Summary Sheet

In order to streamline airline operations, save cost and time, and ultimately ensure a better consumer experience, this paper aims to determine and present the best boarding and disembarking methods for various aircraft and capacity levels.

First, we isolated the key factors affecting boarding and disembarking times for each individual passenger. These were luggage interference time, t_{il}, seat interference time, t_{is}, and aisle time, t_{ai}. We evaluated models based on the maximum time taken by the last passenger, as representative of how long the entire process took. We accounted for the different speeds of passengers, modelling them on a normal distribution curve, as well as likely human physiology which affects space taken to store luggage. We additionally considered the number of carry-ons, modelled by Poisson distribution, as factors affecting the velocity of the luggage handler.

Based on this, we coded simulations of five different boarding methods and evaluated them for speed and consistency. We found boarding by column to be the theoretical optimal, with reverse pyramid boarding the most effective after accounting for real-world discrepancies.

We additionally isolated factors most important for disembarking, namely time taken to retrieve carry-ons and time taken to exit after retrieval, and qualitatively concluded that disembarking by column would be the fastest way to disembark a narrow-body aircraft.

We modified these methods for Flying Wing and Two-Entrance, Two-Aisle aircrafts. A Flying Wing aircraft was modelled as 4 modified narrow-body aircrafts. We also account for a new factor: waiting time to arrive at the interference point (point where two queues intersect).

We approached modifications for a Two-Entrance, Two-Aisle aircraft by splitting the plane column-wise into 3 groups, taking into account first-class passengers' priority.

We have also taken into consideration necessary social distancing measures in light of the COVID-19 pandemic, and changed our ideal boarding and disembarking methods accordingly. We considered the necessity of reducing the number of strangers each passenger comes into contact with, as well as possible real-world social-distanced seating plans and their effect on total boarding and disembarking time (e.g. leaving one seat in between two passengers).

Letter to Airline Executive

Dear Sir/Madam:

Thank you for this opportunity to improve the boarding and disembarking processes for airline passengers. After extensive modelling, using Python-coded simulations, we have compiled recommendations for narrow-body aircraft, Flying Wing aircraft, Two-Entrance, Two-Aisle aircraft, as well as modifications in light of the pandemic situation. Optimization of boarding and disembarking methods can save passengers time and airlines' cost, creating a win-win situation. We have determined the optimal boarding and disembarking methods based on efficiency, consistency, and how sensitive they are to changes, such as passengers that do not follow the instructions.

For a narrow-body aircraft, we recommend the reverse pyramid boarding method. Passengers are grouped by seat position, with dividing lines creating an image that resembles a pyramid. Details attached in Appendix E. Reverse pyramid boarding has proved to be the most efficient and reasonably consistent, with a practical advantage of allowing boarding in general groups, without rigidly fixing passenger order. However, disembarking by columns does not have the exact same limitations as boarding by columns, as passengers are already arranged in their columns. Disembarking by columns involves aisle passengers exiting in order, then middle passengers exiting in order, then window passengers exiting in order. Disembarking by columns has qualitatively been assessed to be the fastest available method. As such, we strongly recommend implementing these methods in your airline's daily operations.

For a Flying Wing aircraft, commonly used for military or research purposes, we recommend a modified boarding by column method, accounting for the change in demographic of likely passengers. This would help drastically reduce the time necessary for boarding. For a Two-Entrance, Two-Aisle aircraft, we also recommend modifying the boarding by column method to account for more columns and exit routes. For both aircraft, we recommend disembarking by columns, with passengers closest to the entrance disembarking first.

We understand that COVID-19 has taken its toll on the airline industry, and we have made adjustments to our model in light of the pandemic situation, to allow for social distancing. At heavily reduced capacity, such as 30%, we recommend boarding by seat or boarding by section on a narrow-body aircraft, depending on the airline's priorities. Boarding by seat will allow faster boarding; boarding by section will help minimise transmission of COVID-19. We propose a modified disembarking by seat method to ensure a swift and safe end to every passenger's flight.

We sincerely hope that our model will prove adaptable to your changing needs and allow you to determine the best boarding and disembarking methods with ease. Thank you!

Yours faithfully,
IMMC team

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0 Introduction

0.1 Background

To save costs in time and money, any airline would try to reduce time taken to board and disembark aircraft as much as possible, while maintaining consumer satisfaction (by, for example, aiming not to break up families with young children, or allowing first-class passengers to board first). This paper will investigate the best way to reduce boarding and disembarking times, by first modelling factors affecting the time taken, and then comparing various potential models.

0.2 Restatement of Problem

We have completed the following tasks:

1. Construct a mathematical model to calculate total aircraft boarding and disembarking times, based on the key factors affecting time taken to get to passengers' seats.
2. Apply said model to a narrow-body aircraft and hence determine the most efficient boarding method. This will account for variables such as differing amounts of carry-on luggage and unruly passengers.
3. Modify our model for Flying Wing aircraft and Two-Entrance, Two-Aisle aircraft and discuss the most efficient boarding method for these aircrafts.
4. Modify the most efficient boarding methods in light of the changing pandemic situation.

0.3 Definition of Variables

Constant	Definition	Working value
V_m	Mean walking speed of passenger	1.34m/s
V_σ	Standard deviation of walking speed of passenger	0.37m/s
N_s	Number of seats to be filled	For narrow-body aircraft: 198
s_w	Distance between seats in same row	0.4454m
s_p	Distance between rows	0.7747m
a_w	Width of aisles (distance between two columns)	0.305m

Variable	Definition
t_{il}	Luggage interference time of passenger i
t_{is}	Seat interference duration of passenger i

Pn	The percentage of people not following the prescribed method
t_{ia}	Time for passenger i to pass the aisle
(x_i, y_i)	Coordinates of the seat of passenger i, y being the row the seat is in and x being the "column" that the seat is in. The entrance would be at (3, 0) The leftmost seat in the first row is at position (0, 0) and rightmost seat in the first row is at position (5, 0) for the narrow-body plane structure
(x_{i0}, y_{i0})	The initial waiting position of passenger i
T_i	Final time taken for passenger i to sit down
T^*	The set that includes all times taken for each passengers to settle down
T_f^*	The set that includes all times taken for each first class passenger to settle
T_e^*	The set that includes all times taken for each economy passenger to settle
T_{max}	The practical maximum boarding time under the boarding method
T_{min}	The practical minimum boarding time under the boarding method
T_{mean}	The average boarding time under the boarding model
V_i	Speed of passenger i
N_i	Number of carry on luggages of passenger i
NA_i	Number of the occurrence of the aisle interference
N_{stow}	Number of relevant stows that contribute to aisle interference of passenger i
N_{shuf}	Number of relevant seat shuffles contributing to aisle interference of passenger i
N_{avg}	The average number of luggage carried by passengers
N_{max}	The maximum number of carry-ons luggage a passenger can have
t_{id}	Time for passenger i to disembark the plane given that he wants to disembark as soon as possible
t'_{id}	Time for passenger i to disembark the plane given that he does not want to disembark as soon as possible

t'_{ia}	Time for passenger i to pass the aisle given that he does not want to disembark as soon as possible
t_{ib}	Time taken for passenger i to get to the aisle he needs to get to before entering the aisle
t_{iw}	The time to wait for entering the lane

0.4 General Assumptions

First, we assume the plane will be full unless otherwise stated (Jan Kamps, 2017). As a full plane maximises profit, overbooking flights is common and largely ensures planes are full. This simplifies our model by not having to account for empty seats, which would affect seat interference time.

Second, the walking speed of all individuals can be plotted on a normal distribution. We assume the mean walking speed of an individual is 1.34m/s and standard deviation of the walking speeds of individuals is 0.37m/s (Buchmüller & Weidmann, 2006) without any luggage. A study from Malaysia has found similar speeds in a rail transit terminal, comparable to an aeroplane due to a similar amount of urgency (to board transportation or arrive at one's seat) and a similar travelling demographic (those who can afford to and have the imperative to travel by train are more likely to also want to travel by air) (Kasehyani et al., 2019, #).

Third, we assume seats are pre-assigned and passengers must stick to their assigned seat. This assumption holds in the real world.

Fourth, we assume the dimensions of seats remain constant at 31.25 inches or 0.7747m by 17.5375 inches or 0.4454m, based on the seat pitch and distance in 2014 (Econlife, 2017). 0.7747m is the distance between each consecutive row. 0.4454m is the width of a seat. As shown in Figure 1 of the narrow-body passenger aircraft as well as Figure 2 of the Flying Wing passenger aircraft, the distance between each row and column (excluding the alsie) is constant, justifying our assumption. This is to simplify our calculation of t_r.

Fifth, we assume that the maximum number of carry-ons a passenger can carry is 5, i.e. This is because most flights will allow passengers to bring a maximum of two pieces of luggage, but passengers may bring additional items such as ladies' handbags, document briefcase or laptop bag.

Sixth, we assume most passengers would follow instructions. The percentage of passengers not following the prescribed method is capped at 20%.

Seventh, all passengers are independent from one another, i.e. they all travel individually. If passengers are travelling in groups, they may disobey instructions in order to queue together, increasing the total time taken. However, travelling in groups may also reduce seat shuffling and hampering as passengers in groups would optimally queue in the order of window, middle and then aisle seats, decreasing the total time taken.

1 Mathematical Model to calculate Boarding and Disembarking times

1.1 Factors to consider for boarding

To measure how long it takes to board the plane, we must consider the journey of each passenger from their initial position to their seat. Ultimately, the total time to seat all passengers will be equal to the total time the last passenger takes to arrive at their seat, assuming a measurement of time taken starts from the time the first passenger begins the process.

There are three main components of a passenger's journey to consider when calculating their total time taken. The i passenger is the passenger that is ith in queue. In the first two (1.1.1 and 1.1.2) we will consider the individual passenger's experience stowing luggage, sitting down, etc. In 1.1.3 we will consider the effects of other passengers' movements on the speed of an individual passenger behind them in the queue.

1.1.1 Luggage interference time, t_{il}

Time taken to stow luggage within given overhead compartment is t_{il}. The main determining factor of t_{il} is the number of carry-ons the passenger has, N_i, and the properties of these carry-ons. We assume N_i is modelled by a Poisson distribution. Each passenger would bring their own carry-on bag independent of others' carry-ons and the average number of luggages in a randomly chosen set of passengers has a constant mean. Most travellers in economy class can bring on board one full-size carry-on bag and one small bag to be stored in the seat in front(Jetstar, 2015).

$N_i \sim Po(N_{avg})$ where parameter N_{avg} can be quantified as 1.0.

$$N_i < N_{max}; N_{max} = 5$$

The more luggage the passengers carry, the greater interference will be caused by stowing luggage. We posit that the time taken to stow bags will follow the Weibull distribution (Lijuan et al., 2021, #).

