# ISYE 6420A/MSA
# Homework 3

Due February 19, 2020

1. Consider the Bayesian model

$$\begin{aligned} y|\theta_1, \theta_2 &\sim N(\theta_1 + \theta_2, 1), \\ \theta_i &\sim^{iid} N(0, 1), \end{aligned}$$

   for $i = 1, 2$. Suppose $y = 1$ is observed. Then, find the marginal posterior distributions of $\theta_1$ and $\theta_2$. (Hint: (i) regression. (ii) If $\boldsymbol{I}_n$ is an $n \times n$ identity matrix and $\boldsymbol{J}_n$ is an $n \times n$ matrix of 1's, then $(\boldsymbol{I}_n + b\boldsymbol{J}_n)^{-1} = \boldsymbol{I}_n - \frac{b}{1+nb}\boldsymbol{J}_n$. (iii) If $\boldsymbol{\theta}$ follows a multivariate normal distribution, then the marginal distribution of $\theta_i$ is a normal distribution with the corresponding mean and variance).

2. Consider the coin example discussed in the class and perform the following simulation. Simulate the weights of 10 coins from $\theta_i \sim N(5.67, .01^2)$ for $i = 1, \cdots, 10$. Simulate 10 measurements from $y_i|\theta_i \sim N(\theta_i, .02^2)$. Compute the total error sum of squares $SSE^{EB} = \sum_{i=1}^{10}(\theta_i - \hat{\theta}_i^{EB})^2$ and $SSE^{MLE} = \sum_{i=1}^{10}(\theta_i - y_i)^2$. Repeat this 1000 times and plot the densities of the two quantities $SSE^{EB}$ and $SSE^{MLE}$, and make comments. (Include your R code with the solutions).

3. Let

$$\begin{aligned} y_i|\theta_i &\sim^{ind} Poisson(\theta_i) \\ \theta_i &\sim^{iid} Exp(\lambda) \end{aligned}$$

   for $i = 1, \ldots, n$. ($p(\theta) = \lambda e^{-\lambda\theta}$). Find the empirical Bayes estimator of $\theta_i$, $i = 1, \ldots, n$.

4. Consider the Bayesian model:

$$\begin{aligned} x_i|\phi_i &\sim^{ind} N(0, \phi_i) \\ \frac{1}{\phi_i} &\sim^{iid} Exp(\lambda), \end{aligned}$$

   for $i = 1, \cdots, n$. Find the empirical Bayes estimator of $\phi_i$, $i = 1, \cdots, n$. Evaluate the expressions as far as possible.