

1 Computational_Geometry

1.1 delaunay

```

1 template<class T>
2 class Delaunay{
3     struct PT:public point<T>{
4         int g[2];
5         PT(const point<T> &p):
6             point<T>(p){ g[0]=g[1]=-1; }
7     };
8     static bool cmp(const PT &a,const PT &b){
9         return a.x<b.x||(a.x==b.x&&a.y<b.y);
10    }
11    struct edge{
12        int v,g[2];
13        edge(int v,int g0,int g1):
14            v(v){g[0]=g0,g[1]=g1;}
15    };
16    vector<PT> S;
17    vector<edge> E;
18    bool convex(int &from,int to,T LR){
19        for(int i=0;i<2;++i){
20            int c = E[S[from].g[i]].v;
21            auto A=S[from]-S[to], B=S[c]-S[to];
22            T v = A.cross(B)*LR;
23            if(v>0||(v==0&&B.abs2()<A.abs2()))
24                return from = c, true;
25        }
26        return false;
27    }
28    void addEdge(int v,int g0,int g1){
29        E.emplace_back(v,g0,g1);
30        E[E.back().g[0]].g[1] = E.size()-1;
31        E[E.back().g[1]].g[0] = E.size()-1;
32    }
33    void climb(int &p, int e, int n, int nl,
34              int nr, int LR){
35        for(int i=E[e].g[LR]; (S[nr]-S[nl]).
36            cross(S[E[i].v]-S[nl])>0;){
37            if(inCircle(S[E[i].v],S[nl],S[nr],S[E[
38                i].g[LR]].v))>0)
39                { p = i; break; }
40            for(int j=0;j<4;++j)
41                E[E[i^j/2].g[j%2^1]].g[j%2] = E[i^j
42                    /2].g[j%2];
43            int j=i; i=E[i].g[LR];
44            E[j].g[0]=E[i].g[1]=E[j^1].g[0]=E[j
45                ^1].g[1]=-1;
46        }
47    }
48    T det3(T a11,T a12,T a13,T a21,T a22,T a23
49        ,T a31,T a32,T a33){
50        return a11*(a22*a33-a32*a23)-a12*(a21*
51            a33-a31*a23)+a13*(a21*a32-a31*a22);
52    }
53    int inCircle(const PT &a, const PT &b,
54                const PT &c, const PT &p){
55        T as = a.abs2(), bs = b.abs2(), cs = c.abs2
56            (), ps = p.abs2();
57        T res = a.x * det3(b.y,bs,1,c.y,cs,1,p.y,ps
58            ,1)
59            -a.y * det3(b.x,bs,1,c.x,cs,1,p.x,ps,1)

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60        +as * det3(b.x,b.y,1,c.x,c.y,1,p.x,p.y,1)
61        -det3(b.x,b.y,bs,c.x,c.y,cs,p.x,p.y,ps);
62        return res<0 ? 1 : (res>0 ? -1 : 0);
63    }
64    void divide(int l, int r){
65        if(l==r)return;
66        if(l+1==r){
67            int A=S[l].g[0]=S[l].g[1]=E.size();
68            E.emplace_back(r,A,A);
69            int B=S[r].g[0]=S[r].g[1]=E.size();
70            E.emplace_back(l,B,B);
71            return;
72        }
73        int mid = (l+r)/2;
74        divide(l,mid), divide(mid+1, r);
75        int nl = mid, nr = mid+1;
76        for(;;){
77            if(convex(nl,nr,1)) continue;
78            if(S[nr].g[0]!=-1&&convex(nr,nl,-1))
79                continue;
80            break;
81        }
82        addEdge(nr,S[nl].g[0],S[nl].g[1]);
83        S[nl].g[1] = E.size()-1;
84        if(S[nr].g[0]==-1){
85            addEdge(nl,E.size(),E.size());
86            S[nr].g[1] = E.size()-1;
87        }else addEdge(nl,S[nr].g[0],S[nr].g[1]);
88        S[nr].g[0] = E.size()-1;
89        int cl = nl, cr = nr;
90        for(;;){
91            int pl=-1, pr=-1, side;
92            climb(pl,E.size()-2,nl,nl,nr,1);
93            climb(pr,E.size()-1,nr,nl,nr,0);
94            if(pl==-1&&pr==-1) break;
95            if(pl==-1||pr==-1) side = pl==-1;
96            else side=inCircle(S[E[pl].v],S[nl],S[
97                nr],S[E[pr].v])<=0;
98            if(side){
99                nr = E[pr].v;
100            }
101            addEdge(nr,E.size()-2,E[E.size()-2].g[1]);
102            addEdge(nl,E[pr^1].g[0],pr^1);
103        }else{
104            nl = E[pl].v;
105            addEdge(nr,pl^1,E[pl^1].g[1]);
106            addEdge(nl,E[E.size()-2].g[0],E.size()-2);
107        }
108    }
109    if(cl==nl&&cr==nr) return; //Collinearity
110    S[nl].g[0] = E.size()-2;
111    S[nr].g[1] = E.size()-1;
112    }
113    public:
114    void solve(const vector<point<T>> &P){
115        S.clear(), E.clear();
116        for(const auto &p:P) S.emplace_back(p);
117        sort(S.begin(),S.end(),cmp);
118        divide(0,int(S.size())-1);
119    }
120    vector<pair<int,int>> getEdge(){
121        vector<pair<int,int>> res;
122        for(size_t i=0;i<E.size();i+=2)
123            if(E[i].g[0]!=-1)
124                res.emplace_back(E[i].v,E[i^1].v);
125        return res;
126    }

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1.2 Geometry

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1 const double PI=atan2(0.0,-1.0);
2 template<typename T>
3 struct point{
4     T x,y;
5     point(){}
6     point(const T&x,const T&y):x(x),y(y){}
7     point operator+(const point &b)const{
8         return point(x+b.x,y+b.y); }
9     point operator-(const point &b)const{
10        return point(x-b.x,y-b.y); }
11     point operator*(const T &b)const{
12        return point(x*b,y*b); }
13     point operator/(const T &b)const{
14        return point(x/b,y/b); }
15     bool operator==(const point &b)const{
16        return x==b.x&&y==b.y; }
17     T dot(const point &b)const{
18        return x*b.x+y*b.y; }
19     T cross(const point &b)const{
20        return x*b.y-y*b.x; }
21     point normal()const{//求法向量
22        return point(-y,x); }
23     T abs2()const{//向量長度的平方
24        return dot(*this); }
25     T rad(const point &b)const{//兩向量的弧度
26        return fabs(atan2(fabs(cross(b)),dot(b))); }
27     T getA()const{//對x軸的弧度
28        T A=atan2(y,x); //超過180度會變負的
29        if(A<=-PI/2)A+=PI*2;
30        return A;
31    }
32 };
33 template<typename T>
34 struct line{
35     line(){}
36     point<T> p1,p2;
37     T a,b,c; //ax+by+c=0
38     line(const point<T>&x,const point<T>&y):p1
39         (x),p2(y){}
40     void pton(){} //轉成一般式
41     a=p1.y-p2.y;
42     b=p2.x-p1.x;
43     c=-a*p1.x-b*p1.y;
44 }
45 T ori(const point<T> &p)const{//點和有向直
46     線的關係，>0左邊、=0在線上、<0右邊
47     return (p2-p1).cross(p-p1);
48 }
49 T btw(const point<T> &p)const{//點投影落在
50     線段上<=0
51     return (p1-p).dot(p2-p);
52 }
53 bool point_on_segment(const point<T>&p)
54     const{//點是否在線段上
55     return ori(p)==0&&btw(p)<=0;
56 }

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57 T dis2(const point<T> &p,bool is_segment
58     =0)const{//點跟直線/線段的距離平方
59     point<T> v=p2-p1,v1=p-p1;
60     if(is_segment){
61         point<T> v2=p-p2;
62         if(v.dot(v1)<=0)return v1.abs2();
63         if(v.dot(v2)>=0)return v2.abs2();
64     }
65     T tmp=v.cross(v1);
66     return tmp*tmp/v.abs2();
67 }
68 T seg_dis2(const line<T> &l)const{//兩線段
69     距離平方
70     return min({dis2(l.p1,1),dis2(l.p2,1),l.
71         dis2(p1,1),l.dis2(p2,1)});
72 }
73 point<T> projection(const point<T> &p)
74     const{//點對直線的投影
75     point<T> n=(p2-p1).normal();
76     return p-n*(p-p1).dot(n)/n.abs2();
77 }
78 point<T> mirror(const point<T> &p)const{
79     //點對直線的鏡射，要先呼叫pton轉成一般式
80     point<T> R;
81     T d=a*a+b*b;
82     R.x=(b*b*p.x-a*a*p.x-2*a*b*p.y-2*a*c)/d;
83     R.y=(a*a*p.y-b*b*p.y-2*a*b*p.x-2*b*c)/d;
84     return R;
85 }
86 bool equal(const line &l)const{//直線相等
87     return ori(l.p1)==0&&ori(l.p2)==0;
88 }
89 bool parallel(const line &l)const{
90     return (p1-p2).cross(l.p1-l.p2)==0;
91 }
92 bool cross_seg(const line &l)const{
93     return (p2-p1).cross(l.p1-p1)*(p2-p1).
94         cross(l.p2-p1)<=0; //直線是否交線段
95 }
96 int line_intersect(const line &l)const{//
97     直線相交情況，-1無限多點、1交於一點、0
98     不相交
99     return parallel(l)?(ori(l.p1)==0?-1:0)
100         :1;
101 }
102 int seg_intersect(const line &l)const{
103     T c1=ori(l.p1), c2=ori(l.p2);
104     T c3=l.ori(p1), c4=l.ori(p2);
105     if(c1==0&&c2==0){ //共線
106         bool b1=btw(l.p1)>0,b2=btw(l.p2)>0;
107         T a3=l.btw(p1),a4=l.btw(p2);
108         if(b1&&b2&&a3==0&&a4==0) return 2;
109         if(b1&&b2&&a3>0&&a4==0) return 3;
110         if(b1&&b2&&a3>0&&a4>0) return 0;
111         return -1; //無限交點
112     }else if(c1*c2<=0&&c3*c4<=0)return 1;
113     return 0; //不相交
114 }
115 point<T> line_intersection(const line &l)
116     const{//直線交點*/
117     point<T> a=p2-p1,b=l.p2-l.p1,s=l.p1-p1;
118     //if(a.cross(b)==0)return INF;
119     return p1+a*(s.cross(b)/a.cross(b));
120 }

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108 point<T> seg_intersection(const line &l) 161
109     const{//線段交點 162
110     int res=seg_intersect(l); 163
111     if(res<=0) assert(0); 164
112     if(res==2) return p1; 165
113     if(res==3) return p2; 166
114     return line_intersection(l); 167
115 } 168
116 template<typename T> 169
117 struct polygon{ 170
118     polygon(){ 171
119     vector<point<T> > p;//逆時針順序 172
120     T area()const{//面積 173
121     T ans=0; 174
122     for(int i=p.size()-1,j=0;j<(int)p.size() 175
123     ;i=j++) 176
124     ans+=p[i].cross(p[j]); 177
125     return ans/2; 178
126 } 179
127 point<T> center_of_mass()const{//重心 180
128     T cx=0,cy=0,w=0; 181
129     for(int i=p.size()-1,j=0;j<(int)p.size() 182
130     ;i=j++){ 183
131     T a=p[i].cross(p[j]); 184
132     cx+=(p[i].x+p[j].x)*a; 185
133     cy+=(p[i].y+p[j].y)*a; 186
134     w+=a; 187
135     } 188
136     return point<T>(cx/3/w,cy/3/w); 189
137 } 190
138 char ahas(const point<T>& t)const{//點是否 191
139     在簡單多邊形內、是的話回傳1、在邊上回 192
140     傳-1、否則回傳0 193
141     bool c=0; 194
142     for(int i=0,j=p.size()-1;i<p.size();j=i 195
143     ++ 196
144     if(line<T>(p[i],p[j]).point_on_segment 197
145     (t))return -1; 198
146     else if((p[i].y>t.y)!=p[j].y>t.y)&& 199
147     t.x<(p[j].x-p[i].x)*(t.y-p[i].y)/(p[j 200
148     ].y-p[i].y)+p[i].x) 201
149     c=1; 202
150     return c; 203
151 } 204
152 char point_in_convex(const point<T>&x) 205
153     const{ 206
154     int l=1,r=(int)p.size()-2; 207
155     while(l<=r){//點是否在凸多邊形內、是的話 208
156     回傳1、在邊上回傳-1、否則回傳0 209
157     int mid=(l+r)/2; 210
158     T a1=(p[mid]-p[0]).cross(x-p[0]); 211
159     T a2=(p[mid+1]-p[0]).cross(x-p[0]); 212
160     if(a1>0&&a2<=0){ 213
161     T res=(p[mid+1]-p[mid]).cross(x-p[ 214
162     mid]); 215
163     return res>0?1:(res>0?-1:0); 216
164     }else if(a1<0)r=mid-1; 217
165     else l=mid+1; 218
166     } 219
167     return 0; 220
168 } 221
169 vector<T> getA()const{//凸包邊對x軸的夾角 222
170     vector<T>res;//一定是遞增的 223
171     for(size_t i=0;i<p.size();++i) 224
172     res.push_back((p[(i+1)%p.size()-p[i]] 225
173     .getA())); 226
174     return res; 227
175 } 228
176 bool line_intersect(const vector<T>&A, 229
177     const line<T> &l)const{//0(logN) 230
178     int f1=upper_bound(A.begin(),A.end(),(l. 231
179     p1-l.p2).getA())-A.begin(); 232
180     int f2=upper_bound(A.begin(),A.end(),(l. 233
181     p2-l.p1).getA())-A.begin(); 234
182     return l.cross_seg(line<T>(p[f1],p[f2])) 235
183     ; 236
184 } 237
185 polygon cut(const line<T> &l)const{//凸包 238
186     對直線切割、得到直線l左側的凸包 239
187     polygon ans; 240
188     for(int n=p.size(),i=n-1,j=0;j<n;i=j++){ 241
189     if(l.ori(p[i])>=0){ 242
190     ans.p.push_back(p[i]); 243
191     if(l.ori(p[j])<0) 244
192     ans.p.push_back(l. 245
193     line_intersection(line<T>(p[i 246
194     ],p[j]))); 247
195     }else if(l.ori(p[j])>0) 248
196     ans.p.push_back(l.line_intersection( 249
197     line<T>(p[i],p[j]))); 250
198     } 251
199     return ans; 252
200 } 253
201 static bool monotone_chain_cmp(const point 254
202     <T>& a,const point<T>& b){//凸包排序函 255
203     數 256
204     return (a.x<b.x)|| (a.x==b.x&&a.y<b.y); 257
205 } 258
206 void monotone_chain(vector<point<T> > &s){ 259
207     //凸包 260
208     sort(s.begin(),s.end(), 261
209     monotone_chain_cmp); 262
210     p.resize(s.size()+1); 263
211     int m=0; 264
212     for(size_t i=0;i<s.size();++i){ 265
213     while(m>2&&(p[m-1]-p[m-2]).cross(s[i 266
214     ]-p[m-2])<=0)--m; 267
215     p[m++]=s[i]; 268
216     } 269
217     for(int i=s.size()-2,t=m+1;i>0;--i){ 270
218     while(m>=t&&(p[m-1]-p[m-2]).cross(s[i 271
219     ]-p[m-2])<=0)--m; 272
220     p[m++]=s[i]; 273
221     } 274
222     T diam(){//直徑 275
223     int n=p.size(),t=1; 276
224     T ans=0;p.push_back(p[0]); 277
225     for(int i=0;i<n;i++){ 278
226     point<T> now=p[i+1]-p[i]; 279
227     while(now.cross(p[t+1]-p[i])>now.cross 280
228     (p[t]-p[i]))t=(t+1)%n; 281
229     ans=max(ans,(p[i]-p[t]).abs2()); 282
230     } 283
231     return p.pop_back(),ans; 284
232 } 285
233 } 286
234 T min_cover_rectangle(){//最小覆蓋矩形 287
235     int n=p.size(),t=1,r=1,l; 288
236     if(n<3)return 0;//也可以做最小周長矩形 289
237     T ans=1e99;p.push_back(p[0]); 290
238     for(int i=0;i<n;i++){ 291
239     point<T> now=p[i+1]-p[i]; 292
240     while(now.cross(p[t+1]-p[i])>now.cross 293
241     (p[t]-p[i]))t=(t+1)%n; 294
242     while(now.dot(p[r+1]-p[i])>now.dot(p[r 295
243     ]-p[i]))r=(r+1)%n; 296
244     if(!i)l=r; 297
245     while(now.dot(p[l+1]-p[i])<=now.dot(p[ 298
246     l]-p[i]))l=(l+1)%n; 299
247     T d=now.abs2(); 300
248     T tmp=now.cross(p[t]-p[i])*(now.dot(p[ 301
249     r]-p[i])-now.dot(p[l]-p[i]))/d; 302
250     ans=min(ans,tmp); 303
251     } 304
252     return p.pop_back(),ans; 305
253 } 306
254 T dis2(polygon &p1){//凸包最近距離平方 307
255     vector<point<T> > &P=p,&Q=p1.p; 308
256     int n=P.size(),m=Q.size(),l=0,r=0; 309
257     for(int i=0;i<n;++i)if(P[i].y<P[l].y)l=i; 310
258     for(int i=0;i<m;++i)if(Q[i].y<Q[r].y)r=i; 311
259     P.push_back(P[0]),Q.push_back(Q[0]); 312
260     T ans=1e99; 313
261     for(int i=0;i<n;++i){ 314
262     while((P[l]-P[l+1]).cross(Q[r+1]-Q[r]) 315
263     <0)r=(r+1)%m; 316
264     ans=min(ans,line<T>(P[l],P[l+1]). 317
265     seg_dis2(line<T>(Q[r],Q[r+1]))); 318
266     l=(l+1)%n; 319
267     } 320
268     return P.pop_back(),Q.pop_back(),ans; 321
269 } 322
270 static char sign(const point<T>&t){ 323
271     return (t.y==0?t.x:t.y)<0; 324
272 } 325
273 static bool angle_cmp(const line<T>& A, 326
274     const line<T>& B){ 327
275     point<T> a=A.p2-A.p1,b=B.p2-B.p1; 328
276     return sign(a)<sign(b)|| (sign(a)==sign(b 329
277     )&&a.cross(b)>0); 330
278 } 331
279 int halfplane_intersection(vector<line<T> 332
280     > &s){//半平面交 333
281     sort(s.begin(),s.end(),angle_cmp);//線段 334
282     左側為該線段半平面 335
283     int L,R,n=s.size(); 336
284     vector<point<T> > px(n); 337
285     vector<line<T> > q(n); 338
286     q[L=R=0]=s[0]; 339
287     for(int i=1;i<n;++i){ 340
288     while(L<R&&s[i].ori(px[R-1])<=0)--R; 341
289     while(L<R&&s[i].ori(px[L])<=0)++L; 342
290     q[++R]=s[i]; 343
291     if(q[R].parallel(q[R-1])){ 344
292     --R; 345
293     if(q[R].ori(s[i].p1)>0)q[R]=s[i]; 346
294     } 347
295     if(L<R)px[R-1]=q[R-1]. 348
296     line_intersection(q[R]); 349
297     } 350
298     while(L<R&&q[L].ori(px[R-1])<=0)--R; 351

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p.clear();
if(R-L<1)return 0;
px[R]=q[R].line_intersection(q[L]);
for(int i=L;i<R;++i)p.push_back(px[i]);
return R-L+1;
};
template<typename T>
struct triangle{
    point<T> a,b,c;
    triangle(){
    triangle(const point<T> &a,const point<T>
    &b,const point<T> &c):a(a),b(b),c(c){}
    T area()const{
    T t=(b-a).cross(c-a)/2;
    return t>0?t:-t;
    }
    point<T> barycenter()const{//重心
    return (a+b+c)/3;
    }
    point<T> circumcenter()const{//外心
    static line<T> u,v;
    u.p1=(a+b)/2;
    u.p2=point<T>(u.p1.x-a.y+b.y,u.p1.y+a.x-
    b.x);
    v.p1=(a+c)/2;
    v.p2=point<T>(v.p1.x-a.y+c.y,v.p1.y+a.x-
    c.x);
    return u.line_intersection(v);
    }
    point<T> incenter()const{//內心
    T A=sqrt((b-c).abs2()),B=sqrt((a-c).abs2
    ()),C=sqrt((a-b).abs2());
    return point<T>(A*a.x+B*b.x+C*c.x,A*a.y+
    B*b.y+C*c.y)/(A+B+C);
    }
    point<T> perpencenter()const{//垂心
    return barycenter()*3-circumcenter()*2;
    }
};
template<typename T>
struct point3D{
    T x,y,z;
    point3D(){
    point3D(const T&x,const T&y,const T&z):x(x
    ),y(y),z(z){}
    point3D operator+(const point3D &b)const{
    return point3D(x+b.x,y+b.y,z+b.z);}
    point3D operator-(const point3D &b)const{
    return point3D(x-b.x,y-b.y,z-b.z);}
    point3D operator*(const T &b)const{
    return point3D(x*b,y*b,z*b);}
    point3D operator/(const T &b)const{
    return point3D(x/b,y/b,z/b);}
    bool operator==(const point3D &b)const{
    return x==b.x&&y==b.y&&z==b.z;}
    T dot(const point3D &b)const{
    return x*b.x+y*b.y+z*b.z;}
    point3D cross(const point3D &b)const{
    return point3D(y*b.z-z*b.y,z*b.x-x*b.z,x
    *b.y-y*b.x);}
    T abs2()const{//向量長度的平方
    return dot(*this);}

