ISYE 6420A

Homework 2

Due February 5, 2020

1. Suppose an engineer is 95% confident that the probability of rejecting a product is going to be $.5 \pm .2$. Use this information to construct a beta prior for θ. (Hint: Use normal approximation for the beta distribution. Note that if $x \sim Beta(\alpha, \beta)$, then $E(x) = \frac{\alpha}{\alpha+\beta}$ and $var(x) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$.)

2. Find the Jeffreys' prior for the parameter α of the Maxwell distribution

$$p(y|\alpha) = \sqrt{\frac{2}{\pi}}\alpha^{3/2}y^2 \exp(-\frac{1}{2}\alpha y^2)$$

   and find a transformation of this parameter in which the corresponding prior is uniform.

3. Jeffreys' prior for multiparameter models is given by

$$p(\theta) \propto \sqrt{det(I(\boldsymbol{\theta}))},$$

   where $I(\boldsymbol{\theta})$ is the Fisher Information matrix whose ijth element is given by $-E(\partial^2 \log p(y|\boldsymbol{\theta})/\partial\theta_i\partial\theta_j)$. Suppose that for $i = 1, \cdots, n$, $y_i \sim p_i(y_i|\theta_i)$ and $\pi_i(\theta_i)$ is the Jeffreys' prior for θ_i. If the y_i's are independent, show that the Jeffreys' prior for $\boldsymbol{\theta} = (\theta_1, \cdots, \theta_n)'$ is $\prod_{i=1}^n \pi_i(\theta_i)$.

4. Suppose $x \sim Binomial(n, \pi)$ and $y \sim Binomial(n, \rho)$ are independent. Find the Bayes rule for estimating $\pi - \rho$ corresponding to the loss function $L(\pi - \rho, a) = (\pi - \rho - a)^2$ under the priors: $\pi \sim Beta(1, 3)$ and $\rho \sim Beta(3, 1)$.

5. Find the Bayes rule for estimating θ corresponding to the loss function $L(\theta, a) = c_1(a - \theta)$ if $a \geq \theta$ and $L(\theta, a) = c_2(\theta - a)$ if $a \leq \theta$.