Frequently-Used Math Common Core State Standards (CCSSs) for Math Student Support Planning					
Grades	**Learning Strand**	**Standard Focus**	**Standard(s)**	**Quantile**	**Page**
Grades 1-5	Number and Operations in Base Ten (NBT)	Addition	1.NBT.4	100Q	10
		Subtraction	1.NBT.6	150Q	14
		Place Value and Rounding	3.NBT.1	200Q	18
		Fractions (with Denominators of 10)	4.NBT.5	400Q	22
		Division with Equations, Arrays, and Area Models	4.NBT.6	400Q	25
		Place Value and Decimals	5.NBT.7	600Q	30
	Operations and Algebraic Thinking (OA)	Addition and Subtraction Word Problems	2.OA.1	200Q	37
		Division with Visual Models	3.OA.2	300Q	42
		Multiplication and Division Word Problems	3.OA.3	300Q	46
		Multiplication and Division Properties of Operations	3.OA.5	300Q	50
		Equations in Word Problems	4.OA.3	500Q	54
		Numerical Expressions [Parentheses, Brackets, Braces]	5.OA.1	500Q	59
	Measurement and Data (MD)	Length Addition and Subtraction	2.MD.5	200Q	64
		Time and Money	2.MD.8	300Q	69
		Time Addition and Subtraction	3.MD.1	200Q	74
		Picture and Bar Graphs	3.MD.3	400Q	78
		Perimeter	3.MD.8	400Q	84
		Measurement Word Problems	4.MD.2	400Q	89
	Number and Operations – Fractions (NF)	Fractions - Part vs. Whole	3.NF.1	200Q	98
		Number Line Fractions	3.NF.2a	200Q	102
		Fraction Comparison	3.NF.3d	400Q	106
			4.NF.2	500Q	113
		Decimals as Fractions	4.NF.6	500Q	118
		Fraction Multiplication	5.NF.4b	500Q	124
			5.NF.5b	600Q	130
	Geometry (G)	Lines and Angles	4.G.1	500Q	137
		The Coordinate Plane	5.G.1	400Q	143

Grades	Learning Strand	Standard Focus	Standard(s)	Quantile	Page
Grades 6-8	Ratios and Proportions (RP)	Ratios	6.RP.1	400Q	150
		Unit Rates	6.RP.2	600Q	156
		Converting to Percents (Decimals, Fractions, Percents)	6.RP.3c	600Q	162
	Equations and Expressions (EE)	Exponents in Algebraic Expressions	6.EE.1	600Q	170
		Variables in Algebraic Expressions	6.EE.2a	700Q	176
		Dependent vs. Independent Variables	6.EE.9	800Q	183
		Equivalent Expressions	7.EE.1	800Q	187
		Interpreting Linear Expressions	7.EE.2	800Q	195
		Powers of 10 and Estimation	8.EE.2	800Q	204
		Linear Equations – Similar Triangles	8.EE.6	100Q	210
	The Number System (NS)	Positive and Negative Numbers	6.NS.5	400Q	220
		Rational Numbers on the Number Line	6.NS.6c	800Q	225
		Opposite Quantities – Addition, Subtraction, and Zero	7.NS.1a	800Q	232
		Multiplication Properties of Operations	7.NS.2a	800Q	240

1.NBT.4 Back to ccss standard

Use place value understanding and properties of operations to add and subtract.

Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose ten.

Skills

1. Decompose any number within one hundred into ten(s) and one(s)

2. Solve addition problems using concrete models and relate it to the written equation

3. Solve addition problems using drawings and relate it to the written equation

4. Solve addition problems using place value and relate it to the written equation

Key Concepts/Vocabulary

Identify – To recognize a particular thing
Value – An amount or number
Digit – A single whole number in a number 10 or larger
Compose – Put together a number from other existing numbers
Decompose – To separate into basic elements (numbers or shapes)
Tens – Sets of 10 ones
Ones – The value of the number in the ones' place
Strategy – A plan or method
Solve – Find an answer
Addition – Finding the total, or sum
Subtraction – Removing objects from a collection
Equation – A statement that value (or group of values) is equal to another value (or other group of values)
Reason – Explain a belief or action

Standard-Specific Resources (1.NBT.4)

- **<u>EngageNY:</u> Grade 1, Module 4, Topic D, Lesson 13 – [Use 'counting on' and the 'make ten' strategy when adding across a ten]**

Concept Development (33 minutes)

Materials: (T) 4 ten-sticks from the personal math toolkit, place value chart drawn on chart paper
(S) 4 ten-sticks from the personal math toolkit, personal white board

Have students sit in semicircle formation in a meeting area with their personal math toolkits.

T: (Show 13 as 1 ten and 3 ones using linking cubes.) How many linking cubes are there?

S: 13 linking cubes.

T: (Add 4 more linking cubes of a different color.) How many linking cubes are there now? Turn and talk to your partner about how you know.

S: There are 17 cubes. I started with 13 and counted on. Thirteeeen, 14, 15, 16, 17. → I added 3 ones and 4 ones. That makes 7 ones. 1 ten and 7 ones is 17. → 4 more than 13 is 17.

T: Nice thinking! Let's try counting on to find our solution.

S: (Point as students count.) Thirteeeen, 14, 15, 16, 17.

T: Now, add the ones first. How many are in the ones place in 13?

S: 3 ones.

T: (Point to 3 cubes.) 3 ones and 4 ones is...?

S: 7 ones.

T: (Snap the ones cubes together to make 7. Write 7 in the ones place in the place value chart.) How many tens do we have?

S: 1 ten.

T: (Write 1 in the tens place in the place value chart.)

T: 1 ten 7 ones is...?

S: 17.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Students love listening to and learning from music. Find a song on iTunes about place value. One suggestion is "The Place Value Song" by Math Fiesta.

Note: Since there were no changes in tens, another option is to write 1 in the tens place first, and then 7 in the ones place.

T: What are some different addition sentences we could use to put together 13 cubes and 4 cubes?

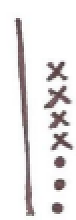
S: 13 + 4 = 17. → 10 + 7 = 17. → 10 + 3 + 4 = 17.

T: Use quick tens to draw the number of linking cubes we started with.

S/T: (Draw 1 quick ten and 3 dots for 3 ones.)

T: Draw to show the number of cubes we added to 13 using Xs in 5-group column formation.

S/T: (Draw 4 Xs above the 3 circles.)



T: Say the number sentence using your drawing.

S: 13 + 4 = 17.

T: Let's use a number bond. (Write 13 + 4.) 13 cubes is 1 ten and 3 ones. (Break 13 apart into 10 and 3.) We next added 3 ones and 4 ones. Use this number bond to solve the problem on your personal white board. Turn and talk to your partner about what you did.

S: First, I added 3 and 4 and got 7. Then, I added 10 and 7 and got 17.

T: Let's record how we added as 2 number sentences. (Write 3 + 4 = 7 and 10 + 7 = 17.) Let's solve another problem. Use your cubes to show 13.

13 + 4

10 3

3 + 4 = 7

10 + 7 = 17

S: (Show 1 ten-stick and 3 individual cubes in a 5-group column.)

T: Using a different color, add 7 more.

S: (Add 7 more cubes using a different color.)

T: How many cubes do you have now? Show your partner what you did, and talk about how you got the answer.

S: I put the 7 cubes next to 13 cubes. I know 3 and 7 is 10. And 10 and 10 is 20. → I stacked 7 cubes on top of the other 3. It made another ten-stick! → Now I see 2 ten-sticks. That's 20.

T: (Model with cubes.) You are right! 3 ones and 7 ones is…?

S: 10 ones.

T: 10 ones is the same as…?

S: 1 ten.

T: How many tens are there now? (Hold up each ten.)

S: 2 tens.

T: Where does the digit 2 go in our place value chart?

S: In the tens place.

T: (Write 2 in the tens place.) Since 3 ones and 7 ones make 1 ten, which we recorded in the tens place (point to place value chart), how many ones do we have now?

S: 0.

T: So we write 0 in the…?

S: Ones place.

T: (Write 0 in the ones place.) Say the number sentence.

S: 13 + 7 = 20.

T: Draw quick tens to show the addition. Explain your drawing to your partner.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Often students learn math concepts in an isolated fashion. Although they may be able to use them with familiar problems, they do not see how to transfer their application to new situations. Be sure to incorporate math at other times in the students' day.

S: I framed my 7 crosses and 3 circles to show that I made a ten. → I drew a long line through my 10 ones to make it look like a quick ten.

T: I love the idea of drawing a line through the new ten to make it look more like a quick ten! (Model.)

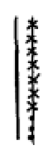
T: Make a number bond to show how you added the ones together.

S: (Write 13 + 7 = 20 by taking apart 13 into 10 and 3.)

T: How does making the number bond help you solve the problem?

S: I can see easily that I can add 3 and 7. That's 10. Then, I add 10 and 10 and get 20.

T: (Write two number sentences.) Great! Now let's try some more!



13 + 7 = 20

10 3

3 + 7 = 10

10 + 10 = 20

Repeat the process using the following sequence: 17 + 2, 18 + 2, 28 + 2, 23 + 6, 33 + 6, 23 + 7, and 33 + 7.

As soon as possible, write the addition expression on the board, and have students use quick ten math drawings and number bonds to solve rather than working with linking cubes. Some students may count on when adding 1 and 2. Counting on becomes less efficient as the second addend increases. When the second addend is larger than 3, encourage students to use Level 3 strategies such as thinking of doubles or using the make ten strategy.

Sample Anchor Chart for 1.NBT.4 – Addition using place value

How would you add 18 and 73 using place value?

Step 1: Look at your first number. What value is in the ones place? This is your number of ones for your first number (8 ones are in 18)

18

8 ones

Step 2: Look at your second number. What value is in the ones place? This is your number of ones for your second number (There are 3 ones in 73)

73

3 ones

Step 3: Look at your first number. What is the value in the tens place? This is your number of tens for your first number (1 ten is in 18)

18

1 ten = 10

Step 4: Look at your second number. What is the value in the tens place? This is your number of tens for your second number (7 tens are in 73)

73

7 tens = 70

Step 6: Add the ones from Steps 1 and 2. (8 ones + 3 ones = 11 ones) If you end up with a 2 digit number, what is the value of the number in the ones place and what is the value of the number in the tens place? (1 one and 1 ten)

Step 7: Add the tens from Steps 3 and 4. (1 ten + 7 tens = 8 tens = 8 x 10s = 80) Add this to the number of tens from step 6 (80 + 1 ten = 80 + 10 = 90). This is the new value for your tens place (a 9 goes in your new tens place)

1 ten (10) + 7 tens (70) = 8 tens (80) + 1 ten (10, from Step 6) = 90

Step 8: Put the ones left over from Step 6 into your ones place, to the right of the new number in your tens place (1 one was left over, so the new number is 91)

90 + 1 = 91 = 9 tens + 1 one

1.NBT.6 Back to ccss standard

Represent and solve problems involving addition and subtraction.

Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g. by using drawings and equations with a symbol for the unknown number to represent the problem.

Skills

1. Identify the value of each digit of a number within 100

2. Solve subtraction problems using concrete models and relate it to the written equation

3. Solve subtraction problems using drawings and relate it to the written equation

4. Solve subtraction problems using place value and relate it to the written equation

Key Concepts/Vocabulary

Identify – To recognize a particular thing

Value – An amount or number

Digit – A single whole number in a number 10 or larger

Subtract – To remove objects from a collection

Multiple – The result of multiplying a number by an integer (not by a fraction)

Range – The difference between the lowest and highest values

Appropriate – The right choice for a particular situation

Strategy – A plan or method

Standard-Specific Resources (1.NBT.6)

- **EngageNY:** **Grade 1, Module 4, Topic C, Lesson 11 - Add and subtract tens from a multiple of 10.**

Concept Development (33 minutes)

Materials: (T) Chart paper (S) Personal white board, number bond/number sentence set (Template)

Students sit in the meeting area in a semicircle formation.

T: (Write 2 + 1 on the chart. Call up two volunteers.) Using your magic counting sticks, show us 2 + 1.

S: (Student A shows 2 fingers. Student B shows 1 finger.)

T: How many fingers are there? Say the number sentence.

S: 2 + 1 = 3.

T: (Complete the number sentence on the chart.)

> **NOTES ON MULTIPLE MEANS OF REPRESENTATION:**
>
> The use of charts in the next few lessons provides students with visual guides to use as resources in the classroom as they learn more about place value. Some students may benefit from having a smaller version of the charts in their personal white boards or folders to refer to as needed.

On their personal white boards, have students write the number sentence, use math drawings to show 2 + 1 = 3, and make a number bond as the teacher records the information in a chart.

T: Let's pretend these circles stand for bananas! Say the number sentence using bananas as the unit.

S: 2 bananas + 1 banana = 3 bananas.

T: (Call for an additional volunteer to join the two volunteers.) Show us 2 tens + 1 ten using your magic counting sticks.

S: (Clasp hands to show 2 tens and 1 ten.)

T: (Help the first two students stand closer together to show 20.)

T: (Point to the first two students.) How many tens do we have here?

S: 2 tens.

T: (Point to the third student.) How many tens do we have here?

S: 1 ten.

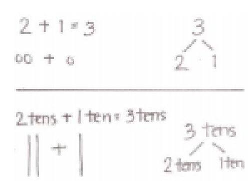
T: How many tens are there in all?

S: 3 tens.

T: Say the number sentence using the unit *tens*. (If students struggle, say, "Say the number sentence starting with 2 *tens*.")

S: 2 tens + 1 ten = 3 tens.

T: (Record the number sentence on the chart.)



> **NOTES ON MULTIPLE MEANS OF REPRESENTATION:**
>
> Students demonstrate a true understanding of math concepts when they apply them in a variety of situations. Some students may not be able to make the connection between different number bonds as seen in this lesson. Their path to abstract thinking may be a little longer than others'. Support these students with use of manipulatives and ample practice on their personal white boards.

Have students write the number sentence, use math drawings, and make a number bond. Chart their responses as shown to the right.

Repeat the process, and record the following suggested [redacted]

[redacted]

may be a little longer than others". Support these students with use of manipulatives and ample practice on their personal white boards.

Repeat the process, and record the following suggested sequence on the chart: 3 tens + 1 ten, 2 tens + 2 tens, and 1 ten + 3 tens. Progress through the units from ones to bananas to tens (e.g., 3 + 1 = 4 → 3 bananas + 1 banana = 4 bananas → 3 tens + 1 ten = 4 tens). Have students write the number sentence, make math drawings, and write the number bond (using the same format from the teacher-generated chart) for each problem. These charts are used later in this lesson.

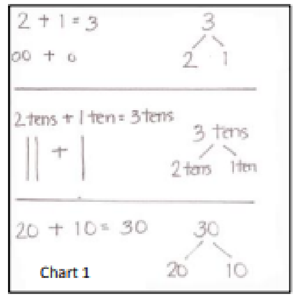


Chart 1

MP.7

T: (Point to the first problem on the chart.) Hmmm, how can knowing 2 + 1 = 3 help us with 2 tens + 1 ten? Turn and talk to your partner.

S: 2 tens + 1 ten = 3 tens is just like 2 + 1 = 3. → It's 2 things and 1 thing make 3 things. 2 circles and 1 circle make 3 circles. 2 bananas and 1 banana make 3 bananas. 2 tens and 1 ten make 3 tens!

T: The numbers stay the same. The numbers, 2 and 1 and 3, stay the same, but the *units* change.

T: (Call up three volunteers to show 2 tens + 1 ten = 3 tens again.) Now, unbundle your magic counting sticks.

S: (Open hands to show 10 fingers.)

T: (Point to the first two students.) What did 2 tens become?

S: 20.

T: (Point to the third student.) What did 1 ten become?

S: 10.

T: What is 20 + 10? Say the number sentence.

S: 20 + 10 = 30.

T: (Write the number sentence on the chart.) When we say 20 + 10 = 30, we'll call this the regular way. When we say the place value units, 2 tens plus 1 ten equals 3 tens, we call this the unit way.

T: Did we change the number of magic counting sticks when we had 2 tens + 1 ten = 3 tens?

S: No.

Elicit responses to make a number bond, and chart responses as shown on Chart 1. Have students fill in the last part of the template on their boards.

Repeat the process by revisiting the previous problems written on the charts, and write them again using only numerals. For example, 1 ten + 3 tens = 4 tens is now written as 10 + 30 = 40.

Next, repeat the process following the suggested sequence for solving subtraction problems as shown on Chart 2: 30 – 10, 30 – 20, 40 – 20, 40 – 40, and 40 – 0. Introduce each expression starting with ones and bananas, then tens, and finally as numerals (e.g., 2 – 1 = 1 → 2 bananas – 1 banana = 1 banana → 2 tens – 1 ten = 1 ten → 20 – 10 = 10).

T: (Write 4 tens – 3 tens on the chart.) What parts of the number bond can we fill in with these numbers?

S: 4 tens on top, with 3 tens as one of the parts. (Show the number bond with 1 ten still missing.)

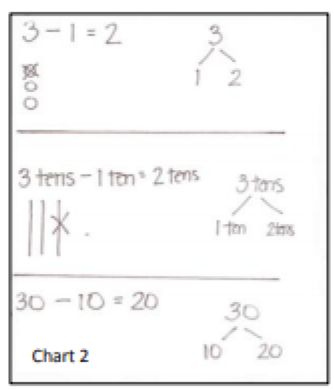


Chart 2

T: What addition sentence can we write to match this number bond? Remember, we can say "unknown" or "mystery number" for the part we don't know yet.

S: 3 tens + "the mystery number" = 4 tens. (Record on the chart.)

T: What is the missing part?

S: 1 ten!

T: (Add the missing part to each section.) Say the subtraction sentence and the related addition sentence we created.

S: 4 tens – 3 tens = 1 ten. 3 tens + 1 ten = 4 tens.

T: Let's say it the regular way, too.

S: 40 – 30 = 10. 30 + 10 = 40.

Repeat the process as needed to support students' understanding.

Sample Anchor Chart for 1.NBT.6 – Add or subtract tens from a multiple of 10

Write a number sentence to match the picture.

Step 1: **Write a *word* sentence to describe the picture, using the symbol "?" for the unknown value**

3 Groups of 10 balloons plus 1 group of 10 balloons equals "?" balloons

Step 2: **Rewrite your word sentence for the new number of groups of 10 objects and write this in place of your '?'**

3 Groups of 10 balloons plus 1 group of 10 balloons equals 4 groups of 10 balloons

Step 3: **Rewrite your word sentence for the *number* of objects in your new number of groups of 10 objects, using repeated addition**

4 groups of 10 balloons equals 10 balloons plus 10 balloons plus 10 balloons plus 10 balloons

Step 4: **Solve your word sentence from Step 3 for the *number* of total objects in the picture**

10 balloons plus 10 balloons plus 10 balloons plus 10 balloons equals 40 balloons

Step 5: **Rewrite your *word* sentence form Step 4 as a *number* sentence, with circles around the original groups of balloons**

10 balloons + 10 balloons + 10 balloons + 10 balloons = 40 balloons

Step 6: **Rewrite your number sentence from Step 6, using the groups you circled to match the original picture.**

30 balloons + 10 balloons = 40 balloons

3.NBT.1 Back to ccss standard

Use place value understanding and properties of operations to perform multi-digit arithmetic.

Use place value understanding to round whole numbers to the nearest 10 or 100.

Skills

1. Understand the rules of rounding in relation to place value

2. Round a whole number to the nearest 10

3. Round a whole number to the nearest 100

4. Complete word problems relating to round numbers to the nearest 10 or 100

Key Concepts/Vocabulary

Round/rounding – Making a number simpler but keeping its value closest to what it was

Place value – The value of where the digit is in a number

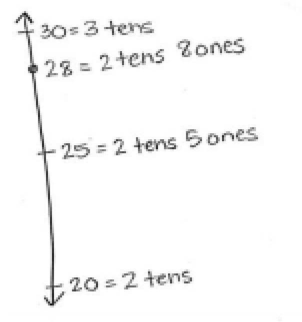
Adding – Finding the total, or sum, by combining two or more numbers

Subtracting – Taking one number away from another

Standard-Specific Resources (3.NBT.1)

- **EngageNY:** **Grade 3, Module 2, Topic C, Lesson 13 – [Round two- and three-digit numbers to the nearest ten on the vertical number line.]**

Concept Development (30 minutes)



Materials: (T) Place value cards (S) Personal white board

Problem 1: Round two-digit measurements to the nearest ten.

T: Let's round 28 minutes to the nearest 10 minutes.

T: How many tens are in 28? (Show place value cards for 28.)

S: 2 tens! (Pull apart the cards to show the 2 tens as 20. Perhaps cover the zero in the ones to clarify the interpretation of 20 as 2 tens.)

T: Draw a tick mark near the bottom of the number line. To the right, label it 20 = 2 tens.

S: (Draw and label 20 = 2 tens.)

T: What is 1 more ten than 2 tens?

S: 3 tens! (Show the place value card for 30 or 3 tens. Again, cover the zero to help clarify.)

T: Draw a tick mark near the top of the number line. To the right, label it 30 = 3 tens.

S: (Draw and label 30 = 3 tens.)

T: What number is halfway between 20 and 30?

S: 25.

T: In unit form, what number is halfway between 2 tens and 3 tens?

S: 2 tens 5 ones.

T: (Show 2 tens 5 ones with the place value cards.) Estimate to draw a tick mark halfway between 20 and 30. Label it 25 = 2 tens 5 ones.

S: (Draw and label 25 = 2 tens 5 ones.)

T: When you look at your vertical number line, is 28 more than halfway or less than halfway between 20 and 30? Turn and talk to a partner about how you know. Then plot it on the number line.

S: 28 is more than halfway between 2 tens and 3 tens. → I know because 28 is more than 25, and 25 is halfway. → I know because 5 ones is halfway, and 8 is more than 5.

T: What is 28 rounded to the nearest ten?

S: 30.

T: Tell me in unit form.

NOTES ON

T: Tell me in unit form.

S: 2 tens 8 ones rounded to the nearest ten is 3 tens.

T: Let's go back to our Application Problem. How would you round to answer the question, "About how long was the ballet recital?" Discuss with a partner.

S: The ballet recital took about 30 minutes. → Rounded to the nearest ten, the ballet recital took 30 minutes.

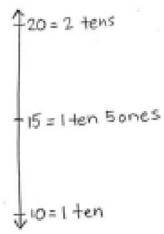
Continue with rounding 17 milliliters to the nearest ten. (Leave the number line used for this on the board. It will be used in Problem 2.)

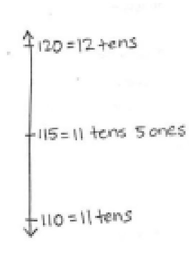
NOTES ON
MULTIPLE MEANS OF ENGAGEMENT:

Alternatively, challenge students who round with automaticity to quickly round 28 minutes to the nearest 10 minutes (without the number line). Students can then write their own word problem for rounding 17 milliliters or 17 minutes.

Problem 2: Round three-digit measurements of milliliters to the nearest ten.

T: To round 17 milliliters to the nearest ten, we drew a number line with **endpoints** 1 ten and 2 tens. How will our endpoints change to round 1 *hundred* 17 to the nearest ten? Turn and talk.





S: Each endpoint has to grow by 1 hundred.

T: How many tens are in 1 hundred? (Show the place value card of 100.)

S: 10 tens.

T: When I cover the ones, we see the 10 tens. (Put your hand over the zero in the ones place.)

T: What is 1 more ten than 10 tens?

S: 11 tens.

T: (Show the place value cards for 10 tens and then 11 tens, that is, 100 and 110.)

T: (Show 117 with the place value cards.)

MP.6

T: How many tens are in 117? Turn and talk about how you know.

S: (Track on fingers.) 10, 20, 30, 40, 50, ..., 110. Eleven tens. → 17 has 1 ten, so 117 has 10 tens, plus 1 ten makes 11 tens. → 110 has 11 tens. → 100 has 10 tens and one more ten is 11 tens.

T: What is 1 more ten than 11 tens?

S: 12 tens.

T: What is the value of 12 tens?

S: 120.

T: What will we label our bottom endpoint on the number line when we round 117 to the nearest ten?

S: 110 = 11 tens.

T: The top endpoint?

S: 120 = 12 tens.

T: (Draw and label endpoints on the vertical number line.)

T: How should we label our halfway point?

S: 115 = 11 tens 5 ones.

T: (Show 11 tens 5 ones with the place value cards.) On your number line, mark and label the halfway point.

S: (Mark and label the halfway point.)

T: Is 117 more or less than halfway between 110 and 120? Tell your partner how you know.

S: It's closer to 120. 17 is only 3 away from 20, but 7 away from 10. → It's more than halfway between 110 and 120.

T: Label 117 on your number line now. (Allow time for students to label 117.) What is 117 rounded to the nearest ten? Use a complete sentence.

S: 117 rounded to the nearest ten is 120.

T: Tell me in unit form with tens and ones.

S: 11 tens 7 ones rounded to the nearest ten is 12 tens.

T: What is 17 rounded to the nearest ten?

S: 20.

T: Again, what is 117 rounded to the nearest ten?

S: 120.

T: Remember from telling time that a number line is continuous. The models we drew to round 17 milliliters and 117 milliliters were the same, even though they showed different portions of the number line; corresponding points are 1 hundred milliliters apart. Discuss the similarities and differences between rounding within those two intervals with your partner.

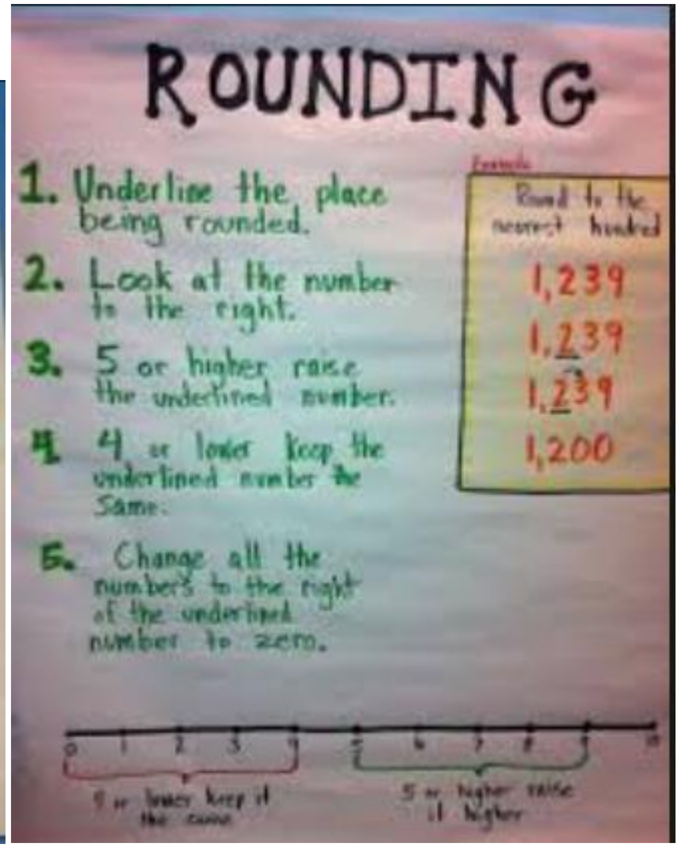
S: All the numbers went in the same place, we just wrote a 1 in front of them all to show they were 1 hundred more. → We still just paid attention to the number of tens. We thought about if 17 was more or less than halfway between 10 and 20.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Reduce the small motor demands of plotting points on a number line by enlarging the number line and offering alternatives to marking with a pencil, such as placing stickers or blocks. Additionally, connect back to yesterday's lesson by using beakers or scales with water or rice.

Continue with rounding the following possible measurements to the nearest ten: 75 mL, 175 mL, 212 g, 315 mL, and 103 kg.

Sample Anchor Charts for 3.NBT.1 – Counting Money with One, Five, and Ten Dollar Bills



4.NBT.5 Back to ccss standard

***Use place value understanding and properties of operations to perform multi-digit arithmetic*.**

Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Skills

1. Multiply a whole number of up to four digits by using place value

2. Multiply a whole number of up to four digits by using properties of operations

3. Multiply two two-digit numbers by using place value and properties of operations

4. Illustrate and explain multiplication calculations by using written equations

5. Illustrate and explain multiplication calculations by using rectangular arrays

6. Illustrate and explain calculations by using area models

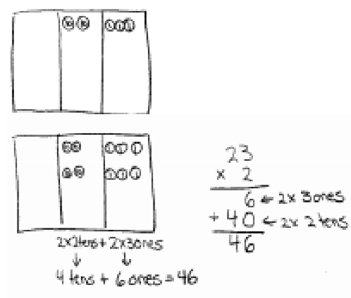
Key Concepts/Vocabulary

Multiply – To repeatedly add the same number
Whole number – Any of the numbers 0, 1, 2, etc. and no fractional part, decimal part, or negative value
Place value – The value of where the digit is in the number
Properties of operations – Commutative, Associative, and Distributive properties
Array – Items (such as objects, numbers, etc.) arranged in rows and columns
Area model – A model for math problems where the length and width are configured using either multiplication, percentage, or fractions to figure out the size of an area

Standard-Specific Resources (4.NBT.5)

- **EngageNY:** **Grade 4, Module 3, Topic C, Lesson 7 – Use place value disks to represent two-digit by one-digit multiplication.**

Concept Development (28 minutes)



Materials: (T) Ten thousands place value chart (Template)
(S) Personal white board, ten thousands place value chart (Template)

Problem 1: Represent 2 × 23 with disks. Write a matching equation, and record the partial products vertically.

T: Use your place value chart and draw disks to represent 23.

T: Draw disks on your place value chart to show 1 more group of 23. What is the total value in the ones?

S: 2 × 3 ones = 6 ones = 6.

T: Write 2 × 3 ones under the ones column. Let's record 2 × 23 vertically.

T: We record the total number for the ones below, just like in addition. (Record the 6 ones as shown above.)

T: Let's look at the tens. What is the total value in the tens?

S: 2 × 2 tens = 4 tens = 40.

T: Write 2 × 2 tens under the tens column. Let's represent our answer in the problem. We write 40 to represent the value of the tens.

T: What is the total value represented by the disks?

S: The total value is 46 because 4 tens + 6 ones = 46.

T: Notice that when we add the values we wrote below the line that they add to 46, the product!

Repeat with 3 × 23.

Problem 2: Model and solve 4 × 54.

T: Draw disks to represent 54 on your place value chart. What is 54 in unit form?

S: 5 tens 4 ones.

T: Draw three more groups of 54 on your chart, and then write the expression 4 × 54 vertically on your personal white board.

T: What is the value of the ones now?

S: 4 × 4 ones = 16 ones.

T: Record the value of the ones. What is the value of the tens?

S: 4 × 5 tens = 20 tens.

T: Record the value of the tens.

T: Add up the **partial products** you recorded. What is the sum?

S: 216.

T: Let's look at our place value chart to confirm.

T: Can we change to make larger units?

MP.4

S: Yes, we can change 10 ones for 1 ten and 10 tens for 1 hundred twice.

T: Show me.

S: (Change 10 smaller units for 1 larger.)

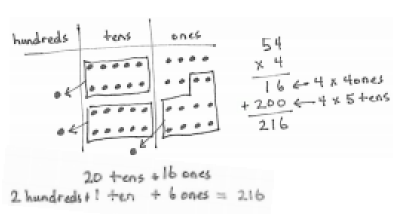
T: What value is represented on the place value chart?

S: 2 hundreds, 1 ten, and 6 ones. That's 216.

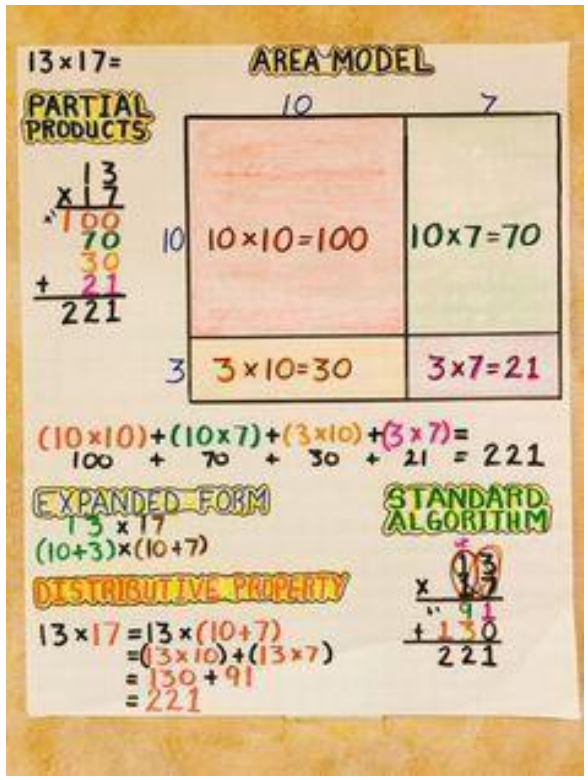
Repeat with 5 × 42.

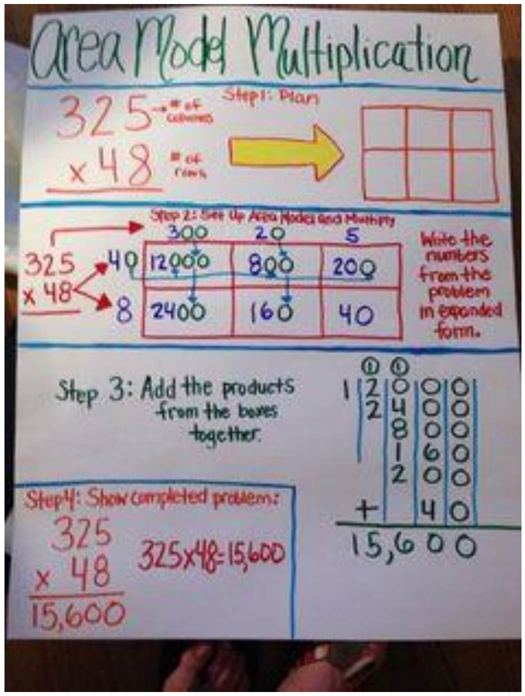
NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Some learners may have difficulty drawing, tracking, and organizing place value disks to represent 4 × 54. A similar demonstration of renaming in the tens and ones place can be shown through 3 × 34. Alternatively, students can model numerals (i.e., writing 4 instead of 4 ones disks).



<u>Sample Anchor Charts for 4.NBT. 5- Multiply Using Place Value, Properties of Operations, and Area Models</u>




Area Model Multiplication
325 → # of columns
x 48 # of rows
Step 1: Plan
Step 2: Set Up Area Model and Multiply
300 20 5
40 12000 800 200
8 2400 160 40
Write the numbers from the problem in expanded form.
Step 3: Add the products from the boxes together.
12000
2400
800
160
200
+ 40
15,600
Step4: Show completed problem:
325
x 48
15,600
325x48=15,600

4.NBT.6 [Back to ccss standard]

Use place value understanding and properties of operations to perform multi-digit arithmetic.

Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Skills

1. Use place value to find whole number quotients and remainders (Partial Quotients)
2. Use properties of operations to find whole number quotients and remainders
3. Illustrate and explain division calculations by using written equations
4. Illustrate and explain division calculations by using rectangular arrays
5. Illustrate and explain division calculations by using area models

Key Concepts/Vocabulary

Divide – To split into equal parts or groups; the result of 'fair sharing'
Quotient – The answer after you divide one number by another
Remainder – The amount left over after dividing one number by another
Dividend – The amount that you want to divide up into equal parts
Divisor – The number you use to divide another number into equal parts
Whole number – Any of the numbers 0, 1, 2, etc. and no fractional part, decimal part, or negative value
Place value – The value of where the digit is in the number
Properties of operations – Commutative, Associative, and Distributive properties
Array – Items (such as objects, numbers, etc.) arranged in rows and columns
Area model – A model for math problems where the length and width are configured using either multiplication, percentage, or fractions to figure out the size of an area

Standard-Specific Resources (4.NBT.6)

- **EngageNY:** **Grade 4, Module 3, Topic E, Lesson 17 – [Represent and solve division problems requiring decomposing a remainder in the tens]()**

Concept Development (34 minutes)

Materials: (T) Tens place value chart (Lesson 16 Template) (S) Personal white board, tens place value chart (Lesson 16 Template)

Problem 1: Divide two-digit numbers by one-digit numbers using place value disks, regrouping in the tens.

3 ones ÷ 2

3 tens ÷ 2

Display 3 ÷ 2 on the board.

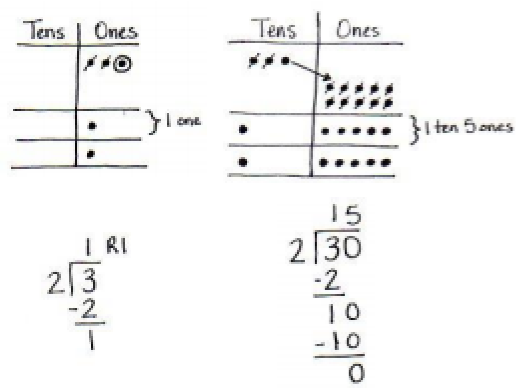
MP.4

T: (Have students model on the place value chart.) 3 ones divided by 2 is...?

S: One with a remainder of 1.

T: Record 3 ÷ 2 as long division.

Students complete the problem. Encourage students to share the relationship of their model to the steps of the algorithm.



Display 30 ÷ 2 on the board.

T: Using mental math, tell your partner the answer to 30 ÷ 2.

S: Thirty divided by 2 is 15.

T: Let's confirm your quotient. Represent 30 on the place value chart. Tell your partner how many groups below are needed.

S: Two. (Draw.)

T: 3 tens divided by 2 is...? Distribute your disks, and cross off what what's been distributed. The answer is...?

S: 1 ten with a remainder of 1 ten. That's an interesting answer.

T: Can we rename the leftover ten?

S: Yes! Change 1 ten for 10 ones.

T: Let's rename 1 ten. Now, rename and distribute the 10 ones with your partner.

S: Our answer is 1 ten 5 ones, or 15.

T: Why didn't we stop when we had a remainder of 1 ten?

S: Because 1 ten is just 10 ones, and you can keep dividing.

T: So, why did we stop when we got a remainder of 1 one?

S: The ones are the smallest unit on our place value chart, so we stopped there and made a remainder.

T: Let's solve 30 ÷ 2 using long division.

T: 3 tens divided by 2?

S: 1 ten.

T: (Record 1 ten. Point to the place value chart.) You recorded 1 ten, twice. Say a multiplication equation that tells that.

S: 1 ten times 2 equals 2 tens.

As students say the multiplication equation, refer to the problem, pointing to 1 ten and the divisor, and record 2 tens.

T: (Point to the place value chart.) We started with 3 tens, distributed 2 tens, and have 1 ten remaining. Tell me a subtraction equation for that.

S: 3 tens minus 2 tens equals 1 ten.

As students say the subtraction equation, refer to the problem, pointing to the tens column, drawing a subtraction line, and recording 1 ten.

T: (Point to the place value chart.) How many ones remain to be divided?

S: 10 ones.

T: Yes. We changed 1 ten for 10 ones. Say a division equation for how you distributed 1 ten or 10 ones.

S: 10 ones divided by 2 equals 5 ones.

As students say the division equation, refer to the problem, pointing to the 10 ones and the divisor, and record 5 ones.

T: (Point to the place value chart.) You recorded 5 ones twice. Say a multiplication equation that tells that.

S: 5 ones times 2 equals 10 ones.

As students say the multiplication equation, refer to the problem, pointing to 5 ones and the divisor, and record 10 ones.

T: (Point to the place value chart.) We renamed 10 ones, distributed 10 ones, and have no ones remaining. Say a subtraction equation for that.

S: 10 ones minus 10 ones equals 0 ones.

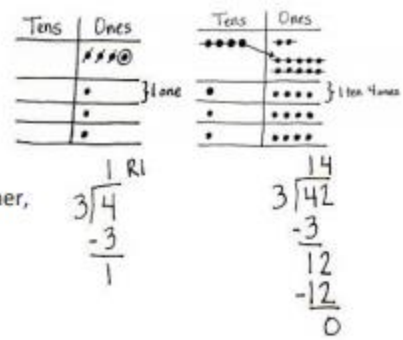
As students say the subtraction equation, refer to the problem, drawing a subtraction line, and record 0 ones.

Have students share with a partner how the model matches the steps of the algorithm. Note that both show equal groups and how both can be used to check their work using multiplication.

Problem 2

4 ones ÷ 3

4 tens 2 ones ÷ 3



Display 4 ÷ 3 on the board.

T: Represent 4 ones on the place value chart. With your partner, solve 4 ÷ 3 using place value disks and long division.

S: The quotient is 1, and the remainder is 1.

Display 42 ÷ 3 on the board.

T: Represent 4 tens 2 ones on the place value chart, and get ready to solve using long division.

T: 4 tens divided by 3 is ...? Distribute your disks, and cross off what is used. The answer is...?

S: 1 ten with a remainder of 1 ten. Oh! I remember from last time, we need to change 1 ten for 10 ones.

T: (With students, draw an arrow to show 1 ten decomposed as 10 ones in the place value chart, and show 12 ones in the algorithm.) How many ones remain?

S: 12.

T: Yes. 10 ones + 2 ones is 12 ones.

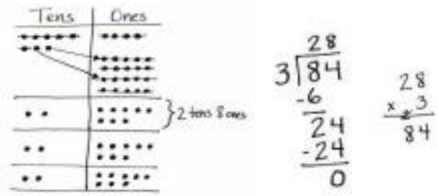
T: Show 12 ones divided by 3. Complete the remaining steps. What is the quotient?

S: Our quotient is 1 ten 4 ones, or 14.

Have students share with a partner how the model matches the steps of the algorithm, paying particular attention to the decomposition of 1 ten and how it is combined with the ones. Note that this is just the same process the students use in subtraction. We decompose a larger unit into smaller units.

Problem 3

8 tens 4 ones ÷ 3



Display 84 ÷ 3 on the board.

T: Solve for 84 ÷ 3 by using place value disks and long division.

S: The quotient is 28.

T: What was different about the place value chart with this problem?

S: There were a lot more disks! → We had to decompose 2 tens this time.

T: How many ones did you have after decomposing your 2 tens?

S: 24 ones.

T: Show your partner where to find 24 ones in the numerical representation.

S: (Students point to the 2 tens remaining that were bundled, as ones, with the 4 ones.)

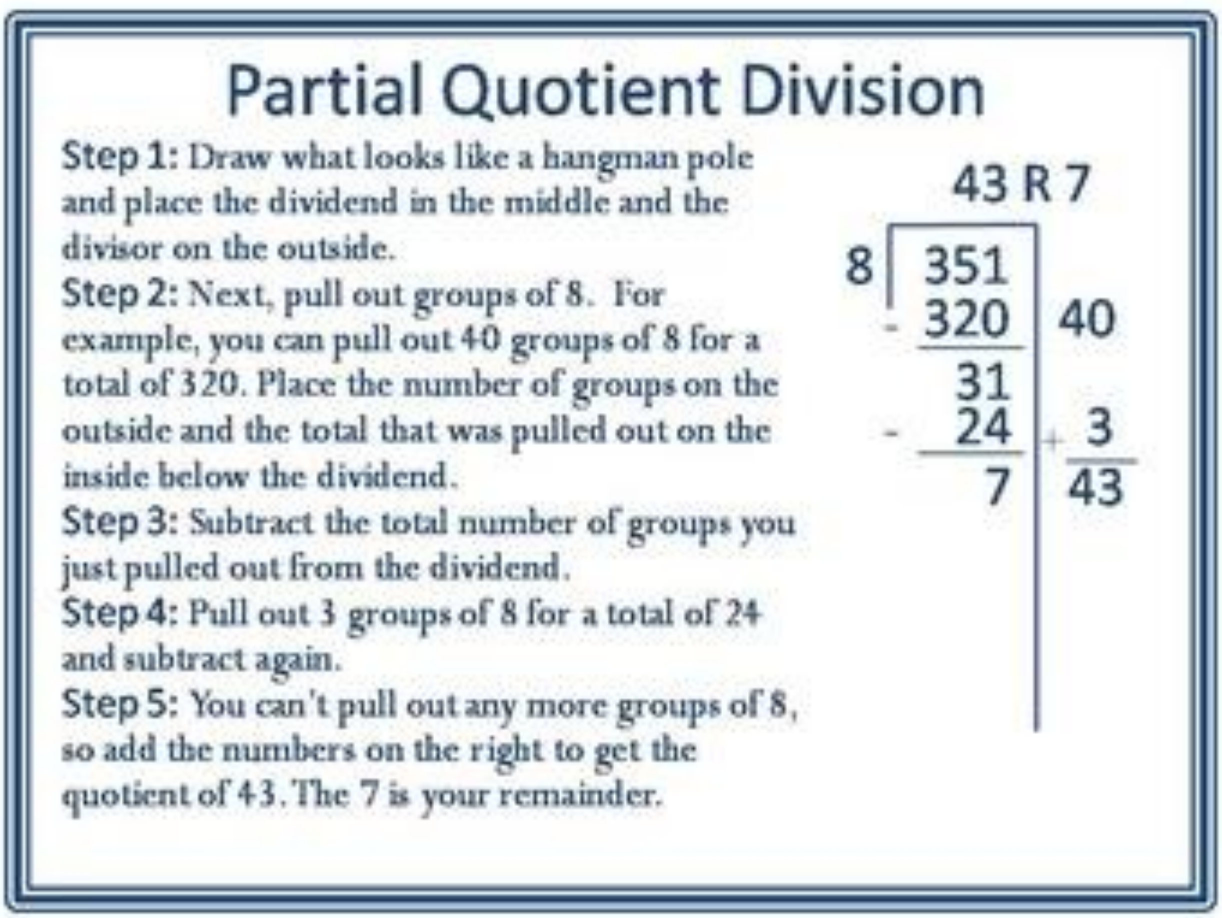
T: Check your answer using multiplication.

S: 28 times 3 is 84. Our answer is right!

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Students working above grade level and others can be encouraged to solve without place value charts to become more efficient at solving long division problems. Allow them to share and explain their method with others.

<u>Sample Anchor Chart for 4.NBT.6 - Division Strategy: Partial Quotients</u>



5.NBT.7

[Back to ccss standard]

Perform operations with multi-digit whole numbers and with decimals to hundredths.

Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Skills

1. Add decimals to hundredths, relating a model to the strategy used
2. **Subtract decimals to hundredths, relating a model to the strategy used**
3. **Multiply decimals to hundredths, relating a model to the strategy used**
4. **Divide decimals to hundredths, relating a model to the strategy used**

Key Concepts/Vocabulary

Decimal – Based on 10, a number that uses a decimal point followed by digits that show a value smaller than one

Whole number – Any of the numbers 0, 1, 2, 3, etc., with no fractional or decimal part, and no negative value

Tenth – One part in 10 equal parts

Hundredth – One part in 100 equal part

Standard-Specific Resources (5.NBT.7)

- **EngageNY:** **Grade 5, Module 1, Topic D, Lesson 10 – Subtract decimals using place value strategies, and relate those strategies to a written method.**

Concept Development (35 minutes)

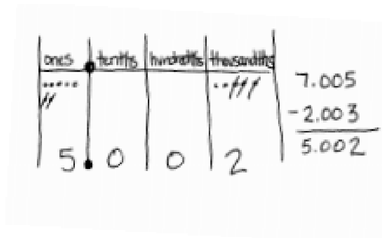
Materials: (S) Hundreds to thousandths place value chart (Lesson 7 Template), personal white board

Problem 1

5 tenths – 3 tenths

7 ones 5 thousandths – 2 ones 3 thousandths

9 hundreds 5 hundredths – 3 hundredths



T: (Write 5 tenths – 3 tenths on the board.) Let's read this expression aloud together. Turn and tell your partner how you'll solve this problem, and then find the difference using your place value chart and disks.

T: Explain your reasoning when solving this subtraction expression.

S: Since the units are alike, we can just subtract. 5 – 3 = 2. → This problem is very similar to 5 ones minus 3 ones, or 5 people minus 2 people. The units may change, but the basic fact 5 – 2 = 3 is the same.

T: (Write 7 ones 5 thousandths – 2 ones 3 thousandths on the board.) Find the difference. Solve this problem with the place value chart and disks. Record your thinking vertically, using the algorithm.

S: (Solve.)

T: What did you have to think about as you wrote the problem vertically?

S: Like units are being subtracted, so my work should also show that. Ones with ones and thousandths with thousandths.

T: (Write 9 hundreds 5 hundredths – 3 hundredths on board.) Solve 9 hundreds 5 hundredths – 3 hundredths. Read carefully, and then tell your neighbor how you'll solve this problem.

S: In word form, these units look similar, but they're not. I'll just subtract 3 hundredths from 5 hundredths.

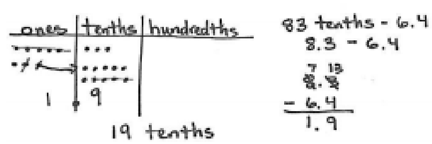
T: Use your place value chart to help you solve, and record your thinking vertically.

> **NOTES ON MULTIPLE MEANS OF ENGAGEMENT:**
>
> Support oral or written responses with sentence frames, such as _____ *is* _______ *hundredths*. Allow the use of place value charts and the sentence frames to scaffold the process of converting units in subtraction. Some students need concrete materials to support their learning, as renaming in various units may not yet be an abstract construct for them.

Problems 2–3

83 tenths – 6.4

9.2 – 6 ones 4 tenths



T: (Write 83 tenths – 6.4 = ____ on the board.) How is this problem different from the problems we've seen previously?

S: This problem involves regrouping.

S: (Solve using disks, recording their work in the standard algorithm.)

T: Share how you solved.

S: We had to regroup before we could subtract tenths from tenths. Then, we subtracted ones from ones using the same process as with whole numbers.

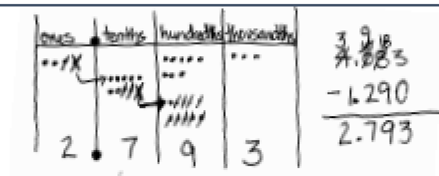
Repeat the sequence with 9.2 – 6 ones 4 tenths. Students may use various strategies to solve. Comparison of strategies makes for interesting discussion.

Problems 4–5

0.831 – 0.292

4.083 – 1.29

6 – 0.48



T: (Write 0.831 – 0.292 on the board.) Use your disks to solve. Record your work vertically using the standard algorithm.

S: (Write and share.)

T: (Write 4.083 – 1.29 on the board.) What do you notice about the thousandths place? Turn and talk.

S: There is no digit in the thousandths place in 1.29. → We can think of 29 hundredths as 290 thousandths. In this case, I don't have to change units because there are no thousandths that must be subtracted.

T: Solve with your disks and record.

NOTES ON: MULTIPLE MEANS OF ENGAGEMENT:

Students may be more engaged with the concept of adding and subtracting decimal fractions when reminded that these are the same skills needed for managing money.

Repeat the sequence with 6 – 0.48. While some students may use a mental strategy to find the difference, others will use disks to regroup in order to subtract. Continue to stress the alignment based on like units when recording vertically. When the ones place is aligned, students will recognize that there are not as many digits in the minuend of 6 wholes as in the subtrahend of 48 hundredths. Ask, "How can we think about 6 wholes in the same units as 48 hundredths?" Then, lead students to articulate the need to record 6 ones as 600 hundredths or 6.00 in order to subtract vertically. Ask, "By decomposing 6 wholes into 600 hundredths, have we changed its value?" (No, we just converted it to smaller units—similar to exchanging six dollars for 600 pennies.)

Sample Anchor Charts for 5.NBT.7 - Operations with Decimals

Multiplying and Dividing by 10, 100 and 1000

10 000	1000	100	10	1 ●	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
				●			

Multiplying

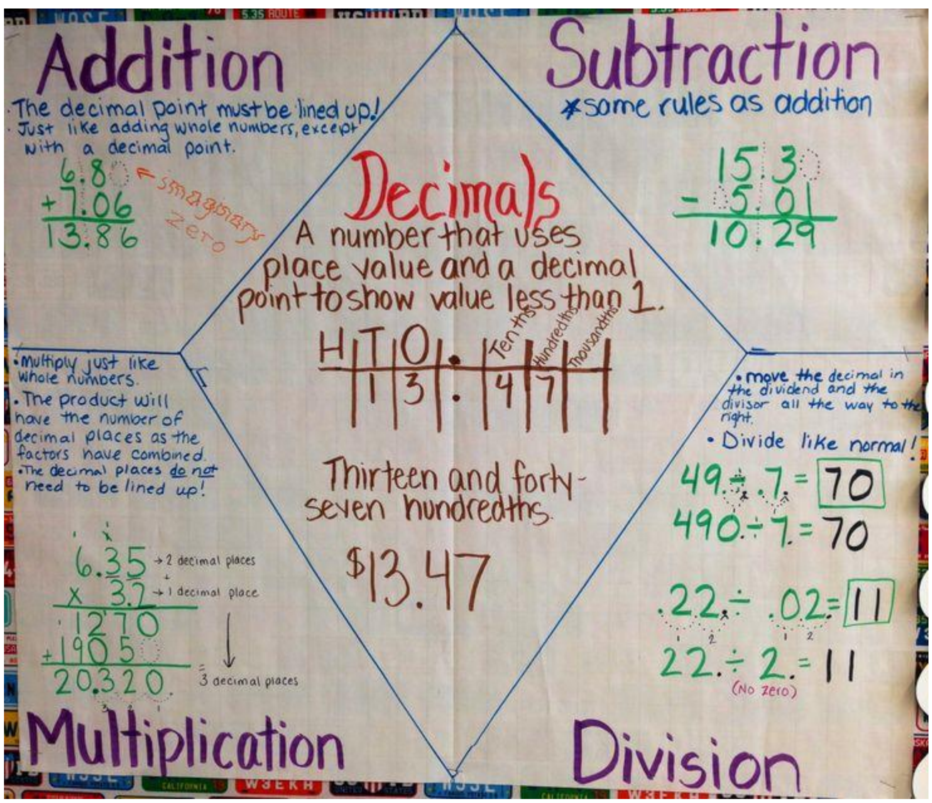
X 10 digits move LEFT **1** space
X 100 digits move LEFT **2** spaces
X 1000 digits move LEFT **3** spaces

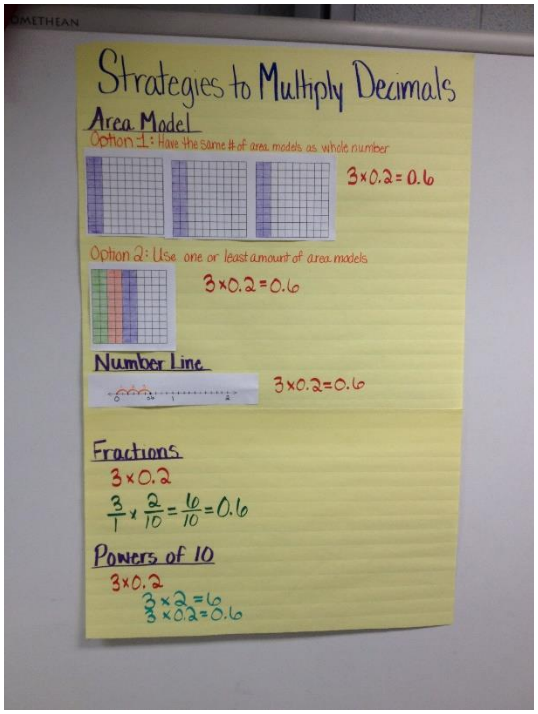
←

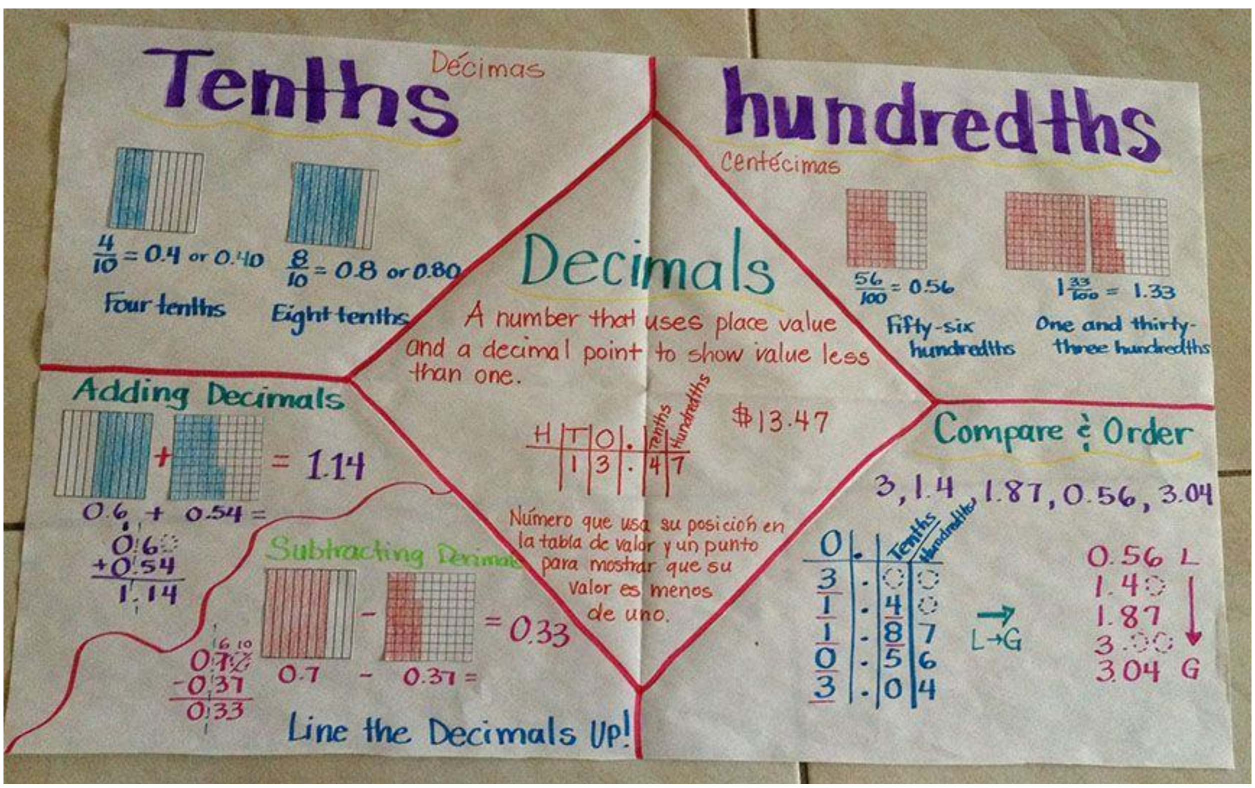
Dividing

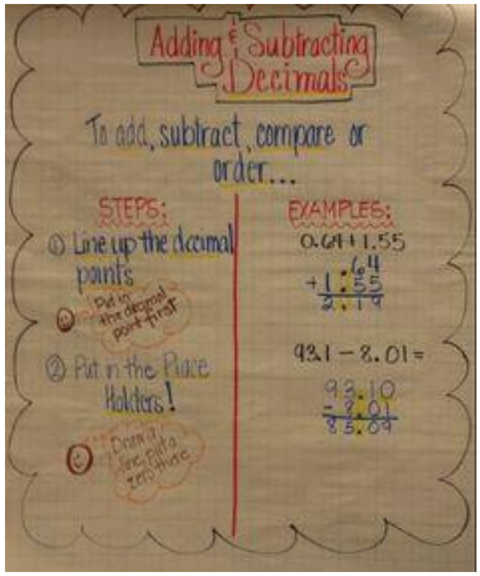
÷ 10 digits move RIGHT **1** space
÷ 100 digits move RIGHT **2** spaces
÷ 1000 digits move RIGHT **3** spaces

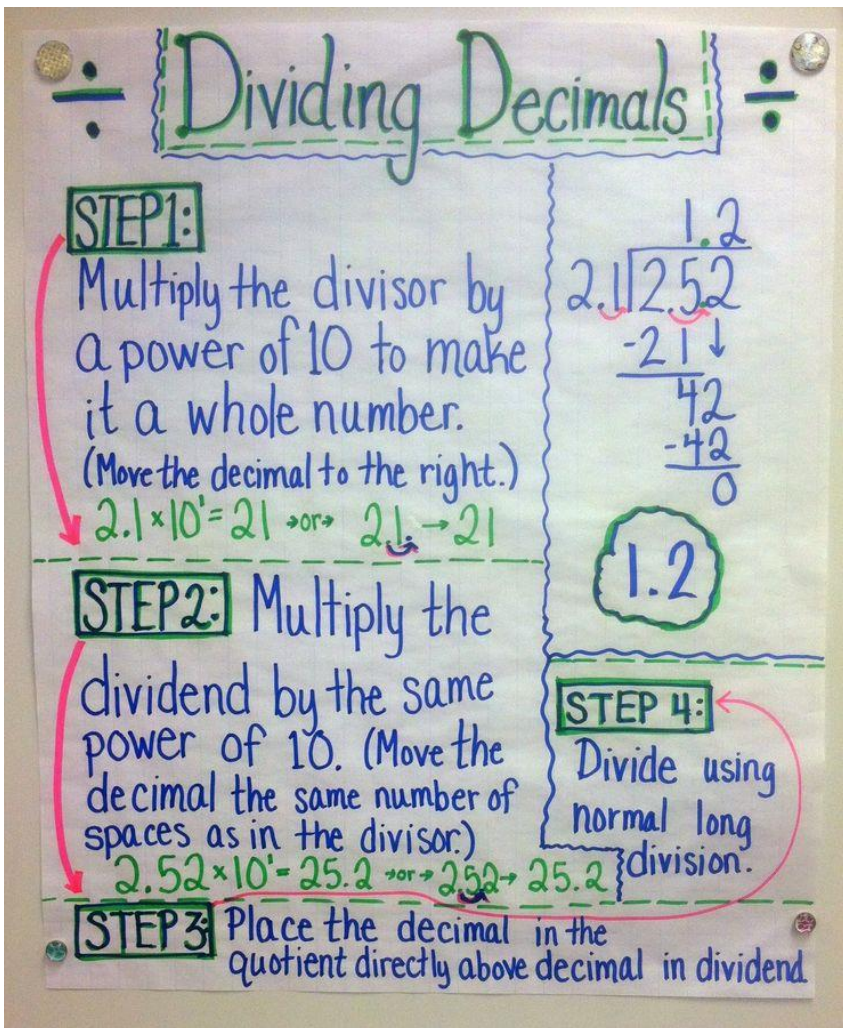
→




Strategies to Multiply Decimals
Area Model
Option 1: Have the same # of area models as whole number
3×0.2=0.6
Option 2: Use one or least amount of area models
3×0.2=0.6
Number Line
3×0.2=0.6
Fractions
3×0.2
3/1 × 2/10 = 6/10 = 0.6
Powers of 10
3×0.2
3×2=6
3×0.2=0.6

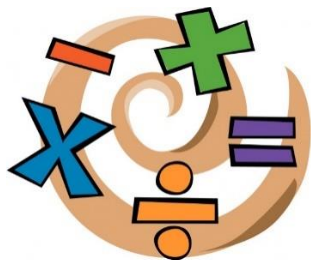

Tenths
Décimas
4/10 = 0.4 or 0.40
8/10 = 0.8 or 0.80
Four tenths
Eight tenths
hundredths
Centécimas
56/100 = 0.56
Fifty-six hundredths
1 33/100 = 1.33
One and thirty-three hundredths
Decimals
A number that uses place value and a decimal point to show value less than one.
H T O . tenths hundredths
1 3 . 4 7
$13.47
Número que usa su posición en la tabla de valor y un punto para mostrar que su valor es menos de uno.
Adding Decimals
= 1.14
0.6 + 0.54 =
0.6
+0.54
1.14
Subtracting Decimals
= 0.33
0.7 - 0.37 =
0.7
-0.37
0.33
Line the Decimals Up!
Compare & Order
3, 1.4, 1.87, 0.56, 3.04
O . Tenths Hundredths
3 .
1 . 4
1 . 8 7
0 . 5 6
3 . 0 4
L→G
0.56 L
1.4
1.87
3.
3.04 G


Adding & Subtracting Decimals
To add, subtract, compare or order...
STEPS:
EXAMPLES:
① Line up the decimal points
0.64+1.55
② Put in the Place Holders!
93.1 − 8.01 =


Dividing Decimals
STEP 1:
Multiply the divisor by a power of 10 to make it a whole number.
(Move the decimal to the right.)
2.1 × 10¹ = 21 →or→ 2.1 → 21
STEP 2:
Multiply the dividend by the same power of 10. (Move the decimal the same number of spaces as in the divisor.)
2.52 × 10¹ = 25.2 →or→ 2.52 → 25.2
STEP 3:
Place the decimal in the quotient directly above decimal in dividend
STEP 4:
Divide using normal long division.
1.2
2.1 | 2.52
-21
42
-42
0
1.2

Operations and Algebraic Thinking

(OA)



2.OA.1 Back to ccss standard

Represent and solve problems involving addition and subtraction.

Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g. by using drawings and equations with a symbol for the unknown number to represent the problem.

Skills

1. Identify the unknown value in a one-step addition word problem and write a subtraction equation to represent the situation

2. Write an addition equation with a symbol for the unknown in a two-step addition word problem

3. Identify the unknown value in a one-step subtraction word problem and write a subtraction equation to represent the situation

2. Write a subtraction equation with a symbol for the unknown in a two-step subtraction word problem

5. Determine operation needed to solve addition and subtraction problems in situations including add to, take from, put together, take apart, and compare

Key Concepts/Vocabulary

Identify – To recognize a particular thing
Unknown – A number that is not known, that must be found
Addition – Finding the total, or sum
Subtraction – Finding the difference, or removing objects from a collection
Equation – A written statement that 2 things, or groups of things, are equal to each other
Symbol – A pattern or image used instead of words
Position – Where something is located, usually in relation to something else
Determine – To come to a decision
Operation – A mathematical process, most often addition, subtraction, multiplication and division
Solve – Figure out the answer, or the value of an unknown number
'Add to' – A given number plus a new amount
'Take from' – A given number minus a new amount
'Put together' – Two given numbers combined through addition
'Take apart' – A total minus a part, or a total minus a new amount
Compare – To examine the difference and similarities between two thing

Standard-Specific Resources (2.OA.1)

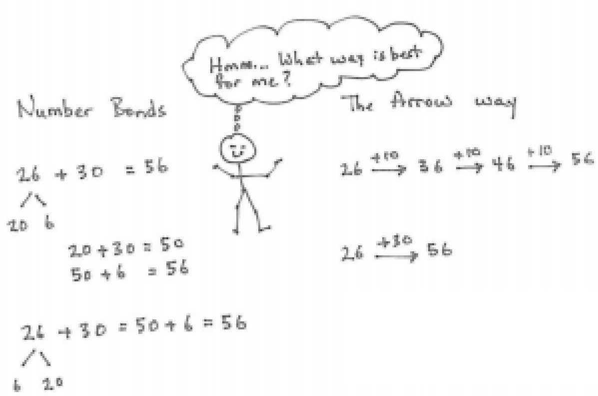
- **<u>EngageNY:</u> Grade 2, Module 4, Topic A, Lesson 2 - [Add and subtract multiples of 10, including counting on to subtract]()**

Concept Development (32 minutes)

Materials: (T) Rekenrek (S) Personal white board

Show 34 beads on the Rekenrek.

T: In Lesson 1, we added and subtracted 1 ten. Today, let's add 2 tens, then 3 tens, and more!
T: How many do you see?
S: 34.
T: The Say Ten way?
S: 3 tens 4.
T: (Add 2 tens.) How many do you see?
S: 5 tens 4.
T: I am going to add 2 more tens. Turn and talk. What will happen to the number when I add 2 tens?
S: The number in the tens place will get bigger by 2. → The number will get bigger by 20. → It will be 74.
T: (Add 2 tens.) What is 54 + 20?
S: 74.
T: The Say Ten way?
S: 7 tens 4.
T: If I asked you to add 3 tens to 26, how could you solve that?
S: Count on by ten three times. → Change the 2 to 5 because 2 tens plus 3 tens is 5 tens. → Add 3 tens on the Rekenrek.
T: Let's show that on the board using both simplifying strategies, the arrow way and number bonds. I know many of you can just do mental math!



T: I can write adding 3 tens the arrow way, as we did yesterday. (Demonstrate and involve the students as you write.) I can also break apart the tens and ones with a number bond, add the tens, and then add the ones. (Demonstrate and involve the students as you write.)
T: No matter which way I write it, when I add tens to a number, the ones stay the same!

Note: The number bond's decomposition is one choice for solving the problem that may not work for some students as a solution strategy but is beneficial for all to understand. Students should be encouraged to make connections between different solution strategies and to choose what works best for a given problem or for their way of thinking.

MP.3

T: Now it's your turn. On your personal white board, solve 18 + 20. Show your board when you have an answer.

Repeat this process using the following possible sequence: 25 + 50, 38 + 40, and 40 + 27.

Show 74 beads on the Rekenrek.

T: Now, let's subtract 2 tens, then 3 tens, and more!

T: How many do you see?

S: 74.

T: The Say Ten way?

S: 7 tens 4.

T: (Subtract 2 tens.) How many do you see?

S: 5 tens 4.

T: I am going to subtract 2 more tens. Turn and talk. What will happen to the number when I subtract 2 tens?

S: The digit in the tens place will get smaller by 2. → The number will get smaller by 20. → It will be 34.

T: (Add 2 tens.) What is 54 – 20?

S: 34.

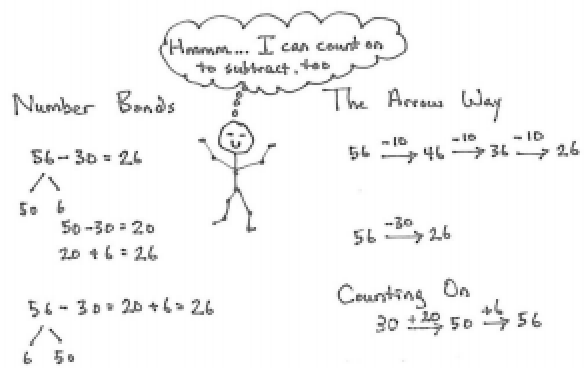
NOTES ON MULTIPLE MEANS OF REPRESENTATION:

When counting up by tens and on by ones, use a number line to provide visual support. For example, when counting from 30 to 42, have students point to the jump between 30 and 40 and then point to 41, 42. The number line correlates very well to the arrow notation.

T: The Say Ten way?

S: 3 tens 4.

T: Okay. Now, subtract 3 tens from 56. Take a moment and work on your personal white board to solve 56 – 30. (Show the work on the board as students work out this first problem using number bonds and the arrow way.)



MP.3

T: (Model both the number bonds and arrow methods from their work.) We have an extra simplifying strategy when we are subtracting. We can count up from the part we know.

T: What is the whole?

S: 56.

T: What is the part we know?

S: 30.

T: How could we show the missing part with an addition problem?

S: 30 + ___ = 56. → ___ + 30 = 56.

T: We can use the arrow way, counting first by either tens or ones. Try it with a partner.

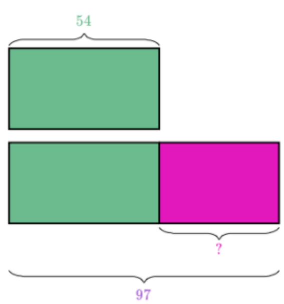
Guide students through this or let them work independently. Starting at 30, they might add 2 tens first and then 6 ones or add 6 ones first and then add 2 tens.

Repeat with 62 – 40, 51 – 20, and 77 – 30.

Sample Anchor Chart #1 for 2.OA.1 – Solving Subtraction and Addition Word Problems with Pictures and Equations

The Animal Shelter has 54 cats and 97 dogs. *How many fewer cats than dogs does the Animal Shelter have?*

<u>Step 1:</u> Draw and label two tape diagrams to represent the amount of cats and dogs, then label the difference with a question mark



<u>Step 2:</u> Write a subtraction number sentence to represent your drawing

To find how many fewer cats than dogs are at the animal shelter, we need the difference, so we subtract.

$97 - 54 = \square$

<u>Step 3:</u> Rewrite your subtraction sentence as an addition sentence to represent your drawing

Or we can solve using addition:

$54 + \square = 97$

<u>Step 4:</u> Solve for the unknown value!

Or we can solve using addition:

$54 + \square = 97$

Sample Anchor Chart #2 for 2.OA.1 – Add and subtract within 100

What number makes this equation true?

$\square + 72 = 95$

Step 1: **Rewrite the number sentence, with a "?" in place of the unknown number (? + 72 = 95)**

$$? + 72 = 95$$

Step 2: **Add groups of 10 to the smaller number (72) until you get close to the larger number (95) *without going over it***

$72 + 10 = 82$

$82 + 10 = 92$

If we add 2 tens, or 20, we reach 92. We cannot add anymore tens without going over 95.

Step 3: **Look at your new number (92) and count how many ones it will take to get to the bigger number (95)**

$$92 + 3 = 95$$

Step 4: **Count how many times you added 10, in (92) 2, (2 10s, or 2 tens = 20) and add this to the number of ones you added in Step 3 (3 1s, or 3 ones)**

We added 2 tens and 3 ones to 72 to get to 95.

$20 + 3 = 23$

Step 6: **Rewrite your number sentence from Step 1, with your answer from Step 4 in place of your symbol (the "?")**

$$23 + 72 = 95$$

Step 7: **Rewrite your number sentence from Step 6, so that only 1 number is on the left and other number is subtracted from the number on the right (23 = 95 – 72)**

Step 8: **Explain how you know that *both* the number sentences in Steps 6 and 7 *are true***

(95 is made up of 23 and 72, so 95 minus 72 is 23, and 95 minus 23 would be 72)

3.OA.2 [Back to ccss standard]

Represent and solve problems involving multiplication and division.

Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. ***For example, describe a context in which a number of shares or a number of groups can be expressed as*** **56 ÷ 8.**

Skills

1. Know what each of the numbers in a division problem represent

2. Explain what division means **using concrete objects**

3. Explain what division means using drawings

4. Explain how division relates to multiplication

5. Interpret quotients as the number of shares per group when a set of objects is into equal groups

6. Interpret quotients as the number of groups when a set object is divided into equal shares

Key Concepts/Vocabulary

Division – To find the total, or sum
Represent – To symbolize or 'stand for'
Relate – Make or show a connection between
Equal – Two things that are the same amount, measure, value, or number
Share – Splitting into equal parts or groups; or, the number of equal parts in each group, for a set of groups
Interpret - Make meaning of; or, explain
Set – A collection of objects, considered an object in itself ('a set')
Quotient – The answer when you divide one number by another number

Standard-Specific Resources (3.OA.2)

- **EngageNY:** **Grade 3, Module 1, Topic B, Lesson 4 – [Understand the meaning of the unknown as the size of the group in division.]**

5

Concept Development (30 minutes)

Materials: (S) Personal white board, 18 counters

Concrete to abstract: Division as fair-share, relate the answer to the unknown factor.

MP.2

T: Yesterday, Mr. Ziegler bought a new pack of 18 markers. He shared them with me by dividing them into 2 equal groups. Now, I have a bunch of new markers for making our charts! Do you want to know how many he gave me?

S: Yes.

T: What are we trying to find, the number of groups or the size of the group?

S: The size of the group.

T: Your 18 counters represent the markers. Divide your 18 counters into 2 equal groups by giving one to Mr. Z, one to me, one to Mr. Z, one to me. (Model partitioning.)

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

This may be students' first time independently dividing in a formal context. Life experience has likely taught them the fair-share strategy of going back and forth to give 1 and 1, 2 and 2, 3 and 3, etc., until there are no more to distribute. Encourage those who are unsure what to do, or who are using a less efficient strategy, toward fair-share.

MP.2

S: (Divide using the fair-share strategy.)

T: Using a complete sentence, tell how many counters are in each group.

S: There are 9 counters in each group.

T: Then, how many markers did Mr. Ziegler give me?

S: 9 markers!

T: Let's write a number sentence to show our work, starting from the beginning. What is our total number of counters?

S: 18 counters.

T: (Write 18 on the board.) We divided our 18 counters into how many equal groups?

S: We divided into 2 equal groups.

T: (Write ÷ 2 = ____ on the board next to the 18.)

T: If 18 is our total and 2 represents our equal groups, then remind me, what does our **unknown** factor represent? (Point to where the answer will go.)

S: The size of the groups.

T: That is?

S: 9.

T: 18 divided by 2 equals 9. (Finish writing as you read 18 ÷ 2 = 9.)

T: How many markers did Mr. Ziegler give me...?

S: 9 markers!

Repeat the process with 15 ÷ 3 = ___: Suppose Mr. Ziegler had 15 markers and shared fairly with 3 teachers. This time, also review that ÷ means to divide.

T: In what ways does dividing remind you of our work with multiplication?

S: It's also about the size of groups and the number of groups, but we used a different symbol. → It still uses factors and a total. → This time the total is not the answer. It's the beginning! → So, the answer has to do with groups, not the total.

T: Right. We multiply when we want to find the total. Here, we divided when we knew the total and wanted to find the size of the groups.

Pictorial to abstract: Analyze a picture to write a division sentence in which the solution tells the size of the group.

T: (Project or draw the following image.) This is how Diana arranges her star stickers.

T: What does 12 represent in the picture?

S: The total number of Diana's star stickers.

T: What does 3 represent?

S: The number of equal groups.

T: What does 4 represent?

S: The size of each group.

T: Write a number sentence to represent Diana's stickers where the answer represents the size of the group.

S: (Write $12 \div 3 = 4$.)

T: (Write $12 \div 3 = 4$ and $12 \div 4 = 3$ on the board, even if students have written the correct number sentence.) What is the difference between these **division** sentences?

S: In the first one, the answer represents the size of each group. In the second one, the answer represents the number of groups.

T: If we're writing a division sentence where the answer represents the size of the group, then which number sentence should we use?

S: $12 \div 3 = 4$.

Abstract to pictorial: Analyze equations for the meaning of the solution and represent the equation with a drawing.

Write $8 \div 4 =$ ____.

T: If 8 is the total and 4 is the number of groups, then what does the unknown factor represent?

S: The size of the groups!

T: Draw a picture on your personal white board to go with my division equation. Use your picture to help you find the unknown factor, then write the complete equation.

S: (Draw various pictures that show $8 \div 4$, then write $8 \div 4 = 2$.)

Repeat the process with $10 \div 2$. While designing examples, keep in mind that Lesson 5 introduces students to division where the unknown factor represents the number of groups.

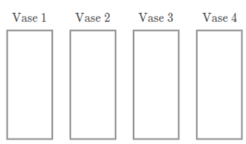
Sample Anchor Chart for 3.OA.2 - Solving Division Problems with Visuals

There are 20 flowers. We want to divide them equally into 4 vases. *How many flowers will be in each vase?*

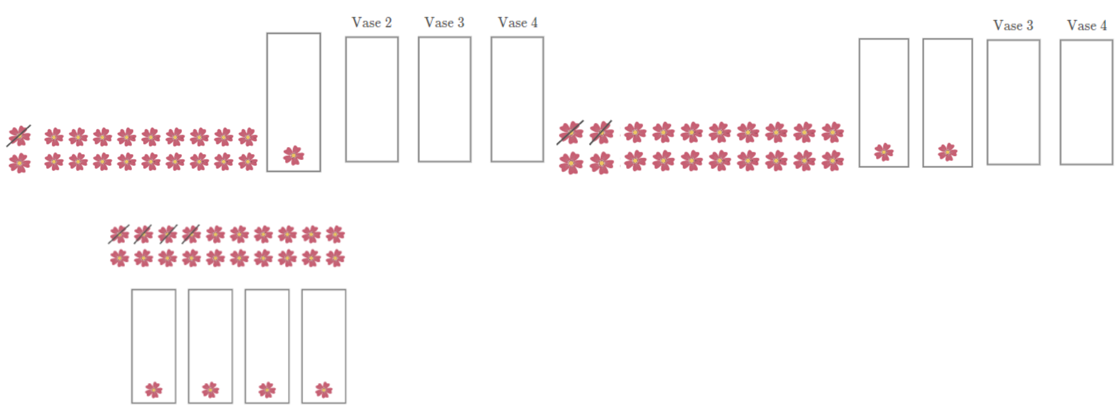
Step 1: Write a number sentence to represent the division problem that you need to solve **(20 ÷ 4 = ?)**.

Step 2: Draw a picture of the *bigger* number (20 flowers)

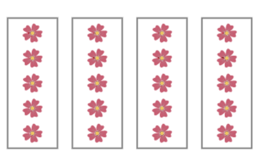
Step 3: Draw a picture of the *smaller* number (4 vases)



Step 4: Cross out 1 flower and redraw it inside the first of your 4 vases. Cross out another flower and redraw it inside the second of your vases.



Step 5: Count how many flowers are now in each vase/the *quotient* (5 flowers in each vase, or 5 flowers per vase).



Step 6: Rewrite your division sentence, replacing the question mark with the quotient you found (the number of flowers in each vase) (20 ÷ 4 = 5, or 20 flowers in 4 vases means there are 5 flowers in each vase).

3.OA.3 [Back to ccss standard]

Represent and solve problems involving multiplication and division.

Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Skills

1. Multiply and divide within 100
2. Solve word problems involving equal groups
3. Solve word problems involving arrays
4. Solve word problems involving measurement quantities
5. Represent a word problem using a picture
6. Represent a word problem using an equation with a symbol for the unknown number

Key Concepts/Vocabulary

Multiply – To repeatedly add the same number
Divide – To break up by a certain number, or number of times
Equal – Two things that are the same amount, measure, value, or number
Group – A number of things that are considered or classed together
Array – Items (such as objects, numbers, etc.) arranged in rows and columns
Measurement – A number that shows the size or amount of something
Quantity – How much there is of something
Represent – To 'act or speak for'
Equation – A written statement that 2 things, or groups of things, are equal to each other
Symbol – A pattern or image used instead of words
Solve – Find an answer
Unknown – A number that is not known, that must be found

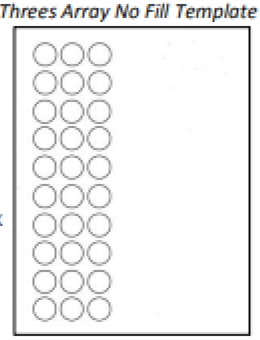
Standard-Specific Resources (3.OA.3)

- **<u>EngageNY:</u> Grade 3, Module 1, Topic C, Lesson 9 – [Find related multiplication facts by adding and subtracting equal groups in array models]()**

Concept Development (35 minutes)

Materials: (S) Personal white board, threes array no fill (Template) (pictured on the right), blank paper

Threes Array No Fill Template



Problem 1: Add two known smaller facts to solve an unknown larger fact.

T: Slip the template into your board. Cover part of the array with blank paper to show 5 rows of 3. Draw a box around the uncovered array. Write and solve a multiplication sentence to describe it.

S: (Cover, then box array, and write 5 × 3 = 15.)

T: Move the paper so the array shows 7 × 3. Shade the rows you added.

S: (Shade 2 rows.)

T: Write and solve a multiplication sentence to describe the shaded part of your array.

S: (Write 2 × 3 = 6.)

T: How many threes are in 5 × 3?

S: 5 threes.

T: How many threes did you add to 5 × 3 to make the array show 7 × 3?

S: 2 threes.

T: (Write 7 threes = 5 threes + 2 threes.) So, 7 threes equals 5 threes plus 2 threes.

T: (Write 7 × 3 = 5 × 3 + 2 × 3 as shown to the right.) Do you agree or disagree?

S: I agree. That's just adding the two parts of the array together. → 7 rows of three is the same as 5 rows of three plus 2 rows of three.

T: We already wrote totals for the two parts of our array. Let's add those to find the total for the whole array. What is the total of 5 × 3?

S: 15.

T: (Write 15 + on the board.) What is the total of 2 × 3?

S: 6.

T: (Add to the board so the equation reads ____ = 15 + 6.) Say the total at the signal. (Signal.)

S: 21.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Decomposing this way naturally relates to the part–whole relationship that students studied in Grades K–2. The vignette implies the relationship, but a more formal connection to prior knowledge may be appropriate for some classes.

Sample Teacher Board

7 threes = 5 threes + 2 threes

7 × 3 = 5 × 3 + 2 × 3

21 = 15 + 6

Provide students with another example. Have them use the template to add the totals of 4 × 3 and 4 × 3 to find the answer to 8 × 3. Teach them to double the total for 4 × 3.

T: Explain how we added to find 7 × 3 = 21 and 8 × 3 = 24.

S: We added the totals of smaller facts together to find the whole. → We used two facts we already knew to find one we didn't know.

Problem 2: Subtract two known smaller facts to solve an unknown larger fact.

T: Draw a box around an array that shows 9 × 3. Notice that 9 × 3 is very close to 10 × 3. 10 × 3 is easier to solve because we can count by tens to get the total. Let's do that now.

S: 10, 20, 30.

T: Let's use 10 × 3 = 30 to help us solve 9 × 3.

T: Use your finger to trace 10 threes.

T: What should we subtract to show 9 threes instead?

S: 1 three!

T: (Write 10 threes – 1 three = _____ on the board.) 10 threes equals?

S: 30.

T: 30 – 3 equals?

S: 27.

Provide another example. Have students subtract to find the answer to 8 × 3. 10 × 3 is the basic fact, so the subtraction to find 8 × 3 is 30 – 6.

T: Tell your partner how we used 10 × 3 to help us find the answer to 9 × 3 and 8 × 3.

S: (Discuss.)

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

The second example for subtraction (8 × 3) is intentionally the same as the second example for addition. Solving the same problem in two ways provides an opportunity for students to compare the strategies. Ask students who benefit from a challenge to analyze the strategies independently or in pairs, and then present their thinking to others during the Debrief.

NOTES ON VOCABULARY:

Introduce the word *distribute* into everyday classroom language. This will help with students' understanding of the distributive property, which is formally introduced in Lesson 16.

For example, "Paper monitors, please distribute the papers to the class."

Sample Anchor Chart for 3.OA.3 - Solving Multiplication and Division Word Problems with Tape Diagrams

On a table at a book fair, there are 3 rows of books with 6 books in each row.

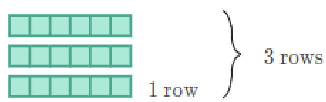
What is the number of books on the table?

Step 1: **Use a tape diagram to draw one of the equal groups of items**

(one row of 6 books) 

Step 2: **Draw the remaining number of equal groups, exactly as you did the first**

(2 more rows of 6 books, making 3 rows of 6 books)



Step 3: **Write a multiplication number sentence to represent what you drew in your tape diagrams, with a question mark for product that you need to find (3 rows x 6 books in each row = ?**

[total # of books], or 3 x 6 = ?)

Step 4: **Solve your multiplication number sentence by counting the total number of items represented in your diagrams**

(6 + 6 + 6 = 3 x 6 = 18)

Step 5: **Rewrite your multiplication number sentence as a division number sentence**

(18 books divided into 3 equal rows = 6 books in each row, or 18 ÷ 3 = 6)

3.OA.5 Back to ccss standard

Understand properties of multiplication and the relationship between multiplication and division. .

Apply properties of operations as strategies to multiply and divide. ***Examples: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known. (Commutative property of multiplication.) 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or by 5 x 2 = 10, then 3 x 10 = 30. (Associative property of multiplication.) Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16 = 56. (Distributive property.)***

Skills

1. Understand and apply the Commutative Property to multiply.
2. Understand and apply the Associative Property to multiply
3. Understand and apply the Distributive Property to multiply
4. Understand and apply the Distributive Property to divide

Key Concepts/Vocabulary

Multiply – To repeatedly add the same number

Divide – To break up by a certain number, or number of times

Operation – A mathematical process

Commutative Laws – For addition and multiplication, when you swap the order of numbers, you still get the same answer (6 +3 = 3 + 6, or (a+b = b+a)

Associative Laws – For addition and multiplication, it doesn't matter *how* you group the numbers (i.e. which are calculated first) [6 + (3 + 4) = (6 + 3) + 4, or a + (b+c) = (a+b) + c]

Distributive Law – For all 4 common operations (+, -, /, *), a number can be 'distributed' across other numbers [3 x (2+4) = 3x2 + 3x4, or a x (b+c) = axb + bxc]

Standard-Specific Resources (3.OA.5)

- **EngageNY: Grade 3, Module 1, Topic F, Lesson 19 – Apply the distributive property to decompose units.**

Concept Development (31 minutes)

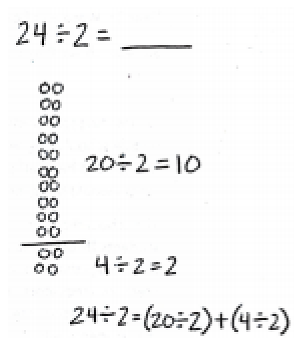
Materials: (S) Personal white board

Problem 1: Model break apart and distribute using an array as a strategy for division.

Draw or project a 12 × 2 array and write 24 ÷ 2 = ____ above it.

T: Let's use the array to help us solve 24 ÷ 2 = ____. There are 24 dots total. (Draw a line after the tenth row.) This shows one way to break apart the array.

T: Write division equations to represent the part of the array above the line and the part of the array below the line.



S: (Write 20 ÷ 2 = 10 and 4 ÷ 2 = 2.)

T: How many twos are above the line?

S: 10 twos.

T: How many twos are below the line?

S: 2 twos.

T: Let's rewrite this as the addition of two quotients. Use my equations.

(____ ÷ 2) + (____ ÷ 2) = _____ ÷ 2

_____ + _____ = _____

S: (Line 1: Fill in totals. Line 2: Write 10 + 2 = 12.)

T: Explain to your partner the process we used to solve 24 ÷ 2.

S: We added the quotients of two smaller facts to find the quotient of a larger one.

Repeat the process with a 13 × 2 array to show 26 ÷ 2. Break it into 20 ÷ 2 and 6 ÷ 2.

Problem 2: Use break apart and distribute as a strategy for division.

T: (Write 27 ÷ 3 = ____.) What are we focused on when we break apart to divide? Breaking up the number of groups (or rows), like in multiplication, or breaking up the total?

S: Breaking up the total.

T: Let's break up 27 into 15 and another number. Fifteen plus what equals 27?

S: 12.

T: Work with a partner to draw an array that shows 27 ÷ 3 where 3 is the number of columns.

S: (Draw a 9 × 3 array.)

T: Box the part of your array that shows a total of 15.

S: (Box the first 5 rows.)

T: Write a division equation for the boxed portion to the right of the array.

S: (Write 15 ÷ 3 = 5.)

T: Box the part of your array that shows a total of 12.

S: (Box the remaining 4 rows.)

T: Now, write a division equation for that part of the array.

S: (Write 12 ÷ 3 = 4.)

T: Tell your partner how you will use the equations to help you solve the original equation, 27 ÷ 3 = __.

S: I'll add the quotients of the two smaller facts.

T: (Write the following.) Complete the following sequence to solve 27 ÷ 3 with your partner.

27 ÷ 3 = (15 ÷ 3) + (12 ÷ 3)

= _____ + ______

= _____

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Add a challenge by asking students to think about other ways of breaking apart 27. A student will most likely choose parts that are not evenly divisible by 3. This will lead to a discussion that gets students to realize that, with division, the strategy relies on the decomposition being such that the dividends must be evenly divisible by the divisor.

Repeat the process with 33 ÷ 3. Students can break apart 33 by using the number pair 30 and 3.

NOTES ON MULTIPLE MEANS

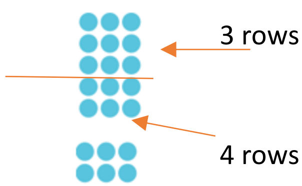
Sample Anchor Chart for 3.OA.5 - Distributive Property of Multiplication

Match the equivalent expressions.

Before Distributive Property	After Distributive Property
7 x 3	(4 x 3) + (4 x 3)
9 x 3	(5 x 3) + (4 x 3)
8 x 3	(5 x 3) + (2 x 3)

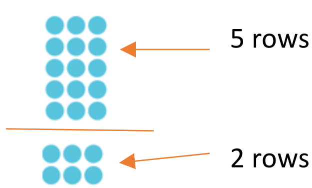
Step 1: Draw an array of circles to represent the first multiplication fact (7 x 3 = 7 rows of 3 circles)

Step 2: Use a line to separate the rows into a set of top rows and a set of bottom rows, and count how many rows are in the top and in the bottom



Step 3: Write a number sentence, including multiplication and addition, to represent the top array of circles added to the bottom array of circles ([3 x 3] + [4 x 3])

Step 4: Check whether your new sentence matches one of the number sentences in the 'after distributive property' column – if not, redraw the original array from the original multiplication fact, this time using the line to separate the circles in a different way (does not match any sentences given, redraw 7 x 3 array of circles, put line somewhere new)



Step 5: Write a new number sentence to represent the top array of circles added to the bottom array of circles, and check to see if it appears in the 'after' column ([5 x 3] + [2 x 3]), yes]

Step 6: Repeat steps 1 through 5 for the remaining 2 multiplication facts

4.OA.3 Back to ccss standard

Use the four operations with whole numbers to solve problems.

Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Skills

1. Divide whole numbers, including division with remainders

2. Represent multi-step word problems using equations with a letter standing in for the unknown quantity

3. Interpret multi-step word problems (including problems in which remainders must be interpreted)

4. Determine the appropriate operation(s) to solve multi-step word problems

5. Assess the reasonableness of an answer in solving a multi-step word problem using mental math

6. Assess the reasonableness of an answer in solving a multi-step word problem using estimation strategies (including rounding)

Key Concepts/Vocabulary

Whole number - Any of the number 0, 1, 2, and beyond, with no fractional or decimal part, and not negative
Division – Something physical, with continuous change
Remainder – The amount left over after dividing one number by another
Equation – A number sentence that says 'This equals that', where the amount to the left of the equal sign is the same as the amount to the right of the equal sign
Unknown – A number that is not known, that must be found
Quantity – How much there is of something
Solve – Find an answer
Estimation – The finding of a value that is close enough to the right answer, usually with some thought or calculation involved
Rounding – Making a number simpler, but keeping its value close to what it was

Standard-Specific Resources (4.OA.3)

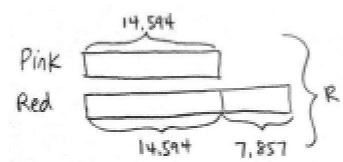
- **EngageNY:** **Grade 4, Module 1, Topic D, Lesson 12 – Solve multistep word problems using the standard addition algorithm modeled with tape diagrams, and assess the reasonableness of answers using rounding**

Concept Development (34 minutes)

Materials: (S) Personal white board

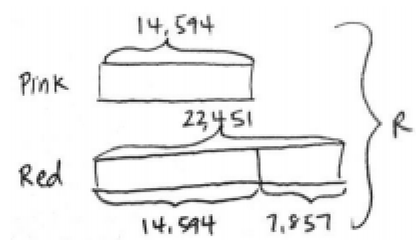
Problem 1: Solve a multi-step word problem using a tape diagram.

The city flower shop sold 14,594 pink roses on Valentine's Day. They sold 7,857 more red roses than pink roses. How many pink and red roses did the city flower shop sell altogether on Valentine's Day? Use a tape diagram to show the work.



MP.2

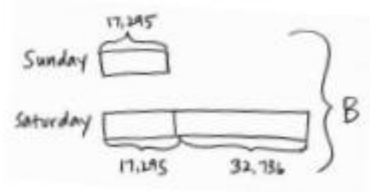
T: Read the problem with me. What information do we know?
S: We know they sold 14,594 pink roses.
T: To model this, let's draw one tape to represent the pink roses. Do we know how many red roses were sold?
S: No, but we know that there were 7,857 more red roses sold than pink roses.
T: A second tape can represent the number of red roses sold. (Model on the tape diagram.)
T: What is the problem asking us to find?
S: The total number of roses.
T: We can draw a bracket to the side of both tapes. Let's label it *R* for pink and red roses.
T: First, solve to find how many red roses were sold.
S: (Solve 14,594 + 7,857 = 22,451.)
T: What does the bottom tape represent?
S: The bottom tape represents the number of red roses, 22,451.
T: (Bracket and label 22,451 to show the total number of red roses.) Now, we need to find the total number of roses sold. How do we solve for *R?*



S: Add the totals for both tapes together. 14,594 + 22,451 = *R*.
T: Solve with me. What does *R* equal?
S: *R* equals 37,045.
T: (Write *R* = 37,045.) Let's write a statement of the answer.
S: (Write: The city flower shop sold 37,045 pink and red roses on Valentine's Day.)

Problem 2: Solve a two-step word problem using a tape diagram, and assess the reasonableness of the answer.

On Saturday, 32,736 more bus tickets were sold than on Sunday. On Sunday, only 17,295 tickets were sold. How many people bought bus tickets over the weekend? Use a tape diagram to show the work.



T: Tell your partner what information we know.

S: We know how many people bought bus tickets on Sunday, 17,295. We also know how many more people bought tickets on Saturday. But we do not know the total number of people that bought tickets on Saturday.

T: Let's draw a tape for Sunday's ticket sales and label it. How can we represent Saturday's ticket sales?

S: Draw a tape the same length as Sunday's, and extend it further for 32,736 more tickets.

T: What does the problem ask us to solve for?

S: The number of people that bought tickets over the weekend.

T: With your partner, finish drawing a tape diagram to model this problem. Use B to represent the total number of tickets bought over the weekend.

T: Before we solve, estimate to get a general sense of what our answer will be. Round each number to the nearest ten thousand.

S: (Write 20,000 + 20,000 + 30,000 = 70,000.) About 70,000 tickets were sold over the weekend.

T: Now, solve with your partner to find the actual number of tickets sold over the weekend.

S: (Solve.)

S: B equals 67,326.

T: (Write B = 67,326.)

T: Now, let's look back at the estimate we got earlier and compare with our actual answer. Is 67,326 close to 70,000?

S: Yes, 67,326 rounded to the nearest ten thousand is 70,000.

T: Our answer is reasonable.

T: Write a statement of the answer.

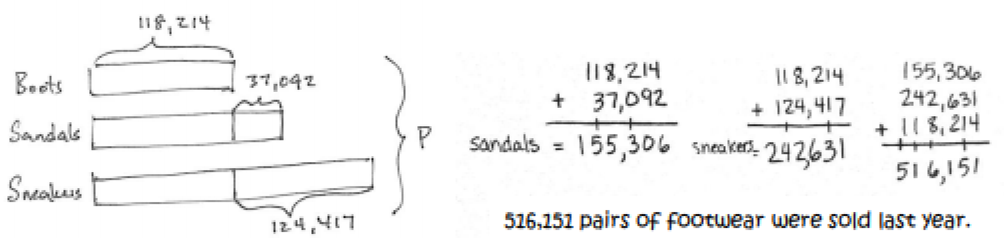
S: (Write: There were 67,326 people who bought bus tickets over the weekend.)

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

English language learners may need direction in creating their answer in the form of a sentence. Direct them to look back at the question and then to verbally answer the question using some of the words in the question. Direct them to be sure to provide a label for their numerical answer.

Problem 3: Solve a multi-step word problem using a tape diagram, and assess reasonableness.

Last year, Big Bill's Department Store sold many pairs of footwear. 118,214 pairs of boots were sold, 37,092 more pairs of sandals than pairs of boots were sold, and 124,417 more pairs of sneakers than pairs of boots were sold. How many pairs of footwear were sold last year?



T: Discuss with your partner the information we have and the unknown information we want to find.

S: (Discuss.)

T: With your partner, draw a tape diagram to model this problem. How do you solve for *P*?

S: The tape shows me I could add the number of pairs of boots 3 times, and then add 37,092 and 124,417. → You could find the number of pairs of sandals, find the number of pairs of sneakers, and then add those totals to the number of pairs of boots.

Have students then round each addend to get an estimated answer, calculate precisely, and compare to see if their answers are reasonable.

Sample Anchor Chart for 4.OA.3 - Estimation Word Problems

Chloe's favorite basketball team scored 77 points in one game, 92 points in their next game, and 104 points in their third game.

Create an equation you can use to estimate the total number of points (*p)* the team scored in all 3 games.

Step 1: **Determine what information you already have**

(Chloe's basketball team's scores for 3 games)

Step 2: **Determine what the question wants you to estimate**

(An equation that represents the total number of points from all 3 games put together, with 'p' to represent the total)

Step 3: **Round the numbers given in the problem**

(77 → 80, 92 → 90, 104 → 100)

1. **Look at the digit furthest to the right**
2. **If the digit is 5 or more, replace it with a 0 and add 1 more to the number to its left**
3. **If the digit is 4 or less, replace it with a 0**

77 ← **7 is more than 5, so it becomes a 0 and the number to its left (also 7) becomes 8 (7 +1 = 8), meaning you should round 77 up to 80**

92 ← **2 is less than 4, so it becomes a 0, meaning you should round 92 down to 90**

104 ← **4 is 4 or less, so it becomes a 0, meaning you should round 104 down to 100**

Step 4: **Use the rounded numbers to find your answer to the 2nd step (p = 80 + 90 + 100 = total points from the 3 games)**

5.OA.1 Back to ccss standard

Write and interpret numerical expressions.

Use parentheses, brackets, or braces in numerical expressions with these symbols.

Skills

1. Use manipulatives to model the solving of expressions containing parentheses and brackets

2. Use visual models to represent the solving of expressions containing parentheses and brackets

3. Solve expressions containing a set of parentheses *inside another set* of parentheses

4. Write and solve expressions containing one or more sets of parentheses or brackets

Key Concepts/Vocabulary

Parentheses – "Round brackets" are the familiar ( ) symbols used to group things together things together.

Brackets– Symbols used in pairs to group things together () [] {} <>

Standard-Specific Resources (5.OA.1)

- **EngageNY: Grade 3, Module 2, Topic A, Lesson 1 – Multiply multi-digit whole numbers and multiples of 10 using place value patterns and the distributive and associative properties.**

Concept Development (32 minutes)

Materials: (S) Personal white board, millions to thousandths place value chart (Template)

Problems 1–4

4 × 30

40 × 30

40 × 300

4,000 × 30

T: (Write 4 × 30. Below it, write 4 × 3 tens = ___.) To find the product, start by multiplying the whole numbers, remembering to state the unit in your product.

S: 12 tens.

T: Show 12 tens on your place value chart. What is 12 tens in standard form?

S: 120.

T: (Write 4 tens × 3 tens = ___.) Solve with a partner.

S: (Solve.)

T: How did you use the previous problem to help you solve 4 tens × 3 tens?

S: The only difference was the place value unit of the first factor, so it was 12 hundreds. → It's the same as 4 threes times 10 times 10, which is 12 hundreds. → I multiplied 4 × 3, which is 12. I then multiplied tens by tens, so my new units are hundreds. Now, I have 12 hundreds, or 1,200.

T: Let me record what I hear you saying. (Write (4 × 3) × 100.)

T: (Write 4 tens × 3 hundreds = ___ on the board.) How is this problem different than the last problem?

S: We are multiplying tens and hundreds, not ones and hundreds or tens and tens.

T: 4 tens is the same as 4 times 10. (Write 4 × 10 on the board). 3 hundreds is the same as 3 times what?

S: 100.

T: (Write 3 × 100 next to 4 × 10 on the board.) So, another way to write our problem would be (4 × 10) × (3 × 100). (Now, write (4 × 3) × (10 × 100) on the board.) Are these expressions equal? Why or why not? Turn and talk.

S: Yes, they are the same. → We can multiply in any order, so they are the same.

T: What is 4 × 3?

S: 12.

T: (Record 12 under 4 × 3.) What is 10 × 100?

S: 1,000.

T: (Record 1,000 under 10 × 100.)

T: What is the product of 12 and 1,000?

S: 12,000.

Problems 5–8

60 × 5

60 × 50

60 × 500

60 × 5,000

MP.7

T: (Write 60 × 5.)

T: (Underneath the expression above, write (6 × 10) × 5 and (6 × 5) × 10.) Are both of these equivalent to 60 × 5? Why or why not? Turn and talk.

T: When we change the order of the factors, we are using the commutative (any-order) property. When we group the factors differently (point to the board), we are using the associative property of multiplication.

T: Let's solve (6 × 5) × 10.

S: (Solve 30 × 10 = 300.)

T: For the next problem, use the properties and what you know about multiplying multiples of 10 to help you solve.

T: (Write 60 × 50 = ____.) Work with a partner to solve, and then explain.

MP.7

S: I thought of 60 as 6 × 10 and 50 as 5 × 10. I rearranged the factors to see (6 × 5) × (10 × 10). I got 30 × 100 = 3,000. → I first multiplied 6 times 5 and got 30. Then, I multiplied by 10 to get 300 and then multiplied by 10 to get 3,000.

T: I notice that in Problems 5–8 the number of zeros in the product was exactly the same as the number of zeros in our factors. That doesn't seem to be the case here. Why is that?

S: Because 6 × 5 is 30, then we have to multiply by 100. So, 30 ones × 100 is 30 hundreds, or 3,000.

T: Think about that as you solve 60 × 500 and 60 × 5,000 independently.

Problems 9–12

451 × 8

451 × 80

4,510 × 80

4,510 × 800

T: Find the product, 451 × 8, using any method.

S: (Solve to find 3,608.)

T: How did you solve?

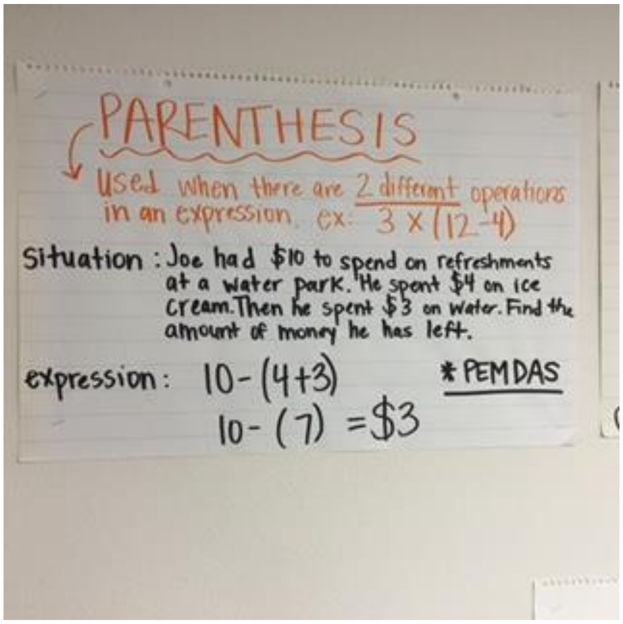
S: I used the vertical algorithm. → I used the distributive property. I multiplied 400 × 8, then 50 × 8, and then 1 × 8. I added those products together.

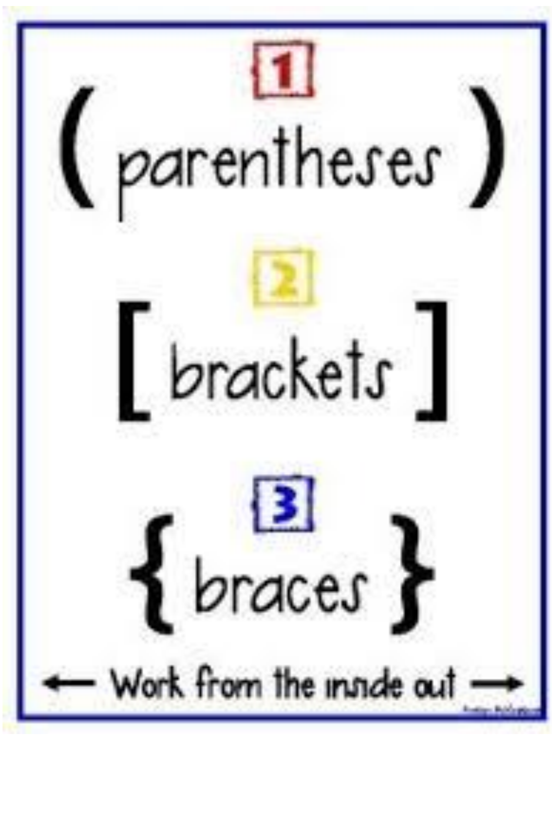
T: What makes the distributive property useful here? Why does it help here when we didn't really use it in our other problems? Turn and talk.

S: There are different digits in three place values instead of all zeros. If I break the number apart by unit, then I can use basic facts to get the products.

T: Turn and talk to your partner about how you can use 451 × 8 to help you solve the 451 × 80, 4,510 × 80, and 4,510 × 800. Then, evaluate these expressions.

Sample Anchor Charts for 5.OA.1 – Parentheses, Brackets, and Braces





Measurement and Data

(MD)

2.MD.5 Back to ccss standard

Relate addition and subtraction to length.

Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

Skills

1. **Add lengths within 100**

2. Solve addition word problems involving lengths that are given in the same units using drawings of rulers

3. Solve addition word problems involving length that have equations with a symbol for the unknown number

4. Subtract lengths within 100

5. Solve subtraction word problems involving lengths that are given in the same units using drawings of rulers

6. Solve subtraction word problems involving lengths that have equations with a symbol for the unknown number

Key Concepts/Vocabulary

Add – To find the total, or sum

Subtract – To find the difference, or remove objects from a collection

Length – Distance, or how far from end to end

Solve – Find an answer

Unit – A quantity used as a standard of measurement

Equation – A statement that a value (or group of values) is equal to another value (or other group of values)

Symbol – A pattern or image used instead of words

Unknown – A number that is not known, that must be found

Standard-Specific Resources (2.MD.5)

- **EngageNY: Grade 2, Module 7, Topic E, Lesson 20 – Solve two-digit addition and subtraction word problems involving length by using tape diagrams and writing equations to represent the problem.**

Concept Development (40 minutes)

Materials: (S) Personal white board, Problem Set

Note: For today's lesson, the Application Problem and the Problem Set are embedded in the Concept Development. The Problem Set is designed so that there is a "we do" and a "you do" portion.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Scaffold the lesson for students who are working below grade level by using adding machine tape or sentence strips to measure, cut, and compare actual lengths. Students can then measure the difference between how long Mr. Ramos wants his scarf to be and the length of what he has knit so far (19 in). Make sure that students line up the zero point as they compare the two lengths.

Part 1: Solve a *difference unknown* type problem.

Mr. Ramos has knitted 19 inches of a scarf he wants to be 1 yard long. How many more inches of scarf does he need to knit? (This is Problem 1 on the Problem Set.)

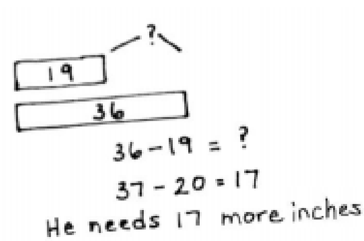
T: Let's read through Problem 1 together.

T/S: (Read aloud.)

T: What can we draw?

S: The scarf now and when he is done. → A tape diagram.

T: Great! I'll give you a minute to draw quietly. When I give the signal, talk to your partner about how your drawing matches the story (as shown on the right).



T: Turn and talk: Look at your drawing. What are you trying to find? Put a question mark to show the part we are trying to figure out.

MP.1

S: (Work.)

T: Why did you put 36 in the tape showing the finished scarf?

S: Because 1 yard is 36 inches. → To find the answer, we have to change 1 yard to 36 inches.

T: Yes! We can compare these lengths just like we compared data using bars in our graphs. (Draw the tape diagram on the board.)

T: Now, write a number sentence and statement to match your work. (Pause while students work.) Explain to your partner how you solved the problem.

S: I wrote 19 + ____ = 36. I counted up 1 to make 20 and then added 16 more to reach 36, and 1 and 16 is 17. → I wrote 36 – 19 = ? I added 1 to both numbers so I wouldn't have to rename. And 37 – 20 = 17.

MP.1

T: Tell your partner the answer in a statement.

S: He needs to knit 17 more inches.

T: Use what we have practiced to complete Problem 2 on your Problem Set by yourself.

Let students work independently on the next problem. Have them compare with a partner when they are finished. Circulate to give support to those students who need it.

Part 2: Solve a two-step problem with a *compare with smaller unknown* type problem as one step.

Frankie has a 64-inch piece of rope and another piece that is 18 inches shorter than the first. What is the total length of both ropes? (This is Problem 3 on the Problem Set.)

T: Let's read this problem together.

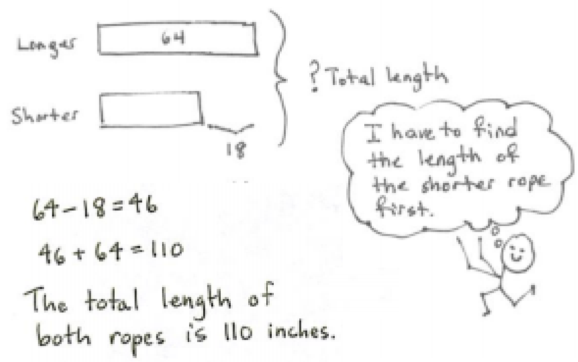
T: Do we know how long each rope is?

S: No. → We know how long one of the ropes is, 64 inches. → We don't know how long the shorter rope is.

T: That's right. Our first step is to find out the length of the shorter rope to answer the question. Then, we can answer the question.

T: What can we draw?

S: The ropes. → A tape diagram.



T: Yes. Let's do this one together. First, let's draw a tape diagram that shows how to find the length of the shorter rope. Remember to put a question mark to show what is missing.

Circulate and guide students to understand that 18 inches is not the length of the shorter rope; rather, it is the difference. Also, guide students to place the question mark not within the tape of the shorter rope but where it shows the total length of both ropes.

T: Turn and talk: How did you label your drawing? Where did you write the 18? Where did you write your question mark?

S: (Share.)

T: Find the length of the shorter rope by writing an equation and telling the answer to your partner in a statement.

S: (Solve using a subtraction strategy, and check the answer with a partner.)

T: What is the length of the shorter rope?

S: 46 inches!

T: Did the problem ask how long the shorter rope is?

S: No.

T: Write a number sentence and statement to answer the question. (Have students share their number sentences and statements once they are finished working.)

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

English language learners might get confused about the difference between *short* and *shorter*. To compare the two words, use sets of objects to illustrate the difference. Have students practice saying the words as they pick out the shorter objects until they are successful.

T: Excellent. The next problem also has two steps. Work on Problem 4 by yourself. When you are done, explain your solution path to your partner.

Let students work independently on the next problem. Have them compare with a partner when they are finished. Circulate to give support to those students who need it.

Part 3: Solve a *put together* problem involving geometry.

The total length of all three sides of a triangle is 96 feet. The triangle has two sides that are the same length. One of the equal sides measures 40 feet. What is the length of the side that is not equal? (This is Problem 5 on the Problem Set.)

T: Let's read this problem together.

T/S: (Read aloud.)

Isosceles Triangle

T: Hmm. This is a lot of information.

T: What can we draw?

S: A triangle!

T: What do we know about this triangle?

S: Two sides are the same length!

T: (Draw a triangle with three very unequal sides.) Did I draw it right?

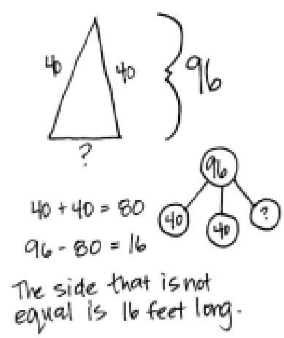
S: No!

T: Why?

S: It has to have two sides that are equal!

T: Is this better? (Draw an isosceles triangle.)

S: Yes.



T: Draw a triangle with two sides that are the same length on your personal white board.

S: (Draw.)

T: Now, let's go back and read the problem and put the information it gives us on our triangle.

T: What does the first part say?

S: All three sides of the triangle put together are 96 feet.

T: Label your drawing to show that the total length of the sides of the triangle is 96 feet. Let's not write the units on our drawing for today. Just label it simply as 96.

S: (Work.)

T: Good. What is the next piece of information?

S: We know the length of one of the equal sides, 40 feet.

T: Yes. Label 40 on your picture. Since we know the length of one equal side, can we add more information to our picture that wasn't written in the problem?

S: Yes! Since the two sides of the triangle are equal, that means their length is equal also, so the other equal side of the triangle is 40 feet, too!

T: Very nice reasoning skills. Sometimes we can figure out more information even if it is not written down in the problem.

T: The last piece of information we have to label on our picture is a question mark to label what we are trying to figure out. What are we trying to figure out?

S: The length of the side of the triangle that is not equal to the others.

T: Good. Do that now.

S: (Work.)

T: What is the length of the missing side of the triangle?

S: 16 feet!

T: What did you do to find that out? Talk to your partner.

S: I subtracted both of the sides we know the length of from 96. → I added the sides we know the length of and then subtracted it from the total 96. → I added the sides and then counted up to 96.

T: Good. All of the solutions I heard involved doing two steps.

T: It's time to try one on your own. Work on Problem 6 on your Problem Set.

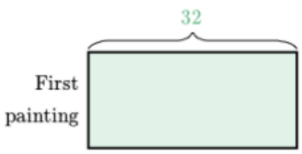
Sample Anchor Chart for 2.MD.5 – Length Word Problems

Anna has a 32-cm painting and another painting that is 12 cm longer than the first painting.

How long would Anna's paintings be if she placed them side by side?

______ cm

Step 1: Draw and label a tape diagram for the length we *do know* from the problem **(32cm = 1st painting)**



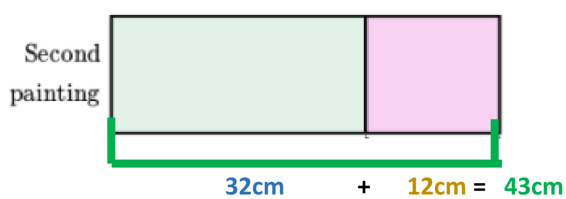
Step 2: Draw and label a tape diagram that represents the length we *don't know* from the problem, including the length of the part of it we do know **(2nd painting = same size as the first, plus 12 more cm)**

Step 3: Write a number sentence to represent the length your second tape diagram (the one where you only know *part* of the length) **(Not necessarily what the question is asking you to find!)**

- **Length of 2nd painting = 32cm + 12cm**

Step 4: Solve your number sentence to find the length of your second tape diagram; label your second tape diagram **(make sure you use your units!)**

- **Length of 2nd painting = 32cm + 12cm = 43cm**



Step 5: Return to the question you were originally asked. What do you need to find?

(The total length of both paintings, if they were lined up end to end)

Step 6: Write and solve a number sentence that represents your answer to Step 5

- **Total Length of Both Paintings, lined up end to end = 1st painting length + 2nd painting length = 32cm + 43cm = 75cm**

2.MD.8 Back to ccss standard

Work with time and money.

Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?

