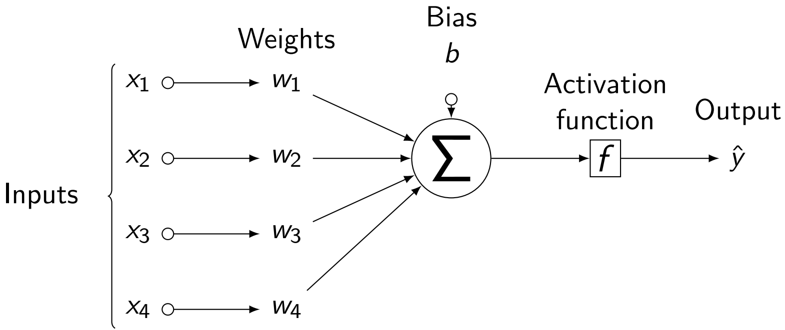
Sigmoid and Tanh Activation Functions



Types of Neural Networks

Single-layer Perceptron

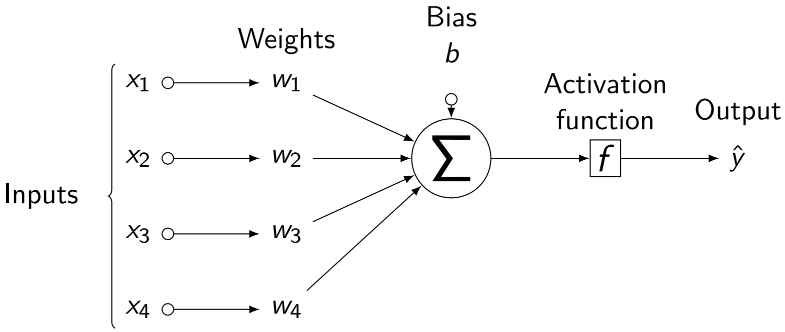
- No hidden layer, a single neuron.



Types of Neural Networks

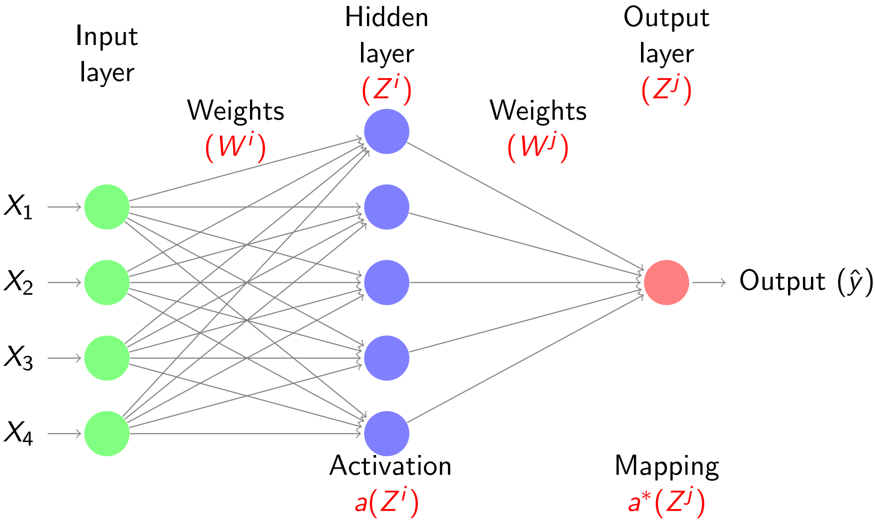
Single-layer Perceptron

- No hidden layer, a single neuron.



$$\hat{y} = b + \sum_{i=1}^{n} x_i w_i$$

Multi-layer perceptron (MLP)



Other types

- Convolutional Neural Network (CNN)
- Recurrent neural networks

Fitting Neural Networks

Weights are the parameters. The generic approach is by **gradient descent**.

- Forward-propagation (feed-forward)
- Backward-propagation

Feed-forward

Consider,

$$Z_1^i = X_1 W_{1,1}^i + X_2 W_{2,1}^i + X_3 W_{3,1}^i + X_4 W_{4,1}^i$$

Feed-forward

Consider,

$$Z_1^i = X_1 W_{1,1}^i + X_2 W_{2,1}^i + X_3 W_{3,1}^i + X_4 W_{4,1}^i$$

$$Z_2^i = X_1 W_{1,2}^i + X_2 W_{2,2}^i + X_3 W_{3,2}^i + X_4 W_{4,2}^i$$

Feed-forward

Consider,

$$Z^i_1 = X_1 W^i_{1,1} + X_2 W^i_{2,1} + X_3 W^i_{3,1} + X_4 W^i_{4,1}$$
$$Z^i_2 = X_1 W^i_{1,2} + X_2 W^i_{2,2} + X_3 W^i_{3,2} + X_4 W^i_{4,2}$$
$$\vdots$$

Feed-forward

Consider,

$$Z_1^i = X_1 W_{1,1}^i + X_2 W_{2,1}^i + X_3 W_{3,1}^i + X_4 W_{4,1}^i$$

$$Z_2^i = X_1 W_{1,2}^i + X_2 W_{2,2}^i + X_3 W_{3,2}^i + X_4 W_{4,2}^i$$

$$\vdots$$

Z^i 'component' is the sum of weighted inputs to each neuron.

