Neural Networks

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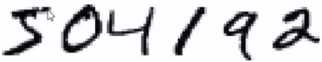
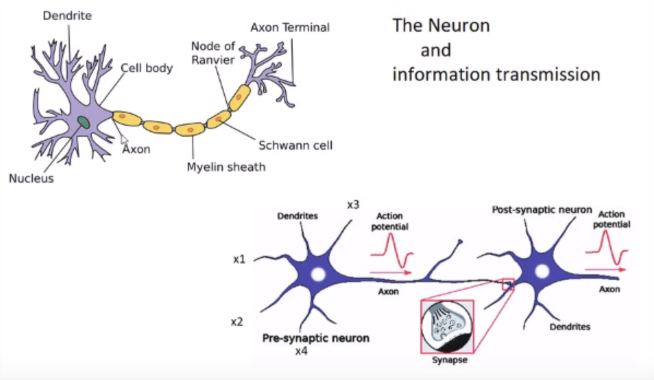
OUTLINE

Why Machine Learning?

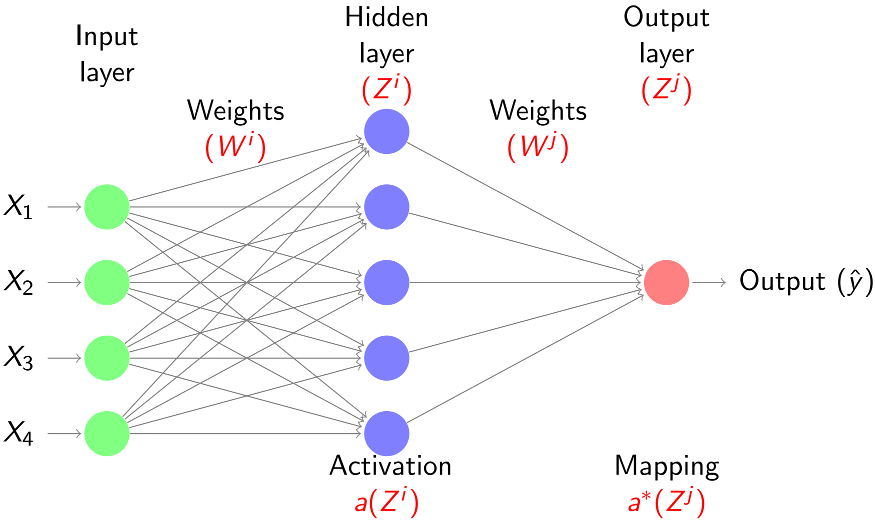
Can we write algorithm to correctly identify each of the objects?



Neural Networks and Human Brain

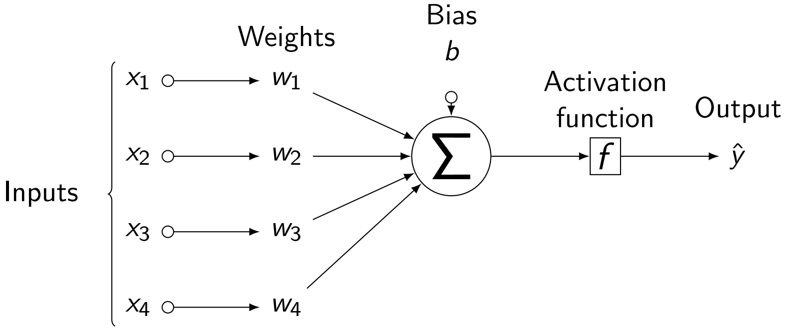


Multi-layer perceptron (MLP)



Single-layer Perceptron

- No hidden layer, a single neuron.



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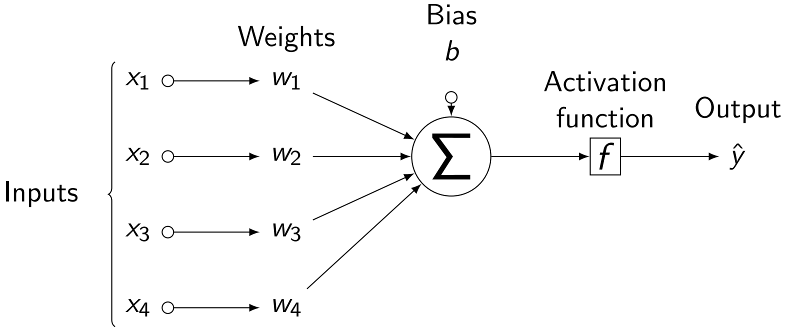


$$\hat{y} = b + \sum_{i=1}^{n} x_i w_i$$

Components

Layers

- **Input nodes**
  No computation
- **Hidden nodes (Neurons)**
  Intermediate processing, computation and transfers to another hidden layer or output.
- **Output nodes**
  Uses a function (not necessarily activation function) to map the input from other layers to desired output format.
  - Sigmoid
  - Softmax

Synapse/Connections

- Transfers the output of neuron i to the input of neuron j.
- Each connection is assigned weight, W_{ij}

Activation function

Introduces nonlinearity into the neuron output.