Skills

1. Identify and recognize the value of dollar bills, quarters, dimes, nickels, and pennies.

2. Identify the $ and ¢ symbol

3. Convert amounts from $ to ¢

4. Convert amounts from ¢ to $

5. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies using $ and ¢ symbols appropriately.

Key Concepts/Vocabulary

Value – How much something is worth
Equivalent – Having the same value
Convert – Change the form of something without changing its value
Symbol – A pattern or image used instead of words
Dollar bill – Paper money with equivalent value of 100 cents, 100 pennies, 4 quarters, 10 dimes, or 20 nickels
Quarter – Coin money with equivalent value of ¼ of a dollar, 5 nickels, or 25 pennies
Dimes – Coin money with equivalent value of 1/10 of a dollar, 2 nickels, or 10 pennies
Nickel – Coin money with equivalent value of 1/20 of a dollar, 1/5 of a quarter, ½ of a dime, or 5 pennies
Penny – Coin money with equivalent value of 1/100 of a dollar, 1/5 of a nickel, or 1/10 of a dime
$ - Dollar symbol
¢ - Cent symbol

Standard-Specific Resources (2.MD.8)

- **EngageNY:** **Grade 2, Module 7, Topic B, Lesson 9 – Solve word problems involving different combinations of coins with the same total value.**

Concept Development (33 minutes)

Materials: (T) 1 dime, 3 nickels, 5 pennies, 2 personal white boards (S) Personal white board, bag with the following coins: 4 quarters, 10 nickels, 10 dimes, 10 pennies

Assign partners before beginning instruction.

Part 1: Manipulate different combinations of coins to make the same total value.

T: (Show 1 dime and 5 pennies on one mat and 3 nickels on another mat.)
T: What is the value of the coins on this mat? (Point to the dime and pennies.)
S: 15 cents!
T: What is the value of the coins on this mat? (Point to the nickels.)
S: 15 cents!
T: So, the values are equal?
S: Yes!

T: How can that be? The coins are different!
S: That one is 10 cents and 5 more. The other is 5 + 5 + 5, so they are both 15 cents. → Three nickels is 15 cents. A dime and 5 pennies is also 15 cents.
T: Aha! So, we used different coins to make the same value?
S: Yes!
T: Let's try that! I will say an amount, and you work with your partner to show the amount in two different ways.
T: With your partner, show 28 cents two different ways.
S: (Arrange the coins on the mats while discussing with their partners.)
T: How did you make 28 cents?
S: I used a quarter and 3 pennies. My partner used 2 dimes and 8 pennies. → I also used a quarter and 3 pennies, but my partner used 2 dimes, 1 nickel, and 3 pennies.

Repeat the above sequence with the following amounts: 56 cents, 75 cents, and 1 dollar.

Part 2: Manipulate different combinations of coins in the context of word problems.

Problem 1: Tony gets 83¢ change back from the cashier at the corner store. What coins might Tony have received?

T: Read the problem to me, everyone.
S: (Read chorally.)
T: Can you draw something?
S: Yes!
T: Do that. (Allow students time to work.)
T: How did you show Tony's change?
S: I drew 8 dimes and 3 pennies. → I made 50¢ using 2 quarters, then added 3 dimes to make 80¢, and then added 3 pennies to make 83¢. → I used 3 quarters, 1 nickel, and 3 pennies.
T: Write your coin combinations and the total value below your drawing. If you used 8 dimes and 3 pennies, write that underneath like this. (Model writing the coin combination with the total value on the board, for example, 8 dimes, 3 pennies is 83 cents.)

> **NOTES ON MULTIPLE MEANS OF ENGAGEMENT:**
>
> Challenge students working above grade level to show 83¢ two ways: using the least number of coins and using the greatest number of coins. Ask students to explain how they came up with their solutions and how it is possible for both solutions to have the same value.

T: Now, pretend that the cashier has run out of quarters. Draw Tony's change in another way without using quarters. Write your coin combination and the total value below.
S: Mine still works! → I traded each of my quarters for 2 dimes and a nickel. Now, I have 7 dimes, 2 nickels, and 3 pennies. → I didn't use a quarter before, but this time I used 6 dimes and 4 nickels instead of 7 dimes and 2 nickels to show 80 cents.

MP.6 Problem 2: Carla has 4 dimes, 1 quarter, and 2 nickels to spend at the snack stand. Peyton has 3 coins, but he has the same amount of money to spend. What coins must Peyton have? How do you know?

T: Read the problem to me, everyone.

S: (Read chorally.)

T: Can you draw something?

S: Yes!

T: Time to draw! (Allow students time to work.)

MP.6

T: What did you draw?

S: 4 dimes, 1 quarter, and 2 nickels. → A tape diagram with one part 40 cents, one part 25 cents, and one part 10 cents.

T: What is the value of Carla's money?

S: 75 cents.

T: Show your partner how you found or can find three coins that make 75¢. (Allow time for sharing.) What coins did Peyton have?

S: 3 quarters.

T: How do you know?

S: We added 25 + 25 + 25 to make 75. → We couldn't make 75¢ with three coins if we used dimes, nickels, or pennies.

Sample Anchor Charts for 2.MD.8 – Counting Money with One, Five, and Ten Dollar Bills

Amy has 6 one dollar bills and 1 ten dollar bill. How much money does Amy have in all?

Step 1: Find the total value of the one dollar bills **(6 one dollar bills)** by adding 1.00 for each dollar bill or by multiplying the number of one dollar bills by 1.00

Dollars	Cents
1	00
1	00
1	00
1	00
1	00
1	00
=6	00

6 dollar bills = 1.00 + 1.00 + 1.00 + 1.00 + 1.00 + 1.00 = 6.00

Or **6 x 1.00 = 6.00**

Step 2: Find the total value of five dollar bills **(3 five dollar bills)** by adding 5.00 for each five dollar bill or by multiplying the number of five dollar bills by 5.00

Dollars	Cents
5	00
5	00
5	00
=15	00

3 five dollar bills = 5.00 + 5.00 + 5.00 = 15.00, *OR* 3 x 5.00 = 15.00

Step 3: Find the total value of ten dollar bills **(1 ten dollar bill)** by adding 10.00 for each ten dollar bill or by multiplying the number of ten dollar bills by 10.00

Dollars	Cents
10	00
=10	00

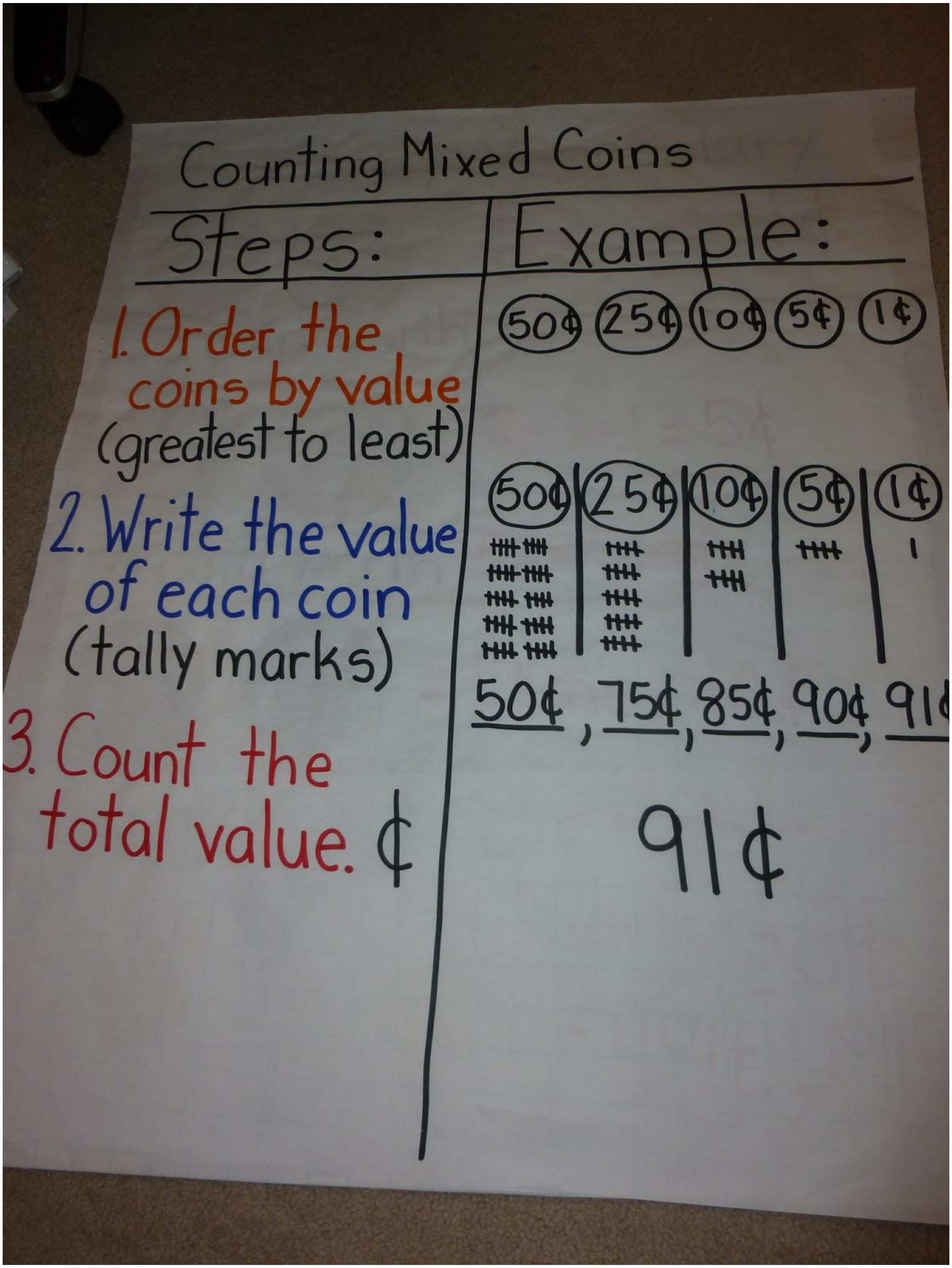
1 ten dollar bill = 10.00

Or **1 x 10.00 = 10.00**

Step 4: Find the total or sum of all the dollars added together by adding your answers to Step 1, Step 2, and Step 3. Make sure you line up all of your decimal points!

Dollars	Cents
6	00
15	00
10	00
=21	00

6.00 + 15.00 + 10.00 = 21.00 = $21.00


Counting Mixed Coins
Steps:
Example:
1. Order the coins by value
(greatest to least)
50¢ 25¢ 10¢ 5¢ 1¢
2. Write the value of each coin
(tally marks)
50¢ 25¢ 10¢ 5¢ 1¢
3. Count the total value. ¢
50¢, 75¢, 85¢, 90¢, 91
91¢

3.MD.1 Back to ccss standard

Solve problems involving measurement and estimation.

Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g. by representing the problem on a number line diagram.

Skills

1. Recognize minute marks on an analog clock face and the minute position on a digital clock face

2. Know how to write and tell time to the minute

3. Compare an analog clock face with a number line diagram

4. Use a number line diagram to add and subtract time intervals in minutes

5. Solve word problems involving addition of time intervals in minutes

6. Solve word problems involving subtraction of time intervals in minutes

Key Concepts/Vocabulary

Minute hand - The large hand on a clock that points to the minutes, that goes once around the clock every hour

Analog – Something physical, with continuous change

Digital – made of numbers

Number line – A line with numbers placed in their correct position

Time Interval – The time between two given points in time

Standard-Specific Resources (3.MD.1)

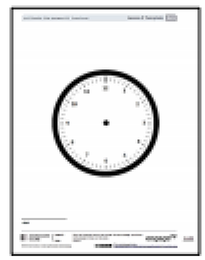
- **EngageNY: Grade 3, Module 2, Topic A, Lesson 4 – Solve word problems involving time intervals within 1 hour by counting backward and forward using the number line and clock.**

Concept Development (33 minutes)

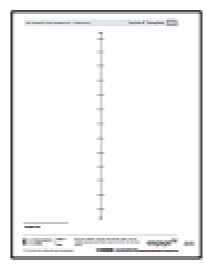
Materials: (T) Analog clock for demonstration (S) Personal white board, number line (Template), clock (Lesson 3 Template)

Problem 1: Count forward and backward using a number line to solve word problems involving time intervals within 1 hour.

Lesson 3 Template



Number Line Template



T: Look back at your work on today's Application Problem. We know that Lilly finished after Patrick. Let's use a number line to figure out how many more minutes than Patrick Lilly took to finish. Slip the number line Template into your personal white board.

T: Label the first tick mark 0 and the last tick mark 60. Label the hours and 5-minute intervals.

T: Plot the times 5:31 p.m. and 5:43 p.m.

T: We could count by ones from 5:31 to 5:43. Instead, discuss with a partner a more efficient way to find the difference between Patrick and Lilly's times.

S: (Discuss.)

T: Work with a partner to find the difference between Patrick's and Lilly's times.

T: How many more minutes than Patrick did it take Lilly to finish her chores?

S: 12 minutes more.

MP.4

T: What strategy did you use to solve this problem?

S: (Share possible strategies, listed below.)

- Count by ones to 5:35, by fives to 5:40, by ones to 5:43.
- Subtract 31 minutes from 43 minutes.
- Count backwards from 5:43 to 5:31.
- Know 9 minutes gets to 5:40 and 3 more minutes gets to 5:43.
- Add a ten and 2 ones.

Repeat the process with other time interval word problems, varying the unknown as suggested below.

- *Result unknown*: Start time and minutes elapsed known, end time unknown. (We started math at 10:15 a.m. We worked for 23 minutes. What time was it when we ended?)
- *Change unknown*: Start time and end time known, minutes elapsed unknown. (Leslie starts reading at 11:24 a.m. She finishes reading at 11:57 a.m. How many minutes does she read?)

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

If appropriate for the class, discuss strategies for solving different problem types (*start unknown, change unknown, result unknown*). Although problem types can be solved using a range of strategies, some methods are more efficient than others depending on the unknown.

- *Start unknown*: End time and minutes elapsed known, start time unknown. (Joe finishes his homework at 5:48 p.m. He worked for 32 minutes. What time did he start his homework?)

NOTES ON PROBLEM TYPES:

Tables 1 and 2 in the Glossary of the *Common Core Learning Standards for Mathematics* provide a quick reference of problem types and examples.

Problem 2: Count forward and backward using a clock to solve word problems involving time intervals within 1 hour.

T: It took me 42 minutes to cook dinner last night. I finished cooking at 5:56 p.m. What time did I start?

T: Let's use a clock to solve this problem. Put the clock template in your board.

T: Work with your partner to draw the hands on your clock to show 5:56 p.m.

T: Talk with your partner, will you count backward or forward on the clock to solve this problem? (Allow time for discussion.)

T: Use an efficient strategy to count back 42 minutes. Write the start time on your personal white board, and as you wait for others, record your strategy.

Circulate as students work and analyze their strategies so that you can select those you would like to have shared with the whole class. Also consider the order in which strategies will be shared.

T: What time did I start making dinner?

S: 5:14 p.m.

T: I would like to ask Nina and Hakop to share their work, in that order.

Repeat the process with other time interval word problems, varying the unknown as suggested below.

- *Result unknown:* Start time and minutes elapsed known, end time unknown. (Henry starts riding his bike at 3:12 p.m. He rides for 36 minutes. What time does he stop riding his bike?)
- *Change unknown*: Start time and end time known, minutes elapsed unknown. (I start exercising at 7:12 a.m. I finish exercising at 7:53 a.m. How many minutes do I exercise?)
- *Start unknown:* End time and minutes elapsed known, start time unknown. (Cassie works on her art project for 37 minutes. She finishes working at 1:48 p.m. What time did she start working?)

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Students who struggle with comprehension may benefit from peers or teachers reading word problems aloud. This accommodation also provides students with the opportunity to ask clarifying questions as needed.

Sample Anchor Chart for 3.MD.1 - Telling Time to the Nearest Minute

What time is it? 

_____ : ______ am

Step 1: Find the 'hour' by looking at the short hand and identifying looking at the smaller number it falls between (The short hand falls between 3 and 4, so the hour is 3)

Step 2: The longer/minute hand starts the hour by pointing straight up at 0. For each little mark on the circle that the minute hand passes, another minute goes by. Label the clock with minutes in multiples of 5, with 0 minutes by the 12, 5 minutes by the 1, 10 minutes by the 2, through the 11

Step 3: Find the 'minutes' by looking at the longer hand and identifying the minute mark it is closest two (The longer/minute had is pointing right at 30, so the minutes are 30)

Step 4: Write the hour first, followed by a ':' (3:)

Step 5: Write the minutes after the ':' (:30)

Step 6: Put the hour and minutes together - this is the time! (3:30am)

3.MD.3 Back to ccss standard

Represent and interpret data.

Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. ***For example, draw a bar graph in which each square in the bar graph might represent 5 pets.***

Skills

1. Identify, analyze, and explain the scale of a graph with a scale greater than one

2. Choose a proper scale for a bar graph

3. Choose a proper scale for a picture graph

4. Interpret a bar or picture graph to solve one- or two-step problems that ask "how many more" and "how many less"

5. Create a scaled picture graph to show data

6. Create a scaled bar graph to show data

Key Concepts/Vocabulary

Data – A collection of facts, such as numbers, words, measurements, observations, or descriptions of things

Scale – The ratio of the length in a drawing (or model) to the length of the real thing

Bar graph – A graph draw using rectangular bars to show how large each value is

Picture graph – A way of showing data using images

Standard-Specific Resources (3.MD.3)

- **EngageNY: Grade 3, Module 6, Topic A, Lesson 3 – [Create scaled bar graphs.]**

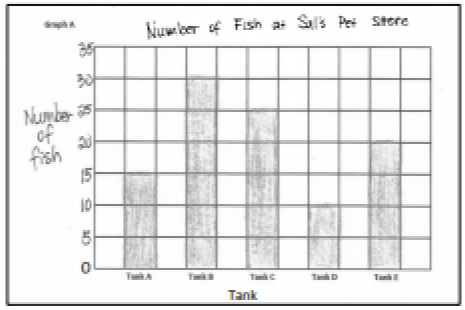
Concept Development (33 minutes)

Materials: (S) Graph A (Template 1) pictured below, Graph B (Template 2) pictured below, colored pencils, straightedge

Problem 1: Construct a scaled bar graph.

T: (Pass out Template 1 pictured to the right.) Draw the vertical tape diagrams from the Application Problem on the grid. (Allow students time to work.) Outline the bars with your colored pencil. Erase the unit labels inside the bar, and shade the entire bar with your colored pencil. (Model an example.)

Template 1 with Student Work



T: What does each square on the grid represent?

S: 5 fish!

MP.6

T: We can show that by creating a scale on our bar graph. (Write 0 where the axes intersect, and then write 5 near the first line on the vertical axis. Point to the next line up on the grid.) Turn and talk to a partner. What number should I write here? How do you know?

S: Ten because you are counting by fives. → Ten because each square has a value of 5, and 2 fives is 10.

MP.6

T: Count by fives to complete the rest of the scale on the graph.

S: (Count and write.)

T: What do the numbers on the scale tell you?

S: The number of fish!

T: Label the scale *Number of Fish*. (Model.) What do the labels under each bar tell you?

S: Which tank the bar is for!

T: What is a good title for this graph?

S: *Number of Fish at Sal's Pet Store*.

T: Write the title *Number of Fish at Sal's Pet Store*. (Model.)

T: Turn and talk to a partner. How is this **scaled bar graph** similar to the vertical tape diagrams in the Application Problem? How is it different?

S: They both show the number of fish in Sal's pet store. → The value of the bars and the tape diagrams is the same. → The way we show the value of the bars changed. In the Application Problem, we labeled each unit. In this graph, we made a scale to show the value.

T: You are right. This scaled bar graph does not have labeled units, but it has a scale we can read to find the values of the bars. (Pass out Template 2, pictured to the right.) Let's create a second bar graph from the data. What do you notice about the labels on this graph?

S: They are switched! → Yeah, the tank labels are on the side, and the *Number of Fish* label is now at the bottom.

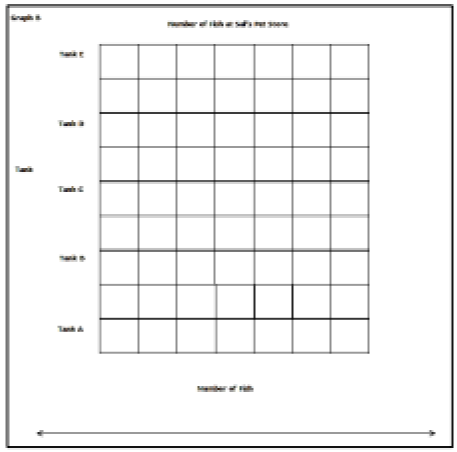
T: Count by fives to label your scale along the horizontal edge. Then, shade in the correct number of squares for each tank. Will your bars be horizontal or vertical?

S: Horizontal. (Label and shade.)

T: Take Graph A and turn it so the paper is horizontal. Compare it with Graph B. What do you notice?

S: They are the same!

Template 2



T: A bar graph can be drawn vertically or horizontally, depending on where you decide to put the labels, but the information stays the same as long as the scales are the same.

T: Marcy buys 3 fish from Tank C. Write a subtraction sentence to show how many fish are left in Tank C.

S: (Write 25 – 3 = 22.)

T: How many fish are left in Tank C?

S: 22 fish!

T: Discuss with a partner how I can show 22 fish on the bar graph.

S: (Discuss.)

T: I am going to erase some of the Tank C bar. Tell me to stop when you think it shows 22 fish. (Erase until students say to stop.) Even though our scale counts by fives, we can show other values for the bars by drawing the bars in between the numbers on the scale.

Problem 2: Plot data from a bar graph on a number line.

T: Let's use Graph B to create a number line to show the same information. There is an empty number line below the graph. Line up a straightedge with each column on the grid to make intervals on the number line that match the scale on the graph. (Model.)

S: (Draw intervals.)

T: Should the intervals on the number line be labeled with the number of fish or with the tanks? Discuss with your partner.

S: The number of fish.

T: Why? Talk to your partner.

S: The number of fish because the number line shows the scale.

T: Label the intervals. (Allow students time to work.) Now, work with a partner to plot and label the number of fish in each tank on the number line.

S: (Plot and label.)

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Assist students with perceptual difficulties, low vision, and others with plotting corresponding points on the number line. To make tick marks, show students how to hold and align the straightedge with the scale at the bottom of the graph, *not the bars*. Precise alignment is desired, but comfort, confidence, accurate presentation of data, and a frustration-free experience are more valuable.

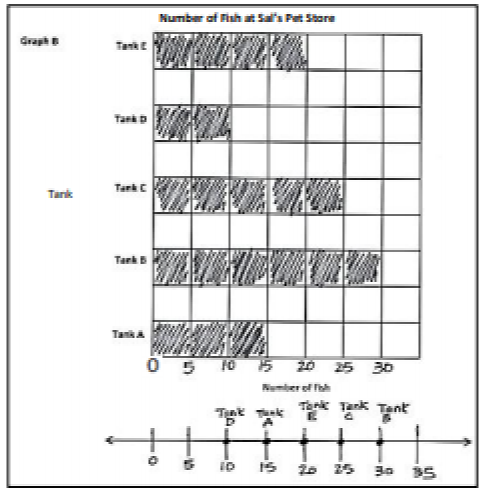
T: Talk to a partner. Compare how the information is shown on the bar graph and the number line.

S: The tick marks on the number line are in the same places as the graph's scale. → The spaces in between the tick marks on the number line are like the unit squares on the bar graph. → On the number line, the tanks are just dots, not whole bars, so the labels look a little different, too.

T: We can read different information from the 2 representations. Compare the information we can read.

S: With a bar graph, it is easy to see the order from least to most fish just by looking at the size of the bars. → The number line shows you how much, too, but you know which is the most by looking for the biggest number on the line, not by looking for the biggest bar.

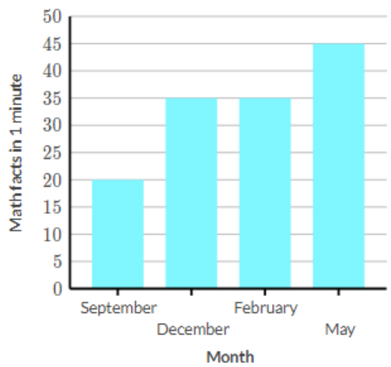
T: Yes. A bar graph allows us to compare easily. A number line plots the information.



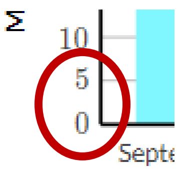
Sample Anchor Chart for 3.MD.3 - Telling Time to the Nearest Minute

Andy timed himself throughout the school year to see how many math facts he could complete in 1 minutes.

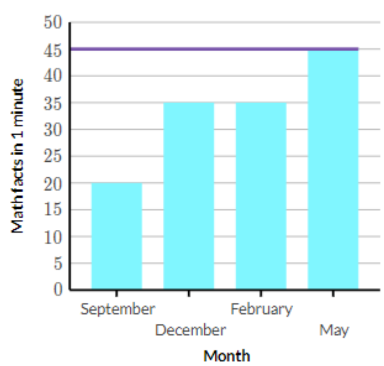
How many more math facts did Andy complete in May than in September?



Step 1: Determine the scale of the graph by looking at the intervals on the left – how much space is there between 0 and the next number? (The scale of the graph is '5' because the first interval after zero is labeled '5')



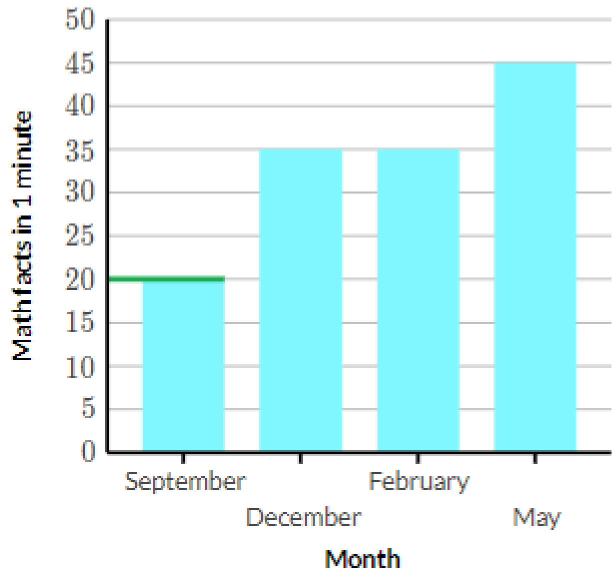
Step 2: Use a straight edge to draw a line from the top of the bar for the first category (May) to the left and identify the value on the left that is closest to the line you drew (45)



(continues on next page)

Step 3: Interpret the value from Step 2 based on the units listed on the left side of the graph ('Math facts in 1 minute'), the category for the bar you're looking at (May), and the question asked in the problem ('In May, Andy completed 45 math facts in 1 minute')

<u>Step 4:</u> Repeat Steps 2 and 3 for the second category mentioned in the problem (September) ('In September, Andy completed 20 math facts in 1 minute')



<u>Step 5:</u> Find the difference between the values (Math facts in 1 minute) for the two categories (May and September) by subtracting the smaller value from the larger value (Math facts in May – Math facts in September = 45 – 20 = 25 more math facts in 1 minute in May than in September)

3.MD.8 Back to ccss standard

Geometric measurement: recognize perimeter.

Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Skills

1. Find the perimeter when given the length of sides

2. Find the perimeter when there is an unknown side length

3. Design/create/draw/model rectangles with the same perimeter but different areas

4. Design/create/draw/model rectangles with the same areas and different perimeters

Key Concepts/Vocabulary

Polygon – A plane shape (2-dimensional) with straight sides

Dimension – A measurement of length in one direction

Perimeter – The distance around a 2-dimensional shape

Length – How far from end to end, or from one point to another

Side – One of the lines that make a flat (2-dimensional) shape

Unknown – A value or number that we don’t know yet

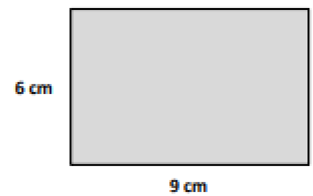
Standard-Specific Resources (3.MD.8)

- **EngageNY: Grade 3, Module 7, Topic C, Lesson 14 – [Determine the perimeter of regular polygons and rectangles when whole number measurements are unknown.](#)**

Concept Development (33 minutes)

Materials: (S) Personal white board

Problem 1: Find the perimeter of rectangles with unknown side lengths.



T: (Project or draw the rectangle as shown.) This shape is a rectangle. Use the given side lengths and what you know about rectangles to label the unknown side lengths.

S: (Label the unknown side lengths.)

T: (Label the unknown side lengths 6 cm and 9 cm.) Check your work against mine, and make changes if you need to. (Allow students time to check their work.) Write an addition sentence that shows the perimeter of the rectangle.

S: (Write 9 cm + 9 cm + 6 cm + 6 cm = 30 cm.)

T: What is the perimeter of the rectangle?

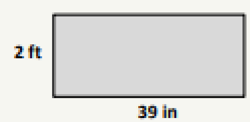
S: 30 centimeters!

T: Talk to a partner. What strategy did you use to add the side lengths?

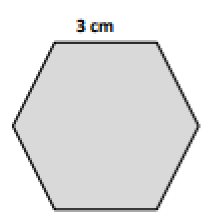
S: I doubled 9 and doubled 6 and then added 18 plus 12 to get 30. → I added 9 plus 6 to get 15 and then doubled 15 to get 30. → I took 1 from each 6 to make tens with the 9's. Then, I added 10 + 5 + 10 + 5. I saw that I had 3 tens, which is 30.

Repeat the process with the suggestions below. Students can sketch the rectangles with the given side lengths, label the unknown side lengths, and then find the perimeter.

- A rectangle with side lengths of 10 inches and 8 inches.
- A rectangle with side lengths of 14 centimeters and 36 centimeters.

> **NOTES ON MULTIPLE MEANS OF ENGAGEMENT:**
>
> Vary the difficulty level of the work in Problem 1 of the Concept Development by adjusting the numbers. Students who understand the concept quickly and calculate perimeter mentally may jump to the last challenge of calculating larger side lengths or side lengths that require conversions, such as the following:
>
> 
>

Problem 2: Find the perimeter of regular polygons with one side length given.



MP.3

T: (Project or draw the hexagon as shown.) This is a regular hexagon. Talk to a partner. How can the labeled side length help you find the unknown side lengths?

S: Since I know it's a regular hexagon, and I know one side length, I know the other side lengths. → Yeah. Since it's a regular hexagon, I know that all the side lengths are equal. So, all 6 sides are each 3 centimeters.

T: That's right. Sketch the hexagon on your personal white board, and label the unknown side lengths.

S: (Sketch and label the unknown side lengths.)

T: Write an addition sentence that shows the perimeter of the hexagon.

S: (Write 3 cm + 3 cm + 3 cm + 3 cm + 3 cm + 3 cm = 18 cm.)

T: What is the perimeter of the hexagon?

S: 18 centimeters!

T: Talk to a partner. Can you write your addition sentence as a multiplication sentence?

S: Yes. It's repeated addition of 3. I can show that with multiplication. → It shows 6 threes. I can write that as 6 × 3.

T: Write a multiplication sentence that shows the perimeter of the hexagon.

S: (Write 6 × 3 = 18.)

T: Discuss with a partner what the factors in this multiplication sentence represent.

S: The 6 is the number of sides on the hexagon, and the 3 is the length of each of those sides.

T: Rewrite your multiplication sentence with units to show 6 sides times the length of each side.

S: (Write 6 × 3 cm = 18 cm.)

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Because *regular* has multiple meanings, a specific reminder to English language learners that it means *equal-sided* when used in the context of *regular* polygon may be helpful.

Repeat the process with the suggestions below. Students write both an addition and a multiplication sentence to find the perimeter of each shape.

- A regular pentagon with side lengths of 7 inches.
- A regular triangle (equilateral triangle) with side lengths of 17 centimeters. (Discuss using the break apart and distribute strategy to solve with multiplication.)

MP.3

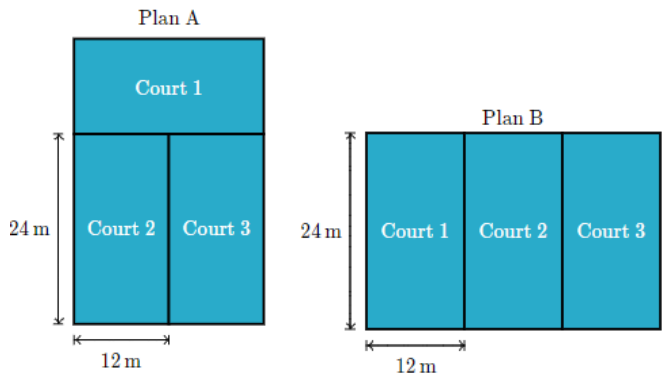
T: Talk to a partner: Which method is more efficient for finding the perimeter of a regular shape, adding or multiplying?

S: I think multiplying is because it's faster than adding. → If the side lengths are small numbers, then multiplying. But if the side lengths were bigger, like 154, I would add instead.

Sample Anchor Charts for 3.MD.8 – Perimeter

Rob plans to build 3 tennis courts in a local park. He can choose between Plan A and Plan B below. He will build a fence around the whole area.

Which plan uses the shortest fence?



Step 1: Use the information given for Plan A to label any missing side lengths - you need to know the length for each the top side, bottom side, left side, and right side

Left Side = 24m + 12m = 36m   Right Side = 24m + 12m = 36m
Bottom Side = 12m + 12m = 24m   Top Side = 12m + 12m = 24m

Step 2: Repeat Step 1, this time for Plan B

Left Side = 24m   Right Side = 24m
Bottom Side = 12m + 12m + 12m = 36m   Top Side = 12m + 12m + 12m = 36m

Step 3: To find the perimeter for Plan A, add the lengths of all of the sides from Step 1 together

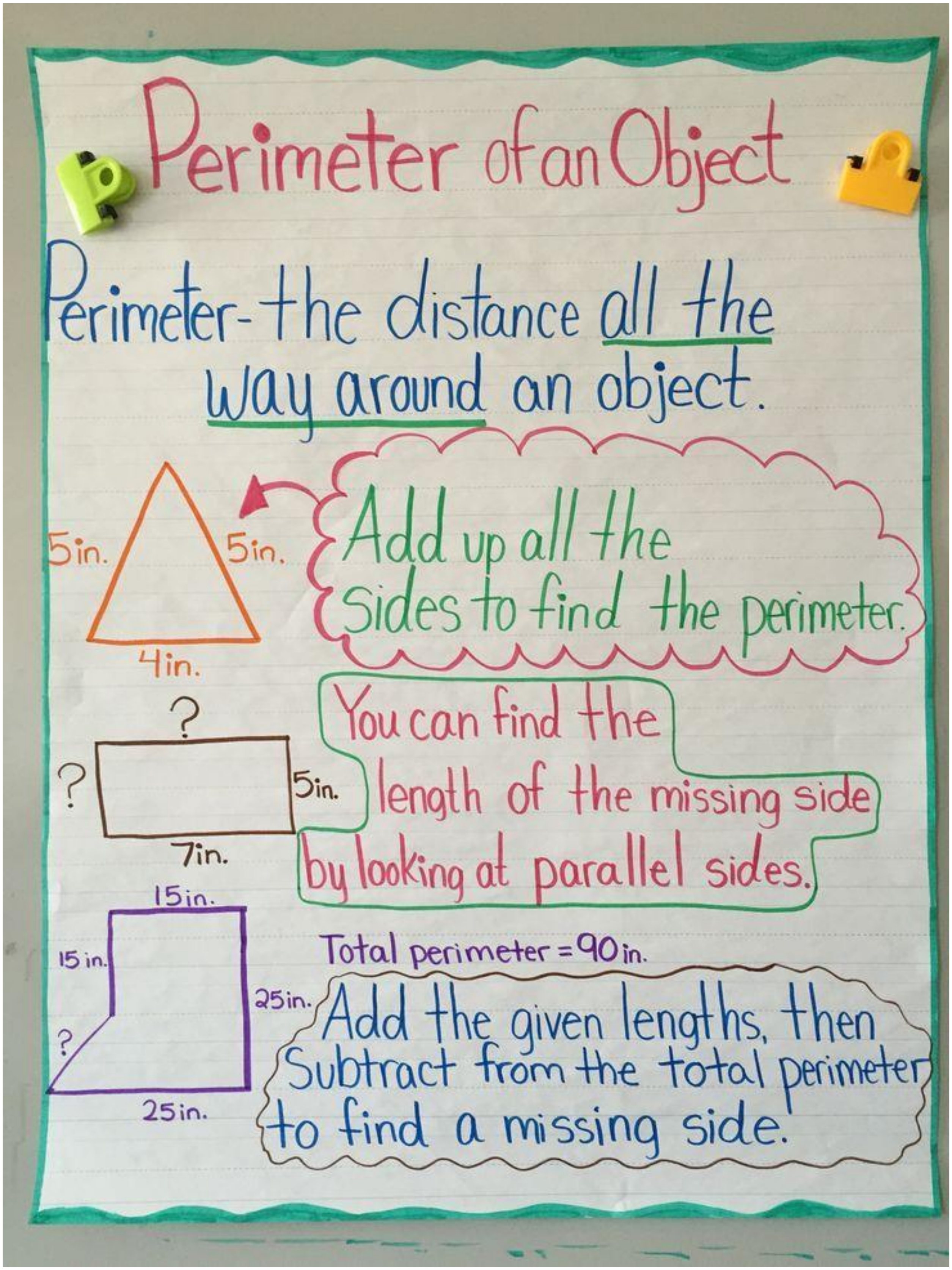
Perimeter of Plan A = Left Side + Right Side + Top Side + Bottom Side = 36m + 36m + 24m + 24m = 120m

Step 4: To find the perimeter for Plan B, add the lengths of all of the sides from Step 2 together

Perimeter of Plan B = Left Side + Right Side + Top Side + Bottom Side = 24m + 24m + 36m + 36m = 120m

Step 5: Determine which Plan has the shortest perimeter

Both Plan A and Plan B have perimeters/fences of 120 m! They are equally as short!


Perimeter of an Object
Perimeter- the distance all the way around an object.
5in.
5in.
4in.
Add up all the sides to find the perimeter.
?
?
5in.
7in.
You can find the length of the missing side by looking at parallel sides.
15in.
15 in.
?
25in.
25in.
Total perimeter = 90 in.
Add the given lengths, then Subtract from the total perimeter to find a missing side.

4.MD.2 Back to ccss standard

Solve problems involving measurement and conversion of measurements.

Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Skills

1. Solve word problems involving distances, including fractions and decimals, using a number line diagram with a measurement scale

2. Solve word problems involving intervals of time, including fractions and decimals, using a number line diagram with a measurement scale

3. Solve word problems involving liquid volumes, including fractions and decimals, using a number line diagram with a measurement scale

4. Solve word problems involving masses of objects, including fractions and decimals, using a number line diagram with a measurement scale

5. Solve word problems involving money, including fractions and decimals, using a number line diagram with a measurement scale

6. Solve word problems that require expressing measurements given in a larger unit in terms of a smaller unit (i.e. conversions)

Key Concepts/Vocabulary

Distance – Length; a measurement of how far through space
Volume – Capacity; the amount of 3-dimensional space an object occupies amount
Mass – A measure of how much matter is in an object; commonly measured by how much something weighs
Fraction – How many equal parts of a whole
Decimal – Based on 10; a number that uses a decimal point followed by digits that show a value smaller than one
Place value – The value of where the digit is in the number

Standard-Specific Resources (4.MD.2)

- **<u>EngageNY:</u> Grade 4, Module 7, Topic B, Lesson 6 – [Solve problems involving mixed units of capacity.]()**

Concept Development (36 minutes)

Materials: (S) Personal white board

Problem 1: Add mixed units of capacity.

MP.7

T: 2 cats + 3 cats is …?
S: 5 cats.
T: 2 fourths + 3 fourths is …?
S: 5 fourths.
T: Express 5 fourths as a mixed number.
S: 1 and 1 fourth.
T: 2 quarts + 3 quarts is …?
S: 5 quarts.
T: Express 5 quarts as gallons and quarts.
S: 1 gallon 1 quart.

T: Here are two different ways of adding 2 quarts and 3 quarts. Analyze them with your partner.

Solution A

$$2\text{ qt} \xrightarrow{+2\text{ qt}} 1\text{ gal} \xrightarrow{+1\text{ qt}} 1\text{ gal } 1\text{ qt}$$

Solution B

$$2\text{ qt} + 3\text{ qt} = \underbrace{5\text{ qt}}_{4\text{ qt} \quad 1\text{ qt}} = 1\text{ gal } 1\text{ qt}$$

MP.7

S: Solution A makes 1 gallon first by adding on 2 quarts. → Solution B adds the quarts together and then takes out 1 gallon from 5 quarts. → Solution A completes a gallon just like if we were adding $\frac{2}{4}$ and $\frac{3}{4}$ and made one by adding $\frac{2}{4}$. → Solution B is like adding $\frac{2}{4}$ and $\frac{3}{4}$, getting $\frac{5}{4}$, and then taking out $\frac{4}{4}$ to get one and 1 fourth.

T: Yes, we can either complete a gallon first and then add on the remaining quarts or add to get 5 quarts and then rename to make 1 gallon and 1 quart.

Allow students to choose a method to solve and express the following sums with mixed units:

- 3 quarts + 3 quarts
- 2 cups + 3 cups
- 3 pints + 4 pints

> **NOTES ON MULTIPLE MEANS OF ENGAGEMENT:**
>
> Today's lesson of partner work and discussion fosters collaboration and communication that is valuable to students working below grade level because it may increase opportunities for one-on-one support and sustained engagement. Some learners may benefit from clear guidance in working effectively with others. Successful engagement comes by providing clear roles and responsibilities for partners or rubrics and norms that communicate partner work expectations.

T: Here are two different ways of adding 5 gallons 2 quarts + 3 quarts. Analyze them with a partner.

Solution C

$$5\text{ gal } 2\text{ qt} \xrightarrow{+2\text{ qt}} 6\text{ gal} \xrightarrow{+1\text{ qt}} 6\text{ gal } 1\text{ qt}$$

Solution D

$$5\text{ gal } 2\text{ qt} + 3\text{ qt} = 5\text{ gal } \underbrace{5\text{ qt}}_{1\text{ gal} \quad 1\text{ qt}} = 6\text{ gal } 1\text{ qt}$$

S: Solution C makes 1 gallon first by counting up 2 quarts to get 6 gallons and then adding on the extra quart. → Solution D adds the quarts together to get 5 gallons 5 quarts and then takes out one gallon from 5 quarts. → It's like adding mixed numbers—we add the like units.

Allow students to choose a method to solve and express the following sums with mixed units:

- 3 gallons 1 quart + 3 quarts
- 17 quarts 3 cups + 3 cups
- 4 gallons 7 pints + 7 pints

T: Here are two different ways of adding 5 gallons 2 quarts + 4 gallons 3 quarts. Analyze them with a partner.

Solution E

$$5\text{ gal } 2\text{ qt} \xrightarrow{+4\text{ gal}} 9\text{ gal } 2\text{ qt} \xrightarrow{+2\text{ qt}} 10\text{ gal} \xrightarrow{+1\text{ qt}} 10\text{ gal } 1\text{ qt}$$

Solution F

$$5\text{ gal } 2\text{ qt} + 4\text{ gal } 3\text{ qt} = 9\text{ gal } \underbrace{5\text{ qt}}_{1\text{ gal } \; 1\text{ qt}} = 10\text{ gal } 1\text{ qt}$$

S: Solution E adds on the gallons first to get 9 gallons, then adds 2 quarts to make another gallon, and finally adds the one left over quart. → Solution F adds gallons first to get 9 gallons and then makes the next gallon to get 10 gallons 1 quart. → It's just like adding mixed numbers! Add the ones and then add the smaller units. → This time, Solution F just added like units to get 9 gallons 5 quarts and then took out the gallon from the 5 quarts.

Allow students to choose a method to solve and express the following sums with mixed units:

- 3 gallons 1 quart + 6 gallons 3 quarts
- 17 quarts 3 cups + 2 quarts 3 cups
- 4 gallons 7 pints + 10 gallons 7 pints

Problem 2: Subtract mixed units of capacity.

T: 4 cats – 3 cats is ...?

S: 1 cat.

T: 4 fourths – 3 fourths is ...?

S: 1 fourth.

T: (Write $1 - \frac{3}{4}$.) 1 minus 3 fourths is ...?

S: 1 fourth.

T: (Directly below, write $8 - \frac{3}{4}$.) $8 - \frac{3}{4}$ is ...?

S: $7\frac{1}{4}$.

T: Here are two different subtraction problems. Solve them with your partner, and then compare how they are similar to each other and to the problems you just solved with the fourths.

Problem 1

1 qt - 3c

Problem 2

8 qt - 3c

7qt 4c

S: 1 quart – 3 cups = 1 cup. 8 quarts – 3 cups = 7 quarts 1 cup. → You have to change 1 quart for 4 cups so you can subtract the cups. → It's like subtracting a fraction from a whole number, too. Actually, cups are like fourths in this problem! It takes 4 cups to make a quart just like it takes 4 fourths to make 1. So, you can change 1 quart to 4 cups just like you change 1 to 4 fourths.

Have students solve the following:

- 1 gallon – 1 pint
- 8 gallons – 1 pint
- 1 quart – 2 cups
- 6 quarts – 2 cups

T: Here are two more subtraction problems. Solve them with your partner, and then compare them. How are they different? How are they the same?

Problem 3

8qt 1c - 3c

7qt 5c

Problem 4

8 qt 1c - 6qt 3c

7qt 5c

S: Problem 3 is a little trickier than Problem 2 because there is an extra cup. So, when you take 4 cups out of 8 quarts and 1 cup, you get 7 quarts and 5 cups because 4 cups + 1 cup is 5 cups. Now, you can subtract 3 cups. → In Problem 4, you have to subtract quarts, too, so just subtract like units. 7 quarts – 6 quarts is 1 quart. 5 cups – 3 cups is 2 cups. The answer is 1 quart 2 cups.

Have students solve the following:

- 9 gallons 2 quarts – 4 quarts
- 12 quarts 1 cup – 5 quarts 2 cups
- 6 gallons 3 pints – 2 gallons 7 pints

Note: Depending on how students are doing with the addition and subtraction of mixed capacity units, introduce compensation and counting up as exemplified below in the context of solving 8 quarts 1 cup – 6 quarts 3 cups. Solution A simply adds a cup to both the subtrahend and minuend (compensation). Solution B shows counting up from the subtrahend to the minuend.

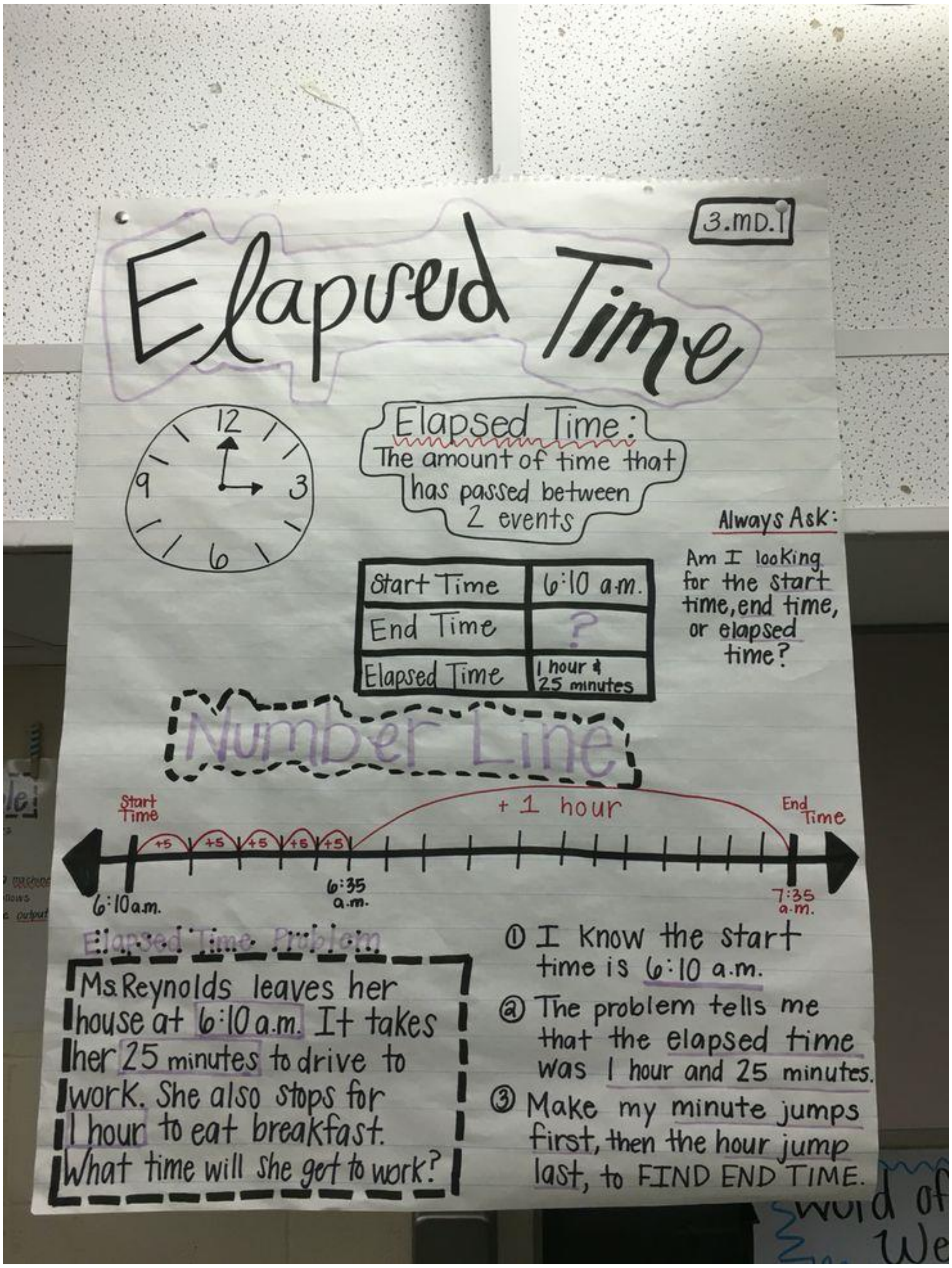
Solution A

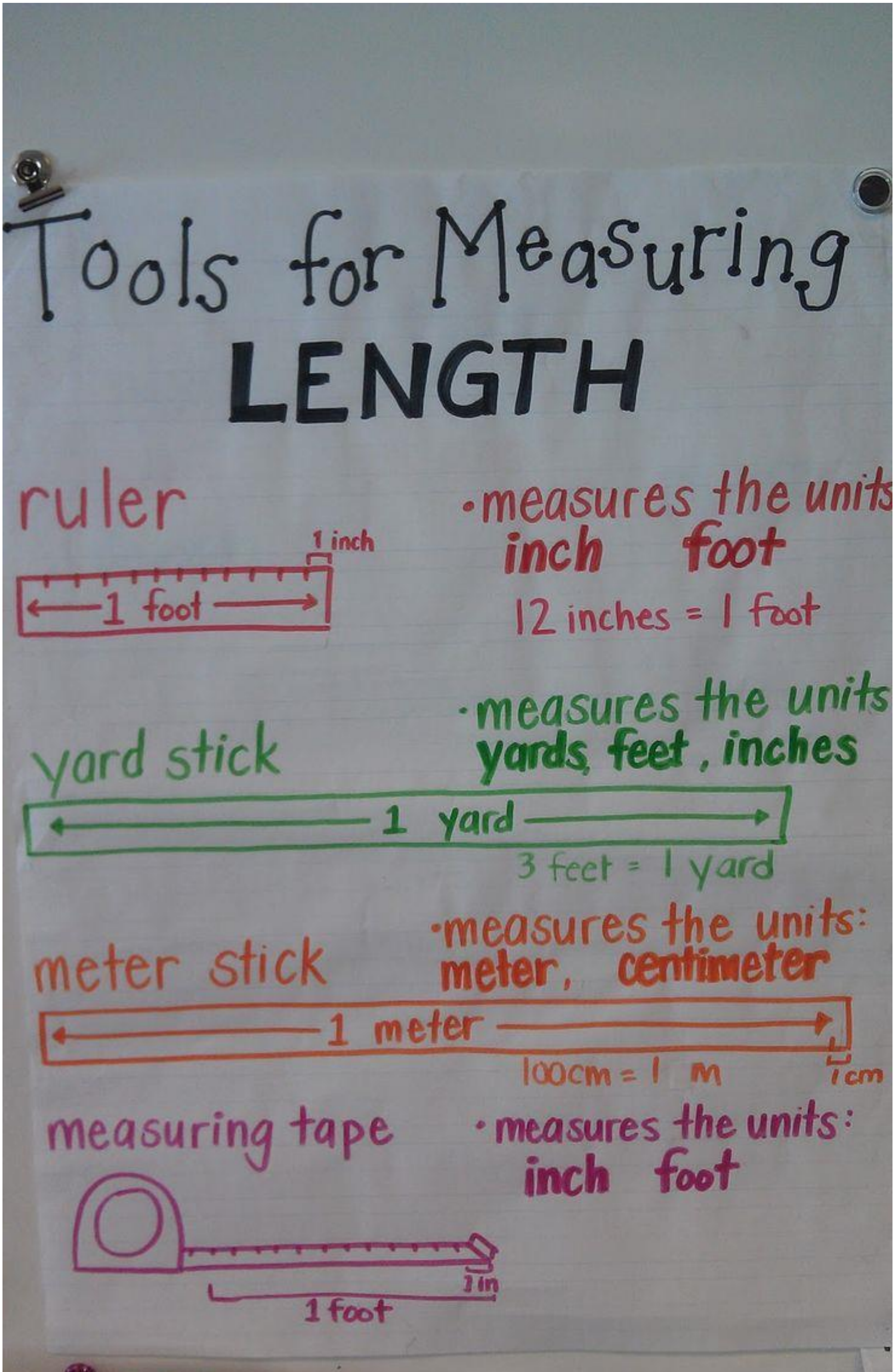
8qt 1c - 6 qt 3c = 8qt 2c - 7qt

Solution B

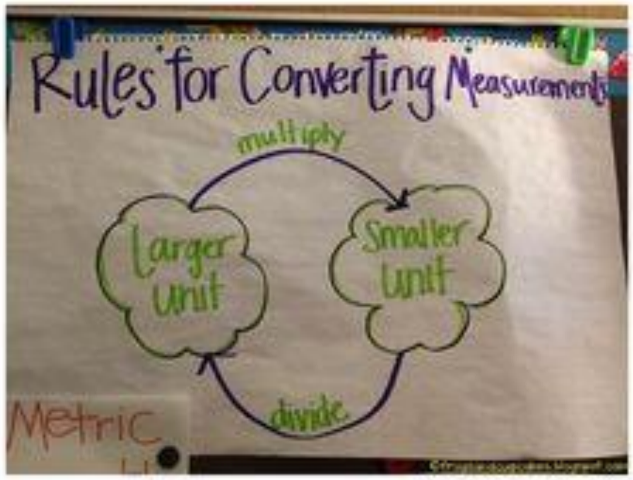
$6\text{qt } 3\text{c} \xrightarrow{+1\text{c}} 7\text{qt} \xrightarrow{1\text{qt } 1\text{c}} 8\text{qt } 1\text{c}$

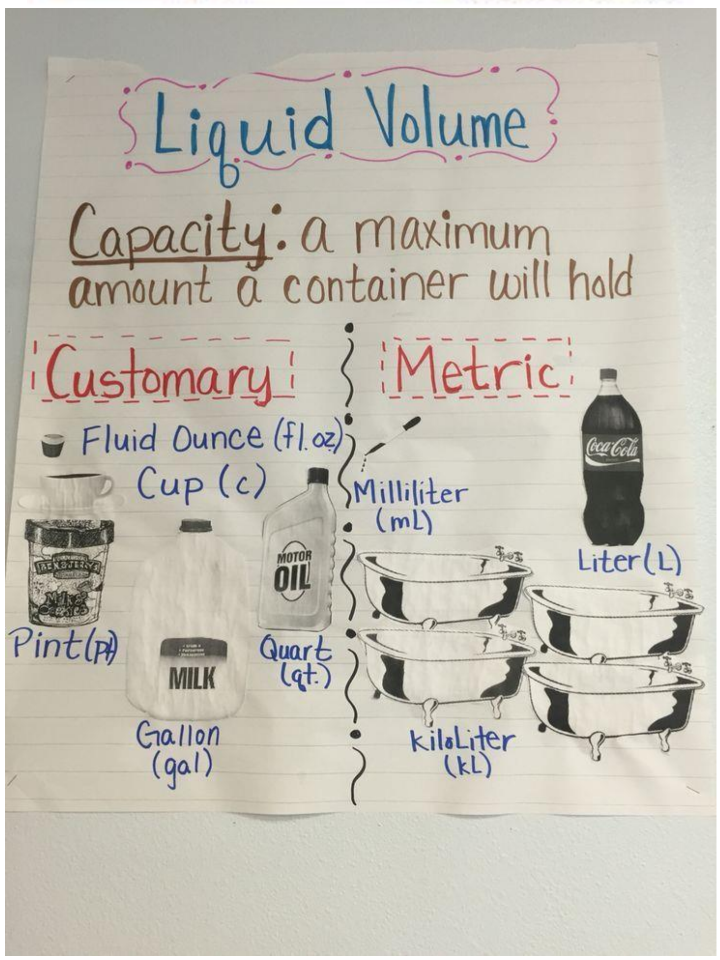
Sample Anchor Charts for 4.MD.2 - Solving Measurement Problems




Tools for Measuring
LENGTH
ruler
1 inch
1 foot
•measures the units
inch foot
12 inches = 1 foot
yard stick
•measures the units
yards, feet, inches
1 yard
3 feet = 1 yard
meter stick
•measures the units:
meter, centimeter
1 meter
100cm = 1 m
1cm
measuring tape
•measures the units:
inch foot
1 in
1 foot

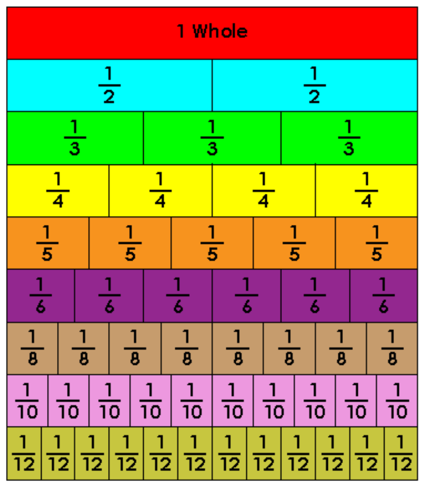

Crafting Connections
When you hear...
GRAM
imagine holding a (small) paperclip.
KILOGRAM
imagine holding a dictionary.
DICTIONARY
Choose the best estimate for each object.
apple 25 grams or 3 kilograms
think...
Would an apple weigh about the same as 25 paperclips (grams) or 3 dictionaries (kilograms)?
bike 8g or 8kg
cat 525g or 6kg
mouse 30g or 2kg
medium pizza 550g or 10kg
baseball 150g or 5kg
watermelon 450g or 3kg
iPhone 6 170g or 1kg
baby (newborn) 575g or 3kg


Rules for Converting Measurements
multiply
Larger Unit
Smaller Unit
divide
Metric


Liquid Volume
Capacity: a maximum amount a container will hold
Customary
Metric
Fluid Ounce (fl.oz)
Cup (c)
Pint (pt)
Gallon (gal)
Quart (qt.)
MOTOR OIL
MILK
Milliliter (mL)
Liter (L)
Coca-Cola
KiloLiter (kL)

Number and Operations - Fractions

(NF)



3.NF.1 Back to ccss standard

Develop understanding of fractions as numbers.