```

```

320 T area2(const point3D &b) const { // 和 b、原點
    圍成面積的平方
321     return cross(b).abs2()/4; }
322 };
323 template<typename T>
324 struct line3D{
325     point3D<T> p1,p2;
326     line3D(){}
327     line3D(const point3D<T> &p1,const point3D<
        T> &p2):p1(p1),p2(p2){}
328     T dis2(const point3D<T> &p,bool is_segment
        =0) const { // 點跟直線/線段的距離平方
329         point3D<T> v=p2-p1,v1=p-p1;
330         if(is_segment){
331             point3D<T> v2=p-p2;
332             if(v.dot(v1)<=0) return v1.abs2();
333             if(v.dot(v2)>=0) return v2.abs2();
334         }
335         point3D<T> tmp=v.cross(v1);
336         return tmp.abs2()/(v.abs2());
337     }
338     pair<point3D<T>,point3D<T>> closest_pair(
        const line3D<T> &l) const {
339         point3D<T> v1=(p1-p2),v2=(l.p1-l.p2);
340         point3D<T> N=v1.cross(v2),ab(p1-l.p1);
341         //if(N.abs2()==0) return NULL; 平行或重合
342         T tmp=N.dot(ab),ans=tmp*tmp/N.abs2(); //
            最近點對距離
343         point3D<T> d1=p2-p1,d2=l.p2-l.p1,D=d1.
            cross(d2),G=l.p1-p1;
344         T t1=(G.cross(d2)).dot(D)/D.abs2();
345         T t2=(G.cross(d1)).dot(D)/D.abs2();
346         return make_pair(p1+d1*t1,l.p1+d2*t2);
347     }
348     bool same_side(const point3D<T> &a,const
        point3D<T> &b) const {
349         return (p2-p1).cross(a-p1).dot((p2-p1).
            cross(b-p1))>0;
350     }
351 };
352 template<typename T>
353 struct plane{
354     point3D<T> p0,n; // 平面上的點和法向量
355     plane(){}
356     plane(const point3D<T> &p0,const point3D<T>
        &n):p0(p0),n(n){}
357     T dis2(const point3D<T> &p) const { // 點到平
        面距離的平方
358         T tmp=(p-p0).dot(n);
359         return tmp*tmp/n.abs2();
360     }
361     point3D<T> projection(const point3D<T> &p)
        const {
362         return p-n*(p-p0).dot(n)/n.abs2();
363     }
364     point3D<T> line_intersection(const line3D<
        T> &l) const {
365         T tmp=n.dot(l.p2-l.p1); // 等於 0 表示平行或
            重合該平面
366         return l.p1+(l.p2-l.p1)*(n.dot(p0-l.p1)/
            tmp);
367     }
368     line3D<T> plane_intersection(const plane &
        p1) const {
369         point3D<T> e=n.cross(p1.n),v=n.cross(e);
370         T tmp=p1.n.dot(v); // 等於 0 表示平行或重合
            該平面
371         point3D<T> q=p0+(v*(p1.n.dot(p1.p0-p0))/
            tmp);
372         return line3D<T>(q,q+e);
373     }
374 };
375 template<typename T>
376 struct triangle3D{
377     point3D<T> a,b,c;
378     triangle3D(){}
379     triangle3D(const point3D<T> &a,const
        point3D<T> &b,const point3D<T> &c):a(a),
        b(b),c(c){}
380     bool point_in(const point3D<T> &p) const { //
        點在該平面上的投影在三角形中
381         return line3D<T>(b,c).same_side(p,a)&&
            line3D<T>(a,c).same_side(p,b)&&
            line3D<T>(a,b).same_side(p,c);
382     }
383 };
384 template<typename T>
385 struct tetrahedron { // 四面體
386     point3D<T> a,b,c,d;
387     tetrahedron(){}
388     tetrahedron(const point3D<T> &a,const
        point3D<T> &b,const point3D<T> &c,
        const point3D<T> &d):a(a),b(b),c(c),d(d){}
389     T volume6() const { // 體積的六倍
390         return (d-a).dot((b-a).cross(c-a));
391     }
392     point3D<T> centroid() const {
393         return (a+b+c+d)/4;
394     }
395     bool point_in(const point3D<T> &p) const {
396         return triangle3D<T>(a,b,c).point_in(p)
            && triangle3D<T>(c,d,a).point_in(p);
397     }
398 };
399 template<typename T>
400 struct convexhull3D{
401     static const int MAXN=1005;
402     struct face{
403         int a,b,c;
404         face(int a,int b,int c):a(a),b(b),c(c){}
405     };
406     vector<point3D<T>> pt;
407     vector<face> ans;
408     int fid[MAXN][MAXN];
409     void build(){
410         int n=pt.size();
411         ans.clear();
412         memset(fid,0,sizeof(fid));
413         ans.emplace_back(0,1,2); // 注意不能共線
414         ans.emplace_back(2,1,0);
415         int ftop = 0;
416         for(int i=3, ftop=1; i<n; ++i, ++ftop){
417             vector<face> next;
418             for(auto &f:ans){
419                 T d=(pt[i]-pt[f.a]).dot((pt[f.b]-pt[
                    f.a]).cross(pt[f.c]-pt[f.a]));
420                 if(d<0) next.push_back(f);
421                 int ff=0;
422                 if(d>0) ff=ftop;
423                 else if(d<0) ff=-ftop;
424                 fid[f.a][f.b]=fid[f.b][f.c]=fid[f.c]
                    ][f.a]=ff;
425             }
426             for(auto &f:ans){
427                 if(fid[f.a][f.b]>0 && fid[f.a][f.b]
                    ]!=fid[f.b][f.a])
428                     next.emplace_back(f.a,f.b,i);
429                 if(fid[f.b][f.c]>0 && fid[f.b][f.c]
                    ]!=fid[f.c][f.b])
430                     next.emplace_back(f.b,f.c,i);
431                 if(fid[f.c][f.a]>0 && fid[f.c][f.a]
                    ]!=fid[f.a][f.c])
432                     next.emplace_back(f.c,f.a,i);
433             }
434             ans=next;
435         }
436         point3D<T> centroid() const {
437             point3D<T> res(0,0,0);
438             T vol=0;
439             for(auto &f:ans){
440                 T tmp=pt[f.a].dot(pt[f.b].cross(pt[f.c]
                    ));
441                 res=res+(pt[f.a]+pt[f.b]+pt[f.c])*tmp;
442                 vol+=tmp;
443             }
444             return res/(vol*4);
445         }
446     };
447 };
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```

2 Data_Structure

2.1 CDQ_DP

1.3 SmallestCircle

```

1 #include<bits/stdc++.h>
2 using namespace std;
3 const int MAXN = 100005;
4 struct node{
5     double a,b,r,k,x,y;
6     int id;
7 } p[MAXN];
8 double DP[MAXN];
9 deque<int> q;
10 bool cmpK(const node &a,const node &b){
11     return a.k>b.k;
12 }
13 bool cmpX(const node &a,const node &b){
14     return a.x<b.x|| (a.x==b.x&&a.y<b.y);
15 }
16 double Slope(int a,int b){
17     if(!b) return -1e20;
18     if(p[a].x==p[b].x) return 1e20;
19     return (p[a].y-p[b].y)/(p[a].x-p[b].x);
20 }
21 void CDQ(int l, int r){
22     if(l==r){
23         DP[l] = max(DP[l],DP[l-1]);
24         p[l].y = DP[l]/(p[l].a*p[l].r+p[l].b);
25         p[l].x = p[l].y*p[l].r;
26         return;
27     }
28     int mid = (l+r)/2;
29     stable_partition(p+l,p+r+1,[&](const node
        &d){return d.id<=mid;});
30     CDQ(l, mid); q.clear();
31     for(int i=l, j; i<=mid; ++i){

```

1.4 最近點對

```

32 while((j=q.size())>1&&Slope(q[j-2],q[j-1])<Slope(q[j-1],i)) q.pop_back();
33 q.push_back(i);
34 q.push_back(0);
35 for(int i=mid+1; i<=r; ++i){
36 while(q.size())>1&&Slope(q[0],q[1])>p[i].k) q.pop_front();
37 DP[p[i].id] = max(DP[p[i].id], p[i].a*p[q[0]].x+p[i].b*p[q[0]].y);
38 }
39 CDQ(mid+1,r);
40 inplace_merge(p+1,p+mid+1,p+r+1,cmpX);
41 }
42 double solve(int n,double S){
43 DP[0] = S;
44 sort(p+1,p+1+n,cmpK);
45 CDQ(1,n);
46 return DP[n];
47 }
48 int main(){
49 int n; double S;
50 scanf("%d%lf",&n,&S);
51 for(int i=1; i<=n; ++i){
52 scanf("%lf%lf%lf",&p[i].a,&p[i].b,&p[i].r);
53 p[i].id = i, p[i].k = -p[i].a/p[i].b;
54 }
55 printf("%.3lf\n",solve(n,S));
56 return 0;
57 }

```

2.2 DLX