Weibull distribution:

$$F(t_{il}, k, m) = \int_0^{t_{li}} \frac{k}{m^k} t_{li}^{\,k-1} e^{-(t_{li}/m)^k} dt_{li} = 1 - e^{-(t_{li}/m)^k}$$

t_{il}, random variable denoting the luggage interference time in seconds

$k > 0$, shape parameter; k =1.7s

$m > 0$, scale parameter; m =16s (Schultz, 2017, #)

Luggage interference time

$t_{il} \sim Weibull(16,\ 1.7)$

1.1.2 Seat interference time, t_{is}

Time taken to take their seat, once they've arrived at the row and placed their luggage, is t_{is} (i.e. after accounting for seat shuffling or hampered movement). Seat interference here refers to the additional step necessary to access a window or middle seat if another passenger is already seated, blocking access to the seats inside, within the row.

There are 3 possible scenarios for an individual attempting to take their seat:

1. They can enter their row unobstructed, and sit down. Speed and time taken will still depend on the position of their seat (aisle, middle, window).
2. There may already be individuals sitting in their row. They can enter their row slower, as previously seated individuals stand up to make space for entry. We have called this hampered movement. The probability of this *given an obstruction in the passenger arriving at their seat* is denoted by $P(hampered)$, which will be taken as 0.5, but can be modified.
3. There may already be individuals sitting in their row. The seated individual may choose to stand up and exit the row, before allowing both passengers to enter/re-enter. For example, if a passenger is in a window seat and the aisle seat is occupied, the passenger in the aisle seat must exit the row, allow the window seat passenger to enter, and then re-enter the row to sit down. We have called this seat shuffling. The probability of this *given an obstruction in the passenger arriving at their seat* is denoted by $P(shuffling)$, which will be taken as 0.5, but can be modified.

We have set $P(hampered) = P(shuffling) = 0.5$ as the two options are based on the taste of individuals, and there is not enough data on the probability of taste leaning in a certain direction.

Seat position of passenger attempting to take their seat (waiting passenger)	Seat position(s) of already seated passenger(s) within the row	Effect
Aisle	None, Middle or Window	Unhindered entry: No additional time is taken. However, time is still taken to enter the row and take their seats.
Middle	None or Window	
Window	None	
Middle	Aisle	Additional time taken due to seat interference. If seat shuffling occurs, this is the three-step process: a) The seated passenger must exit b) The waiting passenger must enter and take their seat c) The seated passenger must enter again.
Window	Aisle	
Window	Middle	

Window	Aisle and Middle	The longest time taken due to seat interference. If seat shuffling occurs, this is the five-step process: a) The aisle passenger must exit b) The window passenger must exit c) The window passenger must enter and take their seat d) The middle passenger must enter e) The aisle passenger must enter.

We understand that exiting or entering a row would not be at a walking speed. In aircrafts, due to cramped space and a difficulty to walk comfortably while getting to one's seat, the speed of the passenger while getting to their seat (after arriving at their row) cannot be approximated to the walking speed of a passenger.

Unobstructed Time Taken

We will approximate unobstructed speed to get to the seat from the aisle to be 75% of their regular speed. Thus,

$t_{is} = \frac{k*0.4454}{0.75v_m}$ where k is either 1, 2 or 3 depending on whether the passenger i goes to the aisle, middle or window seat respectively

Hampered Time Taken

We will approximate the hampered speed to get to the seat from the aisle to be 50% of their regular speed if there is one passenger blocking. Thus,

$t_{is} = \frac{k*0.4454}{0.5v_m}$ where k is either 2 or 3 depending on whether the passenger i goes to the middle or window seat respectively

We will approximate the hampered speed to get to the seat from the aisle to be 25% of their regular speed if there are two passengers blocking. Thus,

$t_{is} = \frac{k*0.4454}{0.25v_m}$ where k is 3 as the passenger i goes to the window seat

We use intervals of 25% to emphasise the non-negligible difference between the different scenarios. This is our own approximation due to a lack of data on comparison of walking speed to speed with obstructions, such as on an aeroplane. Nonetheless, these intervals can be easily modified if further research is provided.

Seat Shuffling Time Taken

We will use a triangular distribution to model the time taken for seat shuffling. For the three-step process, we will set minimum time as 1.8s, mode as 2.4s and maximum time as 3.0s (Schultz, 2017, #).

Triangular distribution:

$$F(t) = \frac{(t-1.8)^2}{(1.2)(0.6)},\ 1.8 \le t \le 2.4$$

$$F(t) = 1 - \frac{(3.0 - t)^2}{(1.2)(0.6)},\ 2.4 \le t \le 3.0$$

Seat interference time:

$$t_{is} \sim Triangular(1.8,\ 2.4,\ 3.0)$$

For the five-step process, we will double each. This is because there is likely to be lag time in all 3 passengers coordinating their exit and entry in the correct order. As such, a triangular distribution with minimum time 3.6s, mode as 4.8s, and maximum time as 6.0s will be used.

Triangular distribution:

$$F(t) = \frac{(t - 3.6)^2}{(2.4)(1.2)},\ 3.6 \le t \le 4.8$$

$$F(t) = 1 - \frac{(6 - t)^2}{(2.4)(1.2)},\ 4.8 \le t \le 6.0$$

Seat interference time:

$$t_{is} \sim Triangular(3.6,\ 4.8,\ 6.0)$$

1.1.3 Aisle time, t_{ia}

We define aisle time as the time taken to travel from waiting position to seating row, t_{ia}. Aisle time is dependent on a passenger's walking speed, the distance they are required to walk to their row, and aisle interference.

A passenger's walking speed will depend on the number of bags they are carrying. Without bags, $V_i = V_m$. A reduction of 6% (male) and 10% (female) was found in walking speeds when carrying luggage (Kasehyani et al., 2019, #). Taking the average, we find an 8% decrease in walking speed when carrying luggage. In China, an average 10% decrease in walking speed was reported, with acknowledgement that size and weight of the luggage will affect total reduction (Liang & Zhao, 2016, #). We have decided to approximate this to a 10% reduction per carry-on bag. Hence, with one bag, $V_i = 0.90V_m$, so on and so forth. As in 1.1.1, the number of bags will be modelled by a Poisson distribution.

Passengers may be required to stop and wait for those in front of them to stow their bags or seat shuffle – these are aisle interferences. Aisle interference is determined by the summation of the interferences occurring directly in front of the passenger i from when he starts waiting in line (i.e. when the first passenger boards the plane) all the way to when the passenger i reaches his row. We can calculate this time based on the time that the person directly in front of him stops. This would occur a) If they have reached their row, and need to stow a bag and b) If the person in front of them has stopped. Every passenger in front of passenger i would stop only due to these two reasons. Thus, we would be naturally accounting for the aisle interference by simulating the movement of the passengers based on the assumptions we've stated and on the luggage and seat interferences of each individual passenger. The number of relevant seat shuffles that contribute to aisle interference, N_{shuf}, depends on the boarding method used and $N_{shuf} \leq i - 1$. The number of relevant stows that contribute to aisle interference, N_{stow}, also depends on the boarding method used and $N_{stow} \leq i - 1$.

$$t_{ia} = V_i(y_i)(0.7747) + N_{stow}(t_{il}) + N_{shuf}(t_{is})$$

1.1.4 Percentage of passengers not following the prescribed method, Pn

The percentage of passengers will have a direct impact on the total boarding time. The larger the percentage of passengers not following the scheme, the longer the boarding time will be. Hence, we may measure the increase in total boarding time T by a factor of Pn, i.e.

$$\Delta T = Pn \times T$$

1.1.5 Conclusion: Total boarding time, T

Timing starts when the first passenger enters the place. The total time taken for each passenger to walk from its initial waiting position to its position, T_i, is measure by formulae

$$T_i = t_{ia} + t_{is} + t_{il}$$
$$T^* = \{T_i | i \in Z^*\}$$

The total time taken for all passengers to settle down is the boarding time for the last person to settle down. Hence, total aircraft boarding time T, can be obtained from the formulae:

$$T = (1 + Pn)max(T^*)$$

1.2 Factors to consider for disembarking

When modelling disembarking, we have renumbered the passengers according to where they sit. From passenger 1 being the person at the front row window seat, and passenger 2 being the person next to him and so on.

1.2.1 Time to retrieve carry-ons

The time taken for a passenger to retrieve his bag is equivalent to t_{il} of that passenger, as retrieval is the opposite process of stowing and should take a comparable amount of time. To retrieve bags, passengers must be standing in the aisle.

We assume most individuals would stand up and retrieve their carry-ons at the first opportunity, for instance, those sitting in aisle seats standing up to retrieve their carry-ons once the seatbelt sign has been switched off. We will model this at 90% probability, due to a lack of concrete statistics. These passengers are <u>expected</u>. Some passengers, however, will wait for a certain number of passengers to have disembarked before retrieving their carry-ons and beginning the disembarking process, out of a desire to make disembarking easier for others and less rushed for themselves. These passengers are <u>outliers</u>. If both aisle passengers in the same row would like to get out of the plane as soon as possible, both have an equal chance of standing up first, with probability = 0.5. If one of the aisle passengers is an outlier, then the other aisle passenger has a 100% chance of getting to the aisle first.

Time taken to exit a passenger's seats, in order to reach the aisle, would be the unobstructed time taken to get to the aisle, i.e. $t_{is} = \frac{k*0.4454}{0.75v_m}$ where k is either 1, 2 or 3 depending on whether the passenger i goes to the aisle, middle or window seat respectively.

If an aisle passenger is an outlier, and the middle passenger is expected, the middle passenger may choose to walk (obstructed) to the aisle anyway. If both aisle passengers are outliers, both middle passengers have an equal chance of getting to the aisle first and so on and so forth. In this situation, we will consider the hampered time taken to reach the aisle.

$t_{is} = \frac{k*0.4454}{0.5v_m}$ where there's only one person obstructing the passenger

$t_{is} = \frac{k*0.4454}{0.25v_m}$ where there are two people obstructing the passenger, for instance, those sitting in aisle seats standing up to retrieve their carry-ons once the seatbelt sign has been switched off. We will model this at 90% probability, due to a lack of concrete statistics. These passengers are expected. Some passengers, however, will wait for a certain number of passengers to have disembarked before retrieving their carry-ons and beginning the disembarking process, out of a desire to make disembarking easier for others and less rushed for themselves. These passengers are outliers. If both aisle passengers in the same row would like to get out of the plane as soon as possible, both have an equal chance of standing up first, with probability = 0.5. If one of the aisle passengers is an outlier, then the other aisle passenger has a 100% chance of getting to the aisle first.

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1.2.2 Time taken to exit after retrieving carry-on

Time taken to exit after retrieving all carry-on bags will be affected by those closer to the exit retrieving their possessions and thus blockading the aisle, restricting the ability for waiting passengers to move forward. We will modify our equation for aisle interference when boarding for aisle interference when disembarking. We assume that during disembarking, no seat shuffling will take place. This is because the motivations of those remaining in their seats are likely to be altruistic and in the interests of speeding up disembarking for other passengers – seat shuffling runs counter to this aim.

$$t_{ia} = V_i(y_i)(0.7747) + N_{stow}(t_{il})$$

Thus, the time for passenger i, given that he wants to get off the plane as soon as possible, to disembark would be $t_{id} = t_{is} + t_{ia}$ and the maximum time this takes (for the last passenger) is $max(t_{id})$.