Understand a fraction *1/b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a/b* as the quantity formed by *a* parts of size *1/b.*

Skills

1. Recognize a unit fraction, such as ¼, as the quantity formed by 1 part when a whole is divided into 4 equal parts

2. Identify a fraction, such as 2/3, and explain how the quantity formed is 2 equal parts of the whole that has been divided into 3 equal parts (e.g., 1/3 and 1/3 of the whole 3/3)

3. Express a fraction as the number of unit fractions (e.g., 2/3 = 2 x 1/3)

4. Use accumulated unit fractions to represent numbers equal to, less than, or greater than 1 (e.g., 1/3 and 1/3 is 2/3; 1/3, 1/3, 1/3, and 1/3 is 4/3)

Key Concepts/Vocabulary

Fraction – How many equal parts of a whole

Unit fraction – A fraction where the top number/numerator is 1

Quantity – How much there is of something

Whole – All of something; complete

Partition – A division into parts; a separation

Equal – Exactly the same in amount or value

Represent – To ‘act or speak for’

Standard-Specific Resources (3.NF.1)

- **EngageNY:** **Grade 3, Module 5, Topic B, Lesson – Partition a whole into equal parts and define the equal parts to identify the unit fraction numerically**

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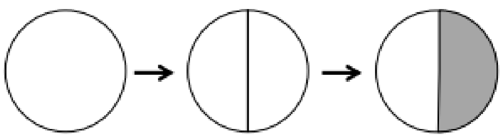
Concept Development (25 minutes)

Materials: (S) Personal white board

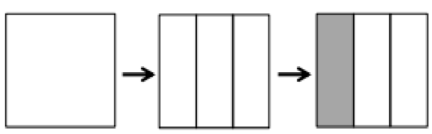
T: (Project or draw a circle, as shown below.) Whisper the name of this shape.
S: Circle.



1 half; $\frac{1}{2}$

> NOTES ON MULTIPLE MEANS OF REPRESENTATION:
>
> While introducing the new terms—*unit form*, *fraction form*, and *unit fraction*—check for student understanding. English language learners may choose to discuss definitions of these terms in their first language with the teacher or their peers.

T: Watch as I partition the whole. (Draw a line to partition the circle into 2 equal parts, as shown.) How many equal parts are there?
S: 2 equal parts.
T: What's the name of each unit?
S: 1 half.
T: (Shade one unit.) What fraction is shaded?
S: 1 half.
T: Just like any number, we can write one half in many ways. This is the **unit form**. (Write *1 half* under the circle.) This is the **fraction form**. (Write $\frac{1}{2}$ under the circle.) Both of these refer to the same number, 1 out of 2 equal units. We call 1 half a **unit fraction** because it names one of the equal parts.
T: (Project or draw a square, as shown below.) What's the name of this shape?
S: It's a square.

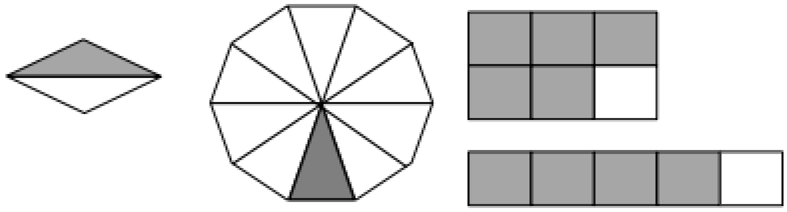


1 third; $\frac{1}{3}$

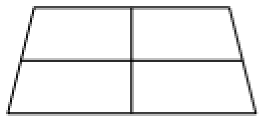
> NOTES ON MULTIPLE MEANS OF ENGAGEMENT:
>
> Students working above grade level may enjoy identifying fractions with an added challenge of each shape representing a *fraction* rather than the whole. For example, ask the following: "If the square is 1 third, name the shaded region" (e.g., $\frac{3}{12}$ or $\frac{1}{4}$).

T: Draw it on your personal white board. (After students draw the square.) Estimate to partition the square into 3 equal parts.
S: (Partition.)
T: What's the name of each unit?
S: 1 third.
T: Shade one unit. Then, write the fraction for the shaded amount in unit form and fraction form on your board.
S: (Shade and write 1 third and $\frac{1}{3}$.)
T: Talk to a partner: Is the number that you wrote to represent the shaded part a unit fraction? Why or why not?
S: (Discuss.)

ontinue the process with more shapes as needed. The following suggested shapes include examples of both haded and non-shaded unit fractions. Alter language accordingly.



T: (Project or draw the following image.) Discuss with your partner: Does the shape have equal parts? How do you know?



MP.6

S: No. The parts are not the same size. → They're also not exactly the same shape. → The parts are not equal because the bottom parts are larger. The lines on the sides lean in at the top.

T: Most agree that the parts are not equal. How could you partition the shape to make the parts equal?

S: I can cut it into 2 equal parts. You have to cut it right down the middle going up and down. The lines aren't all the same length like in a square.

T: Turn and talk: If the parts are not equal, can we call these fourths? Why or why not?

S: (Discuss.)

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Review personal goals with students. For example, if students working below grade level chose to solve one word problem (per lesson) last week, encourage them to work toward completing two word problems by the end of this week.

T: Box the part of your array that shows a total of 12.

S: (Box the remaining 4 rows.)

T: Now, write a division equation for that part of the array.

S: (Write $12 \div 3 = 4$.)

T: Tell your partner how you will use the equations to help you solve the original equation, $27 \div 3 =$ __.

S: I'll add the quotients of the two smaller facts.

T: (Write the following.) Complete the following sequence to solve $27 \div 3$ with your partner.

$$27 \div 3 = (15 \div 3) + (12 \div 3)$$
$$= \_\_\_\_\_ + \_\_\_\_\_\_$$
$$= \_\_\_\_\_$$

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Add a challenge by asking students to think about other ways of breaking apart 27. A student will most likely choose parts that are not evenly divisible by 3. This will lead to a discussion that gets students to realize that, with division, the strategy relies on the decomposition being such that the dividends must be evenly divisible by the divisor.

Repeat the process with $33 \div 3$. Students can break apart 33 by using the number pair 30 and 3.

NOTES ON MULTIPLE MEANS

__Sample Anchor Chart for 3.NF.1 - Recognize and Write Fractions__

What fraction is shaded blue? 

__Step 1:__ Count how many equal parts there are, in total, of all colors - This is your *denominato.* (6 equal parts)

__Step 2:__ Count how many equal parts are colored blue - this is your *numerator* (2 equal parts colored blue)

__Step 3:__ Prepare your fraction by drawing a horizontal line

__Step 4:__ Write the *denominator* __under__ the line

$$\frac{}{6}$$

__Step 5:__ Write the *numerator* __over__ the line

$$\frac{2}{6}$$

__Step 6:__ Write your fraction in words (two sixths)

3.NF.2A Back to ccss standard

Develop understanding of fractions as number.

Understand a fraction on the number line; represent fractions on a number line diagram:

Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.

Skills

1. Define the interval from 0 to 1 on a number line as a whole

2. Divide a whole number line into equal parts

3. Represent each equal part on the number line with a fraction

4. Explain that the endpoint of each equal part represents the total number of equal parts

Key Concepts/Vocabulary

Interval – What is between 2 points or values

Divide – A division into parts; a separation

Equal – Exactly the same in amount or value

Represent – To 'act or speak for'

Fraction – The number of parts of an equally-divided whole

Endpoint – Any of the two furthest points on a line segment

Total – The result of adding, sum

Standard-Specific Resources (3.NF.2a)

- **EngageNY:** **Grade 3, Module 5, Topic D, Lesson 14 – [Place fractions on a number line with endpoints 0 and 1.](#)**

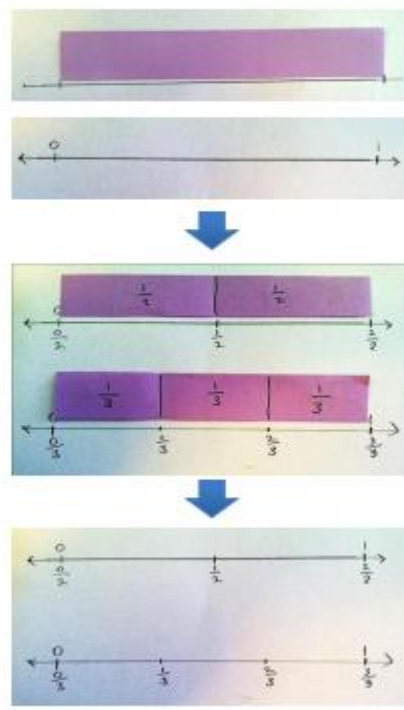
Concept Development (33 minutes)

Materials: (T) Board space, yardstick, large fraction strip for modeling (S) Fraction strips, blank paper, ruler

Part 1: Measure a line of length 1 whole.

T: (Model the steps below as students follow along on their personal white boards.)

1. Draw a horizontal line with your ruler that is a bit longer than one of your fraction strips.
2. Place a whole fraction strip just above the line you drew.
3. Make a small mark on your line that is even with the left end of your strip.
4. Label that mark 0 above the line. This is where we start measuring the length of the strip.
5. Make a small mark on your line that is even with the right end of your strip.
6. Label that mark 1 above the line. If we start at 0, the 1 tells us when we've travelled 1 whole length of the strip.



MP.7

Part 2: Measure the fractions.

T: (Model the steps below as students follow along on their boards.)

1. Place your fraction strip with halves above the line.
2. Make a mark on the number line at the right end of 1 half. This is the length of 1 half of the fraction strip.
3. Label that mark $\frac{1}{2}$. Label 0 halves and 2 halves.
4. Repeat the process to measure and label other fractional numbers on a number line.

T: Look at your number line with thirds. Read the numbers on this line to a partner.

S: 0, 1. → I think it's 0, $\frac{1}{3}$, $\frac{2}{3}$, 1. → What about $\frac{0}{3}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{3}$? → Are fractions numbers?

T: Some of you read the whole numbers, and others read whole numbers and fractions. Fractions are numbers. Let's read the numbers from least to greatest, and let's say 0 thirds and 3 thirds for now rather than zero and one.

S: (Read numbers, $\frac{0}{3}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{3}$.)

T: Let's read again and this time say zero and 1 rather than 0 thirds and 3 thirds.

S: (Read numbers, 0, $\frac{1}{3}$, $\frac{2}{3}$, 1 .)

Part 3: Draw number bonds to correspond with the number lines.

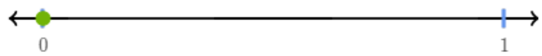
Once students have become excellent at making and labeling fractions on number lines using strips to measure, have them draw number bonds to correspond. Use questioning while circulating to help them see similarities and differences between the bonds, fraction strips, and fractions on the number line. Guide students to recognize that placing fractions on the number line is analogous to placing whole numbers on the number line. If preferred, the following suggestions can be used:

- What do both the number bond and number line show?
- Which model best shows how big the unit fraction is in relation to the whole? Explain how.
- How do your number lines help you make number bonds?

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

This lesson gradually leads students from the concrete level (fraction strips) to the pictorial level (number lines).

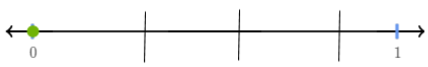
__Sample Anchor Chart for 3.NF.2a - Using Unit Fractions on the Number Line__



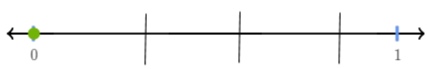
Label where ¾ is on the number line.

__Step 1:__ **Identify the number of equal parts/denominator in the fraction you need to find (3/4 → number of equal parts = 4 = denominator)**

__Step 2:__ **Divide the number line, from 0 to 1, into the number of equal parts from Step** 1 **(Divide the line into 4 equal parts)**



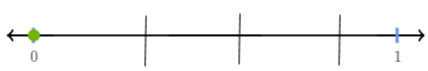
__Step 3:__ **Identify the whole (or, '1') as a fraction with number from Step 1 in *both* the numerator and denominator**



4/4 = 1

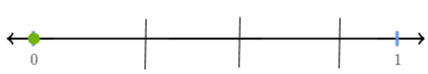
__Step 4:__ **For the first equal part, label it 1/# of number equal parts in the whole (1st part = ¼, since it's the first part of a whole divided into 4 equal parts)**

¼



__Step 5:__ **Label the next equal part as 2/# of equal parts in the whole and the next equal part as 3/# of equal parts in the whole (2nd part = 2/4, since it's the second part of a whole divided into 4 equal parts, and the 3rd part = 3/4)**

¼ 2/4 ¾ 4/4



__Step 6:__ **Draw a dot on the fraction the problem asked you to find! (Draw a dot at ¾)**

3.NF.3D   Back to ccss standard

Develop understanding of fractions as numbers.

Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g. by using a visual fraction model.

Skills

1. Explain what the numerator and denominator in a fraction each represent and where each is located

2. Determine if comparisons of fractions can be made (are parts of the same whole)

3. Compare two fractions with the same numerator by reasoning about their size, recording the results of comparisons using the symbols >, <, or =

4. Compare two fractions with the same denominator by reasoning about their size, recording the results of comparisons using the symbols >, <, or =

Key Concepts/Vocabulary

Fraction – How many equal parts of a whole that's been divided into equal parts

Numerator – The top number in a fraction; how many equal parts you're talking about

Denominator – The bottom number in the fraction; how many total equal parts are in 1 whole

Compare – Determining whether a value is smaller than, greater than, or equal to another value

Equal to (=)

Less than (<)

Greater than (>)

Standard-Specific Resources (3.NF.3d)

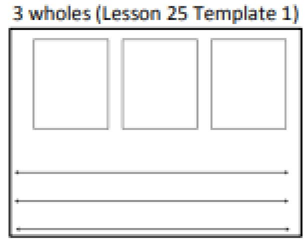
- **EngageNY: Grade 3, Module 5, Topic F, Lesson 29 – Compare fractions with the same numerator using <, >, or =, and use a model to reason about their size.**

Concept Development (30 minutes)

Materials: (S) Personal white board, 3 wholes (Lesson 25 Template 1)

Seat students in pairs facing each other in a large circle around the room. 3 wholes should be in their personal white boards.

3 wholes (Lesson 25 Template 1)



T: Today, we'll only use the first rectangle. At my signal, draw and shade a fraction less than $\frac{1}{2}$, and label it below the rectangle. (Signal.)

S: (Draw and label.)

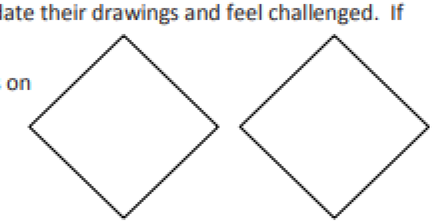
T: Check your partner's work to make sure it's less than $\frac{1}{2}$.

S: (Check.)

T: This is how we're going to play a game today. For the next round, we'll see which partner is quicker but still accurate. As soon as you finish drawing, raise your personal white board. If you are quicker, then you are the winner of the round. If you are the winner of the round, you will stand up, and your partner will stay seated. If you are standing, you will then move to partner with the person on your right, who is still seated. Ready? Erase your boards. At my signal, draw and label a fraction that is greater than $\frac{1}{2}$. (Signal.)

S: (Draw and label.)

The student who goes around the entire circle and arrives back at his original place faster than the other students wins the game. The winner can also be the student who has moved the furthest if it takes too long to play all the way around. Move the game at a brisk pace. Use a variety of fractions, and mix it up between greater than and less than so that students constantly need to update their drawings and feel challenged. If preferred, mix it up by calling out *equal to*.



T: (Draw or show the images on the right.) Draw my shapes on your board. Make sure they match in size like mine.

S: (Draw.)

T: Partition both shapes into sixths.

S: (Partition.)

T: Partition the second shape to show double the number of units in the same whole.

S: (Partition.)

T: What fractional units do we have?

S: Sixths and twelfths.

T: Shade in 4 units of each shape, and label the shaded fraction below each shape.

S: (Shade and label.)

T: Whispering to your partner, say a sentence comparing the fractions using the words *greater than*, *less than*, or *equal to*.

S: $\frac{4}{6}$ is greater than $\frac{4}{12}$.

T: Now, write the comparison as a number sentence with the correct symbol between the fractions.

S: (Write $\frac{4}{6} > \frac{4}{12}$.)

T: (Draw or show the images on the right.) Draw my rectangles on your board. Make sure they match in size like mine.

S: (Draw.)

T: Partition the first rectangle into sevenths and the second one into fifths.

S: (Partition.)

T: Shade in 3 units of each rectangle, and label the shaded fraction below each rectangle.

S: (Shade and label.)

T: Whispering to your partner, say a sentence comparing the fractions using the words *greater than*, *less than*, or *equal to*.

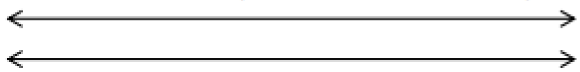
S: $\frac{3}{7}$ is less than $\frac{3}{5}$.

T: Now, write the comparison as a number sentence with the correct symbol between the fractions.

S: (Write $\frac{3}{7} < \frac{3}{5}$.)

Do other examples, if necessary, using a variety of shapes and units.

T: Draw 2 number lines on your board, and label the endpoints 0 and 1.



S: (Draw and label.)

T: Partition the first number line into eighths and the second one into tenths.

S: (Partition.)

T: On the first number line, label $\frac{8}{8}$.

S: (Label.)

T: On the second number line, label 2 copies of $\frac{5}{10}$.

S: (Label.)

T: Whispering to your partner, say a sentence comparing the fractions using the words *greater than*, *less than*, or *equal to*.

S: Wait, they're the same! $\frac{8}{8}$ is equal to $\frac{10}{10}$.

T: How do you know?

S: Because they have the same point on the number line. That means they're equivalent.

T: Now, write the comparison as a number sentence with the correct symbol between the fractions.

S: (Write $\frac{8}{8} = \frac{10}{10}$.)

Do other examples with the number line. In subsequent examples that use smaller units or units that are farther apart, move to using a single number line.

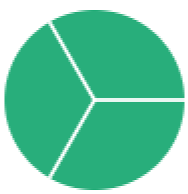
Sample Anchor Charts for 3.NF.3d – Comparing Fractions

Compare.

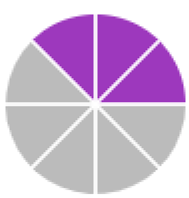
$\frac{3}{3} \_ \frac{3}{8}$

Step 1: Look at the numerator of each fraction to determine how many pieces of each whole you need to shade in **(Both fractions have a numerator of 3)**

Step 2: Draw a circle to represent the first fraction and divide it into the number of equal parts shown in the denominator, and shade in the number of equal parts from Step 1 **(3/3 has a denominator of 3, with all 3 of its parts shaded in)**



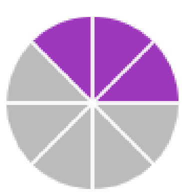
Step 3: Draw a circle to represent the second fraction and divide it into the number of equal parts shown in the denominator, and shade in the number of equal parts from Step 1 **(3/8 has a denominator of 8, with 3 of its parts shaded in)**



Step 4: To compare the two fractions, determine which is larger by finding the one that has the most of its whole shaded in **(3/3 has the entire whole shaded in, while 3/8 has less than half of its whole shaded in)**

Step 5: Write out the final fraction comparison using the appropriate > (greater than) or < (less than) symbol

3/3 > 3/8

Or  **>** 

COMPARING FRACTIONS

 $\frac{1}{2} > \frac{1}{4}$ 

One half **is greater than** one fourth.

 $\frac{3}{6} < \frac{5}{6}$ 

Three sixths **is less than** five sixths.

Tip: Fill in the inequality means to compare the fractions using one of the symbols below.

- Less than (<)
- Greater than (>)
- Equivalent to (=)

$\frac{1}{3}$

Numerator

The number **shaded** or being **counted**

$\frac{1}{3}$

Denominator

The **TOTAL** number of equal parts that represent a **whole**

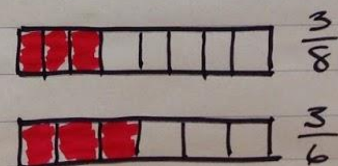
Comparing Fractions

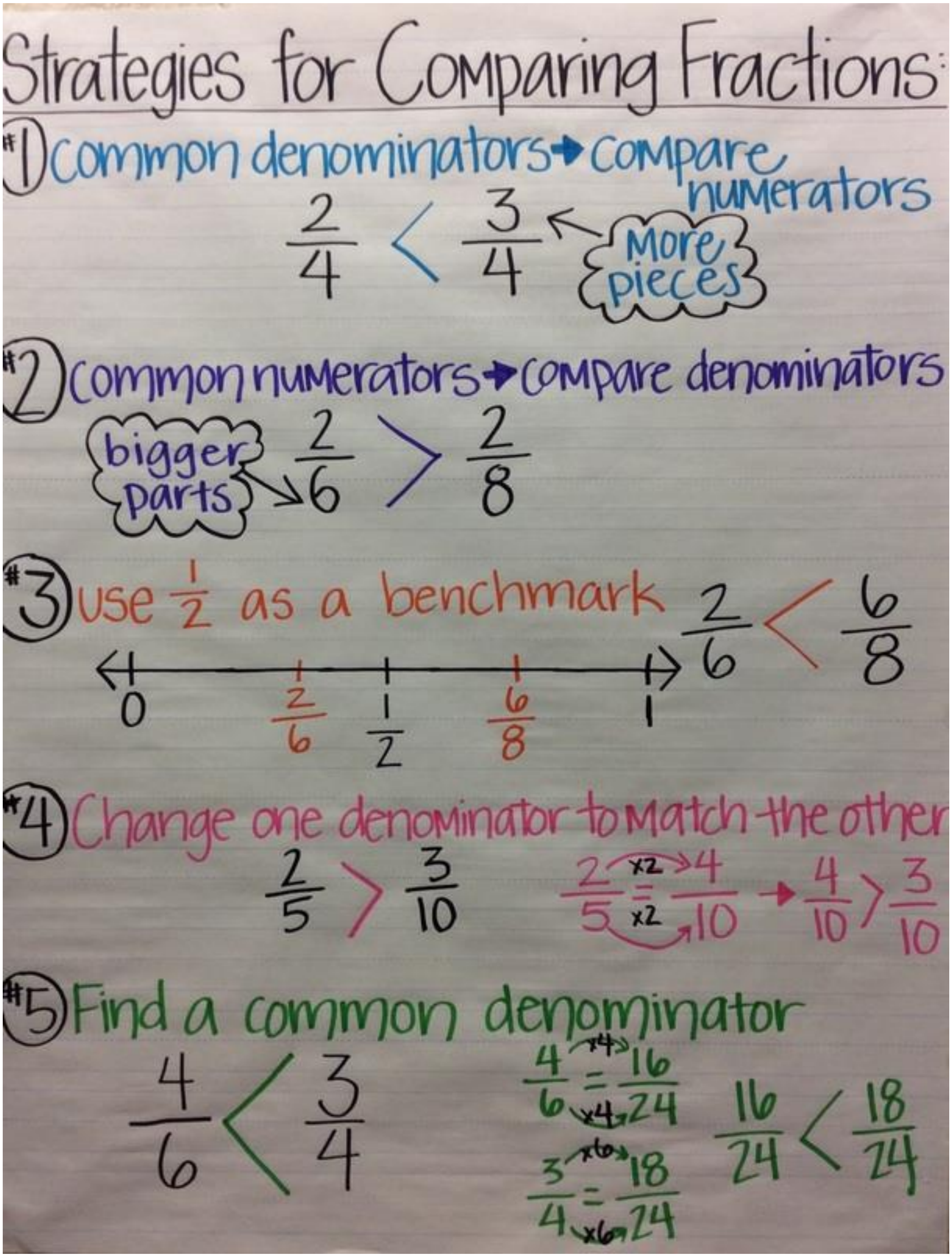
1. If denominator is the same, the larger numerator is the larger fraction

$$\frac{3}{8} < \frac{5}{8}$$

2. If the denominator is different, but the numerator is the same, the fraction with the smaller denominator is larger!

$$\frac{3}{8} < \frac{3}{6}$$




Strategies for Comparing Fractions:
#1 common denominators → compare numerators
2/4 < 3/4
More pieces
#2 common numerators → compare denominators
bigger parts
2/6 > 2/8
#3 use 1/2 as a benchmark
2/6 < 6/8
0
2/6
1/2
6/8
1
#4 Change one denominator to match the other
2/5 > 3/10
2/5 ×2 = 4/10 → 4/10 > 3/10
#5 Find a common denominator
4/6 < 3/4
4/6 ×4 = 16/24
3/4 ×6 = 18/24
16/24 < 18/24

4.NF.2 Back to ccss standard

Extend understanding of fraction equivalence and ordering.

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Skills

1. Recognize fractions as being greater than, less than, or equal to other fractions

2. Record comparison results with symbols: >, <, =

3. Use benchmark fractions such as ½ for comparison purposes

4. Make comparisons based on parts of the same whole

5. Compare two fractions with different numerators and justify the conclusion

6. Compare two fractions with different denominators and justify the conclusion

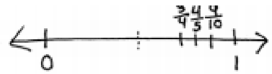
Key Concepts/Vocabulary

Whole number - Any of the number 0, 1, 2, and beyond, with no fractional or decimal part, and not negative
Fraction – The number of equal parts out of a whole that has been divided into equal parts
Numerator – The top number in a fraction; the number of equal parts you have
Denominator – The bottom number in a fraction; the total number of equal parts in one whole
< - less than
> - greater than
= - equal to
Benchmark fraction – A common fraction (such as ½, ¼, etc.) that you can use to compare other fractions

Standard-Specific Resources (4.NF.2)

- **EngageNY: Grade 4, Module 5, Topic C, Lesson 14 – [Find common units or number of units to compare two fractions]**

Concept Development (33 minutes)



Materials: (S) Personal white board

Problem 1: Reason about fraction size using unit language.

T: Which is greater—1 apple or 3 apples?

S: 3 apples!

T: (Write 3 apples > 1 apple.)

T: Which is greater—1 fourth or 3 fourths?

S: 3 fourths!

T: (Write 3 fourths > 1 fourth.)

T: What do you notice about these two statements?

3 apples > 1 apple.

3 fourths > 1 fourth.

S: The units are the same in each. One is apples, and the other is fourths. → We can compare the number of fourths like we compare the number of apples. → It is easy to compare when the units are the same!

MP.7

> **NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:**
>
> To accurately compare two fractions using a tape diagram, both tape diagrams must be the same length and aligned precisely. Providing a template of two blank parallel tape diagrams of equal length may be helpful in assisting students.

T: Which is greater—1 fourth or 1 fifth?

S: 1 fourth.

T: (Write 1 fourth > 1 fifth.)

T: How do you know?

S: I can draw two tape diagrams to compare. I can partition a whole into fourths on one tape diagram and into fifths on the other. There are more fifths than fourths, so each fourth is going to be bigger than a fifth. → $\frac{1}{5}$ is less than $\frac{1}{4}$ because fifths are smaller than fourths.

T: (Write $\frac{1}{4} > \frac{1}{5}$.)

T: Which is greater—2 fourths or 2 sixths?

S: 2 fourths is greater than 2 sixths.

T: (Write $\frac{2}{4} > \frac{2}{6}$.)

T: What do you notice about these statements?

$\frac{1}{4} > \frac{1}{5}$ $\frac{2}{4} > \frac{2}{6}$

S: Fourths are greater than fifths and sixths. → In each comparison, the numerators are the same.

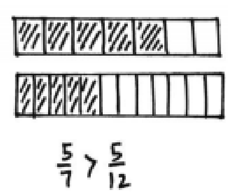
T: Which would be greater—2 inches or 2 feet?

S: 2 feet! I know feet are greater than inches.

MP.7

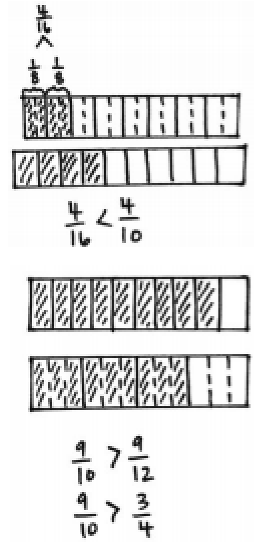
T: In the same way, 2 fourths is greater than 2 sixths because fourths are greater than sixths.

T: When the numerator is the same, we look at the denominator to reason about which fraction is greater. The greater the denominator, the smaller the fractional unit. Explain why $\frac{5}{7}$ is greater than $\frac{5}{12}$ of the same whole.



S: Sevenths are greater fractional units than twelfths. 5 sevenths are greater than 5 twelfths because 1 seventh is greater than 1 twelfth. → The sum of 5 larger units is going to be greater than the sum of 5 smaller units.

Problem 2: Compare fractions with related numerators.



T: (Display $\frac{2}{8}$ and $\frac{4}{10}$.) Draw a tape diagram to show each.

T: Partition the eighths in half. What fraction is now shown?

S: $\frac{4}{16}$. The numerators are the same! → The number of shaded units is the same.

T: Compare $\frac{4}{16}$ and $\frac{4}{10}$.

S: $\frac{4}{16}$ is less than $\frac{4}{10}$ since sixteenths are smaller units than tenths. I can compare the size of the units because the numerators are the same.

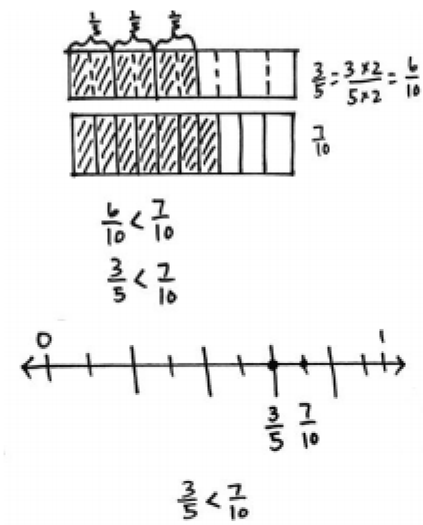
T: Compare $\frac{2}{8}$ and $\frac{4}{10}$.

S: $\frac{2}{8}$ is less than $\frac{4}{10}$.

T: (Display $\frac{9}{10}$ and $\frac{3}{4}$.) Discuss a strategy for comparing these two fractions with your partner.

S: Let's make a common numerator of 9. $\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$. → $\frac{9}{10}$ is greater than $\frac{9}{12}$. → $\frac{9}{10}$ is greater than $\frac{3}{4}$. → $\frac{9}{10} + \frac{1}{10} = 1$, and $\frac{3}{4} + \frac{1}{4} = 1$. 1 tenth is less than 1 fourth, so 9 tenths is greater.

Problem 3: Compare fractions having related denominators where one denominator is a factor of the other.



T: (Display $\frac{7}{10}$ and $\frac{3}{5}$.) Model each fraction using a tape diagram. Can we make a common numerator?

S: No. We can't multiply 3 by a number to get 7. → We could make them both numerators of 21.

T: Finding a common numerator does not work easily here. Consider the denominators. Can we make like units, or **common denominators**?

S: Yes. We can partition each fifth in half to make tenths. → $\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10}$.

T: Compare $\frac{6}{10}$ and $\frac{7}{10}$.

S: $\frac{6}{10}$ is less than $\frac{7}{10}$. → That means that $\frac{3}{5}$ is less than $\frac{7}{10}$.

T: Draw a number line to show 3 fifths. Decompose the line into tenths to show 7 tenths. $\frac{3}{5}$ is equal to how many tenths?

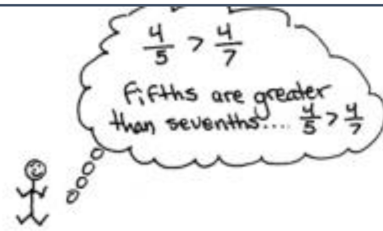
S: $\frac{6}{10}$.

T: Compare $\frac{6}{10}$ and $\frac{7}{10}$.

S: $\frac{6}{10}$ is less than $\frac{7}{10}$, so $\frac{3}{5} < \frac{7}{10}$.

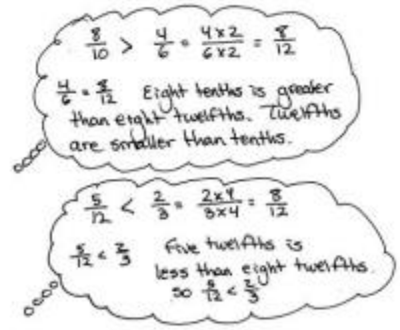
Problem 4: Compare fractions using different methods of reasoning.

T: Think about the strategies that we have learned. What strategy would you use to compare $\frac{4}{5}$ and $\frac{4}{7}$? Discuss with your partner. Defend your reasoning.



S: The numerators are the same. $\frac{4}{5}$ is greater than $\frac{4}{7}$. → There are 4 fifths and 4 sevenths. Since fifths are greater than sevenths, $\frac{4}{5}$ is greater than $\frac{4}{7}$. → 4 fifths is a lot more than 1 half. 4 sevenths is a little more than 1 half.

T: Compare $\frac{8}{10}$ and $\frac{4}{6}$.



S: It looks like we can make numerators that are the same because 8 is a multiple of 4. $\frac{4}{6}$ is the same as $\frac{8}{12}$. $\frac{8}{12}$ is less than $\frac{8}{10}$. So, $\frac{4}{6}$ is less than $\frac{8}{10}$. → $\frac{8}{10} + \frac{2}{10} = 1$, and $\frac{4}{6} + \frac{2}{6} = 1$. I know that 2 tenths is less than 2 sixths, so 8 tenths is greater.

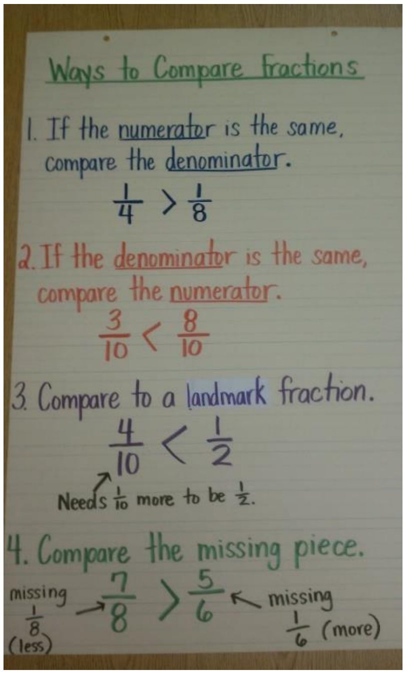
T: Compare $\frac{5}{12}$ and $\frac{2}{3}$.

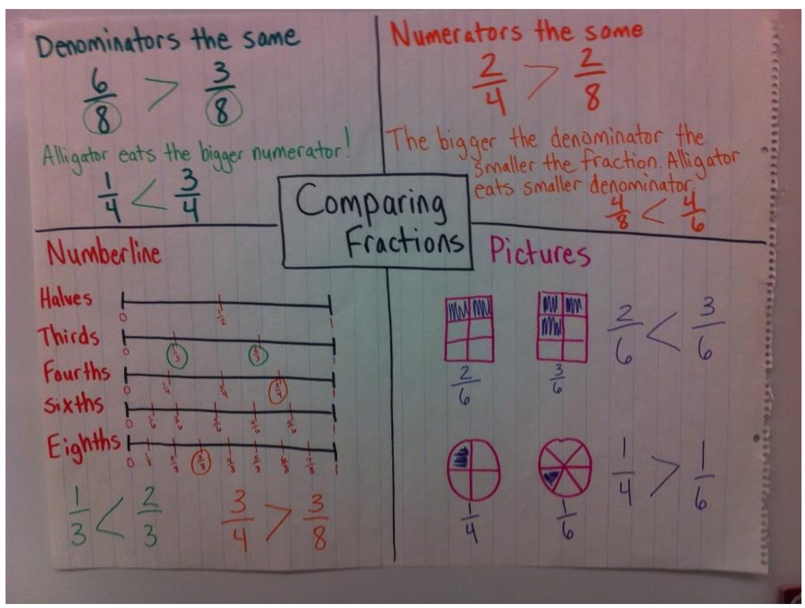
S: The units are different! Twelfths are not thirds, but we can decompose thirds to make twelfths! We can make like denominators. $\frac{2}{3}$ is the same as $\frac{8}{12}$. $\frac{8}{12}$ is more than $\frac{5}{12}$. $\frac{2}{3} > \frac{5}{12}$. → I wouldn't try to make the same number of units, because 5 is not a multiple of 2, but it might be possible. → 5 twelfths is less than a half, and 2 thirds is more than a half.

T: How might we use what we know to compare $1\frac{2}{5}$ and $1\frac{6}{8}$? Share your thoughts with your partner.

S: I see that the whole numbers are the same, so we can just compare the fractions. Let's compare $\frac{2}{5}$ and $\frac{6}{8}$. The numerators are related. 6 is a multiple of 2, so we can make fractions that have equal numerators. $\frac{2}{5}$ is the same as $\frac{6}{15}$, which is smaller than $\frac{6}{8}$. So, $1\frac{2}{5}$ is less than $1\frac{6}{8}$. → 2 fifths is less than half. 6 eighths is greater than half, so $1\frac{6}{8}$ is greater.

<u>Sample Anchor Charts for 4.NF.2 - Comparing Fractions</u>





4.NF.6 Back to ccss standard

Understand decimal notation for fractions, and compare decimal fractions.

Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

Skills

1. Represent fractions with denominators of 10 with pictures and decimal notation

2. Represent fractions with denominators of 100 with pictures and decimal notation

3. Convert decimals to fractions with denominators of 10 or 100

4. Compare decimal fractions with denominators of 10

5. Compare decimal fractions with denominators of 100

6. Explain how decimals and fractions relate to each other

Key Concepts/Vocabulary

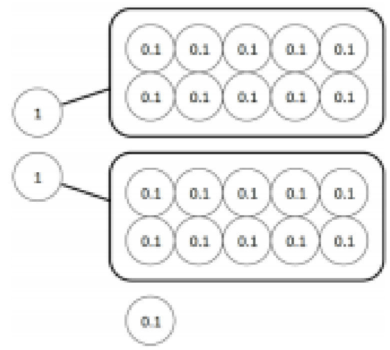
Decimal - "based on 10"
Decimal number – a number that uses a decimal point followed by digits that show a value smaller than one
Fraction – How many equal parts of a whole
Numerator – The top number in a fraction; the number of equal parts we have
Denominator – The bottom number in a fraction; how many equal parts the whole is divided into
< - less than
> - greater than
= - equal to

Standard-Specific Resources (4.NF.6)

- **<u>EngageNY:</u> Grade 4, Module 6, Topic A, Lesson 3 –** [**Represent mixed numbers with units of tens, ones, and tenths with place value disks, on the number line, and in expanded form.**]

Concept Development (35 minutes)

Materials: (T/S) Whole number place value disks (tens and ones), decimal place value disks (tenths), personal white board, tenths on a number line (Template)



Problem 1: Make groups of 10 tenths to rename as ones. Write the number in decimal form.

T: With a partner, use place value disks to show 21 units of 1 tenth in five-group formation.

S: (Lay out 21 disks, all tenths, in five-group formation, as shown.)

T: Talk with your partner. Is there any way we can use *fewer* disks to show this same value?

S: We can bundle 10 tenths to make one. → There are 2 groups of 10 tenths, so we can show 21 tenths as 2 ones 1 tenth. → In the five-groups, I can see 2 groups of 10 disks. 10 tenths is 1 whole. We have 1 (circling group with finger), 2 (circling group with finger) groups that make 2 ones, and then 1 tenth (touching final 0.1 disk.)

T: Let's group 10 tenths together and trade them for…?

S: 1 one.

T: How many times can we do this?

S: 1 more time. → 2 times.

T: What disks do we have now?

S: 2 ones and 1 tenth.

T: Express this number in decimal form.

S: (Write 2.1.)

T: How many more tenths would we have needed to have 3 ones?

S: 9 tenths more. → 0.9.

Repeat the process using disks to model 17 tenths. Then, continue the process having the students draw disks for 24 tenths. Have students circle the disks being bundled.

NOTES ON MULTIPLE MEANS FOR ACTION AND EXPRESSION:

Be sure to enunciate */th/* at the end of *tenths* to help English language learners distinguish *tenths* and *tens*. Try to speak more slowly, pause mor frequently, or couple language with a tape diagram. Check for student understanding and correct pronunciation of fraction names.

Problem 2: Represent mixed numbers with units of tens, ones, and tenths in expanded form.

T: Hold up a place value disk with a value of 1 ten. We say the value of this disk is...?

S: 1 ten. → Ten.

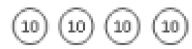
T: (Draw or show 4 tens disks.) The total value of 4 of these is...?

S: 4 tens. → Forty.

T: 4 tens written as a multiplication expression is?

S: 4 × 1 ten. → 4 × 10.

T: (Write the expression below the disks, as pictured to the right.) 4 × 10 is...?



$4 \times 10 = 40$

S: 40.

T: (Complete the number sentence. Draw or show 2 ones disks.) The total value of these 2 disks is...?

S: 2 ones. → Two.

T: 2 ones written as a multiplication expression is...?

S: 2 × 1.

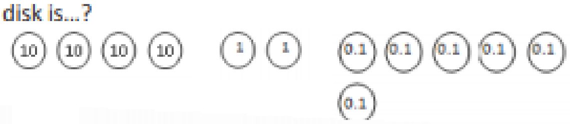
T: (Write the expression below the disks, as pictured to the right.) (4 × 10) + (2 × 1) is...?

$(4 \times 10) + (2 \times 1) = 42$

S: 42.

T: (Complete the number sentence. Draw or show a tenth disk.) This place value disk says zero point one on it. We say the value of this disk is...?

S: 1 tenth.



$(4 \times 10) + (2 \times 1) + (6 \times \frac{1}{10}) = 42\frac{6}{10}$

$(4 \times 10) + (2 \times 1) + (6 \times 0.1) = 42.6$

T: (Draw or show 6 one-tenth disks in five-group formation.) The total value of 6 of these disks is ...?

S: 6 tenths.

T: 6 tenths written as a multiplication expression is...?

S: $6 \times \frac{1}{10}$.

T: (Write the expression below the disks, as pictured above.) Discuss the total value of the number represented by the disks with your partner.

S: Do what is in the parentheses first, and then find the sum. $40 + 2 + \frac{6}{10}$ is $42\frac{6}{10}$. → 4 tens, 2 ones, 6 tenths. → It is like expanded form.

T: We have written $42\frac{6}{10}$ in expanded form, writing each term as a multiplication expression. Just like with whole numbers, the expanded form allows us to see the place value unit for each digit.

T: (Point to $(4 \times 10) + (2 \times 1) + (6 \times \frac{1}{10}) = 42\frac{6}{10}$.) Talk with your partner. How could you write this using **decimal expanded form** instead of **fraction expanded form**? Explain how you know.

S: (Work with partners, and write (4 × 10) + (2 × 1) + (6 × 0.1) = 42.6.) I know that 1 tenth can be written as zero point one, and 42 and 6 tenths can be written as forty-two point six. → We looked at our disks. We had 4 tens, 2 ones, and 6 disks that had 0.1 on them. → We knew it was 42 + 0.6, so that helped us rewrite $42\frac{6}{10}$ as 42.6.

Continue the process of showing a mixed number with place value disks, and then writing the expanded fraction form and expanded decimal form for the following numbers: 24 ones 6 tenths, 13 ones 8 tenths, and 68 ones 3 tenths. Challenge students to think how much each number needs to get to the next *one*.

Problem 3: Use the number line to model mixed numbers with units of ones and tenths.

T: (Distribute the Lesson 3 Template, tenths on a number line, and insert it into personal white boards.) Label the larger intervals from 0 to 5.

T: The segment between each whole number is divided up into how many equal parts?

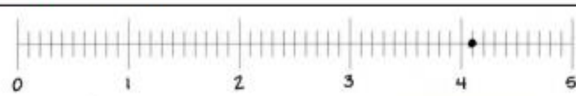
S: 10 equal parts.

T: Plot a point on the number line to represent 4 and 1 tenth.

T: In the chart below your number line, let's plot the same number on a shorter number line partitioned into tenths. What will the endpoints of this shorter number line be?

S: 4 and 5.

T: (Fill out the chart to show 4.1 plotted on a number line between 4 and 5, in decimal form, as a mixed number, and in expanded form.)



Point	Number Line	Decimal Form	Mixed Number (ones and fraction form)	Expanded Form (fraction or decimal form)	How much more is needed to get to the next one?
a.	4 5	4.1	$4\frac{1}{10}$	$(4\times1)+(1\times\frac{1}{10})=4\frac{1}{10}$ $(4\times1)+(1\times0.1)=4.1$	$\frac{9}{10}$ or 0.9
b.	32 33	32.5	$32\frac{5}{10}$	$(3\times10)+(2\times1)+(5\times\frac{1}{10})=32\frac{5}{10}$ $(3\times10)+(2\times1)+(5\times0.1)=32.5$	$\frac{5}{10}$ or 0.5
c.	40 41	40.7	$40\frac{7}{10}$	$(4\times10)+(7\times\frac{1}{10})=40\frac{7}{10}$ $(4\times10)+(7\times0.1)=40.7$	$\frac{3}{10}$ or 0.3
d.	90 91	90.9	$90\frac{9}{10}$	$(9\times10)+(9\times\frac{1}{10})=90\frac{9}{10}$ $(9\times10)+(9\times0.1)=90.9$	$\frac{1}{10}$ or 0.1

tenths on a number line

S: (Write 4 ones and 1 tenth, 4.1, $4\frac{1}{10}$, $(4 \times 1) + (1 \times 0.1) = 4.1$. → $(4 \times 1) + (1 \times \frac{1}{10}) = 4\frac{1}{10}$.)

T: How many more tenths are needed to get to 5? Explain to your partner how you know, and complete the final column of the chart.

S: 9 tenths. → $\frac{9}{10}$. → 0.9. → I know because it takes 10 tenths to make a one. If we have 1 tenth, we need 9 more tenths to make 1.

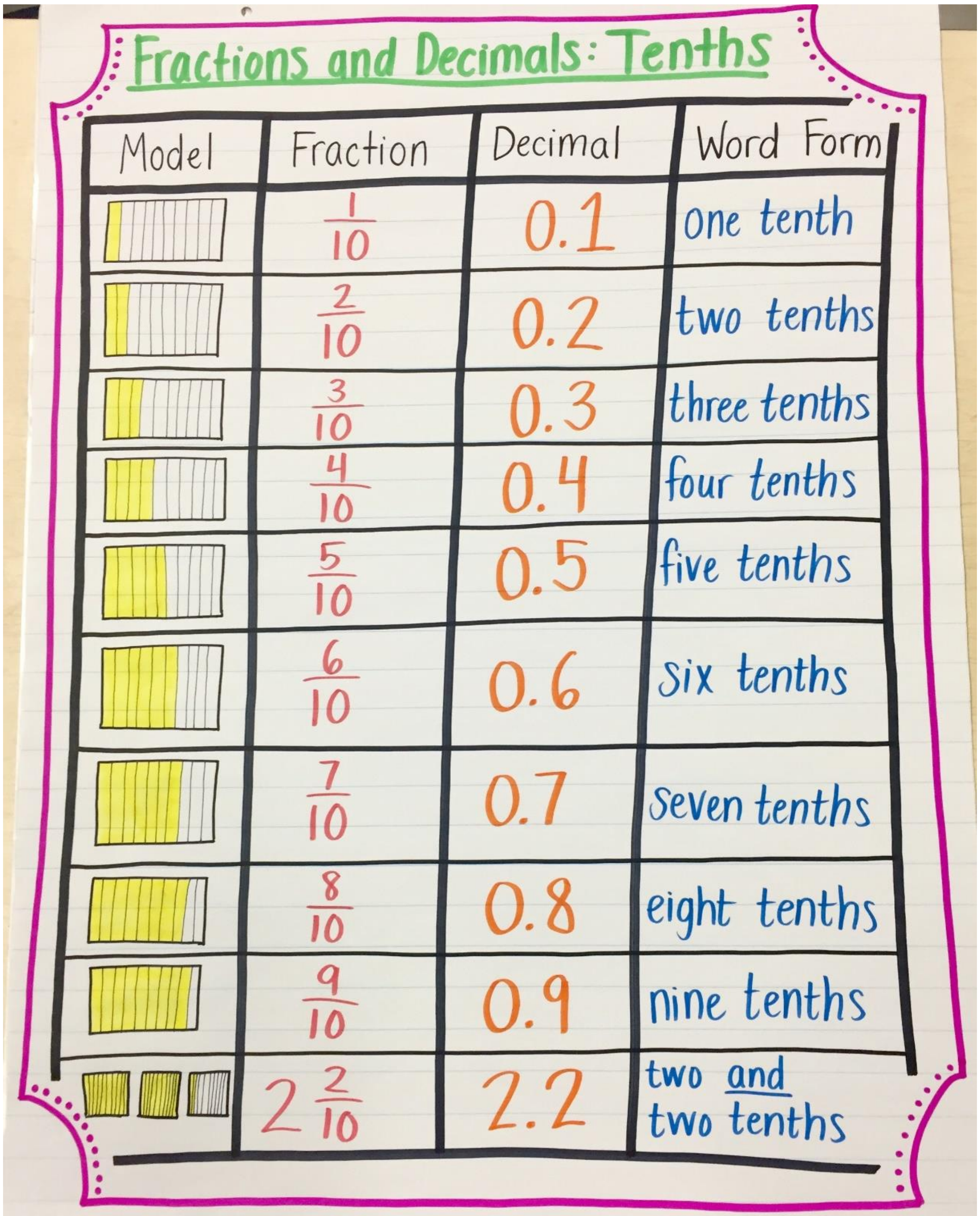
Repeat the process by naming the following points for students to plot. Then, have them complete and share their charts. The longer number line with 5 whole number intervals can either be relabeled to show a broader range of numbers than those included in the chart or omitted for parts (b)–(d) below.

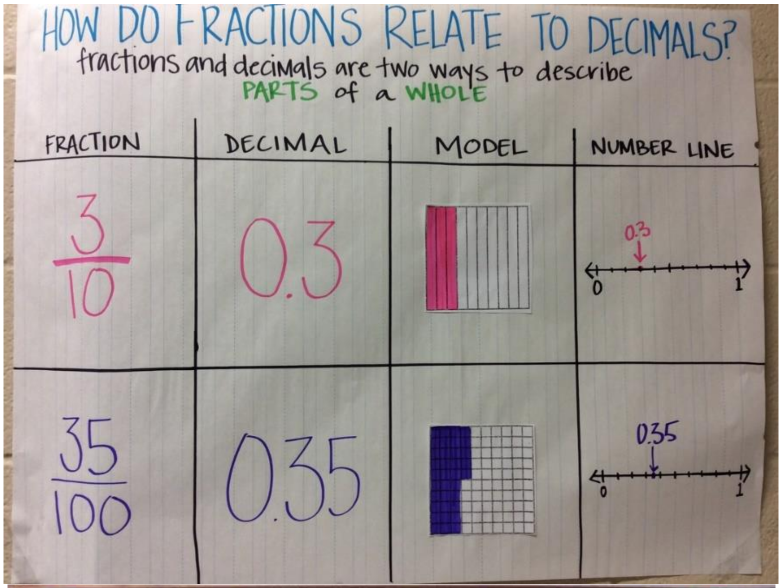
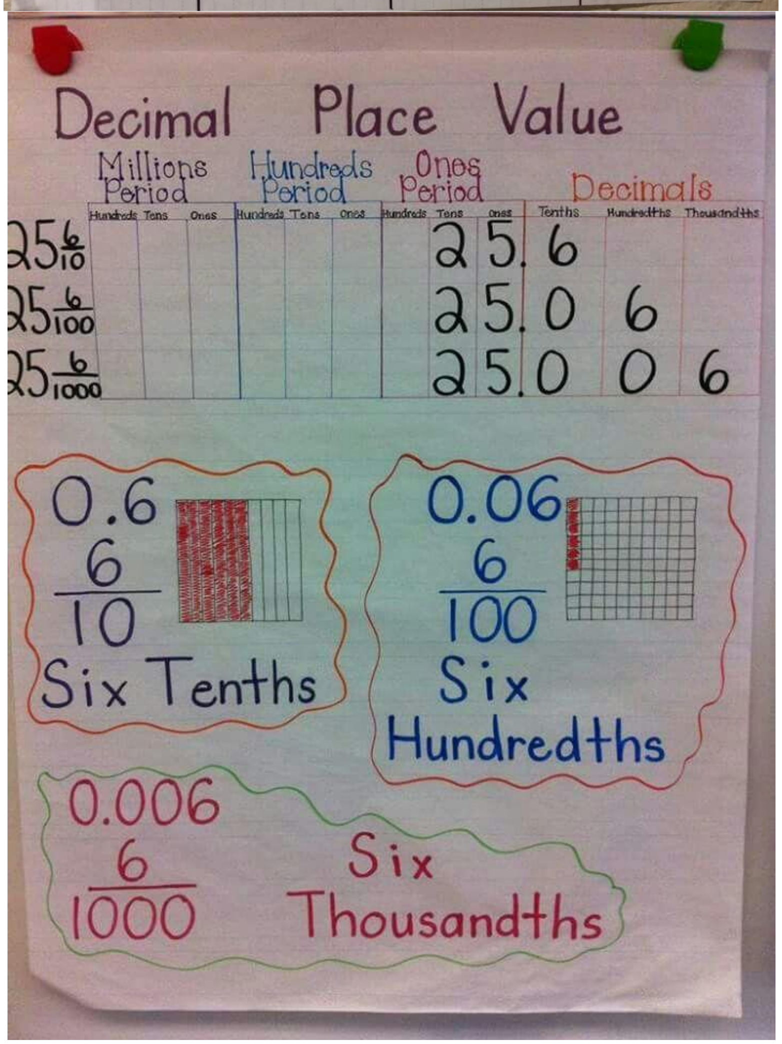
b. 3 tens 2 ones and 5 tenths

c. 4 tens 7 tenths

d. 9 tens 9 tenths

<u>Sample Anchor Charts for 4.NF.6 - Fractions, Decimals, and Place Value</u>




HOW DO FRACTIONS RELATE TO DECIMALS?
fractions and decimals are two ways to describe
PARTS of a WHOLE
FRACTION
DECIMAL
MODEL
NUMBER LINE
3/10
0.3
0.3
0
1
35/100
0.35
0.35
0
1

Decimal Place Value
Millions Period
Hundreds Period
Ones Period
Decimals
Hundreds Tens Ones
Hundreds Tens Ones
Hundreds Tens Ones
Tenths Hundredths Thousandths
25 6/10
25.6
25 6/100
25.06
25 6/1000
25.006
0.6
6/10
Six Tenths
0.06
6/100
Six Hundredths
0.006
6/1000
Six Thousandths

5.NF.4B Back to ccss standard

Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

Interpret the product *(a/b) x q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a x q ÷ b. For example, use a visual fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation. Do the same with (2/3) x (4/5) = 8/15. (In general, (a/b) x (c/d) = (ac)/(bed).)*

Skills

1. Find the area of a rectangle with fractional side lengths by tiling with unit squares of the appropriate unit fraction side lengths

2. Find the area of a rectangle with fractional side lengths by multiplying side lengths

3. Represent the product of fractions as rectangular areas

4. Justify that multiplying fractional side lengths to find the area is the same as tiling a rectangle with unit squares of the appropriate unit fraction side lengths

Key Concepts/Vocabulary

Area – The size of a surface
Rectangle – a plan figure with four straight sides and four right angles, especially one with unequal adjacent sides, in contrast to a square
Fractional side length – side lengths that are fractions, rather than whole numbers
Tiling – overlay of individual tiles with no gaps or overlaps
Unit squares – Squares whose sides have a length of 1
Unit fraction – a rational number written as a fraction where the numerator is one and the denominator is a positive integer. A unit fraction is, therefore, the reciprocal of a positive integer, 1/n. Examples are 1/1, ½, 1/3, 1/4, 1/5, etc.
Multiplying – Repeated addition of one number by as many times as the second number.
Product – the result of multiplying, or an expression that identifies factors to be multiplied. For instance, 6 is the product of 2 and 3.
Rectangular area – The number of square units inside a rectangle (length x width)
Justify – To show or prove to be right or reasonable
Model – a representation of a thing

Standard-Specific Resources (5.NF.4b)

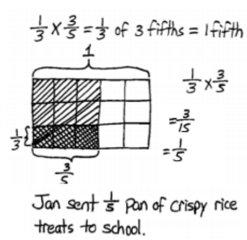
- **<u>EngageNY:</u> Grade 5, Module 4, Topic E, Lesson 14 – [Multiply unit fractions by non-unit fractions.]()**

Concept Development (32 minutes)

Materials: (S) Personal white board

Problem 1

Jan had $\frac{3}{5}$ pan of crispy rice treats. She sent $\frac{1}{3}$ of the treats to school. What fraction of the whole pan did she send to school?



T: (Write Problem 1 on the board.) How is this problem different from the ones we solved yesterday? Turn and talk.

S: Yesterday, Jan always had 1 unit fraction of treats. She had 1 half or 1 third or 1 fourth. Today, she has 3 fifths. This one has a 3 in one of the numerators. → We only multiplied unit fractions yesterday.

T: In this problem, what are we finding $\frac{1}{3}$ of?

S: 3 fifths of a pan of treats.

T: Before we find $\frac{1}{3}$ of Jan's $\frac{3}{5}$, visualize this. If there are 3 bananas, how many would $\frac{1}{3}$ of the bananas be? Turn and talk.

S: Well, if you have 3 bananas, one-third of that is just 1 banana. → One-third of 3 of any unit is just one of those units. → 1 third of 3 is always 1. It doesn't matter what the unit is.

T: What is $\frac{1}{3}$ of 3 *pens*?

S: 1 pen.

T: What is $\frac{1}{3}$ of 3 *books*?

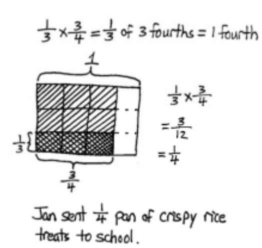
S: 1 book.

Problem 2

Jan had $\frac{3}{4}$ pan of crispy rice treats. She sent $\frac{1}{3}$ of the treats to school. What fraction of the whole pan did she send to school?

T: What are we finding $\frac{1}{3}$ of this time?

S: $\frac{1}{3}$ of 3 fourths.



T: (Write $= \frac{1}{3}$ of 3 fourths.) Based on what we learned in the previous problem, what do you think the answer will be for $\frac{1}{3}$ of 3 fourths? Whisper and tell a partner.

S: Just like $\frac{1}{3}$ of 3 apples is equal to 1 apple, and $\frac{1}{3}$ of 3 fifths is equal to 1 fifth. We know that $\frac{1}{3}$ of 3 fourths is equal to 1 fourth. → We are taking 1 third of 3 units again. The units are fourths this time, so the answer is 1 fourth.

T: Work with a neighbor to solve one-third of 3 fourths. One of you can draw the rectangular fraction model, while the other writes a matching number sentence.

S: (Work and share.)

T: In your area model, when you partitioned each of the fourths into 3 equal parts, what new unit did you create?