```

1 const int MAXN=4100, MAXM=1030, MAXND=16390;
2 struct DLX{
3 int n,m,sz,ansd; //高是n · 寬是m的稀疏矩陣
4 int S[MAXN],H[MAXN];
5 int row[MAXN],col[MAXN]; //每個節點代表的列與行
6 int L[MAXN],R[MAXN],U[MAXN],D[MAXN];
7 vector<int> ans,anst;
8 void init(int _n,int _m){
9 n=_n,m=_m;
10 for(int i=0;i<=m;++i){
11 U[i]=D[i]=i,L[i]=i-1,R[i]=i+1;
12 S[i]=0;
13 }
14 R[m]=0,L[0]=m;
15 sz=m,ansd=INT_MAX; //ansd存最優解的個數
16 for(int i=1;i<=n;++i)H[i]=-1;
17 }
18 void add(int r,int c){
19 ++S[col[++sz]=c];
20 row[sz]=r;
21 D[sz]=D[c],U[D[c]]=sz,U[sz]=c,D[c]=sz;
22 if(H[r]<0)H[r]=L[sz]=R[sz]=sz;
23 else R[sz]=R[H[r]],L[R[H[r]]]=sz,L[sz]=H[r],R[H[r]]=sz;
24 }
25 #define DFOR(i,A,s) for(int i=A[s];i!=s;i=
A[i])

```

```

26 void remove(int c){ //刪除第c行和所有當前覆蓋到第c行的列
27 L[R[c]]=L[c],R[L[c]]=R[c]; //這裡刪除第c行 · 若有些行不需要處理可以在開始時呼叫他
28 DFOR(i,D,c)DFOR(j,R,i){U[D[j]]=U[j],D[U[j]]=D[j],--S[col[j]];}
29 }
30 void restore(int c){ //恢復第c行和所有當前覆蓋到第c行的列 · remove的逆操作
31 DFOR(i,U,c)DFOR(j,L,i){++S[col[j]],U[D[j]]=j,D[U[j]]=j;}
32 L[R[c]]=c,R[L[c]]=c;
33 }
34 void remove2(int nd){ //刪除nd所在的行當前所有點(包括虛擬節點) · 只保留nd
35 DFOR(i,D,nd)L[R[i]]=L[i],R[L[i]]=R[i];
36 }
37 void restore2(int nd){ //刪除nd所在的行當前所有點 · 為remove2的逆操作
38 DFOR(i,U,nd)L[R[i]]=R[L[i]]=i;
39 }
40 bool vis[MAXN];
41 int h(){ //估價函數 for IDA*
42 int res=0;
43 memset(vis,0,sizeof(vis));
44 DFOR(i,R,0)if(!vis[i]){
45 vis[i]=1;
46 ++res;
47 DFOR(j,D,i)DFOR(k,R,j)vis[col[k]]=1;
48 }
49 return res;
50 }
51 bool dfs(int d){ //for精確覆蓋問題
52 if(d+h())>=ansd return 0; //找最佳解用 · 找任意解可以刪掉
53 if(!R[0]){ansd=d;return 1;}
54 int c=R[0];
55 DFOR(i,R,0)if(S[i]<S[c])c=i;
56 remove(c);
57 DFOR(i,D,c){
58 ans.push_back(row[i]);
59 DFOR(j,R,i)remove(col[j]);
60 if(dfs(d+1))return 1;
61 ans.pop_back();
62 DFOR(j,L,i)restore(col[j]);
63 }
64 restore(c);
65 return 0;
66 }
67 void dfs2(int d){ //for最小重複覆蓋問題
68 if(d+h())>=ansd return 0;
69 if(!R[0]){ansd=d;ans=anst;return;}
70 int c=R[0];
71 DFOR(i,R,0)if(S[i]<S[c])c=i;
72 DFOR(i,D,c){
73 anst.push_back(row[i]);
74 remove2(i);
75 DFOR(j,R,i)remove2(j),--S[col[j]];
76 dfs2(d+1);
77 anst.pop_back();
78 DFOR(j,L,i)restore2(j),++S[col[j]];
79 restore2(i);

```

```

80 }
81 }
82 bool exact_cover(){ //解精確覆蓋問題
83 return ans.clear(), dfs(0);
84 }
85 void min_cover(){ //解最小重複覆蓋問題
86 anst.clear(); //暫存用 · 答案還是存在ans裡
87 dfs2(0);
88 }
89 #undef DFOR
90 };

```

2.3 Dynamic_KD_tree

```

1 template<typename T,size_t kd> //有kd個維度
2 struct kd_tree{
3 struct point{
4 T d[kd];
5 T dist(const point &x)const{
6 T ret=0;
7 for(size_t i=0;i<kd;++i)ret+=abs(d[i]-x.d[i]);
8 return ret;
9 }
10 bool operator==(const point &p){
11 for(size_t i=0;i<kd;++i)
12 if(d[i]!=p.d[i])return 0;
13 return 1;
14 }
15 bool operator<(const point &b)const{
16 return d[0]<b.d[0];
17 }
18 };
19 private:
20 struct node{
21 node *l,*r;
22 point pid;
23 int s;
24 node(const point &p):l(0),r(0),pid(p),s(1){}
25 ~node(){delete l;delete r;}
26 void up(){s=(l?l->s:0)+1+(r?r->s:0);}
27 }*root;
28 const double alpha,loga;
29 const T INF; //記得要給INF · 表示極大值
30 int maxn;
31 struct __cmp{
32 int sort_id;
33 bool operator()(const node*x,const node*y)const{
34 return operator()(x->pid,y->pid);
35 }
36 bool operator()(const point &x,const point &y)const{
37 if(x.d[sort_id]!=y.d[sort_id])
38 return x.d[sort_id]<y.d[sort_id];
39 for(size_t i=0;i<kd;++i)
40 if(x.d[i]!=y.d[i])return x.d[i]<y.d[i];
41 return 0;
42 }
43 }cmp;

```

```

44 int size(node *o){return o?o->s:0;}
45 vector<node*> A;
46 node* build(int k,int l,int r){
47 if(l>r) return 0;
48 if(k==kd) k=0;
49 int mid=(l+r)/2;
50 cmp.sort_id = k;
51 nth_element(A.begin()+l,A.begin()+mid,A.begin()+r+1,cmp);
52 node *ret=A[mid];
53 ret->l = build(k+1,l,mid-1);
54 ret->r = build(k+1,mid+1,r);
55 ret->up();
56 return ret;
57 }
58 bool isbad(node*o){
59 return size(o->l)>alpha*o->s||size(o->r)>alpha*o->s;
60 }
61 void flatten(node *u,typename vector<node*>::iterator &it){
62 if(!u)return;
63 flatten(u->l,it);
64 *it=u;
65 flatten(u->r,++it);
66 }
67 void rebuild(node*&u,int k){
68 if((int)A.size()<u->s)A.resize(u->s);
69 auto it=A.begin();
70 flatten(u,it);
71 u=build(k,0,u->s-1);
72 }
73 bool insert(node*&u,int k,const point &x,int dep){
74 if(!u) return u=new node(x), dep<=0;
75 ++u->s;
76 cmp.sort_id=k;
77 if(insert(cmp(x,u->pid)?u->l:u->r,(k+1)%kd,x,dep-1)){
78 if(!isbad(u))return 1;
79 rebuild(u,k);
80 }
81 return 0;
82 }
83 node *findmin(node*o,int k){
84 if(!o)return 0;
85 if(cmp.sort_id==k)return o->l?findmin(o->l,(k+1)%kd):o;
86 node *l=findmin(o->l,(k+1)%kd);
87 node *r=findmin(o->r,(k+1)%kd);
88 if(l&&r)return cmp(l,o)?l:o;
89 if(!l&&r)return cmp(r,o)?r:o;
90 if(l&&r)return o;
91 if(cmp(l,r))return cmp(l,o)?l:o;
92 return cmp(r,o)?r:o;
93 }
94 bool erase(node *&u,int k,const point &x){
95 if(!u)return 0;
96 if(u->pid==x){
97 if(u->r);
98 else if(u->l) u->r=u->l, u->l=0;
99 else return delete(u),u=0, 1;
100 --u->s;
101 cmp.sort_id=k;
102 u->pid=findmin(u->r,(k+1)%kd)->pid;
103 return erase(u->r,(k+1)%kd,u->pid);

```



```

104 }
105 cmp.sort_id=k;
106 if(erase(cmp(x,u->pid)?u->l:u->r,(k+1)%
107     kd,x))
108     return --u->s, 1;
109 return 0;
110 }
111 T heuristic(const T h[])const{
112     T ret=0;
113     for(size_t i=0;i<kd;++i)ret+=h[i];
114     return ret;
115 }
116 int qM;
117 priority_queue<pair<T,point>> pQ;
118 void nearest(node *u,int k,const point &x,
119     T *h,T &mndist){
120     if(u==0||heuristic(h)>=mndist)return;
121     T dist=u->pid.dist(x),old=h[k];
122     /*mndist=std::min(mndist,dist);*/
123     if(dist<mndist){
124         pQ.push(std::make_pair(dist,u->pid));
125         if((int)pQ.size()==qM+1)
126             mndist=pQ.top().first,pQ.pop();
127     }
128     if(x.d[k]<u->pid.d[k]){
129         nearest(u->l,(k+1)%kd,x,h,mndist);
130         h[k] = abs(x.d[k]-u->pid.d[k]);
131         nearest(u->r,(k+1)%kd,x,h,mndist);
132     }else{
133         nearest(u->r,(k+1)%kd,x,h,mndist);
134         h[k] = abs(x.d[k]-u->pid.d[k]);
135         nearest(u->l,(k+1)%kd,x,h,mndist);
136     }
137     h[k]=old;
138 }
139 vector<point>in_range;
140 void range(node *u,int k,const point&mi,
141     const point&ma){
142     if(!u)return;
143     bool is=1;
144     for(int i=0;i<kd;++i)
145         if(u->pid.d[i]<mi.d[i]||ma.d[i]<u->pid
146             .d[i])
147             { is=0;break; }
148     if(is) in_range.push_back(u->pid);
149     if(mi.d[k]<u->pid.d[k])range(u->l,(k+1)
150         %kd,mi,ma);
151     if(ma.d[k]>u->pid.d[k])range(u->r,(k+1)
152         %kd,mi,ma);
153 }
154 public:
155 kd_tree(const T &INF,double a=0.75):
156     root(0),alpha(a),loga(log2(1.0/a)),INF(INF
157         ),maxn(1){}
158 ~kd_tree(){delete root;}
159 void clear(){delete root,root=0,maxn=1;}
160 void build(int n,const point *p){
161     delete root,A.resize(maxn=n);
162     for(int i=0;i<n;++i)A[i]=new node(p[i]);
163     root=build(0,0,n-1);
164 }
165 void insert(const point &x){
166     insert(root,0,x,__lg(size(root))/loga);
167     if(root->s>maxn)maxn=root->s;
168 }
169 bool erase(const point &p){

```

```

163 bool d=erase(root,0,p);
164 if(root&&root->s<alpha*maxn)rebuild();
165 return d;
166 }
167 void rebuild(){
168     if(root)rebuild(root,0);
169     maxn=root->s;
170 }
171 T nearest(const point &x,int k){
172     qM=k;
173     T mndist=INF,h[kd]={};
174     nearest(root,0,x,h,mndist);
175     mndist=pQ.top().first;
176     pQ = priority_queue<pair<T,point>>();
177     return mndist;//回傳離x第k近的點的距離
178 }
179 const vector<point> &range(const point&mi,
180     const point&ma){
181     in_range.clear();
182     range(root,0,mi,ma);
183     return in_range;//回傳介於mi到ma之間的點
184     vector
185 }
186 int size(){return root?root->s:0;}
187 };

```

2.4 kd_tree_replace_segment

```

1 struct node{//kd樹代替高維線段樹
2     node *l,*r;
3     point pid,mi,ma;
4     int s, data;
5     node(const point &p,int d):l(0),r(0),pid(p
6         ),mi(p),ma(p),s(1),data(d),dmin(d),
7         dmax(d){}
8     void up(){
9         mi=ma=pid;
10        s=1;
11        if(l){
12            for(int i=0;i<kd;++i){
13                mi.d[i]=min(mi.d[i],l->mi.d[i]);
14                ma.d[i]=max(ma.d[i],l->ma.d[i]);
15            }
16            s+=l->s;
17        }
18        if(r){
19            for(int i=0;i<kd;++i){
20                mi.d[i]=min(mi.d[i],r->mi.d[i]);
21                ma.d[i]=max(ma.d[i],r->ma.d[i]);
22            }
23            s+=r->s;
24        }
25    }
26    void up2(){/*其他懶惰標記向上更新*/}
27    void down(){/*其他懶惰標記下推*/}
28 }*root;
29 //檢查區間包含用的函數
30 bool range_include(node *o,const point &L,
31     const point &R){
32     for(int i=0;i<kd;++i)
33         if(L.d[i]>o->ma.d[i]||R.d[i]<o->mi.d[i])
34             return 0;
35     return 1;
36 }

```

```

31 }//(L,R)區間有和o的區間有交集就回傳true
32 return 1;
33 }
34 bool range_in_range(node *o,const point &L,
35     const point &R){
36     for(int i=0;i<kd;++i){
37         if(L.d[i]>o->mi.d[i]||o->ma.d[i]>R.d[i])
38             return 0;
39     }
40 }//(L,R)區間完全包含o的區間就回傳true
41 return 1;
42 }
43 bool point_in_range(node *o,const point &L,
44     const point &R){
45     for(int i=0;i<kd;++i){
46         if(L.d[i]>o->pid.d[i]||R.d[i]<o->pid.d[i]
47             )return 0;
48     }
49 }//(L,R)區間完全包含o->pid這個點就回傳true
50 return 1;
51 }
52 //單點修改 · 以單點改值為例
53 void update(node *u,const point &x,int data,
54     int k=0){
55     if(!u)return;
56     u->down();
57     if(u->pid==x){
58         u->data=data;
59         u->up2();
60         return;
61     }
62     cmp.sort_id=k;
63     update(cmp(x,u->pid)?u->l:u->r,x,data,(k
64         +1)%kd);
65     u->up2();
66 }
67 //區間修改
68 void update(node *o,const point &L,const
69     point &R,int data){
70     if(!o)return;
71     o->down();
72     if(range_in_range(o,L,R)){
73         //區間懶惰標記修改
74         o->down();
75         return;
76     }
77     if(point_in_range(o,L,R)){
78         //這個點在(L,R)區間 · 但是他的左右子樹不
79         //一定在區間中
80         //單點懶惰標記修改
81     }
82     if(o->l&&range_include(o->l,L,R))update(o
83         ->l,L,R,data);
84     if(o->r&&range_include(o->r,L,R))update(o
85         ->r,L,R,data);
86     o->up2();
87 }
88 //區間查詢 · 以總和為例
89 int query(node *o,const point &L,const point
90     &R){
91     if(!o)return 0;
92     o->down();
93     if(range_in_range(o,L,R))return o->sum;
94     int ans=0;
95     if(point_in_range(o,L,R))ans+=o->data;
96     if(o->l)ans+=query(o->l,L,R);
97     if(o->r)ans+=query(o->r,L,R);
98     return ans;
99 }

```

2.5 reference_point

