ISyE 6416 Computational Statistics Homework 5

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Problem (1) Nonlinear regression using spline

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
%matplotlib inline
```

```
df = np.loadtxt("/Users/jim/Dropbox (GaTech)/Courses/ISyE6416/Homework/Homework5/copper-new.txt")
```

```
y = df[:, [0]]
X = df[:, [1]]
```

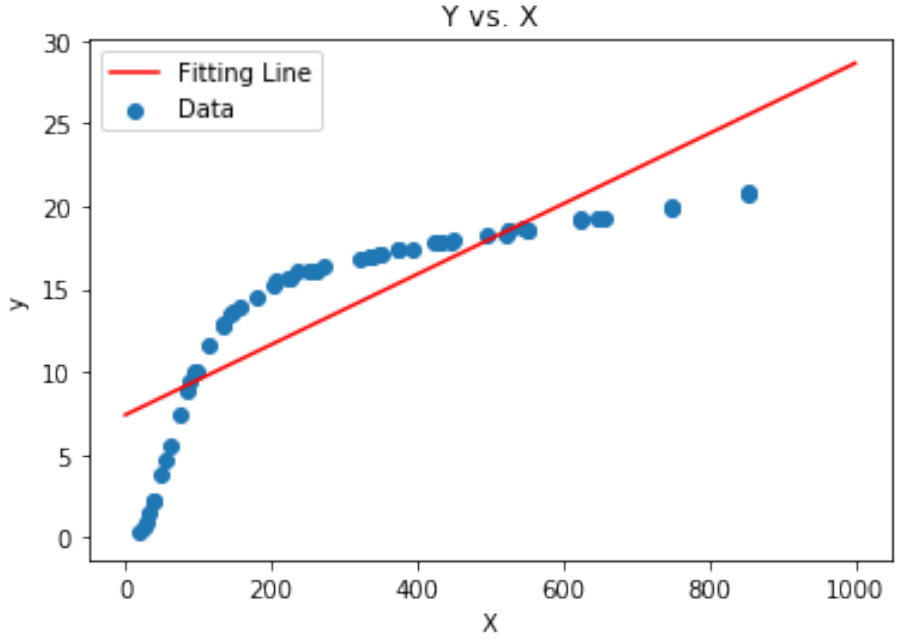
(a) Perform linear regression on the data. Report the fitted model and the fitting error.

```
from numpy.linalg import inv
# add intercept
ones = np.ones((len(X), 1))
#combined
X_withIntercept = np.concatenate((ones, X), axis = 1)
#betahat
betahat = inv(X_withIntercept.T @ X_withIntercept) @ X
_withIntercept.T @ y
betahat
```

```
array([[7.38412739],
       [0.02128314]])
```

```
elements = [i for i in np.arange(0, 1000)]
y_loop= []
for i in elements:
    y_loop.append(7.38412739 + i * 0.02128314)

plt.plot(elements, y_loop, c='r', label = "Fitting Lin
e")
plt.scatter(X, y, label = "Data")
plt.xlabel('X')
plt.ylabel('y')
plt.legend()
plt.title("Y vs. X")
```

```
Text(0.5, 1.0, 'Y vs. X')
```



```
X_elements = df[:, 1]
true_y = df[:, 0]
RSS = []
for i in range(len(X_elements)):
    fitted = 7.38412739 + X_elements[i] * 0.02128314
    RSS.append((fitted - true_y[i]) ** 2)

print("Linear Model: y = {0} + {1} * x".format(betahat
[0][0], betahat[1][0]))
print("Fitting Error:", sum(RSS))
```

```
Linear Model: y = 7.384127393045498 + 0.021283144237618158
* x
Fitting Error: 620.7208889638787
```

From the above, the linear model of part(a) is

$$y = 7.384127393045498 + 0.0212831442376181$$

And, the fitting error is 620 where the formula is

Residual Sum of Squares $\sum_{i=1}^{n}(x - \bar{x})^2$

(b) Perform nonlinear regression with spline function (i.e., using all data points).

Use GCV for find λ. Report the fitting error.

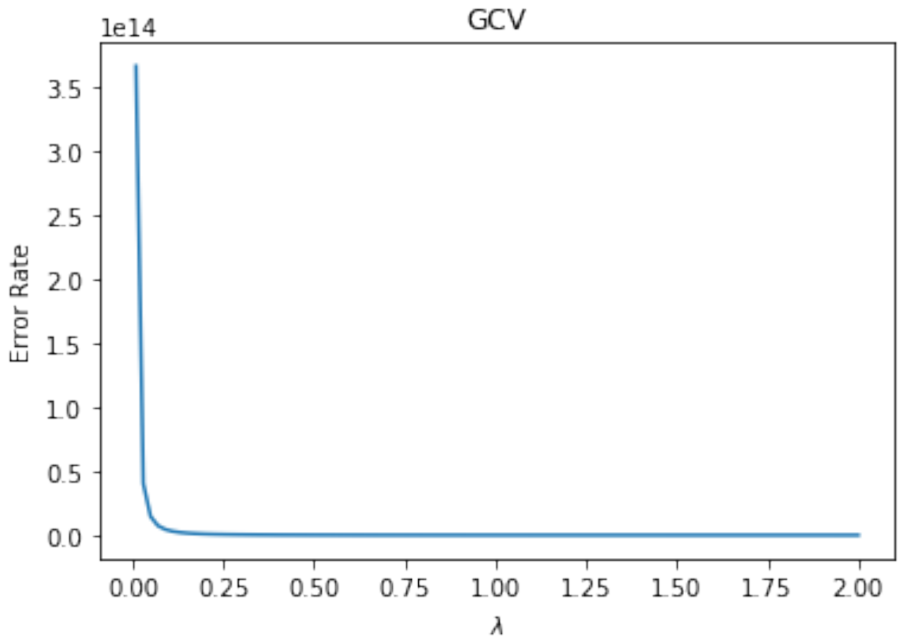
```
len(X_elements)
```

59

```
from numpy import linalg as LA
# Data
X_elements = df[:, 1]
true_y = df[:, [0]]

num_elements = len(X_elements)

h = (max(X_elements) - min(X_elements)) / (len(X_elements) - 1) # total distance divide by number of knots - 1 (59knots -> 58 intervals)
weights = np.eye(len(X_elements)) #number of weights used for cross validation
Q_Matrix = np.zeros((num_elements - 2, num_elements))# Q (57 * 59 matrix)
#create Q matrix with h
for i in range(Q_Matrix.shape[0]):
    for j in range(3):
        if j % 2 == 1:
            Q_Matrix[i][i + j] = -2 / h
        else:
            Q_Matrix[i][i + j] = 1 / h

identity = np.eye(num_elements - 2) # M (57 * 57 matrix)
M_Matrix = np.multiply(identity, 2 / 3 * h)
#  Create M matrix with h
for i in range(identity.shape[0] - 1):
    M_Matrix[i][i + 1] = h / 6
    M_Matrix[i + 1][i] = h / 6
```

```
# Compute for GCV(lambda)
lambda_it = [i for i in np.linspace(0.01, 2, 100)]
GCV = []
for i in range(len(lambda_it)):
    lamda = lambda_it[i]
    QMQ = np.multiply(Q_Matrix.T @ inv(M_Matrix) @ Q_M
atrix, lamda)
    Slambda = np.multiply(inv(np.add(weights, QMQ)), w
eights)
    a_top = np.power(LA.norm(true_y - np.multiply(Slam
bda, true_y)), 2)
    b_below = np.power(1 - np.divide(np.trace(Slambda)
, num_elements), 2)
    GCV.append(np.divide(a_top, b_below))
```

