ECE661 HW1
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1. The homogeneous coordinate of origin in the physical space is (0, 0, 1).
   Therefore, all points in the representational space form the w-axis except the origin point in the representational space, by taking any scale k.

2. Points at infinity in the physical plane are not the same.
   For two parallel lines $\mathbf{l_1}$ = (a, b, c) and $\mathbf{l_2}$ = (a, b, c'), their intersection point is (b, -a, 0). The point is located at infinity in the physical plane, with a specific direction controlled by a and b. All points at infinity in the physical plane have different (a, b) values and hence are not the same.

3. The matrix of a degenerate conic is given by $\mathbf{C} = \mathbf{l}\mathbf{m}^T + \mathbf{m}\mathbf{l}^T$. Since the rank of an outer product matrix is always 1, the rank of matrix **C** can never exceed 2.

4. 5 points define a conic in $\mathbf{R}^2$.
   Given the form of a conic: $ax^2 + bxy + cy^2 + dx + ey + f = 0$, by counting the dimensions, if we have 5 points, then the 5 equations can form a system that solves the 6 parameters (a, b, c, d, e, f). Therefore, 5 points define a conic in a 2D plane.

5. $\mathbf{l_1}$ = (0, 0, 1) × (1, 2, 1) = (-2, 1, 0)
   $\mathbf{l_2}$ = (3, 4, 1) × (5, 6, 1) = (-2, 2, -2)
   The HC of the intersection point is (-2, 1, 0) × (-2, 2, -2) = (-2, -4, -2) = (1, 2, 1), hence the point is (1, 2).

   For the second line passed through (7, -8) and (-7, 8),
   $\mathbf{l_1}$ = (0, 0, 1) × (1, 2, 1) = (-2, 1, 0)
   $\mathbf{l_2}$ = (7, -8, 1) × (-7, 8, 1) = (-16, -14, 0)
   Since both $\mathbf{l_1}$ and $\mathbf{l_2}$ pass through the origin, the intersection point is the origin (0, 0). Only 2 steps are needed.

6. $\mathbf{l_1}$ = (-4, 0, 1) × (-2, 8, 1) = (-8, 2, -32) = (-4, 1, -16)
   $\mathbf{l_2}$ = (0, -2, 1) × (4, 14, 1) = (-16, 4, 8) = (-4, 1, 2)
   The HC of the intersection point is (-4, 1, -16) × (-4, 1, 2) = (18, 72, 0).
   The intersection point is an ideal point, since $\mathbf{l_1}$ and $\mathbf{l_2}$ are two parallel lines.

7. Since the two lines are x = 1 and y = -1, the intersection point is (1, -1).

8. From the eclipse equation, we have

$$\frac{(x-2)^2}{0.5^2}+\frac{(y-3)^2}{1^2}=1$$

Turn into the implicit from we get $4x^2 + y^2 - 16x - 6y + 24 = 0$

The conic matrix C = $\begin{bmatrix} 4 & 0 & -8 \\ 0 & 1 & -3 \\ -8 & -3 & 24 \end{bmatrix}$

The origin **p** = $(0\ 0\ 1)^T$

The polar line **l** = **Cp** = $(-8\ -3\ 24)^T$

The intersections point of the polar line and x-axis is (3, 0)

The intersections point of the polar line and y-axis is (0, 8)