1.2.3 Conclusion: Total disembarking time, t'_{id}

We would get the time taken for the last person to get off the plane accounting for the people who would want to get off the plane after the last instruction-following person has left would be $t'_{id} = max(t_{id}) + t'_{ia} + t_{is}$ where t'_{ia} is the new aisle interference after the people that want to leave as soon as possible have left. renumbered the passengers according to where they sat. From passenger 1 being the person at the front row window seat, and passenger 2 being the person next to him and so on.

1.2.4 Justification of optimal disembarking method

Disembarking by column is the optimal method. Passengers in one of the aisle columns should retrieve their carry-ons immediately after the seatbelt sign is lifted and leave promptly when disembarking begins. As the aisle passengers are moving up the aisle, those seated in the other aisle should begin retrieving their carry-ons, and join the queue to leave once they are able to. This is followed by middle seaters, and finally, window seaters. This would maximise the total amount of luggage retrieved at the same time, minimising the total luggage interference time. As aisle interference is dependent on the luggage interference time of others, the aisle time is significantly reduced, minimising time necessary for disembarking.

2 Standard narrow-body aircraft

Random boarding method, boarding by section method and boarding by seat method are some common boarding strategies used in real life. We have developed a Python programme based on the simulation framework by Gaurav Deshmukh to calculate the boarding time for the aforementioned methods respectively. Then, we use the Monte Carlo method to estimate the cumulative probability distribution of the boarding time in different models, and determine the average, practical maximum and practical minimum of the aforementioned models. To determine which is more significant in affecting the boarding time, T, we implement a sensitivity analysis on constant N_{avg}, N_{max} and Pn. All of our data and sensitivity analysis can be found in **Appendix A**.

2.1 Random (Unstructured) Boarding
We assume all seats are assigned, as most airlines have phased out unassigned seating. With random boarding, passengers are let into the plane in any order they please. In real life, this usually means passengers queue up based on a first-come-first-serve basis, with luck playing the largest role in determining a passenger's position in the queue.

2.2 Boarding by Section
Passengers are divided into three areas: row 23-33 (Aft), row 12-22 (Middle) and row 1-11 (Bow). The passengers with larger row numbers (e.g. Aft) can enter the airplane earlier, as they are seated further away from the entrance. Passengers enter in groups, but ordering within said groups is still random.

2.3 Boarding by Seat
In the boarding by seat method, passengers queue in the order of window seaters (A and F) first, middle seaters (B and E) in the middle and aisle seaters (C and D) last, i.e. Window seaters enter first when all passengers follow the prescribed method.

2.4 Boarding by Column
This is referenced by Gaurav Deepkesh, who termed it "Super Ideal Not Practical" – but for this report, we have renamed it "Boarding by Column". The first in queue will be a window seater from the last row of the plane, say from column A. The second in queue will be a window seater from the second-last row of the plane, from the same column, A. This ordering system continues until all passengers from column A are in queue. The next in queue is then the other window seater from the last row of the plane, seated in column F. This ordering system continues until all window seaters are in queue (from column A, and then column F). A similar ordering process continues, with middle seaters queuing in order (column B and E), and then aisle seaters entering the queue (column C and D). A practical limitation of this is that passengers travelling together, such as families in children, would be unwilling to remain separated.

2.5 Reverse Pyramid Boarding
Reverse pyramid boarding method involves grouping passengers by sections, with the groups looking like a "reverse pyramid". This was developed by Arizona State University and America West Airline, and is described as a "hybrid between traditional back-to-front boarding and outside-inside boarding used by other airlines" (van den Briel et al., 2005, #). Passengers can queue however they like within their respective sections.

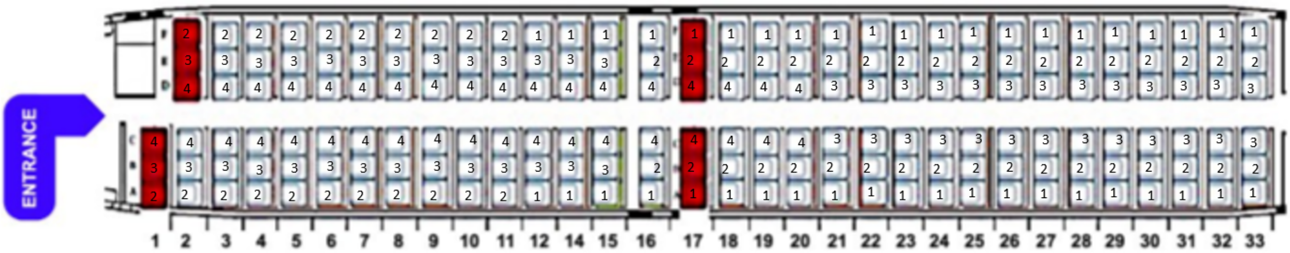

Fig 2.1: Illustration of the order of reverse boarding pyramid

2.6 Comparison of potential models

The best model will be decided on two metrics.

1) Mean time, to determine which boarding method is the fastest in normal circumstances.
2) The difference between the practical maximum and practical minimum, to evaluate which of the boarding methods is the most consistent.

The following table shows the results of our simulations:

Method	Random	Boarding by section	Boarding by seat	Boarding by column	Reverse Pyramid boarding
Average time	1132	1087	1105	1028	1028
Practical Minimum	1083	1061	1060	996	995
Practical maximum	1177	1114	1148	1047	1047

2.6.1 Fastest

The fastest model is a tie between the reverse pyramid boarding and boarding by column methods. These models are followed by boarding by section, then boarding by seat and lastly, randomised boarding. This may be due to both methods maximising the number of passengers stowing luggage at each instance of time. Both methods also reduce seat shuffling by ensuring that passengers sitting furthest from the aisle enter first.

Boarding by section is more efficient than the boarding by seat method. This is because in the boarding by seat method, passengers can still stall those behind them in the queue as passengers are not arranged by row. However, the boarding by section method does try to limit aisle interference in the form of arranging the queue in sections as such, more people can enter the plane at any particular time than the boarding by section method. Randomised boarding is the most inefficient as it does not optimise any of the interferences.

2.6.2 Most Consistent

The most consistent method based on our data is boarding by column method, followed by boarding by section and reverse pyramid boarding, then boarding by seat and lastly, randomised boarding. This is likely because the aisle interference is causing the most deviation in the timings (being a summation term and thus, being very sensitive) and as such, methods which attempted to minimise the aisle interferences, such as boarding by column, section and reverse pyramid boarding, were more consistent.

2.6.3 Accounting for allowance in the change in luggage amounts and disobedience rate

By comparing the change in the average time to board the plane, based on our sensitivity analysis, we find out that boarding by seat is most sensitive, decreasing by 2.30% when the average number of carry-on luggages decreased from 2 to 1, followed by boarding by section, decreasing by 1.27% and finally boarding randomly, with a decrease of 0.789%.

We also find out that boarding by seat is less sensitive to disobedient people, increasing by 8.83% when the percentage increases from 10% to 20%, than boarding by section, increasing by 9.05%. Boarding randomly wouldn't be affected by any disobedience in the queue as the arrangement does not matter in a random queue.

Lastly, to find out what happens when the passengers bring more carry-ons than usual, we will compare the increase in average time when bringing on 5 carry-ons instead of only 4. We find that boarding by seat has the largest increase in average time of 21.8%, then boarding randomly of 20.8% and lastly, boarding by section of 18.8%.

2.7 Most Optimal Boarding

We recommend using the reverse pyramid boarding method. From our analysis of the mean, practical maximum and practical minimum, we can conclude that the most optimal method to use would be the boarding by column method as it is the most efficient and the most consistent method, quantitatively determined by our model. The difference between both methods is negligible. As the boarding by column method requires each individual to be queued up in a specific order, to minimise logistical difficulty, we can employ the reverse pyramid method because it allows for some leniency with the boarding process, allowing passengers to order themselves within the section they are assigned to (1, 2, 3, or 4) in the queue. As such, this method might be more practical in real life situations.

3 Modification for other aircrafts

In this section, we adapt our model to fit other aircraft.. The structures we'll be modifying our model to are the Flying Wing Passenger Aircraft and the "Two-Entrance, Two Aisle" Passenger Aircraft. Luggage interference time will remain constant, with only the path each passenger takes changing. The aisle width, a_w is 12 inches (HighSkyFlying, 2022), or 0.305m which needs to be taken into account as the passengers need to travel past the aisles as well.

3.1 Flying Wing boarding model modifications

First, the passengers would enter from the bow of the aircraft, at the top left-hand side. We must account for time taken to reach the beginning of their respective aisles, t_{ib} .

$$t_{ib} = \frac{3(0.4454)}{V_i} \text{ for } x_i = 1,\ 2,\ 3,\ 4,\ 5,\ 6$$
$$t_{ib} = \frac{9(0.4454)+3(0.7747)+0.305}{V_i} \text{ for } x_i = 7,\ 8,\ 9,\ 10,\ 11,\ 12$$
$$t_{ib} = \frac{15(0.4454)+3(0.7747)+2(0.305)}{V_i} \text{ for } x_i = 13,\ 14,\ 15,\ 16,\ 17,\ 18$$
$$t_{ib} = \frac{18(0.4454)+6(0.7747)+3(0.305)}{V_i} \text{ for } x_i = 19,\ 20,\ 21,\ 22,\ 23,\ 24$$

Second, the number of rows in each column is different. A Flying Wing aircraft can be modelled by 4 narrow-body plane structures of different sizes. There are 2 11-row blocks of 3 columns, and 3 14-row blocks of 6 columns. This will not affect the equation for luggage interference time or aisle time for each individual passenger, but overall interference should decrease.

As such,

$$T_i = t_{ia} + t_{is} + t_{il} + t_{ib}$$
$$T^* = \{T_i | \, i \in Z^*\}$$

$T = (1 + Pn)max(T^*)$, Pn being the percentage of passengers not following the prescribed method

3.1.1 Most optimal method to board Flying Wing

In a narrow-body aircraft, reverse pyramid and boarding by column methods were efficient as they maximised the number of passengers stowing luggage at each instance of time. This maximisation is easier on a Flying Wing aircraft as there are multiple aisles and points of entry to a row.

We understand that Flying Wing planes are most commonly used for military or research purposes, rather than used for the general public. In these circumstances, passengers are unlikely to disobey instructions, especially if they are military-trained soldiers. Passengers will be willing to employ time-saving practices such as not bringing a large carry-on bag that requires stowing – this is in order to arrive at their destination as fast as possible, with a trust that all checked-in luggage will arrive safely. Passengers will be willing to split up, streamlining the modelling process and ensuring easy use of the boarding by column method.