```

1 template<typename T>
2 struct _RefC{
3     T data;
4     int ref;
5     _RefC(const T&d=0):data(d),ref(0){}
6 };
7 template<typename T>
8 struct _rp{
9     _RefC<T> *p;
10    T *operator->(){return &p->data;}
11    T &operator*(){return p->data;}
12    operator _RefC<T>*(){return p;}
13    _rp &operator=(const _rp &t){
14        if(p&&!--p->ref)delete p;
15        p=t.p,p&&+p->ref;
16        return *this;
17    }
18    _rp(_RefC<T> *t=0):p(t){p&&+p->ref;}
19    _rp(const _rp &t):p(t.p){p&&+p->ref;}
20    ~_rp(){if(p&&!--p->ref)delete p;}
21 };
22 template<typename T>
23 inline _rp<T> new_rp(const T&d){
24     return _rp<T>(new _RefC<T>(d));
25 }

```

2.6 skew_heap

```

1 node *merge(node *a,node *b){
2     if(!a||!b) return a?a:b;
3     if(b->data<a->data) swap(a,b);
4     swap(a->l,a->r);
5     a->l=merge(b,a->l);
6     return a;
7 }

```

2.7 undo_disjoint_set

```

1 struct DisjointSet {
2     // save() is like recursive
3     // undo() is like return
4     int n, fa[MXN], sz[MXN];
5     vector<pair<int*,int>> h;
6     vector<int> sp;
7     void init(int tn) {
8         n=tn;
9         for (int i=0; i<n; i++) sz[fa[i]=i]=1;
10    }

```

```

10 sp.clear(); h.clear();
11 }
12 void assign(int *k, int v) {
13     h.PB({k, *k});
14     *k=v;
15 }
16 void save() { sp.PB(SZ(h)); }
17 void undo() {
18     assert(!sp.empty());
19     int last=sp.back(); sp.pop_back();
20     while (SZ(h)!=last) {
21         auto x=h.back(); h.pop_back();
22         *x.F=x.S;
23     }
24 }
25 int f(int x) {
26     while (fa[x]!=x) x=fa[x];
27     return x;
28 }
29 void uni(int x, int y) {
30     x=f(x); y=f(y);
31     if (x==y) return;
32     if (sz[x]<sz[y]) swap(x, y);
33     assign(&sz[x], sz[x]+sz[y]);
34     assign(&fa[y], x);
35 }
36 }djs;

```

2.8 整體二分

```

1 void totBS(int L, int R, vector<Item> M){
2     if(Q.empty()) return; //維護全域B陣列
3     if(L==R) 整個M的答案=r, return;
4     int mid = (L+R)/2;
5     vector<Item> mL, mR;
6     do_modify_B_with_divide(mid,M);
7     //讓B陣列在遞迴的時候只會保留[L~mid]的資訊
8     undo_modify_B(mid,M);
9     totBS(L,mid,mL);
10    totBS(mid+1,R,mR);
11 }

```

3 Flow

3.1 dinic

```

1 template<typename T>
2 struct DINIC{
3     static const int MAXN=105;
4     static const T INF=INT_MAX;
5     int n, LV[MAXN], cur[MAXN];
6     struct edge{
7         int v,pre;
8         T cap,r;
9         edge(int v,int pre,T cap):v(v),pre(pre),
10            cap(cap),r(cap){}
11 };

```

```

11 int g[MAXN];
12 vector<edge> e;
13 void init(int _n){
14     memset(g,-1,sizeof(int)*((n=_n)+1));
15     e.clear();
16 }
17 void add_edge(int u,int v,T cap,bool
18     directed=false){
19     e.push_back(edge(v,g[u],cap));
20     g[u]=e.size()-1;
21     e.push_back(edge(u,g[v],directed?0:cap));
22     g[v]=e.size()-1;
23 }
24 int bfs(int s,int t){
25     memset(LV,0,sizeof(int)*(n+1));
26     memcpy(cur,g,sizeof(int)*(n+1));
27     queue<int> q;
28     q.push(s);
29     LV[s]=1;
30     while(q.size()){
31         int u=q.front();q.pop();
32         for(int i=g[u];~i;i=e[i].pre){
33             if(!LV[e[i].v]&&e[i].r){
34                 LV[e[i].v]=LV[u]+1;
35                 q.push(e[i].v);
36                 if(e[i].v==t)return 1;
37             }
38         }
39     }
40     return 0;
41 }
42 T dfs(int u,int t,T CF=INF){
43     if(u==t)return CF;
44     T df;
45     for(int &i=cur[u];~i;i=e[i].pre){
46         if(LV[e[i].v]==LV[u]+1&&e[i].r){
47             if(df=dfs(e[i].v,t,min(CF,e[i].r))){
48                 e[i].r-=df;
49                 e[i^1].r+=df;
50                 return df;
51             }
52         }
53     }
54     return LV[u]=0;
55 }
56 T dinic(int s,int t,bool clean=true){
57     if(clean)for(size_t i=0;i<e.size();++i)
58         e[i].r=e[i].cap;
59     T ans=0, f=0;
60     while(bfs(s,t))while(f=dfs(s,t))ans+=f;
61     return ans;
62 };

```

3.2 Gomory_Hu

```

1 //最小割樹+求任兩點間最小割
2 //0-base, root=0
3 LL e[MAXN][MAXN]; //任兩點間最小割
4 int p[MAXN]; //parent
5 ISAP D; // original graph

```

```

6 void gomory_hu(){
7     fill(p, p+n, 0);
8     fill(e[0], e[n], INF);
9     for( int s = 1; s < n; ++s ) {
10         int t = p[s];
11         ISAP F = D;
12         LL tmp = F.min_cut(s, t);
13         for( int i = 1; i < s; ++i )
14             e[s][i] = e[i][s] = min(tmp, e[t][i]);
15         for( int i = s+1; i <= n; ++i )
16             if( p[i] == t && F.vis[i] ) p[i] = s;
17     }
18 }

```

3.3 ISAP_with_cut

```

1 template<typename T>
2 struct ISAP{
3     static const int MAXN=105;
4     static const T INF=INT_MAX;
5     int n;//點數
6     int d[MAXN],gap[MAXN],cur[MAXN];
7     struct edge{
8         int v,pre;
9         T cap,r;
10        edge(int v,int pre,T cap):v(v),pre(pre),
11           cap(cap),r(cap){}
12 };
13 int g[MAXN];
14 vector<edge> e;
15 void init(int _n){
16     memset(g,-1,sizeof(int)*((n=_n)+1));
17     e.clear();
18 }
19 void add_edge(int u,int v,T cap,bool
20     directed=false){
21     e.push_back(edge(v,g[u],cap));
22     g[u]=e.size()-1;
23     e.push_back(edge(u,g[v],directed?0:cap));
24     g[v]=e.size()-1;
25 }
26 T dfs(int u,int s,int t,T CF=INF){
27     if(u==t)return CF;
28     T tf=CF,df;
29     for(int &i=cur[u];~i;i=e[i].pre){
30         if(e[i].r&&d[u]==d[e[i].v]+1){
31             df=dfs(e[i].v,s,t,min(tf,e[i].r));
32             e[i].r-=df;
33             e[i^1].r+=df;
34             if(!(tf-=df)||d[s]==n)return CF-tf;
35         }
36     }
37     int mh=n;
38     for(int i=cur[u]=g[u];~i;i=e[i].pre){
39         if(e[i].r&&d[e[i].v]<mh)mh=d[e[i].v];
40     }
41     return CF-tf;
42 }
43 T isap(int s,int t,bool clean=true){
44     memset(d,0,sizeof(int)*(n+1));

```

```

45     memset(gap,0,sizeof(int)*(n+1));
46     memcpy(cur,g,sizeof(int)*(n+1));
47     if(clean) for(size_t i=0;i<e.size();++i)
48         e[i].r=e[i].cap;
49     T MF=0;
50     for(gap[0]=n;d[s]<n;)MF+=dfs(s,s,t);
51     return MF;
52 }
53 vector<int> cut_e;//最小割邊集
54 bool vis[MAXN];
55 void dfs_cut(int u){
56     vis[u]=1;//表示u屬於source的最小割集
57     for(int i=g[u];~i;i=e[i].pre)
58         if(e[i].r>0&&!vis[e[i].v])dfs_cut(e[i].v);
59 }
60 T min_cut(int s,int t){
61     T ans=isap(s,t);
62     memset(vis,0,sizeof(bool)*(n+1));
63     dfs_cut(s);
64     for(int u=0;u<n;++u)if(vis[u])
65         for(int i=g[u];~i;i=e[i].pre)
66             if(!vis[e[i].v])cut_e.push_back(i);
67     return ans;
68 }
69 };

```

3.4 MinCostMaxFlow

```

1 template<typename TP>
2 struct MCMF{
3     static const int MAXN=440;
4     static const TP INF=999999999;
5     struct edge{
6         int v,pre;
7         TP r,cost;
8         edge(int v,int pre,TP r,TP cost):v(v),
9            pre(pre),r(r),cost(cost){}
10 };
11 int n,S,T;
12 TP dis[MAXN],PIS,ans;
13 bool vis[MAXN];
14 vector<edge> e;
15 int g[MAXN];
16 void init(int _n){
17     memset(g,-1,sizeof(int)*((n=_n)+1));
18     e.clear();
19 }
20 void add_edge(int u,int v,TP r,TP cost,
21     bool directed=false){
22     e.push_back(edge(v,g[u],r,cost));
23     g[u]=e.size()-1;
24     e.push_back(
25         edge(u,g[v],directed?0:r,-cost));
26     g[v]=e.size()-1;
27 }
28 TP augment(int u,TP CF){
29     if(u==T||!CF)return ans+=PIS*CF,CF;
30     vis[u]=1;
31     TP r=CF,d;
32     for(int i=g[u];~i;i=e[i].pre){
33         if(e[i].r&&e[i].cost&&!vis[e[i].v]){

```

```

32     d=augment(e[i].v,min(r,e[i].r));
33     e[i].r=d;
34     e[i^1].r+=d;
35     if(!(r==d))break;
36 }
37 return CF-r;
38 }
39 bool modlabel(){
40     for(int u=0;u<n;++u)dis[u]=INF;
41     static deque<int>q;
42     dis[T]=0,q.push_back(T);
43     while(q.size()){
44         int u=q.front();q.pop_front();
45         TP dt;
46         for(int i=g[u];~i;i=e[i].pre){
47             if(e[i^1].r&&(dt=dis[u]-e[i].cost)<
48                 dis[e[i].v]){
49                 if((dis[e[i].v]=dt)<=dis[q.size()])
50                     q.front():S){
51                         q.push_front(e[i].v);
52                     }else q.push_back(e[i].v);
53             }
54         }
55     }
56     for(int u=0;u<n;++u)
57         for(int i=g[u];~i;i=e[i].pre)
58             e[i].cost+=dis[e[i].v]-dis[u];
59     return PIS+=dis[S], dis[S]<INF;
60 }
61 TP mincost(int s,int t){
62     S=s,T=t;
63     PIS=ans=0;
64     while(modlabel()){
65         do memset(vis,0,sizeof(bool)*(n+1));
66         while(augment(S,INF));
67     }return ans;
68 }

```

4 Graph

4.1 Augmenting_Path

```

1 #define MAXN1 505
2 #define MAXN2 505
3 int n1,n2;//n1個點連向n2個點
4 int match[MAXN2];//屬於n2的點匹配了哪個點
5 vector<int> g[MAXN1];//圖 0-base
6 bool vis[MAXN2];//是否走訪過
7 bool dfs(int u){
8     for(int v:g[u]){
9         if(vis[v]) continue;
10        vis[v]=1;
11        if(match[v]==-1||dfs(match[v]))
12            return match[v]=u, 1;
13    }
14    return 0;
15 }
16 int max_match(){

```

```

17     int ans=0;
18     memset(match,-1,sizeof(int)*n2);
19     for(int i=0;i<n1;++i){
20         memset(vis,0,sizeof(bool)*n2);
21         if(dfs(i)) ++ans;
22     }
23     return ans;
24 }

```

4.2 Augmenting_Path_multiple

```

1 #define MAXN1 1005
2 #define MAXN2 505
3 int n1,n2;
4 //n1個點連向n2個點，其中n2個點可以匹配很多邊
5 vector<int> g[MAXN1];//圖 0-base
6 size_t c[MAXN2];
7 //每個屬於n2點最多可以接受幾條匹配邊
8 vector<int> matchs[MAXN2];
9 //每個屬於n2的點匹配了那些點
10 bool vis[MAXN2];
11 bool dfs(int u){
12     for(int v:g[u]){
13         if(vis[v])continue;
14         vis[v] = 1;
15         if(matchs[v].size()<c[v]){
16             return matchs[v].push_back(u), 1;
17         }else for(size_t j=0;j<matchs[v].size()
18             ;++j){
19             if(dfs(matchs[v][j]))
20                 return matchs[v][j]=u, 1;
21         }
22     }
23     return 0;
24 }
25 int max_match(){
26     for(int i=0;i<n2;++i) matchs[i].clear();
27     int cnt=0;
28     for(int u=0;u<n1;++u){
29         memset(vis,0,sizeof(bool)*n2);
30         if(dfs(u))++cnt;
31     }
32     return cnt;

```

4.3 blossom_matching

```

1 #define MAXN 505
2 int n; //1-base
3 vector<int> g[MAXN];
4 int MH[MAXN]; //output MH
5 int pa[MAXN],st[MAXN],S[MAXN],v[MAXN],t;
6 int lca(int x,int y){
7     for(++t;;swap(x,y)){
8         if(!x) continue;
9         if(v[x]==t) return x;
10        v[x] = t;
11        x = st[pa[MH[x]]];
12    }

```

```

13 }
14 #define qpush(x) q.push(x),S[x]=0
15 void flower(int x,int y,int l,queue<int>&q){
16     while(st[x]!=1){
17         pa[x]=y;
18         if(S[y==MH[x]]==1)qpush(y);
19         st[x]=st[y]=1, x=pa[y];
20     }
21 }
22 bool bfs(int x){
23     iota(st+1, st+n+1, 1);
24     memset(S+1,-1,sizeof(int)*n);
25     queue<int>q; qpush(x);
26     while(q.size()){
27         x=q.front();q.pop();
28         for(int y:g[x]){
29             if(S[y]==-1){
30                 pa[y]=x,S[y]=1;
31                 if(!MH[y]){
32                     for(int lst;x=y=lst,x=pa[y])
33                         lst=MH[x],MH[x]=y,MH[y]=x;
34                     return 1;
35                 }
36                 qpush(MH[y]);
37             }else if(!S[y]&&st[y]!=st[x]){
38                 int l=lca(y,x);
39                 flower(y,x,l,q),flower(x,y,l,q);
40             }
41         }
42     }
43     return 0;
44 }
45 int blossom(){
46     memset(MH+1,0,sizeof(int)*n);
47     int ans=0;
48     for(int i=1;i<n;++i)
49         if(!MH[i]&&bfs(i)) ++ans;
50     return ans;
51 }

```

4.4 BronKerbosch

```

1 struct maximalCliques{
2     using Set = vector<int>;
3     size_t n; //1-base
4     vector<Set> G;
5     static Set setUnion(const Set &A, const
6         Set &B){
7         Set C(A.size() + B.size());
8         auto it = set_union(A.begin(),A.end(),B.
9             begin(),B.end(),C.begin());
10        C.erase(it, C.end());
11        return C;
12    }
13    static Set setIntersection(const Set &A,
14        const Set &B){
15        Set C(min(A.size(), B.size()));
16        auto it = set_intersection(A.begin(),A.
17            end(),B.begin(),B.end(),C.begin());
18        C.erase(it, C.end());
19        return C;
20    }

```