```

# plot GCV VS lambda
plt.plot(lambda_it, GCV)
plt.ylabel("Error Rate")
plt.xlabel("$\lambda$")
plt.title("GCV")
plt.show()
print("Minimum Lambda:", lambda_it[GCV.index(min(GCV))
])
```



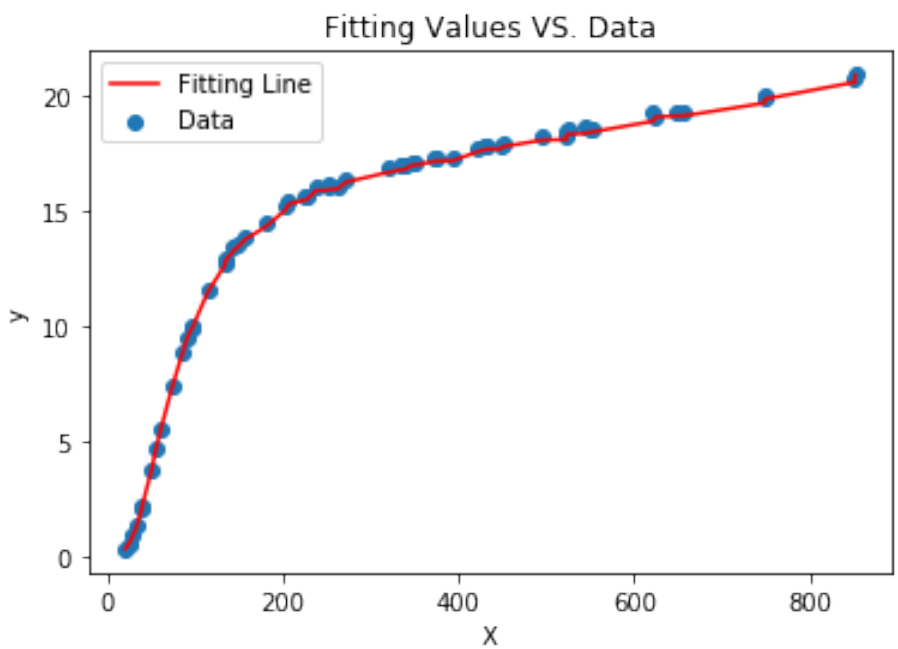

```
Minimum Lambda: 2.0
```

```
QMQ_new = np.multiply(Q_Matrix.T @ inv(M_Matrix) @ Q_Matrix, 2.0)
Slambda_new = np.multiply(inv(np.add(weights, QMQ_new)), weights)
y_hat = np.multiply(Slambda_new, true_y)
y_hat_to_array = []
for i in range(len(y_hat)):
    y_hat_to_array.append(y_hat[i][i])
y_hat_array = np.array(y_hat_to_array)
```

```
s = sorted(y_hat_array)
x_sort = sorted(X_elements)
```

```
plt.scatter(X_elements, true_y, label = "Data")
plt.plot(x_sort,s , c = 'r', label = "Fitting Line")
plt.title("Fitting Values VS. Data")
plt.xlabel("X")
plt.ylabel('y')
plt.legend()

y_array = df[:, 0]
RSS = []
for i in range(len(y_hat_array)):
    RSS.append((y_array[i] - y_hat_array[i]) ** 2)
print("Fitting error:", sum(RSS))
```

```
Fitting error: 1.189281188415568
```



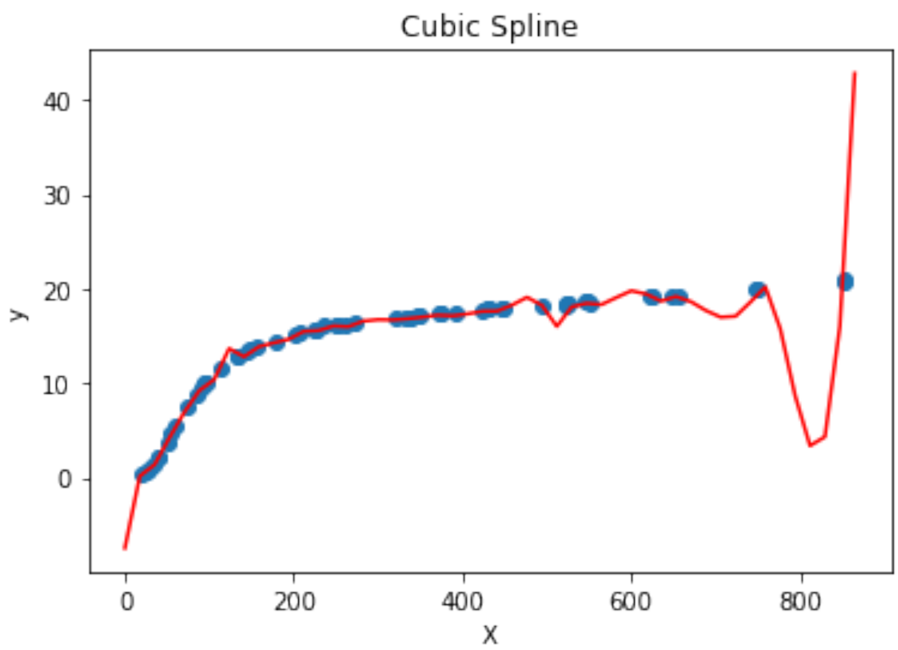
From the above codes, for part(b) in problem(1), we chose $\lambda = 2$ as our tuning parameters and the fitting error is around 1.19.

(c) Predict the coefficient at 400 degree Kelvin.

```
to_sorted = [[i, j] for i, j in zip(X_elements, y_hat_
array)]
sorted_X = sorted(to_sorted)
X_sort = [sorted_X[i][0] for i in range(len(sorted_X))
]
y_sort = [sorted_X[i][1] for i in range(len(sorted_X))
]
```

```
from scipy.interpolate import CubicSpline
cs = CubicSpline(X_sort, y_sort)
xs = np.linspace(0, 865)
ys = cs(xs)
plt.plot(xs, ys, c = 'r')
plt.scatter(X_elements, true_y)
plt.title("Cubic Spline")
plt.xlabel("X")
plt.ylabel("y")
print("Cubic Spline at 400 degrees Kelvin:", cs(400))
print("Linear Model at 400 degrees Kelvin:", 7.3841273
9 + 400 * 0.02128314)
```

```
Cubic Spline at 400 degrees Kelvin: 17.252860307873554
Linear Model at 400 degrees Kelvin: 15.89738339
```



There is an interesting thing to think that when x is over 750. There is an big upside and down, and the reason might be that there are fewer points in this area.

From the above codes, for part(c) in problem 1:

- Linear Model Prediction at 400 degrees Kelvin is 15.89738339.
- Cubic Spline Prediction at 400 degrees Kelvin is 17.25286030.

Problem(2) PCA for face recognition.

(a) Perform data analysis on the Yale face dataset for subject 14.

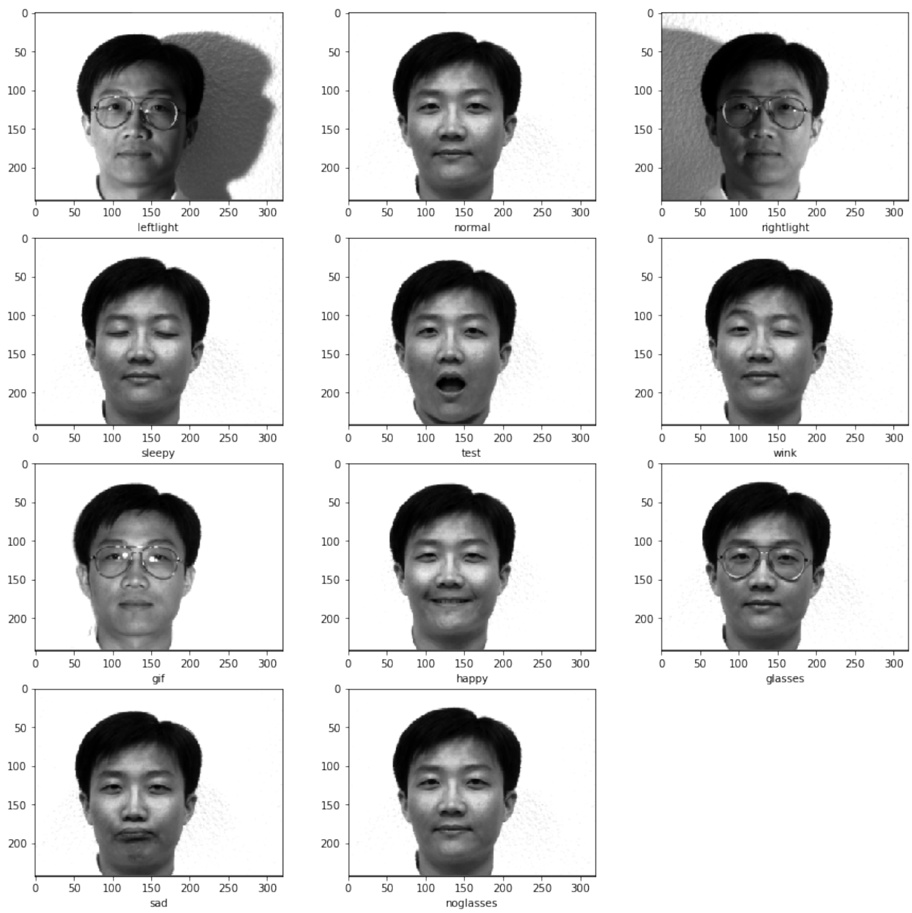
Plot the mean face and the first 6 eigenfaces for subject 14.