To modify the boarding by column method for a Flying Wing aircraft, we would need to consider the new type of block – 6-column, 14-row blocks, of which there are 3. In this, the two seats at the very middle of the block might be treated as “window seats” as they are furthest from the aisle. The seats directly next to “window seats” will be treated as “middle seats”, and those directly next to the aisle will be treated as “aisle seats”. With this reclassification of seats, we can board passengers in window-middle-aisle order.

3.2 Flying Wing disembarking model modifications

In disembarking, we must additionally account for hindrance due to blocking of the lane to get to the entrance. The lane refers to the horizontal strip for walking towards the entrance, and is connected to each aisle. For a person to get into the lane, a spot must be clear for them. This delay will be counted as aisle interference, as those queuing in the aisle aren't able to move onto the lane. When a spot is available, there are usually 2 passengers headed towards the entrance available to contest it – by taking it, moving ahead of the other passenger, and thereby making a beeline for the exit.

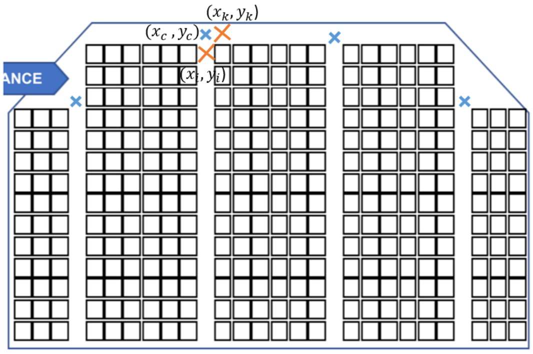

Fig 3.1: Crosses on the flying wing model show positions of (x_i, y_i), (x_k, y_k) and (x_c, y_c)

We must account for a new variable: time taken to wait for entering the lane, t_{iw}. Let (x_i, y_i) and (x_k, y_k) be waiting positions, with (x_c, y_c) denoting the interference point. Passenger i from queue m at (x_i, y_i) with speed v_{im} and passenger k from queue n with speed v_{kn} are two passengers meeting at the interference point (for example, the blue cross on the diagram, with the orange crosses representing the waiting positions). If $v_{im} > v_{kn}$, passenger i can move to (x_k, y_k), thus the waiting time for passenger i t_{iw} is 0. After passenger i exits, passenger k from queue n can proceed to interference point and the waiting time t_{iw} of passenger k is s_w/v_{im}. After passenger i and passenger k pass the interference point, the passengers after them can proceed to the waiting position and their respective t_{iw} will be calculated. From the diagram, there are 3 interference points in the flying wing plane. The t_{iw} will be calculated when the passenger is going to enter the interference point.

The time taken for passenger i to disembark can be calculated by $t_{id} = t_{is} + t_{ia} + t_{iw}$. Thus, the total disembarking time $t'_{id} = max(t_{id}) + t'_{ia} + t_{is}$

We want to minimise the chance that the last passenger will be stationary for a long period of time. Instead, the last passenger should be walking for as large a proportion of their journey as possible. Those in columns closer to the entrance/exit should thus disembark first, to allow for each passenger to be walking more than they are waiting. While those close to the entrance disembark, those further from the entrance can walk closer to the entrance. For each of the narrow-body sections in a Flying Wing aircraft (of which there are 4), the disembarking by column method, as mentioned in section 1.2.3, should be employed.

3.3 Two-Entrance, Two-Aisle boarding model modifications

The Two-Entrance, Two-Aisle plane has 2 seperate sections (termed “first class” and “economy”) with two entrances and two aisles to walk down. The plane is effectively split column-wise into 3 groups: 2 columns of seats at each side, 3 columns of seats in the middle. Since most airline companies provide first class seating before economy as a perk of being a first class passenger (Carrick, 2021), we can add both of the time taken for first class passengers and economy passengers to board the plane separately. We’ll assume the first class seat pitch is 80 inches or 2.03m, and 21 inches or 0.533 the Air China first class seat pitch (SeatGuru, 2022).

We have modified the coordinate system to account for first class passengers. This will split the plane into 3 distinct groups: first class, and 2 groups of economy class (determined by seat position). For the first class, seat A3 is to be (1, 2), D3 to be (3, 1) and H3 to be (5, 2). For the economy class, the first coordinate system is from A12 to K25. Seat A12 is (1, 1), D12 is (3, 2) and H12 is (5, 1). The other economy class coordinate system is from A47 to K26. Seat A47 is (1, 1).

3.3.1 First Class Passengers

As a general rule, if a passenger is sitting at row D, they would go for the aisle closer to the entrance and if a passenger is sitting at row F, they would go for the aisle further from the entrance. This is to allow easy access to their seats.

$$t_{is} = \frac{k*0.544}{V_i} \text{ with } k = 1 \text{ when } x_i = 2,\ 3,\ 4,\ 5 \text{ and } k = 2 \text{ when } x_i = 1,\ 6$$

Luggage interference time remains the same for first class passengers, but we neglect effects of seat shuffling and hampering due to large seat width. Aisle interference still occurs as the aisle is narrow.

$$t_{ia} = V_i(y_i)(2.03) + N_{stow}(t_{il})$$

To travel to the aisle they are required to go,

$$t_{ib} = 2(0.533) \text{ for } x_i = 1,\ 2,\ 3$$

$$t_{ib} = 4(0.533) + 0.305 \text{ for } x_i = 4,\ 5,\ 6$$

As such, for the first class passengers,

3.3.2 Economy Class Passengers

For economy class passengers, we must additionally take into account seat interference, assuming that the dimensions of the aisle and the seats are the same as the narrow-body aircraft. Changes to our original model will be made to account for differing entrances and aisles of the passengers.

Assuming the passengers would want to be as efficient as possible, passengers sitting at A12 to K25 would enter from the front entrance and the passengers sitting at A26 to K47 would enter from the back entrance, as it minimises the distance they need to travel to get to their seat. As an airline, this split will allow easier modelling and is the first step towards reducing boarding time. For passengers sitting at Row A to Row E, they would enter the aisle closer to the entrance and for passengers sitting at Row F to K, they would enter the other aisle.

$$t_{ib} = \frac{2(0.4454)}{V_i} \text{ for } x_i = 1,\ 2,\ 3,\ 4$$
$$t_{ib} = \frac{5(0.4454)+0.305}{V_i} \text{ for } x_i = 5,\ 6,\ 7$$

After arriving at their respective aisles, passengers' movements can be modelled similar to the narrow-body aircraft model. All interferences are calculated in the same way, however, for the seat shuffling, there's no more middle seating so $k = 1 \ or \ 2$ only.
Thus,

$$T_i = t_{ia} + t_{il} + t_{ib} \ , T_i = t_{ia} + t_{is} + t_{il} + t_{ib}$$
$$T_f^* = \{T_i | \ i \in Z^*\} \ , T_e^* = \{T_i | \ i \in Z^*\}$$
$$T_f = (1 + Pn) max(T_f^*) \ , T_e = (1 + Pn) max(T^*)$$

3.3.3 To Conclude
To calculate total time taken:

$$T = T_f + T_e$$

3.3.4 Most optimal method to board Two-Entrance, Two-Aisle
We still recommend using the boarding by column method, but modify it for the modified aeroplane structure.

Aisle interferences are naturally minimised as there is more than 1 path to go down. Seat interference has also significantly decreased, as the maximum number of passengers in front of the last passenger has decreased, reducing seat shuffling. However, there is an increase in the number of aisles, which will increase time taken if boarding by column is used. As such, we recommend a modified reverse pyramid boarding method.Passengers of row E should enter the aircraft first. Passengers in Row D and F should then enter together in an alternating pattern from back to front. Row C and H, and then, Row A and K, should enter in the same pattern.

3.4 Two-Entrance, Two-Aisle disembarking model modifications
The model of the disembarking time for the Two-Entrance, Two-Aisle plane is similar to the model for the disembarking of the Flying Wing plane, just that, now, there are only two aisles and that we only need to consider the time taken for the passengers to disembark through the entrance that majority of the passengers would go through. We would also need to do the

same addition as what we did for boarding, considering the first class and economy class passengers separately.

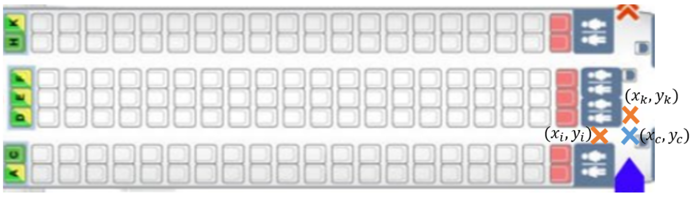

Fig 3.2: "Two-Entrance, Two Aisle" model to illustrate coordinate system

We would only be considering the portion of the economy passengers that are shown in the figure above that are disembarking as they would take more time than the other portion of the economy passengers, as it holds more passengers. We simply have to account for t_{iw} as shown in section 3.2. As such, the time taken for first class passengers to disembark, $t_{ifd} = t_{ifa} + t_{ifs} + t_{ifw}$ and for economy class passengers $t_{ied} = t_{iea} + t_{ies} + t_{iew}$ (with subscript, f denoting all for the first class passengers and e denoting all the economy class passengers). Then, $t_{id} = max(t_{ifd}) + max(t_{ied})$ and lastly, $t'_{id} = t_{id} + t'_{ia} + t_{is}$.

3.5 Most optimal method to disembark Two-Entrance, Two-Aisle

We recommend using a disembarking by column method. Coordinating disembarking in an alternating fashion may pose problems as passengers are unlikely to digest the instructions and follow them, especially when their main aim is to disembark as fast as possible. This would proceed with Row H disembarking first, then C, then K, then A, then D then F and finally, Row E. The passenger would get off at the same entrance as they got on. We choose to disembark using this method to reduce aisle and seat interferences by eliminating obstructions for passengers to get to the aisle. Everyone in the column is already ready to get off as soon as the last column has left the plane so there is little aisle interference.

4 Changes to prescribed boarding and disembarking methods in light of Covid-19

We will qualitatively explain necessary changes to our boarding and disembarking methods, to account for social distancing or limited capacity.

4.1 Boarding

First, the decrease in capacity would have a significant effect on aisle interference as there are fewer passengers to stall those behind them.

Second, having fewer passengers boarding also implies that there would be less seat shuffling, as there is a lower chance of a passenger being obstructed by another passenger. Taking into account social distancing, we understand that it would be optimal to provide each passenger with their own row (to truly minimise contact with strangers). For narrow-body aircraft, this would fill approximately 33% of capacity, and completely eliminate seat shuffling. To fill approximately 66% of capacity, passengers would be seated at window and aisle seats. This does not eliminate seat shuffling if aisle seaters are ahead of window seaters in queue.

For an individual passenger, aisle interference would experience a greater decrease than seat interference, as aisle interference is calculated by summation while seat interference is dependent on an individual passenger's experience. At 70% capacity, aisle interference on a narrow-body aircraft may not be significantly reduced, and we should still aim to minimise it. As such, a reverse pyramid boarding method is still recommended. The Two-Entrance, Two-Aisle plane can still use the modified boarding by column to board the plane. As social-distancing cannot be adequately ensured for all passengers at 70% capacity, we assume a low probability of transmission to begin with, and therefore do not account for possible transmission while in queue.