S: Twelfths.

T: How many twelfths represent 1 third of 3 fourths?

S: 3 twelfths.

T: Say 3 twelfths in its simplest form.

S: 1 fourth.

T: So, $\frac{1}{3}$ of 3 fourths is equal to what?

S: 1 fourth.

T: Look back at the two problems we just solved. If $\frac{1}{3}$ of 3 fifths is 1 fifth and $\frac{1}{3}$ of 3 fourths is 1 fourth, what then is $\frac{1}{3}$ of 3 eighths?

S: 1 eighth.

T: $\frac{1}{3}$ of 3 tenths?

S: 1 tenth.

T: $\frac{1}{3}$ of 3 hundredths?

S: 1 hundredth.

T: Based on what you've just learned, what is $\frac{1}{4}$ of 4 fifths?

S: 1 fifth.

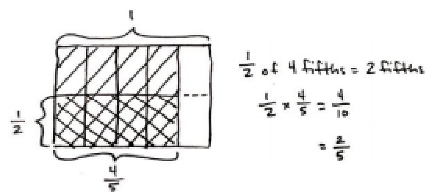
T: $\frac{1}{2}$ of 2 fifths?

S: 1 fifth.

T: $\frac{1}{4}$ of 4 sevenths?

S: 1 seventh.

Problem 3: $\frac{1}{2} \times \frac{4}{5}$



T: We need to find 1 half of 4 fifths. If this were 1 half of 4 bananas, how many bananas would we have?

S: 2 bananas.

T: How can you use this thinking to help you find 1 half of 4 fifths? Turn and talk.

S: It's half of 4, so it must be 2. This time, it's 4 fifths, so half would be 2 fifths. → Half of 4 is always 2. It doesn't matter that it is fifths. The answer is 2 fifths.

T: It sounds like we agree that 1 half of 4 fifths is 2 fifths. Let's draw a rectangular fraction model to confirm our thinking. Work with your partner.

S: (Draw.)

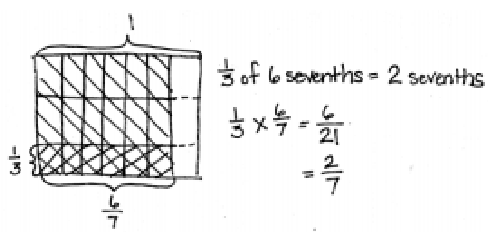
T: I notice that our rectangular fraction model shows that the product is 4 tenths, but we said a moment ago that our product was 2 fifths. Did we make a mistake? Why or why not?

S: No. 4 tenths is just another name for 2 fifths. → I can see 5 groups of 2 tenths, but only 2 of them are double-shaded. Two out of 5 groups is another way to say 2 fifths.

Repeat this sequence with $\frac{1}{3} \times \frac{6}{7}$.

T: What patterns do you notice in our multiplication sentences? Turn and talk.

T: What patterns do you notice in our multiplication sentences? Turn and talk.



S: I notice that the denominator in the product is the product of the two denominators in the factors before we simplified. → I notice that you can just multiply the numerators and then multiply the denominators to get the numerator and denominator in the final answer. → When you split the amount in the second factor into thirds, it's like tripling the units, so it's just like multiplying the first unit by 3. But the units become smaller, so you have the same amount that you started with.

T: As we are modeling the rest of our problems, let's see if this pattern continues.

Problem 4

$\frac{3}{4}$ of Benjamin's garden is planted in vegetables. Carrots are planted in $\frac{1}{2}$ of his vegetable section of the garden. How much of Benjamin's garden is planted in carrots?

T: Write a multiplication expression to represent the amount of his garden planted in carrots.

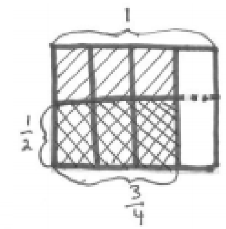
S: $\frac{1}{2} \times \frac{3}{4}$. → $\frac{1}{2}$ of $\frac{3}{4}$.

T: I'll write this in unit form. (Write $\frac{1}{2}$ of 3 fourths on the board.) Compare this problem to the previous ones. Turn and talk.

S: This one seems trickier because all the others were easy to halve. They were all even numbers of units. → This is half of 3. I know that's 1 and 1 half, but the unit is fourths, and I don't know how to say $1\frac{1}{2}$ fourths.

T: Could we name 3 fourths of Benjamin's garden using another unit that makes it easier to halve? Turn and talk with your partner, and then write the amount in unit form.

S: We need a unit that lets us name 3 fourths with an even number of units. We could use 6 eighths. → 6 eighths is the same amount as 3 fourths, and 6 is a multiple of 2.



$\frac{1}{2}$ of 3 fourths = ? fourths

$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

$\frac{3}{8}$ of Benjamin's garden is planted in Carrots.

T: What is 1 half of 6?

S: 3.

T: So, what is 1 half of 6 eighths?

S: 3 eighths.

T: Let's draw our rectangular fraction model to confirm our thinking. (Allow students time to draw.)

T: Looking at our model, what was the new unit that we used to name the parts of the garden?

S: Eighths.

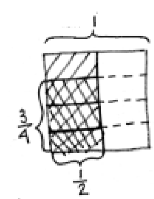
T: How much of Benjamin's garden is planted in carrots?

S: 3 eighths.

Problem 5: $\frac{3}{4}$ of $\frac{1}{2}$

T: (Post Problem 5 on the board.) Solve this by drawing a **rectangular fraction model** and writing a multiplication sentence. (Allow students time to work.)

T: Compare this model to the one we drew for Benjamin's garden. Turn and talk.



$\frac{3}{4} \times \frac{1}{2}$

$= \frac{3}{8}$

S: It's similar. The fractions are the same, but when you draw this one, you have to start with 1 half and then chop that into fourths. → The model for this problem looks like what we drew for Benjamin's garden, except it's been turned on its side. → When we wrote the multiplication sentence, the factors were switched around. → This time, we're finding 3 fourths of a half, not a half of 3 fourths. → If this were another garden, less of the garden is planted in vegetables overall. Last time, it was 3 fourths of the garden; this time, it would be only half. The fraction of the whole garden that is carrots is the same, but now, there is only 1 eighth of the garden planted in other vegetables. Last time, 3 eighths of the garden would have had other vegetables.

T: I hear you saying that $\frac{1}{2}$ of $\frac{3}{4}$ and $\frac{3}{4}$ of $\frac{1}{2}$ are equivalent expressions. (Write $\frac{1}{2} \times \frac{3}{4} = \frac{3}{4} \times \frac{1}{2}$.) Can you give an equivalent expression for $\frac{1}{2} \times \frac{3}{5}$?

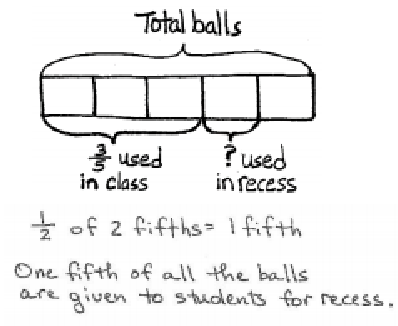
S: $\frac{1}{2}$ of $\frac{3}{5}$. → $\frac{3}{5}$ of $\frac{1}{2}$. → $\frac{3}{5} \times \frac{1}{2}$.

T: Show me another pair of equivalent expressions that involve fraction multiplication.

S: (Work and share.)

Problem 6

Mr. Becker, the gym teacher, uses $\frac{3}{5}$ of his kickballs in class. Half of the remaining balls are given to students for recess. What fraction of all the kickballs is given to students for recess?



T: (Post Problem 6, and read it aloud with students.) This time, let's solve using a tape diagram.

S/T: (Draw a tape diagram.)

T: What fraction of the balls does Mr. Becker use in class?

S: 3 fifths. (Partition the diagram into fifths, and label $\frac{3}{5}$ *used in class*.)

T: What fraction of the balls is remaining?

S: 2 fifths.

T: How many of those balls are given to students for recess?

S: One-half of them.

T: What is one-half of 2?

S: 1.

T: What's one-half of 2 fifths?

S: 1 fifth.

T: Write a number sentence, and make a statement to answer the question.

S: $\frac{1}{2}$ of 2 fifths = 1 fifth. One-fifth of Mr. Becker's kickballs are given to students to use at recess.

Repeat this sequence using $\frac{1}{3} \times \frac{3}{5}$.

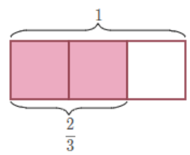
Return to CCSS Standards

Sample Anchor Chart for 5.NF.4b – Multiplying Fractions by Fractions with Visual Models

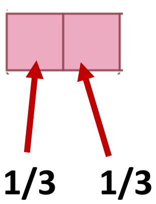
What is $\frac{1}{2}$ of $\frac{2}{3}$? ***Give your answer as a fraction in the lowest terms.***

<u>Step 1:</u> Draw a model of a fraction strip and divide the whole into the equal number of <u>parts in the denominator of the second number</u> (A fraction strip with 3 equal parts)

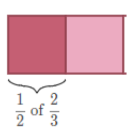
<u>Step 2:</u> Color in the number of equal parts <u>in the numerator of the second number</u> and label the whole and the fraction you colored in (Label the whole strip '1' and the colored-in fraction '2/3')



<u>Step 3:</u> Divide the part you colored in into the number of equal <u>parts in the denominator of the first number</u>, labeling each part (Divide the colored '2/3' into 2 equal parts)



<u>Step 4:</u> DOUBLE-Color in the number of equal parts <u>in the numerator of the first number</u> and label the fraction of the fraction you colored in (Label the double-colored part '1/3 is ½ of 2/3')



<u>Step 5:</u> Solve the problem mathematically by changing the 'of' to 'times' and multiplying

$$\frac{1}{2} \text{ of } \frac{2}{3} = \frac{1}{2} \times \frac{2}{3}$$

$$= \frac{1 \times 2}{2 \times 3}$$

$$= \frac{1 \times \cancel{2}^{\,1}}{\cancel{2}_{\,1} \times 3}$$

Instead of simplifying our answer at the end, we can divide the numerator and denominator by a common factor before multiplying. This makes our multiplication work easier!

We can divide the 2 in the numerator and the 2 in the denominator by their common factor of 2.

$$\frac{1 \times \cancel{2 \div 2}^{\,1}}{\cancel{2 \div 2}_{\,1} \times 3}$$

$$= \frac{1 \times 1}{1 \times 3}$$

$$= \frac{1}{3}$$

5.NF.5B

Back to ccss standard

Apply and extend previous understandings of multiplication and division.

Interpret multiplication as scaling (resizing), by:

Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

Skills

1. Using a visual model, multiply whole numbers by benchmark fractions (e.g. by ½, 1/3, ¼, etc.)

2. Using a visual model, multiply a fraction greater than one by a whole number and understand that the product will always be greater than the original whole number (e.g. 1 ¼ x 2 = 2 ½ and 2 ½ > 1 ¼)

3. Using visual models, multiply a fraction by the number 1 and understand that the product will always be equivalent to the original fraction (e.g. ¾ x 1 = ¾)

4. Using visual models, multiply a fraction by a fraction and understand that the product will always be smaller than either of the original fractions (e.g. ½ of ½ = ¼, or ½ x ½ = ¼)

Key Concepts/Vocabulary

Multiplying/multiply – Repeated addition
Whole number – A number with no fractional parts, no decimals, and no negatives
Fraction – How many parts of a whole
Product – The result of multiplying factors
Factor – Numbers that can be multiplied together to get another number/to get a product
Equivalent – Having the same value
Greater than (>)
Less than (<)
Equal to (=)

Standard-Specific Resources (5.NF.5b)

- **EngageNY: Grade 5, Module 4, Topic F, Lesson 22 – [Compare the size of the product to the size of the factors.]**

Concept Development (32 minutes)

Materials: (T) 12-inch string (S) Personal white board

Problem 1: a. $\frac{4}{4} \times 12$ inches   b. $\frac{3}{4} \times 12$ inches   c. $\frac{5}{4} \times 12$ inches

T: (Post Problem 1(a–c) on the board.) Find the products of these expressions.

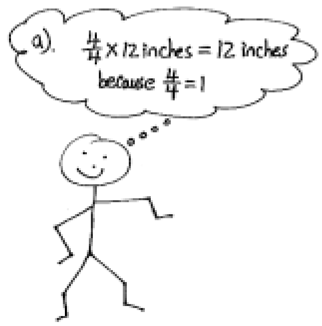
S: (Work.)

T: Let's compare the size of the products you found to the size of this factor. (Point to 12 inches.) Did multiplying 12 inches by 4 fourths change the length of this string? (Hold up the string.) Why or why not? Turn and talk.

S: The product is equal to 12 inches. → We multiplied and got 48 fourths, but that's just another name for 12 using a different unit. → It's 4 fourths of the string, all of it. → Multiplying by 1 means just 1 copy of the number, so it stays the same. → The other factor just named 1 as a fraction, but it is still just multiplying by 1, so the size of 12 won't change.

T: (Write $\frac{4}{4} \times 12 = 12$ under Problem 1(a).) Next consider Problem 1(b). Did multiplying by 3 fourths change the size of our other factor—12 inches? If so, how? Turn and talk.

S: The string became shorter because we only took 3 of 4 parts of it. → We got almost all of 12 inches, but not quite. We wanted 3 fourths of it rather than 4 fourths, so the factor became smaller after we multiplied. → We got 9 inches this time instead of 12 inches.



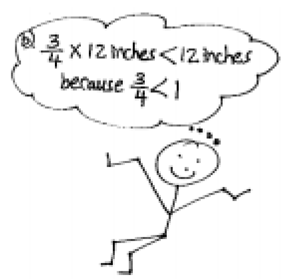
NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Whenever students calculate problems involving measurements, they benefit from having established mental benchmarks of each increment. For example, students should be able to think about 12 inches not just as a foot, but also as a specific length (perhaps a length just a little longer than a sheet of paper). Although teachers can give benchmarks for specific increments, it is probably better if students discover benchmarks on their own. Establishing mental benchmarks may be essential for English language learners' understanding.

T: (Write $\frac{3}{4} \times 12 < 12$ under Problem 1(b).) I hear you saying that 12 inches was shortened—resized to 9 inches. How can it be that multiplying made 12 inches smaller when I thought multiplication always made numbers become larger? Turn and talk.

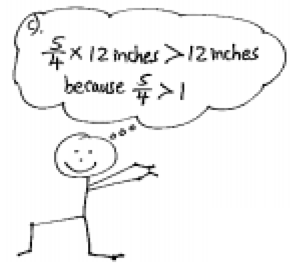
S: We took only part of 12 inches. When you take just a part of something, it is smaller than what you start with. → We ended up with 3 of the 4 parts, not the whole thing. → Adding $\frac{3}{4}$ twelve times is going to be smaller than adding one the same number of times.

T: So, 9 inches is 3 fourths as much as 12 inches. True or false?



S: True.

T: Let's consider our last expression—Problem 1(c). How did multiplying by 5 fourths change or not change the size of the other factor, 12 inches? How would it change the length of the string? Turn and talk.



MP.2

S: The answer to this one was greater than 12 inches because it's more than 4 fourths of it. → The product was greater than 12 inches. → $\frac{5}{4} \times 12 = \frac{60}{4} = 15$. → We copied a number greater than 1 twelve times. The answer had to be greater than copying 1 the same number of times. → 5 fourths of the string would be 1 fourth longer than the string is now.

T: (Write $\frac{5}{4} \times 12 > 12$ under Problem 1(c).) So, 15 inches is 5 fourths as much as 12 inches. True or false?

S: True.

T: 15 inches is 1 and $\frac{1}{4}$ times as much as 12 inches. True or false?

S: True.

T: We've compared our products to one factor, 12 inches, in each of these expressions. We explained the changes we noticed by thinking about the other factor. We can call that other factor a *scaling factor*. A scaling factor can change the size of the other factor. Let's look at the relationships in these expressions one more time. (Point to Problem 1(a).) When we multiplied 12 inches by a scaling factor equal to 1, what happened to the 12 inches?

S: 12 inches didn't change. → The product was the same size as 12 inches, even after we multiplied it.

T: (Point to Problem 1(b).) In this expression, $\frac{3}{4}$ was the scaling factor. Was this scaling factor more than or less than 1? How do you know?

S: Less than 1 because 4 fourths is 1.

T: What happened to the length of the string?

S: It became shorter.

T: (Point to Problem 1(c).) Also, in our last expression, what was the scaling factor?

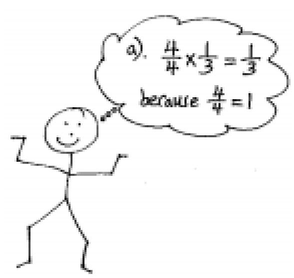
S: 5 fourths.

T: Was 5 fourths more or less than 1?

S: More than 1.

T: What happened to the length of the string?

S: It became longer. → The product was larger than 12 inches.



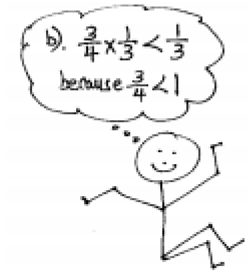
Problem 2: **a.** $\frac{4}{4} \times \frac{1}{3}$ **b.** $\frac{3}{4} \times \frac{1}{3}$ **c.** $\frac{5}{4} \times \frac{1}{3}$

T: (Post Problem 2 (a–c) on the board.) Considering the relationships that we've just noticed between our products and factors, evaluate these expressions.

S: (Work.)

T: Let's compare the products that you found to this factor. (Point to $\frac{1}{3}$.) What is the product of $\frac{1}{3}$ and $\frac{4}{4}$?

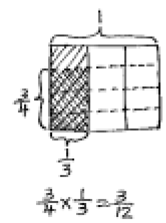
S: $\frac{4}{12}$.



T: Did the size of $\frac{1}{3}$ change when we multiplied it by a scaling factor equal to 1?

S: No.

T: (Write $\frac{4}{4}\times\frac{1}{3}=\frac{1}{3}$ under Problem 2(a).) Since we are comparing our product to 1 third, what is the scaling factor in the second expression? (Point to Problem 2(b).)



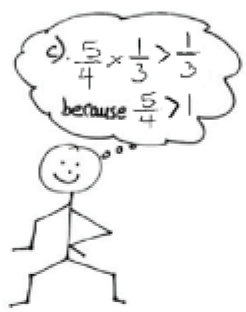
S: $\frac{3}{4}$.

T: Is this scaling factor more than or less than 1?

S: Less than 1.

T: What happened to the size of $\frac{1}{3}$ when we multiplied it by a scaling factor less than 1? Why? Turn and share.

S: The product was 3 twelfths. That is less than 1 third, which is 4 twelfths. → We only wanted part of 1 third this time, so the answer had to be smaller than 1 third. → When you multiply by less than 1, the product is smaller than what you started with.



T: (Write $\frac{3}{4}\times\frac{1}{3}<\frac{1}{3}$ under Problem 2(b).) In the last expression, $\frac{5}{4}$ is the scaling factor. Is the scaling factor more than or less than 1?

S: More than 1.

T: Say the product of $\frac{5}{4}\times\frac{1}{3}$.

S: $\frac{5}{12}$.

T: Is 5 twelfths more than, less than, or equal to $\frac{1}{3}$?

S: More than $\frac{1}{3}$.

T: (Write $\frac{5}{4}\times\frac{1}{3}>\frac{1}{3}$ under Problem 2(c).) Explain why the product of $\frac{1}{3}$ and $\frac{5}{4}$ is more than $\frac{1}{3}$.

S: (Share.)

Problem 3: $\frac{1}{2}\times\frac{5}{5}$ **b.** $\frac{1}{2}\times\frac{3}{5}$ **c.** $\frac{1}{2}\times\frac{9}{5}$

T: I'm going to show you some multiplication expressions where we start with $\frac{1}{2}$. The expressions will have different scaling factors. Think about what will happen to the size of 1 half when it is multiplied by the scaling factor. Tell whether the product will be equal to $\frac{1}{2}$, more than $\frac{1}{2}$, or less than $\frac{1}{2}$. Ready? (Show $\frac{1}{2}\times\frac{5}{5}$.)

S: Equal to $\frac{1}{2}$.

T: Tell a neighbor why.

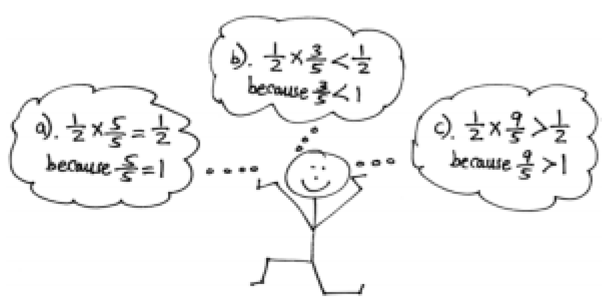
S: The scaling factor is equal to 1.

T: (Show $\frac{1}{2}\times\frac{3}{5}$.)

S: Less than $\frac{1}{2}$.

T: Tell a neighbor why.

S: The scaling factor is less than 1.



T: (Show $\frac{1}{2} \times \frac{9}{5}$.)

S: More than $\frac{1}{2}$.

T: Tell a neighbor why.

S: The scaling factor is more than 1.

Repeat the questioning with the following possible problems: $\frac{1}{2} \times \frac{2}{3}$, $\frac{1}{2} \times \frac{1}{2}$, $\frac{1}{2} \times \frac{4}{3}$, and $\frac{1}{2} \times \frac{8}{8}$.

Problem 4:

At the book fair, Vald spends all of his money on new books. Pamela spends $\frac{2}{3}$ as much as Vald. Eli spends $\frac{4}{3}$ as much as Vald. Who spent the most? The least?

T: (Post Problem 4 on the board, and read it aloud with the students.) Read the first sentence again out loud.

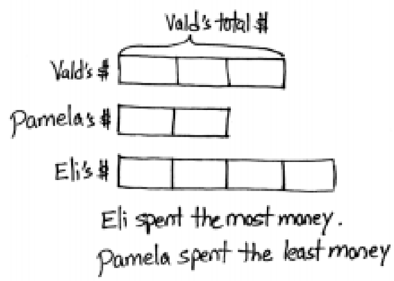
S: (Read.)

T: Before we begin drawing, to whose money will we make the comparisons?

S: Vald's money.

T: What can we draw from the first sentence?

S: We can make a tape diagram. → We should label a tape diagram *Vald's money*.



T: Vald spent all of his money at the book fair. I'll draw a tape diagram and label it *Vald's money*. (Write *Vald's $*.) Read the next sentence aloud.

S: (Read.)

T: What can we draw from this sentence?

S: We can draw another tape that is shorter than Vald's.

T: Let me record that. (Draw a shorter tape representing Pamela's money.) How will we know how much shorter to draw it? Turn and talk.

S: We know she spent $\frac{2}{3}$ of the same amount. Since Pamela's units are thirds, we can split Vald's tape into 3 equal units, and then draw a tape below it that is 2 units long and label it *Pamela's money*. → I know Pamela's has 2 units, and those 2 units are 2 out of the 3 that Vald spent. I'll draw 2 units for Pam, and then make Vald's 1 unit longer than hers.

T: I'll record that. Thinking of $\frac{2}{3}$ as a scaling factor, did Pamela spend more or less than Vald? How do you know? Does our tape diagram show it?

S: Pamela spent less than Vald. If you think of $\frac{2}{3}$ as a scaling factor, it's less than 1, so she spent less than Vald. That's how we drew it. → She spent less than Vald. She only spent a part of the same amount as Vald. → Vald spent all his money, or $\frac{3}{3}$ of his money. Pamela only spent $\frac{2}{3}$ as much as Vald. You can see that in the diagram.

T: Read the third sentence and discuss what you can draw from this information.

S: (Read and discuss.)

T: Eli spent $\frac{4}{3}$ as much as Vald. If we think of $\frac{4}{3}$ as a scaling factor, what does that tell us about how much money Eli spent?

S: Eli spent more than Vald because $\frac{4}{3}$ is more than 1. → Again, Vald spent all of his money, or $\frac{3}{3}$ of it. $\frac{4}{3}$ is more than $\frac{3}{3}$, so Eli spent more than Vald. We have to draw a tape diagram that is one-third more than Vald's.

T: Since the scaling factor $\frac{4}{3}$ is more than 1, I'll draw a third tape diagram for Eli that is longer than Vald's money. What is the question we have to answer?

S: Who spent the most and least money at the book fair?

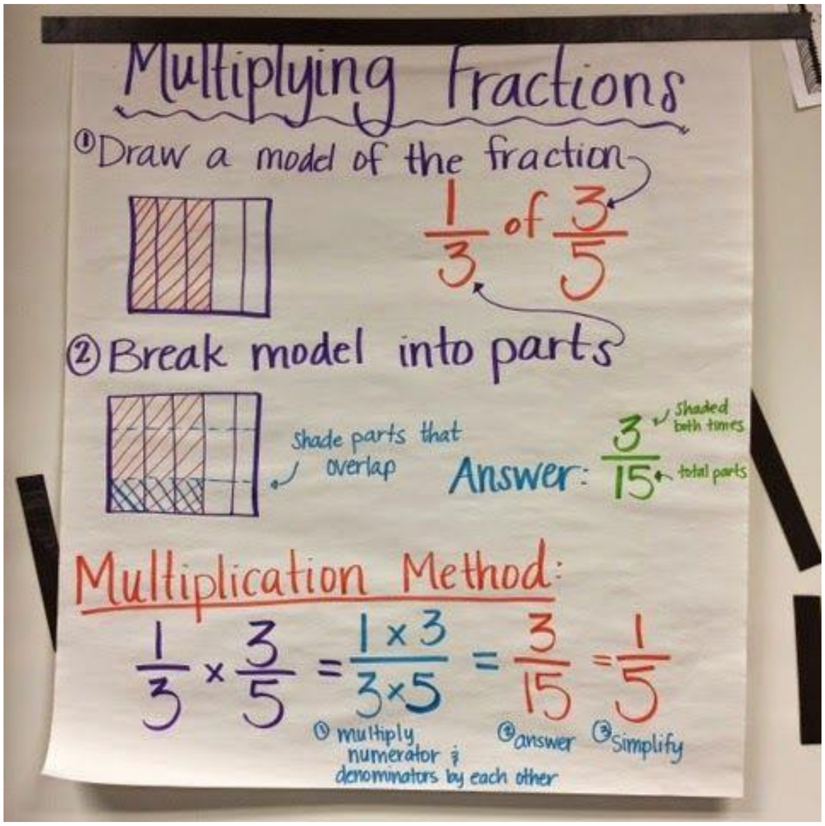
T: Does our tape diagram show enough information to answer this question?

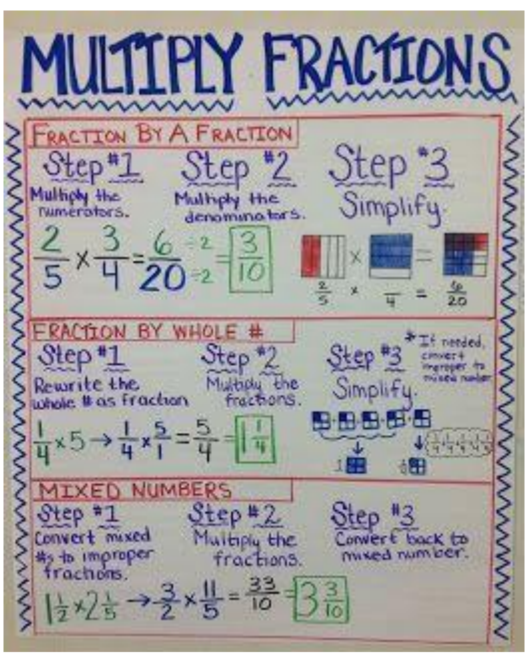
S: Yes, it's very easy to see whose tape diagram is the longest and shortest. → Even though we don't know exactly how much Vald spent, we can still answer the question. Since the scaling factors are more than 1 and less than 1, we know who spent the most and least amount of money.

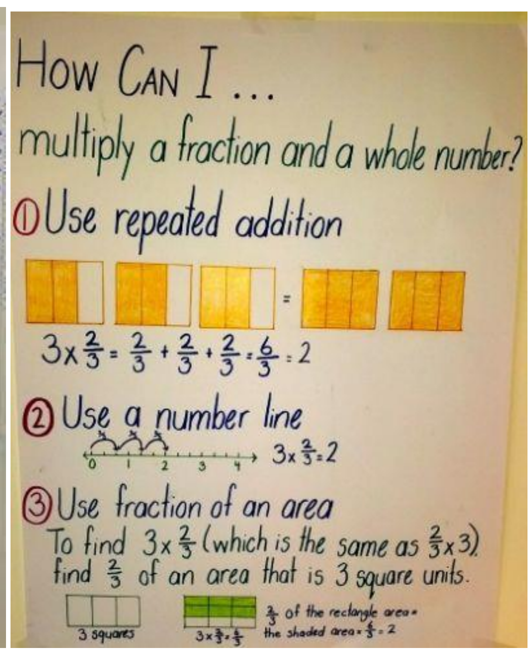
T: Answer the question in a complete sentence.

S: Eli spent the most money. Pamela spent the least money.

Sample Anchor Chart for 5.NF.5b – Multiplying Fractions

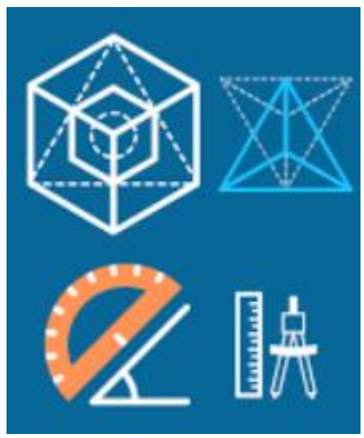






Geometry

(G)



4.G.1 Back to ccss standard

Draw and identify lines and angles, and classify shapes by properties of their lines angles.

Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

Skills

1. Draw and label points, lines, line segments, and rays

2. Analyze two- dimensional figures to identify points, lines, line segments, and rays

3. Draw, and label angles (right, acute, obtuse)

4. Analyze dimensional figures to angles (right, acute, obtuse)

5. Draw and label lines (perpendicular and parallel)

6. Analyze two-dimensional figures to identify lines (perpendicular and parallel)

Key Concepts/Vocabulary

Point – An exact location; has no size, only position
Line segment – The part of a line that connects 2 points
Ray – A portion of a line which starts at a point and goes off in a particular direction to infinity
Right angle – An angle that has a measure of 90 degrees, or one quarter of a circular revolution
Acute angle – An angle that has a measure of less than 90 degrees
Obtuse angle – An angle that has a measure more than 90 degree but less than 180 degrees
Perpendicular lines – Lines that are at right angles (90 degrees) to each other; lines that create right angles when they cross one another
Parallel lines – Two lines that will never meet – they are always the same distance apart from one another
Two-dimensional figure – A shape with only width and height, but no thickness

Standard-Specific Resources (4.G.1)

- **EngageNY: Grade 4, Module 4, Topic A, Lesson 2 – Use right angles to determine whether angles are equal to, greater than, or less than right angles. Draw right, obtuse, and acute angles.**

Concept Development (34 minutes)

Materials: (T/S) Paper, straightedge, angles (Template)

Note: The following activity and images for the paper-folding activity are modeled using a large circle. Any sized paper and any shaped paper is sufficient for this activity. Include a variety of papers for this activity. Students find that any paper folded twice results in a right angle template.

Problem 1: Create right angles through a paper folding activity.

T: Everyone, hold your circle, and fold it in half like this. (Demonstrate.)

T: Then, fold it in half again, like this. (Demonstrate.)

T: Do you notice any angles in our folded circle?

S: Yes! This corner right here!

T: Yes, that shared endpoint is where these two lines meet to form an angle.

T: Now, trace both lines with your fingers, starting at their shared endpoint.

T: Point to the angle we formed. This is called a **right angle**.

T: Using your folded circle as a reference, look around the room for right angles. With your partner, create a list of objects that have right angles.

S: Door, book, desk, floor tile, window, paper, and white board.

T: Use the words *equal to* for describing the relationship between your right angle template and the other right angles you found around the room.

S: The angles on the corners of the floor tile are equal to the right angle on my folded paper. → The corner of the door is equal to a right angle.

Problem 2: Determine whether angles are equal to, greater than, or less than a right angle.

T: Use your right angle template to find all of the right angles on the angles template. How will you know if it's a right angle?

S: The sides of the right angle template will match exactly with the sides of the angles. (Find the right angles on the angles template.)

T: Let's identify the right angles with a symbol. We put a square in the corner of the angle, or the **vertex**, to show that it is a right angle (demonstrate). It's your turn.

Students identify each right angle by putting a right angle symbol at the vertex.

T: What do you notice about the other angles on the angles template?

S: They are not right angles. → Some are less than right angles. → Some are greater than right angles.

T: But what if one looks *almost, but not quite like* a right angle?

S: It would be hard to tell. → We can use our right angle template!

T: Place your right angle template on $\angle B$ so that the corner of the template and one of the sides lines up with the corner and side of the angle. What do you notice?

S: The two rays make an opening that is smaller than the right angle. → I can only see one ray of the angle. → This angle fits inside the right angle.

T: Find the other angles that are less than a right angle. Write *less* next to them.

Students identify other angles that are less than a right angle.

MP.5

T: Are the remaining angles greater or less than a right angle?

S: Greater!

T: Place your right angle template on $\angle C$ so that the corner of the template and one of the sides lines up with the corner and side of the angle. What do you notice?

S: My right angle fits inside of it. → When I line up my right angle along this side, the other side of the angle is outside my right angle. → It's greater than a right angle.

T: Verify that each of the other remaining angles is greater than a right angle using your template. Write *greater* next to each angle.

T: We just identified three groups of angles. What are they?

S: Some are right angles. Some are less than right angles. Some are greater than right angles.

T: $\angle A$, $\angle E$, and $\angle G$ are right angles. $\angle B$, $\angle D$, and $\angle F$ are examples of another type of angle. We call them **acute angles**. Describe an acute angle.

S: An acute angle is an angle that is less than a right angle.

T: Look around the classroom for acute angles.

S: I see one by the flagpole.

T: What two objects represent the rays or sides of your acute angle?

S: The flagpole and the wall.

T: When we align the right angle template against the wall and follow the flagpole, it goes inside the interior of the right angle. (Demonstrate.)

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

To assist with building math vocabulary for English language learners and other students, point to a picture of acute, right, and obtuse angles each time they are mentioned during today's lesson. Consider building into the instruction additional checks for understanding. Additionally, learners may benefit from adding these new terms and corresponding pictures to their personal math dictionaries before or after the lesson.

T: $\angle C$, $\angle H$, $\angle I$, and $\angle J$ are examples of another type of angle. We call them **obtuse angles**. Describe an obtuse angle.

S: An obtuse angle is an angle that is greater than a right angle.

T: Look around the classroom for obtuse angles.

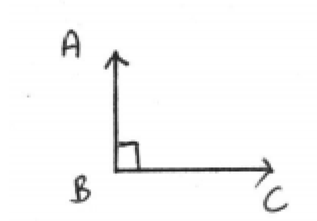
S: The door is creating an obtuse angle right now.

T: What two objects represent the sides composing your obtuse angle?

S: The wall and the bottom of the door.

Problem 3: Draw right, acute, and obtuse angles.

T: Using your straightedge, draw one ray. Use your right angle template as a guide. Then, draw a second ray, creating a right angle, $\angle ABC$. Will you label the two rays' shared endpoint A, B, or C?



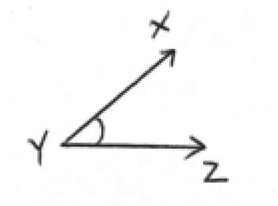
S: The shared endpoint should be labeled B because it is $\angle ABC$. Point B is in the middle.

T: When you are finished drawing your angle, use your template to check your partner's angle. Do everyone's right angles look exactly the same?

S: Not all of them. → Our angles are facing different directions, but the angle looks exactly the same.

T: Right angles are represented with a little square in the angle. (Demonstrate). Add one to your angle.

T: Next, using the same process, draw an acute angle labeled $\angle XYZ$. When you are finished, check your partner's angle.

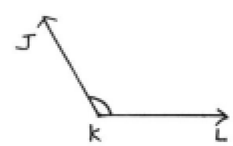


T: What did you notice?

S: This time, they all look different. → I notice that our angles are facing different directions, but also, the sizes of the angles look different. → All are different sizes, but all are less than a right angle. → Right angles are exactly the same, but acute angles can be anything less than a right angle, so there are a lot of them.

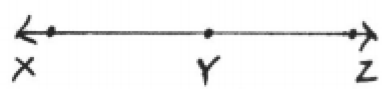
T: *Acute* indicates less than a right angle, so everyone in our class may have drawn a different angle!

T: For all angles that are not equal to a right angle, we can draw an arc to show the angle. (Demonstrate.) Add one to your angle.



T: Lastly, draw an obtuse angle labeled $\angle JKL$, and draw an arc to show the angle.

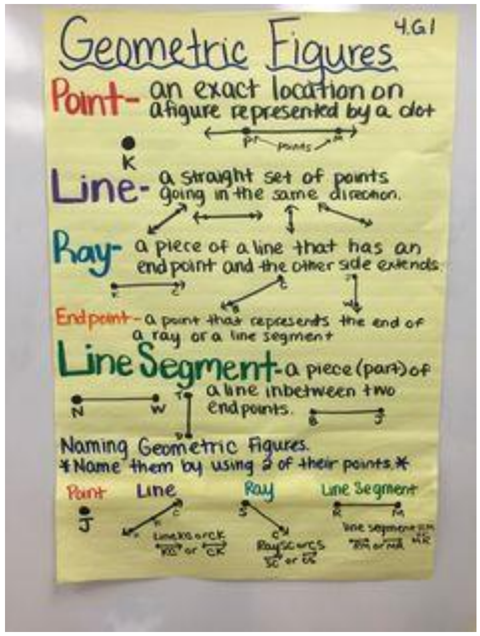
T: (Draw a straight line and label points X, Y, and Z on the line.) Identify this angle.

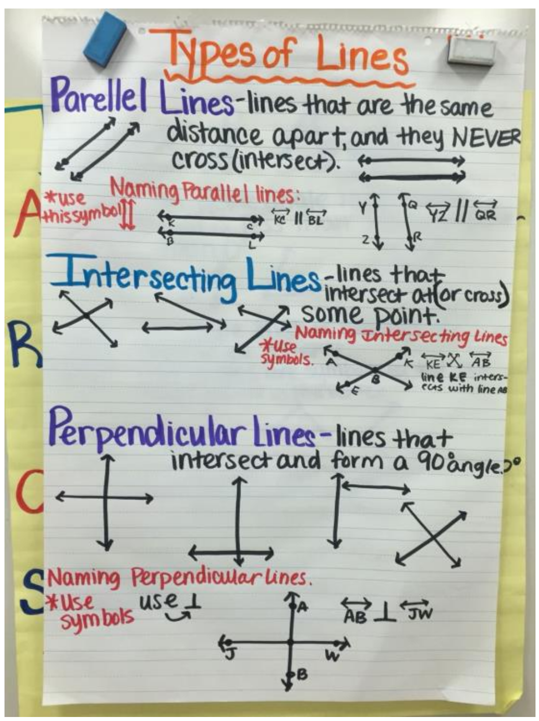


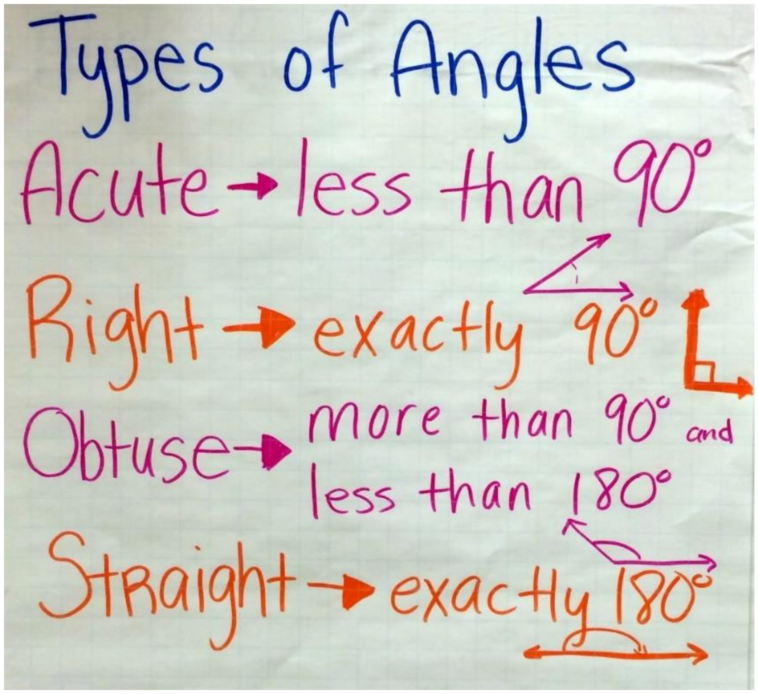
S: I don't see an angle. → Isn't it just a line? Line XYZ.

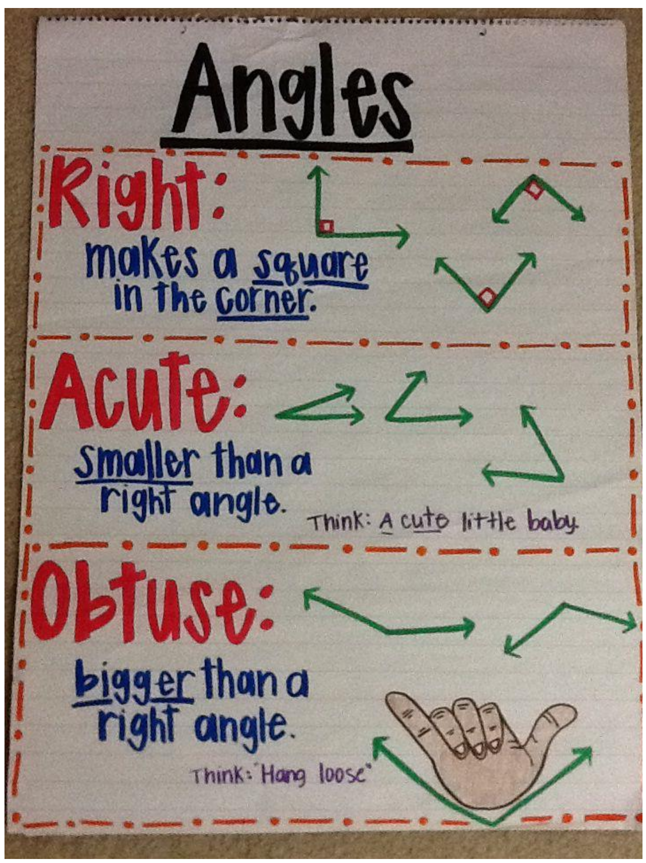
T: There are two rays, $\overrightarrow{YX}$ and $\overrightarrow{YZ}$. So yes, it is $\angle XYZ$. However, since all three points lie on a line, we have a special angle. We call this a **straight angle**. Obtuse angles are smaller than a straight angle, but larger than a right angle. Check your partner's work. Use your right angle template and straightedge as guides.

Sample Anchor Charts for 4.G.1 - Solving Measurement Problems






Types of Angles
Acute → less than 90°
Right → exactly 90°
Obtuse → more than 90° and less than 180°
Straight → exactly 180°


Angles
Right:
makes a square in the corner.
Acute:
smaller than a right angle.
Think: A cute little baby.
Obtuse:
bigger than a right angle.
Think: "Hang loose"

5.G.1 Back to ccss standard

Graph points on the coordinate plane to solve real-world and mathematical problems.

Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).

Skills

1. Understand and draw the origin, x-axis, and y-axis on a coordinate system

2. Identify and draw coordinates of points that have positive x and y values on the coordinate system

3. Identify and draw coordinates of points that have combinations of positive or negative values for the x or y coordinates

4. Draw shapes on the coordinate system and identify the coordinates of points on those shapes

5. Play 'Battleship', but using the coordinate plane to 'guess' another person's chosen coordinate points

Key Concepts/Vocabulary

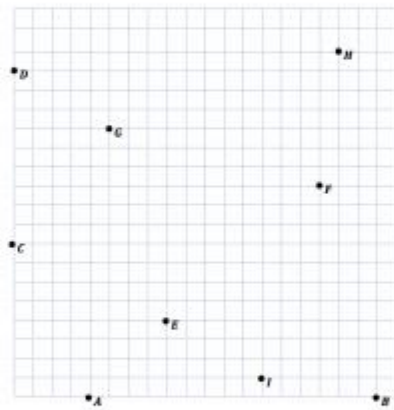
Coordinate system – A two-dimensional surface created by two intersecting and perpendicular number lines on which points are plotted and located by their x and y coordinates
x-axis – the horizontal (left-right) number line in the coordinator system/plane
y-axis – the vertical (up-down) number line in the coordinate system/plane
Origin – where the x-axis meets the y-axis, also known as the coordinate point (0,0)
Coordinates of a point – the value where the point aligns with the x-axis and y-axis (x,y)
Ordered pair – how the coordinates of a point are written, with the x value on the left of the comma, and the y value on the right of the comma, e.g. (2,3) → x = 2, y = 3

Standard-Specific Resources (5.G.1)

- **<u>EngageNY:</u> Grade 5, Module 6, Topic A, Lesson 3 – [Name points using coordinate pairs, and use the coordinate pairs to plot points.]**

Concept Development (32 minutes)

Materials: (S) Ruler, unlabeled coordinate plane (Template 2)



Problem 1: Construct a coordinate plane.

MP.6

T: (Distribute a copy of the unlabeled coordinate plane template to each student.) Use your ruler to draw an x-axis so that it goes through points A and B, and label it the x-axis. (Model on the board.)

S: (Draw and label the x-axis.)

T: Use your ruler to draw the y-axis so that it goes through points C and D, and label it the y-axis.

S: (Draw and label the y-axis.)

MP.6

T: Label 0 at the origin.

S: (Label the origin.)

T: On the x-axis, we are going to label the whole numbers only. The length of one square on the grid represents 1 fourth. How many whole numbers can we label? Turn and talk.

S: I counted 20 grid lengths, or 20 fourths, which is 5. We can label the whole numbers 0 through 5. → Each grid length is 1 fourth, so every 4 grid lengths is a whole number. → Point A is at 4 fourths, or 1, and there is room for 4 more groups of 4 fourths.

T: Count by fourths with me as we label the whole number grid lines. One fourth .... (Move along the x-axis while counting, and label every whole number grid line.)

T/S: 2 fourths, 3 fourths, 1 (label 1), 1 and 1 fourth, 1 and 2 fourths, 1 and 3 fourths, 2 (label 2). (Label the whole number grid lines.)

T: What is the x-coordinate of A?

S: 1.

T: B?

S: $4\frac{3}{4}$.

T: Label the y-axis in the same way.

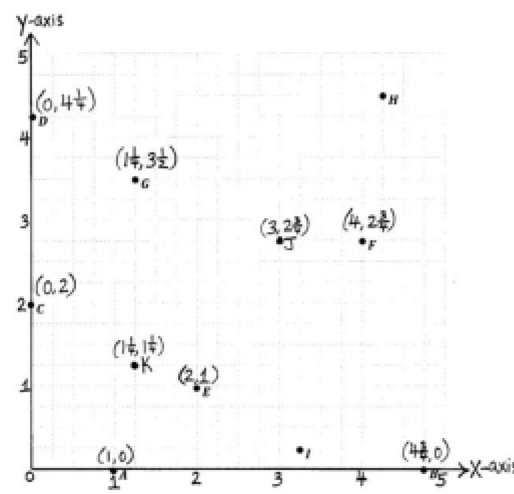
S: (Label the whole number grid lines.)

T: What is the y-coordinate of C?

S: 2.

T: D?

S: $4\frac{1}{4}$.



Problem 2: Use coordinate pairs to name and plot points.

T: Put your finger on E. How do we find the x-coordinate of E? Turn and talk.

S: I can just follow the grid line down from E to the x-axis, and it falls at a distance of 2 from the origin. So, the x-coordinate is 2. → E is directly above 2 on the x-axis, so its x-coordinate is 2. → Start at the origin, and move along the x-axis to the x-coordinate of E.

T: What is the x-coordinate of E?

S: 2.

T: Show me that x-coordinate as part of a coordinate pair.

S: (Show (2, ____).)

T: Find the y-coordinate of E. (Pause.) Show me the coordinate pair for E.

S: (Show (2, 1).)

T: Write that coordinate pair above point E on your plane. Work with a partner to name the coordinate pair for F.

S: (Share and show the coordinate pair for F as $(4, 2\frac{3}{4})$.)

Repeat for points G, B, and C, respectively, $(1\frac{1}{4}, 3\frac{1}{2})$, $(4\frac{3}{4}, 0)$, (0, 2).

T: Name the point located at (1, 0).

S: A.

T: Name the point located at $(0, 4\frac{1}{4})$.

S: D.

T: I want to name the point whose distance from the y-axis is $4\frac{1}{4}$. How is this question different from the other questions I have asked you about points in this plane? Turn and talk.

S: You are asking us about the distance from the whole line, not the distance from the origin on x. → We are looking at the distance away from the y-axis, rather than going a distance down the x-axis.

T: Work with a neighbor to name the point whose distance from the y-axis is $4\frac{1}{4}$.

S: H.

T: Which point lies at a distance of $\frac{1}{4}$ from the x-axis?

S: I.

T: Plot a point J at $(3, 2\frac{3}{4})$. Have a neighbor check your work.

S: (Work and share.)

T: Turn and tell a partner how to find the distance between J and F.

S: Since they both have a y-coordinate of $2\frac{3}{4}$, I can just count the number of 1-fourth lengths on the x-axis from J to F. → It's just like finding the distance between 3 and 4 on a ruler. It's just 1 unit away.

T: What is the distance between J and F? (Gesture between the points.)

S: One unit.

T: Yes. Now, plot a point K so that the x- and y-coordinates are both $1\frac{1}{4}$, and then find the distance between K and G.

S: (Work.)

T: Say the distance between K and G.

S: $2\frac{1}{4}$ units.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

This module has many new vocabulary words. Here are a few strategies to help students make these new words their own:

- Have students tap and whisper a new word three times.
- Allow students to explore online vocabulary builders such as *Word2Word*, an online collection of dictionaries of multiple languages.
- Have students continue to add to their collection of math words on 3″ × 5″ cards held together by a metal ring.
- Have students continue building their illustrated glossary.

(The last two options assume students have been using these tools all year, which may not be the case.)

- **<u>EngageNY:</u> Grade 5, Module 6, Topic A, Lesson 4 – [BATTLESHIP - Points on the Coordinate Plane]()**

> **NOTES ON MULTIPLE MEANS OF ENGAGEMENT:**
>
> One possible extension of today's Concept Development would be to have students write a handbook for winning at Battleship. To write such a guide, students must articulate strategic thinking, which gives them an opportunity to use critical thinking and communication skills.

Concept Development (34 minutes)

Materials: (S) Problem Set (1 per student/per game), red pencil or crayon (1 per student), black pencil or crayon (1 per student), folder (1 per pair of students)

Note: Today, students are playing a version of the board game Battleship. Depending on the level of experience students have with this game, the following suggested discussion might be modified.

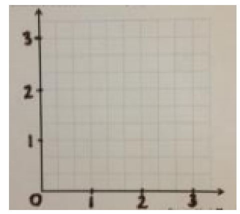
T: Raise your hand if you have heard of, or have ever played, Battleship.

T: (Distribute a copy of the Problem Set to each student.) Take four minutes to read and talk about Battleship Rules with a partner.

S: (Read and share.)

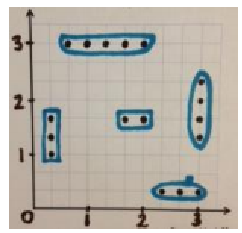
T: Find your My Ships coordinate plane, and hold it up.

S: (Hold up the paper.)



T: Once we get started, one of the first things you'll do with your opponent is label the axes using halves, thirds, fourths, or fifths. (Display the image on the board.) This is an example of a coordinate plane that has already been prepared for play. What fractional unit is designated by the grid lengths? Turn and talk.

S: Thirds!



T: The next step is the fun part. You get to secretly select locations for your fleet on the coordinate plane. How many ships does each player get?

S: 5.

T: Exactly, and some ships are small, such as the patrol boat, while others are large, such as the aircraft carrier. Let's look at an example of how a fleet might be set up on the coordinate plane. (Display the image on the board.)

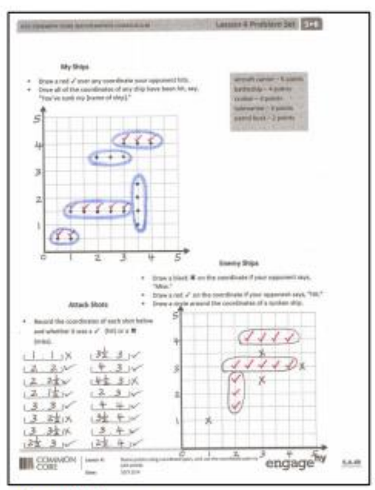
MP.2

T: Then, once both of you have your ships secretly placed on your My Ships plane, you will take turns guessing attack shots, attempting to hit your enemy's boats. Work with a neighbor to show a coordinate pair that would *hit* the submarine on this plane.

S: (Share and show.)

T: Jasmine, I saw you named the location (2,). What would her opponent have to say if Jasmine guessed these coordinates?

S: Hit!



T: That's right! Then, Jasmine would record those coordinates on her paper and mark a red check on her Enemy Ships plane. What would the opponent have to do?

S: Mark a red check on the hit coordinate of the submarine.

T: You got it! Then, it is Jasmine's opponent's turn to make an attack shot. When does the game end? How do you win?

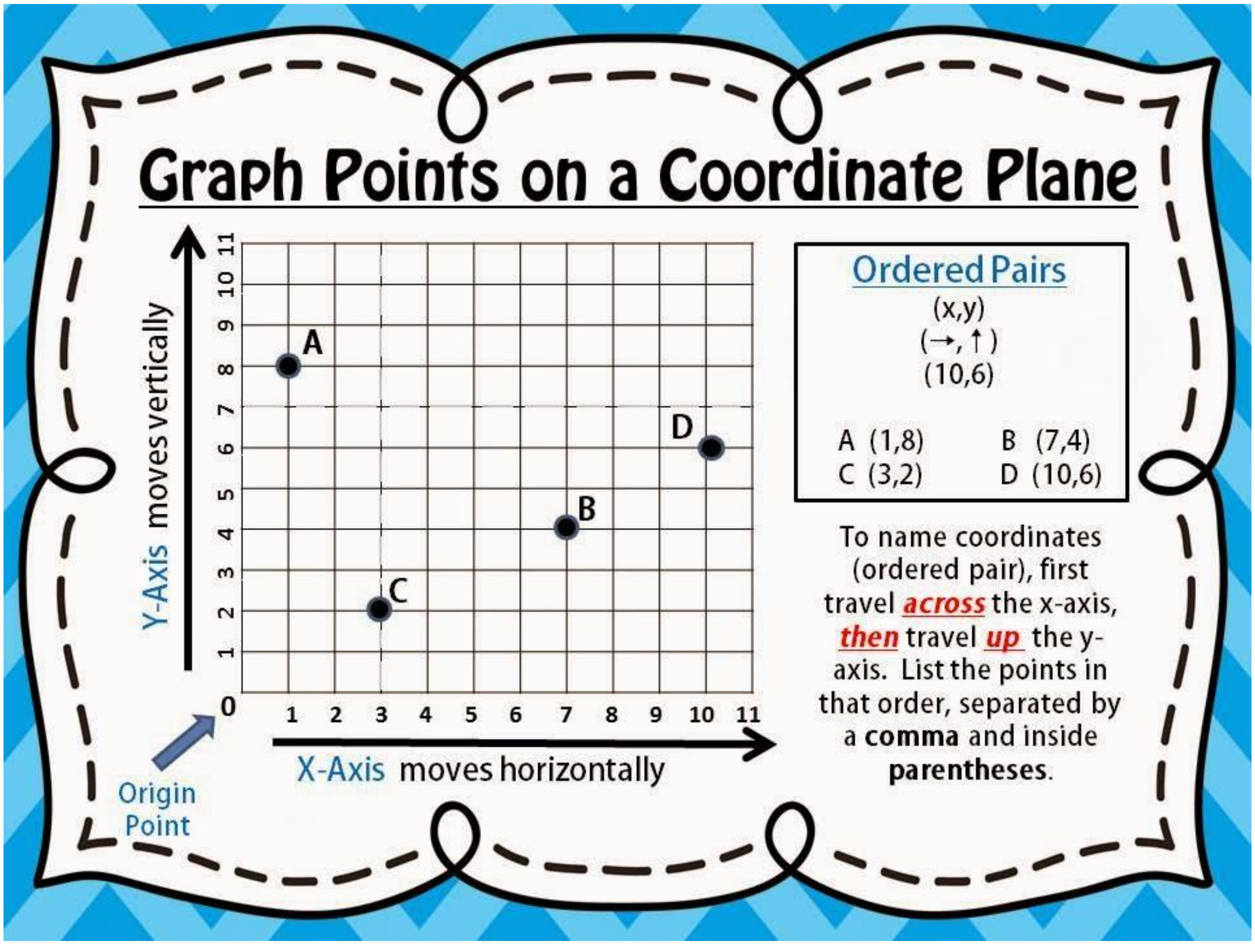
S: The game ends when one person sinks all of the opponent's ships!

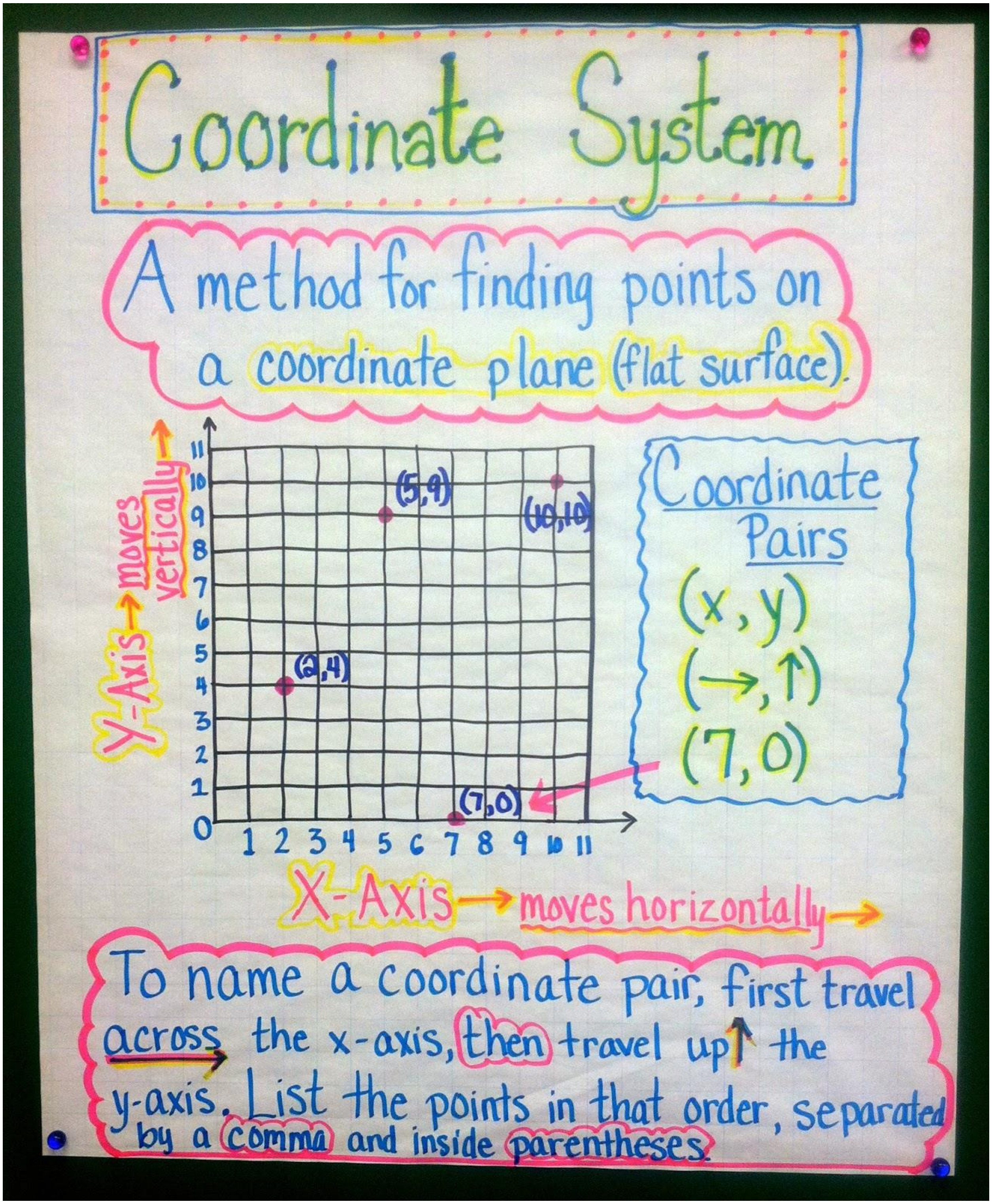
T: Or, when time is up, the winner is the player who has sunk the most ships. Let's play!

