ECE661: Homework 1

Fall 2022

Due Date: 1:30pm, Sep 01, 2022

Turn in typed solutions via BrightSpace. Additional instructions can be found at BrightSpace.

The following notation conventions are used for representing vector, matrix, and scalar variables. Boldface lowercase letters are used to represent vectors and boldface uppercase letters are used to represent matrices. Lowercase letters (without any special typeface) are used to represent scalars.

1. What are all the points in the representational space $\mathbb{R}^3$ that are the homogeneous coordinates of the origin in the physical space $\mathbb{R}^2$? (10 pts)

2. Are all points at infinity in the physical plane $\mathbb{R}^2$ the same? Justify your answer. (10 pts)

3. Argue that the matrix rank of a degenerate conic can never exceed 2. (10 pts)

4. A line in $\mathbb{R}^2$ is defined by two points. That raises the question – how many points define a conic in $\mathbb{R}^2$? Justify your answer. (10 pts)

5. Derive in just 3 steps the intersection of two lines $\mathbf{l}_1$ and $\mathbf{l}_2$ with $\mathbf{l}_1$ passing through the points $(0, 0)$ and $(1, 2)$, and with $\mathbf{l}_2$ passing through the points $(3, 4)$ and $(5, 6)$. How many steps would take you if the second line passed through $(7, -8)$ and $(-7, 8)$? (15 pts)

6. Let $\mathbf{l}_1$ be the line passing through points $(-4, 0)$ and $(-2, 8)$ and $\mathbf{l}_2$ be the line passing through points $(0, -2)$ and $(4, 14)$. Find the intersection between these two lines. Comment on your answer. (10 pts)

7. Find the intersection of two lines whose equations are given by $x = 1$ and $y = -1$. (10 pts)

8. As you know, when a point $\mathbf{p}$ is on a conic $\mathbf{C}$, the tangent to the conic at that point is given by $\mathbf{l} = \mathbf{C}\mathbf{p}$. That raises the question as to what $\mathbf{C}\mathbf{p}$ would correspond to when $\mathbf{p}$ was outside the conic. As you'll see later in class, when $\mathbf{p}$ is outside the conic, $\mathbf{C}\mathbf{p}$ is the line that joins the two points of contact if you draw tangents to $\mathbf{C}$ from the point $\mathbf{p}$. This line is referred to as the *polar line*. Now let our conic $\mathbf{C}$ be an ellipse that is centered at the coordinates $(2,3)$, with $a = 1/2$ and $b = 1$, where a and b, respectively, are the lengths of semi-minor and semi-major axes. For simplicity, assume that the minor axis is parallel to $\mathbf{x}$-axis and the major axis is parallel to $\mathbf{y}$-axis. Let $\mathbf{p}$ be the origin of the $\mathbb{R}^2$ physical plane. Find the intersections points of the polar line with $\mathbf{x}$- and $\mathbf{y}$-axes. (25 pts)