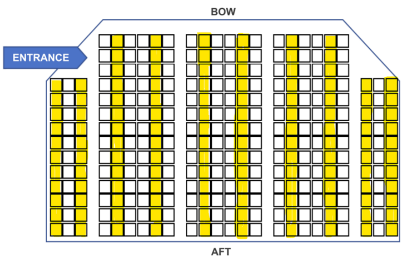

Fig 4.1: Seats to be used for social distancing, based on provided Flying Wing Passenger Aircraft diagram

By only utilising the coloured seats (as pictured), social distancing can be enforced on a Flying Wing aircraft. This is approximately 41.7% capacity. A boarding by column or section method can effectively be used for the 3 blocks in the middle, without any seat shuffling, with negligible difference due to already minimised aisle interference. For the two blocks at the side, boarding by column is recommended to eliminate seat shuffling.

For 50% or 30% capacity, we change two factors. First, we deprioritize optimising for aisle interference, as this variable is already minimised by low capacity. Instead, we focus on minimising seat interference. Second, we understand that propensity for transmission might be high, and want to minimise interaction with different strangers. There are 2 recommended methods, depending on the priority of the airline. First, as the boarding by seat method provides more liberty than the reverse pyramid boarding while still eliminating seat shuffling, we would recommend this as an efficient, logistically-easy option. On a Flying Wing aircraft, boarding by seat can also be easily used. On a Two-Entrance, Two-Aisle plane, our boarding by seat modified method can be employed, where passengers going to different aisles are alternated, to reduce time delay to reach the aisle. Then, we have them board in the order of Row A and K, Row E, Row C and H and lastly, Row D and F. This is ideal if the airline aims first and foremost to reduce seat interference. Second, boarding by section, as this may reduce transmission probability. Given passengers seated in the same section are most likely to spread viruses to each other, it is prudent to have these passengers queue together too, such that if any passenger has COVID-19, the number of people they infect is reduced. This is possible for all types of aircraft, and small sections are ideal to minimise the number of individuals infected.

4.2 Disembarking

For disembarking of the narrow-body, Flying Wing, and Two-Entrance, Two-Aisle plane at 70% capacity, there is no change made to the recommended boarding method that was mentioned earlier in the respective sections, as a 70% reduction in capacity would not cause a large difference in factors to be considered.

For disembarking of a narrow-body plane at 50% and 30% capacity, we propose disembarking by seat, with aisle seaters disembarking first, followed by middle seaters, and lastly, window seaters. Due to reduced capacity, not all aisle and middle seats may be filled. As such, if there is no aisle/middle seater obstructing the path of a middle/window seater (respectively), they may move towards the aisle and begin disembarking. Same goes for the Flying Wing plane and the Two-Entrance, Two-Aisle however, with a slight adjustment that the aisle closest from the entrance gets to disembark first as they can reach the entrance/exit faster.

5 Evaluation of Model and Future Work

In this section, we evaluate the strengths and weaknesses of our model and discuss future work that could reduce the impact of such weaknesses.

5.1 Strengths of Model

First, our model takes into account physiological differences amongst different passengers. We modelled the velocities of the passengers based on a normal distribution of walking speed of pedestrians, rather than taking the mean. This increases accuracy in accounting for real-world differences in passenger speeds.

Second, our model is adaptable. With a basis in Python code, we can change factors like mean number of carry-ons, or percentage of disobedient people, with relative ease. The code is also adaptable to different models of aircraft.. As such, airlines can use our code in a very cost effective manner to test out different boarding strategies on multiple aircraft.

5.2 Limitations of Model and Solutions to such Limitations

First, our code does not take into account the people travelling in groups. Families travelling together might protest separation during the boarding and disembarking process. Given more time and access to experimental conditions, we would investigate frequency of group travelling, and plot a Poisson distribution to determine the distribution of group sizes within the plane. With this, we would be able to group families together in the queue, seat them in the same row, and account for special considerations with children boarding and disembarking. For disembarking, we can reduce resistance to the disembarking by column method, through providing spaces outside the aircraft for family members to wait for each other and airline attendants to aid passengers unable to find this space.

Second, our model also does not account for the time delay caused by checking tickets and passports before boarding. This might cause gaps within the queue from the very beginning of the boarding process, implying that those with greater velocity could travel a further distance before their path is blocked by those with lower velocity. This would decrease the time taken to settle in the plane as the duration of aisle interference is lower. Given more time, we could have modelled the gap between each passenger on a normal distribution.

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Appendix A: Simulation Results and Sensitivity Analysis Data

Sensitivity Analysis:
Method: Random boarding
Changing average number of carry-ons:

N_{avg}	**1**	**2**	**3**
$\Delta N_{avg}/N_{avg} \times 100\%$	**-50%**	**0**	**50%**
T_{mean}	1132	1141	1189
$\Delta T_{mean}/T_{mean} \times 100\%$	-0.789%	0	4.21%
T_{min}	1098	1098	1117
$\Delta T_{min}/T_{min} \times 100\%$	0	0	1.73%
T_{max}	1177	1182	1231
$\Delta T_{max}/T_{max} \times 100\%$	-0.423%	0	4.15%

In situation where passengers have more carry-ons than usual:

N_{max}	**4**	**5**	**6**
$\Delta N_{avg}/N_{avg} \times 100\%$	**0**	**25%**	**50%**
T_{mean}	**936**	**1131**	**1324**
$\Delta T_{mean}/T_{mean} \times 100\%$	**0**	**20.8%**	**41.4%**
T_{min}	**898**	**1083**	**1275**
$\Delta T_{min}/T_{min} \times 100\%$	**0**	**20.6%**	**42.0%**
T_{max}	**974**	**1176**	**1376**
$\Delta T_{max}/T_{max} \times 100\%$	**0**	**20.7%**	**41.3%**

Method: Boarding by Section
Changing average number of carry-ons:

N_{avg}	**1**	**2**	**3**
$\Delta N_{avg}/N_{avg} \times 100\%$	**-50%**	**0**	**50%**
T_{mean}	**1087**	**1101**	**1144**
$\Delta T_{mean}/T_{mean} \times 100\%$	**-1.27%**	**0**	**3.76%**
T_{min}	**1061**	**1075**	**1102**
$\Delta T_{min}/T_{min} \times 100\%$	**-1.30%**	**0**	**6.79%**
T_{max}	**1114**	**1130**	**1181**
$\Delta T_{max}/T_{max} \times 100\%$	**1.42%**	**0**	**4.51%**

Changing percentage of disobedient people:

Pn	0	10%	20%
$\Delta Pn/Pn \times 100\%$	**-100%**	**0**	**100%**
T_{mean}	**1087**	**1200**	**1306**
$\Delta T_{mean}/T_{mean} \times 100\%$	**-9.42%**	**0**	**8.83%**
T_{min}	**1061**	**1169**	**1274**
$\Delta T_{min}/T_{min} \times 100\%$	**-9.25%**	**0**	**8.98%**
T_{max}	**1114**	**1225**	**1337**
$\Delta T_{max}/T_{max} \times 100\%$	**-9.06%**	**0**	**9.14%**

In situation where passengers have more carry-ons than usual:

N_{max}	4	5	6
$\Delta N_{avg}/N_{avg} \times 100\%$	**0**	**25%**	**50%**
T_{mean}	**915**	**1087**	**1298**
$\Delta T_{mean}/T_{mean} \times 100\%$	**0**	**18.8%**	**41.9%**
T_{min}	**890**	**1061**	**1240**
$\Delta T_{min}/T_{min} \times 100\%$	**0**	**19.2%**	**39.3%**
T_{max}	**944**	**1114**	**1298**
$\Delta T_{max}/T_{max} \times 100\%$	**0**	**18.0%**	**37.5%**

Method: Boarding by Seat
Changing average number of carry-ons:

N_{avg}	**1**	**2**	**3**
$\Delta N_{avg}/N_{avg} \times 100\%$	**-50%**	0	**50%**
T_{mean}	**1105**	1131	**1118**
$\Delta T_{mean}/T_{mean} \times 100\%$	**-2.30%**	0	**-1.15%**
T_{min}	**1060**	1069	**1079**
$\Delta T_{min}/T_{min} \times 100\%$	**-0.842%**	0	**0.935%**
T_{max}	**1148**	1152	**1163**
$\Delta T_{max}/T_{max} \times 100\%$	**-0.347%**	0	**0.935%**

Changing percentage of disobedient people:

Pn	**0**	**10%**	**20%**
$\Delta Pn/Pn \times 100\%$	**-100%**	**0**	**100%**
T_{mean}	**1105**	**1216**	**1326**
$\Delta T_{mean}/T_{mean} \times 100\%$	**-9.23%**	**0**	**9.05%**
T_{min}	**1060**	**1172**	**1275**
$\Delta T_{min}/T_{min} \times 100\%$	**-9.56%**	**0**	**8.79%**
T_{max}	**1148**	**1258**	**1375**
$\Delta T_{max}/T_{max} \times 100\%$	**-8.74%**	**0**	**9.30%**

In situation where passengers have more carry-ons than usual:

N_{max}	**4**	**5**	**6**
$\Delta N_{avg}/N_{avg} \times 100\%$	**0**	**25%**	**50%**
T_{mean}	**911**	**1105**	**1294**
$\Delta T_{mean}/T_{mean} \times 100\%$	**0**	**21.3%**	**42.4%**
T_{min}	**875**	**1060**	**1250**
$\Delta T_{min}/T_{min} \times 100\%$	**0**	**21.1%**	**42.9%**
T_{max}	**945**	**1147**	**1346**
$\Delta T_{max}/T_{max} \times 100\%$	**0**	**21.4%**	**42.4%**

