ISYE 6402 – Fall 2020 - Georgia Institute of Technology

Final Exam

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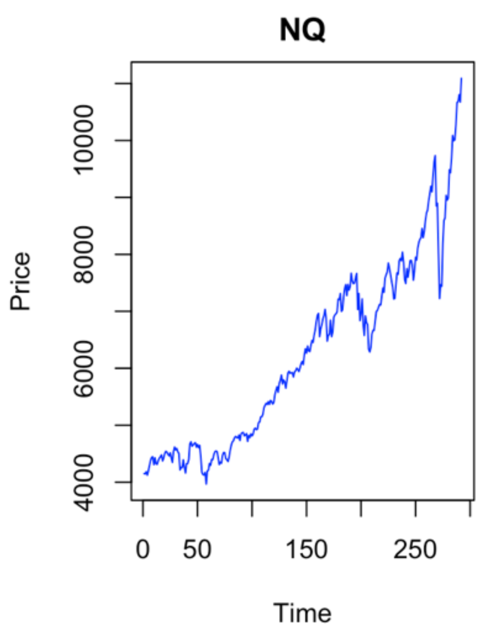
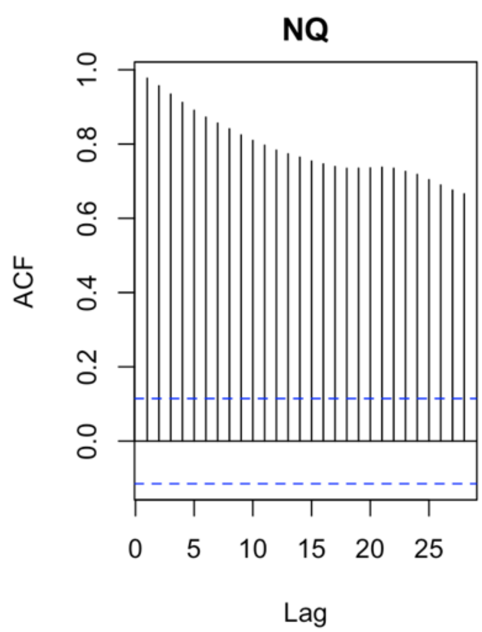
Question 1 Data Exploration and Simple Modeling - 15 Points

Note: ACF plots below built using library that starts charts at 1st lag

1. Use graphical analysis on each dataset as well as its first difference. Comment on any relevant features of the three time series. Are there any similarities?

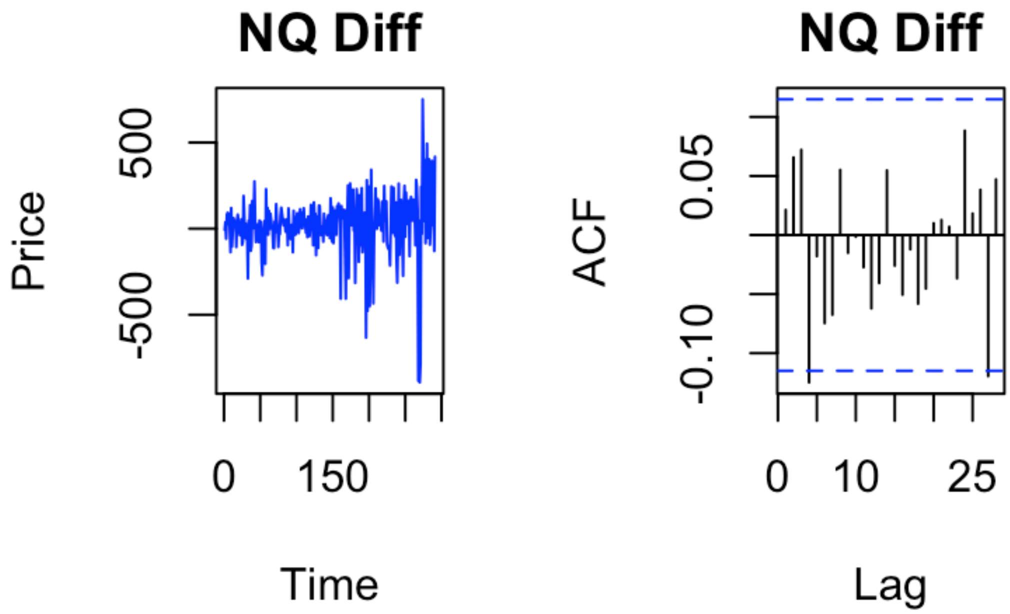
NQ:





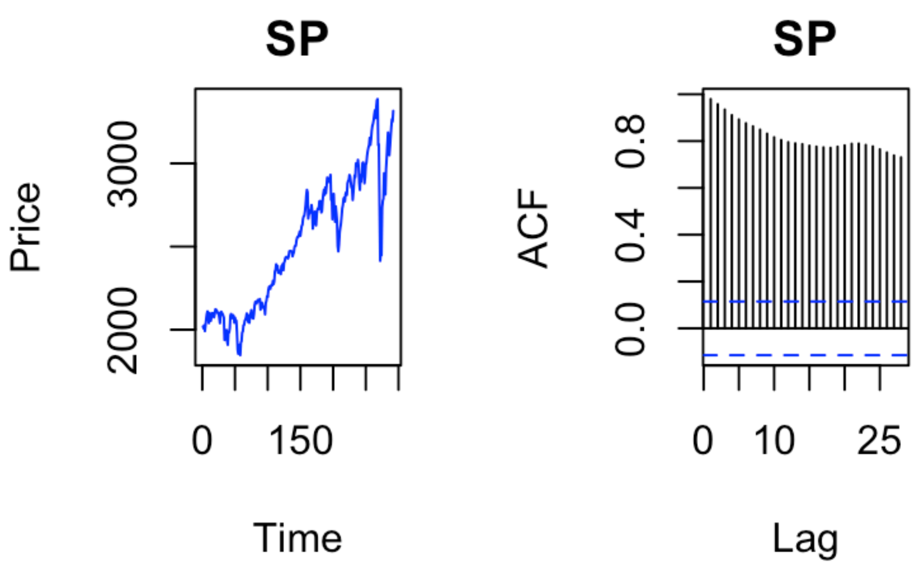
There is a trend and the variance is not constant based on the plots.
From the time series plot, we could see around 260, the variance becomes larger and the mean is not constant. We could also see that ACF has an decreasing trend. Therefore, it violates assumptions of constant mean and variance.

NQ Diff:



Here there is some possibility for heteroscedasticity at the end, but ACF results inconclusive.

SP



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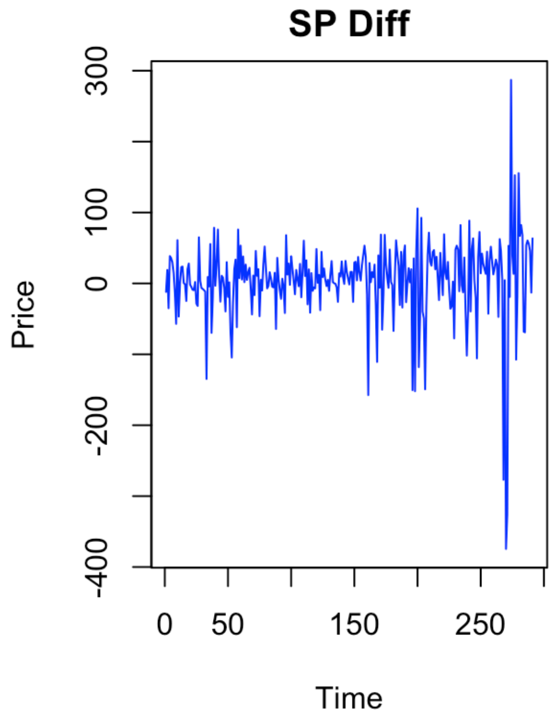
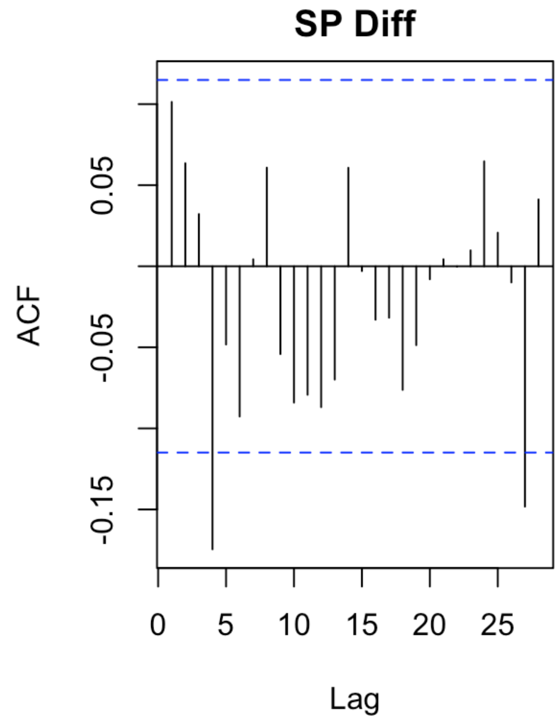
From the time series plot, we could see around 260, the variance becomes larger and the mean is not constant. We could also see that ACF has an decreasing trend. Therefore, it violates assumptions of constant mean and variance.