- Sigmoid (Logistic Activation Function)

$$a(z) = \frac{1}{1 + exp(-z)}$$

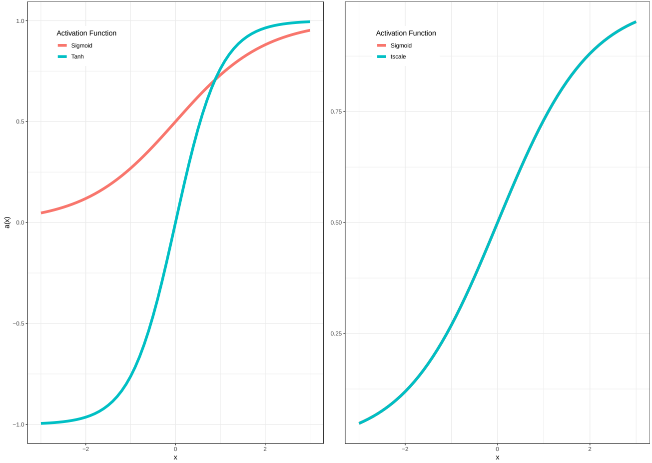
- Tanh (hyperbolic tangent Activation Function)

$$a(z) = tanh(z) = \frac{2}{1 + exp(-2z)} - 1 = 2sigmoid(2z) - 1$$

- ReLU (Rectified Linear Unit Activation Function)

$$a(z) = max(0, z)$$

Sigmoid and Tanh Activation Functions



Fitting Neural Networks

Weights are the parameters. The generic approach is by **gradient descent**.

- Forward-propagation (feed-forward)
- Backward-propagation

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\begin{aligned}
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$$\vdots$$

Z^i 'component' is the sum of weighted inputs to each neuron.

$$Z^i = XW^i \tag{1}$$

Feed-forward

Apply activation function to 1

$$a^i = a(Z^i) \tag{2}$$

Propagate 2 to the output layer

$$Z^j = a^i W^j \tag{3}$$

$$\implies \hat{y} = a^j = a^*(Z^j) \tag{4}$$

Back propagation

- Aim is to estimate weights that ensures the model fits the training data well.
- Calculate the error at the output nodes and propagate them back to the network.

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- Compute the gradient; $\frac{\partial J}{\partial W^i}$ and $\frac{\partial J}{\partial W^j}$
- Adjust the weights using optimization method such as **Gradient Descent**.

Gradient descent

$$W_{t+1} = W_t - \gamma \Delta J(W_t)$$

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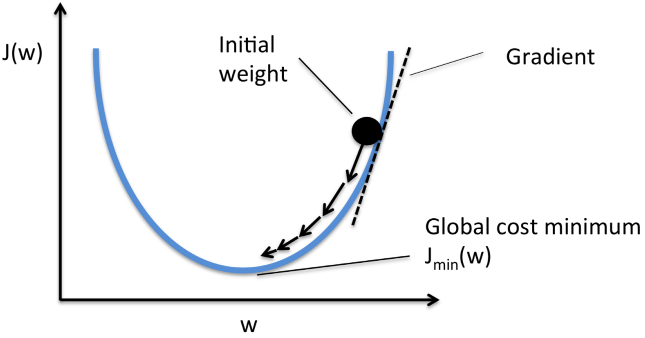

Some Issues in Training Neural Networks

1. Starting values
   - Starting weights are random numbers near zero.
   - However, near zero weights collapses NN into approximately linear model.
   - Exactly zero weights leads to zero derivatives and perfect symmetry.
   - Large weights lead to poor results.

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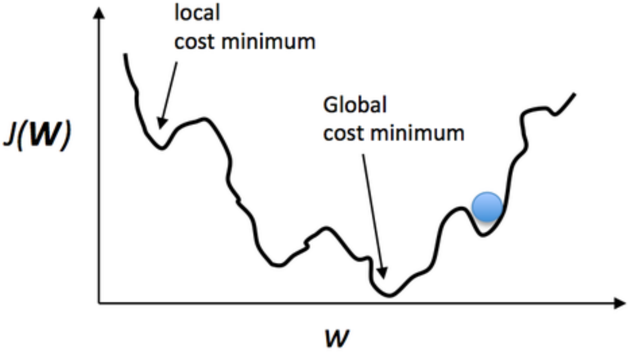
2. Overfitting and Stopping Criterion
   - Reduce training error to some predetermined threshold $\longrightarrow$ overfitting.
   - Regularization by *weight decay* (analogous to ridge regression)

$$J(W) = \sum \frac{1}{2}(y - \hat{y})^2/n + \frac{1}{2}\lambda\left(\sum W_1^2 + \sum W_2^2\right)$$

     $\lambda \geq 0$ - Is tuning parameter. Larger values of λ shrinks weights toward zero.
   - Cross-validation is used to estimate λ.

3. Convergence at the Local Minima



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[1] Trevor, H., Robert, T., & JH, F. (2009). The elements of statistical learning: data mining, inference, and prediction. *Springer series in statistics*. Second Edition

[2] Tom M. Mitchell. (1997). Machine Learning *McGraw-Hill International Editions*.

[3] Internet sources (2019).