```

17     static Set setDifference(const Set &A,
18         const Set &B){
19         Set C(min(A.size(), B.size()));
20         auto it = set_difference(A.begin(),A.end
21             (),B.begin(),B.end(),C.begin());
22         C.erase(it, C.end());
23         return C;
24     }
25 void BronKerbosch1(Set R, Set P, Set X){
26     if(P.empty()&&X.empty()){
27         // R form an maximal clique
28         return;
29     }
30     for(auto v: P){
31         BronKerbosch1(setUnion(R,{v}),
32             setIntersection(P,G[v]),
33             setIntersection(X,G[v]));
34         P = setDifference(P,{v});
35         X = setUnion(X,{v});
36     }
37 }
38 void init(int _n){
39     G.clear();
40     G.resize((n = _n) + 1);
41 }
42 void addEdge(int u, int v){
43     G[u].emplace_back(v);
44     G[v].emplace_back(u);
45 }
46 void solve(int n){
47     Set P;
48     for(int i=1;i<n;++i){
49         sort(G[i].begin(), G[i].end());
50         G[i].erase(unique(G[i].begin(), G[i].end()),
51             G[i].end());
52         P.emplace_back(i);
53     }
54     BronKerbosch1({}, P, {});
55 }

```

4.5 graphISO

```

1 const int MAXN=1005,K=30;//K要夠大
2 const long long A=3,B=11,C=2,D=19,P=0
3     xdefaced;
4 long long f[K+1][MAXN];
5 vector<int> g[MAXN],rg[MAXN];
6 int n;
7 void init(){
8     for(int i=0;i<n;++i){
9         f[0][i]=1;
10        g[i].clear(), rg[i].clear();
11    }
12 }
13 void add_edge(int u,int v){
14     g[u].push_back(v), rg[v].push_back(u);
15 }
16 long long point_hash(int u){//O(N)
17     for(int t=1;t<=K;++t){
18         for(int i=0;i<n;++i){
19             f[t][i]=f[t-1][i]*A%P;

```

```

19   for(int j:g[i])f[t][i]=(f[t][i]+f[t
    -1][j]*B%P)%P;
20   for(int j:rg[i])f[t][i]=(f[t][i]+f[t
    -1][j]*C%P)%P;
21   if(i==u)f[t][i]+=D; //如果圖太大的話，
    把這行刪掉，執行一次後f[K]就會是所
    有點的答案
22   f[t][i]%=P;
23 }
24 }
25 return f[K][u];
26 }
27 vector<long long> graph_hash(){
28   vector<long long> ans;
29   for(int i=0;i<n;++i)ans.push_back(
    point_hash(i)); //O(N^2)
30   sort(ans.begin(),ans.end());
31   return ans;
32 }

```

4.6 KM

```

1 #define MAXN 405
2 #define INF 0x3f3f3f3f3f3f3f3f
3 int n; // 1-base，0表示沒有匹配
4 LL g[MAXN][MAXN]; //input graph
5 int My[MAXN],Mx[MAXN]; //output match
6 LL lx[MAXN],ly[MAXN],pa[MAXN],Sy[MAXN];
7 bool vx[MAXN],vy[MAXN];
8 void augment(int y){
9   for(int x,z;y;z){
10     x=pa[y],z=Mx[x];
11     My[y]=x,Mx[x]=y;
12   }
13 }
14 void bfs(int st){
15   for(int i=1;i<n;++i)
16     Sy[i] = INF, vx[i]=vy[i]=0;
17   queue<int> q; q.push(st);
18   for(;;){
19     while(q.size()){
20       int x=q.front(); q.pop();
21       vx[x]=1;
22       for(int y=1;y<n;++y) if(!vy[y]){
23         LL t = lx[x]+ly[y]-g[x][y];
24         if(t==0){
25           pa[y]=x;
26           if(!My[y]){augment(y);return;}
27           vy[y]=1,q.push(My[y]);
28         }else if(Sy[y]>t) pa[y]=x,Sy[y]=t;
29       }
30     }
31     LL cut = INF;
32     for(int y=1;y<n;++y)
33       if(!vy[y]&&cut>Sy[y]) cut=Sy[y];
34     for(int j=1;j<n;++j){
35       if(vx[j]) lx[j] -= cut;
36       if(vy[j]) ly[j] += cut;
37       else Sy[j] -= cut;
38     }
39     for(int y=1;y<n;++y){
40       if(!vy[y]&&Sy[y]==0){

```

```

41       if(!My[y]){augment(y);return;}
42       vy[y]=1, q.push(My[y]);
43     }
44   }
45 }
46 }
47 LL KM(){
48   memset(My,0,sizeof(int)*(n+1));
49   memset(Mx,0,sizeof(int)*(n+1));
50   memset(ly,0,sizeof(LL)*(n+1));
51   for(int x=1;x<=n;++x){
52     lx[x] = -INF;
53     for(int y=1;y<=n;++y)
54       lx[x] = max(lx[x],g[x][y]);
55   }
56   for(int x=1;x<=n;++x) bfs(x);
57   LL ans = 0;
58   for(int y=1;y<=n;++y) ans+=g[My[y]][y];
59   return ans;
60 }

```

4.7 MaximumClique

```

1 struct MaxClique{
2   static const int MAXN=105;
3   int N,ans;
4   int g[MAXN][MAXN],dp[MAXN],stk[MAXN][MAXN]
    ];
5   int sol[MAXN],tmp[MAXN]; //sol[0~ans-1]為答
    案
6   void init(int n){
7     N=n; //0-base
8     memset(g,0,sizeof(g));
9   }
10  void add_edge(int u,int v){
11    g[u][v]=g[v][u]=1;
12  }
13  int dfs(int ns,int dep){
14    if(!ns){
15      if(dep>ans){
16        ans=dep;
17        memcpy(sol,tmp,sizeof tmp);
18        return 1;
19      }else return 0;
20    }
21    for(int i=0;i<n;++i){
22      if(dep+ns-i<=ans)return 0;
23      int u=stk[dep][i],cnt=0;
24      if(dep+dp[u]<=ans)return 0;
25      for(int j=i+1;j<n;++j){
26        int v=stk[dep][j];
27        if(g[u][v])stk[dep+1][cnt++]=v;
28      }
29      tmp[dep]=u;
30      if(dfs(cnt,dep+1))return 1;
31    }
32    return 0;
33  }
34  int clique(){
35    int u,v,ns;
36    for(ans=0,u=N-1;u>=0;--u){
37      for(ns=0,tmp[0]=u,v=u+1;v<N;++v)

```

```

38      if(g[u][v])stk[1][ns++]=v;
39      dfs(ns,1),dp[u]=ans;
40    }
41    return ans;
42  }
43 };

```

4.8 MinimumMeanCycle

```

1 #include<cstdio> //for DBL_MAX
2 int dp[MAXN][MAXN]; // 1-base,0(NM)
3 vector<tuple<int,int,int>> edge;
4 double mmc(int n){ //allow negative weight
5   const int INF=0x3f3f3f3f;
6   for(int t=0;t<n;++t){
7     memset(dp[t+1],0,sizeof(dp[t+1]));
8     for(const auto &e:edge){
9       int u,v,w;
10      tie(u,v,w) = e;
11      dp[t+1][v]=min(dp[t+1][v],dp[t][u]+w);
12    }
13  }
14  double res = DBL_MAX;
15  for(int u=1;u<=n;++u){
16    if(dp[n][u]==INF) continue;
17    double val = -DBL_MAX;
18    for(int t=0;t<n;++t)
19      val=max(val,(dp[n][u]-dp[t][u])*1.0/(n
    -t));
20    res=min(res,val);
21  }
22  return res;
23 }

```

4.9 Rectilinear_MST

```

1 //平面曼哈頓最小生成樹構造圖(去除非必要邊)
2 #define T int
3 #define INF 0x3f3f3f3f
4 struct point{
5   T x,y;
6   int id; //從0開始編號
7   point(){}
8   T dist(const point &p)const{
9     return abs(x-p.x)+abs(y-p.y);
10  }
11 };
12 bool cmpx(const point &a,const point &b){
13   return a.x<b.x|| (a.x==b.x&&a.y<b.y);
14 }
15 struct edge{
16   int u,v;
17   T cost;
18   edge(int u,int v,T c):u(u),v(v),cost(c){}
19   bool operator<(const edge&e)const{
20     return cost<e.cost;
21   }
22 };
23 struct bit_node{

```

```

24   T mi;
25   int id;
26   bit_node(const T&mi=INF,int id=-1):mi(mi),
    id(id){}
27 };
28 vector<bit_node> bit;
29 void bit_update(int i,const T&data,int id){
30   for(;;i=i&(-i)){
31     if(data<bit[i].mi)bit[i]=bit_node(data,
    id);
32   }
33 }
34 int bit_find(int i,int m){
35   bit_node x;
36   for(i<=m;i+=i&(-i)) if(bit[i].mi<x.mi)x=
    bit[i];
37   return x.id;
38 }
39 vector<edge> build_graph(int n,point p[]){
40   vector<edge> e; //edge for MST
41   for(int dir=0;dir<4;++dir){ //4種座標變換
42     if(dir%2) for(int i=0;i<n;++i) swap(p[i]
    .x,p[i].y);
43     else if(dir==2) for(int i=0;i<n;++i) p[i]
    .x=-p[i].x;
44     sort(p,p+n,cmpx);
45     vector<T> ga(n), gb;
46     for(int i=0;i<n;++i)ga[i]=p[i].y-p[i].x;
47     gb=ga, sort(gb.begin(),gb.end());
48     gb.erase(unique(gb.begin(),gb.end()),gb.
    end());
49     int m=gb.size();
50     bit=vector<bit_node>(m+1);
51     for(int i=n-1;i>=0;--i){
52       int pos=lower_bound(gb.begin(),gb.end
    (),ga[i])-gb.begin()+1;
53       int ans=bit_find(pos,m);
54       if(~ans)e.push_back(edge(p[i].id,p[ans]
    .id,p[i].dist(p[ans])));
55       bit_update(pos,p[i].x+p[i].y,i);
56     }
57   }
58   return e;
59 }

```

4.10 treeISO

```

1 const int MAXN=100005;
2 const long long X=12327,P=0xdefaced;
3 vector<int> g[MAXN];
4 bool vis[MAXN];
5 long long dfs(int u){ //hash ver
6   vis[u]=1;
7   vector<long long> tmp;
8   for(auto v:g[u])if(!vis[v])tmp.pb(dfs(v));
9   if(tmp.empty())return 177;
10  long long ret=4931;
11  sort(tmp.begin(),tmp.end());
12  for(auto v:tmp)ret=((ret*X)^v)%P;
13  return ret;
14 }
15 //-----
16 string dfs(int x,int p){

```



```

17 vector<string> c;
18 for(int y:g[x])
19     if(y!=p)c.emplace_back(dfs(y,x));
20 sort(c.begin(),c.end());
21 string ret("(");
22 for(auto &s:c)ret+=s;
23 ret+=")";
24 return ret;
25 }

```

4.11 一般圖最小權完美匹配

```

1 struct Graph {
2     // Minimum General Weighted Matching (
3     Perfect Match) 0-base
4     static const int MXN = 105;
5     int n, edge[MXN][MXN];
6     int match[MXN], dis[MXN], onstk[MXN];
7     vector<int> stk;
8     void init(int _n) {
9         n = _n;
10        for (int i=0; i<n; i++)
11            for (int j=0; j<n; j++)
12                edge[i][j] = 0;
13    }
14    void add_edge(int u, int v, int w) {
15        edge[u][v] = edge[v][u] = w;
16    }
17    bool SPFA(int u){
18        if (onstk[u]) return true;
19        stk.push_back(u);
20        onstk[u] = 1;
21        for (int v=0; v<n; v++){
22            if (u != v && match[u] != v && !onstk[v]){
23                int m = match[v];
24                if (dis[m] > dis[u] - edge[v][m] +
25                    edge[u][v]){
26                    dis[m] = dis[u] - edge[v][m] +
27                        edge[u][v];
28                    onstk[v] = 1;
29                    stk.push_back(v);
30                    if (SPFA(m)) return true;
31                    stk.pop_back();
32                    onstk[v] = 0;
33                }
34            }
35        }
36        onstk[u] = 0;
37        stk.pop_back();
38        return false;
39    }
40    int solve() {
41        // find a match
42        for (int i=0; i<n; i+=2){
43            match[i] = i+1, match[i+1] = i;
44        }
45        for(;;){
46            int found = 0;
47            for (int i=0; i<n; i++) dis[i] = onstk[i] = 0;
48            for (int i=0; i<n; i++){
49                stk.clear();

```

```

47         if (!onstk[i] && SPFA(i)){
48             found = 1;
49             while (stk.size()>2){
50                 int u = stk.back(); stk.pop_back();
51                 int v = stk.back(); stk.pop_back();
52                 match[u] = v;
53                 match[v] = u;
54             }
55             if (!found) break;
56         }
57         int ret = 0;
58         for (int i=0; i<n; i++)
59             ret += edge[i][match[i]];
60         ret /= 2;
61         return ret;
62     }
63 }graph;

```

4.12 全局最小割

```

1 const int INF=0x3f3f3f3f;
2 template<typename T>
3 struct stoer_wagner{// 0-base
4     static const int MAXN=150;
5     T g[MAXN][MAXN], dis[MAXN];
6     int nd[MAXN], n, s, t;
7     void init(int _n){
8         n=_n;
9         for(int i=0; i<n; ++i)
10             for(int j=0; j<n; ++j)g[i][j]=0;
11    }
12    void add_edge(int u, int v, T w){
13        g[u][v]=g[v][u]+=w;
14    }
15    T min_cut(){
16        T ans=INF;
17        for(int i=0; i<n; ++i)nd[i]=i;
18        for(int ind, tn=n; tn>1; --tn){
19            for(int i=1; i<tn; ++i)dis[nd[i]]=0;
20            for(int i=1; i<tn; ++i){
21                ind=i;
22                for(int j=i; j<tn; ++j){
23                    dis[nd[j]]+=g[nd[i-1]][nd[j]];
24                    if(dis[nd[ind]]<dis[nd[j]])ind=j;
25                }
26                swap(nd[ind], nd[i]);
27            }
28            if(ans>dis[nd[ind]])ans=dis[t=nd[ind]];
29            s=nd[ind-1];
30            for(int i=0; i<tn; ++i)
31                g[nd[ind-1]][nd[i]]=g[nd[i]][nd[ind-1]]+=g[nd[i]][nd[ind]];
32        }
33        return ans;
34    }

```

4.13 弦圖完美消除序列