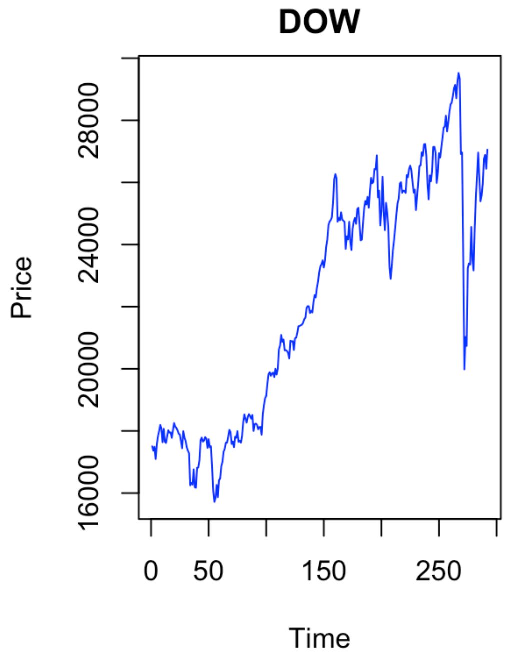
SP DIF

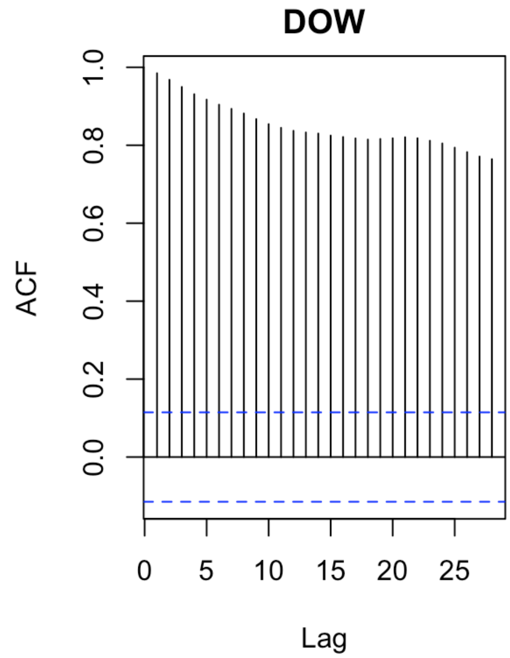




Clear heteroscedasticity near the end of the recorded series but mean appears constant. ACF results inconclusive.

Dow





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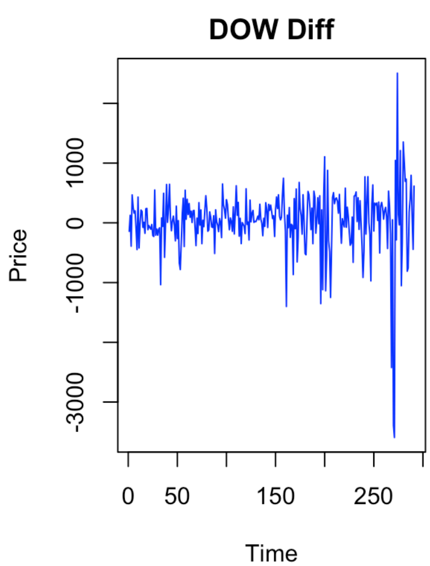
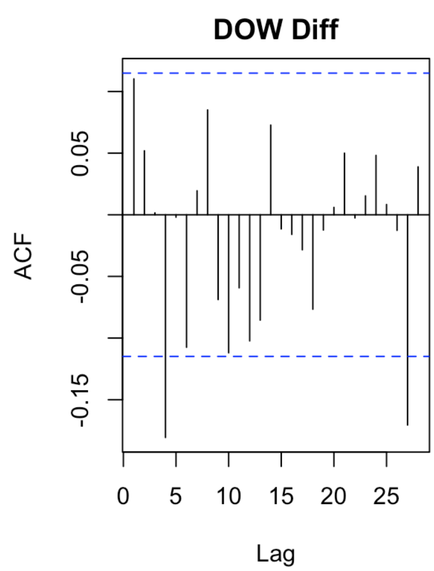
From the time series plot, we could see around 260, the variance becomes larger and the mean is not constant. We could also see that ACF has an decreasing trend. Therefore, it violates assumptions of constant mean and variance.

Dow Diff

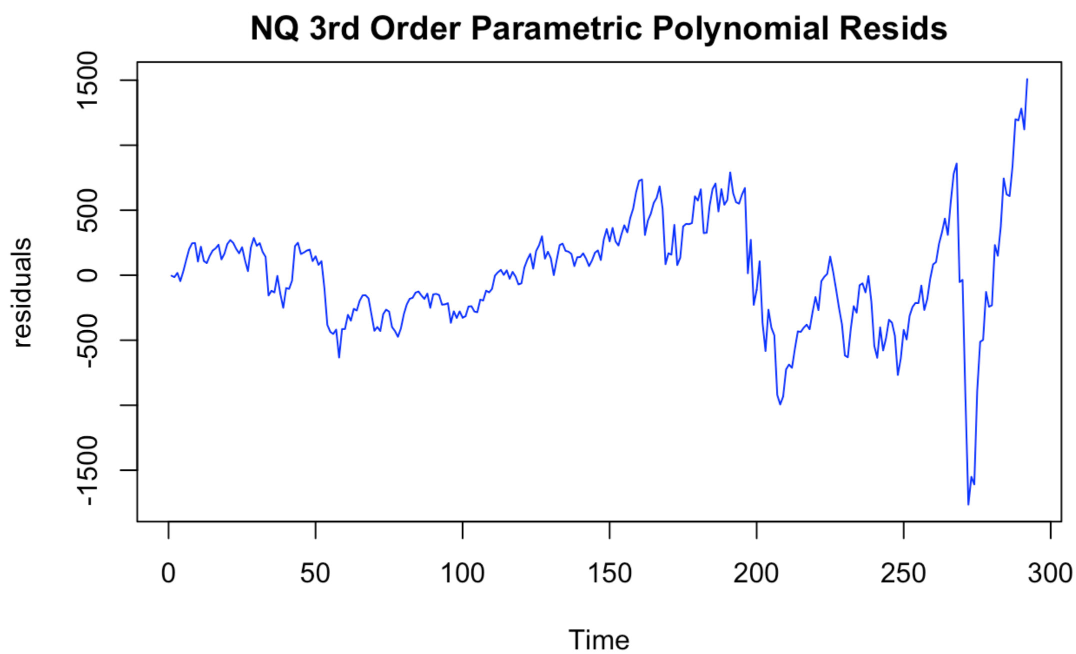


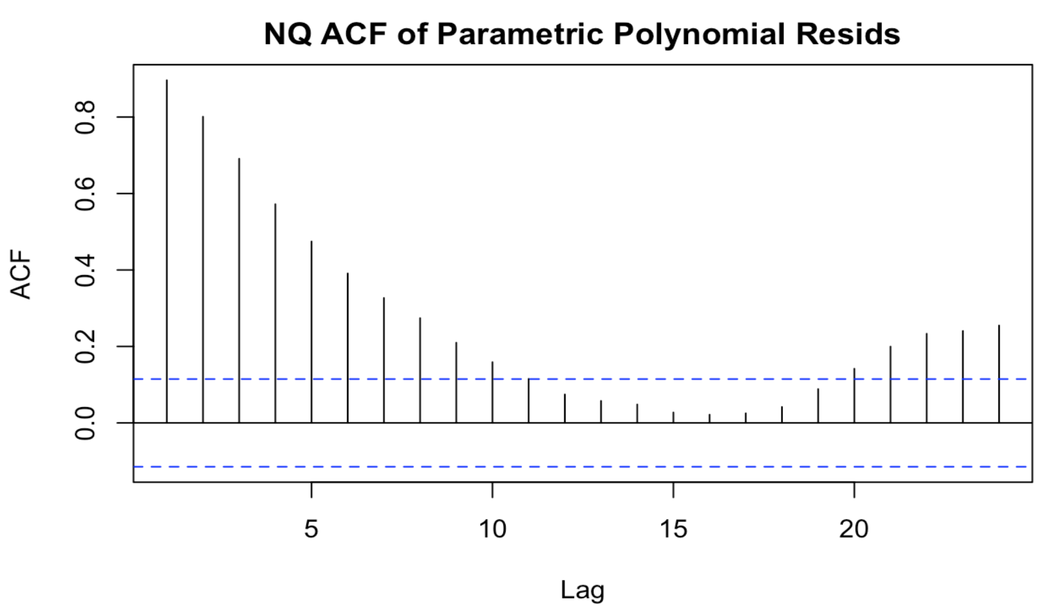


Clear heteroscedasticity near the end of the recorded series but mean appears constant. ACF results inconclusive.