```python
import os

# List all files in a directory using os.listdir
basepath = '/Users/jim/Dropbox (GaTech)/Courses/ISyE6416/Homework/Homework5/yalefaces'
collections = []
for entry in os.listdir(basepath):
    if os.path.isfile(os.path.join(basepath, entry)):
        if "14" in entry:
            collections.append(entry)
collections[:5]
```

```
['subject14.leftlight.gif',
 'subject14.normal.gif',
 'subject14.rightlight.gif',
 'subject14.sleepy.gif',
 'subject14.test.gif']
```

In the beginning, we try to print out every image to know more about facial structures.

```python
import matplotlib.pyplot as plt
import matplotlib.image as mpimg
index = 1
plt.figure(figsize=(15, 15))
for i in range(len(collections)):
    img = mpimg.imread("/Users/jim/Dropbox (GaTech)/Courses/ISyE6416/Homework/Homework5/yalefaces/{}".format(collections[i]))
    plt.subplot(4, 3, index)
    index += 1
    plt.imshow(img, cmap = 'Greys_r')
    plt.xlabel("{}".format(collections[i].split('.')[1]))
plt.show()
```



```
# mean face
from PIL import Image

arrays = []
for i in range(11):
    im = Image.open("/Users/jim/Dropbox (GaTech)/Courses/ISyE6416/Homework/Homework5/yalefaces/{}".format(collections[i]))
    arr = mpimg.pil_to_array(im)
    arrays.append(arr)
nparrays = np.array(arrays)
mean_face_array = np.zeros(shape = (243, 320))

row = 0
column = 0
for i in range(mean_face_array.shape[0]):
    for j in range(mean_face_array.shape[1]):
        mean_pixel = 0
        for index in range(len(collections)):
            mean_pixel += nparrays[index][i][j]
        mean_face_array[i][j] = mean_pixel / len(collections)
```

To create a set of eigenfaces, one must:

1. Prepare a training set of face images (11 images -> test image out -> 10 images) Each image is treated as one training sample (i.e. (243 *320)* 10 matrix)
2. Subtract the mean.
3. Calculate the eigenvectors and eigenvalues of the covariance matrix
4. Choose the principal components
5. Get descending eigenvalues of first six components.

```
#Helper functions
from scipy import linalg as LA
from scipy.sparse.linalg import svds, eigs
# pca function
def faceSVD(X):
    '''
    X: matrix that is unnormalized for a non-square matrix
    '''
    # mean: each row
    mean = np.mean(X, axis = 1).reshape(len(X), 1)
    # standard derivation
    #std = np.std(X, axis = 1).reshape(len(X), 1)
    # normalized -> not normalized since it will create many nans or infs
    minusmean = np.subtract(X, mean)
    #X = np.divide(minusmean, std)
    # SVD
    #u, d, v = LA.svd(X, overwrite_a = True,lapack_driver = 'gesvd')
```

```
ver = 'gesvd')

    u, d, v = svds(minusmean)
    return u, d, v

# downsample image
def downsample(X, factor):
    '''
    X: matrix representing images
    factor: an integer that we want to downsample to f
actor * factor image
    ----------
    image: 243 * 320
    downsampled image: 16 * 4860 (243 * 320 -> downsca
le to 15 * 20)
    '''
    height = 0
    width = 0
    if (X.shape[0] % factor is not 0 or X.shape[1] % f
actor is not 0):
        height = int((X.shape[0] -  (X.shape[0] % fact
or)) /  factor)
        width = int((X.shape[1] -  (X.shape[1] % facto
r)) /  factor)
    else:
        height = int(X.shape[0] / factor)
        width = int(X.shape[1] / factor)
    # patches
    patch = np.zeros((factor * factor, height * width)
)
    test = X
    for ii in range(height):
        for jj in range(width):
            tmp = test[ii * factor: (ii + 1) * factor,
jj * factor: (jj + 1) * factor]
            patch[:, ii * width + jj] = tmp.flatten()
    return patch
```

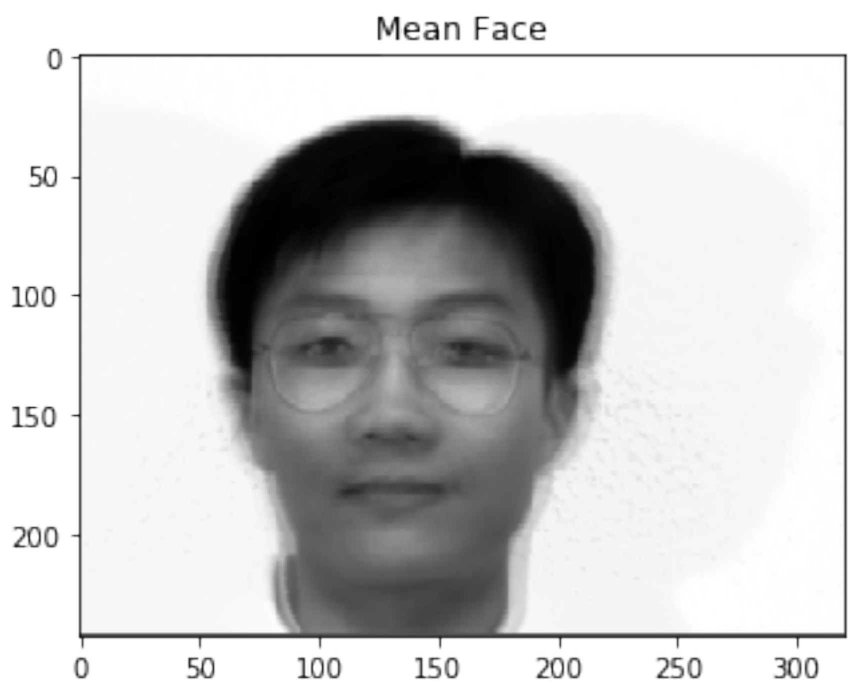
```python
import os
train_imgs = np.zeros((0, 243 * 320))

# List all files in a directory using os.listdir
basepath = '/Users/jim/Dropbox (GaTech)/Courses/ISyE6416/Homework/Homework5/yalefaces'
train_images = []
test_image = []
for entry in os.listdir(basepath):
    if os.path.isfile(os.path.join(basepath, entry)):
        if "14" in entry and "test" not in entry:
            train_images.append(entry)
        elif "test" in entry:
            test_image.append(entry)

for i in range(len(train_images)):
    im = Image.open("/Users/jim/Dropbox (GaTech)/Courses/ISyE6416/Homework/Homework5/yalefaces/{}".format(train_images[i]))
    copy = train_imgs.copy()
    arr = mpimg.pil_to_array(im)
    train_imgs = np.concatenate((copy, arr.reshape(1, 243 * 320)), axis = 0)

# mean
mean_train = np.mean(train_imgs, axis = 0)
mean_train = mean_train.reshape(mean_train.shape[0], 1)
# eigenv, eigenvalues
s_diff, d_diff, v_diff = faceSVD(train_imgs)
s_diff, d_diff, v_diff = s_diff[::-1], d_diff[::-1], v_diff[::-1]
```

(1) Mean Face

Reference is from [Face Recognition Using Eigenfaces (https://www.cin.ufpe.br/~rps/Artigos/Face%20Recogni

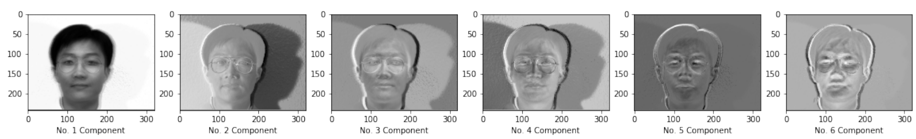
```
plt.imshow(mean_train.reshape(243, 320), cmap = 'Greys
_r')
plt.title('Mean Face')
```

```
Text(0.5, 1.0, 'Mean Face')
```



(2) First Six Eigenfaces

```
v_diff.T.shape
plt.figure(figsize = (20,20))
plot_index = 1

for i in range(6):
    rec = v_diff.T[:,[i]]
    rec = rec.reshape(243, 320)
    plt.subplot(1, 6, plot_index)
    plot_index += 1
    plt.xlabel("No. "+ str(i + 1)+ " Component")
    plt.imshow(rec, cmap = 'Greys_r')
```



From first six eigenfaces, we can know the first eigneface can nearly represent a person. And, with larger eigenfaces, gradually, the infomation is not derived so much. Maybe the second component represents shadow and the third one represents light.

(b) Now use subject14.test.gif to perform face recognition using the following procedure.

Outline of Face Recognition: Given an test image, Γ,

1. $\Gamma' = \Gamma - \Psi \; (mean \; face)$
2. $w_1 = \mu_1^T \Gamma'$

The weights form a vctor w that represents

contribution of each eigenface in representing the input image. So it transform input image into eigenspace.

For the question, are we able to recognize the person correctly using the first principle component? The answer is Yes.

Since the first eigenface represents larger variance that means we have more infomation by projection onto first eigenface, we could clearly know the scores of the weight.

For Facial Recognition, the simplest method for determining which face class provides the best description of an input face image is based on facial images in database. Then we project these images onto the first component to get their weights. Then,

we use Eucleadian distance, $||w - w_{database}|| < \delta$.

If the distance is below δ, we classify this image as

similar. Otherwise, it is assigned as Unknown.