```

1 struct chordal{
2     static const int MAXN=1005;
3     int n;// 0-base
4     vector<int>G[MAXN];
5     int rank[MAXN], label[MAXN];
6     bool mark[MAXN];
7     void init(int _n){n=_n;
8         for(int i=0; i<n; ++i)G[i].clear();
9     }
10    void add_edge(int u, int v){
11        G[u].push_back(v);
12        G[v].push_back(u);
13    }
14    vector<int> MCS(){
15        memset(rank, -1, sizeof(int)*n);
16        memset(label, 0, sizeof(int)*n);
17        priority_queue<pair<int, int>> pq;
18        for(int i=0; i<n; ++i)pq.push(make_pair(0, i));
19        for(int i=n-1; i>=0; --i)for(;;){
20            int u=pq.top().second; pq.pop();
21            if(~rank[u])continue;
22            rank[u]=i;
23            for(auto v:G[u])if(rank[v]==-1){
24                pq.push(make_pair(++label[v], v));
25            }
26            break;
27        }
28        vector<int> res(n);
29        for(int i=0; i<n; ++i)res[rank[i]]=i;
30        return res;
31    }
32    bool check(vector<int> ord){//弦圖判定
33        for(int i=0; i<n; ++i)rank[ord[i]]=i;
34        memset(mark, 0, sizeof(bool)*n);
35        for(int i=0; i<n; ++i){
36            vector<pair<int, int>> tmp;
37            for(auto u:G[ord[i]])if(!mark[u])
38                tmp.push_back(make_pair(rank[u], u));
39            sort(tmp.begin(), tmp.end());
40            if(tmp.size()){
41                int u=tmp[0].second;
42                set<int> S;
43                for(auto v:G[u])S.insert(v);
44                for(size_t j=1; j<tmp.size(); ++j)
45                    if(!S.count(tmp[j].second))return 0;
46            }
47            mark[ord[i]]=1;
48        }
49        return 1;
50    }
51 };

```

4.14 最小斯坦納樹 DP

```

1 //n個點，其中r個要構成斯坦納樹
2 //答案在max(dp[(1<r)-1][k]) k=0~n-1
3 //p表示要構成斯坦納樹的點集

```

```

4 //O( n^3 + n*3^r + n^2*2^r )
5 #define REP(i,n) for(int i=0;i<(int)n;++i)
6 const int MAXN=30, MAXM=8; // 0-base
7 const int INF=0x3f3f3f3f;
8 int dp[1<MAXN][MAXN];
9 int g[MAXN][MAXN]; //圖
10 void init(){memset(g, 0x3f, sizeof(g));}
11 void add_edge(int u, int v, int w){
12     g[u][v]=g[v][u]=min(g[v][u], w);
13 }
14 void steiner(int n, int r, int *p){
15     REP(k,n)REP(i,n)REP(j,n)
16         g[i][j]=min(g[i][j], g[i][k]+g[k][j]);
17     REP(i,n)g[i][i]=0;
18     REP(i,r)REP(j,n)dp[1<i][j]=g[p[i]][j];
19     for(int i=1; i<(1<r); ++i){
20         if(!(i&(i-1)))continue;
21         REP(j,n)dp[i][j]=INF;
22         REP(j,n){
23             int tmp=INF;
24             for(int s=i&(i-1); s<=i&(s-1))
25                 tmp=min(tmp, dp[s][j]+dp[i^s][j]);
26             REP(k,n)dp[i][k]=min(dp[i][k], g[j][k]+tmp);
27         }
28     }
29 }

```

4.15 最小樹形圖 — 朱劉

```

1 template<typename T>
2 struct zhu_liu{
3     static const int MAXN=110, MAXM=10005;
4     struct node{
5         int u, v;
6         T w, tag;
7         node *l, *r;
8         node(int u=0, int v=0, T w=0):u(u), v(v), w(w), tag(0), l(0), r(0){}
9     }
10    void down(){
11        w+=tag;
12        if(l)l->tag+=tag;
13        if(r)r->tag+=tag;
14        tag=0;
15    }
16    mem[MAXN]; //靜態記憶體
17    node *pq[MAXN*2], *E[MAXN*2];
18    int st[MAXN*2], id[MAXN*2], m;
19    void init(int n){
20        for(int i=1; i<n; ++i){
21            pq[i]=E[i]=0, st[i]=id[i]=i;
22            m=0;
23        }
24        node *merge(node *a, node *b){//skew heap
25            if(!a||!b)return a?a:b;
26            a->down(), b->down();
27            if(b->w<a->w)return merge(b, a);
28            swap(a->l, a->r);
29            a->l=merge(b, a->l);
30            return a;
31        }
32    void add_edge(int u, int v, T w){

```

```

32 if(u!=v)pq[v]=merge(pq[v],&mem[m++]=
   node(u,v,w));
33 }
34 int find(int x,int *st){
35     return st[x]==x?x:st[x]=find(st[x],st);
36 }
37 T build(int root,int n){
38     T ans=0;int N=n,all=n;
39     for(int i=1;i<=N;++i){
40         if(i==root||!pq[i])continue;
41         while(pq[i]){
42             pq[i]->down(),E[i]=pq[i];
43             pq[i]=merge(pq[i]->l,pq[i]->r);
44             if(find(E[i]->u,id)!=find(i,id))
               break;
45         }
46         if(find(E[i]->u,id)==find(i,id))
           continue;
47         ans+=E[i]->w;
48         if(find(E[i]->u,st)==find(i,st)){
49             if(pq[i]pq[i]->tag==E[i]->w;
50             pq[++N]=pq[i];id[N]=N;
51             for(int u=find(E[i]->u,id);u!=i;u=
               find(E[u]->u,id)){
52                 if(pq[u]pq[u]->tag==E[u]->w;
53                 id[find(u,id)]=N;
54                 pq[N]=merge(pq[N],pq[u]);
55             }
56             st[N]=find(i,st);
57             id[find(i,id)]=N;
58             }else st[find(i,st)]=find(E[i]->u,st)
               ,--all;
59         }
60     return all==1?ans:-INT_MAX;//圖不連通就
       無解
61 }
62 };

```

4.16 穩定婚姻模板

```

1 queue<int> Q;
2 for ( i : 所有考生 ) {
3     設定在第0志願;
4     Q.push(考生i);
5 }
6 while(Q.size()){
7     當前考生=Q.front();Q.pop();
8     while ( 此考生未分發 ) {
9         指標移到下一志願;
10        if ( 已經沒有志願 or 超出志願總數 )
           break;
11        計算該考生在該科系加權後的總分;
12        if ( 不符合科系需求 ) continue;
13        if ( 目前科系有餘額 ) {
14            依加權後分數高低順序將考生id加入科系錄
              取名單中;
15            break;
16        }
17        if ( 目前科系已額滿 ) {
18            if ( 此考生成績比最低分數還高 ) {

```

```

19        依加權後分數高低順序將考生id加入科系
              錄取名單;
20        Q.push(被踢出的考生);
21    }
22 }
23 }
24 }

```

5 Linear_Programming

5.1 simplex

```

1 /*target:
2   max \sum_{j=1}^n A_{0,j}*x_j
3 condition:
4   \sum_{j=1}^n A_{i,j}*x_j <= A_{i,0} | i=1~m
5   x_j >= 0 | j=1~n
6 VDB = vector<double>*/
7 template<class VDB>
8 VDB simplex(int m,int n,vector<VDB> a){
9     vector<int> left(m+1), up(n+1);
10    iota(left.begin(), left.end(), n);
11    iota(up.begin(), up.end(), 0);
12    auto pivot = [&](int x, int y){
13        swap(left[x], up[y]);
14        auto k = a[x][y]; a[x][y] = 1;
15        vector<int> pos;
16        for(int j = 0; j <= n; ++j){
17            a[x][j] /= k;
18            if(a[x][j] != 0) pos.push_back(j);
19        }
20        for(int i = 0; i <= m; ++i){
21            if(a[i][y]==0 || i == x) continue;
22            k = a[i][y], a[i][y] = 0;
23            for(int j : pos) a[i][j] -= k*a[x][j];
24        }
25    };
26    for(int x,y;){
27        for(int i=x+1; i <= m; ++i)
28            if(a[i][0]<a[x][0]) x = i;
29        if(a[x][0]>=0) break;
30        for(int j=y+1; j <= n; ++j)
31            if(a[x][j]<a[x][y]) y = j;
32        if(a[x][y]>=0) return VDB();//infeasible
33        pivot(x, y);
34    }
35    for(int x,y;){
36        for(int j=y+1; j <= n; ++j)
37            if(a[0][j] > a[0][y]) y = j;
38        if(a[0][y]<=0) break;
39        x = -1;
40        for(int i=1; i<=m; ++i) if(a[i][y] > 0)
41            if(x == -1 || a[i][0]/a[i][y]
42               < a[x][0]/a[x][y]) x = i;
43        if(x == -1) return VDB();//unbounded
44        pivot(x, y);
45    }
46    VDB ans(n + 1);
47    for(int i = 1; i <= m; ++i)
48        if(left[i] <= n) ans[left[i]] = a[i][0];

```

```

49 ans[0] = -a[0][0];
50 return ans;
51 }

```

6 Number_Theory

6.1 basic

```

1 template<typename T>
2 void gcd(const T &a,const T &b,T &d,T &x,T &
   y){
3     if(!b) d=a,x=1,y=0;
4     else gcd(b,a%b,d,y,x), y-=x*(a/b);
5 }
6 long long int phi[N+1];
7 void phiTable(){
8     for(int i=1;i<=N;i++)phi[i]=i;
9     for(int i=1;i<=N;i++)for(x=i*2;x<=N;x+=i)
       phi[x]-=phi[i];
10 }
11 void all_divdown(const LL &n){// all n/x
12     for(LL a=1;a<=n;a=n/(n/(a+1))) {
13         // dosomething;
14     }
15 }
16 const int MAXPRIME = 1000000;
17 int iscom[MAXPRIME], prime[MAXPRIME],
   primecnt;
18 int phi[MAXPRIME], mu[MAXPRIME];
19 void sieve(void){
20     memset(iscom,0,sizeof(iscom));
21     primecnt = 0;
22     phi[1] = mu[1] = 1;
23     for(int i=2;i<MAXPRIME;++i) {
24         if(!iscom[i]) {
25             prime[primecnt++] = i;
26             mu[i] = -1;
27             phi[i] = i-1;
28         }
29         for(int j=0;j<primecnt;++j) {
30             int k = i * prime[j];
31             if(k>MAXPRIME) break;
32             iscom[k] = prime[j];
33             if(i%prime[j]==0) {
34                 mu[k] = 0;
35                 phi[k] = phi[i] * prime[j];
36                 break;
37             } else {
38                 mu[k] = -mu[i];
39                 phi[k] = phi[i] * (prime[j]-1);
40             }
41         }
42     }
43 }
44 bool g_test(const LL &g, const LL &p, const
   vector<LL> &v) {
45     for(int i=0;i<v.size();++i)
46         if(modexp(g,(p-1)/v[i],p)==1)
47             return false;
48     return true;

```

```

50 }
51 LL primitive_root(const LL &p) {
52     if(p==2) return 1;
53     vector<LL> v;
54     Factor(p-1,v);
55     v.erase(unique(v.begin(), v.end()), v.end()
       ());
56     for(LL g=2;g<p;++g)
57         if(g_test(g,p,v))
58             return g;
59     puts("primitive_root NOT FOUND");
60     return -1;
61 }
62 int Legendre(const LL &a, const LL &p) {
63     return modexp(a%p,(p-1)/2,p); }
64 LL inv(const LL &a, const LL &n) {
65     LL d,x,y;
66     gcd(a,n,d,x,y);
67     return d==1 ? (x+n)%n : -1;
68 }
69 int inv[maxn];
70 LL invtable(int n,LL P){
71     inv[1]=1;
72     for(int i=2;i<n;++i)
73         inv[i]=(P-(P/i))*inv[P%i]%P;
74 }
75 LL log_mod(const LL &a, const LL &b, const
   LL &p) {
76     // a ^ x = b ( mod p )
77     int m=sqrt(p+.5), e=1;
78     LL v=inv(modexp(a,m,p), p);
79     map<LL,int> x;
80     x[1]=0;
81     for(int i=1;i<m;++i) {
82         e = LLMul(e,a,p);
83         if(!x.count(e)) x[e] = i;
84     }
85     for(int i=0;i<m;++i) {
86         if(x.count(b)) return i*m + x[b];
87         b = LLMul(b,v,p);
88     }
89     return -1;
90 }
91 LL Tonelli_Shanks(const LL &n, const LL &p)
   {
92     // x^2 = n ( mod p )
93     if(n==0) return 0;
94     if(Legendre(n,p)!=1) while(1) { puts("SQRT
       ROOT does not exist"); }
95     int S = 0;
96     LL Q = p-1;
97     while( !(Q&1) ) { Q>>=1; ++S; }
98     if(S==1) return modexp(n%p,(p+1)/4,p);
99     LL z = 2;
100    for(;Legendre(z,p)!=-1;++z)
101        LL c = modexp(z,Q,p);
102    LL R = modexp(n%p,(Q+1)/2,p), t = modexp(n
       %p,Q,p);
103    int M = S;
104    while(1) {
105        if(t==1) return R;
106        LL b = modexp(c,1L<<(M-i-1),p);

```

```

110 R = LLMul(R,b,p);
111 t = LLMul( LLMul(b,b,p), t, p);
112 c = LLMul(b,b,p);
113 M = i;
114 }
115 return -1;
116 }
117
118 template<typename T>
119 T Euler(T n){
120     T ans=n;
121     for(T i=2;i*i<=n;++i){
122         if(n%i==0){
123             ans=ans/i*(i-1);
124             while(n%i==0)n/=i;
125         }
126     }
127     if(n>1)ans=ans/n*(n-1);
128     return ans;
129 }
130
131 //Chinese_remainder_theorem
132 template<typename T>
133 T pow_mod(T n,T k,T m){
134     T ans=1;
135     for(n=(n>m?n%m:n);k;k>>=1){
136         if(k&1)ans=ans*n%m;
137         n=n*n%m;
138     }
139     return ans;
140 }
141
142 template<typename T>
143 T crt(vector<T> &m,vector<T> &a){
144     T M=1,tM,ans=0;
145     for(int i=0;i<(int)m.size();++i)M*=m[i];
146     for(int i=0;i<(int)a.size();++i){
147         tM=M/m[i];
148         ans=(ans+(a[i]*tM%M)*pow_mod(tM,Euler(m[i])-1,m[i])%M)%M;
149         /*如果m[i]是質數·Euler(m[i])-1=m[i]-2·
150         就不用算Euler了*/
151     }
152     return ans;
153 }
154
155 //java code
156 //求sqrt(N)的連分數
157 public static void Pell(int n){
158     BigInteger N,p1,p2,q1,q2,a0,a1,a2,g1,g2,h1
159         ,h2,p,q;
160     g1=q2=p1=BigInteger.ZERO;
161     h1=q1=p2=BigInteger.ONE;
162     a0=a1=BigInteger.valueOf((int)Math.sqrt
163         (1.0*n));
164     BigInteger ans=a0.multiply(a0);
165     if(ans.equals(BigInteger.valueOf(n))){
166         System.out.println("No solution!");
167         return ;
168     }
169     while(true){
170         g2=a1.multiply(h1).subtract(g1);
171         h2=N.subtract(g2.pow(2)).divide(h1);
172         a2=g2.add(a0).divide(h2);
173         p=a1.multiply(p2).add(p1);
174         q=a1.multiply(q2).add(q1);

```

```

171     if(p.pow(2).subtract(N.multiply(q.pow
172         (2))).compareTo(BigInteger.ONE)==0)
173         break;
174     g1=g2;h1=h2;a1=a2;
175     p1=p2;p2=p;
176     q1=q2;q2=q;
177 }
178 System.out.println(p+" "+q);
179 }

```

6.2 bit_set

```

1 void sub_set(int S){
2     int sub=S;
3     do{
4         //對某集合的子集合的處理
5         sub=(sub-1)&S;
6     }while(sub!=S);
7 }
8
9 void k_sub_set(int k,int n){
10    int comb=(1<<k)-1,S=1<<n;
11    while(comb<S){
12        //對大小為k的子集合的處理
13        int x=comb&-comb,y=comb+x;
14        comb=((comb&y)/x>>1)|y;
15    }
16 }

```

6.3 cantor_expansion