2. For each original time series of the three financial indices, fit a third order parametric polynomial. Use graphical methods to perform residual analysis and comment on the fit.

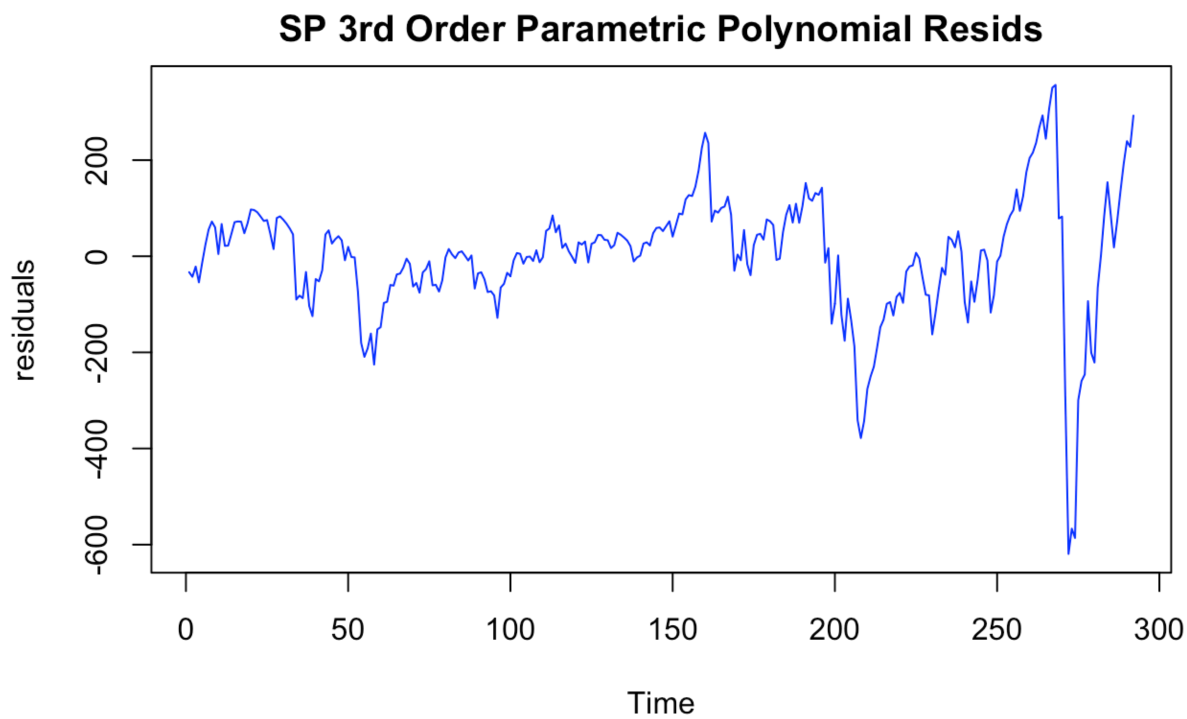
NQ:

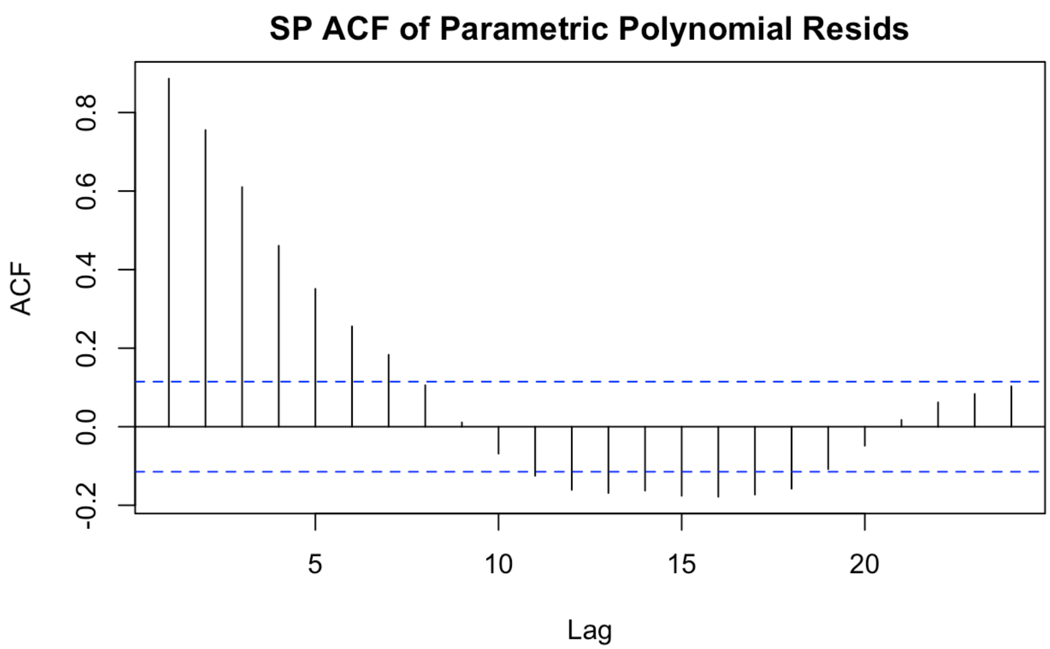




From the above plots, it is clear the residual process for the 3rd order parametric polynomial fit on NQ is non-stationary with a clearly present non-zero trend. And, the variance is not constant as well, since we could clearly see in some periods, there is larger variance.