Game Play (20 minutes)

Students should select or be assigned an opponent and begin play. Early finishers may choose to play a rematch or be assigned another opponent. Please note that a new copy of the Problem Set is needed for each game. However, the grid sheets can be inserted into page protectors for multiple uses.

Sample Anchor Charts for 5.G.1 – Points on a Coordinate Plane





Coordinate System
A method for finding points on a coordinate plane (flat surface).
Y-Axis → moves vertically →
X-Axis → moves horizontally →
0 1 2 3 4 5 6 7 8 9 10 11
(5,9)
(10,10)
(2,4)
(7,0)
Coordinate Pairs
(x, y)
(→,↑)
(7,0)
To name a coordinate pair, first travel across the x-axis, then travel up↑ the y-axis. List the points in that order, separated by a comma and inside parentheses.

Ratios and Proportions

(RP)

$$a : b = c : d$$

IF $\dfrac{a}{b} = \dfrac{c}{d}$

THEN $ad = bc$

6.RP.1 Back to ccss standard

Understand ratio concepts and use ratio reasoning to solve problems.

Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. ***For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."***

Skills

1. Read and write ratios that represent a given comparison

2. Create equivalent ratios

3. Solve problems using part-to-part ratios

4. Solve problems using part-to-whole ratios

5. Represent fractions using ration notation

Key Concepts/Vocabulary

Ratio notation - **__:__, __ to __, __/__**

Ratio – Shows the relative sizes of two or more values

Quantity – How much there is of something

Unit of measure – A quantity used as a standard of measurement; how much makes up ‘1’ of the measurement

Part-to-whole – The ratio, or relationship, between one part of a whole to the entirety of the whole

Part-to-part – The ratio, or relationship, between one equal part of a whole to another part of the whole

Rate – How much of something per 1 unit of something else

Standard-Specific Resources (6.RP.1)

- **<u>EngageNY:</u> Grade 6, Module 1, Topic A, Lesson 7 – [Students understand the relationships between ratios and fractions and describe the fraction A/B associated with the ratio A:B as the value of the ratio A to B.]**

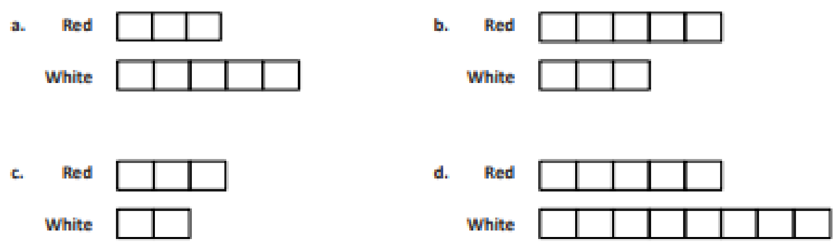
Classwork

Example 1 (2 minutes)

Direct students to select an answer to the question posed by Example 1 in their student materials.

Example 1

Which of the following correctly models that the number of red gumballs is $\frac{5}{3}$ the number of white gumballs?

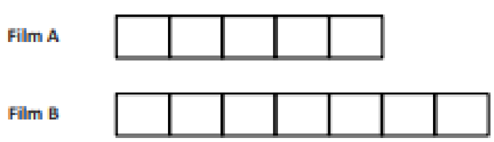


Poll students, and host a discussion encouraging students to express their reasoning about their choices. Ideally, students can come to a consensus that (b) is the correct answer without teacher direction. Provide an additional example if needed before moving on.

Example 2 (5 minutes)

Example 2

The duration of two films are modeled below.



a. The ratio of the length of Film A to the length of Film B is <u>5: 7</u>.

b. The length of Film A is $\frac{\boxed{5}}{\boxed{7}}$ of the length of Film B.

c. The length of Film B is $\frac{\boxed{7}}{\boxed{5}}$ of the length of Film A.

Exercise 1 (10 minutes)

Have students work the following problem independently and then compare their answers with a neighbor's answer. Encourage discussion among pairs of students or among students who arrived at different answers.

Exercise 1

Sammy and Kaden went fishing using live shrimp as bait. Sammy brought 8 more shrimp than Kaden brought. When they combined their shrimp they had 32 shrimp altogether.

a. **How many shrimp did each boy bring?**

Kaden brought 12 shrimp. Sammy brought 20 shrimp.

b. **What is the ratio of the number of shrimp Sammy brought to the number of shrimp Kaden brought?**

$20:12$

c. **Express the number of shrimp Sammy brought as a fraction of the number of shrimp Kaden brought.**

$\frac{20}{12}$

d. **What is the ratio of the number of shrimp Sammy brought to the total number of shrimp?**

$20:32$

e. **What fraction of the total shrimp did Sammy bring?**

$\frac{20}{32}$

Exercise 2 (20 minutes)

Exercise 2

A food company that produces peanut butter decides to try out a new version of its peanut butter that is extra crunchy, using twice the number of peanut chunks as normal. The company hosts a sampling of its new product at grocery stores and finds that 5 out of every 9 customers prefer the new extra crunchy version.

a. **Let's make a list of ratios that might be relevant for this situation.**

   i. **The ratio of number preferring new extra crunchy to total number surveyed is** *5 to 9*.

   ii. **The ratio of number preferring regular crunchy to the total number surveyed is** *4 to 9*.

   iii. **The ratio of number preferring regular crunchy to number preferring new extra crunchy is** *4 to 5*.

   iv. **The ratio of number preferring new extra crunchy to number preferring regular crunchy is** *5 to 4*.

b. **Let's use the value of each ratio to make multiplicative comparisons for each of the ratios we described here.**

   i. **The number preferring new extra crunchy is** $\frac{5}{9}$ **of the total number surveyed.**

   ii. **The number preferring regular crunchy is** $\frac{4}{9}$ **of the total number surveyed.**

   iii. **The number preferring regular crunchy is** $\frac{4}{5}$ **of those preferring new extra crunchy.**

   iv. **The number preferring new extra crunchy is** $\frac{5}{4}$ **of those preferring regular crunchy.**

c. **If the company is planning to produce $90,000$ containers of crunchy peanut butter, how many of these containers should be the new extra crunchy variety, and how many of these containers should be the regular crunchy peanut butter? What would be helpful in solving this problem? Does one of our comparison statements above help us?**

The company should produce $50,000$ containers of new crunchy peanut butter and $40,000$ containers of regular crunchy peanut butter.

Discuss whether it is appropriate to assume that the company will still sell the same amount of regular crunchy peanut butter or whether the $90,000$ containers will simply be split between the two kinds of peanut butter.

- What would be helpful in solving this problem? Does one of our comparison statements above help us?

Guide students to the recognition that if we assume $90,000$ is the total number of containers sold for both types, then the number of new extra crunchy containers should be $\frac{5}{9}$ of that number.

Allow students to try solving the following three scenarios:

Try these next scenarios:

d. If the company decides to produce $2,000$ containers of regular crunchy peanut butter, how many containers of new extra crunchy peanut butter would it produce?

$2,500$ *new extra crunchy peanut butter containers*

e. If the company decides to produce $10,000$ containers of new extra crunchy peanut butter, how many containers of regular crunchy peanut butter would it produce?

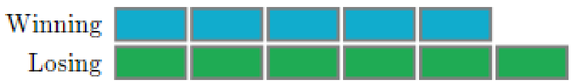
$8,000$ *regular crunchy peanut butter containers*

f. If the company decides to only produce $3,000$ containers of new extra crunchy peanut butter, how many containers of regular crunchy peanut butter would it produce?

$2,400$ *regular crunchy peanut butter containers*

Sample Anchor Charts for 6.RP.1 – Ratios with Tape Diagrams

Boden is making a prize wheel for the school fair. The ratio of winning spaces to losing spaces is shown in the diagram.



The table shows the number of winning and losing spaces that could be on the wheel.

Based on the ratio, complete the missing values in the table.

Winning	Losing
10	
	18

<u>Step 1:</u> Write in words what information is learned from the tape diagrams

There will be 5 winning spaces for every 6 losing spaced

<u>Step 2:</u> Write in ratio notation what you wrote for Step 1

(winning : losing = 5 : 6)

<u>Step 3:</u> Identify the multiplicative relationship between the original first value in the ratio to the number in the first column of the table

2 x 5 = 10

<u>Step 4:</u> Identify the multiplicative relationship between the original second value in the ratio to the number in second column of the table

3 x 6 = 8

<u>Step 5:</u> Use the factor you identified in Step 3 to multiply by the <u>second</u> number in the original ration. This is the missing number from the <u>second</u> column of the table

2 x 6 = 12

<u>Step 6:</u> Use the factor you identified in Step 4 to multiply by the <u>first</u> number in the original ration. This is the missing number from the <u>first</u> column of the table

3 x 5 = 15

6th Grade Math 6.RP.A.1.

Find Equivalent Ratios

Ratio of Boys to Girls: 3 to 5 or $\frac{3}{5}$

Boys	3	6	9	12
Girls	5	10	15	20

Mult. by 2: $\frac{3}{5} \cdot \frac{2}{2} = \frac{6}{10}$

Mult. by 3: $\frac{3}{5} \cdot \frac{3}{3} = \frac{9}{15}$

Mult. by 4: $\frac{3}{5} \cdot \frac{4}{4} = \frac{12}{20}$

Evan saves $2 of every $5 he earns mowing lawns.

$ Saved	2			8	10	20		
$ Spent	3	6	9		15		60	
Total $ Earned	5	10			25			150

How much will Evan have saved when he has earned $150?

Finding Equivalent Ratios

6th Grade Math 6.RP.A.1.

Simplify the Ratio 16 : 12

Divide both our number values by the GCF of 4.

$$16 : 12 \xrightarrow{\div 4} 4 : 3$$

The simplified Ratio Answer is 4 : 3 ✓

Eddie baked cookies with 4 cups of chocolate chips and 8 cups of sugar.
What is the ratio of chocolate chips to sugar?

4 : 8

2 : 4

1 : 2

Simplifying Ratios

6.RP.2 Back to ccss standard

Understand ratio concepts and use ratio reasoning to solve problems.

Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. ***For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."***

Skills

1. Identify the unit rate for a given ratio

2. Find the ratio for a given rate

3. Interpret a rate as division of two quantities

4. Interpret a rate as a fraction

5. Convert measurement units using rates

Key Concepts/Vocabulary

Ratio – Shows the relative sizes of two or more values

Unit rate – How much of something per 1 unit of something else

Rate – A measure or quantity measured against some other quantity or measure

Standard-Specific Resources (6.RP.2)

- **EngageNY: Grade 6, Module 1, Topic C, Lesson 17 – [From rates to ratios.]**

Classwork

> Given a rate, you can calculate the unit rate and associated ratios. Recognize that all ratios associated with a given rate are equivalent because they have the same value.

Example 1 (4 minutes)

> **Example 1**
>
> Write each ratio as a rate.
>
> a. The ratio of miles to the number of hours is 434 to 7.
>
> *Miles to hour:* $434:7$
>
> *Student responses:* $\frac{434}{7}\,\frac{\text{miles}}{\text{hours}} = 62$ *miles/hour*
>
> b. The ratio of the number of laps to the number of minutes is 5 to 4.
>
> *Laps to minute:* $5:4$
>
> *Student responses:* $\frac{5}{4}\,\frac{\text{laps}}{\text{minutes}} = \frac{5}{4}$ *laps/min*

Example 2 (15 minutes)

Demonstrate how to change a ratio to a unit rate then to a rate by recalling information students learned the previous day. Use Example 1, part (b).

> **Example 2**
>
> a. Complete the model below using the ratio from Example 1, part (b).

Ratio	Unit Rate	Rate
↓	↓	↓
	$\frac{\square}{\square}$	$\frac{\square}{\square}$ laps/minute
Ratio: $5:4$	*Unit Rate:* $\frac{5}{4}$	*Rate:* $\frac{5}{4}$ *laps/minute*

Rates to Ratios: Guide students to complete the next flow map where the rate is given. Students identify the unit rate and ratio.

> b. Complete the model below now using the rate listed below.

Ratio	Unit Rate	Rate
↓	↓	↓
		6 ft/sec
Ratios: Answers may vary $6:1$, $60:10$, $12:2$, *etc.*	*Unit Rate:* 6	

Discussion

- Will everyone have the same exact ratio to represent the given rate? Why or why not?
  - *Possible Answer: Not everyone's ratios will be exactly the same because there are many different equivalent ratios that could be used to represent the same rate.*
- What are some different examples that could be represented in the ratio box?
  - *Answers will vary: All representations represent the same rate:* $12:2$, $18:3$, $24:4$.
- Will everyone have the same exact unit rate to represent the given rate? Why or why not?
  - *Possible Answer: Everyone will have the same unit rate for two reasons. First, the unit rate is the value of the ratio, and each ratio only has one value. Second, the second quantity of the unit rate is always* 1, *so the rate will be the same for everyone.*
- Will everyone have the same exact rate when given a unit rate? Why or why not?
  - *Possible Answer: No, a unit rate can represent more than one rate. A rate of* $\frac{18}{3}$ *feet/second has a unit rate of* 6 *feet/second.*

Examples 3–6 (20 minutes)

Students work on one problem at a time. Have students share their reasoning. Provide opportunities for students to share different methods on how to solve each problem.

Examples 3–6

3. Dave can clean pools at a constant rate of $\frac{3}{5}$ pools/hour.

a. What is the ratio of the number of pools to the number of hours?

$3:5$

b. How many pools can Dave clean in 10 hours?

Pools	2	2	2			$=6$ *pools*
Hours	2	2	2	2	2	$=10$ *hours*

Dave can clean 6 pools in 10 hours.

c. How long does it take Dave to clean 15 pools?

Pools	5	5	5			$=15$ *pools*
Hours	5	5	5	5	5	$=25$ *hours*

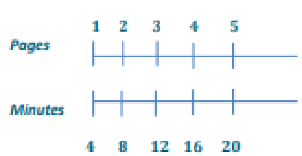
It will take Dave 25 hours to clean 15 pools.

4. Emeline can type at a constant rate of $\frac{1}{4}$ pages/minute.

a. What is the ratio of the number of pages to the number of minutes?

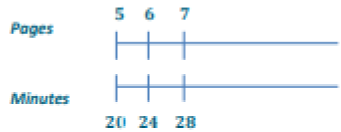
$1:4$

b. Emeline has to type a 5-page article but only has 18 minutes until she reaches the deadline. Does Emeline have enough time to type the article? Why or why not?



No, Emeline will not have enough time because it will take her 20 minutes to type a 5-page article.

c. Emeline has to type a 7-page article. How much time will it take her?



It will take Emeline 28 minutes to type a 7-page article.

5. Xavier can swim at a constant speed of $\frac{5}{3}$ meters/second.

a. What is the ratio of the number of meters to the number of seconds?

$5:3$

b. Xavier is trying to qualify for the National Swim Meet. To qualify, he must complete a 100-meter race in 55 seconds. Will Xavier be able to qualify? Why or why not?

Meters	*Seconds*
5	3
10	6
100	60

Xavier will not qualify for the meet because he would complete the race in 60 seconds.

c. Xavier is also attempting to qualify for the same meet in the 200-meter event. To qualify, Xavier would have to complete the race in 130 seconds. Will Xavier be able to qualify in this race? Why or why not?

Meters	*Seconds*
100	60
200	120

Xavier will qualify for the meet in the 200 meter race because he would complete the race in 120 seconds.

6. The corner store sells apples at a rate of 1.25 dollars per apple.

a. What is the ratio of the amount in dollars to the number of apples?

$1.25:1$

b. Akia is only able to spend $10 on apples. How many apples can she buy?

8 apples

c. Christian has $6 in his wallet and wants to spend it on apples. How many apples can Christian buy?

Christian can buy 4 apples and would spend $5.00. Christian cannot buy 5 apples because it would cost $6.25, and he only has $6.00.

Sample Anchor Charts for 6.RP.2 – Rates

A roller coaster can take 162 passengers around the track in 9 minutes. The roller coaster operates at a constant rate.

How many passengers can the roller coaster take around the track per minute?

______ passengers

1/4 The ratio of number of passengers to minutes is $162 : 9$.

Let's find an equivalent ratio that shows how many passengers the roller coaster takes in 1 minute.

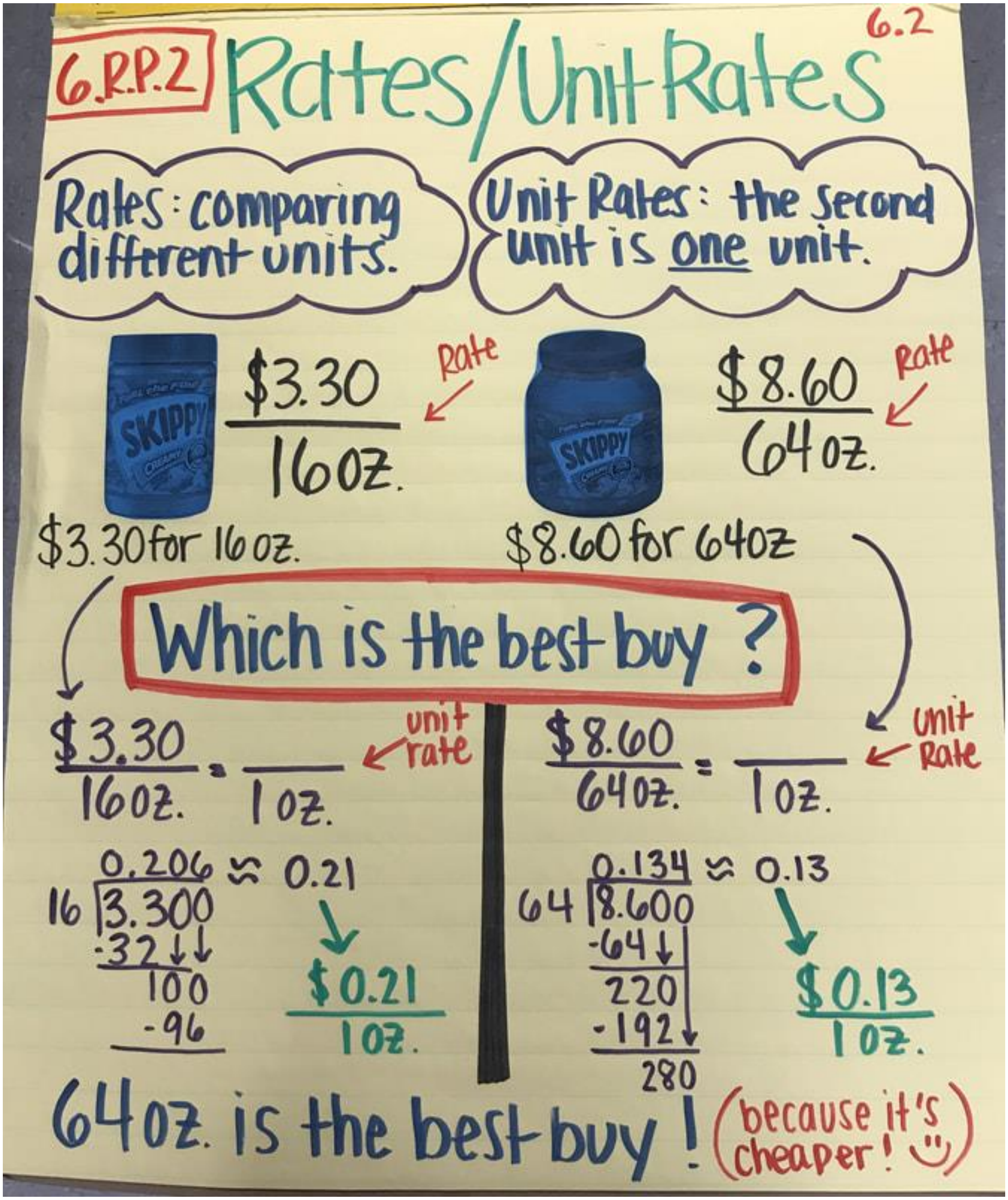
2/4 A ratio where one of the terms is 1 is called a unit rate. We can divide the number of minutes by 9 to get to 1 minute.

$$
\begin{array}{ccccc}
 & \textbf{passengers} & \longrightarrow & \textbf{minutes} & \\
\div 9 & 162 & \longrightarrow & 9 & \div 9 \\
 & ? & \longrightarrow & 1 &
\end{array}
$$

3/4 $162 \div 9 = 18$

$$
\begin{array}{ccccc}
 & \textbf{passengers} & \longrightarrow & \textbf{minutes} & \\
\div 9 & 162 & \longrightarrow & 9 & \div 9 \\
 & 18 & \longrightarrow & 1 &
\end{array}
$$

4/4 **The roller coaster can take 18 passengers around the track per minute.**


6.R.P.2
Rates/Unit Rates
6.2
Rates: comparing different units.
Unit Rates: the second unit is one unit.
SKIPPY
$3.30 / 16oz.
Rate
SKIPPY
$8.60 / 64oz.
Rate
$3.30 for 16oz.
$8.60 for 64oz
Which is the best buy?
$3.30 / 16oz. = ___ / 1oz.
unit rate
$8.60 / 64oz. = ___ / 1oz.
unit Rate
0.206 ≈ 0.21
16 ⟌3.300
-32
100
-96
$0.21 / 1oz.
0.134 ≈ 0.13
64 ⟌8.600
-64
220
-192
280
$0.13 / 1oz.
64oz. is the best buy!
(because it's cheaper!)

6.RP.3C

Back to ccss standard

Understand ratio concepts and use ratio reasoning to solve problems.

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g. by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Find a percent of a quantity as a rate per 100 (e.g. 30% of a quantity means 30/100 times the quantity); solve problems involving the whole, given a part and the percent.

Skills

1. Know that a percent is a ratio of a number in relation to 100

2. Find a % of a number as a rate per 100.

3. Solve real-world problems involving finding the whole, given a part and a percent

4. Apply ratio reasoning to convert measurements units by multiplying in real-world mathematical problems

5. Apply ratio reasoning to convert measurement units by dividing in real-world and mathematical problems

Key Concepts/Vocabulary

Percent – The probability of two or more events happening at the same time

Ratio – The relation of sizes between two or more values

Rate – A special ratio in which the two terms are in different units

Convert – A change in the form of a measurement [different units] without a change in the size or amount

Unit – A quantity used as a standard of measurement

Standard-Specific Resources (6.RP.3c)

- **EngageNY: Grade 6, Module 1, Topic D, Lesson 24 – Percent and Rates per 100**

Example 1 (5 minutes)

Begin class with a discussion to gather prior knowledge and to show a relationship to real-world applications.

- Imagine that you are shopping. You want to purchase an item for $100, but today it is 20% off. What does this mean?
  - *It means that $20 out of every $100 dollars will be subtracted from the total.*
- How can this situation be modeled?
  - *We could use a tape diagram that represents $100 divided into ten sections of $10. Two of the sections represent the discount, and eight of the sections represent the amount paid for the item. It could also be shown on a 10×10 grid, where 20 of the squares represent the discount, and 80 squares represent the amount paid.*
- How are percent problems related to the types of problems we have been working with involving ratios and rates?
  - *Answers will vary depending on prior knowledge. Some students may see that 20% of $100 is $20 off. Other students may see that we are trying to find part of a whole.*

Use the following website on a projector to visually explore percents in a 10×10 grid model.

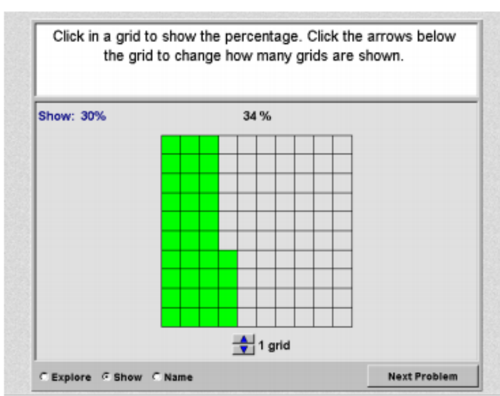
http://nlvm.usu.edu/en/nav/frames_asid_333_g_3_t_1.html?from category_g_3_t_1.html

Click the explore button on the website to be able to show 20%. This provides students with the visual for making the connection that 20% means 20 out of 100.

- What does this grid show?
  - 100 *blocks*
- How many are shaded in?
  - 20 *blocks*
- How many are not shaded in?
  - 80 *blocks*
- How can we use this model to help us think through 20% off of $100?
  - *From the grid, I can see that when I save 20%, I am paying 80% of the original value.*

Now they can see that since each block represents $1, they would be saving the 20 and spending the 80 when a $100 item is 20% off the original price.

Here is an example of what the website will look like:



If time allows, add more grids to model percents greater than 100% so that students further build an understanding.

Exercises 1–2 (8 minutes)

Solve the following two exercises with student input in order to model the process of working with percents. Students will need coloring utensils in order to complete the remaining activities.

Exercise 1

B	B	G	G	G	G	G	P	P	P
B	B	G	G	G	G	G	P	P	P
B	B	G	G	G	G	G	P	P	P
B	B	G	G	G	G	G	P	P	P
B	B	G	G	G	G	G	P	P	P
B	B	B	G	G	G	G	P	P	P
B	B	B	G	G	G	G	P	P	P
B	B	B	G	G	G	G	P	P	P
B	B	B	G	G	G	G	P	P	P
B	B	B	G	G	G	G	P	P	P

Robb's Fruit Farm consists of 100 acres on which three different types of apples grow. On 25 acres, the farm grows Empire apples. McIntosh apples grow on 30% of the farm. The remainder of the farm grows Fuji apples. Shade in the grid below to represent the portion of the farm each apple type occupies. Use a different color for each type of apple. Create a key to identify which color represents each type of apple.

	Color Key	Part-to-Whole Ratio
Empire	*Black (B)*	$25:100$
McIntosh	*Purple (P)*	$30:100$
Fuji	*Green (G)*	$45:100$

Exercise 2

The shaded portion of the grid below represents the portion of a granola bar remaining.

What percent does each block of granola bar represent?

1% *of the granola bar*

What percent of the granola bar remains?

80%

What other ways can we represent this percent?

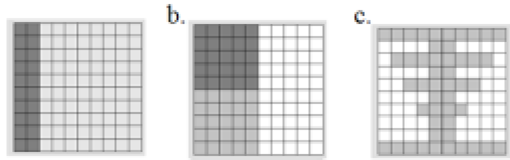
$\frac{80}{100}, \frac{8}{10}, \frac{4}{5}, \frac{16}{20}, \frac{32}{40}, \frac{64}{80}, 0.8$

0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

In this example, the teacher can discuss how 0.01 is related to $\frac{1}{100}$ and 1%. There are many examples that could be used to represent this percent in the last question. Students should list several examples.

Exercise 3

a. b. c.



a. For each figure shown, represent the gray shaded region as a percent of the whole figure. Write your answer as a decimal, fraction, and percent.

Picture (a)	Picture (b)	Picture (c)
20% is shaded darker than the rest, 0.20, $\frac{20}{100}$.	*Answers will vary. Sample answer (colored compared to total):* 50%, 0.50, $\frac{50}{100}$ *(Students could also compare darker shading to total, lighter shading to total, light shading to darker shading, darker shading to lighter shading, etc.)*	48%, 0.48, $\frac{48}{100}$

b. What ratio is being modeled in each picture?

Picture (a): Answers will vary. One example is the ratio of darker gray to the total is 20 *to* 100.

Picture (b): 50 *to* 100*, or a correct answer for whichever description they chose.*

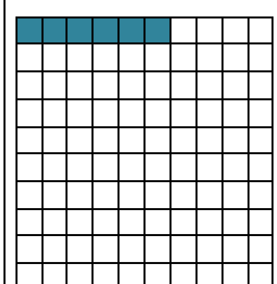
Picture (c): The ratio of gray to the total is 48 *to* 100.

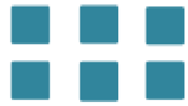
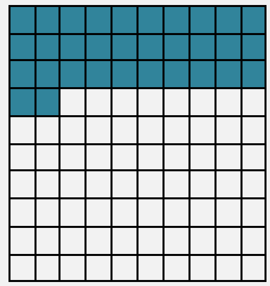
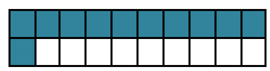
c. How are the ratios and percents related?

Answers will vary.

Exercise 4

Each relationship below compares the shaded portion (or part) to the entire figure (the whole). Complete the table.

Percentage	Decimal	Fraction	Ratio	Model
6%	0.06	$\frac{6}{100}$	$6:100$	

60%	0.6	$\frac{60}{100}, \frac{6}{10}$	$60:100$	
600%	6	$\frac{600}{100} = \frac{6}{1}$	$6:1$	 *6 wholes*
32%	0.32	$\frac{32}{100}$	$32:100$	
55%	0.55	$\frac{55}{100}, \frac{11}{20}$	$11:20$	
90%	0.9	$\frac{9}{10}$	$9:10$	
70%	0.7	$\frac{7}{10}, \frac{70}{100}$	$7:10$	

Exercise 5

Mr. Brown shares with the class that 70% of the students got an A on the English vocabulary quiz. If Mr. Brown has 100 students, create a model to show how many of the students received an A on the quiz.

$70\% \rightarrow \frac{70}{100} = \frac{7}{10}$

What fraction of the students received an A on the quiz?

$\frac{7}{10}$ *or* $\frac{70}{100}$

How could we represent this amount using a decimal?

0.7 *or* 0.70

How are the decimal, the fraction, and the percent all related?

The decimal, fraction, and percent all show 70 *out of* 100.

Exercise 6

Marty owns a lawn mowing service. His company, which consists of three employees, has 100 lawns to mow this week. Use the 10×10 grid to model how the work could have been distributed between the three employees.

Students choose how they want to separate the workload. The answers will vary. Below is a sample response.

G	G	G	P	P	P	P	P	B	B
G	G	G	P	P	P	P	P	B	B
G	G	G	P	P	P	P	P	B	B
G	G	G	P	P	P	P	P	B	B
G	G	G	P	P	P	P	P	B	B
G	G	G	P	P	P	P	P	B	B
G	G	G	P	P	P	P	P	B	B
G	G	G	P	P	P	P	P	B	B
G	G	G	P	P	P	P	P	B	B
G	G	G	P	P	P	P	P	B	B

Worker	Percentage	Fraction	Decimal
Employee 1 (G)	30%	$\frac{30}{100}$	0.30
Employee 2 (P)	50%	$\frac{50}{100}$	0.50
Employee 3 (B)	20%	$\frac{20}{100}$	0.20

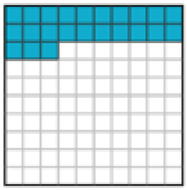
Closing (12 minutes)

Students present their work. Each group presents a problem or a part of a problem in order for all groups to respond.

Students complete this closing activity.

- What are three things you learned about in this lesson?
- Share two ways that you can write 2%.
- What is one thing that you still want to know about from the lesson?

Sample Anchor Chart for 6.RP.3c – Relate Fractions, Decimals, and Percents



The large square represents one whole. <u>Express the shaded area as a Fraction, a Decimal, and a Percent.</u> **Fraction = ____ Decimal = ____ Percent = _____**

<u>Step 1:</u> Find the <u>fraction</u> by putting the number of colored tiles in the numerator (23) and the value of the whole (100) in the denominator

The square is split into 100 equal pieces.

23 of the 100 pieces are shaded.

We can write the fraction as $\frac{23}{100}$.

<u>Step 2:</u> Rewrite the fraction from Step 1 as the numerator (23) next to the spelled-out denominator (hundredths)

$\frac{23}{100}$ can also be expressed as 23 hundredths.

<u>Step 3:</u> Create a place value chart to write Step 2 (23 hundredths) as a <u>decimal</u> (0.23)

Tens	Ones	.	Tenths	Hundredths
	0	.	2	3

We can write the decimal as 0.23.

<u>Step 4:</u> Find the <u>ratio</u> of colored squares (23) to <u>all</u> the squares (100). (23:100)

<u>Step 5:</u> Given that 'percent', or 'per cent', means 'per hundred', write the number of colored squares (from your ratio in Step 4) as "____%" (23%)

<u>Step 6:</u> You can now complete the table with the equal fraction, decimal, and percent for the shaded area.

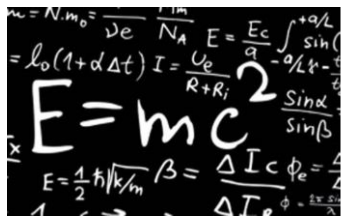
Fraction: $\frac{23}{100}$

Decimal: 0.23

Percent: 23%

Equations and Expressions

(EE)



6.EE.1 Back to ccss standard

***Apply and extend previous understandings of arithmetic to algebraic expressions*.**

Write and evaluate numerical expressions involving whole-number exponents.

Skills

1. Read numerical expressions involving number exponents (Ex. 3^4 = 3x3x3x3)

2. Write numerical expressions involving whole number exponents (Ex. 3^4 = 3x3x3x3)

3. Evaluate numerical expressions involving whole number exponents (Ex. 3^4 = 3x3x3x3 = 81)

4. Solve order of operation problems that contain exponents (Ex. $3 + 2^2 – (2 + 3) = 2$)

Key Concepts/Vocabulary

Numerical expression – A mathematical sentence involving only numbers and one or more operation symbols

Whole number – Any of the number 0, 1, 2, 3, etc., with no fractional or decimal part, and no negative value

Exponent – The exponent of a number says how many times to use that number in multiplication of itself, with a small number to the right and above the base number (e.g. $8^2 = 8 \times 8 = 64$)

Order of operations – The rules that say which calculation comes first in an expression, a.k.a. 'PEMDAS', or parentheses → exponents → multiplication/division (left to right) → addition/subtraction (left to right)

Standard-Specific Resources (6.EE.1)

- **<u>EngageNY:</u> Grade 5, Module 1, Topic D, Lesson 10** – **Students discover that $3x = x + x + x$ is not the same thing as x*x*x. Students understand that a base number can be represented with a positive whole number, positive fraction, or positive decimal and that for any number a^m is defined as the product of *m* factors of *a*. The number *a* is the base, and *m* is called the exponent or power of *a*.**

Opening Exercise (2 minutes)

> **Opening Exercise**
>
> **As you evaluate these expressions, pay attention to how you arrive at your answers.**
>
> $4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4$
>
> $9 + 9 + 9 + 9 + 9$
>
> $10 + 10 + 10 + 10 + 10$

Discussion (15 minutes)

- How many of you solved the problems by *counting on*? That is, starting with 4, you counted on 4 more each time ($5, 6, 7, \mathbf{8}, 9, 10, 11, \mathbf{12}, 13, 14, 15, \mathbf{16}, \ldots$).
- If you did not find the answer that way, could you have done so?
  - *Yes, but it is time consuming and cumbersome.*
- Addition is a faster way of counting on.
- How else could you find the sums using addition?
  - *Count by* 4, 9, *or* 10.
- How else could you solve the problems?
  - *Multiply* 4 *times* 10*; multiply* 9 *times* 5*; or multiply* 10 *times* 5.
- Multiplication is a faster way to add numbers when the addends are the same.
- When we add five groups of 10, we use an abbreviation and a different notation, called *multiplication*. $10 + 10 + 10 + 10 + 10 = 5 \times 10$
- If multiplication is a more efficient way to represent addition problems involving the repeated addition of the same addend, do you think there might be a more efficient way to represent the repeated multiplication of the same factor, as in $10 \times 10 \times 10 \times 10 \times 10 = ?$

Allow students to make suggestions; some may recall this from previous lessons.

$$10 \times 10 \times 10 \times 10 \times 10 = 10^5$$

- We see that when we add five groups of 10, we write 5×10, but when we multiply five copies of 10, we write 10^5. So, multiplication by 5 in the context of addition corresponds exactly to the exponent 5 in the context of multiplication.

Make students aware of the correspondence between addition and multiplication because what they know about *repeated addition* helps them learn exponents as *repeated multiplication* going forward.

- The little 5 we write is called an *exponent* and is written as a *superscript*. The numeral 5 is written only half as tall and half as wide as the 10, and the bottom of the 5 should be halfway up the number 10. The top of the 5 can extend a little higher than the top of the zero in 10. Why do you think we write exponents so carefully?
  - *It reduces the chance that a reader will confuse* 10^5 *with* 105.

> *Scaffolding:*
>
> When teaching students how to write an exponent as a *superscript*, compare and contrast the notation with how to write a *subscript*, as in the molecular formula for water, H_2O, or carbon dioxide, CO_2. Here the number is again half as tall as the capital letters, and the top of the 2 is halfway down it. The bottom of the subscript can extend a little lower than the bottom of the letter. Ignore the meaning of a chemical subscript.

Examples 1–5 (5 minutes)

Work through Examples 1–5 as a group; supplement with additional examples if needed.

Examples 1–10

Write each expression in exponential form.

1. $5 \times 5 \times 5 \times 5 \times 5 = 5^5$

2. $2 \times 2 \times 2 \times 2 = 2^4$

Write each expression in expanded form.

3. $8^3 = 8 \times 8 \times 8$

4. $10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10$

5. $g^3 = g \times g \times g$

- The repeated factor is called the *base*, and the exponent is also called the *power*. Say the numbers in Examples 1–5 to a partner.

Check to make sure students read the examples correctly:

- *Five to the fifth power, two to the fourth power, eight to the third power, ten to the sixth power, and g to the third power.*

Go back to Examples 1–4, and use a calculator to evaluate the expressions.

1. $5 \times 5 \times 5 \times 5 \times 5 = 5^5 = 3,125$

2. $2 \times 2 \times 2 \times 2 = 2^4 = 16$

3. $8^3 = 8 \times 8 \times 8 = 512$

4. $10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1,000,000$

What is the difference between $3g$ and g^3?

$3g = g + g + g$ *or* 3 *times* g; $g^3 = g \times g \times g$

Take time to clarify this important distinction.

- The base number can also be written in decimal or fraction form. Try Examples 6, 7, and 8. Use a calculator to evaluate the expressions.

Examples 6–8 (4 minutes)

6. **Write the expression in expanded form, and then evaluate.**

$(3.8)^4 = 3.8 \times 3.8 \times 3.8 \times 3.8 = 208.5136$

7. **Write the expression in exponential form, and then evaluate.**

$2.1 \times 2.1 = (2.1)^2 = 4.41$

8. **Write the expression in exponential form, and then evaluate.**

$0.75 \times 0.75 \times 0.75 = (0.75)^3 = 0.421875$

The base number can also be a fraction. Convert the decimals to fractions in Examples 7 and 8 and evaluate. Leave your answer as a fraction. Remember how to multiply fractions!

Example 7:

$$\frac{21}{10} \times \frac{21}{10} = \left(\frac{21}{10}\right)^2 = \frac{441}{100} = 4\frac{41}{100}$$

Example 8:

$$\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \left(\frac{3}{4}\right)^3 = \frac{27}{64}$$

Note to teacher:

If students need additional help multiplying fractions, refer to the first four modules of Grade 5.

Examples 9–10 (1 minute)

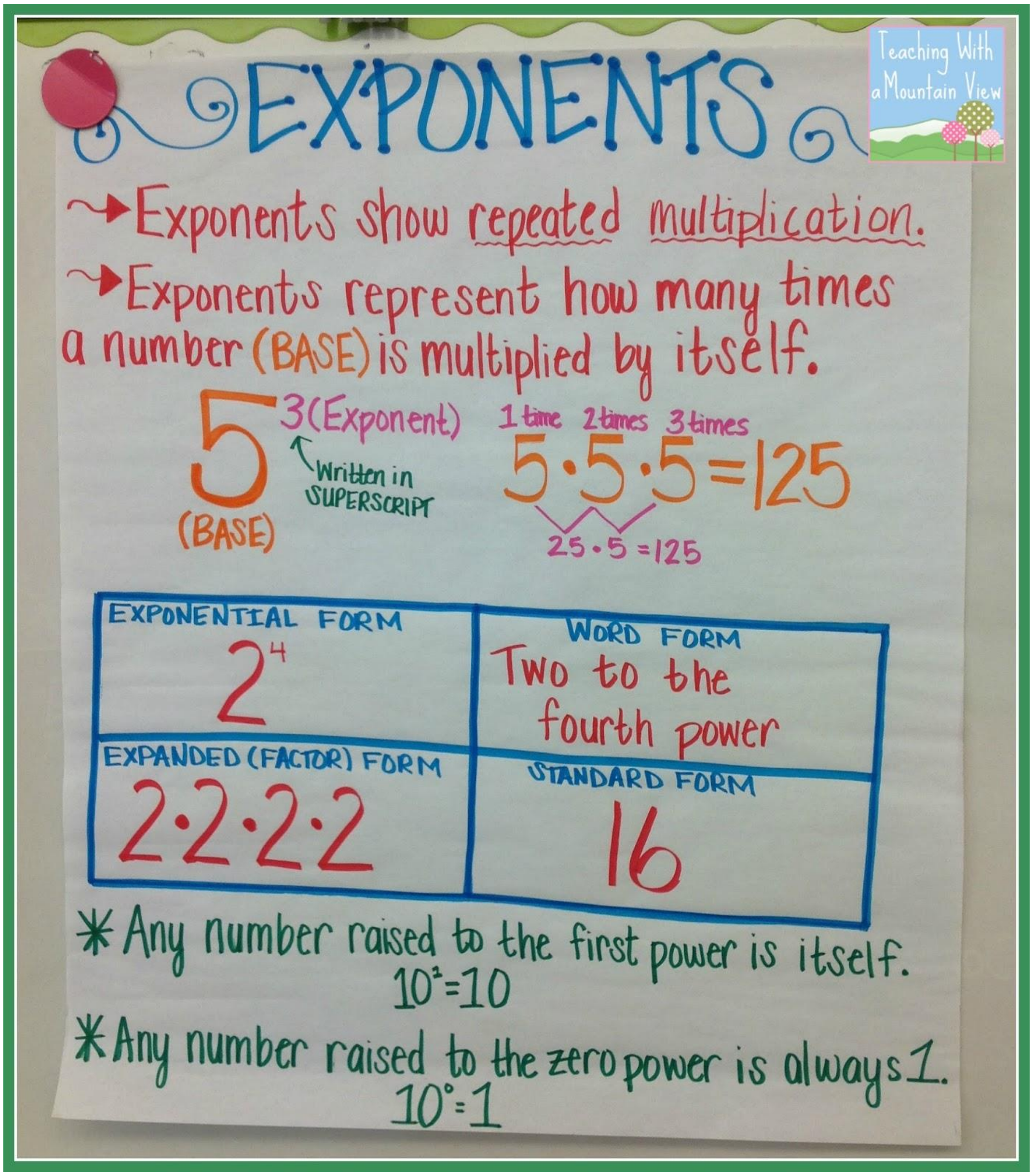
9. **Write the expression in exponential form, and then evaluate.** $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\left(\frac{1}{2}\right)^3=\frac{1}{8}$	**10.** **Write the expression in expanded form, and then evaluate.** $\left(\frac{2}{3}\right)^2=\frac{2}{3}\times\frac{2}{3}=\frac{4}{9}$

- There is a special name for numbers raised to the second power. When a number is raised to the second power, it is called *squared*. Remember that in geometry, squares have the same two dimensions: length and width. For $b > 0$, b^2 is the area of a square with side length b.
- What is the value of 5 squared?
  - 25
- What is the value of 7 squared?
  - 49
- What is the value of 8 squared?
  - 64
- What is the value of 1 squared?
  - 1

A multiplication chart is included at the end of this lesson. Post or project it as needed.

- Where are square numbers found on the multiplication table?
  - *On the diagonal*
- There is also a special name for numbers raised to the third power. When a number is raised to the third power, it is called *cubed*. Remember that in geometry, cubes have the same three dimensions: length, width, and height. For $b > 0$, b^3 is the volume of a cube with edge length b.
- What is the value of 1 cubed?
  - $1 \times 1 \times 1 = 1$
- What is the value of 2 cubed?
  - $2 \times 2 \times 2 = 8$
- What is the value of 3 cubed?
  - $3 \times 3 \times 3 = 27$
- In general, for any number x, $x^1 = x$, and for any positive integer $n > 1$, x^n is, by definition, $x^n = \underbrace{(x \cdot x \cdots x)}_{n \text{ times}}$.
- What does the x represent in this equation?
  - *The x represents the factor that will be repeatedly multiplied by itself.*
- What does the n represent in this expression?
  - *n represents the number of times x will be multiplied.*

Sample Anchor Charts for 6.EE.1 - Exponents



LAWS of EXPONENTS

1) Product Rule $a^m \cdot a^n = a^{m+n}$

EX: $(3x^6)(2x^4) = 3 \cdot 2 \cdot x^6 \cdot x^4 = 6x^{10}$

2) Quotient Rule $\frac{a^m}{a^n} = a^{m-n}$

EX: $\frac{m^5 n^7}{m^4 n^{10}} = \frac{mmmmm\,nnnnnnn}{mmmm\,nnnnnnnnnn} = \frac{m}{n^3}$

3) Power Rule $(a^m b^m)^n = a^{mn} b^{mn}$

$(-2xy^7z^3)^4 = -2 \cdot 2 \cdot 2 \cdot 2\, x^4 y^{28} z^{12} = 16x^4 y^{28} z^{12}$

4) Zero Power Rule $a^0 = 1$

ANY # to the zero power = 1 $(4m^2n^3)^0 = 1$

5) Negative Power Rule $a^{-n} = \frac{1}{a^n}$ $\frac{1}{b^{-m}} = b^m$

$3x^{-4} = \frac{3}{x^4}$ elevator down

$\frac{2}{m^{-5}} = 2m^5$

6.EE.2A [Back to ccss standard]

Apply and extend previous understandings of arithmetic to algebraic expressions.

Write, read, and evaluate expressions in which letters stand for numbers. Write expressions that record operations with numbers and with letters standing for numbers. ***For example, express the calculation 'Subtract y from 5' as 5 – y.***

Skills

1. Use numbers and variables to represent desired operations

2. Write expressions that record operations with numbers and with letters standing for numbers

3. Translate written phrases into algebraic expressions

4. Translate algebraic expressions into written phrases

Key Concepts/Vocabulary

Variable – A symbol for a number we don't know yet. It is usually a letter like x or y.

Operation – A mathematical process. +, -, x, /, squaring, square root, etc.

Expression – Numbers, symbols, and operators (such as + and x) grouped together to show the value of something

Standard-Specific Resources (6.EE.2a)

- **EngageNY:** Grade 6, Module 4, Topic D, Lesson 9 – [Students write expressions that record addition and subtraction operations with numbers.]

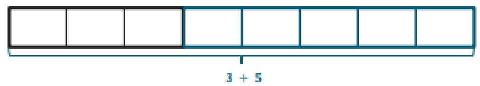
Classwork

Example 1 (3 minutes)

MP.2

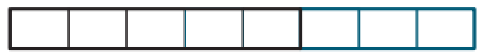
Example 1

Create a bar diagram to show 3 plus 5.



How would this look if you were asked to show 5 plus 3?

There would be 5 tiles and then 3 tiles.



Are these two expressions equivalent?

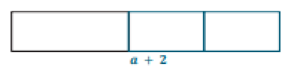
Yes. Both $3 + 5$ and $5 + 3$ have a sum of 8.

Example 2 (3 minutes)

Example 2

How can we show a number increased by 2?

$a + 2$ or $2 + a$



Can you prove this using a model? If so, draw the model.

Yes. I can use a bar diagram.

Example 3 (3 minutes)

Example 3

Write an expression to show the sum of m and k.

$m + k$ or $k + m$

Which property can be used in Examples 1–3 to show that both expressions given are equivalent?

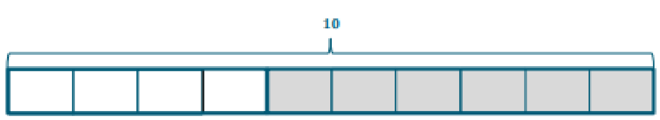
The commutative property of addition

Example 4 (3 minutes)

Example 4

How can we show 10 minus 6?

- Draw a bar diagram to model this expression.



- What expression would represent this model?

  $10 - 6$

- Could we also use $6 - 10$?

  No. If we started with 6 and tried to take 10 away, the models would not match.

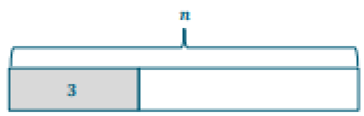
Example 5 (3 minutes)

Example 5

How can we write an expression to show 3 less than a number?

- **Start by drawing a diagram to model the subtraction. Are we taking away from the 3 or the unknown number?**

  We are taking 3 away from the unknown number.



- **What expression would represent this model?**

  The expression is $n - 3$.

Example 6 (3 minutes)

Example 6

How would we write an expression to show the number c being subtracted from the sum of a and b?

- **Start by writing an expression for "the sum of a and b."**

  $a + b$ or $b + a$

- **Now, show c being subtracted from the sum.**

  $a + b - c$ or $b + a - c$

Example 7 (3 minutes)

Example 7

Write an expression to show c minus the sum of a and b.

$c - (a + b)$

Why are parentheses necessary in this example and not the others?

Without the parentheses, only a is being taken away from c, where the expression says that $a + b$ should be taken away from c.

Replace the variables with numbers to see if $c - (a + b)$ is the same as $c - a + b$.

If students do not see the necessity for the parentheses, have them replace the variables with numbers to see whether $c - (a + b)$ is the same as $c - a + b$.

Here is a sample of what they could try:

$$a = 1, b = 2, c = 3$$

$3 - (1 + 2)$		$3 - 1 + 2$
$3 - 3$	or	$2 + 2$
0		4

Exercises (12 minutes)

These questions can be done on the worksheet. However, if white boards, small chalkboards, or some other personal board is available, the teacher can give instant feedback as students show their boards after each question.

Exercises

1. Write an expression to show the sum of 7 and 1.5.

   $7 + 1.5$ *or* $1.5 + 7$

2. Write two expressions to show w increased by 4. Then, draw models to prove that both expressions represent the same thing.

   $w + 4$ *and* $4 + w$

3. Write an expression to show the sum of a, b, and c.

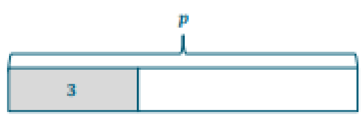
   Answers will vary. Below are possible answers.

   $a + b + c$ $\quad$ $b + c + a$ $\quad$ $c + b + a$

   $a + c + b$ $\quad$ $b + a + c$ $\quad$ $c + a + b$

4. Write an expression and a model showing 3 less than p.

   $p - 3$



5. Write an expression to show the difference of 3 and p.

   $3 - p$

6. Write an expression to show 4 less than the sum of g and 5.

   $g + 5 - 4$ *or* $5 + g - 4$

7. Write an expression to show 4 decreased by the sum of g and 5.

   $4 - (g + 5)$ *or* $4 - (5 + g)$

8. Should Exercises 6 and 7 have different expressions? Why or why not?

   The expressions are different because one includes the word "decreased by," and the other has the words "less than." The words "less than" give the amount that was taken away first, whereas the word "decreased by" gives us a starting amount and then the amount that was taken away.

Anchor Charts and Conceptual Development – 6.EE.2a - Writing Expressions with Variables

Remember when you were in elementary school, and you were learning your addition? The teacher would hand you worksheets that said things like:

$\square + 2 = 5$; fill in the box.

Variables are the same thing. Now we say:

$x + 2 = 5$; solve for x.

Why did we switch from boxes to letters? Because letters are better. Boxes come in only a few shapes, but letters come in many varieties, and letters can *stand* for something. For instance, the formula from geometry for finding a circle's circumference is:

$\mathrm{C} = 2\,\pi\, r$

This formula makes more sense than, say:

$\square = 2\,\pi\,\triangle$

The two formulae say exactly the same thing, but using "C" for "circumference" and "r" for "radius" is more useful than using "square" and "triangle, respectively. Boxes are fine, but letters are better.

In the above discussion, I illustrated both uses of variables:

In the equation "$x + 2 = 5$", x can only have a value of 3. The statement (the equation) is not true for any other value. That is to say, the value of x is "fixed"; we just have to figure out what that fixed value is. In this context (that is, when the variable "holds" a fixed value that you can find by solving), the variable may also be called "the unknown".

On the other hand, in the equation "$\mathrm{C} = 2\pi r$", the radius r can be any non-negative number we choose — we get to pick! — and then we get to figure out what the circumference C is. We got to put the value in ourselves.

In this context (that is, where you get to plug a value into one letter, and then can find the value for another letter), the variable whose value you pick (in this case, the r) is called the "independent" variable, because you, independently of others, got to pick the value; and the variable for which you can then find the value (in this case, the C) is called the "dependent" variable, because its value was entirely dependent upon what you picked for the value of r.

Now that we have variables, what do we do with them? Go back in your mind again to elementary school: Your teacher would have you add "2 apples plus 6 apples is 8 apples". The same rules apply to variables:

"2 boxes plus 6 boxes is 8 boxes"

...or, using variables:

"$2x + 6x = 8x$"

Or:

"A box and another box is two boxes"

...which translates as:

"$x + x = 2x$"

Using negatives:

"Two dollars, less the ten that you owe to your friend, means that you're eight dollars in the red"

or:

"$2x - 10x = -8x$".

But note: "2 apples plus 6 oranges" is just 2 apples and 6 oranges; they might make a nice fruit salad, but they're not 8 of anything. In the same way, "$2x + 6y$" is just $2x + 6y$; you can't combine the two variables into one. The variable portions of the things being added (the "terms") aren't the same — in mathematical parlance, the terms are "unlike" — so they cannot be "combined".

> By the way, this form of multiplication notation, where a number and a letter are put next to each other, is called "multiplication by juxtaposition" ("juhx-tuh-po-ZIH-shun"), because the number and the letter are "juxtaposed" (that is, they're placed right next to each other). Any time you see a number and a variable, or two or more variables, placed right next to each other like this, it means that the number and the variable, or the many variables, are to be multiplied together.

When multiplying, we use exponents. For instance, $(5)(5) = 5^2$. Of course, we can simplify this as $5^2 = 25$. Similarly, $(x)(x) = x^2$. But, until we know what value to put in for x, we cannot simplify this any further.

> Warning: Don't confuse multiplication and addition: $(x)(x)$ does not equal $2x$, just as $(5)(5)$ does not equal $(2)(5)$; instead, $(x)(x)$ equals x^2.

By the way, take note of the technique I just used: I used plain old numbers, with which I was well familiar before I ever took algebra, in order to example-fy what is going on in the world of algebra. You can do the same thing in your own studies. If you're not sure what to do with the variables, try using regular numbers, where you know what to do. Then, whatever you did with the numbers, try doing the same thing with the variables.

When evaluating variable expression, it is important to pay attention to the fact that the variable is a "box" into which you're plugging a value. Any multipliers, powers, or other things are happening *outside* of that box. For instance:

Evaluate x^2 when $x = -3$

Sometimes people will write the following as their evaluation expression:

Wrong way!
$-3^2 = -(3)(3) = -9$

But this moves the squaring inside the box, and takes the "minus" outside of the box. This is backwards. Instead, the evaluation expression should be:

Right way:
$(-3)^2 = (-3)(-3) = 9$

There are often difficulties when the "minus" is outside of the box. For instance:

Evaluate $-x^2$ for $x = -3$

There tend to be two errors that are the result of the above. Either people will do this:

Wrong way!
$-(-(3^2)) = +3^2 = 9$

...or this:

Wrong way!
$(-(-3))^2 = (+3)^2 = 9$

However, the correct evaluation expresson is the following:

Right way:
$-(-3)^2 = -(9) = -9$

6.EE.9

Back to ccss standard

Represent and analyze quantitative relationships between dependent and independent variables.

Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.

Skills

1. Define independent and dependent variables
2. Use variables to represent two quantities in a real-world problem that change in relationship to one another
3. Write an equation to express one quantity (dependent) in terms of the other quantity (independent)
4. Analyze the relationship between the dependent variable and independent variable using tables
5. Analyze the relationship between the dependent variable and independent variable using graphs
6. Relate the data in a graph and table to the corresponding equation.

Key Concepts/Vocabulary

Variable – A symbol for a number that we don't learn yet
Independent variable – An 'input' value of a function
Dependent variable – The 'output' value of a function
Quantity – How much there is of something
Equation – A number sentence that says that two things are equal
Expression – Numbers, symbols, and operators (such as + and x) grouped together that show the value of something
Table – Information (such as numbers and descriptions) arranged in rows and columns
Graph – A diagram of values, usually shown as lines or bars
Data – A collection of facts, such as numbers, words, measurements, observations or even just descriptions of things

Standard-Specific Resources (6.EE.9)

- **EngageNY:** Grade 6, Module 4, Topic H, Lesson 31 – Students analyze an equation in two variables to choose an independent variable and a dependent variable. Students determine whether or not the equation is solved for the second variable in terms of the first variable or vice versa. They then use the information to determine which variable is the independent variable and which is the dependent variable.

Classwork

Example 1 (10 minutes)

Example 1

Marcus reads for 30 minutes each night. He wants to determine the total number of minutes he will read over the course of a month. He wrote the equation $t = 30d$ to represent the total amount of time that he has spent reading, where t represents the total number of minutes read and d represents the number of days that he read during the month. Determine which variable is independent and which is dependent. Then, create a table to show how many minutes he has read in the first seven days.