```

1 int factorial[MAXN];
2 void init(){
3     factorial[0]=1;
4     for(int i=1;i<=MAXN;++i)factorial[i]=
5         factorial[i-1]*i;
6 }
7
8 int encode(const vector<int> &s){
9     int n=s.size(),res=0;
10    for(int i=0;i<n;++i){
11        int t=0;
12        for(int j=i+1;j<n;++j)
13            if(s[j]<s[i])++t;
14        res+=t*factorial[n-i-1];
15    }
16    return res;
17 }
18
19 vector<int> decode(int a,int n){
20     vector<int> res;
21     vector<bool> vis(n,0);
22     for(int i=n-1;i>=0;--i){
23         int t=a/factorial[i];j;
24         for(j=0;j<n;++j)
25             if(!vis[j]){
26                 if(t==0)break;
27                 --t;
28             }
29         res.push_back(j);
30         vis[j]=1;
31         a%=factorial[i];

```

```

29 }
30 return res;
31 }

```

6.4 FFT

```

1 template<typename T,typename VT=vector<
2     complex<T> >>
3 struct FFT{
4     const T pi;
5     FFT(const T pi=acos((T)-1)):pi(pi){}
6     unsigned bit_reverse(unsigned a,int len){
7         a=((a&0x55555555U)<<1)|((a&0xAAAAAAAAU)>>1);
8         a=((a&0x33333333U)<<2)|((a&0xCCCCCCCCU)>>2);
9         a=((a&0x0F0F0F0FU)<<4)|((a&0xFF0F0F0FU)>>4);
10        a=((a&0x00FF00FFU)<<8)|((a&0xFFFF0000U)>>8);
11        a=((a&0x0000FFFFU)<<16)|((a&0xFFFF0000U)>>16);
12        return a>>(32-len);
13    }
14    void fft(bool is_inv,VT &in,VT &out,int N)
15    {
16        int bitlen=__lg(N),num=is_inv?-1:1;
17        for(int i=0;i<N;++i)out[bit_reverse(i,
18            bitlen)]=in[i];
19        for(int step=2;step<=N;step<=1){
20            const int mh=step>>1;
21            for(int i=0;i<mh;++i){
22                complex<T> wi=exp(complex<T>(0,i*num
23                    *pi/mh));
24                for(int j=i;j<N;j+=step){
25                    int k=j+mh;
26                    complex<T> u=out[j],t=wi*out[k];
27                    out[j]=u+t;
28                    out[k]=u-t;
29                }
30            }
31        }
32        if(is_inv)for(int i=0;i<N;++i)out[i]/=N;
33    }
34 }

```

6.5 find_real_root

```

1 // a*x^n + ... + a1x + a0 = 0;
2 int sign(double x){
3     return x < -eps ? -1 : x > eps;
4 }
5
6 double get(const vector<double>&coef, double
7     x){
8     double e = 1, s = 0;
9     for(auto i : coef) s += i*e, e *= x;
10    return s;
11 }
12
13 double find(const vector<double>&coef, int n
14     , double lo, double hi){
15     double sign_lo, sign_hi;

```

```

14 if( !(sign_lo = sign(get(coef,lo))) )
15     return lo;
16 if( !(sign_hi = sign(get(coef,hi))) )
17     return hi;
18 if(sign_lo * sign_hi > 0) return INF;
19 for(int stp = 0; stp < 100 && hi - lo >
20     eps; ++stp){
21     double m = (lo+hi)/2.0;
22     int sign_mid = sign(get(coef,m));
23     if(!sign_mid) return m;
24     if(sign_lo*sign_mid < 0) hi = m;
25     else lo = m;
26 }
27 return (lo+hi)/2.0;
28 }
29
30 vector<double> cal(vector<double>coef, int n
31 )){
32     vector<double>res;
33     if(n == 1){
34         if(sign(coef[1])) res.pb(-coef[0]/coef
35             [1]);
36         return res;
37     }
38     vector<double>dcoef(n);
39     for(int i = 0; i < n; ++i) dcoef[i] = coef
40         [i+1]*(i+1);
41     vector<double>droot = cal(dcoef, n-1);
42     droot.insert(droot.begin(), -INF);
43     droot.pb(INF);
44     for(int i = 0; i+1 < droot.size(); ++i){
45         double tmp = find(coef, n, droot[i],
46             droot[i+1]);
47         if(tmp < INF) res.pb(tmp);
48     }
49     return res;
50 }
51
52 int main () {
53     vector<double>ve;
54     vector<double>ans = cal(ve, n);
55     // 視情況把答案 +eps · 避免 -0
56 }

```

6.6 FWT

```

1 vector<int> F_OR_T(vector<int> f, bool
2     inverse){
3     for(int i=0; (2<<i)<=f.size(); ++i)
4         for(int j=0; j<f.size(); j+=2<<i)
5             f[j+k*(1<<i)] += f[j+k]*(inverse
6                 ?-1:1);
7     return f;
8 }
9
10 vector<int> rev(vector<int> A) {
11     for(int i=0; i<A.size(); i+=2)
12         swap(A[i],A[i^(A.size()-1)]);
13     return A;
14 }
15
16 vector<int> F_AND_T(vector<int> f, bool
17     inverse){
18     return rev(F_OR_T(rev(f), inverse));
19 }

```

```

15 }
16 vector<int> F_XOR_T(vector<int> f, bool
    inverse){
17     for(int i=0; (2<<i)<=f.size(); ++i)
18         for(int j=0; j<f.size(); j+=2<<i)
19             for(int k=0; k<(1<<i); ++k){
20                 int u=f[j+k], v=f[j+k+(1<<i)];
21                 f[j+k+(1<<i)] = u-v, f[j+k] = u+v;
22             }
23     if(inverse) for(auto &a:f) a/=f.size();
24     return f;
25 }

```

6.9 Matrix

6.7 LinearCongruence

```

1 pair<LL,LL> LinearCongruence(LL a[],LL b[],
    LL m[],int n) {
2     // a[i]*x = b[i] ( mod m[i] )
3     for(int i=0;i<n;++i) {
4         LL x, y, d = extgcd(a[i],m[i],x,y);
5         if(b[i]%d!=0) return make_pair(-1LL,0LL);
6         m[i] /= d;
7         b[i] = LLmul(b[i]/d,x,m[i]);
8     }
9     LL lastb = b[0], lastm = m[0];
10    for(int i=1;i<n;++i) {
11        LL x, y, d = extgcd(m[i],lastm,x,y);
12        if((lastb-b[i])%d!=0) return make_pair
            (-1LL,0LL);
13        lastb = LLmul((lastb-b[i])/d,x,(lastm/d)
            )*m[i];
14        lastm = (lastm/d)*m[i];
15        lastb = (lastb+b[i])%lastm;
16    }
17    return make_pair(lastb<0?lastb+lastm:lastb
        ,lastm);
18 }

```

6.8 Lucas

```

1 ll C(ll n, ll m, ll p){// n!/m!/(n-m)!
2     if(n<m) return 0;
3     return f[n]*inv(f[m],p)%p*inv(f[n-m],p)%p;
4 }
5 ll L(ll n, ll m, ll p){
6     if(!m) return 1;
7     return C(n%p,m%p,p)*L(n/p,m/p,p)%p;
8 }
9 ll Wilson(ll n, ll p){ // n!%p
10    if(!n)return 1;
11    ll res=Wilson(n/p, p);
12    if((n/p)%2) return res*(p-f[n%p])%p;
13    return res*f[n%p]%p; //(p-1)!%p=-1
14 }

```

```

1 template<typename T>
2 struct Matrix{
3     using rt = std::vector<T>;
4     using mt = std::vector<rt>;
5     using matrix = Matrix<T>;
6     int r,c;
7     mt m;
8     Matrix(int r,int c):r(r),c(c),m(r,rt(c)){
9         rt& operator[](int i){return m[i];}
10    matrix operator+(const matrix &a){
11        matrix rev(r,c);
12        for(int i=0;i<r;++i)
13            for(int j=0;j<c;++j)
14                rev[i][j]=m[i][j]+a.m[i][j];
15        return rev;
16    }
17    matrix operator-(const matrix &a){
18        matrix rev(r,c);
19        for(int i=0;i<r;++i)
20            for(int j=0;j<c;++j)
21                rev[i][j]=m[i][j]-a.m[i][j];
22        return rev;
23    }
24    matrix operator*(const matrix &a){
25        matrix rev(r,a.c);
26        matrix tmp(a.c,a.r);
27        for(int i=0;i<a.r;++i)
28            for(int j=0;j<a.c;++j)
29                tmp[j][i]=a.m[i][j];
30        for(int i=0;i<r;++i)
31            for(int j=0;j<a.c;++j)
32                for(int k=0;k<c;++k)
33                    rev.m[i][j]+=m[i][k]*tmp[j][k];
34        return rev;
35    }
36    bool inverse(){
37        Matrix t(r,r+c);
38        for(int y=0;y<r;y++){
39            t.m[y][c+y] = 1;
40            for(int x=0;x<c;++x)
41                t.m[y][x]=m[y][x];
42        }
43        if(!t.gas())
44            return false;
45        for(int y=0;y<r;y++){
46            for(int x=0;x<c;++x)
47                m[y][x]=t.m[y][c+x]/t.m[y][y];
48        return true;
49    }
50    T gas(){
51        vector<T> lazy(r,1);
52        bool sign=false;
53        for(int i=0;i<r;++i){
54            if(m[i][i]==0){
55                int j=i+1;
56                while(j<r&&!m[j][i])j++;
57                if(j==r)continue;
58                m[i].swap(m[j]);
59                sign=!sign;
60            }
61            for(int j=0;j<r;++j){
62                if(i==j)continue;
63                lazy[j]=lazy[j]*m[i][i];

```

```

64        T mx=m[j][i];
65        for(int k=0;k<c;++k)
66            m[j][k]=m[j][k]*m[i][i]-m[i][k]*mx;
67    }
68    T det=sign?-1:1;
69    for(int i=0;i<r;++i){
70        det = det*m[i][i];
71        det = det/lazy[i];
72        for(auto &j:m[i])j/=lazy[i];
73    }
74    return det;
75 }
76 }
77 };

```

6.10 MillerRobin

```

1 ULL LLMul(ULL a, ULL b, const ULL &mod) {
2     LL ans=0;
3     while(b) {
4         if(b&1) {
5             ans+=a;
6             if(ans>=mod) ans-=mod;
7         }
8         a<<=1, b>>=1;
9         if(a>=mod) a-=mod;
10    }
11    return ans;
12 }
13 ULL mod_mul(ULL a,ULL b,ULL m){
14    a%=m,b%=m;/* fast for m < 2^58 */
15    ULL y=(ULL)((double)a*b/m+0.5);
16    ULL r=(a*b-y*m)%m;
17    return r<0?r+m:r;
18 }
19 template<typename T>
20 T pow(T a,T b,T mod){//a^b%mod
21    T ans=1;
22    for(;a=mod_mul(a,a,mod),b>>=1)
23        if(b&1)ans=mod_mul(ans,a,mod);
24    return ans;
25 }
26 int sprp[3]={2,7,61};//int範圍可解
27 int llsprp
    [7]={2,325,9375,28178,450775,9780504,
28    1795265022};//至少 unsigned long long範圍
29 template<typename T>
30 bool isprime(T n,int *sprp,int num){
31    if(n==2)return 1;
32    if(n<2||n%2==0)return 0;
33    int t=0;
34    T u=n-1;
35    for(;u%2==0;++t)u>>=1;
36    for(int i=0;i<num;++i){
37        T a=sprp[i]%n;
38        if(a==0||a==1||a==n-1)continue;
39        T x=pow(a,u,n);
40        if(x==1||x==n-1)continue;
41        for(int j=0;j<t;++j){
42            x=mod_mul(x,x,n);
43            if(x==1)return 0;

```

```

44        if(x==n-1)break;
45    }
46    if(x==n-1)continue;
47    return 0;
48 }
49 return 1;
50 }

```

6.11 NTT

```

1 2615053605667*(2^18)+1,3
2 15*(2^27)+1,31
3 479*(2^21)+1,3
4 7*17*(2^23)+1,3
5 3*3*211*(2^19)+1,5
6 25*(2^22)+1,3
7 template<typename T,typename VT=vector<T> >
8 struct NTT{
9     const T P,G;
10    NTT(T p=(1<<23)*7*17+1,T g=3):P(p),G(g){}
11    unsigned bit_reverse(unsigned a,int len){
12        //look FFT.cpp
13    }
14    T pow_mod(T n,T k,T m){
15        T ans=1;
16        for(n=(n>=m?n%m:n);k>>=1){
17            if(k&1)ans=ans*n%m;
18            n=n*n%m;
19        }
20        return ans;
21    }
22    void ntt(bool is_inv,VT &in,VT &out,int N)
23    {
24        int bitlen=__lg(N);
25        for(int i=0;i<N;++i)out[bit_reverse(i,
26            bitlen)]=in[i];
27        for(int step=2,id=1;step<=N;step<=1,++
28            id){
29            T wn=pow_mod(G,(P-1)>>id,P),wi=1,u,t;
30            const int mh=step>>1;
31            for(int i=0;i<mh;++i){
32                for(int j=i;j<N;j+=step){
33                    u=out[j],t=wi*out[j+mh]%P;
34                    out[j]=u+t;
35                    out[j+mh]=u-t;
36                    if(out[j]>=P)out[j]-=P;
37                    if(out[j+mh]<0)out[j+mh]+=P;
38                }
39                wi=wi*wn%P;
40            }
41        }
42        if(is_inv){
43            for(int i=1;i<N/2;++i)swap(out[i],out[
44                N-i]);
45            T invn=pow_mod(N,P-2,P);
46            for(int i=0;i<N;++i)out[i]=out[i]*invn
47                %P;
48        }
49    }
50 };

```


6.12 Simpson

```

1 double simpson(double a,double b){
2     double c=a+(b-a)/2;
3     return (F(a)+4*F(c)+F(b))*(b-a)/6;
4 }
5 double asr(double a,double b,double eps,
6     double A){
7     double c=a+(b-a)/2;
8     double L=simpson(a,c),R=simpson(c,b);
9     if( abs(L+R-A)<15*eps )
10        return L+R+(L+R-A)/15.0;
11    return asr(a,c,eps/2,L)+asr(c,b,eps/2,R);
12 }
13 double asr(double a,double b,double eps){
14    return asr(a,b,eps,simpson(a,b));
15 }

```

6.13 外星模運算

```

1 //a[0]^a[1]^a[2]^...
2 #define maxn 1000000
3 int euler[maxn+5];
4 bool is_prime[maxn+5];
5 void init_euler(){
6     is_prime[1]=1; // --不是質數
7     for(int i=1;i<=maxn;i++)euler[i]=i;
8     for(int i=2;i<=maxn;i++){
9         if(!is_prime[i]){ //是質數
10             euler[i]--;
11             for(int j=i<<1;j<=maxn;j+=i){
12                 is_prime[j]=1;
13                 euler[j]=euler[j]/i*(i-1);
14             }
15         }
16     }
17 }
18 LL pow(LL a,LL b,LL mod){ //a^b%mod
19     LL ans=1;
20     for(;b;a=a%mod,b>>=1)
21         if(b&1)ans=ans*a%mod;
22     return ans;
23 }
24 bool isless(LL *a,int n,int k){
25     if(*a==1)return k<1;
26     if(--n==0)return *a<k;
27     int next=0;
28     for(LL b=1;b<k;next++)
29         b*=*a;
30     return isless(a+1,n,next);
31 }
32 LL high_pow(LL *a,int n,LL mod){
33     if(*a==1||--n==0)return *a%mod;
34     int k=0,r=euler[mod];
35     for(LL tma=1;tma!=pow(*a,k+r,mod);++k)
36         tma=tma*(a%mod);
37     if(isless(a+1,n,k))return pow(*a,high_pow(
38         a+1,n,k),mod);
39     int tmd=high_pow(a+1,n,r), t=(tmd-k+r)%r;
40     return pow(*a,k+t,mod);
41 }

```