```
first_eigenface = v_diff[0].reshape(77760, 1)
```

```
im = Image.open("/Users/jim/Dropbox (GaTech)/Courses/ISyE6416/Homework/Homework5/yalefaces/subject14.test.gif")
arr = mpimg.pil_to_array(im)

# difference
mean_train_reshape = mean_train.reshape(243, 320)
diff = np.subtract(arr, mean_train_reshape).reshape(77760, 1)

#weights
weights = np.abs(first_eigenface.T @ diff)
weights
```

```
array([[898.12671473]])
```

From the score 898.1267, we use this to compare each image's weight. If we find the distance below δ, we will detect this image. Otherwise, this image is unknown. For example, if the weight of an image in our database is 800 and we define δ as 100. So we could know these two images are similar.

Problem (3) Recommender systems

```
df_movie = pd.read_excel('Movie.xlsx')
```

Data Clearning

```
df_movie.head()
```

	For favourites 1 identifies the choice(s). For ratings 5 is the best and 1 is the worst. (I have flipped the results from the survey to be consistent.) I left unfilled ones as 0. -H	Unnamed: 1	Unnamed: 2	Unnamed: 3	Unnam
0	Gender	NaN	Male	Male	Male
1	Favourite Colour	NaN	Blue	Orange	Black
2	Favourite Genres	NaN	NaN	NaN	NaN
3	Drama	NaN	1	0	0
4	Action	NaN	0	1	0

5 rows × 48 columns

```
df_movie.rename(columns = {'For favourites 1 identifies the choice(s). For ratings 5 is the best and 1 is the worst. (I have flipped the results from the survey to be consistent.) I left unfilled ones as 0.  -H':'Ratings'}, inplace = True)
```

```
userList = ["User{}".format(i + 1) for i in range(df_movie.iloc[:36, 1:].shape[1])]
```

```
df_movie.set_index("Ratings", inplace = True)
```

```
subDF = df_movie.iloc[:36, 1:]
dict_user = {i : j for i, j in zip(subDF.columns, userList)}
subDF = subDF.rename(columns = dict_user).drop(['Favourite Genres', 'Movie Ratings'])
```

Strategy for using User-based Collaboritive Filter:

1. in this dataset, 0 means null values that users leave this block blank. But when we calculate these values will affect our similarity functions.
2. for favorite genres, 0 means dislike or no preference that users do not have inclination toward this kind of genre. So in this field, 0 has meaning and is not a null value. So we would not change values of this one.
3. for movie ratings, 0s means null values, but it will create misunderstanding between other zero values that have meaning. So in this situation, we will change 0 to -1 to differentiate.
4. calculate similarity scores to get similar scores\
   And for missing values, we will not use this data point to get similar score if each of them has -1 value.
5. We will select top 1 user who is similar to the picked user. If the top 1 user has null values same as the picked user's. We will pick the second highest user, and so on.

```python
row_list = list(subDF.T.columns)
preprocess = subDF.copy()
gender = pd.get_dummies(preprocess.iloc[0,:])
colors = pd.get_dummies(preprocess.iloc[1])
movies = preprocess.iloc[14:, :]
movies_numpy = movies.to_numpy()
for i in range(movies_numpy.shape[0]):
    for j in range(movies_numpy.shape[1]):
        if movies_numpy[i][j] == 0:
            movies_numpy[i][j] = -1
```

```python
middle = preprocess.append(gender.T).append(colors.T)
updated_columns = list(middle.T.columns)
for i in range(34):
    updated_columns[i] = row_list[i]
middle.set_axis(updated_columns, axis = 0)
rearrange = middle.T.columns
index_list = []
for i in range(len(rearrange)):
    if rearrange[i] == "Female":
        index_list.append(i)
    if i == len(rearrange) - 1:
        index_list.append(i)
long_list = rearrange[34:].append(rearrange[2:index_li
st[0]])
long_list
final = middle.reindex(long_list)
final = final.drop(['Female']).rename(index = {'Male':
"Gender"})
```

```
/opt/anaconda3/lib/python3.7/site-packages/ipykernel_launch
er.py:5: FutureWarning: set_axis currently defaults to oper
ating inplace.
This will change in a future version of pandas, use inplace
=True to avoid this warning.
  """
```

```
final.head()
```

	User1	User2	User3	User4	User5	User6	User7
Gender	1	1	1	1	1	1	0
Black	0	0	1	0	0	0	0
Blue	1	0	0	1	1	0	1
Green	0	0	0	0	0	1	0
Orange	0	1	0	0	0	0	0

5 rows × 46 columns

Introduction to three similar functions

(1) L-2 norm: $||x_i - x||_2 = (\sum_{i=1}^{n}(x_i - x)^2)^{\frac{1}{2}}$ \ (2) L-1 norm: $||x_i - x||_1 = \sum_{i=1}^{n} |x_i - x|$\ (3) L-0 norm(Hamming distance): $||x_i - x||_0 = 1\ , if\ x_i \neq x\ \ , 0\ otherwise.$

```
from numpy import linalg as LA
## handle missing values to deal with similarity
def sim1(u, v):
    '''
    L2-Norm
    '''
    diff = np.zeros(len(u))
    for i in range(len(diff)):
        if u[i] == -1 or v[i] == -1:
            diff[i] = -1
```

```
            diff[i] = -1

        else:
            diff[i] = abs(u[i] - v[i])
    diff = diff[diff >= 0]
    return np.exp(-(LA.norm(diff) ** 2))

def sim2(u, v):
    '''
    L1-Norm
    '''
    u = np.array(u)
    v = np.array(v)
    diff = np.zeros(len(u))
    for i in range(len(diff)):
        if u[i] == -1 or v[i] == -1:
            diff[i] = -1
        else:
            diff[i] = abs(u[i] - v[i])
    diff = diff[diff >= 0]
    return np.exp(-sum(diff) ** 2)

def sim3(u, v):
    '''
    L0-Norm
    '''
    diff = np.zeros(len(u))
    for i in range(len(u)):
        if u[i] == -1 or v[i] == -1:
            diff[i] = -1
        elif u[i] - v[i] == 0:
            diff[i] = 0
        else:
            diff[i] = 1
    return np.exp(- diff.sum() ** 2)
```

(a) User-based collaborative filter

```
final.tail()
```

	User1	User2	User3	User4	User5	User6	Use
The Matrix	5	5	2	3	3	1	2
Goodfellas	2	4	3	-1	-1	2	-1
One Flew over the Cuckoo's Nest	1	4	-1	4	-1	-1	-1
Seven Samural	-1	4	-1	2	-1	-1	-1
Interstellar	5	4	3	-1	5	3	-1

5 rows × 46 columns

(1) Similarity Metric (L-2 Norm)

```
users_similar1 = {}
for i in final.columns:
    users_similar1[i] = {}
    for j in final.drop(columns = [i]).columns:
        users_similar1[i][j] = 0
```

```python
for i in users_similar1.keys():
    for j in users_similar1[i]:
        users_similar1[i][j] = sim1(final.loc[:, i], f
inal.loc[:, j])
```

```python
sortedusers1 = {}
for i in users_similar1.keys():
    sortedusers1[i] = sorted(users_similar1[i].items()
, key = lambda x: x[1], reverse = True)
```

```python
movies_list = final.T.columns[20:]
users_list = final.columns
prediction_sim1 = {i: [] for i in sortedusers1.keys()}
```

```python
# helper function to find index
def findIndex(x):
    l = []
    for i in range(len(x)):
        if x[i] < 0:
            l.append(i)
    return l
```

```
# find ratings
for i in sortedusers1.keys():
    cond = True
    updated_list = list(final[i][20:])
    user_index = 0
    while cond:
        cond = False
        #list users
        first_user = sortedusers1[i][user_index][0]
        # get their ratings
        first_ratings = final[first_user][20:]
        first_ratings_list = first_ratings.values
        #index
        index_list = findIndex(updated_list)
        if len(index_list) > 0:
            for j in range(len(index_list)):
                index = index_list[j]
                updated_list[index] = first_ratings_li
st[index]
            if len(findIndex(updated_list)) > 0:
                cond = True
                user_index = user_index + 1
            else:
                prediction_sim1[i] = updated_list
                break
        else:
            prediction_sim1[i] = updated_list
            break
```

