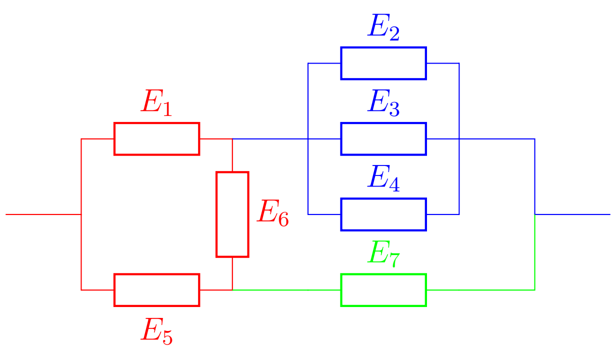
Problem 1

Answer to the problem goes here.



1. Find the probability that the circuit is operational during time interval T.
   First, we discuss two scenarios. The key component for this problem is whether E_6 is on or off.
   Given $S \in$ circuits work and $p_1, \ldots, p_6$ represent $E_1, \ldots, E_6$ work, and $q_1, \ldots, q_6$ represent failing.

   - $H_1 - E_6$ on:

   $$\begin{aligned} P(S|H_1) &= (1 - q_1 q_5)[1 - (q_2 q_3 q_4)(q_7)] \\ &= (1 - 0.5 \times 0.1)[1 - (0.3 \times 0.7 \times 0.6)(0.3)] \\ &= 0.95 \times 0.9622 \\ &= 0.91409 \end{aligned}$$

   - $H_2 - E_6$ off:

   $$\begin{aligned} P(S|H_2) &= 1 - [1 - p_1(1 - q_2 q_3 q_4)](1 - p_5 p_7) \\ &= 1 - [1 - 0.5(1 - 0.3 \times 0.7 \times 0.6)(1 - 0.9 \times 0.7)] \\ &= 1 - 0.563 \times 0.37 \\ &= 0.79169 \end{aligned}$$

   Finally, by Total Probability,

   $$\begin{aligned} P(S) &= P(S|H_1)P(H_1) + P(S|H_2)P(H_2) \\ &= 0.5 \times 0.91409 + 0.5 \times 0.79169 \\ &= 0.85289 \end{aligned}$$

2. If the circuit was found operational at the time T, what is the probability that the element E_6 was operational.
   We know $E_6 \subset S$,

$$\begin{aligned} P(E_6|S) &= P(E_6 \cap S)/P(S) \\ &= (1 - q_1 q_5)(1 - (q_2 q_3 q_4) q_7)/0.85289 \text{ by } (1) \\ &= 0.457045/0.85289 \\ &= 0.53587801 \end{aligned}$$

Problem 2

Answer to the problem goes here.
An approach to this problem is to use Bayesian Learning. We observe the first trial and then use this trail as our prior to make an inference about the second trial.

- First Trail: this product is conforming.
  Given H_1, H_2 represent as two batches, A as conforming.

$$\begin{aligned} P(H_1) &= 0.5, P(H_2) = 0.5 \\ P(H_1|A) &= P(A|H_1)P(H_1)/P(A) \\ &= 1 \times \frac{1}{2} / \frac{1}{2} \times 1 + \frac{1}{2} \times 0.9 \\ &= \frac{10}{19} \\ P(H_2|A) &= P(A|H_2)P(H_2)/p(A) \\ &= 0.9 \times 0.5 / \frac{1}{2} \times 1 + \frac{1}{2} \times 0.9 \\ &= \frac{0.45}{0.95} \\ &= \frac{9}{19} \end{aligned}$$

- Second Trail:

$$\begin{aligned} P(A^c) &= P(A^c|H_1)P(H_1) + P(A^c|H_2)P(H_2) \\ &= 0 \times \frac{10}{19} + 0.1 \times \frac{9}{19} \\ &= \frac{0.9}{19} \\ &= \frac{9}{190} \end{aligned}$$

Problem 3

Answer to the problem goes here.

1. What is the probability that the machine will fail? Evaluate this probability for $p = 0.4$. Given A, B, C, D as machines where D fails with probability 1/2.
   The approach to solve this problem is to calculate the probability when no one or only one works.
   H_0 : no one works, that is A, B, C, D fail at the same time H_1 : only one works, that is only A works, only B works, only C works and only D works

$$\begin{aligned}
P(H_0) &= q \times q \times q \times \frac{1}{2} = \frac{1}{2}q^3 \\
P(H_1) &= p \times q \times q \times \frac{1}{2} + q \times p \times q \times \frac{1}{2} + q \times q \times p \times \frac{1}{2} + q \times q \times q \times \frac{1}{2} \\
&= \frac{3}{2}pq^2 + \frac{1}{2}q^3 \\
P(M) &= \frac{3}{2}pq^2 + \frac{1}{2}q^3 + \frac{1}{2}q^3 \\
&= \frac{3}{2}pq^2 + q^3 \\
&= 0.432
\end{aligned}$$

2. If the machine failed, what is the probability that the component which fails with probability 1/2 actually failed.
   The strategy to solve this problem is to use conditional probability. That is, $P(D \text{ fails}|M \text{ fails})$ By Baye's rule, $P(D|M) = P(M|D)P(D)/P(M)$ where $P(M|D) = q^3 + \binom{3}{1}qp^2$

$$\begin{aligned}
P(D|M) &= 0.5(q^3 + \binom{3}{1}pq^2)/\frac{3}{2}pq^2 + q^3 \\
&= 0.5q^3 + 1.5pq^2/q^3 + 1.5pq^2 \\
&= 0.5 + p/1 + p
\end{aligned}$$