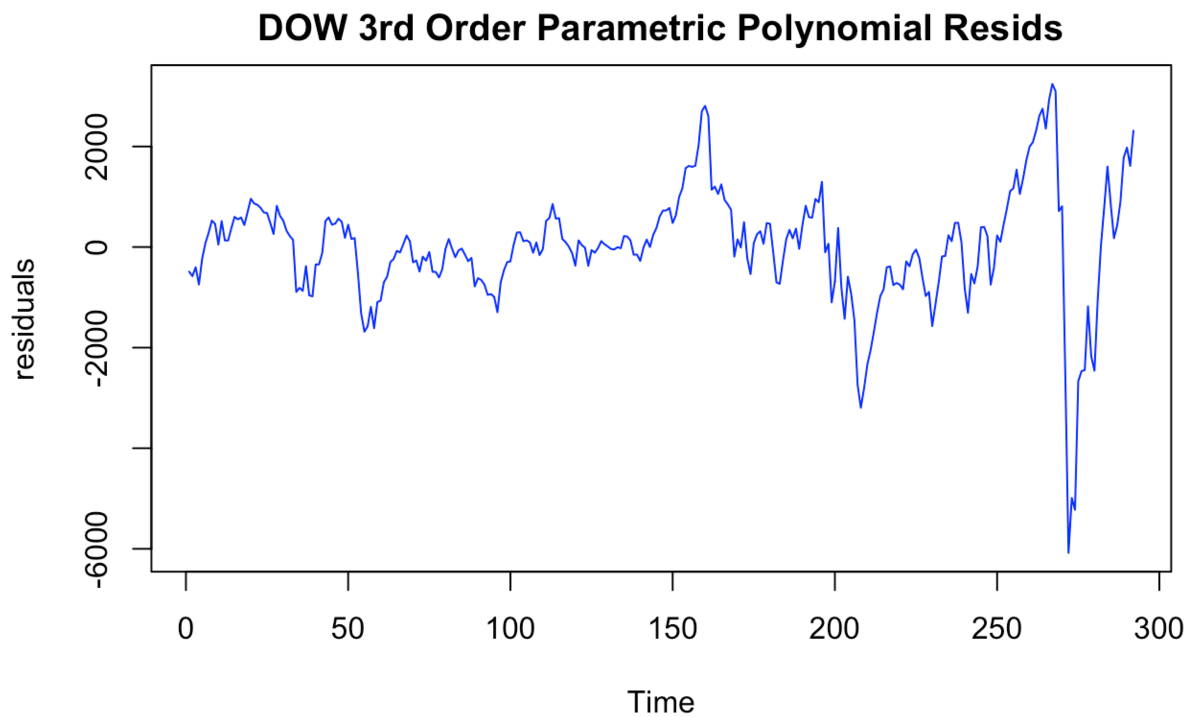
SP

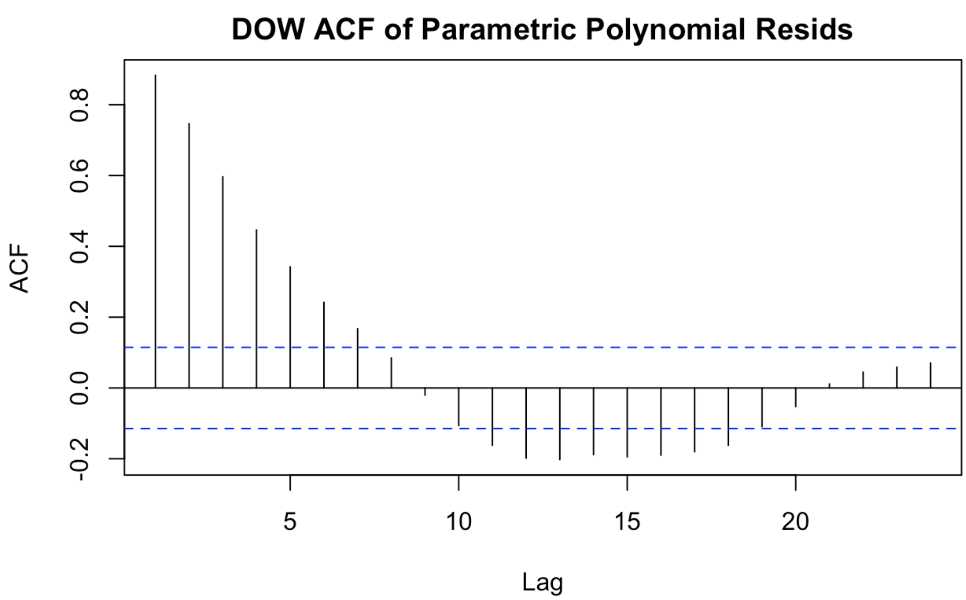




From the above plots, it is clear the residual process for the 3rd order parametric polynomial fit on SP is non-stationary with a clearly present non-zero trend. And, the variance is not constant as well, since we could clearly see in some periods, there is larger variance.

DOW





From the above plots, it is clear the residual process for the 3rd order parametric polynomial fit on DOW is non-stationary with a clearly present non-zero trend. And, the variance is not constant as well, since we could clearly see in some periods, there is larger variance.

3. Calculate Precision Measures (PM) and Mean Absolute Percentage Error (MAPE) on the fit of each model and compare them to one another in terms of model performance

NQ:

```
PM
[1] 0.06598024
MAPE
[1] 0.05064186
```

SP:

```
PM
[1] 0.1024034
MAPE
[1] 0.03378732
```

DOW:

```
PM
[1] 0.08984547
MAPE
[1] 0.03423548
```

Based on 3rd order parametric polynomial fit on NQ, SP and Dow, we could see in MAPEs, SP has smallest values and in PM, we could see NQ has smallest values. But, we could not just use smallest values to compare each model since MAPE and PM have different smallest values from each time series. It is hard to make a conclusion which model is best.

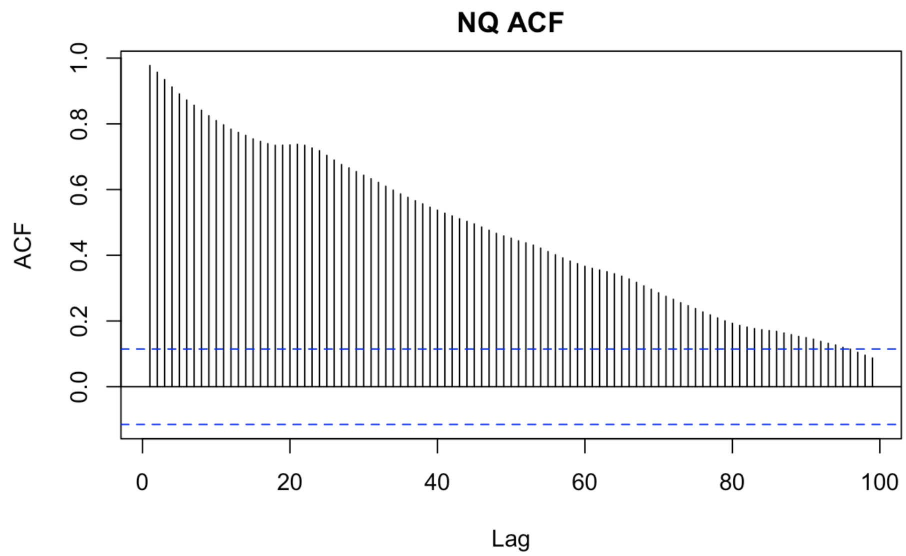
4. Does the simple parametric approach appear to sufficiently capture main trend for all three time series? If not, comment on what some limitations this approach may have.

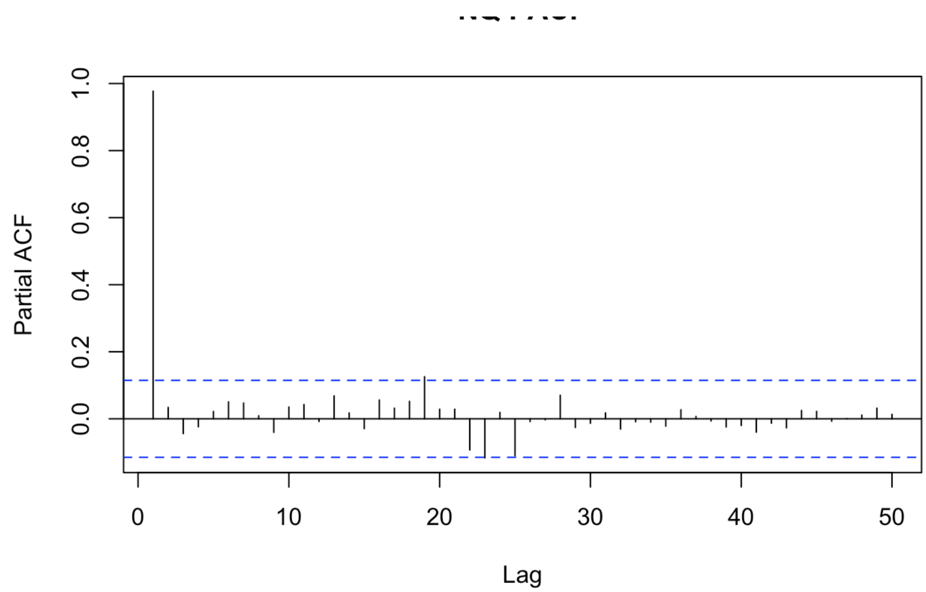
No. In the time series plots and ACF, we could still see there is non-constant variance and mean. So we directly use models on them which does not give us good evaluation since the plots doesn't seem to follow the assumptions of stationary process.

Question 2 ARIMA Modeling - 20 Points

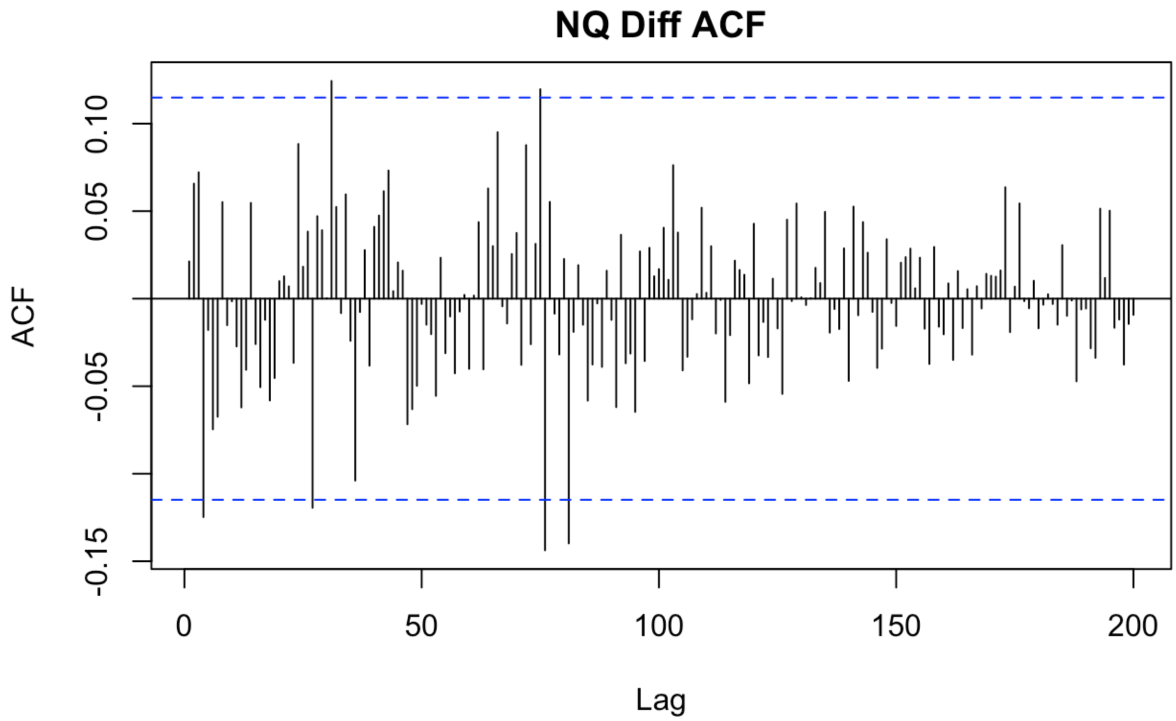
1. For each dataset, use graphical approaches to attempt to assess possible orders p,d,q for an ARIMA model. State what orders you can infer (if any) using this method.