```
userbase_simlar1 = pd.DataFrame(prediction_sim1, index
= movies_list)
```

```
userbase_simlar1
```

	User1	User2	User3	User4	User5	User6	U
The Shawkshank Redemption	3	4	1	5	5	1	5
The Godfather	2	4	4	4	3	2	5
The Dark Knight	3	5	2	3	5	2	5
The Godfather Part II	2	4	1	2	5	3	5
The Lord of the Rings III	2	4	4	3	5	1	5
Pulp Fiction	1	4	2	4	5	1	5
Schindler's List	2	5	2	3	5	2	5
The Good, the Bad and the Ugly	1	3	4	3	4	4	5
12 Angry Men	4	4	4	1	4	5	4
Inception	4	4	1	2	4	1	5
Fight Club	4	4	3	1	3	1	4
The Lord of the Rings I	2	4	1	3	5	1	5
Forrest Group	4	5	2	3	5	2	5
Star Wars V the Empire Strikes Back	2	3	3	2	4	3	3
The Lord of the Rings II	2	3	4	3	5	1	5
The Matrix	5	5	2	3	3	1	2
Goodfellas	2	4	3	3	5	2	5
One Flew over the Cuckoo's Nest	1	4	3	4	4	3	4

Seven Samural	3	4	3	2	3	3	2
Interstellar	5	4	3	5	5	3	5

20 rows × 46 columns

(2) Similarity Metric (L-1 Norm)

```
users_similar2 = {}
for i in final.columns:
    users_similar2[i] = {}
    for j in final.drop(columns = [i]).columns:
        users_similar2[i][j] = 0
```

```
for i in users_similar2.keys():
    for j in users_similar2[i]:
        users_similar2[i][j] = sim2(final.loc[:, i], f
inal.loc[:, j])
```

```
sortedusers2 = {}
for i in users_similar2.keys():
    sortedusers2[i] = sorted(users_similar2[i].items()
, key = lambda x: x[1], reverse = True)
```

```
movies_list = final.T.columns[20:]
users_list = final.columns
prediction_sim2 = {i: [] for i in sortedusers2.keys()}
```

```
# find ratings
for i in sortedusers2.keys():
    cond = True
    updated_list = list(final[i][20:])
    user_index = 0
    while cond:
        cond = False
        #list users
        first_user = sortedusers2[i][user_index][0]
        # get their ratings
        first_ratings = final[first_user][20:]
        first_ratings_list = first_ratings.values
        #index
        index_list = findIndex(updated_list)
        if len(index_list) > 0:
            for j in range(len(index_list)):
                index = index_list[j]
                updated_list[index] = first_ratings_li
st[index]
            if len(findIndex(updated_list)) > 0:
                cond = True
                user_index = user_index + 1
            else:
                prediction_sim2[i] = updated_list
                break
        else:
            prediction_sim2[i] = updated_list
            break
```

```
userbase_simlar2 = pd.DataFrame(prediction_sim2, index
= movies_list)
```

```
userbase_simlar2
```

	User1	User2	User3	User4	User5	User6	U
The Shawkshank Redemption	3	4	1	5	5	1	5
The Godfather	2	4	4	4	3	2	5
The Dark Knight	3	5	2	3	5	2	5
The Godfather Part II	2	4	1	2	5	3	5
The Lord of the Rings III	2	4	4	3	5	1	5
Pulp Fiction	1	4	2	4	5	1	5
Schindler's List	2	5	2	3	5	2	5
The Good, the Bad and the Ugly	1	3	4	3	4	4	5
12 Angry Men	4	4	4	4	4	5	4
Inception	4	4	1	4	4	1	5
Fight Club	4	4	3	3	3	1	4
The Lord of the Rings I	2	4	1	3	5	1	5
Forrest Group	4	5	2	3	5	2	5
Star Wars V the Empire Strikes Back	2	3	4	2	4	3	3
The Lord of the Rings II	2	3	5	3	5	1	5
The Matrix	5	5	2	3	3	1	2
Goodfellas	2	4	3	3	5	2	5
One Flew over the Cuckoo's Nest	1	4	3	4	3	3	4

Seven Samural	3	4	3	2	3	3	3
Interstellar	5	4	3	5	5	3	5

20 rows × 46 columns

(3) Similarity Metric (L-0 Norm)

```
users_similar3 = {}
for i in final.columns:
    users_similar3[i] = {}
    for j in final.drop(columns = [i]).columns:
        users_similar3[i][j] = 0
```

```
for i in users_similar3.keys():
    for j in users_similar3[i]:
        users_similar3[i][j] = sim3(final.loc[:, i], f
inal.loc[:, j])
```

```
sortedusers3 = {}
for i in users_similar3.keys():
    sortedusers3[i] = sorted(users_similar3[i].items()
, key = lambda x: x[1], reverse = True)
```

```
movies_list = final.T.columns[20:]
users_list = final.columns
prediction_sim3 = {i: [] for i in sortedusers3.keys()}
```

```python
# find ratings
for i in sortedusers3.keys():
    cond = True
    updated_list = list(final[i][20:])
    user_index = 0
    while cond:
        cond = False
        #list users
        first_user = sortedusers3[i][user_index][0]
        # get their ratings
        first_ratings = final[first_user][20:]
        first_ratings_list = first_ratings.values
        #index
        index_list = findIndex(updated_list)
        if len(index_list) > 0:
            for j in range(len(index_list)):
                index = index_list[j]
                updated_list[index] = first_ratings_list[index]
            if len(findIndex(updated_list)) > 0:
                cond = True
                user_index = user_index + 1
            else:
                prediction_sim3[i] = updated_list
                break
        else:
            prediction_sim3[i] = updated_list
            break
```

```python
userbase_simlar3 = pd.DataFrame(prediction_sim3, index = movies_list)
```

```python
userbase_simlar3
```

	User1	User2	User3	User4	User5	User6	U
The Shawkshank Redemption	3	4	3	5	5	1	5
The Godfather	2	4	2	4	3	2	5
The Dark Knight	3	5	3	3	5	4	1
The Godfather Part II	2	4	2	2	3	3	5
The Lord of the Rings III	2	4	2	3	5	1	5
Pulp Fiction	1	4	1	4	4	1	1
Schindler's List	2	5	2	3	4	2	5
The Good, the Bad and the Ugly	1	3	4	3	4	4	1
12 Angry Men	4	4	4	4	3	5	1
Inception	4	4	4	5	5	1	5
Fight Club	4	4	4	5	5	1	1
The Lord of the Rings I	2	4	1	3	4	1	5
Forrest Group	4	5	2	3	5	2	1
Star Wars V the Empire Strikes Back	2	3	2	2	4	3	1
The Lord of the Rings II	2	3	2	3	5	1	5
The Matrix	5	5	2	3	3	1	2
Goodfellas	2	4	3	5	3	2	1
One Flew over the Cuckoo's Nest	1	4	1	4	3	4	3

Seven Samural	3	4	3	2	3	3	3
Interstellar	5	4	3	5	5	3	3

20 rows × 46 columns

(b) Item-based collaborative filter

Item-based recommendation to recommend 5 movies to each user. try using the above three metrics respectively.

Strategy for Item-based Collaborative filter

For item-based recommender systems, we won't use user's favorite color, gender or movie genre becasue those things are specific to user's personality. For item-based tasks, we only use ratings from our products and then try to predict a rating of a movie.