Appendix B: Code (modified from the simulation framework of Gaurav Deshmukh)

```
import scipy as sci
import numpy as np
import matplotlib.pyplot as plt
time_set = []
nu = int(input('no of times:'))
max_luggage = int(input("max luggage:"))
disobedience_rate = float(input("Disobedience rate"))
av_luggage = int(input('average luggages:'))
boarding_method = input("intended boarding method")
for shu in range(nu):
    print(shu + 1) #check the progress
################################################################################
####
####part 1: Initialize
    #Define number of rows and columns
    n_rows=33
    n_cols=6
    #Calculate number of passengers
    n_pass=n_rows*n_cols
    #Create seat matrix
    seats=np.zeros((n_rows,n_cols))
    seats[:,:]=-1
    #Create aisle array
    aisle_q=np.zeros(n_rows)
    aisle_q[:]=-1
    #Create initial passenger number queue
    pass_q=[int(i) for i in range(n_pass)]
    pass_q=np.array(pass_q)
    #Create array for seat nos
    row_q_init=np.zeros(n_pass)
    col_q_init=np.zeros(n_pass)
    #Let's create moveto arrays
    moveto_loc=np.zeros(n_pass)
    moveto_time=np.zeros(n_pass)
    moveto_loc_dict={i:j for i in pass_q for j in moveto_loc}
    moveto_time_dict={i:j for i in pass_q for j in moveto_time}
################################################################################
####
    ###part 2
    def AssignSeats(rq,cq,assign_type,n_pass=n_pass,n_rows=n_rows):
        ##method 1: random boarding
        if(assign_type=="Random"):
```

```
    #Initialize possible row positions
    av_rows=np.arange(0,n_rows,1)
    #Make as many copies of these positions as the number of columns
    av_rows=np.tile(av_rows,(n_cols,1))
    av_rows=av_rows.T.flatten()

    #Initialize possible column positions
    av_cols=np.arange(0,n_cols,1)
    #Make as many copies of these positions as the number of rows
    av_cols=np.tile(av_cols,(n_rows,1)).flatten()

    #Create list of all possible seat positions
    av_seats=np.zeros((n_pass,2))
    for i in range(n_pass):
        av_seats[i]=[av_rows[i],av_cols[i]]

    #Randomize seat positions
    sci.random.shuffle(av_seats)
    rq=av_seats[:,0]
    cq=av_seats[:,1]

##method 2:boarding by seats method
if(assign_type=="BBS"):
#Initialize initial and final positions
    i=0
    f=n_rows
#Define column seating positions
    c=[0,5,1,4,2,3]
#Define iteration counter
    count=0
#Assign queue
    while(f<=n_pass):
        cq[i:f]=[c[count]]*n_rows
        i+=n_rows
        f+=n_rows
        count+=1
    #Initialize possible row positions
    av_rows = np.arange(0,n_rows,1)
    np.random.shuffle(av_rows)
    #Make as many copies of these positions as the number of columns
    av_rows=np.tile(av_rows,(n_cols,1)).flatten()
    rq = av_rows

##method 3:boarding by section method
```

```
    if(assign_type=="BBA"):
        #Initialize possible row positions
        av_rows=np.arange(0,n_rows,1)
        #Make as many copies of these positions as the number of columns
        av_rows=np.tile(av_rows,(n_cols,1))
        av_rows=av_rows.T.flatten()
        av_rows = np.sort(av_rows)[::-1]
        #Initialize possible column positions
        av_cols=np.arange(0,n_cols,1)
        #Make as many copies of these positions as the number of rows
        av_cols=np.tile(av_cols,(n_rows,1)).flatten()
        #Create list of all possible seat positions
        av_seats=np.zeros((n_pass,2))
        for i in range(n_pass):
            av_seats[i]=[av_rows[i],av_cols[i]]
        #Randomize seat positions
        rq=av_seats[:,0]
        cq=av_seats[:,1]

    ##method 4:boarding by columns
    if(assign_type=="SINP"):
    #Initialize initial and final positions
        i=0
        f=n_rows
    #Define column seating positions
        c=[0,5,1,4,2,3]
    #Define iteration counter
        count=0
    #Assign queue
        while(f<=n_pass):
            rq[i:f]=list(reversed(range(0,n_rows)))
            cq[i:f]=[c[count]]*n_rows
            i+=n_rows
            f+=n_rows
            count+=1

    ##method 5 reverse pyramid method
    if(assign_type == 'Reverse_Pyramid'):
        cq,rq = np.loadtxt('sequence.csv',delimiter = ',', usecols = (0,1), unpack = True)

    return rq,cq

##################################################################################
#part 3: assign factors
```

```
row_q,col_q=AssignSeats(row_q_init,col_q_init,boarding_method)

def norm_distribute(a,b,c):
    import random
    lst = []
    i = 0
    while i < c:
        rannum = random.normalvariate(a,b)
        if rannum > 0 and rannum < 1.5:
            lst.append(rannum)
            i += 1
        else:
            continue
    return lst
mean_velocity= 1.34
stddev_velocity= 0.37
velocity_q=norm_distribute(mean_velocity,stddev_velocity,n_pass)

def weibull_dis(a, b, c):
    import random
    lst = []
    i = 0
    while i < c:
        rannum = random.weibullvariate(a,b)
        if rannum > 0 and rannum < max_luggage:
            lst.append(rannum)
            i += 1
        else:
            continue
    return lst
time_l = weibull_dis(16,1.7, n_pass)
time_q = np.array(time_l)

pass_dict={}
time_dict={}
velocity_dict = {}
seat_nos=np.column_stack((row_q,col_q))
NL = []
shu2 =0
while shu2 <= n_pass:
    N = list(np.random.poisson(av_luggage,1))
    if N[0] < 10:
        NL += N
        shu2 +=1
```

```
        else:
            continue
    NL = np.array(NL)


    for i in range(n_pass):
        pass_dict[i]=seat_nos[i]
    for i in range(n_pass):
        time_dict[i]=time_q[i]
    for i in range(n_pass):
        velocity_dict[i] = velocity_q[i]*(1-NL[i]*0.1)

    #Create sum time array
    sum_time=np.zeros(n_pass)
    for i in range(n_pass):
        sum_time[i]=sum(time_q[:i+1])
##############################################################################
    ##part 4: function to move passengers into aircraft
    def MoveToAisle(t,aisle_q,pass_q,sum_time):
        if(t>sum_time[0]):
            if(aisle_q[0]==-1):
                aisle_q[0]=pass_q[0].copy()
                pass_q=np.delete(pass_q,0)
                sum_time=np.delete(sum_time,0)
        return aisle_q,pass_q,sum_time

##############################################################################
    ##part 5: simulation of boarding method
    time=0
    time_step= 0.4454/1.34 #speed = velocity_dict[passg] #distance of each step = 0.4454
    exit_sum=np.sum(pass_q)
    pass_sum=np.sum(seats)

    while(pass_sum!=exit_sum):
        if(pass_q.size!=0):
            aisle_q,pass_q,sum_time=MoveToAisle(time,aisle_q,pass_q,sum_time)
        for passg in aisle_q:
            if(passg!=-1):
                row=int(np.where(aisle_q==passg)[0]
                if(moveto_time_dict[passg]!=0:
                    if(time>moveto_time_dict[passg]:
                        if(moveto_loc_dict[passg]=="a"):
                            #If move is in the aisle, check if position ahead is empty
                            if(aisle_q[row+1]==-1:
```

```
                    aisle_q[row+1]=passg
                    aisle_q[row]=-1
                    #Set moves to 0 again
                    moveto_loc_dict[passg]=0
                    moveto_time_dict[passg]=0
                elif(moveto_loc_dict[passg]=="s"):
                    #If move is to the seat,
                    #Find seat row and column of passenger
                    passg_row=int(pass_dict[passg][0])
                    passg_col=int(pass_dict[passg][1])
                    #Set seat matrix position to the passenger number
                    seats[passg_row,passg_col]=passg
                    #Free the aisle
                    aisle_q[row]=-1
            elif(moveto_time_dict[passg]==0):
                #If move hasn't been assigned to passenger
                #Check passenger seat location
                passg_row=int(pass_dict[passg][0])
                passg_col=int(pass_dict[passg][1])
                if(passg_row==row):
                    #If passenger at the row where his/her seat is,
                    #Designate move type as seat
                    moveto_loc_dict[passg]="s"
                    #Check what type of seat: aisle, middle or window
                    #Depending upon seat type, designate when it is time to move
                    #tis=k*0.4454/0.75vm
                    if(passg_col==0):
                        if(seats[passg_row,1]!=-1 and seats[passg_row,2]!=-1):
                            moveto_time_dict[passg]=time+ time_dict[passg] + 3*0.445/
(0.25*velocity_dict[passg])
                        elif(seats[passg_row,1]!=-1):
                            moveto_time_dict[passg]=time+  time_dict[passg] + 3*0.445/
(0.5*velocity_dict[passg])
                        elif(seats[passg_row,2]!=-1):
                            moveto_time_dict[passg]=time+ time_dict[passg] + 3*0.445/
(0.5*velocity_dict[passg])
                        else:
                            moveto_time_dict[passg]=time+ time_dict[passg] +
3*0.445/((0.75*velocity_dict[passg]))
                    elif(passg_col==5):
                        if(seats[passg_row,4]!=-1 and seats[passg_row,3]!=-1):
                            moveto_time_dict[passg]=time+ time_dict[passg] + 3*0.445/
(0.25*velocity_dict[passg])
                        elif(seats[passg_row,4]!=-1):
```

```
                            moveto_time_dict[passg]=time+ time_dict[passg] + 3*0.445/
(0.5*velocity_dict[passg])
                        elif(seats[passg_row,3]!=-1):
                            moveto_time_dict[passg]=time+ time_dict[passg] + 3*0.445/
(0.5*velocity_dict[passg])
                        else:
                            moveto_time_dict[passg]=time+ time_dict[passg] +
3*0.445/(0.75*velocity_dict[passg])
                    elif(passg_col==1):
                        if(seats[passg_row,2]!=-1):
                            moveto_time_dict[passg]=time+ time_dict[passg] + 2*0.445/(
(0.5*velocity_dict[passg]))
                        else:
                            moveto_time_dict[passg]=time+ time_dict[passg] +
2*0.445/((0.75*velocity_dict[passg]))
                    elif(passg_col==4):
                        if(seats[passg_row,3]!=-1):
                            moveto_time_dict[passg]=time+ time_dict[passg] + 2*0.445/(
(0.5*velocity_dict[passg]))
                        else:
                            moveto_time_dict[passg]=time+ time_dict[passg] +
2*0.445/((0.75*velocity_dict[passg]))
                    elif(passg_col==2 or passg_col==3):
                        moveto_time_dict[passg]=time+ time_dict[passg] +
1*0.445/(0.75*velocity_dict[passg])
                elif(passg_row!=row):
                    #If passenger is not at the row where his/her seat is,
                    #Designate movement type as aisle
                    moveto_loc_dict[passg]="a"
                    #Designate time to move
                    moveto_time_dict[passg]=time+time_dict[passg]
##############################################################################
####
#part 6:calculate total time
        #Iteration timekeeping
        time += time_step
        pass_sum=np.sum(seats)
    time = time*(1 + disobedience_rate)
    time_set.append(time)
##############################################################################
###
#part 7: output
time_array = np.array(time_set)
data = np.array([np.arange(nu), time_array])
```

```
#save data into the data_random file
np.savetxt("data.csv", data.T, fmt = '%d',delimiter = ',')
```

Appendix C: Table of data (part of data)

	Rando m	Boarding By Seat	Boarding By Section	Reverse Pyramid	Boarding By Column
No.	Time/s	Time/s	Time/s	Time/s	Time/s
1	1137	1096	1083	1010	1015
2	1102	1112	1093	1013	1013
3	1106	1081	1104	994	1019
4	1136	1122	1094	1016	983
5	1166	1068	1097	1035	1009
6	1105	1147	1081	1007	1024
7	1051	1128	1064	1017	1004
8	1131	1130	1154	1033	1005
9	1077	1096	1086	1016	1027
10	1156	1111	1069	1017	1033
11	1143	1100	1072	1029	1018
12	1163	1087	1099	1005	1025
13	1130	1108	1069	1022	1023
14	1105	1066	1071	1013	1023