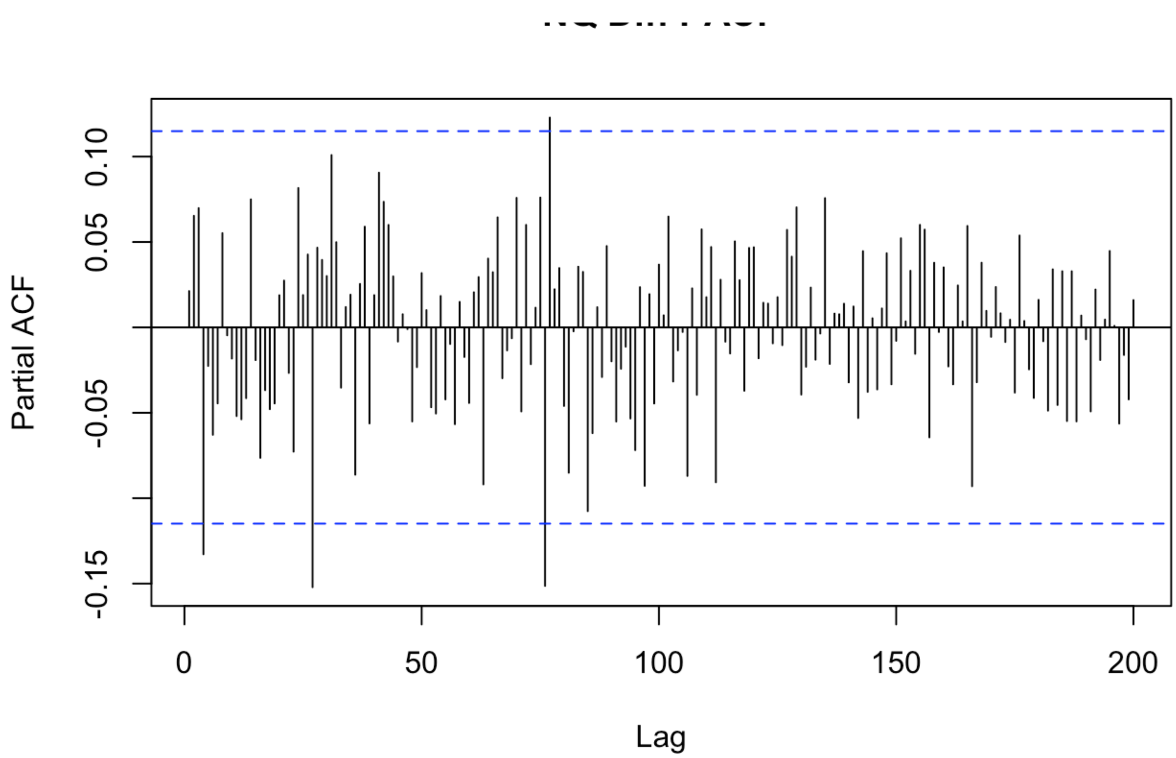
NQ:





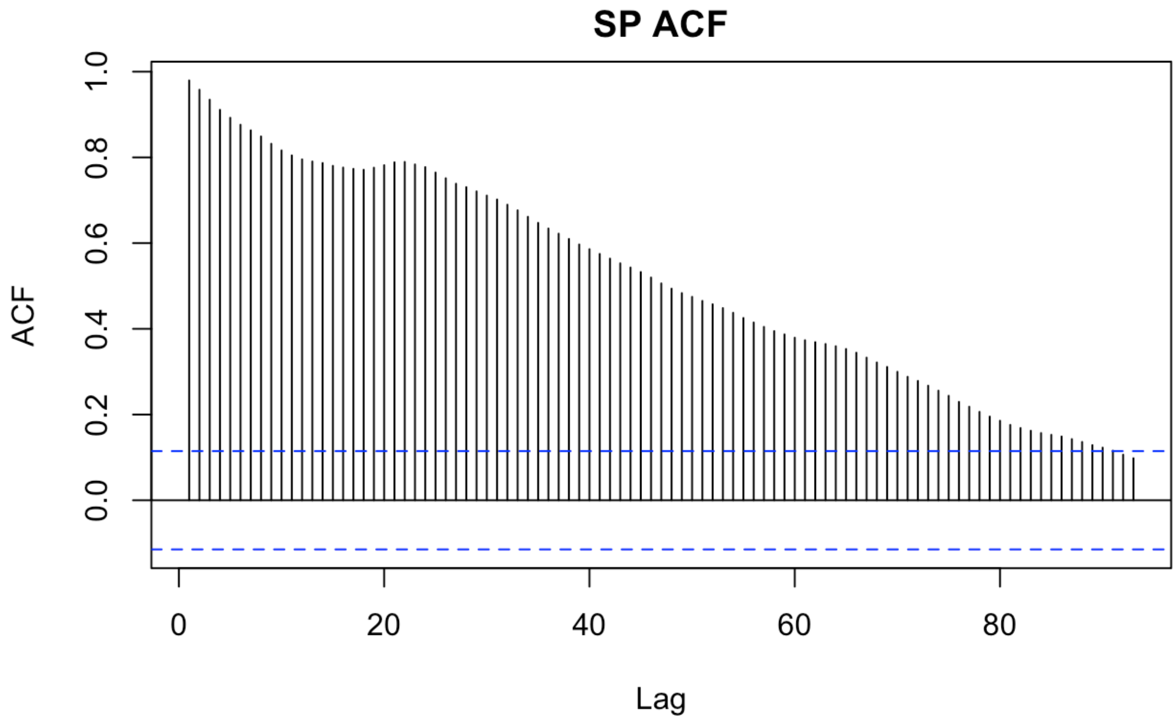
MA(q) determines q by ACF and AR(p) determines p by PACF. Based on the above plots, q=99, p = 25.