```
itembaseDF = final.T.iloc[:, 20:]
```

```python
itembaseDF.head()
```

	The Shawkshank Redemption	The Godfather	The Dark Knight	The Godfather Part II	The Lord of the Rings III
User1	3	2	3	2	2
User2	4	4	5	4	4
User3	-1	-1	-1	-1	-1
User4	5	4	3	2	3
User5	-1	3	5	-1	5

(1) Similarity Metric (L-2 Norm) For Item-Based

```python
users_similar1_item = {}
for i in itembaseDF.columns:
    users_similar1_item[i] = {}
    for j in itembaseDF.drop(columns = [i]).columns:
        users_similar1_item[i][j] = 0
```

```python
for i in users_similar1_item.keys():
    for j in users_similar1_item[i]:
        users_similar1_item[i][j] = sim1(itembaseDF.lo
c[:, i], itembaseDF.loc[:, j])
```

```
sortedusers1_item = {}
for i in users_similar1_item.keys():
    sortedusers1_item[i] = sorted(users_similar1_item[i].items(), key = lambda x: x[1], reverse = True)
```

```
movies_list_item = itembaseDF.columns
users_list_item = itembaseDF.T.columns
prediction_sim1_item = {i: [] for i in sortedusers1_item.keys()}
```

```
# find ratings
for i in sortedusers1_item.keys():
    cond = True
    updated_list = list(itembaseDF[i])
    movie_index = 0
    while cond:
        cond = False
        #list movie
        first_moive = sortedusers1_item[i][movie_index
][0]
        # get their ratings
        first_ratings = itembaseDF[first_moive]
        first_ratings_list = first_ratings.values
        #index
        index_list = findIndex(updated_list)
        if len(index_list) > 0:
            for j in range(len(index_list)):
                index = index_list[j]
                updated_list[index] = first_ratings_li
st[index]
            if len(findIndex(updated_list)) > 0:
                cond = True
                movie_index = movie_index + 1
            else:
                prediction_sim1_item[i] = updated_list
                break
        else:
            prediction_sim1_item[i] = updated_list
            break
```

```
itembase_simlar1 = pd.DataFrame(prediction_sim1_item,
index = users_list_item)
```

```
itembase_simlar1
```

	The Shawkshank Redemption	The Godfather	The Dark Knight	The Godfather Part II	The Lord of the Rings II
User1	3	2	3	2	2
User2	4	4	5	4	4
User3	3	3	3	3	1
User4	5	4	3	2	3
User5	3	3	5	3	5
User6	1	2	1	3	1
User7	5	5	5	5	5
User8	4	5	4	5	3
User9	5	5	5	5	3
User10	4	4	5	4	4
User11	2	2	4	2	3
User12	5	5	5	5	5
User13	5	5	5	3	5
User14	2	2	2	1	6
User15	2	1	2	1	3
User16	5	5	5	5	5
User17	5	5	5	5	5
User18	5	5	4	5	4
User19	3	3	4	4	5
User20	5	5	5	5	5
User21	4	4	3	4	3
User22	3	5	5	4	5

User23	5	3	4	3	4
User24	5	4	5	4	3
User25	2	2	3	2	2
User26	3	5	5	5	5
User27	5	5	5	5	4
User28	1	2	2	2	4
User29	3	2	1	3	2
User30	1	1	2	1	4
User31	5	5	4	5	3
User32	5	5	1	5	4
User33	1	1	2	1	1
User34	1	2	2	2	2
User35	5	4	5	4	4
User36	5	4	4	4	5
User37	5	5	1	5	4
User38	1	1	2	1	1
User39	5	5	5	5	5
User40	5	5	4	5	5
User41	2	2	3	3	2
User42	4	4	5	3	4
User43	3	3	4	3	4
User44	5	5	5	5	5
User45	5	3	4	3	4
User46	5	5	5	4	5

(2) Similarity Metric (L-1 Norm) For Item-Based

```
users_similar2_item = {}
for i in itembaseDF.columns:
    users_similar2_item[i] = {}
    for j in itembaseDF.drop(columns = [i]).columns:
        users_similar2_item[i][j] = 0
```

```
for i in users_similar2_item.keys():
    for j in users_similar2_item[i]:
        users_similar2_item[i][j] = sim2(itembaseDF.lo
c[:, i], itembaseDF.loc[:, j])
```

```
sortedusers2_item = {}
for i in users_similar2_item.keys():
    sortedusers2_item[i] = sorted(users_similar2_item[
i].items(), key = lambda x: x[1], reverse = True)
```

```
movies_list_item = itembaseDF.columns
users_list_item = itembaseDF.T.columns
prediction_sim2_item = {i: [] for i in sortedusers2_it
em.keys()}
```

```
# find ratings
for i in sortedusers2_item.keys():
    cond = True
    updated_list = list(itembaseDF[i])
    movie_index = 0
    while cond:
        cond = False
        #list movie
        first_moive = sortedusers2_item[i][movie_index
][0]
        # get their ratings
        first_ratings = itembaseDF[first_moive]
        first_ratings_list = first_ratings.values
        #index
        index_list = findIndex(updated_list)
        if len(index_list) > 0:
            for j in range(len(index_list)):
                index = index_list[j]
                updated_list[index] = first_ratings_li
st[index]
            if len(findIndex(updated_list)) > 0:
                cond = True
                movie_index = movie_index + 1
            else:
                prediction_sim2_item[i] = updated_list
                break
        else:
            prediction_sim2_item[i] = updated_list
            break
```

```
itembase_simlar2 = pd.DataFrame(prediction_sim2_item,
index = users_list_item)
```

```
itembase_simlar2
```

	The Shawkshank Redemption	The Godfather	The Dark Knight	The Godfather Part II	The Lord of the Rings II
User1	3	2	3	2	2
User2	4	4	5	4	4
User3	3	3	3	3	1
User4	5	4	3	2	3
User5	4	3	5	3	5
User6	1	2	1	3	1
User7	5	5	5	5	5
User8	4	5	4	5	3
User9	3	5	5	5	3
User10	4	4	5	4	4
User11	2	2	4	4	3
User12	5	5	5	5	5
User13	5	5	5	3	5
User14	2	2	2	1	6
User15	2	1	2	1	3
User16	5	5	5	5	5
User17	5	5	5	5	5
User18	5	5	4	5	4
User19	3	3	4	5	5
User20	5	5	4	5	5
User21	4	4	3	4	3
User22	3	5	5	4	5

User23	5	3	4	3	4
User24	5	4	5	4	3
User25	2	2	3	2	2
User26	3	5	5	5	5
User27	5	5	5	5	4
User28	1	2	2	1	4
User29	3	2	1	3	2
User30	1	1	2	1	4
User31	2	5	2	2	3
User32	5	5	4	5	4
User33	1	1	2	1	1
User34	1	2	2	2	2
User35	5	4	5	4	4
User36	5	4	4	4	5
User37	5	5	1	5	4
User38	1	1	2	1	1
User39	5	5	5	5	5
User40	5	5	4	5	5
User41	2	2	3	3	2
User42	4	4	5	3	4
User43	3	3	4	3	4
User44	5	5	5	5	5
User45	5	3	4	3	4
User46	5	5	5	4	5

(3) Similarity Metric (L-0 Norm) For Item-Based

```python
users_similar3_item = {}
for i in itembaseDF.columns:
    users_similar3_item[i] = {}
    for j in itembaseDF.drop(columns = [i]).columns:
        users_similar3_item[i][j] = 0
```

```python
for i in users_similar3_item.keys():
    for j in users_similar3_item[i]:
        users_similar3_item[i][j] = sim3(itembaseDF.loc[:, i], itembaseDF.loc[:, j])
```

```python
sortedusers3_item = {}
for i in users_similar3_item.keys():
    sortedusers3_item[i] = sorted(users_similar3_item[i].items(), key = lambda x: x[1], reverse = True)
```

```python
movies_list_item = itembaseDF.columns
users_list_item = itembaseDF.T.columns
prediction_sim3_item = {i: [] for i in sortedusers3_item.keys()}
```

```
# find ratings
for i in sortedusers3_item.keys():
    cond = True
    updated_list = list(itembaseDF[i])
    movie_index = 0
    while cond:
        cond = False
        #list movie
        first_moive = sortedusers3_item[i][movie_index
][0]
        # get their ratings
        first_ratings = itembaseDF[first_moive]
        first_ratings_list = first_ratings.values
        #index
        index_list = findIndex(updated_list)
        if len(index_list) > 0:
            for j in range(len(index_list)):
                index = index_list[j]
                updated_list[index] = first_ratings_li
st[index]
            if len(findIndex(updated_list)) > 0:
                cond = True
                movie_index = movie_index + 1
            else:
                prediction_sim3_item[i] = updated_list
                break
        else:
            prediction_sim3_item[i] = updated_list
            break
```

```
itembase_simlar3 = pd.DataFrame(prediction_sim3_item,
index = users_list_item)
```

```
itembase_simlar3
```

	The Shawkshank Redemption	The Godfather	The Dark Knight	The Godfather Part II	The Lord of the Rings II
User1	3	2	3	2	2
User2	4	4	5	4	4
User3	2	2	2	2	2
User4	5	4	3	2	3
User5	3	3	5	5	5
User6	1	2	1	3	1
User7	5	2	5	5	5
User8	4	5	4	5	3
User9	4	5	5	5	3
User10	4	4	5	5	3
User11	2	2	4	4	2
User12	5	5	5	5	5
User13	4	3	5	3	5
User14	2	2	2	1	6
User15	2	1	2	1	3
User16	5	5	5	5	5
User17	5	5	5	5	5
User18	5	5	5	5	5
User19	1	3	4	3	5
User20	5	5	5	5	5
User21	4	4	4	4	4
User22	3	5	5	4	5

User23	5	3	4	3	4
User24	5	4	5	3	3
User25	2	3	3	3	3
User26	5	5	5	5	5
User27	5	5	5	5	4
User28	1	1	2	1	4
User29	3	2	1	3	2
User30	3	1	2	1	4
User31	5	5	3	5	3
User32	5	5	4	4	4
User33	1	1	2	1	1
User34	2	1	1	1	1
User35	5	4	5	3	4
User36	5	4	5	4	5
User37	5	5	1	5	4
User38	1	1	2	1	1
User39	5	4	5	5	4
User40	5	5	4	5	5
User41	2	2	3	3	2
User42	4	4	5	3	4
User43	3	3	4	3	4
User44	5	5	5	5	5
User45	5	3	4	3	4
User46	4	5	4	4	5

(C) Soft-Impute

Using matrix completion algorithm based on soft-impute ([R package (https://cran.r-project.org/web/packages/softImpute/softImpute.pdf)](https://cran.r-project.org/web/packages/softImpute/softImpute.pdf)) to fill out missing entries to recommend 5 movies to each user.