$$Z^i = XW^i \tag{1}$$

Feed-forward

Apply activation function to 1

$$a^i = a(Z^i) \tag{2}$$

Propagate 2 to the output layer

$$Z^j = a^i W^j \tag{3}$$

$$\implies \hat{y} = a^j = a^*(Z^j) \tag{4}$$

Back propagation

- Estimate weights that ensures the model fits the training data well.

$$J = \sum \frac{1}{2}(y - \hat{y})^2 \tag{5}$$

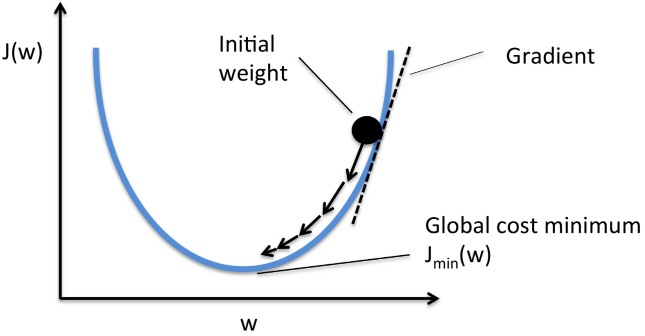
Back propagation

- Estimate weights that ensures the model fits the training data well.

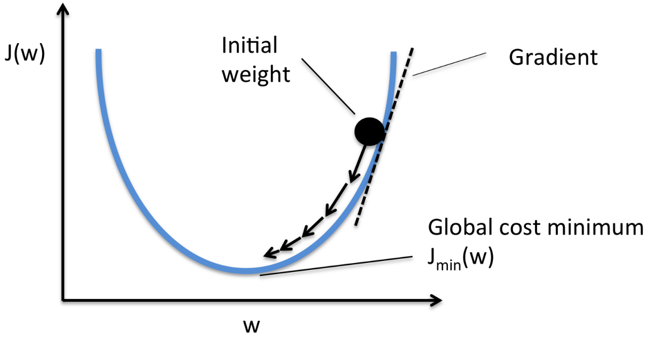
$$J = \sum \frac{1}{2}(y - \hat{y})^2 \tag{5}$$

$$J(W) = \frac{1}{2}\sum \left(y - a(a(XW^i)W^j)\right)^2 \tag{6}$$

Gradient descent



Gradient descent



$$W_{t+1} = W_t - \gamma \Delta J(W_t)$$

Gradient descent

Compute $\frac{\partial J}{\partial W^i}$ and $\frac{\partial J}{\partial W^j}$

$$\frac{\partial J}{\partial W^j} \approx -(y - \hat{y})\frac{\partial \hat{y}}{\partial W^j}$$
$$\implies \frac{\partial J}{\partial W^j} = -a^{iT}(y - \hat{y})a'(Z^j)$$

$$
\begin{aligned}
\frac{\partial J}{\partial W^i} &\approx -(y-\hat{y})\frac{\partial \hat{y}}{\partial W^i} \\
\implies \frac{\partial J}{\partial W^i} &= -X^T(y-\hat{y})a'(Z^j)W^{jT}a'(Z^i)
\end{aligned}
$$

Some Issues in Training Neural Networks

1. Starting values
   - Starting weights are random numbers near zero.
   - However, near weights collapses NN into approximately linear model.
   - Exactly zero weights leads to zero derivatives and perfect symmetry.
   - Large weights lead to poor results.

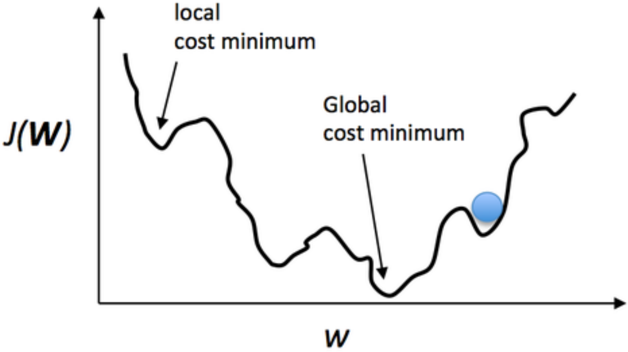
Some Issues in Training Neural Networks

1. Starting values
   - Starting weights are random numbers near zero.
   - However, near weights collapses NN into approximately linear model.
   - Exactly zero weights leads to zero derivatives and perfect symmetry.
   - Large weights lead to poor results.
2. Overfitting and Stopping Criterion
   - Reduce training error to some predetermined threshold $\longrightarrow$ overffiting.
   - Regularization by *weight decay* (analogous to ridge regression)

$$J(W) = \sum \frac{1}{2}(y - \hat{y})^2/n + \frac{1}{2}\lambda\left(\sum W_1^2 + \sum W_2^2\right)$$

     $\lambda \geq 0$ - Is tuning parameter. Larger values of λ shrinks weights toward zero.
   - Cross-validation is used to estimate λ.

3. Convergence at the Local Minima



References

[1] Trevor, H., Robert, T., & JH, F. (2009). The elements of statistical learning: data mining, inference, and prediction. *Springer series in statistics*. Second Edition

[2] Tom M. Mitchell. (1997). Machine Learning *McGraw-Hill International Editions*.

[3] Internet sources (2019).