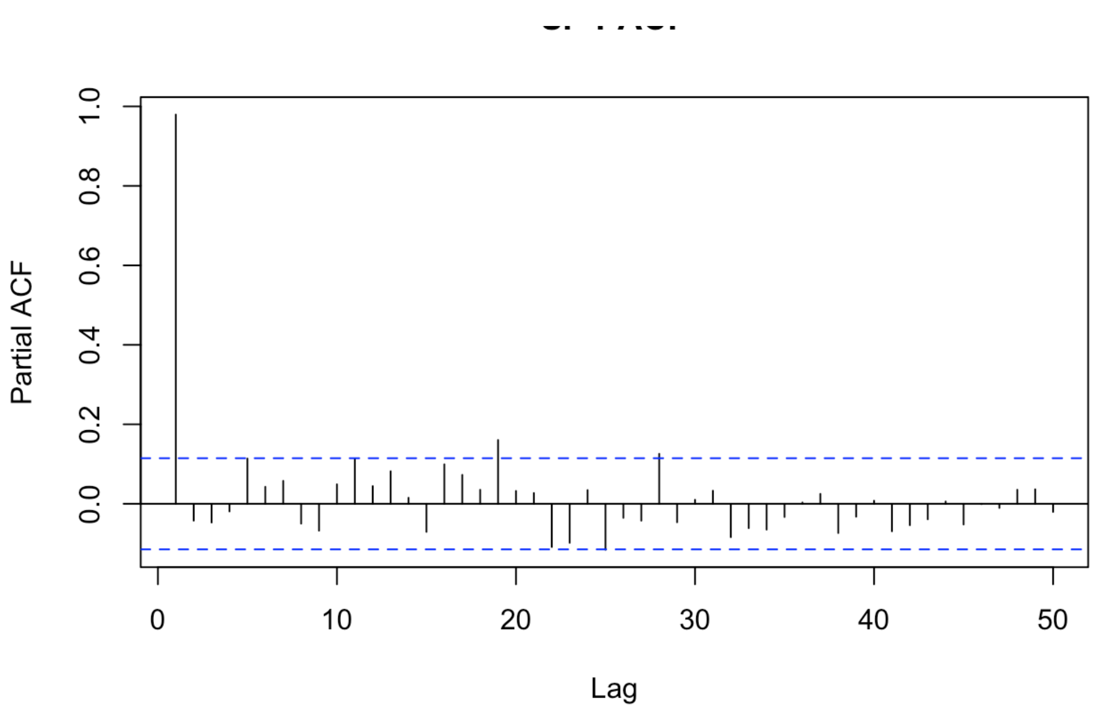




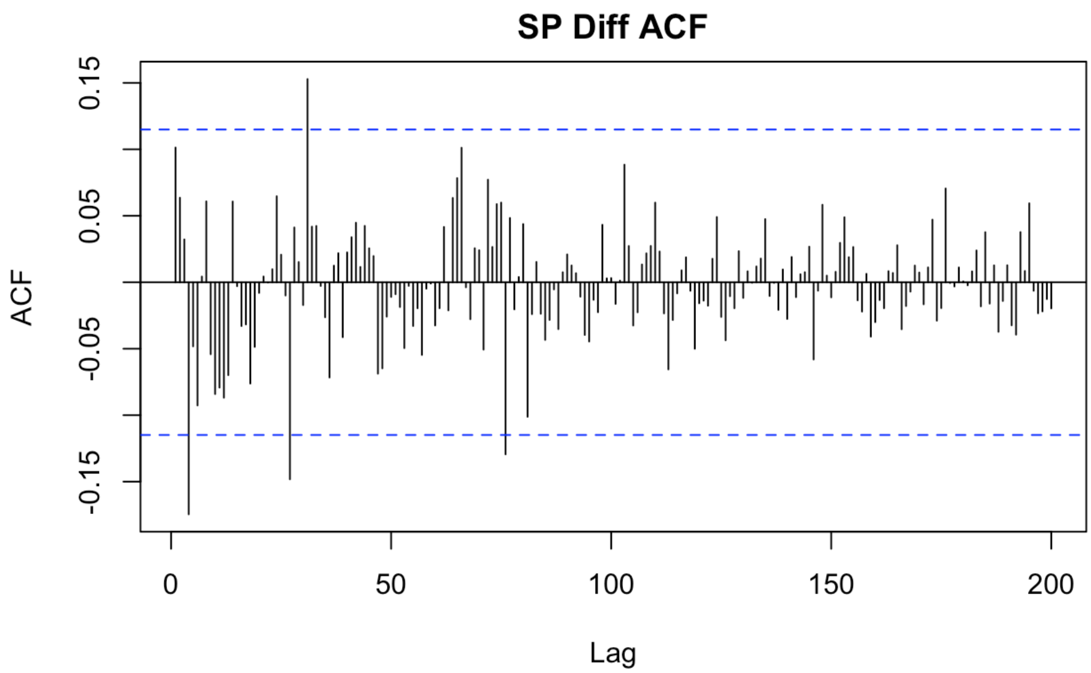
We used difference data to infer orders. Based on the above plots, q=80, p = 80.

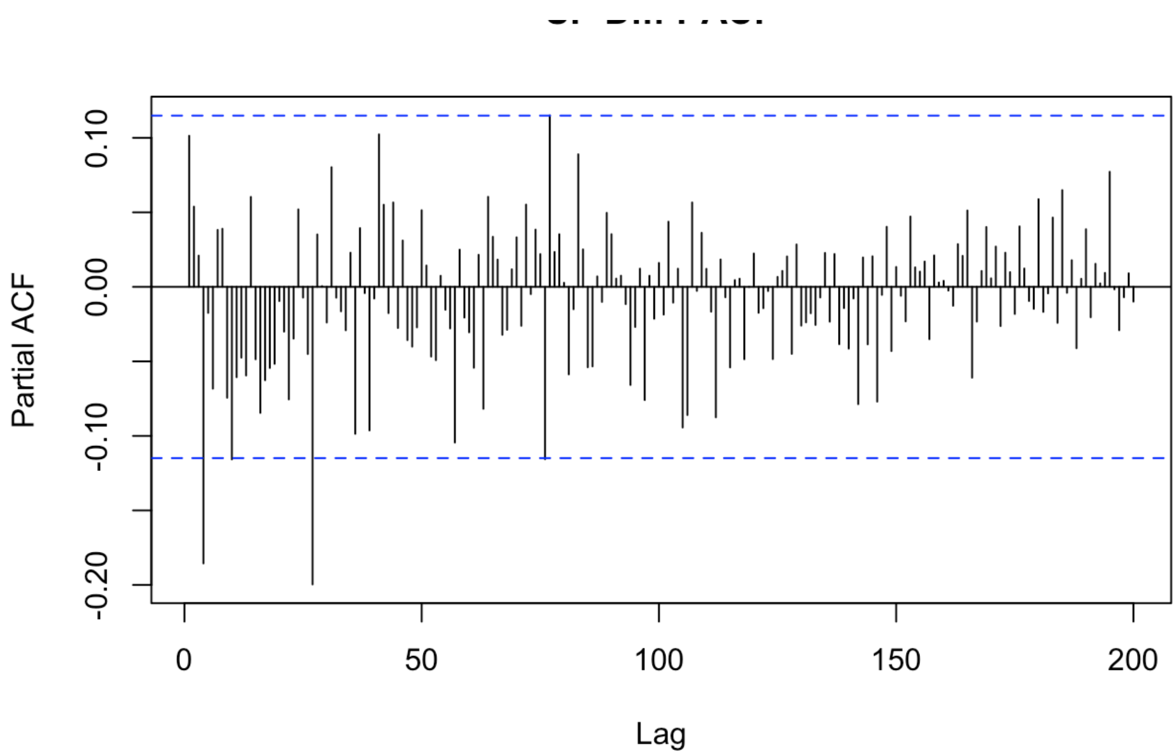
SP:





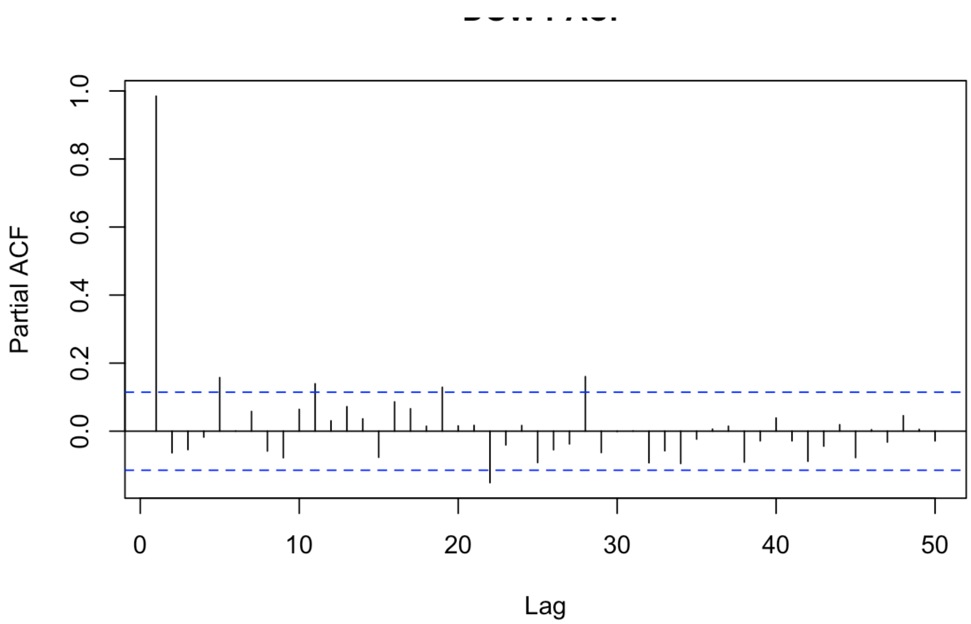
Based on the above plots, q=93, p = 28.