```
from fancyimpute import SoftImpute
```

```
soft_imputeDF = subDF.iloc[14:, :].copy()
```

```
soft_imputeDF.head()
```

Ratings	User1	User2	User3	User4	User5	User6	U
The Shawkshank Redemption	3	4	0	5	0	1	5
The Godfather	2	4	0	4	3	2	0
The Dark Knight	3	5	0	3	5	0	0
The Godfather Part II	2	4	0	2	0	3	0
The Lord of the Rings III	2	4	0	3	5	1	5

5 rows × 46 columns

```
soft_impute_numpy = soft_imputeDF.to_numpy().astype(float)
for i in range(soft_impute_numpy.shape[0]):
    for j in range(soft_impute_numpy.shape[1]):
        if soft_impute_numpy[i][j] == 0:
            soft_impute_numpy[i][j] = np.nan
        else:
            soft_impute_numpy[i][j] = int(soft_impute_numpy[i][j])
```

```
X_filled_softimpute = SoftImpute().fit_transform(soft_impute_numpy)
```

```
[SoftImpute] Max Singular Value of X_init = 67.040454
[SoftImpute] Iter 1: observed MAE=0.189442 rank=20
[SoftImpute] Iter 2: observed MAE=0.192229 rank=20
[SoftImpute] Iter 3: observed MAE=0.194827 rank=20
```

```
[SoftImpute] Iter 4: observed MAE=0.197333 rank=20
[SoftImpute] Iter 5: observed MAE=0.199469 rank=19
[SoftImpute] Iter 6: observed MAE=0.199661 rank=19
[SoftImpute] Iter 7: observed MAE=0.200599 rank=19
[SoftImpute] Iter 8: observed MAE=0.201076 rank=18
[SoftImpute] Iter 9: observed MAE=0.201310 rank=18
[SoftImpute] Iter 10: observed MAE=0.201876 rank=18
[SoftImpute] Iter 11: observed MAE=0.202606 rank=18
[SoftImpute] Iter 12: observed MAE=0.203259 rank=17
[SoftImpute] Iter 13: observed MAE=0.202965 rank=17
[SoftImpute] Iter 14: observed MAE=0.202931 rank=17
[SoftImpute] Iter 15: observed MAE=0.202683 rank=16
[SoftImpute] Iter 16: observed MAE=0.202351 rank=16
[SoftImpute] Iter 17: observed MAE=0.202299 rank=16
[SoftImpute] Iter 18: observed MAE=0.202346 rank=16
[SoftImpute] Iter 19: observed MAE=0.202483 rank=16
[SoftImpute] Iter 20: observed MAE=0.202636 rank=16
[SoftImpute] Iter 21: observed MAE=0.202762 rank=16
[SoftImpute] Iter 22: observed MAE=0.202859 rank=16
[SoftImpute] Iter 23: observed MAE=0.202911 rank=16
[SoftImpute] Iter 24: observed MAE=0.202982 rank=16
[SoftImpute] Iter 25: observed MAE=0.202911 rank=15
[SoftImpute] Iter 26: observed MAE=0.202547 rank=15
[SoftImpute] Iter 27: observed MAE=0.202351 rank=15
[SoftImpute] Iter 28: observed MAE=0.202301 rank=15
[SoftImpute] Iter 29: observed MAE=0.202310 rank=15
[SoftImpute] Iter 30: observed MAE=0.202310 rank=15
[SoftImpute] Iter 31: observed MAE=0.202297 rank=15
[SoftImpute] Iter 32: observed MAE=0.202281 rank=15
[SoftImpute] Iter 33: observed MAE=0.202240 rank=15
[SoftImpute] Iter 34: observed MAE=0.202181 rank=15
[SoftImpute] Iter 35: observed MAE=0.202112 rank=15
[SoftImpute] Iter 36: observed MAE=0.202067 rank=15
[SoftImpute] Iter 37: observed MAE=0.202051 rank=15
[SoftImpute] Iter 38: observed MAE=0.202034 rank=15
[SoftImpute] Iter 39: observed MAE=0.202010 rank=15
[SoftImpute] Iter 40: observed MAE=0.201994 rank=15
[SoftImpute] Iter 41: observed MAE=0.201993 rank=15
[SoftImpute] Iter 42: observed MAE=0.202005 rank=15
[SoftImpute] Iter 43: observed MAE=0.202007 rank=15
[SoftImpute] Iter 44: observed MAE=0.202005 rank=15
[SoftImpute] Iter 45: observed MAE=0.202024 rank=15
[SoftImpute] Iter 46: observed MAE=0.202053 rank=15
[SoftImpute] Iter 47: observed MAE=0.202071 rank=15
[SoftImpute] Iter 48: observed MAE=0.202082 rank=15
```

```
[SoftImpute] Iter 49: observed MAE=0.202084 rank=15
[SoftImpute] Iter 50: observed MAE=0.202078 rank=15
[SoftImpute] Iter 51: observed MAE=0.202065 rank=15
[SoftImpute] Iter 52: observed MAE=0.202050 rank=15
[SoftImpute] Iter 53: observed MAE=0.202046 rank=15
[SoftImpute] Iter 54: observed MAE=0.202049 rank=15
[SoftImpute] Iter 55: observed MAE=0.202059 rank=15
[SoftImpute] Iter 56: observed MAE=0.202064 rank=15
[SoftImpute] Iter 57: observed MAE=0.202068 rank=15
[SoftImpute] Iter 58: observed MAE=0.202087 rank=15
[SoftImpute] Iter 59: observed MAE=0.202109 rank=15
[SoftImpute] Iter 60: observed MAE=0.202130 rank=15
[SoftImpute] Iter 61: observed MAE=0.202147 rank=15
[SoftImpute] Iter 62: observed MAE=0.202161 rank=15
[SoftImpute] Iter 63: observed MAE=0.202172 rank=15
[SoftImpute] Iter 64: observed MAE=0.202180 rank=15
[SoftImpute] Iter 65: observed MAE=0.202187 rank=15
[SoftImpute] Iter 66: observed MAE=0.202192 rank=15
[SoftImpute] Iter 67: observed MAE=0.202196 rank=15
[SoftImpute] Iter 68: observed MAE=0.202197 rank=15
[SoftImpute] Iter 69: observed MAE=0.202197 rank=15
[SoftImpute] Iter 70: observed MAE=0.202196 rank=15
[SoftImpute] Iter 71: observed MAE=0.202193 rank=15
[SoftImpute] Iter 72: observed MAE=0.202189 rank=15
[SoftImpute] Iter 73: observed MAE=0.202184 rank=15
[SoftImpute] Iter 74: observed MAE=0.202178 rank=15
[SoftImpute] Iter 75: observed MAE=0.202176 rank=15
[SoftImpute] Iter 76: observed MAE=0.202176 rank=15
[SoftImpute] Iter 77: observed MAE=0.202180 rank=15
[SoftImpute] Iter 78: observed MAE=0.202184 rank=15
[SoftImpute] Iter 79: observed MAE=0.202188 rank=15
[SoftImpute] Iter 80: observed MAE=0.202190 rank=15
[SoftImpute] Iter 81: observed MAE=0.202192 rank=15
[SoftImpute] Iter 82: observed MAE=0.202193 rank=15
[SoftImpute] Iter 83: observed MAE=0.202194 rank=15
[SoftImpute] Iter 84: observed MAE=0.202193 rank=15
[SoftImpute] Iter 85: observed MAE=0.202193 rank=15
[SoftImpute] Iter 86: observed MAE=0.202191 rank=15
[SoftImpute] Iter 87: observed MAE=0.202191 rank=15
[SoftImpute] Iter 88: observed MAE=0.202190 rank=15
[SoftImpute] Iter 89: observed MAE=0.202188 rank=15
[SoftImpute] Iter 90: observed MAE=0.202187 rank=15
[SoftImpute] Iter 91: observed MAE=0.202184 rank=15
[SoftImpute] Iter 92: observed MAE=0.202182 rank=15
[SoftImpute] Iter 93: observed MAE=0.202179 rank=15
```