15	1152	1076	1106	1016	1020
16	1149	1080	1101	1014	1025
17	1119	1123	1091	998	1004
18	1127	1128	1111	1007	1036
19	1129	1102	1093	1007	1030
20	1118	1167	1077	1011	1035
21	1110	1138	1094	1005	1025
22	1104	1057	1118	1021	1005
23	1122	1120	1097	1008	1022
24	1108	1051	1102	1028	977
25	1107	1147	1081	1017	1030
26	1145	1149	1086	1011	1004
27	1114	1101	1105	1016	1009
28	1101	1115	1073	989	1024
29	1115	1128	1096	1031	1015
30	1121	1128	1081	1014	1016
31	1116	1140	1093	1023	1031

32	1139	1068	1102	1022	1029
33	1140	1100	1215	1020	1020
34	1132	1047	1092	1020	1023
35	1102	1133	1126	1014	1025
36	1077	1138	1073	1000	1033
37	1105	1150	1087	1012	1022
38	1145	1074	1071	1012	1033
39	1112	1113	1110	1020	1017
40	1121	1134	1093	992	1022
41	1100	1106	1111	1025	1014
42	1068	1107	1085	1008	1014
43	1186	1065	1086	1022	1028
44	1146	1110	1078	1009	1046
45	1093	1063	1085	1049	1020
46	1112	1086	1087	1005	1000
47	1132	1096	1148	1044	1026
48	1095	1072	1115	1024	1017

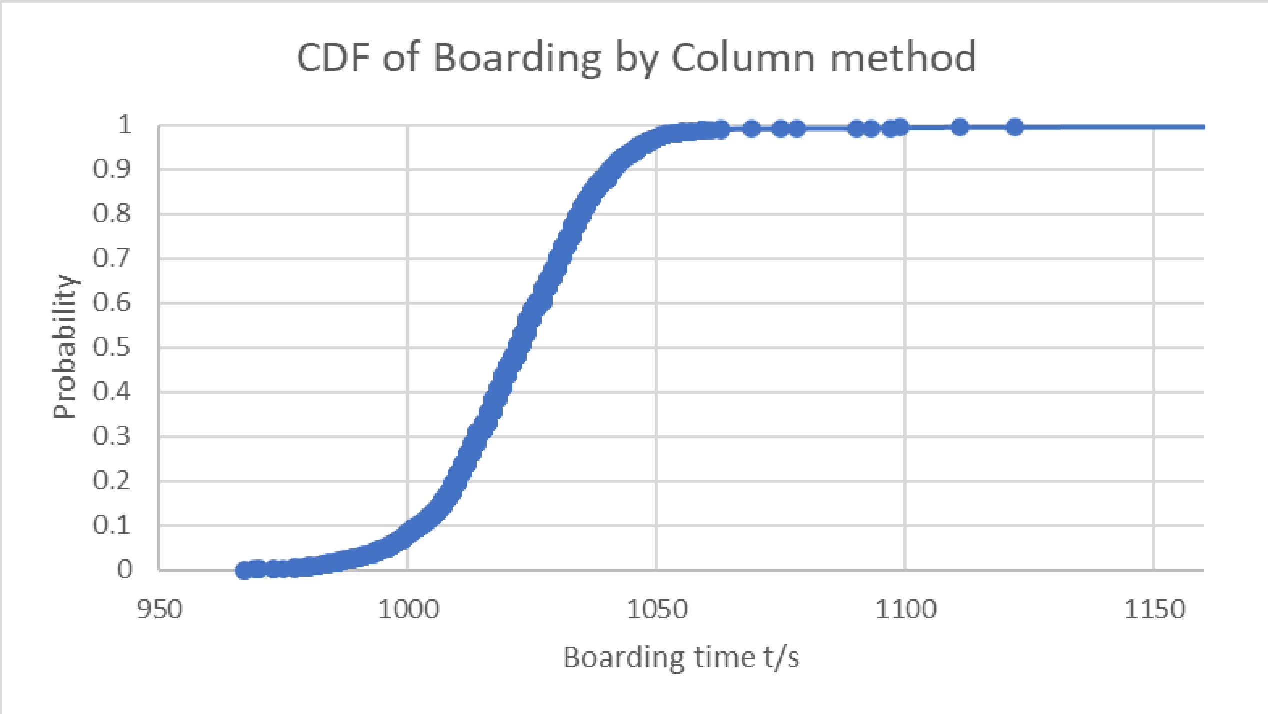
49	1144	1104	1090	1033	1004
50	1131	1078	1097	996	1010
51	1120	1109	1079	1012	1029
52	1127	1123	1109	1009	1023
53	1117	1070	1049	1040	1042
54	1127	1149	1087	1012	1029
55	1125	1122	1075	1047	1033
56	1133	1092	1072	998	1010
57	1169	1135	1076	1005	1026
58	1110	1123	1093	1012	1041
59	1131	1062	1093	975	1028
60	1136	1113	1082	989	1015
61	1145	1104	1093	1009	984
62	1185	1145	1082	1024	1044
63	1143	1117	1092	1045	1040
64	1097	1166	1100	1025	1031
65	1095	1090	1103	1012	1034

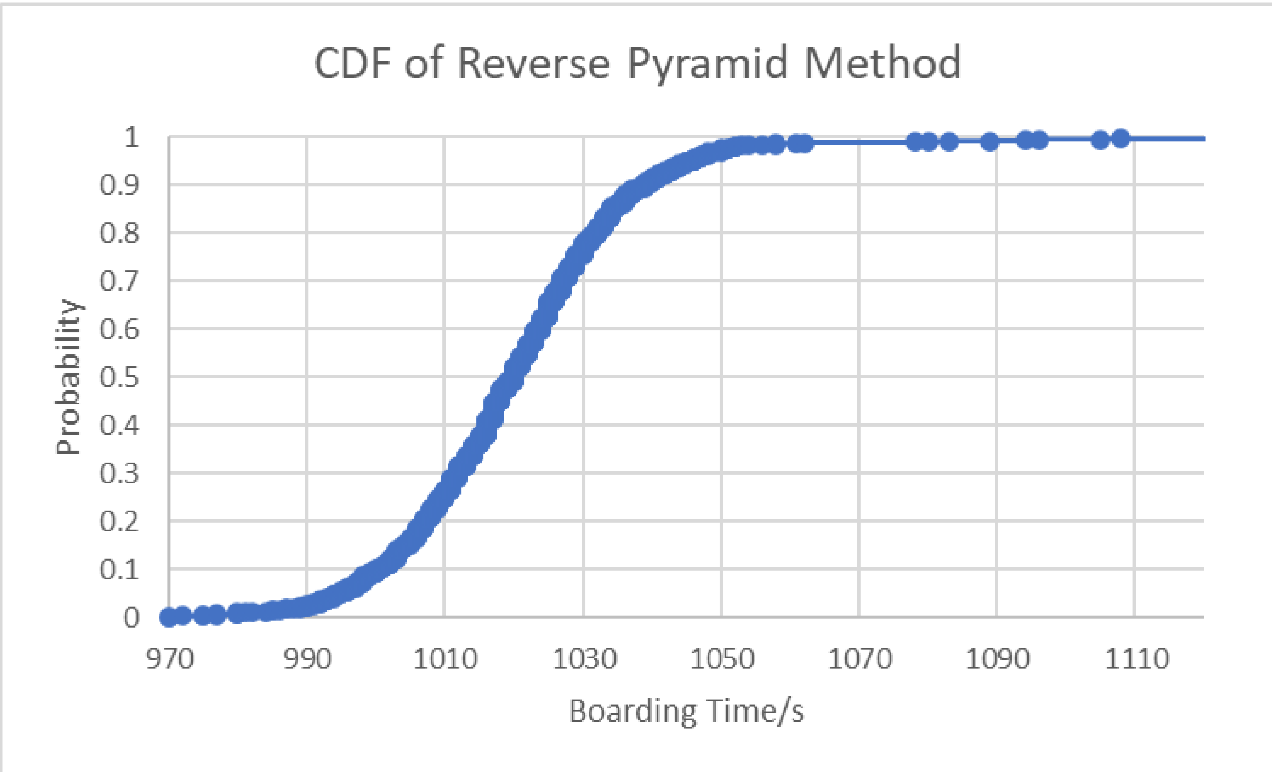
66	1135	1110	1096	998	997
67	1182	1113	1091	1020	997
68	1151	1062	1088	1009	1033
69	1120	1078	1103	1033	1024
70	1142	1093	1088	1003	1009
71	1145	1116	1097	998	1026
72	1131	1073	1100	1018	1014
73	1135	1058	1058	1013	1015
74	1157	1059	1091	1019	1027
75	1098	1083	1116	1021	1033
76	1117	1039	1092	1004	1037
77	1203	1105	1081	1044	1041
78	1097	1053	1094	1024	1013
79	1086	1078	1094	1028	1036
80	1156	1082	1110	1025	1023
81	1096	1099	1102	1026	1030
82	1119	1152	1094	999	1002

