ISYE 6420 A/MSA

Homework 1

Due January 24, 2020

1. 10% of the emails I receive are spam. The automatic spam filter in my email system correctly classifies an email with probability 0.99. What percentage of my emails in the spam folder are expected to be genuine (non-spam)?

2. Suppose $\theta \sim log - normal(0, 1)$, that is, $\log \theta \sim N(0, 1)$. Find the 95% equal-tail and HPD credible intervals of θ. Compute the width of the two intervals. Which one do you think is a better interval? (Hint: for HPD calculation, use the R package *HDInterval*).

3. Let $y_i|\theta \sim^{iid} Exp\ (\theta)$, for $i = 1, \ldots, n$. $(p(y|\theta) = \theta e^{-\theta y})$. Assume the prior distribution for θ to be $Gamma(a, b)$, that is, $p(\theta) = b^a/\Gamma(a)\theta^{a-1} \exp\{-b\theta\}$. Find the posterior distribution of θ.

4. Find the posterior predictive distribution of a future observation in problem 3.

5. Let $y_i|\theta \sim^{iid} Uniform\ (0, \theta)$, for $i = 1, \ldots, n$. Assume the prior distribution for θ to be $Pareto(a, b)$, where $p(\theta) = ba^b/\theta^{b+1}$ for $\theta > a$ and 0 otherwise. Find the posterior distribution of θ.