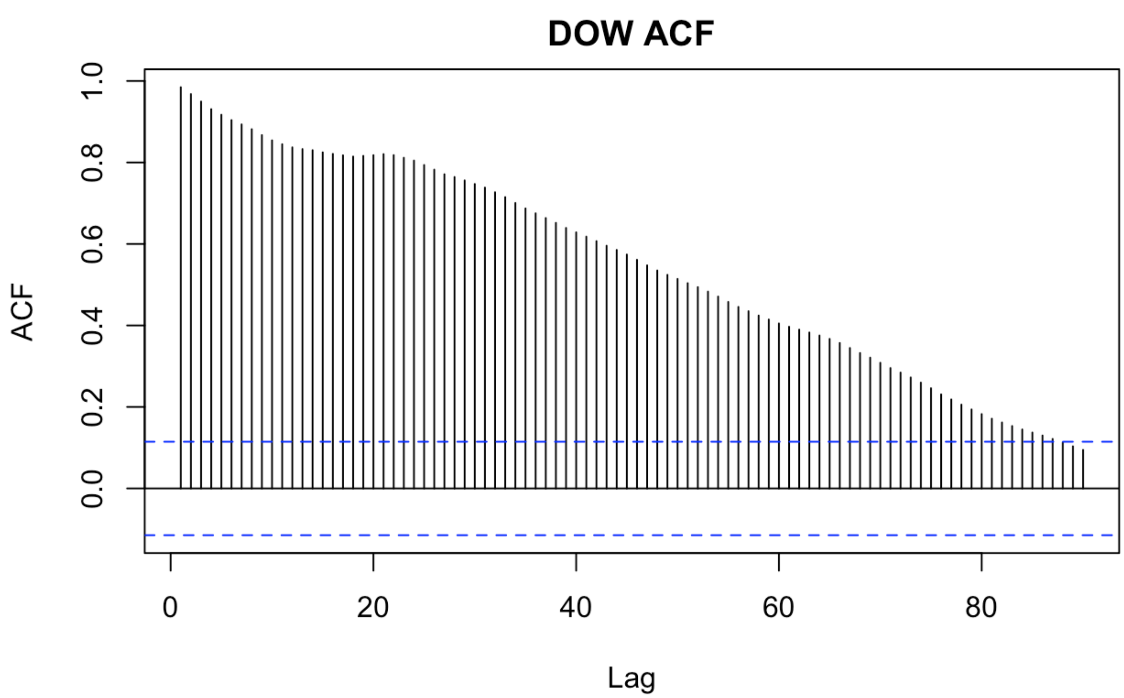




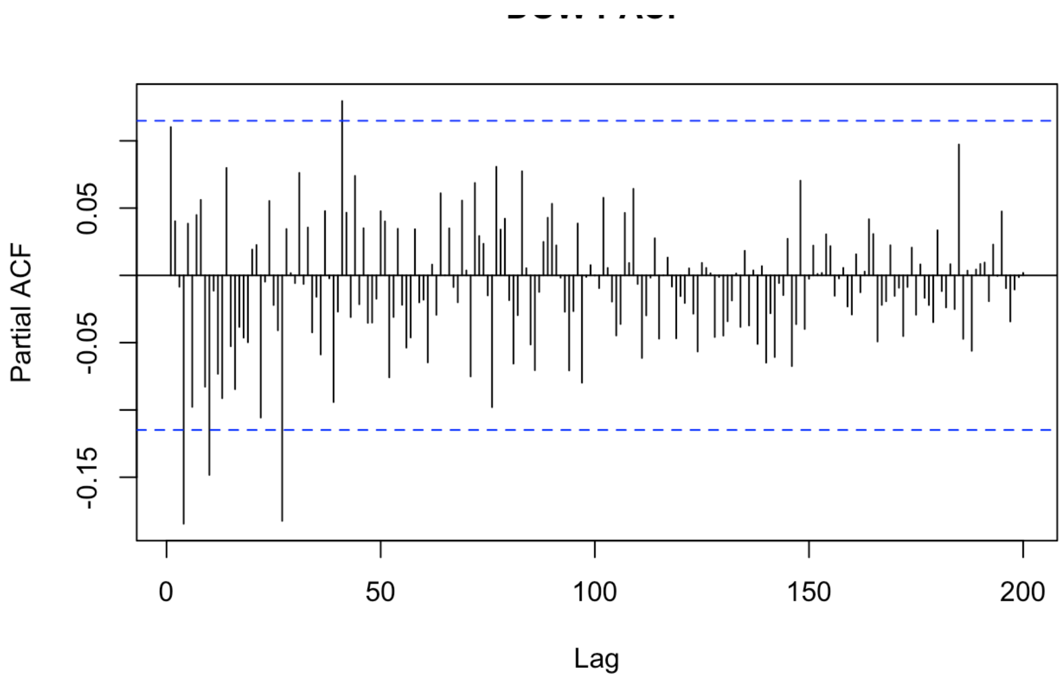
Based on the above plots, q=70, p = 70.

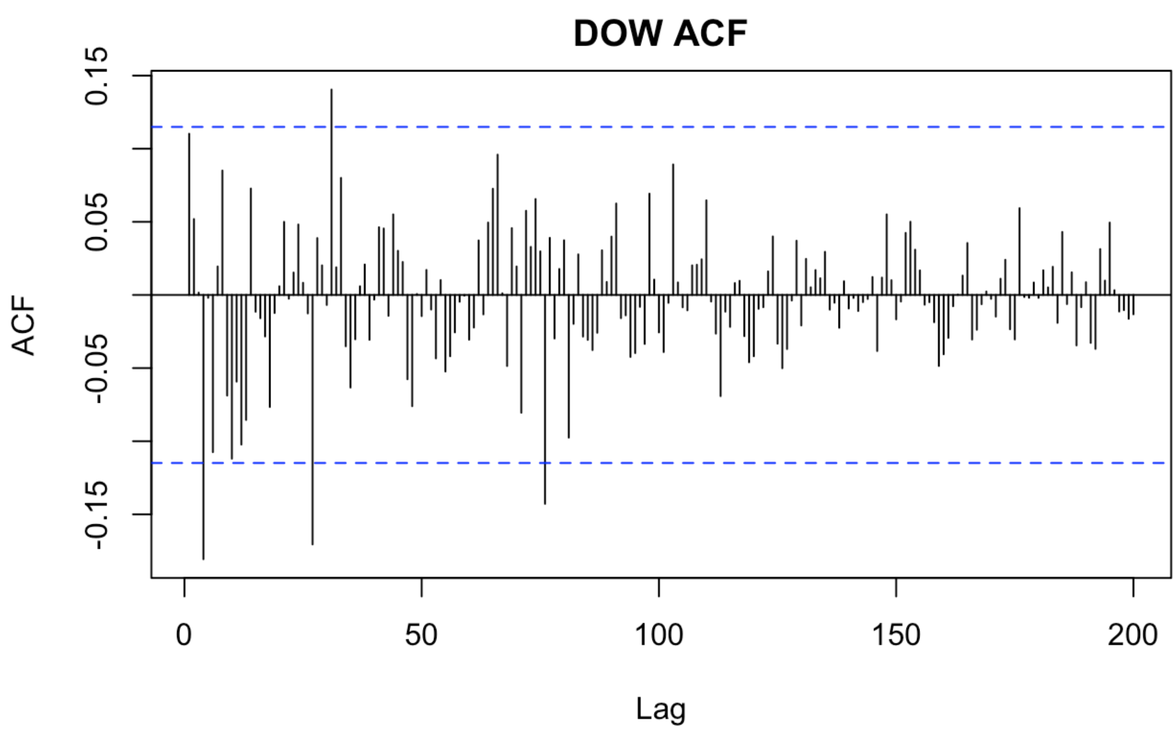
Dow





Based on the above plots, q=90, p = 28.





Based on the above plots, q=75, p = 50.

2. For each differenced time series of the three financial indices, use the iterative AIC minimization approach with a max potential order of (4,2,4) to select and fit the ARIMA model with the selected orders.

NQ – ARIMA(2, 1, 3) AIC – 3825.957

	p	d	q	AIC
	<dbl>	<dbl>	<dbl>	<dbl>
70	4	1	3	3829.789
14	0	2	2	3829.581
74	4	2	2	3829.076
8	0	1	1	3829.067
68	4	1	1	3828.999
40	2	1	3	3825.957

SP – ARIMA(1, 0, 4) AIC – 3176.858

	p	d	q	AIC
	<dbl>	<dbl>	<dbl>	<dbl>
66	4	0	4	3179.262
70	4	1	3	3178.843
64	4	0	2	3178.819
56	3	1	4	3178.400
36	2	0	4	3177.956
21	1	0	4	3176.858

DOW – ARIMA(2, 1, 4) AIC- 4468

	p	d	q	AIC
	<dbl>	<dbl>	<dbl>	<dbl>
68	4	1	1	4475.175
71	4	1	4	4474.482
51	3	0	4	4473.218
56	3	1	4	4473.057
40	2	1	3	4472.801
41	2	1	4	4468.330

3. Extract the roots for each model of the three time series (rounded to third decimal place) and comment what can be inferred from the root analysis.

NQ Roots

```
AR
[1] 0.662 1.712
MA
[1] 1.018 1.007 1.018
```

One AR root is within unit circle. Other roots are outside the unit circle.

The process is causal, not invertible, and stationary.

SP Roots

```
AR
[1] 1.218
MA
[1] 1.000 1.802 1.629 1.629
```

One MA root within unit circle. Other roots are outside the unit circle. Process is stationary but neither causal nor invertible.

DOW Roots

```
AR
[1] 0.580 1.778
MA
[1] 1.000 1.000 1.000 4.894
```

One AR root is within the circle. And two MA roots are on the unit circle. Process is stationary but neither causal nor invertible.

4. Forecast ahead 4 future data points (the last month) for each time series and calculate the prediction PM and MAPE measures. Additionally, perform Box-Pierce tests for each model and comment on what the results tell you.

NQ:

```
MAPE
[1] 0.6134139
PM
[1] 1.403608
```

```
Box-Pierce test

data:  resid(nq.arima)
X-squared = 4.4407, df = 1, p-value = 0.03509
```

The p-value is larger than 0.025, so we could not reject the null hypothesis. That is, residuals are uncorrelated.

SP

```
MAPE
[1] 2.298801
PM
[1] 3.379614

Box-Pierce test

data:  resid(sp.arima)
X-squared = 1.5747, df = 1, p-value = 0.2095
```

The p-value is larger than 0.025, so we could not reject the null hypothesis. That is, residuals are uncorrelated.

DOW

```
MAPE
[1] 0.7238216
PM
[1] 1.67264

       Box-Pierce test

data:  resid(dow.arima)
X-squared = 9.5301, df = 1, p-value = 0.002021
```

The p-value is smaller than 0.025, so we could reject the null hypothesis. That is, residuals are correlated.