Number of Days (d)	*Total Minutes Read ($30d$)*
1	30
2	60
3	90
4	120
5	150
6	180
7	210

Independent variable *Number of days*

Dependent variable *Total minutes read*

MP.1

- When setting up a table, we want the independent variable in the first column and the dependent variable in the second column.
- What do independent and dependent mean?
  - *The independent variable changes, and when it does, it affects the dependent variable. So, the dependent variable depends on the independent variable.*
- In this example, which would be the independent variable, and which would be the dependent variable?
  - *The dependent variable is the total number of minutes read because it depends on how many days Marcus reads. The independent variable is the number of days that Marcus reads.*
- How could you use the table of values to determine the equation if it had not been given?
  - *The number of minutes read shown in the table is always 30 times the number of days. So, the equation would need to show that the total number of minutes read is equal to the number of days times 30.*

Example 2 (5 minutes)

Example 2

Kira designs websites. She can create three different websites each week. Kira wants to create an equation that will give her the total number of websites she can design given the number of weeks she works. Determine the independent and dependent variables. Create a table to show the number of websites she can design over the first 5 weeks. Finally, write an equation to represent the number of websites she can design when given any number of weeks.

Independent variable *# of weeks worked*

Dependent variable *# of websites designed*

# of Weeks Worked (w)	*# of Websites Designed (d)*
1	3
2	6
3	9
4	12
5	15

Equation *$d = 3w$, where w is the number of weeks worked and d is the number of websites designed.*

- How did you determine which is the dependent variable and which is the independent variable?
    - *Because the number of websites she can make depends on how many weeks she works, I determined that the number of weeks worked was the independent variable, and the number of websites designed was the dependent variable.*
- Does knowing which one is independent and which one is dependent help you write the equation?
    - *I can write the equation and solve for the dependent variable by knowing how the independent variable will affect the dependent variable. In this case, I knew that every week 3 more websites could be completed, so then I multiplied the number of weeks by 3.*

.1

Example 3 (5 minutes)

Example 3

Priya streams movies through a company that charges her a $5 monthly fee plus $1.50 per movie. Determine the independent and dependent variables, write an equation to model the situation, and create a table to show the total cost per month given that she might stream between 4 and 10 movies in a month.

Independent variable *# of movies watched per month*

Dependent variable *Total cost per month, in dollars*

Equation $c = 1.5m + 5$ *or* $c = 1.50m + 5$

# of Movies (m)	*Total Cost Per Month, in dollars (c)*
4	11
5	12.50
6	14
7	15.50
8	17
9	18.50
10	20

- Is the flat fee an independent variable, a dependent variable, or neither?
    - *The $5 flat fee is neither. It is not causing the change in the dependent value, and it is not changing. Instead, the $5 flat fee is a constant that is added on each month.*
- Why isn't the equation $c = 5m + 1.50$?
    - *The $5 fee is only paid once a month. m is the number of movies watched per month, so it needs to be multiplied by the price per movie, which is $1.50.*

Exercises (15 minutes)

Students work in pairs or independently.

Exercises

1. **Sarah is purchasing pencils to share. Each package has 12 pencils. The equation $n = 12p$, where n is the total number of pencils and p is the number of packages, can be used to determine the total number of pencils Sarah purchased. Determine which variable is dependent and which is independent. Then, make a table showing the number of pencils purchased for 3–7 packages.**

   The number of packages, p, is the independent variable.

   The total number of pencils, n, is the dependent variable.

# of Packages (p)	*Total # of Pencils ($n = 12p$)*
3	36
4	48
5	60
6	72
7	84

2. **Charlotte reads 4 books each week. Let b be the number of books she reads each week, and let w be the number of weeks that she reads. Determine which variable is dependent and which is independent. Then, write an equation to model the situation, and make a table that shows the number of books read in under 6 weeks.**

   The number of weeks, w, is the independent variable.

   The number of books, b, is the dependent variable.

   $b = 4w$

# of Weeks (w)	*# of Books ($b = 4w$)*
1	4
2	8
3	12
4	16
5	20

Sample Anchor Chart for 6.EE.9 – Equations

Complete the table for the given rule.

Rule: $y = x + 8$

x	y
0	
2	
4	

1 / 4 The equation, $x + 8$ means 8 is added to x.

2 / 4 To find each value of y, we need to take each value of x and add 8.

For example, when $x = 0$ we can add $0 + 8 = 8$.

So, when $x = 0$, then $y = 8$.

3 / 4 Let's complete the rest of the table:

x	y
0	$0 + 8 = 8$
2	$2 + 8 = 10$
4	$4 + 8 = 12$

4 / 4 The answer is:

x	y
0	8
2	10
4	12

7.EE.1 Back to ccss standard

Use properties of operations to generate equivalent expressions.

Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

Skills

1. Combine like terms with rational coefficients

2. Factor and expand linear expressions with rational coefficients using the distributive property

3. Apply properties of operations as strategies to add and expand linear expressions with rational coefficients

4. Apply properties of operations as strategies to subtract and expand linear expressions with rational coefficients

5. Apply properties of operations as strategies to factor and expand linear expressions with rational coefficients

Key Concepts/Vocabulary

Rational coefficient – a rational number that is used to multiply a variable

Factor – numbers that can be multiplied by another number to get a different number

Expand – when you remove the parentheses () from an expression

Linear expression – an expression in which each term is either a constant or the product of a constant and a single variable

Distributive property – multiplying the sum by a number gives the same result as first multiplying each addend by the number and then adding the products (3 x [2 + 4] = [3x2] + [3x4])

Variable – a symbol (such as a letter) that represents a number (i.e., it is a placeholder for a number)

Standard-Specific Resources (7.EE.1)

- **EngageNY:** Grade 7, Module 3, Topic A, Lesson 1 – Students generate equivalent expressions using the fact that addition and multiplication can be done in any order (commutative property) and any grouping (associative property).

Materials

Prepare a classroom set of manila envelopes (non-translucent). Cut and place four triangles and two quadrilaterals in each envelope (provided at the end of this lesson). These envelopes are used in the Opening Exercise of this lesson.

Classwork

Opening Exercise (15 minutes)

MP.2

This exercise requires students to represent unknown quantities with variable symbols and reason mathematically with those symbols to represent another unknown value.

As students enter the classroom, provide each one with an envelope containing two quadrilaterals and four triangles; instruct students not to open their envelopes. Divide students into teams of two to complete parts (a) and (b).

> **Opening Exercise**
>
> **Each envelope contains a number of triangles and a number of quadrilaterals. For this exercise, let t represent the number of triangles, and let q represent the number of quadrilaterals.**
>
> a. **Write an expression using t and q that represents the total number of sides in your envelope. Explain what the terms in your expression represent.**
>
> *$3t + 4q$. Triangles have 3 sides, so there will be 3 sides for each triangle in the envelope. This is represented by $3t$. Quadrilaterals have 4 sides, so there will be 4 sides for each quadrilateral in the envelope. This is represented by $4q$. The total number of sides will be the number of triangle sides and the number of quadrilateral sides together.*
>
> b. **You and your partner have the same number of triangles and quadrilaterals in your envelopes. Write an expression that represents the total number of sides that you and your partner have. If possible, write more than one expression to represent this total.**
>
> $3t + 4q + 3t + 4q$; $2(3t + 4q)$; $6t + 8q$

> *Scaffolding:*
>
> To help students understand the given task, discuss a numerical expression, such as
> $$2 \times 3 + 6 \times 4,$$
> as an example where there are two triangles and six quadrilaterals.

MP.8

Discuss the variations of the expressions in part (b) and whether those variations are equivalent. This discussion helps students understand what it means to combine like terms; some students have added their number of triangles together and number of quadrilaterals together, while others simply doubled their own number of triangles and quadrilaterals since the envelopes contain the same number. This discussion further shows how these different forms of the same expression relate to each other. Students then complete part (c).

> c. **Each envelope in the class contains the same number of triangles and quadrilaterals. Write an expression that represents the total number of sides in the room.**
>
> *Answer depends on the number of students in the classroom. For example, if there are 12 students in the classroom, the expression would be $12(3t + 4q)$, or an equivalent expression.*

Next, discuss any variations (or possible variations) of the expression in part (c), and discuss whether those variations are equivalent. Are there as many variations in part (c), or did students use multiplication to consolidate the terms in their expressions? If the latter occurred, discuss students' reasoning.

Choose one student to open an envelope and count the numbers of triangles and quadrilaterals. Record the values of t and q as reported by that student for all students to see. Next, students complete parts (d), (e), and (f).

d. **Use the given values of t and q and your expression from part (a) to determine the number of sides that should be found in your envelope.**

$3t + 4q$
$3(4) + 4(2)$
$12 + 8$
20

There should be 20 sides contained in my envelope.

e. **Use the same values for t and q and your expression from part (b) to determine the number of sides that should be contained in your envelope and your partner's envelope combined.**

Variation 1	*Variation 2*	*Variation 3*
$2(3t + 4q)$	$3t + 4q + 3t + 4q$	$6t + 8q$
$2\big(3(4) + 4(2)\big)$	$3(4) + 4(2) + 3(4) + 4(2)$	$6(4) + 8(2)$
$2(12 + 8)$	$12 + 8 + 12 + 8$	$24 + 16$
$2(20)$	$20 + 20$	40
40	40	

My partner and I have a combined total of 40 sides.

f. **Use the same values for t and q and your expression from part (c) to determine the number of sides that should be contained in all of the envelopes combined.**

Answer will depend on the seat size of your classroom. Sample responses for a class size of 12:

Variation 1	*Variation 2*	*Variation 3*
$12(3t + 4q)$	$\overbrace{3t + 4q}^{1} + \overbrace{3t + 4q}^{2} + \cdots + \overbrace{3t + 4q}^{12}$	$36t + 48q$
$12\big(3(4) + 4(2)\big)$	$\overbrace{3(4) + 4(2)}^{1} + \overbrace{3(4) + 4(2)}^{2} + \cdots + \overbrace{3(4) + 4(2)}^{12}$	$36(4) + 48(2)$
$12(12 + 8)$	$\overbrace{12 + 8}^{1} + \overbrace{12 + 8}^{2} + \cdots + \overbrace{12 + 8}^{12}$	$144 + 96$
$12(20)$	$\overbrace{20}^{1} + \overbrace{20}^{2} + \cdots + \overbrace{20}^{12}$	240
240	240	

For a class size of 12 students, there should be 240 sides in all of the envelopes combined.

Have all students open their envelopes and confirm that the number of triangles and quadrilaterals matches the values of t and q recorded after part (c). Then, have students count the number of sides on the triangles and quadrilaterals from their own envelopes and confirm with their answers to part (d). Next, have partners count how many sides they have combined and confirm that number with their answers to part (e). Finally, total the number of sides reported by each student in the classroom and confirm this number with the answer to part (f).

g. **What do you notice about the various expressions in parts (e) and (f)?**

The expressions in part (e) are all equivalent because they evaluate to the same number: 40. The expressions in part (f) are all equivalent because they evaluate to the same number: 240. The expressions themselves all involve the expression $3t + 4q$ in different ways. In part (e), $3t + 3t$ is equivalent to $6t$, and $4q + 4q$ is equivalent to $8q$. There appear to be several relationships among the representations involving the commutative, associative, and distributive properties.

When finished, have students return their triangles and quadrilaterals to their envelopes for use by other classes.

Example 1 (10 minutes): Any Order, Any Grouping Property with Addition

This example examines how and why we combine numbers and other like terms in expressions. An expression that is written as sums (and/or differences) of products whose factors are numbers, variables, or variables raised to whole number powers is said to be in *expanded form*. A single number, variable, or a single product of numbers and/or variables is also considered to be in expanded form. Examples of expressions in expanded form include 324, $3x$, $5x + 3 - 40$, $x + 2x + 3x$, etc.

Each summand of an expression in expanded form is called a *term*, and the number found by multiplying just the numbers in a term together is called the *coefficient of the term*. After defining the word *term*, students can be shown what it means to *combine like terms* using the distributive property. Students saw in the Opening Exercise that terms sharing exactly the same letter could be combined by adding (or subtracting) the coefficients of the terms:

$$3t + 3t = \overbrace{(3+3)}^{\text{coefficients}} \cdot t = 6t, \qquad \text{and} \qquad 4q + 4q = \overbrace{(4+4)}^{\text{coefficients}} \cdot q = 8q.$$

An expression in expanded form with all its like terms collected is said to be in *standard form*.

Example 1: Any Order, Any Grouping Property with Addition

a. **Rewrite $5x + 3x$ and $5x - 3x$ by combining like terms.**

Write the original expressions and expand each term using addition. What are the new expressions equivalent to?

$$5x + 3x = \underbrace{\overbrace{x+x+x+x+x}^{5x} + \overbrace{x+x+x}^{3x}}_{8x} = 8x$$

$$5x - 3x = \overbrace{x+x+\underbrace{x+x+x}_{3x}}^{5x} = 2x$$

Scaffolding:

Refer students to the triangles and quadrilaterals from the Opening Exercise to understand why terms containing the same variable symbol x can be added together into a single term.

- Because both terms have the common factor of x, we can use the distributive property to create an equivalent expression.

$$5x + 3x = (5+3)x = 8x \qquad\qquad 5x - 3x = (5-3)x = 2x$$

Scaffolding:

Note to the teacher: The distributive property was covered in Grade 6 (**6.EE.A.3**) and is reviewed here in preparation for further use in this module starting with Lesson 3.

Ask students to try to find an example (a value for x) where $5x + 3x \neq 8x$ or where $5x - 3x \neq 2x$. Encourage them to use a variety of positive and negative rational numbers. Their failure to find a counterexample helps students realize what equivalence means.

In Example 1, part (b), students see that the commutative and associative properties of addition are regularly used in consecutive steps to reorder and regroup like terms so that they can be combined. Because the use of these properties does not change the value of an expression or any of the terms within the expression, the commutative and associative properties of addition can be used simultaneously. The simultaneous use of these properties is referred to as the *any order, any grouping property*.

Scaffolding:

Teacher may also want to show the expression as $\underbrace{x+x+1}_{2x+1} + \underbrace{x+x+x+x+x}_{5x}$ in the same manner as in part (a).

b. **Find the sum of $2x + 1$ and $5x$.**

$(2x+1)+5x$	*Original expression*
$2x+(1+5x)$	*Associative property of addition*
$2x+(5x+1)$	*Commutative property of addition*
$(2x+5x)+1$	*Associative property of addition*
$(2+5)x+1$	*Combined like terms (the distributive property)*
$7x+1$	*Equivalent expression to the given problem*

With a firm understanding of the commutative and associative properties of addition, students further understand that these steps can be combined.

- Why did we use the associative and commutative properties of addition?
  - *We reordered the terms in the expression to group together like terms so that they could be combined.*
- Did the use of these properties change the value of the expression? How do you know?
  - *The properties did not change the value of the expression because each equivalent expression includes the same terms as the original expression, just in a different order and grouping.*
- If a sequence of terms is being added, the *any order, any grouping* property allows us to add those terms in any order by grouping them together in any way.
- How can we confirm that the expressions $(2x + 1) + 5x$ and $7x + 1$ are equivalent expressions?
  - *When a number is substituted for the x in both expressions, they both should yield equal results.*

The teacher and student should choose a number, such as 3, to substitute for the value of x and together check to see if both expressions evaluate to the same result.

Given Expression	Equivalent Expression?
$(2x + 1) + 5x$	$7x + 1$
$(2 \cdot 3 + 1) + 5 \cdot 3$	$7 \cdot 3 + 1$
$(6 + 1) + 15$	$21 + 1$
$(7) + 15$	22
22	

The expressions both evaluate to 22; however, this is only one possible value of x. Challenge students to find a value for x for which the expressions do not yield the same number. Students find that the expressions evaluate to equal results no matter what value is chosen for x.

- What prevents us from using any order, any grouping in part (c), and what can we do about it?
  - *The second expression, $(5a - 3)$, involves subtraction, which is not commutative or associative; however, subtracting a number x can be written as adding the opposite of that number. So, by changing subtraction to addition, we can use any order and any grouping.*

c. **Find the sum of $-3a + 2$ and $5a - 3$.**

$(-3a + 2) + (5a - 3)$	*Original expression*
$-3a + 2 + 5a + (-3)$	*Add the opposite (additive inverse)*
$-3a + 5a + 2 + (-3)$	*Any order, any grouping*
$2a + (-1)$	*Combined like terms (Stress to students that the expression is not yet simplified.)*
$2a - 1$	*Adding the inverse is subtracting.*

- What was the only difference between this problem and those involving all addition?
  - *We first had to rewrite subtraction as addition; then, this problem was just like the others.*

Example 2 (3 minutes): Any Order, Any Grouping with Multiplication

Students relate a product to its expanded form and understand that the same result can be obtained using any orde any grouping since multiplication is also associative and commutative.

Example 2: Any Order, Any Grouping with Multiplication

Find the product of $2x$ and 3.

$2x \cdot 3$		
$2 \cdot (x \cdot 3)$	*Associative property of multiplication (any grouping)*	*With a firm understanding of the commutative and associative properties of multiplication, students further understand that these steps can be combined.*
$2 \cdot (3 \cdot x)$	*Commutative property of multiplication (any order)*	
$6x$	*Multiplication*	

MP.7

- Why did we use the associative and commutative properties of multiplication?
  - *We reordered the factors to group together the numbers so that they could be multiplied.*
- Did the use of these properties change the value of the expression? How do you know?
  - *The properties did not change the value of the expression because each equivalent expression includes the same factors as the original expression, just in a different order or grouping.*
- If a product of factors is being multiplied, the *any order, any grouping* property allows us to multiply those factors in any order by grouping them together in any way.

Example 3 (9 minutes): Any Order, Any Grouping in Expressions with Addition and Multiplication

Students use any order, any grouping to rewrite products with a single coefficient first as terms only, then as terms within a sum, noticing that any order, any grouping cannot be used to mix multiplication with addition.

Example 3: Any Order, Any Grouping in Expressions with Addition and Multiplication

Use any order, any grouping to write equivalent expressions.

a. $3(2x)$

$(3 \cdot 2)x$

$6x$

Ask students to try to find an example (a value for x) where $3(2x) \neq 6x$. Encourage them to use a variety of positive and negative rational numbers because in order for the expressions to be equivalent, the expressions must evaluate to equal numbers for *every* substitution of numbers into all the letters in both expressions. Again, the point is to help students recognize that they cannot find a value—that the two expressions are equivalent. Encourage students to follow the order of operations for the expression $3(2x)$: multiply by 2 first, then by 3.

b. $4y(5)$

$(4 \cdot 5)y$

$20y$

c. $4 \cdot 2 \cdot z$

$(4 \cdot 2)z$

$8z$

d. $3(2x) + 4y(5)$

$3(2x) + 4y(5) = \overbrace{2x + 2x + 2x}^{6x} + \overbrace{4y + 4y + 4y + 4y + 4y}^{20y}$

$(3 \cdot 2)x + (4 \cdot 5)y$

$6x + 20y$

e. $3(2x) + 4y(5) + 4 \cdot 2 \cdot z$

$3(2x) + 4y(5) + 4 \cdot 2 \cdot z = \overbrace{2x + 2x + 2x}^{6x} + \overbrace{4y + 4y + 4y + 4y + 4y}^{20y} + \overbrace{z + z + z + z + z + z + z + z}^{8z}$

$(3 \cdot 2)x + (4 \cdot 5)y + (4 \cdot 2)z$

$6x + 20y + 8z$

f. **Alexander says that $3x + 4y$ is equivalent to $(3)(4) + xy$ because of any order, any grouping. Is he correct? Why or why not?**

Encourage students to substitute a variety of positive and negative rational numbers for x and y because in order for the expressions to be equivalent, the expressions must evaluate to equal numbers for *every* substitution of numbers into all the letters in both expressions.

Alexander is incorrect; the expressions are not equivalent because if we, for example, let $x = -2$ and let $y = -3$, then we get the following:

$3x + 4y$	$(3)(4) + xy$
$3(-2) + 4(-3)$	$12 + (-2)(-3)$
$-6 + (-12)$	$12 + 6$
-18	18

$-18 \neq 18$, so the expressions cannot be equivalent.

- What can be concluded as a result of part (f)?
  - *Any order, any grouping cannot be used to mix multiplication with addition. Numbers and letters that are factors within a given term must remain factors within that term.*

Sample Anchor Charts for 7.EE.1 – Equivalent Expressions

Which expressions are equivalent to $-7+3(-4e-3)$?

Choose all answers that apply:

(A) $-4(3e+4)$

(B) $12e$

(C) None of the above

1/4 We can play around with the expressions in the answer choices to see if they are the same as $-7+3(-4e-3)$.

2/4 Let's see if the first expression is equivalent to $-7+3(-4e-3)$.

$$-4(3e+4) \stackrel{?}{=} -7+3(-4e-3)$$

$$-4(3e+4) \stackrel{?}{=} -7+(3)(-4e)-(3)3$$

$$-4(3e+4) \stackrel{?}{=} -7+(-12e)-9$$

$$-4(3e+4) \stackrel{?}{=} -12e-16$$

$$(-4)3e+(-4)4 \stackrel{?}{=} -12e-16$$

$$-12e+(-16) \stackrel{?}{=} -12e-16$$

$$-12e-16 = -12e-16$$

3/4 Let's see if the second expression is equivalent to $-7+3(-4e-3)$.

$$12e \stackrel{?}{=} -7 + 3(-4e - 3)$$

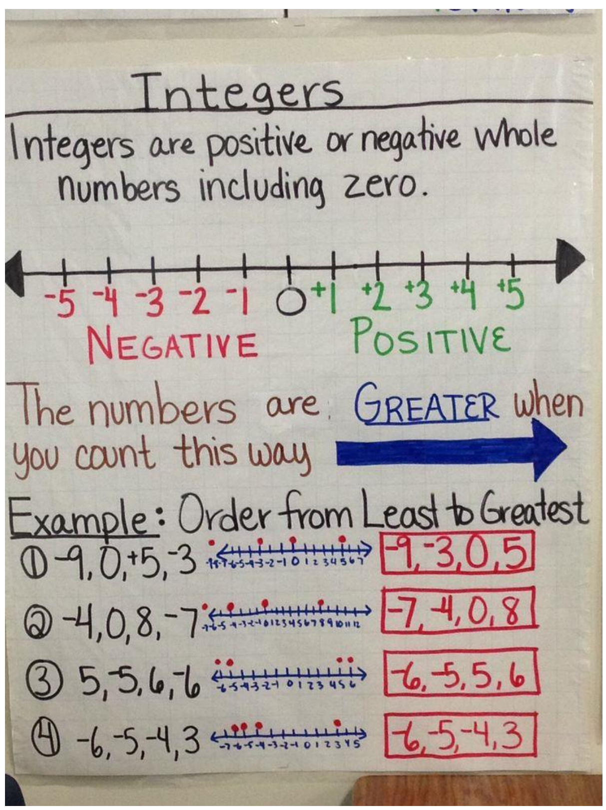
$$12e \stackrel{?}{=} -7 + (3)(-4e) - (3)3$$

$$12e \stackrel{?}{=} -7 + (-12e) - 9$$

$$12e \neq -12e - 16$$

4 / 4 The following expression is equivalent to $-7 + 3(-4e - 3)$:

- $-4(3e + 4)$



7.EE.2 Back to ccss standard

Use properties of operations to generate equivalent expressions.

Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. ***For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."***

Skills

1. Write equivalent expressions with integers

2. Write equivalent expressions with fractions

3. Write equivalent expressions with decimals

4. Write equivalent expressions with percents

5. Rewrite an expression in an equivalent form in order to provide insight about how quantities are related in a real-world problem context

Key Concepts/Vocabulary

Equivalent expression – expressions that are the same, even though they may look different

Fraction – how many parts of a whole

Decimal – based on 10, a number that uses a decimal point followed by digits that show a value smaller than one

Percent – parts per 100

Integer – a number with no fractional part

Quantity – how much there is of something

Standard-Specific Resources (7.EE.2)

- **EngageNY: Grade 7, Module 3, Topic A, Lesson 6 – Students rewrite rational number expressions by collecting like terms and combining them by repeated use of the distributive property.**

Example 1 (4 minutes)

Scaffolding:

Students who need a challenge could tackle the following problem:

$$\frac{1}{2}a + 2\frac{2}{3}b + \frac{1}{5} - \frac{1}{4}a - 1\frac{1}{2}b + \frac{3}{5} + \frac{3}{4}a - 4 - \frac{4}{5}b$$

Example 1

Rewrite the expression in standard form by collecting like terms.

$\frac{2}{3}n - \frac{3}{4}n + \frac{1}{6}n + 2\frac{2}{9}n$

$\frac{24}{36}n - \frac{27}{36}n + \frac{6}{36}n + 2\frac{8}{36}n$

$2\frac{11}{36}n$

- What are various strategies for adding, subtracting, multiplying, and dividing rational numbers?
  - *Find common denominators; change from mixed numbers and whole numbers to improper fractions, and then convert back.*

Exercise 1

For the following exercises, predict how many terms the resulting expression will have after collecting like terms. Then, write the expression in standard form by collecting like terms.

a. $\frac{2}{5}g - \frac{1}{6} - g + \frac{3}{10}g - \frac{4}{5}$

There will be two terms.

$\frac{2}{5}g - 1g + \frac{3}{10}g - \frac{1}{6} - \frac{4}{5}$

$\left(\frac{2}{5} - 1 + \frac{3}{10}\right)g - \left(\frac{1}{6} + \frac{4}{5}\right)$

$-\frac{3}{10}g - \frac{29}{30}$

b. $i + 6i - \frac{3}{7}i + \frac{1}{3}h + \frac{1}{2}i - h + \frac{1}{4}h$

There will be two terms.

$\frac{1}{3}h + \frac{1}{4}h - h + i - \frac{3}{7}i + 6i + \frac{1}{2}i$

$\left(\frac{1}{3} + \frac{1}{4} + (-1)\right)h + \left(1 - \frac{3}{7} + 6 + \frac{1}{2}\right)i$

$-\frac{5}{12}h + 7\frac{1}{14}i$

Example 2 (5 minutes)

Read the problem as a class and give students time to set up their own expressions. Reconvene as a class to address each expression.

Example 2

At a store, a shirt was marked down in price by $\$10.00$. A pair of pants doubled in price. Following these changes, the price of every item in the store was cut in half. Write two different expressions that represent the new cost of the items, using s for the cost of each shirt and p for the cost of a pair of pants. Explain the different information each one shows.

For the cost of a shirt:

$\frac{1}{2}(s-10)$ *The cost of each shirt is* $\frac{1}{2}$ *of the quantity of the original cost of the shirt, minus* 10.

$\frac{1}{2}s-5$ *The cost of each shirt is half off the original price, minus* 5*, since half of* 10 *is* 5.

For the cost of a pair of pants:

$\frac{1}{2}(2p)$ *The cost of each pair of pants is half off double the price.*

p *The cost of each pair of pants is the original cost because* $\frac{1}{2}$ *is the multiplicative inverse of* 2.

- Describe a situation in which either of the two expressions in each case would be more useful.
  - *Answers may vary. For example,* p *would be more useful than* $\frac{1}{2}(2p)$ *because it is converted back to an isolated variable, in this case the original cost.*

Exercise 2 (3 minutes)

Exercise 2

Write two different expressions that represent the total cost of the items if tax was $\frac{1}{10}$ of the original price. Explain the different information each shows.

For the cost of a shirt:

$\frac{1}{2}(s-10)+\frac{1}{10}s$ *The cost of each shirt is* $\frac{1}{2}$ *of the quantity of the original cost of the shirt, minus* 10*, plus* $\frac{1}{10}$ *of the cost of the shirt.*

$\frac{3}{5}s-5$ *The cost of each shirt is* $\frac{3}{5}$ *of the original price (because it is* $\frac{1}{2}s+\frac{1}{10}s=\frac{6}{10}s$*), minus* 5*, since half of* 10 *is* 5.

For the cost of a pair of pants:

$\frac{1}{2}(2p)+\frac{1}{10}p$ *The cost of each pair of pants is half off double the price plus* $\frac{1}{10}$ *of the cost of a pair of pants.*

$1\frac{1}{10}p$ *The cost of each pair of pants is* $1\frac{1}{10}$ *(because* $1p+\frac{1}{10}p=1\frac{1}{10}p$*) times the number of pair of pants.*

Example 3 (4 minutes)

Example 3

Write this expression in standard form by collecting like terms. Justify each step.

$$5\frac{1}{3}-\left(3\frac{1}{3}\right)\left(\frac{1}{2}x-\frac{1}{4}\right)$$

$\frac{16}{3}+\left(-\frac{10}{3}\right)\left(\frac{1}{2}x\right)+\left(-\frac{10}{3}\right)\left(-\frac{1}{4}\right)$	*Write mixed numbers as improper fractions, then distribute.*
$\frac{16}{3}+\left(-\frac{5}{3}x\right)+\frac{5}{6}$	*Any grouping (associative) and arithmetic rules for multiplying rational numbers*
$-\frac{5}{3}x+\left(\frac{32}{6}+\frac{5}{6}\right)$	*Commutative property and associative property of addition, collect like terms*
$-\frac{5}{3}x+\frac{37}{6}$	*Apply arithmetic rule for adding rational numbers*

- A student says he created an equivalent expression by first finding this difference: $5\frac{1}{3}-3\frac{1}{3}$. Is he correct? Why or why not?
  - *Although they do appear to be like terms, taking the difference would be incorrect. In the expression $\left(3\frac{1}{3}\right)\left(\frac{1}{2}x-\frac{1}{4}\right)$, $3\frac{1}{3}$ must be distributed before applying any other operation in this problem.*

- How should $3\frac{1}{3}$ be written before being distributed?
  - *The mixed number can be rewritten as an improper fraction $\frac{10}{3}$. It is not necessary to convert the mixed number, but it makes the process more efficient and increases the likelihood of getting a correct answer.*

Exercise 3 (5 minutes)

Walk around as students work independently. Have students check their answers with a partner. Address any unresolved questions.

Exercise 3

Rewrite the following expressions in standard form by finding the product and collecting like terms.

a. $-6\frac{1}{3}-\frac{1}{2}\left(\frac{1}{2}+y\right)$

$-6\frac{1}{3}+\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)+\left(-\frac{1}{2}\right)y$

$-6\frac{1}{3}+\left(-\frac{1}{4}\right)+\left(-\frac{1}{2}y\right)$

$-\frac{1}{2}y-\left(6\frac{1}{3}+\frac{1}{4}\right)$

$-\frac{1}{2}y-\left(6\frac{4}{12}+\frac{3}{12}\right)$

$-\frac{1}{2}y-6\frac{7}{12}$

b. $\frac{2}{3}+\frac{1}{3}\left(\frac{1}{4}f-1\frac{1}{3}\right)$

$\frac{2}{3}+\frac{1}{3}\left(\frac{1}{4}f\right)+\frac{1}{3}\left(-\frac{4}{3}\right)$

$\frac{2}{3}+\frac{1}{12}f-\frac{4}{9}$

$\frac{1}{12}f+\left(\frac{6}{9}-\frac{4}{9}\right)$

$\frac{1}{12}f+\frac{2}{9}$

Example 4 (5 minutes)

Example 4

Model how to write the expression in standard form using rules of rational numbers.

$$\frac{x}{20}+\frac{2x}{5}+\frac{x+1}{2}+\frac{3x-1}{10}$$

- What are other equivalent expressions of $\frac{x}{20}$? How do you know?
  - *Other expressions include $\frac{1x}{20}$ and $\frac{1}{20}x$ because of the arithmetic rules of rational numbers.*
- What about $\frac{1}{20x}$? How do you know?
  - *It is not equivalent because if $x = 2$, the value of the expression is $\frac{1}{40}$, which does not equal $\frac{1}{10}$.*
- How can the distributive property be used in this problem?
  - *For example, it can be used to factor out $\frac{1}{20}$ from each term of the expression.*
  - *Or, for example, it can be used to distribute $\frac{1}{10}$: $\frac{3x-1}{10}=\frac{1}{10}(3x-1)=\frac{3x}{10}-\frac{1}{10}$.*

Below are two solutions. Explore both with the class.

$\frac{x}{20}+\frac{4(2x)}{20}+\frac{10(x+1)}{20}+\frac{2(3x-1)}{20}$	$\frac{1}{20}x+\frac{2}{5}x+\frac{1}{2}x+\frac{1}{2}+\frac{3}{10}x-\frac{1}{10}$
$\frac{x+8x+10x+10+6x-2}{20}$	$\left(\frac{1}{20}+\frac{2}{5}+\frac{1}{2}+\frac{3}{10}\right)x+\left(\frac{1}{2}-\frac{1}{10}\right)$
$\frac{25x+8}{20}$	$\left(\frac{1}{20}+\frac{8}{20}+\frac{10}{20}+\frac{6}{20}\right)x+\left(\frac{5}{10}-\frac{1}{10}\right)$
$\frac{5}{4}x+\frac{2}{5}$	$\frac{5}{4}x+\frac{2}{5}$

Ask students to evaluate the original expression and the answers when $x = 20$ to see if they get the same number.

Evaluate the original expression and the answers when $x = 20$. Do you get the same number?

$\frac{x}{20}+\frac{2x}{5}+\frac{x+1}{2}+\frac{3x-1}{10}$	$\frac{5}{4}x+\frac{2}{5}$
$\frac{20}{20}+\frac{2(20)}{5}+\frac{20+1}{2}+\frac{3(20)-1}{10}$	$\frac{5}{4}(20)+\frac{2}{5}$
$1+8+\frac{21}{2}+\frac{59}{10}$	$25+\frac{2}{5}$
$9+\frac{105}{10}+\frac{59}{10}$	$25\frac{2}{5}$
$9+\frac{164}{10}$	
$9+16\frac{4}{10}$	
$25\frac{2}{5}$	

Important: After students evaluate both expressions for $x = 20$, ask them which expression was easier. (The expression in standard form is easier.) Explain to students: When you are asked on a standardized test to *simplify an expression*, you must put the expression in standard form because standard form is often much simpler to evaluate and read. This curriculum is specific and will often tell you the form (such as standard form) it wants you to write the expression in for an answer.

Exercise 4 (3 minutes)

Allow students to work independently.

Exercise 4

Rewrite the following expression in standard form by finding common denominators and collecting like terms.

$$\frac{2h}{3}-\frac{h}{9}+\frac{h-4}{6}$$

$$\frac{6(2h)}{18}-\frac{2(h)}{18}+\frac{3(h-4)}{18}$$

$$\frac{12h-2h+3h-12}{18}$$

$$\frac{13h-12}{18}$$

$$\frac{13}{18}h-\frac{2}{3}$$

Example 5 (Optional, 5 minutes)

Give students a minute to observe the expression and decide how to begin rewriting it in standard form.

Example 5

Rewrite the following expression in standard form.

$$\frac{2(3x-4)}{6}-\frac{5x+2}{8}$$

- How can we start to rewrite this problem?
  - *There are various ways to start rewriting this expression, including using the distributive property, renaming $\frac{2}{6}$, rewriting the subtraction as an addition, distributing the negative in the second term, rewriting each term as a fraction (e.g., $\frac{2}{6}(3x-4)-\left(\frac{5x}{8}+\frac{2}{8}\right)$), or finding the lowest common denominator.*

Method 1:	Method 2a:	Method 2b:	Method 3:
$\frac{1(3x-4)}{3}-\frac{5x+2}{8}$	$\frac{6x-8}{6}-\frac{5x+2}{8}$		$\frac{1}{3}(3x-4)-\left(\frac{5x}{8}+\frac{1}{4}\right)$
$\frac{8(3x-4)}{24}-\frac{3(5x+2)}{24}$	$\frac{4(6x-8)}{24}-\frac{3(5x+2)}{8}$	$\frac{6}{6}x-\frac{8}{6}-\frac{5}{8}x-\frac{2}{8}$	$x-\frac{4}{3}-\frac{5}{8}x-\frac{1}{4}$
$\frac{((24x-32)-(15x+6))}{24}$	$\frac{(24x-32-15x-6)}{24}$	$x-\frac{4}{3}-\frac{5x}{8}-\frac{1}{4}$	$1x-\frac{5}{8}x-\frac{4}{3}-\frac{1}{4}$
$\frac{(24x-32-15x-6)}{24}$	$\frac{9x-38}{24}$	$1x-\frac{5}{8}x-\frac{4}{3}-\frac{1}{4}$	$\frac{3}{8}x-\frac{16}{12}-\frac{3}{12}$
$\frac{9x-38}{24}$	$\frac{9x}{24}-\frac{38}{24}$	$\frac{3}{8}x-\frac{16}{12}-\frac{3}{12}$	$\frac{3}{8}x-\frac{19}{12}$
$\frac{9x}{24}-\frac{38}{24}$	$\frac{3}{8}x-\frac{19}{12}$	$\frac{3}{8}x-\frac{19}{12}$	
$\frac{3}{8}x-\frac{19}{12}$			

- Which method(s) keep(s) the numbers in the expression in integer form? Why would this be important to note?
  - *Finding the lowest common denominator would keep the number in integer form; this is important because working with the terms would be more convenient.*
- Is one method better than the rest of the methods?
  - *No, it is by preference; however, the properties of addition and multiplication must be used properly.*
- Are these expressions equivalent: $\frac{3}{8}x$, $\frac{3x}{8}$, and $\frac{3}{8x}$? How do you know?
  - *The first two expressions are equivalent, but the third one, $\frac{3}{8x}$, is not. If you substitute a value other than zero or one (such as $x=2$), the values of the first expressions are the same, $\frac{2}{3}$. The value of the third expression is $\frac{3}{16}$.*
- What are some common errors that could occur when rewriting this expression in standard form?
  - *Some common errors may include distributing only to one term in the parentheses, forgetting to multiply the negative sign to all the terms in the parentheses, incorrectly reducing fractions, and/or adjusting the common denominator but not the numerator.*

Exercise 5 (Optional, 3 minutes)

Allow students to work independently. Have students share the various ways they started to rewrite the problem.

Exercise 5

Write the following expression in standard form.

$$\frac{2x-11}{4}-\frac{3(x-2)}{10}$$
$$\frac{5(2x-11)}{20}-\frac{2\cdot 3(x-2)}{20}$$
$$\frac{(10x-55)-6(x-2)}{20}$$
$$\frac{10x-55-6x+12}{20}$$
$$\frac{4x-43}{20}$$
$$\frac{1}{5}x-2\frac{3}{20}$$

Sample Anchor Chart for 7.EE.2 – Interpreting Linear Expressions

Mr. Golv is practicing his jiu-jitsu drill where he does 5 guard passes and 2 kimura arm locks. A guard pass takes G seconds, and a kimura arm lock takes K seconds.

Which expressions can we use to describe the number of seconds it takes Mr. Golv to complete his jiu-jitsu drill 7 times?

Choose 2 answers:

(A) $7(K+G)$

(B) $2K+5G$

(C) $7(5G-2K)$

(D) $7(5G+2K)$

(E) $14K+35G$

1 / 6 One way to find how long Mr. Golv spends on his drills is to multiply the number of drills from the $\text{number of seconds per drill}$.

Mr. Golv practiced his drill 7 times.

$7(\text{number of seconds per drill})$

2 / 6 During a drill, Mr. Golv does 5 guard passes, and each guard pass takes G seconds. So he spends $5G$ seconds on guard passes per drill.

During a drill, Mr. Golv does 2 kimura arm locks, and each arm lock takes K seconds. So he spends $2K$ seconds on kimura arm locks per drill.

$5G+2K$ is an expression for the $\text{number seconds per drill}$.

$7(5G+2K)$

One expression to find how many seconds it takes Mr. Golv to complete his jiu-jitsu drill 7 times is $7(5G+2K)$.

3 / 6 Let's see if there is another way to represent how many seconds Mr. Golv's drills take.

We could add the time he spends on arm locks to the time he spends on guard passes.

$\text{seconds on arm locks} + \text{seconds on guard passes}$

(continued on next page)

4/6 Since he takes $5G$ seconds to finish the guard passes per drill, he spends $7 \cdot 5G$ seconds, which equals $35G$ seconds, to finish the guard passes in all 7 drills.

Likewise, he takes $2K$ seconds to finish the kimura arm locks per drill, so he spends $7 \cdot 2K$ seconds, which equals $14K$ seconds, to finish the kimura arm locks in all 7 drills.

$$14K + 35G$$

We can also use the expression $14K + 35G$ to find the number of seconds it takes Mr. Golv to complete his jiu-jitsu drill 7 times.

5/6 These expressions are equivalent.

$$\begin{aligned} 7(5G + 2K) &= 7 \cdot 5G + 7 \cdot 2K \\ &= 35G + 14K \\ &= 14K + 35G \end{aligned}$$

6/6 **We can use the expression $7(5G + 2K)$ or $14K + 35G$ to describe the number of seconds it takes Mr. Golv to complete his jiu-jitsu drill 7 times.**

8.EE.2 Back to ccss standard

Expressions and Equations: Work with radicals and integer exponents.

Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.

Skills

1. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number

2. Evaluate square roots of small perfect squares

3. Evaluate cube roots of small perfect cubes

4. Explain why the square root of 2 is irrational.

Key Concepts/Vocabulary

Square root – a value that, when multiplied by itself, gives the number

Cube root – a value that, when used in a multiplication three times, gives that number

Symbol – a pattern or image used instead of words

X^2 **–** x times x, or x(x), or x multiplied by itself

X^3 **–** x times x times x, or x(x)(x), or x multiplied by itself and multiplied by itself again

Perfect square – a number made by squaring a whole number

Rational **Number –** a number that can be made by dividing 2 integers (an integer has no fractions)

Irrational **–** a real number that canNOT be made by dividing 2 integers (an integer has no fractions)

Standard-Specific Resources (8.EE.2)

- **EngageNY: Grade 8, Module 7, Topic A, Lesson 5 – Students find the positive solutions to equations algebraically equivalent to equations of the form x^2 =p and x^3=p.**

Example 1

Example 1

$$x^3 + 9x = \frac{1}{2}(18x + 54)$$

MP.1

- Now that we know about square roots and cube roots, we can combine that knowledge with our knowledge of the properties of equality to begin solving nonlinear equations like $x^3 + 9x = \frac{1}{2}(18x + 54)$. Transform the equation until you can determine the positive value of x that makes the equation true.

Challenge students to solve the equation independently or in pairs. Have students share their strategy for solving the equation. Ask them to explain each step.

- *Sample response:*

$$\begin{aligned} x^3 + 9x &= \frac{1}{2}(18x + 54) \\ x^3 + 9x &= 9x + 27 \\ x^3 + 9x - 9x &= 9x - 9x + 27 \\ x^3 &= 27 \\ \sqrt[3]{x^3} &= \sqrt[3]{27} \\ x &= \sqrt[3]{3^3} \\ x &= 3 \end{aligned}$$

Scaffolding:
Consider using a simpler version of the equation (line 2, for example):
$x^3 + 9x = 9x + 27$.

- Now, we verify our solution is correct.

$$\begin{aligned} 3^3 + 9(3) &= \frac{1}{2}(18(3) + 54) \\ 27 + 27 &= \frac{1}{2}(54 + 54) \\ 54 &= \frac{1}{2}(108) \\ 54 &= 54 \end{aligned}$$

- Since the left side is the same as the right side, our solution is correct.

Example 2

Example 2

$$x(x - 3) - 51 = -3x + 13$$

- Let's look at another nonlinear equation. Find the positive value of x that makes the equation true: $x(x - 3) - 51 = -3x + 13$.

Provide students with time to solve the equation independently or in pairs. Have students share their strategy for solving the equation. Ask them to explain each step.

- *Sample response:*

$$\begin{aligned} x(x - 3) - 51 &= -3x + 13 \\ x^2 - 3x - 51 &= -3x + 13 \\ x^2 - 3x + 3x - 51 &= -3x + 3x + 13 \\ x^2 - 51 &= 13 \\ x^2 - 51 + 51 &= 13 + 51 \\ x^2 &= 64 \\ \sqrt{x^2} &= \pm\sqrt{64} \\ x &= \pm\sqrt{64} \\ x &= \pm 8 \end{aligned}$$

- Now we verify our solution is correct.

Provide students time to check their work.

- Let $x = 8$.
$$8(8-3)-51=-3(8)+13$$
$$8(5)-51=-24+13$$
$$40-51=-11$$
$$-11=-11$$
Let $x=-8$.
$$-8(-8-3)-51=-3(-8)+13$$
$$-8(-11)-51=24+13$$
$$88-51=37$$
$$37=37$$

- Now it is clear that the left side is exactly the same as the right side, and our solution is correct.

Exercises (20 minutes)

Students complete Exercises 1–7 independently or in pairs. Although we are asking students to find the positive value of x that makes each equation true, we have included in the exercises both the positive and the negative values of x so that the teacher can choose whether to use them.

Exercises

Find the positive value of x that makes each equation true, and then verify your solution is correct.

1.

a. Solve $x^2 - 14 = 5x + 67 - 5x$.

$$x^2-14=5x+67-5x$$
$$x^2-14=67$$
$$x^2-14+14=67+14$$
$$x^2=81$$
$$\sqrt{x^2}=\pm\sqrt{81}$$
$$x=\pm\sqrt{81}$$
$$x=\pm 9$$

Check:

$$9^2-14=5(9)+67-5(9)$$
$$81-14=45+67-45$$
$$67=67$$

$$(-9)^2-14=5(-9)+67-5(-9)$$
$$81-14=-45+67+45$$
$$67=67$$

b. Explain how you solved the equation.

To solve the equation, I had to first use the properties of equality to transform the equation into the form of $x^2 = 81$. Then, I had to take the square root of both sides of the equation to determine that $x = 9$ since the number x is being squared.

2. Solve and simplify: $x(x-1) = 121 - x$.

$$x(x-1)=121-x$$
$$x^2-x=121-x$$
$$x^2-x+x=121-x+x$$
$$x^2=121$$
$$\sqrt{x^2}=\pm\sqrt{121}$$
$$x=\pm\sqrt{121}$$
$$x=\pm 11$$

Check:

$$11(11-1)=121-11$$
$$11(10)=110$$
$$110=110$$

$$-11(-11-1)=121-(-11)$$
$$-11(-12)=121+11$$
$$132=132$$

3. A square has a side length of $3x$ inches and an area of $324\ \text{in}^2$. What is the value of x?

$$(3x)^2=324$$
$$3^2x^2=324$$
$$9x^2=324$$
$$\frac{9x^2}{9}=\frac{324}{9}$$
$$x^2=36$$
$$\sqrt{x^2}=\sqrt{36}$$
$$x=6$$

Check:

$$(3(6))^2=324$$
$$18^2=324$$
$$324=324$$

A negative number would not make sense as a length, so $x = 6$.

4. $-3x^3 + 14 = -67$

$$-3x^3 + 14 = -67$$
$$-3x^3 + 14 - 14 = -67 - 14$$
$$-3x^3 = -81$$
$$\frac{-3x^3}{-3} = \frac{-81}{-3}$$
$$x^3 = 27$$
$$\sqrt[3]{x^3} = \sqrt[3]{27}$$
$$x = 3$$

Check:

$$-3(3)^3 + 14 = -67$$
$$-3(27) + 14 = -67$$
$$-81 + 14 = -67$$
$$-67 = -67$$

5. $x(x+4) - 3 = 4(x + 19.5)$

$$x(x+4) - 3 = 4(x + 19.5)$$
$$x^2 + 4x - 3 = 4x + 78$$
$$x^2 + 4x - 4x - 3 = 4x - 4x + 78$$
$$x^2 - 3 = 78$$
$$x^2 - 3 + 3 = 78 + 3$$
$$x^2 = 81$$
$$\sqrt{x^2} = \pm\sqrt{81}$$
$$x = \pm 9$$

Check:

$$9(9+4) - 3 = 4(9 + 19.5)$$
$$9(13) - 3 = 4(28.5)$$
$$117 - 3 = 114$$
$$114 = 114$$

$$-9(-9+4) - 3 = 4(-9 + 19.5)$$
$$-9(-5) - 3 = 4(10.5)$$
$$45 - 3 = 42$$
$$42 = 42$$

6. $216 + x = x(x^2 - 5) + 6x$

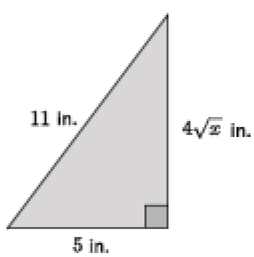
$$216 + x = x(x^2 - 5) + 6x$$
$$216 + x = x^3 - 5x + 6x$$
$$216 + x = x^3 + x$$
$$216 + x - x = x^3 + x - x$$
$$216 = x^3$$
$$\sqrt[3]{216} = \sqrt[3]{x^3}$$
$$6 = x$$

Check:

$$216 + 6 = 6(6^2 - 5) + 6(6)$$
$$222 = 6(31) + 36$$
$$222 = 186 + 36$$
$$222 = 222$$

7.

a. What are we trying to determine in the diagram below?



We need to determine the value of x so that its square root, multiplied by 4, satisfies the equation

$$5^2 + \left(4\sqrt{x}\right)^2 = 11^2.$$

b. Determine the value of x, and check your answer.

$$5^2 + \left(4\sqrt{x}\right)^2 = 11^2$$
$$25 + 4^2\left(\sqrt{x}\right)^2 = 121$$
$$25 - 25 + 4^2\left(\sqrt{x}\right)^2 = 121 - 25$$
$$16x = 96$$
$$\frac{16x}{16} = \frac{96}{16}$$
$$x = 6$$

Check:

$$5^2 + \left(4\sqrt{6}\right)^2 = 11^2$$
$$25 + 16(6) = 121$$
$$25 + 96 = 121$$
$$121 = 121$$

The value of x is 6.

Sample Anchor Chart for 8.EE.2 – Equations with Square Roots and Cube Roots

Which of the following equations has *both* -2 and 2 as possible values of b?

Choose all answers that apply:

(A) $b^2 = 4$

(B) $b^3 = 8$

(C) None of the above

1 / 4 Let's solve the first equation and see if *both* -2 and 2 are possible values of b.

2 / 4

$$b^2 = 4$$

$$\sqrt{b^2} = \sqrt{4}$$

$$b = \pm 2$$

Yes, both -2 and 2 are possible values of b for the first equation!

3 / 4 Let's do the same for the second equation.

$$b^3 = 8$$

$$\sqrt[3]{b^3} = \sqrt[3]{8}$$

$$b = 2$$

No, both -2 and 2 are not possible values of b for the second equation.

4 / 4 The following equation has *both* -2 and 2 as possible values of b:

- $b^2 = 4$

8.EE.6 Back to ccss standard

Understand the connections between proportional relationships, lines, and linear equations.

Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation *y* = *mx* + *b* for a line intercepting the vertical axis at *b*.

Skills

1. Identify characteristics of similar triangles

2. Find the slope of a line and determine its y-intercept

3. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane

4. Analyze patterns for points on a line through the origin and for points on a line that do not pass through or include the origin

5. Derive an equation of the form y = mx for a line through the origin.

6. Derive an equation of the form y=mx + b for a line intercepting the vertical axis at b (the y-intercept)

Key Concepts/Vocabulary

Similar Triangle – two triangles are similar if the only difference between them is size (and possibly the need to turn or flip one around)

Slope – how steep a straight line is; also called a ‘gradient’

Y-Intercept – where a line or curve crosses the y-axis of a graph; the y value when x = 0

Coordinate Plane – a two-dimensional surface created by two intersecting and perpendicular number lines, on which points are plotted and located by their x and y coordinates

Standard-Specific Resources (8.EE.6)

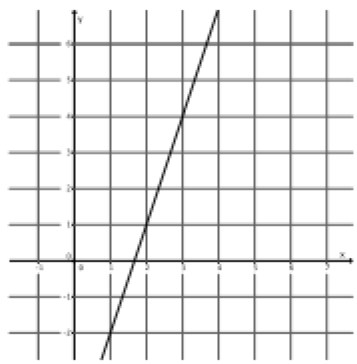
- **EngageNY:** **Grade 8, Module 4, Topic C, Lesson 16 – Students use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane.**

Example 1 (1 minute)

This example requires students to find the slope of a line where the horizontal distance between two points with integer coordinates is fixed at 1.

Example 1

Using what you learned in the last lesson, determine the slope of the line with the following graph.



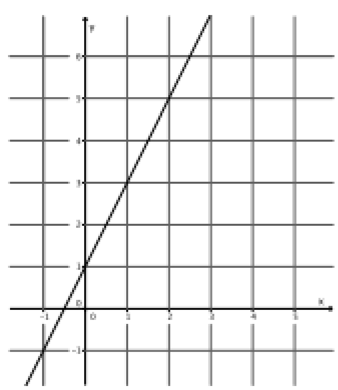
The slope of the line is 3.

Example 2 (1 minute)

This example requires students to find the slope of a line where the horizontal distance between two points with integer coordinates is fixed at 1.

Example 2

Using what you learned in the last lesson, determine the slope of the line with the following graph.



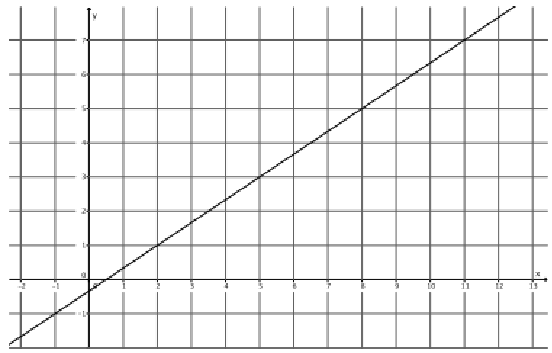
The slope of this line is 2.

Example 3 (3 minutes)

This example requires students to find the slope of a line where the horizontal distance can be fixed at one, but determining the slope is difficult because it is not an integer. The point of this example is to make it clear to students that they need to develop a strategy that allows them to determine the slope of a line no matter what the horizontal distance is between the two points that are selected.

Example 3

What is different about this line compared to the last two examples?



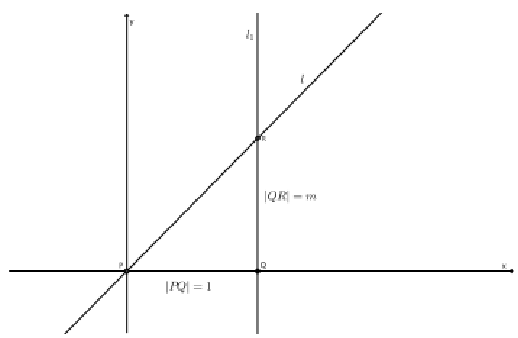
This time, if we choose two points on the line that have a horizontal distance at 1, we cannot precisely determine the slope of the line because the vertical change is not an integer. It is some fractional amount.

- Make a conjecture about how you could find the slope of this line.

Have students write their conjectures and share their ideas about how to find the slope of the line in this example; then, continue with the Discussion that follows.

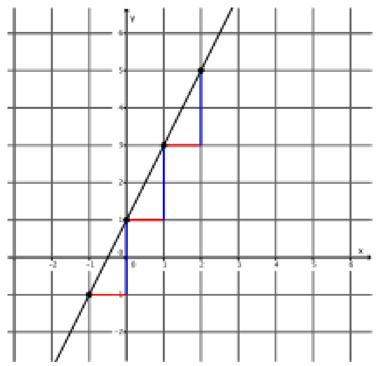
Discussion (10 minutes)

- In the last lesson, we found a number that described the slope or rate of change of a line. In each instance, we were looking at a special case of slope because the horizontal distance between the two points used to determine the slope, P and Q, was always 1. Since the horizontal distance was 1, the difference between the y-coordinates of points Q and R was equal to the slope or rate of change. For example, in the following graph, we thought of point Q as zero on a vertical number line and noted how many units point R was from point Q.

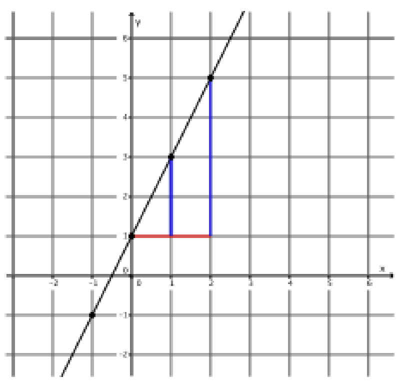


- Also in the last lesson, we found that the unit rate of a problem was equal to the slope. Using that knowledge, we can say that the slope or rate of change of a line $m = \frac{|QR|}{|PQ|}$.
- Now the task is to determine the rate of change of a non-vertical line when the distance between points P and Q is a number other than 1. We can use what we know already to guide our thinking.

- Let's take a closer look at Example 2. There are several points on the line with integer coordinates that we could use to help us determine the slope of the line. Recall that we want to select points with integer coordinates because our calculation of slope will be simpler. In each instance, from one point to the next, we have a horizontal distance of 1 unit noted by the red segment and the difference in the y-values between the two points, which is a distance of 2, noted by the blue segments. When we compare the change in the y-values to the change in the x-values, or more explicitly, when we compare the height of the slope triangle to the base of the slope triangle, we have a ratio of $2:1$ with a value of $\frac{2}{1}$ or just 2, which is equal to the slope of the line.



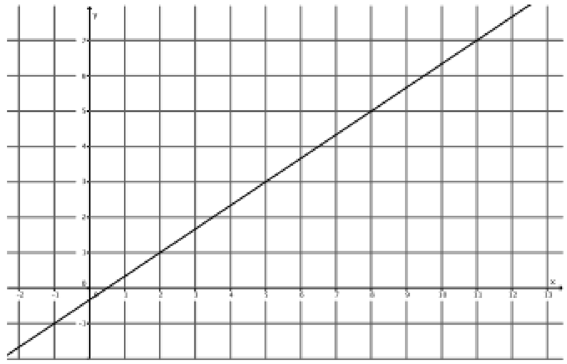
Each of the "slope triangles" shown have values of their ratios equal to $\frac{2}{1}$. Using the same line, let's look at a different pair of "slope triangles."



- What is the ratio of the larger slope triangle?
  - *The value of the ratio of the larger slope triangle is* $\frac{4}{2}$.
- What do you notice about the ratio of the smaller slope triangle and the ratio of the larger slope triangle?
  - *The values of the ratios are equivalent:* $\frac{2}{1} = \frac{4}{2} = 2$.
- We have worked with triangles before where the ratios of corresponding sides were equal. We called them *similar* triangles. Are the slope triangles in this diagram similar? How do you know?
  - *Yes. The triangles are similar by the AA criterion. Each triangle has a right angle (at the intersection of the blue and red segments), and both triangles have a common angle (the angle formed by the red segment and the line itself).*
- When we have similar triangles, we know that the ratios of corresponding side lengths must be equal. That is the reason that both of the slope triangles result in the same number for slope. Notice that we still got the correct number for the slope of the line even though the points chosen did not have a horizontal distance of 1. We can now find the slope of a line given any two points on the line. The horizontal distance between the points does not have to be 1.

Acknowledge any students who may have written or shared this strategy for finding slope from their work with Example 3.

- Now let's look again at Example 3. We did not have a strategy for finding slope before, but we do now. What do you think the slope of this line is? Explain.



  - *The slope of this line is* $\frac{2}{3}$.

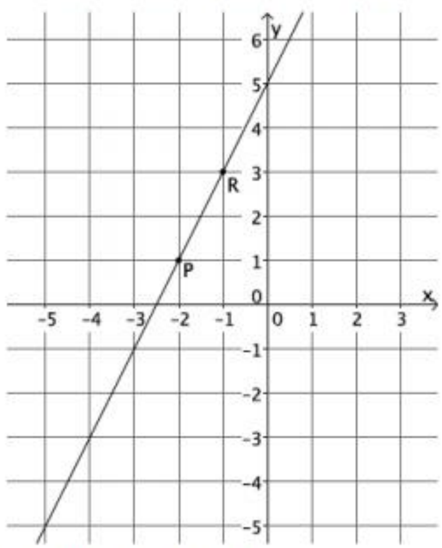
Ask students to share their work and explanations with the class. Specifically, have them show the slope triangle they used to determine the slope of the line. Select several students to share their work; ideally, students will pick different points and different slope triangles. Whether they do or not, have a discussion similar to the previous one that demonstrates that all slope triangles that could be drawn are similar and that the ratios of corresponding sides are equal.