```
[SoftImpute] Iter 94: observed MAE=0.202180 rank=15
[SoftImpute] Iter 95: observed MAE=0.202179 rank=15
[SoftImpute] Iter 96: observed MAE=0.202179 rank=15
[SoftImpute] Iter 97: observed MAE=0.202178 rank=15
[SoftImpute] Iter 98: observed MAE=0.202177 rank=15
[SoftImpute] Iter 99: observed MAE=0.202176 rank=15
[SoftImpute] Iter 100: observed MAE=0.202175 rank=15
[SoftImpute] Stopped after iteration 100 for lambda=1.34080
9
```

```
X_filled_softimpute.shape
```

```
(20, 46)
```

```
softimputeDF = pd.DataFrame(X_filled_softimpute, index
= movies_list_item, columns = users_list_item)
```

```
softimputeDF.T
```

	The Shawkshank Redemption	The Godfather	The Dark Knight	The Godfather Part II	T Lord t Rin
User1	3.000000	2.000000	3.000000	2.000000	2.000
User2	4.000000	4.000000	5.000000	4.000000	4.000
User3	2.293802	1.684446	2.430176	2.072020	1.630
User4	5.000000	4.000000	3.000000	2.000000	3.000
User5	4.308896	3.000000	5.000000	3.073260	5.000
User6	1.000000	2.000000	2.337693	3.000000	1.000

User7	5.000000	4.156609	3.439809	3.342418	5.000
User8	4.000000	5.000000	4.000000	5.000000	3.000
User9	4.097706	5.000000	5.000000	5.000000	3.000
User10	4.000000	4.000000	5.000000	3.436901	3.636
User11	2.438638	2.154873	4.000000	2.110646	1.884
User12	5.000000	4.672840	5.000000	4.365906	4.089
User13	3.998227	4.075333	5.000000	3.379923	5.000
User14	2.000000	2.000000	2.000000	1.000000	6.000
User15	2.000000	1.000000	2.000000	1.000000	2.229
User16	5.000000	5.000000	5.395027	5.000000	4.877
User17	5.000000	5.000000	5.230700	4.102511	4.306
User18	5.000000	4.209962	4.450872	3.603366	3.806
User19	3.368895	4.106221	4.000000	3.396451	5.000
User20	4.306480	5.000000	4.175956	4.064943	5.000
User21	4.000000	4.000000	3.719360	3.526581	3.220
User22	3.000000	5.000000	5.000000	4.000000	5.000
User23	5.000000	3.000000	4.000000	3.000000	4.000
User24	5.000000	4.000000	5.000000	4.031797	3.000
User25	2.000000	1.997692	3.000000	2.073027	1.887
User26	4.104244	4.697483	5.000000	5.000000	5.000
User27	5.006686	5.000000	5.053753	5.000000	4.000
User28	1.000000	1.752216	2.000000	1.775376	4.000
User29	3.000000	2.000000	1.000000	3.000000	2.000
User30	2.309786	1.000000	2.000000	1.000000	4.000
User31	3.995215	3.436156	3.978688	3.037577	3.000
User32	3.246343	5.000000	3.129920	3.557035	4.000
User33	1.000000	1.073446	2.000000	1.028290	1.000
User34	1.328752	1.188661	1.793580	1.179554	1.254
User35	5.000000	4.000000	5.000000	3.644435	4.000
User36	5.000000	4.000000	3.939746	3.369916	5.000
User37	5.000000	5.000000	1.000000	5.000000	4.000

User38	1.000000	1.136482	1.700287	1.263552	1.196
User39	5.000000	3.874364	5.000000	3.435772	3.818
User40	5.000000	5.000000	4.000000	5.000000	5.000
User41	2.000000	2.000000	3.000000	3.000000	2.000
User42	4.000000	4.000000	5.000000	3.000000	4.032
User43	3.000000	3.533603	4.000000	2.975863	4.000
User44	5.000000	5.000000	5.000000	5.000000	5.000
User45	5.000000	3.000000	4.000000	3.000000	4.000
User46	4.683009	5.000000	5.249357	4.000000	5.000

Summary: Recommendation Movies to Users

From the three methods, sometimes L2 and L1 norm have same values. But L0 norm has more different values. And, for the soft impute method, approximate values are not exact integers.

```
## to get excel workbook each sheet
columns = []
for i in range(5):
    columns.append("Recommended Movie {}".format(i + 1
))
    columns.append('Score {}'.format(i + 1))
user_list = users_list_item.copy()
```

(a) User-based Recommender Systems

```
## first l2 norm
l2_excel = {}
for i in range(len(user_list)):
    l = []
    for index, value in userbase_simlar1.T.iloc[i, :].
sort_values(ascending=False)[:5].items():
        l.append(index)
        l.append(value)
    l2_excel[user_list[i]] = l

l1_excel = {}
for i in range(len(user_list)):
    l = []
    for index, value in userbase_simlar2.T.iloc[i, :].
sort_values(ascending=False)[:5].items():
        l.append(index)
        l.append(value)
    l1_excel[user_list[i]] = l

l0_excel = {}
for i in range(len(user_list)):
    l = []
    for index, value in userbase_simlar3.T.iloc[i, :].
sort_values(ascending=False)[:5].items():
        l.append(index)
        l.append(value)
    l0_excel[user_list[i]] = l
```

(b) Item-based Recommender Systems

```
## first l2 norm
l2_excel_i = {}
for i in range(len(user_list)):
    l = []
    for index, value in itembase_simlar1.iloc[i, :].so
rt_values(ascending=False)[:5].items():
        l.append(index)
        l.append(value)
    l2_excel_i[user_list[i]] = l

l1_excel_i = {}
for i in range(len(user_list)):
    l = []
    for index, value in itembase_simlar2.iloc[i, :].so
rt_values(ascending=False)[:5].items():
        l.append(index)
        l.append(value)
    l1_excel_i[user_list[i]] = l

l0_excel_i = {}
for i in range(len(user_list)):
    l = []
    for index, value in itembase_simlar3.iloc[i, :].so
rt_values(ascending=False)[:5].items():
        l.append(index)
        l.append(value)
    l0_excel_i[user_list[i]] = l
```

(c) Soft-Impute

Since there are some values in columns that are over 5, this does not make sense since those values are from one to five. So in this situation, if the values are over 5, we will define it as 5.

```
excel_soft = {}
for i in range(len(user_list)):
    l = []
    for index, value in softimputeDF.T.iloc[i, :].sort
_values(ascending=False)[:5].items():
        l.append(index)
        if round(value, 2) > 5:
            l.append(5)
        else:
            l.append(round(value, 2))
    excel_soft[user_list[i]] = l
```

Finally, put all sheets into one excel workbook

```
df1_u = pd.DataFrame(l2_excel, index = columns)
df2_u = pd.DataFrame(l1_excel, index = columns)
df3_u = pd.DataFrame(l0_excel, index = columns)
df1_u_T = df1_u.T.copy()
df2_u_T = df2_u.T.copy()
df3_u_T = df3_u.T.copy()
```

```
combined_u = df1_u_T.append(df2_u_T).append(df3_u_T)
```

```
df1_i = pd.DataFrame(l2_excel_i, index = columns)
df2_i = pd.DataFrame(l1_excel_i, index = columns)
df3_i = pd.DataFrame(l0_excel_i, index = columns)
df1_i_T = df1_i.T.copy()
df2_i_T = df2_i.T.copy()
df3_i_T = df3_i.T.copy()
combined_i = df1_i_T.append(df2_i_T).append(df3_i_T)
```

```
df_soft = pd.DataFrame(excel_soft, index = columns)
```

```
df_soft_T = df_soft.T.copy()
```

```
with pd.ExcelWriter('RecommendSystems.xlsx') as writer
:
    combined_u.to_excel(writer, sheet_name='User_Based
')
    combined_i.to_excel(writer, sheet_name='Item_Based
')
    df_soft_T.to_excel(writer, sheet_name='Soft Impute
')
```