5. Does ARIMA modelling seem appropriate for these data? Why or why not? An ideal answer will include references to results as well as theory.

In the root analysis, we could see there are some roots are on the unit circle which does not have a good property like Invertible and Causal and Box-pierce test tells us Dow time series has correlated residuals, which is not appropriate for analysis.

Question 3 **Multivariate Modeling - 15 Points**

1. Using both the differenced and undifferenced time series, fit VAR models with orders selected by minimzing AIC. For each model use appropriate tests to assess the following residual assumptions: Constant Variance, Normality, Non-Correlation. Does either fit seem notably better?

Undifferenced

```
AIC(n)  HQ(n)  SC(n) FPE(n)
     8      3      1      8

       ARCH (multivariate)

data:  Residuals of VAR object mod
Chi-squared = 516.77, df = 180, p-value < 2.2e-16

$JB

       JB-Test (multivariate)

data:  Residuals of VAR object mod
Chi-squared = 534.24, df = 6, p-value < 2.2e-16


$Skewness

       Skewness only (multivariate)

data:  Residuals of VAR object mod
Chi-squared = 124.63, df = 3, p-value < 2.2e-16


$Kurtosis

       Kurtosis only (multivariate)

data:  Residuals of VAR object mod
Chi-squared = 409.61, df = 3, p-value < 2.2e-16



       Portmanteau Test (asymptotic)
```

```
data:  Residuals of VAR object mod
Chi-squared = 86.903, df = 72, p-value = 0.1112
```

Differenced

```
AIC(n)  HQ(n)  SC(n) FPE(n) 
     7      2      1      7 

	ARCH (multivariate)

data:  Residuals of VAR object mod.dif
Chi-squared = 608.07, df = 180, p-value < 2.2e-16

$JB

	JB-Test (multivariate)

data:  Residuals of VAR object mod.dif
Chi-squared = 676.25, df = 6, p-value < 2.2e-16


$Skewness

	Skewness only (multivariate)

data:  Residuals of VAR object mod.dif
Chi-squared = 115.83, df = 3, p-value < 2.2e-16


$Kurtosis

	Kurtosis only (multivariate)

data:  Residuals of VAR object mod.dif
Chi-squared = 560.42, df = 3, p-value < 2.2e-16




	Portmanteau Test (asymptotic)

data:  Residuals of VAR object mod.dif
Chi-squared = 105.07, df = 81, p-value = 0.0374
```

Constant Variance, Normality, Non-Correlation. Does either fit seem notably better?

Arch test:

All p-values are smaller than 0.025. So both models reject the null hypothesis. There is non-constant variance in these series.

Normality Test (JB test)

All p-values are smaller than 0.025. So both models reject the null hypothesis. There is non-normality in these series.

Non-correlation:

All p-values are larger than 0.025. So both models do not reject the null hypothesis. The residuals might have uncorrelated errors.

It is hard to tell which model is best. They both have same situations in these tests.

2. For both models, calculate the forecasting Prediction PM and MAPE on the appropriate training data for each time series. Compare the two models as well as compare to the ARIMA models

NQ:

```
MAPE
[1] 0.01621698
PM
[1] 0.3600216
```

NQ Diff:

```
MAPE
[1] 1.738116
PM
[1] 2.243331
```

SP:

```
MAPE
[1] 0.01887416
PM
[1] 1.173442
```

SP Diff:

```
MAPE
[1] 6.259673
PM
[1] 6.822153
```

Dow

```
MAPE
[1] 0.02406944
PM
[1] 3.350554
```

Dow Diff

```
MAPE
[1] 1.766613
PM
[1] 5.200405
```

It seems that they perform better than just univariate arima models. We could see the values become smaller. As for differenced and undifferenced data, we could see undifferenced data perform better since differenced data have higher MAPE and PM.

3. Citing both your results and relevant theory, does this dataset seem to benefit from a multivariate modelling method as opposed to univariate modeling?

Yes. VAR consider other time series and incorporate these series to build a model. VAR models give us better forecasting capacity. And all variables are endogenous.

Question 4 **Heteroskedasticity Modeling - 20 Points**

1. Now using just the differenced Dow Jones data, use the iterative approach via BIC minimization (select only non-trivial orders) to select and fit the 'best' ARMA-GARCH (Max (4,4)x(2,2), start from ARMA(3,3)).

Stepwise results using provided A-G Order Selection Code:

Initial GARCH Order: 2,0 BIC = 14.94394

	m	n	BIC
	<dbl>	<dbl>	<dbl>
5	1	0	15.02864
7	1	2	14.99312
9	2	1	14.99237
10	2	2	14.98839
6	1	1	14.95735
8	2	0	14.94394

ARMA Update: 3,3 BIC = 14.94394

	p	q	BIC
	<dbl>	<dbl>	<dbl>
10	1	3	14.96513
7	1	0	14.96465
9	1	2	14.96226
3	0	1	14.95937
2	0	0	14.95145
20	3	3	14.94394

Final GARCH: 2,0 BIC = 14.94394

	m	n	BIC
	<dbl>	<dbl>	<dbl>
5	1	0	15.02864
7	1	2	14.99312
9	2	1	14.99237
10	2	2	14.98839
6	1	1	14.95735
8	2	0	14.94394

Final A-G Order: (3,3)x(2,0)

2. Print the coefficients, comment on their significance, and write out the model equation in full. Additionally, assess residual assumptions (Hint: Fit using garchFit method and check summary).

```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~arma(3, 3) + garch(2, 0), data = train.dif$Dow,
    trace = FALSE)

Mean and Variance Equation:
 data ~ arma(3, 3) + garch(2, 0)
<environment: 0x12b9d7740>
 [data = train.dif$Dow]
Conditional Distribution:
 norm

Coefficient(s):
         mu          ar1          ar2          ar3          ma1
 4.0048e+01  -3.6127e-01   4.7995e-02   6.0458e-01   2.1868e-01
        ma2          ma3        omega       alpha1       alpha2
-1.7697e-01  -8.3265e-01   5.4793e+04   3.6640e-01   6.6977e-01

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu      4.005e+01   1.243e+01    3.222 0.001273 **
ar1    -3.613e-01   1.074e-01   -3.365 0.000765 ***
ar2     4.800e-02   7.518e-02    0.638 0.523212
ar3     6.046e-01   8.709e-02    6.942 3.87e-12 ***
ma1     2.187e-01   7.933e-02    2.757 0.005839 **
ma2    -1.770e-01   4.422e-02   -4.002 6.28e-05 ***
ma3    -8.327e-01   6.284e-02  -13.251  < 2e-16 ***
omega   5.479e+04   1.523e+04    3.598 0.000321 ***
alpha1  3.664e-01   1.116e-01    3.282 0.001032 **
alpha2  6.698e-01   1.856e-01    3.609 0.000307 ***
```

```
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:
 -2163.865    normalized:  -7.43596

Description:
 Sun Dec  6 17:37:04 2020 by user:


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  120.5303  0
 Shapiro-Wilk Test  R    W      0.9587738 2.456702e-07
 Ljung-Box Test     R    Q(10)  13.57279  0.1933845
 Ljung-Box Test     R    Q(15)  20.23643  0.1630493
 Ljung-Box Test     R    Q(20)  24.44591  0.2234563
 Ljung-Box Test     R^2  Q(10)  3.897023  0.9518729
 Ljung-Box Test     R^2  Q(15)  7.400586  0.9455661
 Ljung-Box Test     R^2  Q(20)  7.935957  0.992283
 LM Arch Test       R    TR^2   3.825805  0.9863784

Information Criterion Statistics:
     AIC      BIC      SIC     HQIC
14.94065 15.06688 14.93839 14.99122
```

Y_t = 4.005e+01 -3.613e-01 Y_t-1 + 4.800e-02 Y_t-2 + 6.046e-01 Y_t-3 + Z_t + 2.187e-01 Z_t-1 -1.770e-01 Z_t-2 -8.327e-01 Z_t-3

$\sigma^2_t = 5.479e+04 + 3.664e-01 * Z^2_{t-1} + 6.698e-01 * Z^2_{t-2}$

3. Now using the selected model order, perform forecasts using the rolling method for the last 48 differenced Dow training values and calculate the Prediction PM and MAPE measures.

```
MAPE
[1] 2.920119
PM
[1] 1.083729
```

4. Do you believe this model is a better fit than the alternatives? Why or why not?

Yes. Based on PM values (5.2 and 1.08), we could see differenced DOW time series has lower values. Since in this time series plots, we could see there is heteroscedasticity, we should perform GRACH model to estimate sigma. Therefore, we could see improved models based on PM values. However, MAPE does not seem to perform well.

Also, based on testing, Arch Test as well as residual and squared residual L-B Tests all have large p values. This indicates no significant evidence for non-constant variance of error and dependence in residuals and squared residuals (weak independence). J-B Test has a small p, indicating significant evidence for non-normality of error.