Exercise (4 minutes)

Students complete the Exercise independently.

Exercise

Let's investigate concretely to see if the claim that we can find slope between any two points is true.



a. **Select any two points on the line to label as P and R.**

   Sample points are selected on the graph.

b. **Identify the coordinates of points P and R.**

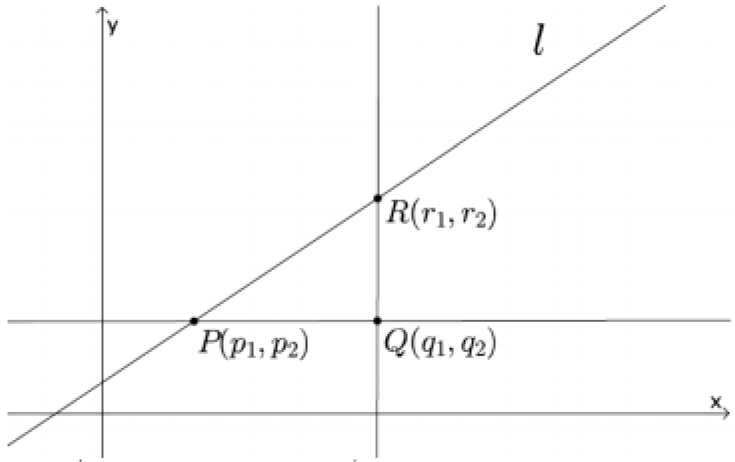
   Sample points are labeled on the graph.

c. **Find the slope of the line using as many different points as you can. Identify your points, and show your work below.**

   Points selected by students will vary, but the slope should always equal 2. *Students could choose to use points* $(0,5)$, $(-1,3)$, $(-2,1)$, $(-3,-1)$, $(-4,-3)$, *and* $(-5,-5)$.

Discussion (10 minutes)

- We want to show that the slope of a non-vertical line l can be found using any two points P and R on the line.
- Suppose we have point $P(p_1, p_2)$, where p_1 is the x-coordinate of point P, and p_2 is the y-coordinate of point P. Also, suppose we have points $Q(q_1, q_2)$ and $R(r_1, r_2)$.



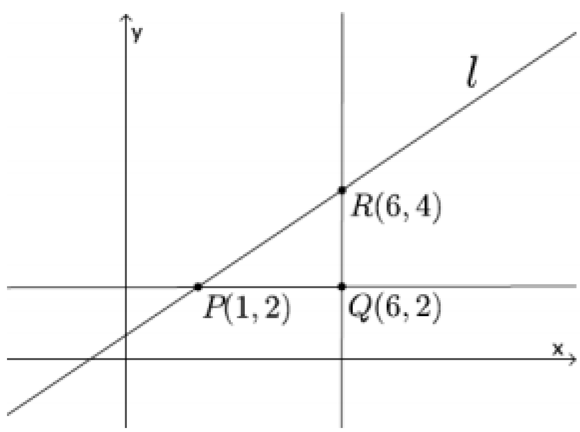
- Then, we claim that the slope m of line l is

$$m = \frac{|QR|}{|PQ|}.$$

- From the last lesson, we found the length of segment QR by looking at the y-coordinate. Without having to translate, we can find the length of segment QR by finding the difference between the y-coordinates of points R and Q (vertical distance). Similarly, we can find the length of the segment PQ by finding the difference between the x-coordinates of P and Q (horizontal distance). We claim

$$m = \frac{|QR|}{|PQ|} = \frac{(q_2 - r_2)}{(p_1 - q_1)}.$$

- We would like to remove any reference to the coordinates of Q, as it is not a point on the line. We can do this by looking more closely at the coordinates of point Q. Consider the following concrete example.



- What do you notice about the y-coordinates of points P and Q?
  - *The y-coordinates of points P and Q are the same:* 2.
- That means that $q_2 = p_2$. What do you notice about the x-coordinates of points R and Q?
  - *The x-coordinates of points R and Q are the same:* 6.
- That means that $q_1 = r_1$. Then, by substitution:

$$m = \frac{|QR|}{|PQ|} = \frac{(q_2 - r_2)}{(p_1 - q_1)} = \frac{(p_2 - r_2)}{(p_1 - r_1)}.$$

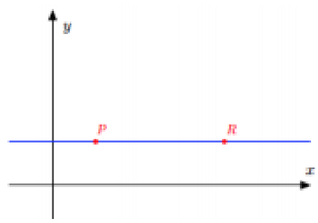
- Then, we claim that the slope can be calculated regardless of the choice of points. Also, we have discovered something called "the slope formula." With the formula for slope, or rate of change, $m = \frac{(p_2 - r_2)}{(p_1 - r_1)}$, the slope of a line can be found using any two points P and R on the line!

Ask students to translate the slope formula into words, and provide them with the traditional ways of describing slope. For example, students may say the slope of a line is the "height of the slope triangle over the base of the slope triangle" or "the difference in the y-coordinates over the difference in the x-coordinates." Tell students that slope can be referred to as "rise over run" as well.

Discussion (3 minutes)

Show that the formula to calculate slope is true for horizontal lines.

- Suppose we are given a horizontal line. Based on our work in the last lesson, what do we expect the slope of this line to be?



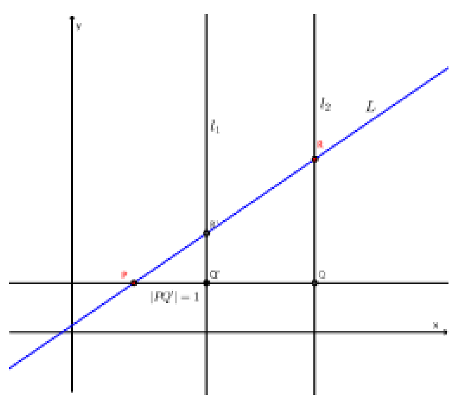
  - *The slope should be zero because if we go one unit to the right of P and then identify the vertical difference between that point and point R, there is no difference. Therefore, the slope is zero.*
- As before, the coordinates of points P and R are represented as $P(p_1, p_2)$ and $R(r_1, r_2)$. Since this is a horizontal line, what do we know about the y-coordinates of each point?
  - *Horizontal lines are graphs of linear equations in the form of $y = c$, where the y-value does not change. Therefore, $p_2 = r_2$.*
- By the slope formula:

$$m = \frac{(p_2 - r_2)}{(p_1 - r_1)} = \frac{0}{p_1 - r_1} = 0.$$

The slope of the horizontal line is zero, as expected, regardless of the value of the horizontal change.

Discussion (7 minutes)

- Now for the general case. We want to show that we can choose any two points P and R to find the slope, not just a point like R', where we have fixed the horizontal distance at 1. Consider the diagram below.



- Now we have a situation where point Q is an unknown distance from point P. We know that if $\triangle\ PQ'R'$ is similar to $\triangle\ PQR$, then the ratio of the corresponding sides will be equal, and the ratios are equal to the slope of the line L. Are $\triangle\ PQ'R'$ and $\triangle\ PQR$ similar? Explain.
  - *Yes, the triangles are similar, i.e., $\triangle\ PQ'R' \sim \triangle\ PQR$. Both triangles have a common angle, $\angle RPQ$, and both triangles have a right angle, $\angle R'Q'P$ and $\angle RQP$. By the AA criterion, $\triangle\ PQ'R' \sim \triangle\ PQR$.*

- Now what we want to do is find a way to express this information in a formula. Because we have similar triangles, we know the following:

$$\frac{|R'Q'|}{|RQ|} = \frac{|PQ'|}{|PQ|} = \frac{|PR'|}{|PR|} = r.$$

- Based on our previous knowledge, we know that $|R'Q'| = m$, and $|PQ'| = 1$. By substitution, we have

$$\frac{m}{|RQ|} = \frac{1}{|PQ|},$$

  which is equivalent to

$$\frac{m}{1} = \frac{|RQ|}{|PQ|}$$
$$m = \frac{|RQ|}{|PQ|}.$$

- We also know from our work earlier that $|RQ| = p_2 - r_2$, and $|PQ| = p_1 - r_1$. By substitution, we have

$$m = \frac{p_2 - r_2}{p_1 - r_1}.$$

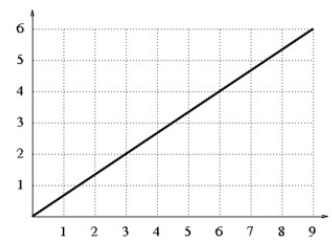
  The slope of a line can be computed using any two points!

Anchor Charts and Conceptual Development for 8.EE.6 – Similar Triangles and Slope of Points on a Line

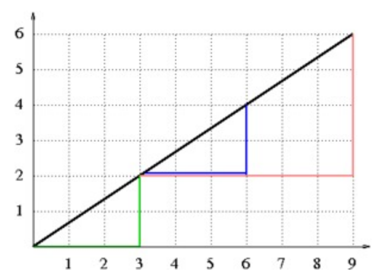
The slope between two points is calculated by finding the change in y-values and dividing by the change in x-values. For example, the slope between the points (7, -15) and (-8, 22) can be computed as follows:

- The difference in the y-values is $-15 - 22 = -37$.
- The difference in the x-values is $7 - (-8) = 15$.
- Dividing these two differences, we find that the slope is $-\frac{37}{15}$.

Eva, Carl, and Maria are computing the slope between pairs of points on the line shown below.



Eva finds the slope between the points (0,0) and (3,2). Carl finds the slope between the points (3,2) and (6,4). Maria finds the slope between the points (3,2) and (9,6). They have each drawn a triangle to help with their calculations (shown below).

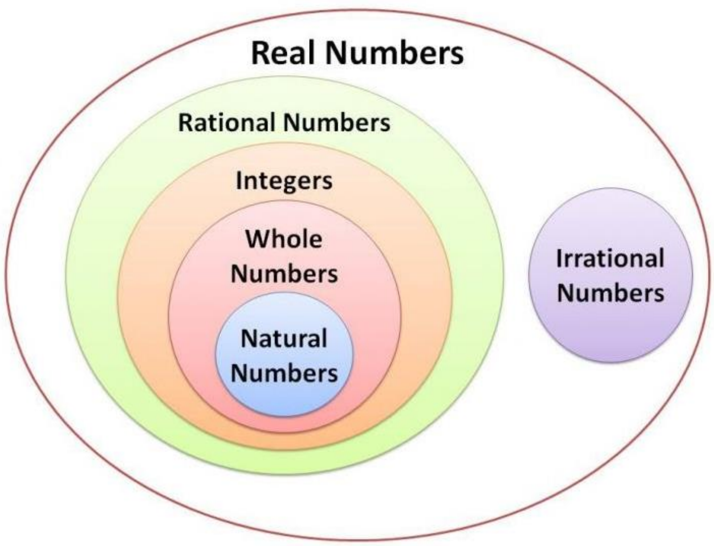


i. Which student has drawn which triangle? Finish the slope calculation for each student. How can the differences in the x- and y-values be interpreted geometrically in the pictures they have drawn?

ii. Consider any two points (x_1, y_1) and (x_2, y_2) on the line shown above. Draw a triangle like the triangles drawn by Eva, Carl, and Maria. What is the slope between these two points? Why should this slope be the same as the slopes calculated by the three students?

The Number System

(NS)



6.NS.5 Back to ccss standard

Apply and extend previous understandings of numbers to the system of rational numbers.

Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning the meaning of 0 in each situation.

Skills

1. Solve problems using positive and negative numbers involving temperature, including explaining the meaning of 0 in this context

2. Solve problems using positive and negative numbers involving elevation, including explaining the meaning of 0 in this context

3. Solve problems using positive and negative numbers involving credits/debits, including explaining the meaning of 0 in this context

4. Solve problems using positive and negative numbers involving positive and negative electrical charge, including explaining the meaning of 0 in this context

Key Concepts/Vocabulary

Positive number – A number greater than zero

Negative number – A number less than zero

Rational number – A number that can be made by dividing two integers (an integer is a number with no fractional part)

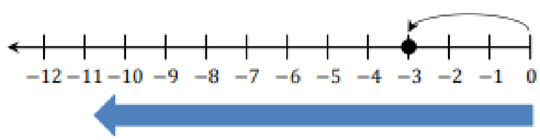
Standard-Specific Resources (6.NS.5)

- **EngageNY: Grade 6, Module 3, Topic A, Lesson 1 – Students extend their understanding of the number line, which includes zero and numbers to the right, that are above zero, and numbers to the left, that are below zero. Students use positive integers to locate negative integers, moving in the opposite direction from zero. Students understand that the set of integers includes the set of positive whole numbers and their opposites, as well as zero. They also understand that zero is its own opposite.**

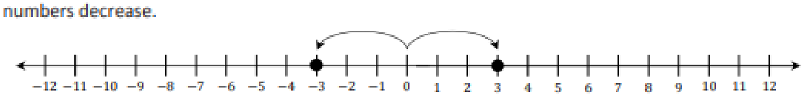
Example 1 (5 minutes): Negative Numbers on the Number Line

Students use their constructions to model the location of a number relative to zero by using a curved arrow starting at zero and pointing away from zero toward the number. Pose questions to students as a whole group, one question at a time.

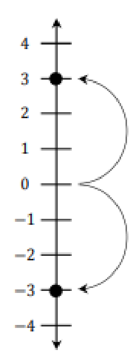
- Starting at 0, as I move to the right on a horizontal number line, the values get larger. These numbers are called *positive numbers* because they are greater than zero. Notice the curved arrow is pointing to the right to show a positive direction.
- How far is the number from zero on the number line?
  - *3 units*
- If 0 was a mirror facing toward the arrow, what would be the direction of the arrow in the mirror?
  - *To the left*
- Would the numbers get larger or smaller as we move to the left of zero?
  - *Smaller*
- Starting at 0, as I move farther to the left of zero on a horizontal number line, the values get smaller. These numbers are called *negative numbers* because they are less than zero. Notice the curved arrow is pointing to the left to show a negative direction. The position of the point is now at negative 3, written as -3.



- Negative numbers are less than zero. As you move to the left on a horizontal number line, the values of the numbers decrease.



- What is the relationship between 3 and -3 on the number line?
  - *3 and -3 are located on opposite sides of zero. They are both the same distance from zero. 3 and -3 are called opposites.*
- As we look farther right on the number line, the values of the numbers increase. For example, $-1 < 0 < 1 < 2 < 3$.
- This is also true for a vertical number line. On a vertical number line, positive numbers are located above zero. As we look upward on a vertical number line, the values of the numbers increase. On a vertical number line, negative numbers are located below zero. As we look farther down on a vertical number line, the values of the numbers decrease.
- The set of whole numbers and their opposites, including zero, are called *integers*. Zero is its own opposite. A number line diagram shows integers listed in increasing order from left to right, or from bottom to top, using equal spaces. For example: $-4, -3, -2, -1, 0, 1, 2, 3, 4$.



Allow students to discuss the example in their groups to solidify their understanding of positive and negative numbers.

Possible discussion questions:

- Where are negative numbers located on a horizontal number line?
  - *Negative numbers are located to the left of 0 on a horizontal number line.*

- Where are negative numbers located on a vertical number line?
  - *Negative numbers are located below 0 on a vertical number line.*
- What is the opposite of 2?
  - -2
- What is the opposite of 0?
  - 0
- Describe the relationship between 10 and -10.
  - *10 and -10 are opposites because they are on opposite sides of 0 and are both 10 units from 0.*

Example 2 (5 minutes): Using Positive Integers to Locate Negative Integers on the Number Line

Have students establish elbow partners, and tell them to move their fingers along their number lines to answer the following set of questions. Students can discuss answers with their elbow partners. Circulate around the room, and listen to the student–partner discussions.

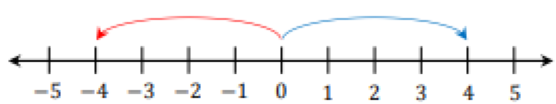
- Describe to your elbow partner how to find 4 on a number line. Describe how to find -4.
  - *To find 4, start at zero, and move right to 4. To find -4, start at zero, and move left to -4.*

Scaffolding:

As an extension activity, have students identify the *unit* differently on different number lines, and ask students to locate two whole numbers other than 1 and their opposites.

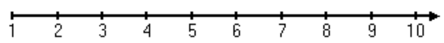
Model how the location of a positive integer can be used to locate a negative integer by moving in the opposite direction.

- Explain and show how to find 4 and the opposite of 4 on a number line.
  - *Start at zero, and move 4 units to the right to locate 4 on the number line. To locate -4, start at zero, and move 4 units to the left on the number line.*
- Where do you start when locating an integer on the number line?
  - *Always start at zero.*
- What do you notice about the curved arrows that represent the location of 4 and -4?
  - *They are the same distance but pointing in opposite directions.*

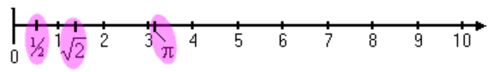


Anchor Charts and Conceptual Development for 6.NS.5 - Positive and Negative Numbers

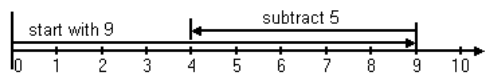
When you first learned your numbers, way back in elementary school, you started with the counting numbers: 1, 2, 3, 4, 5, 6, and so on. Your number line looked something like this:



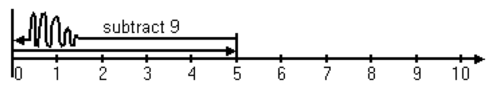
Later on, you learned about zero, fractions, decimals, square roots, and other types of numbers, so your number line started looking something like this:



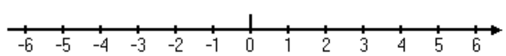
Addition, multiplication, and division always made sense – as long as you didn't try to divide by zero – but sometimes subtraction didn't work. If you had "9 – 5", you got 4:



...but what if you had "5 – 9"? You just couldn't do this subtraction, because there wasn't enough "space" on the number line to go back nine units:



You can solve this "space" problem by using negative numbers. The "whole" numbers start at zero and count off to the right; these are the positive integers. The negative integers start at zero and count off to the left:



Note the arrowhead on the far right end of the number line above. That arrow tells you the direction in which the numbers are getting bigger. In particular, that arrow also tells you that the negatives are getting *smaller* as they move off to the left. That is, –5 is *smaller* than –4.

How do we use this expanded number line? Well, for starters, we can now do the subtraction "5 – 9". From zero, we go five units to the right, and then we subtract nine units to the left:

We end up four units to the left of zero, so we now know that 5 – 9 = –4.

Negative numbers might seem a bit weird at first, but that's okay; negatives take some getting used to. Let's look at a few inequalities, to practice your understanding. Refer to the number line above, as necessary.

- **Complete the following inequality: 3 _____ 6**

  Look at the number line above. Since 6 is to the right of 3, then 6 is larger, so the correct inequality is:

  3 <u><</u> 6

- **Complete the following inequality: –3 ______ 6**

  Look at (or think of) the number line again. Every positive number is to the right of every negative number, so the correct inequality is:

  –3 < 6

- **Complete the following inequality: –3 ______ –6**

  Think of the number line again. Since –6 is to the left of –3, then –3, being further to the right, is actually the larger number. So the correct inequality is:

  –3 > –6

- **Complete the following inequality: 0 ______ 1**

  Zero is less than any positive number, so:

  0 < 1

- **Complete the following inequality: 0 ______ –1**

  Zero is greater than any negative number, so:

  0 > –1

6.NS.6C

Back to ccss standard

Apply and extend previous understandings of numbers to the system of rational numbers.

Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

Skills

1. Identify the opposite value of rational numbers using a number line

2. Identify the opposite of the opposite value of rational numbers using a number line

3. Identify the absolute value of rational numbers

4. Interpret absolute value as magnitude for a positive or negative quantity in a real-world situation

Key Concepts/Vocabulary

Absolute value – How far a number is away from zero, regardless of whether the number is positive or negative

Rational number – A number that can be made by dividing two integers (an integer is a number with no fractional part)

Magnitude – Size

Positive quantity – A quantity greater than zero

Negative quantity – A quantity less than zero

Standard-Specific Resources (6.NS.6c)

- **EngageNY: Grade 6, Module 3, Topic A, Lesson 6 – Students use number lines that extend in both directions and use 0 and 1 to locate integers and rational numbers on the number line. Students know that the sign of a nonzero rational number is positive or negative, depending on whether the number is greater than zero (positive) or less than zero (negative) and use an appropriate scale when graphing rational numbers on the number line.**

Classwork

Opening Exercise (5 minutes)

Students work independently for 5 minutes to review fractions and decimals.

Opening Exercise

a. Write the decimal equivalent of each fraction.

i. $\frac{1}{2}$

0.5

ii. $\frac{4}{5}$

0.8

iii. $6\frac{7}{10}$

6.70

b. Write the fraction equivalent of each decimal.

i. 0.42

$\frac{42}{100} = \frac{21}{50}$

ii. 3.75

$3\frac{75}{100} = 3\frac{3}{4}$

iii. 36.90

$36\frac{90}{100} = 36\frac{9}{10}$

Scaffolding:

- Use polling software to elicit immediate feedback from the class to engage all learners.
- Display each problem one at a time, and use personal white boards for kinesthetic learners.

Scaffolding:

- Use edges of square tiles on the floor as a number line to illustrate how to connect segments of equal length for visual and kinesthetic learners.
- Provide green and red pencils to help with modeling the example for visual learners.

Example 1 (10 minutes): Graphing Rational Numbers

The purpose of this example is to show students how to graph non-integer rational numbers on a real number line. Students complete the example by following along with the teacher.

- Locate and graph the number $\frac{3}{10}$ and its opposite on a number line.

Before modeling the example, review graphing a fraction on the number line to the whole class by first reviewing fraction definitions with respect to the number line.

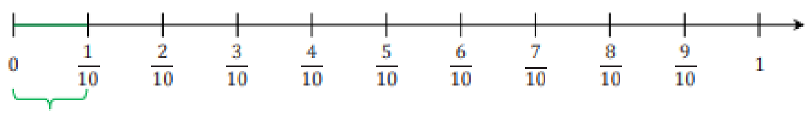
> **Example 1: Graphing Rational Numbers**
>
> **If b is a nonzero whole number, then the unit fraction $\frac{1}{b}$ is located on the number line by dividing the segment between 0 and 1 into b segments of equal length. One of the b segments has 0 as its left end point; the right end point of this segment corresponds to the unit fraction $\frac{1}{b}$.**

- Since the number is a **rational number**, a number that can be represented as a fraction, determine how the number line should be scaled.[1]
  - First, divide the number line into two halves to represent positive and negative numbers.

  Have students complete this task on their student pages.

  - Next, divide the right half of the number line segment between 0 and 1 into ten segments of equal length; each segment has a length of $\frac{1}{10}$.

  Students divide their number lines into ten equal segments as shown. Check for accuracy.



  - There are 10 equal segments. Each segment has a length of $\frac{1}{10}$. The first segment has 0 as its left end point, and the right end point corresponds to $\frac{1}{10}$.

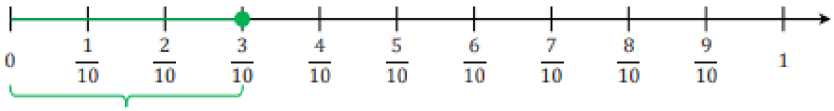
---

[1]Supplemental Exercise:

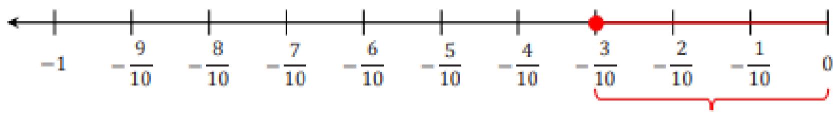
- Have four students each stand in a square floor tile forming a straight line facing the class. Give each student a number to tie around his neck: $0, \frac{1}{10}, \frac{2}{10}$, or $\frac{3}{10}$. (Use index cards or construction paper.)
- Ask a fifth student to assist by giving one end of a ball of string to the person at 0. This person holds one end of the string and passes the rest to the person to the left. (So the class sees it moving to the right.)
- As the string gets passed down the line, each person announces her number, "$\frac{1}{10}, \frac{2}{10}, \frac{3}{10}$," stopping at $\frac{3}{10}$.
- The assistant cuts the string at $\frac{3}{10}$ and gives that end of the string to the person holding $\frac{3}{10}$, making one segment of length $\frac{3}{10}$.
- Have students turn over their numbers to reveal their opposites and rearrange themselves to represent the opposite of $\frac{3}{10}$ using the same process. This time, students pass the string to the right. (So the class sees it moving to the left.)

The fraction $\frac{a}{b}$ is located on the number line by joining a segments of length $\frac{1}{b}$ so that (1) the left end point of the first segment is 0, and (2) the right end point of each segment is the left end point of the next segment. The right end point of the last segment corresponds to the fraction $\frac{a}{b}$.

- To locate the number $\frac{a}{b}$ on a number line, students should divide the interval between zero and 1 into b equal parts. Starting at 0, move along the number line a number of times.

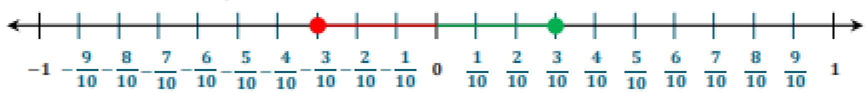


- There are ten equal segments. Each segment has a length of $\frac{1}{10}$. The first segment has a 0 as its left end point, and the right end point of the third segment corresponds to $\frac{3}{10}$. The point is located at $\frac{3}{10}$.
- The opposite of $\frac{3}{10}$ is located the same distance from zero as $\frac{3}{10}$ but in the opposite direction or to the left. Using your knowledge of opposites, what rational number represents the opposite of $\frac{3}{10}$?
  - $-\frac{3}{10}$
- To locate the opposite of $\frac{3}{10}$ on the number line, divide the interval between zero and -1 into ten equal segments. Starting at zero, how far would we move to locate the opposite of $\frac{3}{10}$, and in what direction?
  - *We would move 3 units to the left of zero because that is the same distance but opposite direction we moved to plot the point $\frac{3}{10}$.*



- There are ten equal segments. Each segment has a length of $\frac{1}{10}$. Three consecutive segments, starting at 0 and moving to the left, would have a total length of $\frac{3}{10}$. The point is located at $-\frac{3}{10}$.
- Counting three consecutive segments of length of $\frac{1}{10}$ from 0 moving to the left and taking the end point of the last segment corresponds to the number $-\frac{3}{10}$. Therefore, the opposite of $\frac{3}{10}$ is $-\frac{3}{10}$.

Locate and graph the number $\frac{3}{10}$ and its opposite on a number line.

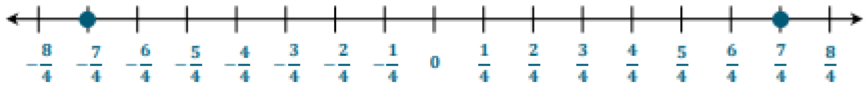


Exercise 1 (5 minutes)

Students work independently to practice graphing a non-integer rational number and its opposite on the number line. Allow 2–3 minutes for review as a whole group.

Exercise 1

Use what you know about the point $-\frac{7}{4}$ and its opposite to graph both points on the number line below. The fraction $-\frac{7}{4}$ is located between which two consecutive integers? Explain your reasoning.



On the number line, each segment will have an equal length of $\underline{\frac{1}{4}}$. **The fraction is located between** $\underline{-1}$ **and** $\underline{-2}$.

Explanation:

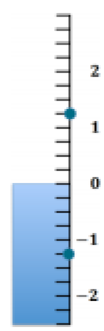
$\frac{7}{4}$ is the opposite of $-\frac{7}{4}$. It is the same distance from zero but on the opposite side of zero. Since $-\frac{7}{4}$ is to the left of zero, $\frac{7}{4}$ is to the right of zero. The original fraction is located between -2 (or $-\frac{8}{4}$) and -1 (or $-\frac{4}{4}$).

Example 2 (7 minutes): Rational Numbers and the Real World

Display the following vertical number line model on the board. Students are to follow along in their student materials to answer the questions. Pose additional questions to the class throughout the example.

Example 2: Rational Numbers and the Real World

The water level of a lake rose 1.25 feet after it rained. Answer the following questions using the number line below.



a. **Write a rational number to represent the situation.**

   1.25 *or* $1\frac{1}{4}$

b. **What two integers is 1.25 between on a number line?**

   1 *and* 2

c. **Write the length of each segment on the number line as a decimal and a fraction.**

   0.25 *and* $\frac{1}{4}$

d. **What will be the water level after it rained? Graph the point on the number line.**

   1.25 feet above the original lake level

e. **After two weeks have passed, the water level of the lake is now the opposite of the water level when it rained. What will be the new water level? Graph the point on the number line. Explain how you determined your answer.**

The water level would be 1.25 feet below the original lake level. If the water level was 1.25, the opposite of 1.25 is -1.25.

f. **State a rational number that is not an integer whose value is less than 1.25, and describe its location between two consecutive integers on the number line.**

Answers will vary. A rational number whose value is less than 1.25 is 0.75. It would be located between 0 and 1 on a number line.

Possible discussion questions:

- What units are we using to measure the water level?
  - *Feet*
- What was the water level after the rain? How do you know?
  - *If zero represents the original water level on the number line, the water level after rain is 1.25 feet. From 0 to 1, there are four equal segments. This tells me that the scale is $\frac{1}{4}$. The top of the water is represented on the number line at one mark above 1, which represents $\frac{5}{4}$ feet or 1.25 feet.*
- What strategy could we use to determine the location of the water level on the number line after it rained?
  - *I started at 0 and counted by $\frac{1}{4}$ for each move. I counted $\frac{1}{4}$ five times to get $\frac{5}{4}$, which is equivalent to $1\frac{1}{4}$ and 1.25. I know the number is positive because I moved up. Since the measurements are in feet, the answer is 1.25 feet.*
- For the fraction $\frac{5}{4}$, what is the value of the numerator and denominator?
  - *The numerator is 5, and the denominator is 4.*
- What do the negative numbers represent on the number line?
  - *They represent the number of feet below the original lake level.*

Exercise 2 (10 minutes)

Students are seated in groups of three or four. Distribute one sheet of grid paper and a ruler to each student. Each group completes the following tasks:

1. Write a real-world story problem using a rational number and its opposite.
2. Create a horizontal or vertical number line diagram to represent your situation.
   a. Determine an appropriate scale, and label the number line.
   b. Write the units of measurement (if needed).
   c. Graph the rational number and its opposite that represent the situation.
3. Describe what points 0 and the opposite number represent on the number line.
4. Name a rational number to the left and right of the rational number you initially chose.

Scaffolding:

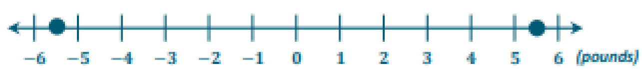
- Project the directions for the example as a way for groups to make sure they are completing all task requirements.
- Have students write their story problems and draw their number lines on large wall grid paper.
- Hang posters around the room to use as a gallery walk for students who finish their Exit Tickets early, or use them as review for the Mid-Module Assessment later in the module.

Exercise 2

Our Story Problem

Answers will vary.

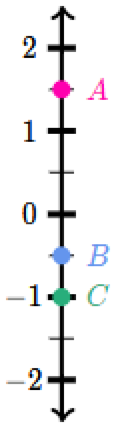
Melissa and Samantha weigh the same amount. Melissa gained 5.5 pounds last month, while Samantha lost the same amount Melissa gained.



- *Our Scale: 1*
- *Our Units: Pounds*
- *Description: On the number line, zero represents Melissa and Samantha's original weight. The point -5.5 represents the change in Samantha's weight. The amount lost is 5.5 pounds.*
- *Other Information: A rational number to the left of 5.5 is 4.5. A rational number to the right of 5.5 is 5.75.*

Sample Anchor Chart for 6.NS.6c – Rational Numbers on the Number Line

Consider this number line:



Point
A
B
C

What is the value of each point?

1/4 The tick marks represent $\frac{1}{2}$ on the number line.

A is located at the 3^{rd} tick mark *above* 0, so $A = \frac{3}{2}$.

2/4 B is located at the 1^{st} tick mark *below* 0, so $B = -\frac{1}{2}$.

3/4 We can see that C is located at -1 on the number line.

4/4 The following cards match each point:

Point	Value
A	$\frac{3}{2}$
B	-0.5
C	-1

7.NS.1A

Back to ccss standard

Apply and extend previous understandings of operations with fractions.

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram: Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged*.

Skills

1. Describe situations in which opposite quantities combine to make 0

2. Apply and extend previous understanding to represent addition problems of rational numbers with a horizontal or vertical number line

3. Apply and extend previous understanding to represent subtraction problems of rational numbers with a horizontal or vertical number line

4. Demonstrate and explain how adding two numbers, $p + q$, if q is positive, the sum of p and q will be $|q|$ spaces to the right of p on the number line

5. Demonstrate and explain how adding two numbers, $p + q$, if q is negative, the sum of p and q will be $|q|$ spaces to the left of p on the number line

6. Identify subtraction of rational numbers as adding the additive inverse property to subtract rational numbers, p-q = p +(-q)

Key Concepts/Vocabulary

Absolute value – how far a number is from zero

Rational number – a number that can be made by dividing two integers (an integer is a number with no fractional part)

Additive Inverse Property – What you add to a number to get zero; the negative of a number

Standard-Specific Resources (7.NS.1a)

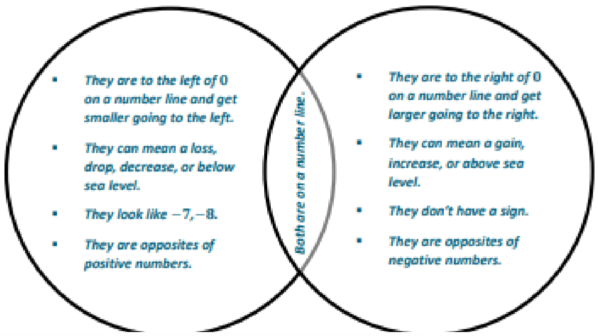
- **<u>EngageNY:</u> Grade 7, Module 2, Topic A, Lesson 1 – [? Students add positive integers by counting up and negative integers by counting down (using curved arrows on the number line.]**

Exercise 1 (3 minutes): Positive and Negative Numbers Review

In pairs, students will discuss "What I Know" about positive and negative integers to access prior knowledge. Have them record and organize their ideas in the graphic organizer in the student materials. At the end of discussion, the teacher will choose a few pairs to share out with the class.

Exercise 1: Positive and Negative Numbers Review

With your partner, use the graphic organizer below to record what you know about positive and negative numbers. Add or remove statements during the whole-class discussion.



Scaffolding:

- Laminate (or place in sheet protectors) 1-page of number lines (vary blank and numbered) for individual use with dry erase markers.
- Create a number line on the floor using painters tape to model the counting on principle.
- Provide a wall model of the number line at the front of the room for visual reinforcement.

Example 1 (5 minutes): Introduction to the Integer Game

Read the Integer Game outline before the lesson. The teacher selects a group of 3 or 4 students to demonstrate to the whole class how to play the Integer Game.[1] The game will be played later in the lesson. The teacher should stress that the object of the game is to get a score of zero.

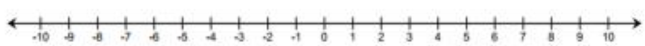
Example 2 (5 minutes): Counting Up and Counting Down on the Number Line

Model a few examples of counting with small curved arrows to locate numbers on the number line, where *counting up* corresponds to positive numbers and *counting down* corresponds to negative numbers.

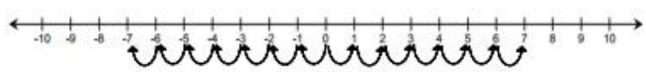
Example 2: Counting Up and Counting Down on the Number Line

Use the number line below to practice counting up and counting down.

- *Counting up starting at* 0 corresponds to ___<u>*positive*</u>___ numbers.
- *Counting down starting at* 0 corresponds to ___<u>*negative*</u>___ numbers.



A negative 7 is 7 units to the left of 0. $|-7| = 7$ *A positive 7 is 7 units to the right of 0.* $|7| = 7$



a. **Where do you begin when locating a number on the number line?**

Start at 0.

b. **What do you call the distance between a number and 0 on a number line?**

The absolute value

c. **What is the relationship between 7 and −7?**

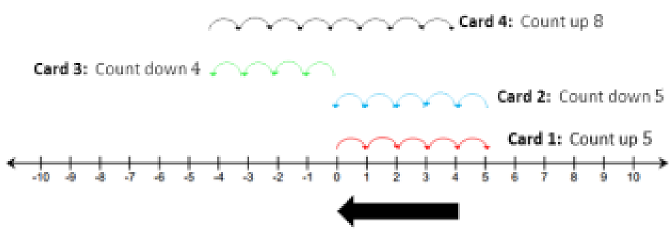
Answers will vary. 7 and −7 both have the same absolute values. They are both the same distance from zero, 0, but in opposite directions; therefore, 7 and −7 are opposites.

Example 3 (5 minutes): Using the Integer Game and the Number Line

The teacher leads the whole class using a number line to model the concept of counting on (addition) to calculate the value of a hand when playing the Integer Game. The hand's value is the sum of the card values.

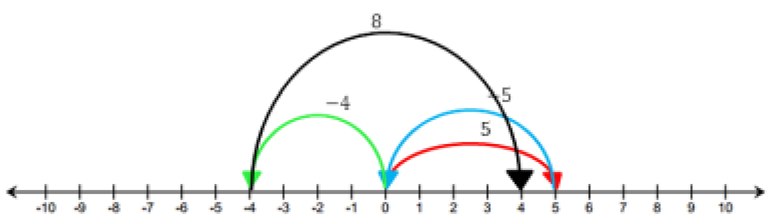
$\boxed{5}$	$\boxed{-5}$	$\boxed{-4}$	$\boxed{8}$
First card: 5	Second Card: -5	Third Card: -4	Fourth Card: 8
Start at 0 and end up at positive 5. This is the first card drawn, so the value of the hand is 5.	Start at 5, the value of the hand after the first card; move 5 units to the left to end at 0.	Start at 0, the value of the hand after the second card; move 4 units to the left.	Start at -4, the value of the hand after the third card; move 8 units to the right.

- What is the final position on the number line?
  - *The final position on the number line is* 4.



Move 4 units to the left to get back to 0.

- What card or combination of cards would you need to get back to 0?
  - *In order to get a score of* 0*, I would need to count down* 4 *units. This means, I would need to draw a* -4 *card or a combination of cards whose sum is* -4*, such as* -1 *and* -3.



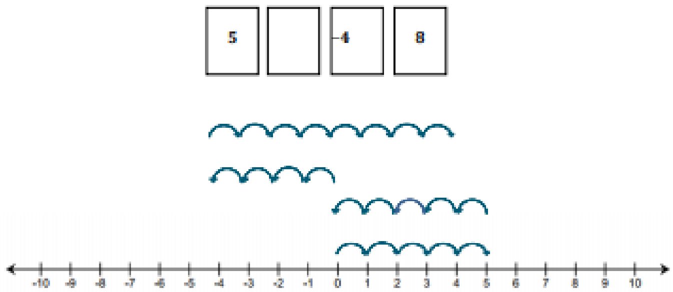
The final position is 4 units to the right of 0.

We can use smaller, curved arrows to show the number of hops or jumps that correspond to each integer. Or, we can use larger, curved arrows to show the length of the hop or jump that corresponds to the distance between the tail and the tip of the arrow along the number line. Either way, the final position is 4 units to the right of zero. Playing the Integer Game will prepare students for integer addition using arrows (vectors) in Lesson 2.

Example 3: Using the Integer Game and the Number Line

What is the sum of the card values shown? Use the counting on method on the provided number line to justify your answer.

4



a. **What is the final position on the number line?** *4*

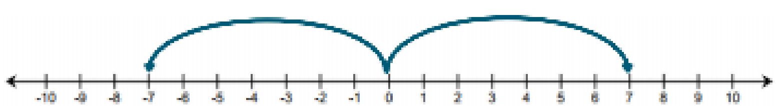
b. **What card or combination of cards would you need to get back to 0?** *-4 or -1 and -3*

Exercise 2 (5 minutes): The Additive Inverse

Before students begin, the teacher highlights part of the previous example where starting at zero and counting up five units and then back down five units brings us back to zero. This is because 5 and -5 are opposites. Students work independently to answer the questions. At the end of the exercise questions, formalize the definition of *additive inverse*.

Exercise 2: The Additive Inverse

Use the number line to answer each of the following questions.



a. **How far is 7 from 0 and in which direction?** *7 units to the right*

b. **What is the opposite of 7?** *-7*

c. **How far is -7 from 0 and in which direction?** *7 units to the left*

d. Thinking back to our previous work, explain how you would use the counting on method to represent the following: While playing the Integer Game, the first card selected is 7, and the second card selected is -7.

I would start at 0 and count up 7 by moving to the right. Then, I would start counting back down from 7 to 0.

e. What does this tell us about the sum of 7 and its opposite, -7?

The sum of 7 and -7 equals 0.

$7 + (-7) = 0$

f. Look at the curved arrows you drew for 7 and -7. What relationship exists between these two arrows that would support your claim about the sum of 7 and -7?

The arrows are both the same distance from 0. They are just pointing in opposite directions.

g. Do you think this will hold true for the sum of any number and its opposite? Why?

I think this will be true for the sum of any number and its opposite because when you start at 0 on the number line and move in one direction, moving in the opposite direction the same number of times will always take you back to zero.

Property: For every number a, there is a number $-a$ so that $a + (-a) = 0$ and $(-a) + a = 0$.

The additive inverse of a number is a number such that the sum of the two numbers is 0. The opposite of a number satisfies this definition: For example, the opposite of 3 is -3, and $3 + (-3) = 0$. Hence, -3 is the additive inverse of 3.

The property above is usually called the existence of additive inverses.

Example 4 (5 minutes): Modeling with Real-World Examples

The purpose of this example is to introduce real-world applications of opposite quantities to make zero. The teacher holds up an Integer Game card, for example -10, to the class and models how to write a story problem.

- How would the value of this card represent a temperature?
  - *-10 could mean 10 degrees below zero.*
- How would the temperature need to change in order to get back to 0 degrees?
  - *The temperature needs to rise 10 degrees.*
- With a partner, write a story problem using money that represents the expression $200 + (-200)$.
  - *Answers will vary. Timothy has \$$200$ in his bank account. He owes \$$200$ to a friend for a bike. In terms of the loan and bank account, how much money is Timothy really worth?*

Students share their responses to the last question with the class.

Exercise 3 (10 minutes): Playing the Integer Game

Exercise 3: Playing the Integer Game

Play the Integer Game with your group. Use a number line to practice counting on.

MP.6 & MP.7

Students play the Integer Game in groups. Students practice counting using their number lines. Let students model addition on the number line. Monitor student understanding by ensuring that the direction of the arrows appropriately represents positive or negative integers.

Sample Anchor Charts for 7.NS.1a – Signs of Sums

What is the sign of $B + A$?

Choose 1 answer:

Ⓐ Positive

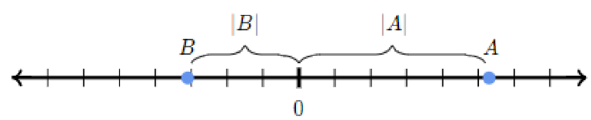
Ⓑ Negative

Ⓒ Neither positive nor negative—the sum is zero.

1/4 We are looking for the sign of the sum of a negative number (B) and a positive number (A).

2/4 The negative number (B) tells us to move to the left on the number line, and the positive number (A) tells us to move to the right. The number that tells us to move farther determines the sign of the sum.

3/4 The absolute value (or *magnitude*) of A is greater than the magnitude of B because A is farther from 0. So, A determines the sign of the sum.



4/4 **The sign of the sum is positive because A is positive.**

What is the sign of $37 + (-37)$?

Choose 1 answer:

Ⓐ Positive

Ⓑ Negative

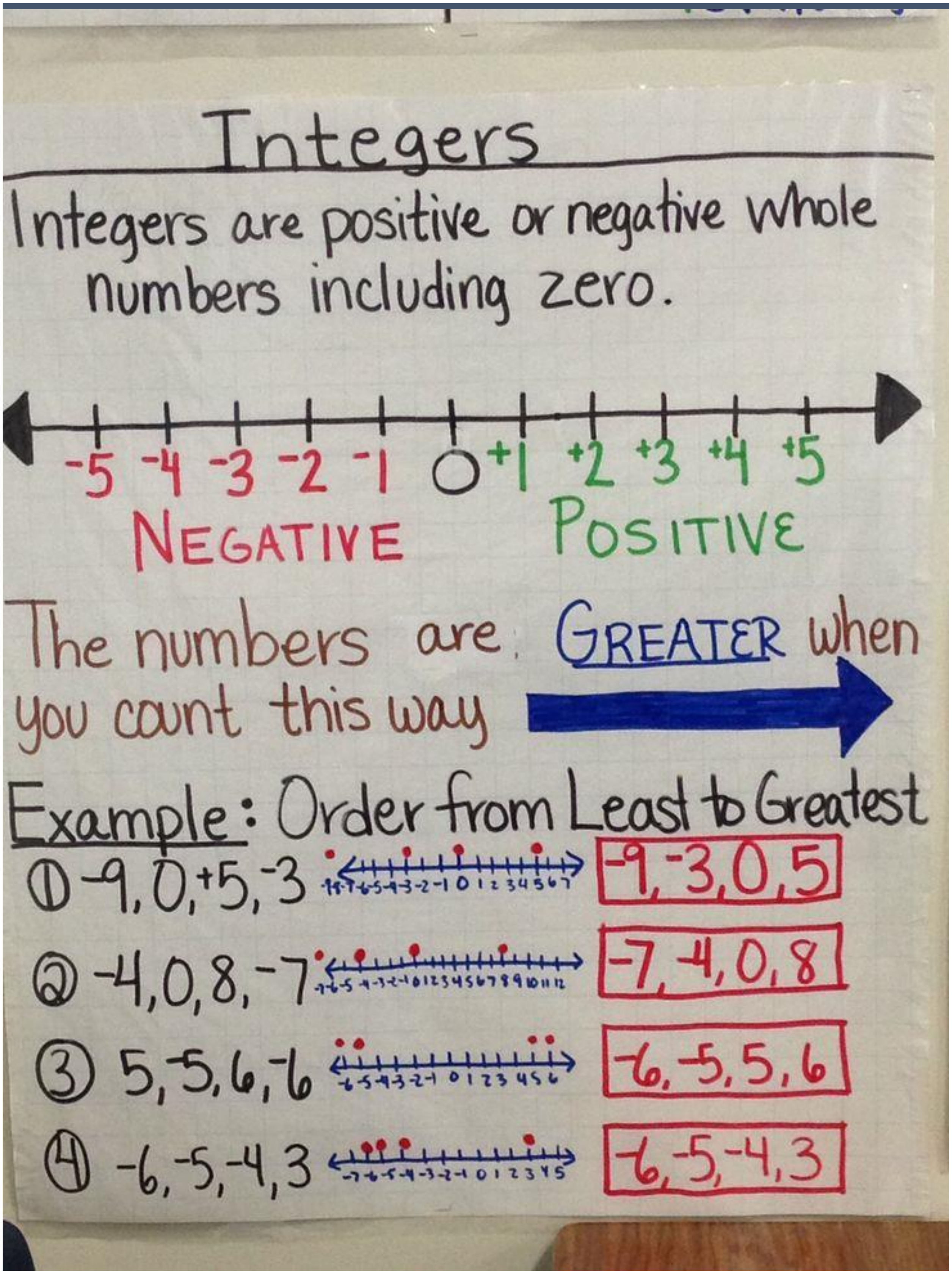
Ⓒ Neither positive nor negative—the sum is zero.

1/4 $37 + (-37)$ tells us to move 37 to the right and 37 to the left on the number line.

2/4

3/4 We moved the same to distance to the right as to the left because 37 has the same absolute value (or *magnitude*) as -37.

4/4 **The sign of the sum is neither positive nor negative, because the sum is zero.**


Integers
Integers are positive or negative whole numbers including zero.
-5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5
NEGATIVE
POSITIVE
The numbers are GREATER when you count this way
Example: Order from Least to Greatest
① -9, 0, +5, -3
-9, -3, 0, 5
② -4, 0, 8, -7
-7, -4, 0, 8
③ 5, -5, 6, -6
-6, -5, 5, 6
④ -6, -5, -4, 3
-6, -5, -4, 3

7.NS.2A

Back to ccss standard

Apply and extend previous understandings of operations with fractions.

Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers:

Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

Skills

1. Know and describe the rules when multiplying signed numbers

2. Know and describe the relationship between multiplication of signed numbers and absolute value using the distributive property

3. Know and describe the relationship between multiplication of signed numbers and absolute value using the additive inverse

4. Interpret the products of rational numbers by describing real-world contexts

Key Concepts/Vocabulary

Rational number – a number that cannot be made by dividing two integers

Integer – a number with no fractional part

Distributive property – multiplying the sum by a number gives the same result as first multiplying each addend by the number and then adding the products (e.g. [3+4]x2 = [3x2] + [4x2])

Product – the answer when two or more numbers are multiplied together

Standard-Specific Resources (7.NS.2a)

- **EngageNY:** Grade 7, Module 2, Topic B, Lesson 11 – Students understand the rules for multiplication of integers and that multiplying the absolute values of integers results in the absolute value of the product. The sign, or absolute value, of the product is positive if the factors have the same sign and negative if they have opposite signs.

Example 1 (17 minutes): Extending Whole Number Multiplication to the Integers

Part A: Students complete only the right half of the table in the student materials. They do this by calculating the total change to a player's score using the various sets of matching cards. Students complete the table with these values to reveal patterns in multiplication.

Students describe, using Integer Game scenarios, what the right quadrants of the table represent and record this in the student materials.

Example 1: Extending Whole Number Multiplication to the Integers

Part A: Complete quadrants I and IV of the table below to show how sets of matching integer cards will affect a player's score in the Integer Game. For example, three 2's would increase a player's score by $0 + 2 + 2 + 2 = 6$ points.

Quadrant II | Quadrant I

What does this quadrant represent? (Quadrant II) *Removing positive value cards*

What does this quadrant represent? (Quadrant I) *Picking up positive value cards*

-25	-20	-15	-10	-5	5	5	10	15	20	25
-20	-16	-12	-8	-4	4	4	8	12	16	20
-15	-12	-9	-6	-3	3	3	6	9	12	15
-10	-8	-6	-4	-2	2	2	4	6	8	10
-5	-4	-3	-2	-1	1	1	2	3	4	5
-5	-4	-3	-2	-1	0	1	2	3	4	5
5	4	3	2	1	-1	-1	-2	-3	-4	-5
10	8	6	4	2	-2	-2	-4	-6	-8	-10
15	12	9	6	3	-3	-3	-6	-9	-12	-15
20	16	12	8	4	-4	-4	-8	-12	-16	-20
25	20	15	10	5	-5	-5	-10	-15	-20	-25

← Number of matching cards (middle row)

What does this quadrant represent? (Quadrant III) *Removing negative value cards*

What does this quadrant represent? (Quadrant IV) *Picking up negative value cards*

Quadrant III | ↑ Integer card values | Quadrant IV

a. **What patterns do you see in the right half of the table?**

The products in quadrant I are positive and the products in quadrant IV are negative. When looking at the absolute values of the products, quadrants I and IV are a reflection of each other with respect to the middle row.

b. **Enter the missing integers in the left side of the middle row, and describe what they represent.**

The numbers represent how many matching cards are being discarded or removed.

-5	-4	-3	-2	-1	0

Part B: Students complete quadrant II of the table.

Students describe, using an Integer Game scenario, what quadrant II of the table represents and record this in the student materials.

Part B: Complete quadrant II of the table.

Quadrant II

What does this quadrant represent? *Removing positive value cards.*

-25	-20	-15	-10	-5	5
-20	-16	-12	-8	-4	4
-15	-12	-9	-6	-3	3
-10	-8	-6	-4	-2	2
-5	-4	-3	-2	-1	1
-5	-4	-3	-2	-1	0

Students answer the following questions:

> **c. What relationships or patterns do you notice between the products (values) in quadrant II and the products (values) in quadrant I?**
>
> *The products in quadrant II are all negative values. Looking at the absolute values of the products, quadrant I and II are a reflection of each other with respect to the center column.*
>
> **d. What relationships or patterns do you notice between the products (values) in quadrant II and the products (values) in quadrant IV?**
>
> *The products in quadrants II and IV are all negative values. Each product of integers in quadrant II is equal to the product of their opposites in quadrant IV.*
>
> **e. Use what you know about the products (values) in quadrants I, II, and IV to describe what quadrant III will look like when its products (values) are entered.**
>
> *The reflection symmetry of quadrant I to quadrants II and IV suggests that there should be similar relationships between quadrant II, III, and IV. The number patterns in quadrants II and IV also suggest that the products in quadrant III are positive values.*

Part C: Discuss the following question. Then instruct students to complete the final quadrant of the table.

- In the Integer Game, what happens to a player's score when he removes a matching set of cards with negative values from his hand?
  - *His score increases because the negative cards that cause the score to decrease are removed.*

Students describe, using an Integer Game scenario, what quadrant III of the table represents and complete the quadrant in the student materials.

> *Scaffolding:*
>
> Create an *anchor poster* showing the quadrants with the new rules for multiplying integers.

> **Part C: Complete quadrant III of the table.**
>
> **Refer to the completed table to help you answer the following questions:**
>
> What does this quadrant represent?
>
> *Removing negative value cards.*
>
> | -5 | -4 | -3 | -2 | -1 | 0 |
> |---|---|---|---|---|---|
> | | | | | | |
> | 5 | 4 | 3 | 2 | 1 | -1 |
> | 10 | 8 | 6 | 4 | 2 | -2 |
> | 15 | 12 | 9 | 6 | 3 | -3 |
> | 20 | 16 | 12 | 8 | 4 | -4 |
> | 25 | 20 | 15 | 10 | 5 | -5 |
>
> ↑
>
> Quadrant III

Students refer to the completed table to answer parts (f) and (g).

> **f. Is it possible to know the sign of a product of two integers just by knowing in which quadrant each integer is located? Explain.**
>
> *Yes, it is possible to know the sign of a product of two integers just by knowing each integer's quadrant because the signs of the values in each of the quadrants are consistent. Two quadrants contain positive values, and the other two quadrants contain negative values.*
>
> **g. Which quadrants contain which values? Describe an Integer Game scenario represented in each quadrant.**
>
> *Quadrants I and III contain all positive values. Picking up three 4's is represented in quadrant I and increases your score. Removing three (-4)'s is represented in quadrant III and also increases your score. Quadrants II and IV contain all negative values. Picking up three (-4)'s is represented in quadrant IV and decreases your score. Removing three 4's is represented in quadrant II and also decreases your score.*

Example 2 (10 minutes): Using Properties of Arithmetic to Explain Multiplication of Negative Numbers

The teacher guides students to verify their conjecture that the product of two negative integers is positive using the distributive property and the additive inverse property.

> **Example 2: Using Properties of Arithmetic to Explain Multiplication of Negative Numbers**

- We have used the Integer Game to explain that adding a number multiple times has the same effect as removing the opposite value the same number of times. What is $(-1) \times (-1)$?
  - *Removing a -1 card is the same as adding a 1 card. So, $(-1) \times (-1) = 1$.*

Scaffolding:

Use color or highlight steps to help students organize and understand the manipulations.

- Why are 1 and -1 called additive inverses? Write an equation that shows this property.
  - *The sum of 1 and -1 is 0; $1 + (-1) = 0$.*

We are now going to show $-1 \times (-1) = 1$ using properties of arithmetic.

- We know $1 + (-1) = 0$ is true.
- We will show that $(-1) \times (-1)$ is the additive inverse of -1, which is 1.

If $-1 \times 0 = 0$	By the zero product property
then $-1 \times \left(1 + (-1)\right) = 0$	By substitution of $\left(1 + (-1)\right)$ for 0
$(-1 \times 1) + \left(-1 \times (-1)\right) = 0$	Distributive property
$-1 + \left(-1 \times (-1)\right) = 0$	Multiplication by 1

- Since -1 and $\left(-1 \times (-1)\right)$ have a sum of zero, they are additive inverses of each other; but, the additive inverse of -1 is 1.
- Because $\left(-1 \times (-1)\right)$ is the additive inverse of -1, we conclude that $(-1) \times (-1) = 1$. This fact can be used to show that $-1 \times a = -a$ for any integer and that $-a \times b = -(a \times b)$ for any integers a and b.

Exercise 1 (8 minutes): Multiplication of Integers in the Real World

Students create real-world scenarios for expressions given in the student materials. Students may use an Integer Game scenario as a reference.

Exercise 1: Multiplication of Integers in the Real World

Generate real-world situations that can be modeled by each of the following multiplication problems. Use the Integer Game as a resource.

a. -3×5

I lost three $\$5$ bills, and now I have $-\$15$.

b. $-6 \times (-3)$

I removed six (-3)'s from my hand in the Integer Game, and my score increased 18 points.

c. $4 \times (-7)$

If I lose 7 lb. per month for 4 months, my weight will change -28 lb. total.

Scaffolding:

For English language learners, create a teacher/student T-chart on which the teacher writes a real-world situation that corresponds with a product, and students write similar situations using different numbers.

Sample Anchor Charts for 7.NS.2a – Multiplying Positive and Negative Numbers

3 years ago, the residents of Planet X found a black hole leading to another galaxy. Since then, they have been leaving Planet X at a rate of 120 residents per year.

The following equation describes this situation:

$$-120 \cdot 3 = -360$$

What does -360 tell us?

Choose 1 answer:

(A) A total of 360 residents left Planet X in the past three years.

(B) 360 residents left Planet X each year.

(C) None of the above

1 / 4 Let's use a model to illustrate this situation.

$$\overbrace{\boxed{-120 \text{ residents}}\boxed{-120 \text{ residents}}\boxed{-120 \text{ residents}}}^{\text{Total change in residents} = -360}$$

2 / 4 There are 360 fewer residents on Planet X than there were three years ago.
The first answer choice is correct.

3 / 4 360 residents did not leave Planet X each year. (120 residents left each year.)
The second answer choice is not correct.

4 / 4 **-360 tells us that there are 360 fewer residents on Planet X than there were three years ago.**

Increases in snow levels are recorded with positive numbers. Decreases in snow levels are recorded wit
negative numbers.

After a winter storm, the depth of the snow on Cherry Street was 10 cm. But then, the snow started me
at a rate of $\frac{1}{3}\text{ cm}$ per hour.

What was the depth of the snow on Cherry Street after 3 hours?

[ ] cm

1/4 The starting depth of the snow was 10 centimeters.

2/4 Then the snow melted. Let's think about what this event means mathematically.

Event	Math
Snow was *melting* at rate of $\frac{1}{3}\text{ cm}$ per hour	$-\frac{1}{3}$
for 3 hours	$\times 3$

As an expression, we get $-\frac{1}{3} \cdot 3$.

3/4 We can use the equation $10 + \left(-\frac{1}{3} \cdot 3\right) = ?$ to find the depth of the snow after 3 hours.

4/4 **The depth of the snow on Cherry Street after 3 hours was 9 cm.**