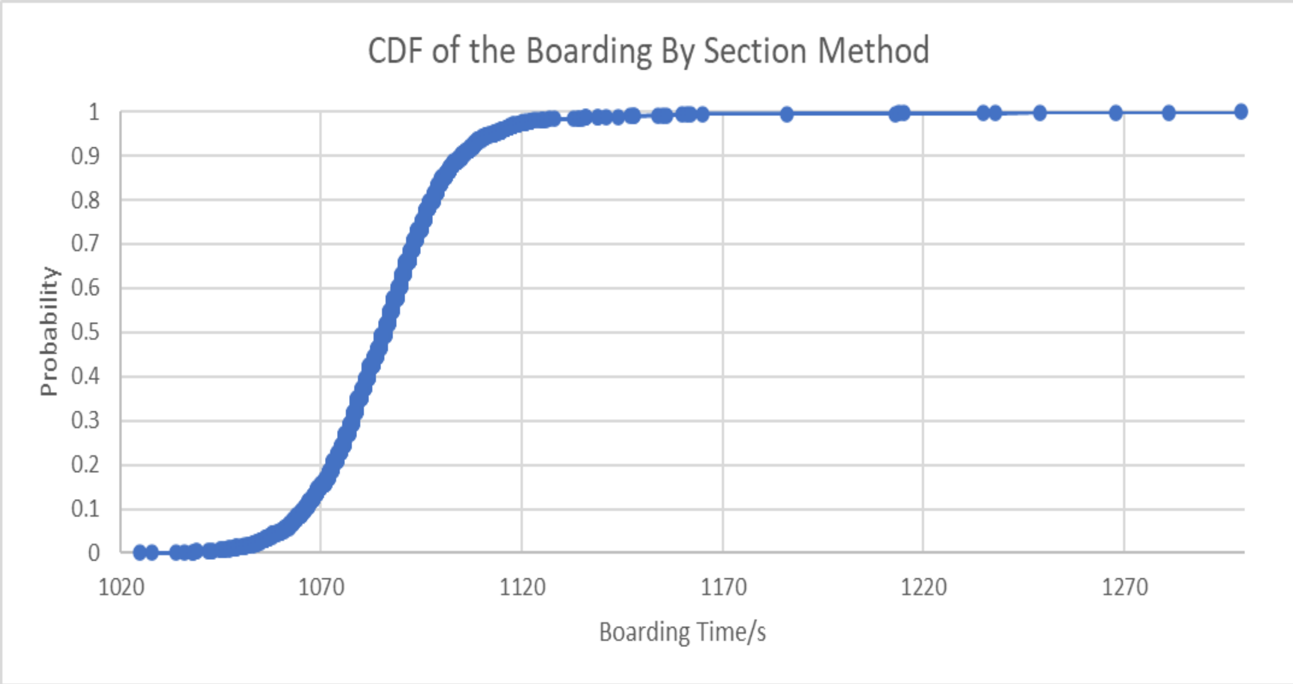
83	1130	1102	1125	1022	1041
84	1133	1462	1072	1014	1040
85	1139	1074	1136	1037	1021
86	1123	1078	1099	1043	1030
87	1109	1133	1078	1028	1029
88	1185	1058	1125	1016	1039
89	1082	1048	1090	1022	1009
90	1107	1089	1093	1012	1031
91	1125	1121	1082	992	1041
92	1120	1122	1069	1012	1028
93	1087	1095	1074	1044	1022
94	1128	1107	1095	1030	1020
95	1108	1102	1064	1034	1015
96	1126	1090	1078	1016	993
97	1105	1139	1067	1021	1043
98	1148	1081	1299	1006	994
99	1116	1128	1094	1000	1009

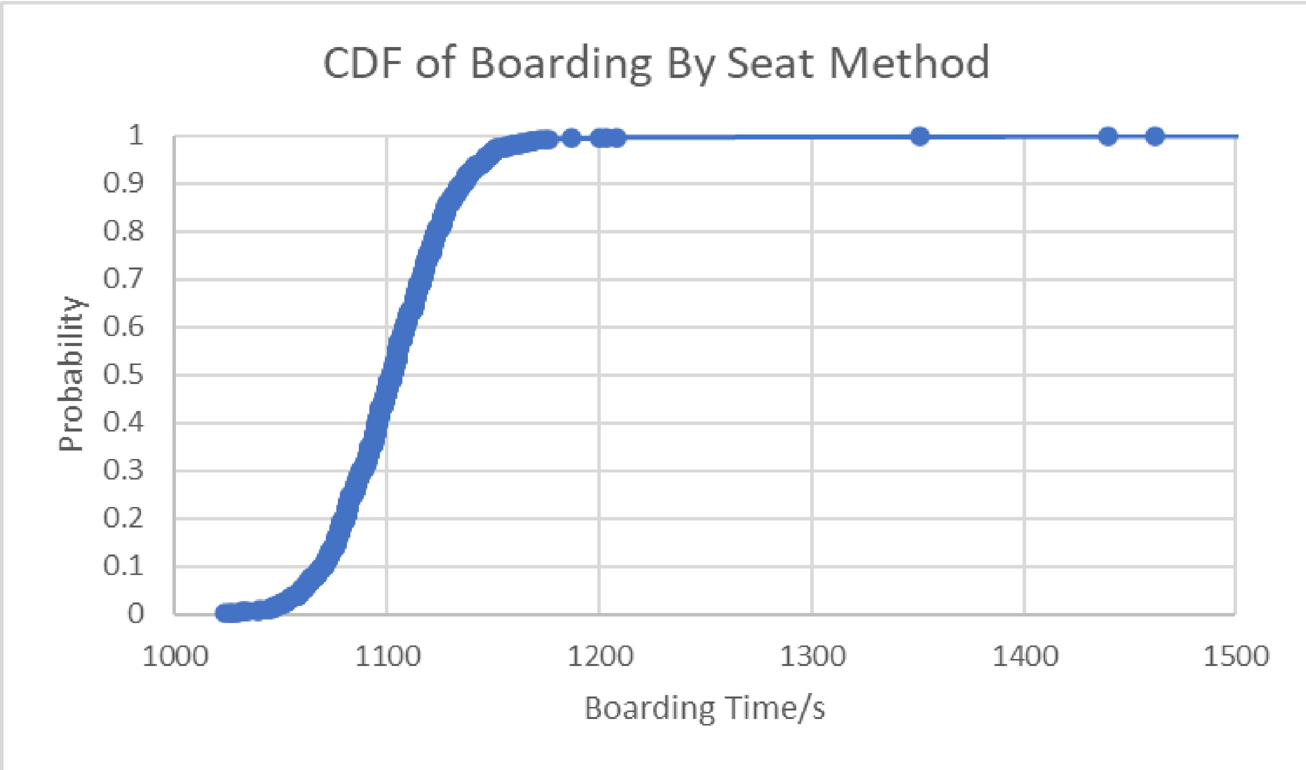
100	1104	1116	1081	1019	1028

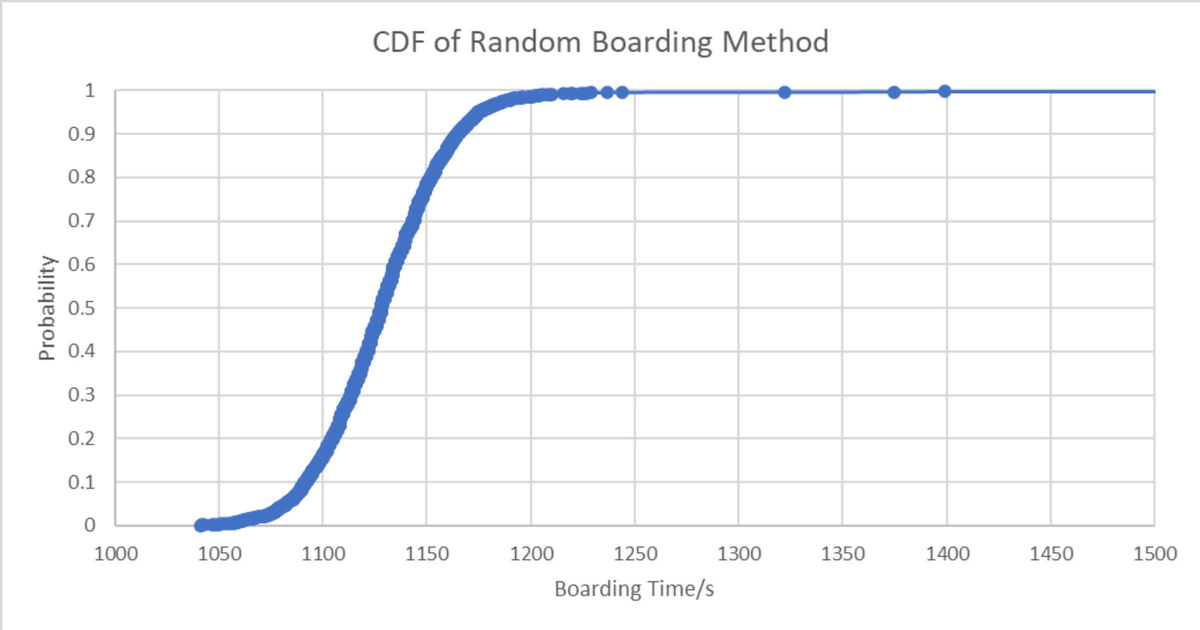
Appendix D: Cumulative Distribution Function diagram






CDF of the Boarding By Section Method
Probability
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1020
1070
1120
1170
1220
1270
Boarding Time/s


CDF of Boarding By Seat Method
Probability
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1000
1100
1200
1300
1400
1500
Boarding Time/s


CDF of Random Boarding Method
Probability
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1000
1050
1100
1150
1200
1250
1300
1350
1400
1450
1500
Boarding Time/s

Appendix E: Sequence of Reverse Pyramid

column, x	row, y
0	32
0	31
0	30
0	29
0	28
0	27
0	26
0	25
0	24
0	23
0	22
0	21
0	20
0	19
0	18
0	17

0	**16**
0	**15**
0	**14**
0	**13**
0	**12**
0	**11**
0	**10**
0	**9**
5	**32**
5	**31**
5	**30**
5	**29**
5	**28**
5	**27**
5	**26**
5	**25**
5	**24**

5	**23**
5	**22**
5	**21**
5	**20**
5	**19**
5	**18**
5	**17**
5	**16**
5	**15**
5	**14**
5	**13**
5	**12**
5	**11**
5	**10**
5	**9**
1	**32**
1	**31**

1	**30**
1	**29**
1	**28**
1	**27**
1	**26**
1	**25**
1	**24**
1	**23**
1	**22**
1	**21**
1	**20**
1	**19**
1	**18**
1	**17**
1	**16**
1	**15**
1	**14**

0	**8**
0	**7**
0	**6**
0	**5**
0	**4**
4	**32**
4	**31**
4	**30**
4	**29**
4	**28**
4	**27**
4	**26**
4	**25**
4	**24**
4	**23**
4	**22**
4	**21**

4	**20**
4	**19**
4	**18**
4	**17**
4	**16**
4	**15**
4	**14**
5	**8**
5	**7**
5	**6**
5	**5**
5	**4**
1	**13**
1	**12**
1	**11**
1	**10**
1	**9**

1	**8**
1	**7**
1	**6**
1	**5**
1	**4**
0	**3**
0	**2**
0	**1**
0	**0**
4	**13**
4	**12**
4	**11**
4	**10**
4	**9**
4	**8**
4	**7**
4	**6**

4	**5**
4	**4**
5	**3**
5	**2**
5	**1**
5	**0**
2	**32**
2	**31**
2	**30**
2	**29**
2	**28**
2	**27**
2	**26**
2	**25**
2	**24**
2	**23**
2	**22**

2	**21**
2	**20**
2	**19**
2	**18**
2	**17**
2	**16**
2	**15**
2	**14**
2	**13**
1	**3**
1	**2**
1	**1**
1	**0**
3	**32**
3	**31**
3	**30**
3	**29**

3	**28**
3	**27**
3	**26**
3	**25**
3	**24**
3	**23**
3	**22**
3	**21**
3	**20**
3	**19**
3	**18**
3	**17**
3	**16**
3	**15**
3	**14**
3	**13**
4	**3**

4	**2**
4	**1**
4	**0**
2	**12**
2	**11**
2	**10**
2	**9**
2	**8**
2	**7**
2	**6**
2	**5**
2	**4**
2	**3**
2	**2**
2	**1**
2	**0**
3	**12**

3	**11**
3	**10**
3	**9**
3	**8**
3	**7**
3	**6**
3	**5**
3	**4**
3	**3**
3	**2**
3	**1**
3	**0**