```

41 LL a[1000005];
42 int t,mod;
43 int main(){
44     init_euler();
45     scanf("%d",&t);
46     #define n 4
47     while(t--){
48         for(int i=0;i<n;++i)scanf("%lld",&a[i]);
49         scanf("%d",&mod);
50         printf("%lld\n",high_pow(a,n,mod));
51     }
52     return 0;
53 }

```

6.14 數位統計

```

1 ll d[65], dp[65][2]; //up區間是不是完整
2 ll dfs(int p,bool is8,bool up){
3     if(!p)return 1; // 回傳0是不是答案
4     if(!up&&dp[p][is8])return dp[p][is8];
5     int mx = up?d[p]:9; //可以用的有那些
6     ll ans=0;
7     for(int i=0;i<=mx;++i){
8         if( is8&&i==7 )continue;
9         ans += dfs(p-1,i==8,up&&i==mx);
10    }
11    if(!up)dp[p][is8]=ans;
12    return ans;
13 }
14 ll f(ll N){
15     int k=0;
16     while(N){ // 把數字先分解到陣列
17         d[++k] = N%10;
18         N/=10;
19     }
20     return dfs(k,false,true);
21 }

```

6.15 質因數分解

```

1 LL func(const LL n,const LL mod,const int c)
2     {
3         return (LLmul(n,n,mod)+c+mod)%mod;
4     }
5 LL pollorro(const LL n, const int c) { //循
6     環節長度
7     LL a=1, b=1;
8     a=func(a,n,c)%n;
9     b=func(b,n,c)%n; b=func(b,n,c)%n;
10    while(gcd(abs(a-b),n)==1) {
11        a=func(a,n,c)%n;
12        b=func(b,n,c)%n; b=func(b,n,c)%n;
13    }
14    return gcd(abs(a-b),n);
15 }
16 void prefactor(LL &n, vector<LL> &v) {
17     for(int i=0;i<12;++i) {

```

```

18     while(n%prime[i]==0) {
19         v.push_back(prime[i]);
20         n/=prime[i];
21     }
22 }
23 }
24 void smallfactor(LL n, vector<LL> &v) {
25     if(n<MAXPRIME) {
26         while(isp[(int)n]) {
27             v.push_back(isp[(int)n]);
28             n/=isp[(int)n];
29         }
30         v.push_back(n);
31     } else {
32         for(int i=0;i<primecnt&&prime[i]*prime[i]
33             ]<=n;++i) {
34             while(n%prime[i]==0) {
35                 v.push_back(prime[i]);
36                 n/=prime[i];
37             }
38         }
39         if(n!=1) v.push_back(n);
40     }
41 }
42 void comfactor(const LL &n, vector<LL> &v) {
43     if(n<1e9) {
44         smallfactor(n,v);
45         return;
46     }
47     if(Isprime(n)) {
48         v.push_back(n);
49         return;
50     }
51     LL d;
52     for(int c=3;++c) {
53         d = pollorro(n,c);
54         if(d!=n) break;
55     }
56     comfactor(d,v);
57     comfactor(n/d,v);
58 }
59 void Factor(const LL &x, vector<LL> &v) {
60     LL n = x;
61     if(n==1) { puts("Factor 1"); return; }
62     prefactor(n,v);
63     if(n==1) return;
64     comfactor(n,v);
65     sort(v.begin(),v.end());
66 }
67 void AllFactor(const LL &n,vector<LL> &v) {
68     vector<LL> tmp;
69     Factor(n,tmp);
70     v.clear();
71     v.push_back(1);
72     int len;
73     LL now=1;
74     for(int i=0;i<tmp.size();++i) {
75         if(i==0 || tmp[i]!=tmp[i-1]) {
76             len = v.size();
77             now = 1;
78         }
79         for(int j=0;j<len;++j)
80             now*=tmp[i];
81     }
82 }

```

```

83     for(int j=0;j<len;++j)
84         v.push_back(v[j]*now);
85 }
86 }

```

7 String

7.1 AC 自動機

```

1 template<char L='a',char R='z'>
2 class ac_automaton{
3     struct joe{
4         int next[R-L+1],fail,efl,ed,cnt_dp,vis;
5         joe():ed(0),cnt_dp(0),vis(0){
6             for(int i=0;i<=R-L;++i)next[i]=0;
7         }
8     };
9     public:
10        std::vector<joe> S;
11        std::vector<int> q;
12        int qs,qe,vt;
13        ac_automaton():S(1),qs(0),qe(0),vt(0){
14            void clear(){
15                q.clear();
16                S.resize(1);
17                for(int i=0;i<=R-L;++i)S[0].next[i]=0;
18                S[0].cnt_dp=S[0].vis=qs=qe=vt=0;
19            }
20            void insert(const char *s){
21                int o=0;
22                for(int i=0,id=s[i];++i){
23                    id=s[i]-L;
24                    if(!S[o].next[id]){
25                        S.push_back(joe());
26                        S[o].next[id]=S.size()-1;
27                    }
28                    o=S[o].next[id];
29                }
30                ++S[o].ed;
31            }
32            void build_fail(){
33                S[0].fail=S[0].efl=-1;
34                q.clear();
35                q.push_back(0);
36                ++qe;
37                while(qs!=qe){
38                    int pa=q[qs++],id,t;
39                    for(int i=0;i<=R-L;++i){
40                        t=S[pa].next[i];
41                        if(!t)continue;
42                        id=S[pa].fail;
43                        while(~id&&S[id].next[i])id=S[id].fail;
44                        S[t].fail=~id?S[id].next[i]:0;
45                        S[t].efl=S[S[t].fail].ed+S[t].fail:S[t].fail;efl;
46                        q.push_back(t);
47                        ++qe;
48                    }
49                }
50            }
51 }

```

```

51 /*DP出每個前綴在字串s出現的次數並傳回所有
   字串被s匹配成功的次數O(N*M)*/
52 int match_0(const char *s){
53     int ans=0,id,p=0,i;
54     for(i=0;s[i];++i){
55         id=s[i]-L;
56         while(!S[p].next[id]&&p==S[p].fail;
57             if(!S[p].next[id])continue;
58             p=S[p].next[id];
59         ++S[p].cnt_dp; /*匹配成功則它所有後綴都
           可以被匹配(DP計算)*/
60     }
61     for(i=qe-1;i>=0;--i){
62         ans+=S[q[i]].cnt_dp*S[q[i]].ed;
63         if(~S[q[i]].fail)S[q[i]].fail.
           cnt_dp+=S[q[i]].cnt_dp;
64     }
65     return ans;
66 }
67 /*多串匹配走efl邊並傳回所有字串被s匹配成功
   的次數O(N*M^1.5)*/
68 int match_1(const char *s)const{
69     int ans=0,id,p=0,t;
70     for(int i=0;s[i];++i){
71         id=s[i]-L;
72         while(!S[p].next[id]&&p==S[p].fail;
73             if(!S[p].next[id])continue;
74             p=S[p].next[id];
75         if(S[p].ed)ans+=S[p].ed;
76         for(t=S[p].efl;~t;t=S[t].efl){
77             ans+=S[t].ed; /*因為都走efl邊所以保證
               匹配成功*/
78         }
79     }
80     return ans;
81 }
82 /*枚舉(s的子字串nA)的所有相異字串各恰一次
   並傳回次數O(N*M*(1/3))*/
83 int match_2(const char *s){
84     int ans=0,id,p=0,t;
85     ++vt;
86     /*把戳記vt+=1. 只要vt沒溢位. 所有S[p].
       vis==vt就會變成false
87     這種利用vt的方法可以O(1)歸零vis陣列*/
88     for(int i=0;s[i];++i){
89         id=s[i]-L;
90         while(!S[p].next[id]&&p==S[p].fail;
91             if(!S[p].next[id])continue;
92             p=S[p].next[id];
93         if(S[p].ed&&S[p].vis!=vt){
94             S[p].vis=vt;
95             ans+=S[p].ed;
96         }
97         for(t=S[p].efl;~t&&S[t].vis!=vt;t=S[t]
           .efl){
98             S[t].vis=vt;
99             ans+=S[t].ed; /*因為都走efl邊所以保證
               匹配成功*/
100     }
101 }
102 return ans;
103 }
104 /*把AC自動機變成真的自動機*/

```

```

105 void evolution(){
106     for(qs=1;qs!=qe;){
107         int p=q[qs++];
108         for(int i=0;i<=R-L;++){
109             if(S[p].next[i]==0)S[p].next[i]=S[p]
               .fail.next[i];
110         }
111     }
112 }

```

7.2 hash

```

1 #define MAXN 1000000
2 #define mod 1073676287
3 /*mod 必須要是質數*/
4 typedef long long T;
5 char s[MAXN+5];
6 T h[MAXN+5]; /*hash陣列*/
7 T h_base[MAXN+5]; /*h_base[n]=(prime^n)%mod*/
8 void hash_init(int len,T prime){
9     h_base[0]=1;
10    for(int i=1;i<=len;++){
11        h[i]=(h[i-1]*prime+s[i-1])%mod;
12        h_base[i]=(h_base[i-1]*prime)%mod;
13    }
14 }
15 T get_hash(int l,int r){ /*閉區間寫法. 設編號
   為0 ~ len-1*/
16     return (h[r+1]-(h[l]*h_base[r-l+1])%mod+
           mod)%mod;
17 }

```

7.3 KMP

```

1 /*產生fail function*/
2 void kmp_fail(char *s,int len,int *fail){
3     int id=-1;
4     fail[0]=-1;
5     for(int i=1;i<len;++){
6         while(~id&&s[id+1]!=s[i])id=fail[id];
7         if(s[id+1]==s[i])++id;
8         fail[i]=id;
9     }
10 }
11 /*以字串B匹配字串A. 傳回匹配成功的數量(用B的
   fail)*/
12 int kmp_match(char *A,int lenA,char *B,int
   lenB,int *fail){
13     int id=-1,ans=0;
14     for(int i=0;i<lenA;++){
15         while(~id&&B[id+1]!=A[i])id=fail[id];
16         if(B[id+1]==A[i])++id;
17         if(id==lenB-1){ /*匹配成功*/
18             ++ans, id=fail[id];
19         }
20     }
21     return ans;
22 }

```

7.4 manacher

```

1 //原字串: asdsasdsa
2 //先把字串變成這樣: @#a#s#d#s#a#s#d#s#a#
3 void manacher(char *s,int len,int *z){
4     int l=0,r=0;
5     for(int i=1;i<len;++){
6         z[i]=r>i?min(z[2*i-l],r-i):1;
7         while(s[i+z[i]]==s[i-z[i]])++z[i];
8         if(z[i]+i>r)r=z[i]+i,l=i;
9     } //ans = max(z)-1
10 }

```

7.5 minimal_string_rotation

```

1 int min_string_rotation(const string &s){
2     int n=s.size(),i=0,j=1,k=0;
3     while(i<n&&j<n&&k<n){
4         int t=s[(i+k)%n]-s[(j+k)%n];
5         ++k;
6         if(t){
7             if(t>0)i+=k;
8             else j+=k;
9             if(i==j)++j;
10            k=0;
11        }
12    }
13    return min(i,j); //最小循環表示法起始位置
14 }

```

7.6 reverseBWT

```

1 const int MAXN = 305, MAXC = 'Z';
2 int ranks[MAXN], tots[MAXC], first[MAXC];
3 void rankBWT(const string &bw){
4     memset(ranks,0,sizeof(int)*bw.size());
5     memset(tots,0,sizeof(tots));
6     for(size_t i=0;i<bw.size();++){
7         ranks[i] = tots[int(bw[i])]++;
8     }
9     void firstCol(){
10        memset(first,0,sizeof(first));
11        int totc = 0;
12        for(int c='A';c<='Z';++c){
13            if(!tots[c]) continue;
14            first[c] = totc;
15            totc += tots[c];
16        }
17    }
18 string reverseBwt(string bw,int begin){
19     rankBWT(bw, firstCol());
20     int i = begin; //原字串最後一個元素的位置
21     string res;
22     do{
23         char c = bw[i];
24         res = c + res;
25         i = first[int(c)] + ranks[i];
26     }while (i != begin);

```

```

27     return res;
28 }

```

7.7 suffix_array_lcp

```

1 #define radix_sort(x,y){\
2     for(i=0;i<A;++){c[i]=0;\
3     for(i=0;i<n;++){c[x[y[i]]]++; \
4     for(i=1;i<A;++){c[i]+=c[i-1]; \
5     for(i=n-1;~i;--i)sa[--c[x[y[i]]]]=y[i]; \
6 }
7 #define AC(r,a,b){\
8     r[a]=r[b]|(a+k>n)?r[a+k]:r[b+k]
9 void suffix_array(const char *s,int n,int *
   sa,int *rank,int *tmp,int *c){
10    int A='z'+1,i,k,id=0;
11    for(i=0;i<n;++){rank[tmp[i]]=i;s[i];
12    radix_sort(rank,tmp);
13    for(k=1;id<n-1;k<=1){
14        for(id=0,i=n-k;i<n;++){tmp[id++]=i;
15        for(i=0;i<n;++){
16            if(sa[i]>=k)tmp[id++]=sa[i]-k;
17        radix_sort(rank,tmp);
18        swap(rank,tmp);
19        for(rank[sa[0]]=id=0,i=1;i<n;++){
20            rank[sa[i]]=id+=AC(tmp,sa[i-1],sa[i]);
21        A=id+1;
22    }
23 }
24 //h:高度數組 sa:後綴數組 rank:排名
25 void suffix_array_lcp(const char *s,int len,
   int *h,int *sa,int *rank){
26     for(int i=0;i<len;++){rank[sa[i]]=i;
27     for(int i=0,k=0;i<len;++){
28         if(rank[i]==0)continue;
29         if(k--<0;
30         while(s[i+k]==s[sa[rank[i]-1]+k])++k;
31         h[rank[i]]=k;
32     }
33     h[0]=0; // h[k]=lcp(sa[k],sa[k-1]);
34 }

```

7.8 Z

```

1 void z_alg(char *s,int len,int *z){
2     int l=0,r=0;
3     z[0]=len;
4     for(int i=1;i<len;++){
5         z[i]=i>r?0:(i-l+z[i-l]<z[i-l]?z[i-l]:r-i
           +1);
6         while(i+z[i]<len&&s[i+z[i]]==s[z[i]])++z
           [i];
7         if(i+z[i]-1>r)r=i+z[i]-1,l=i;
8     }
9 }

```

8 Tarjan

8.1 dominator_tree

```

1 struct dominator_tree{
2     static const int MAXN=5005;
3     int n; // 1-base
4     vector<int> G[MAXN], rG[MAXN];
5     int pa[MAXN], dfn[MAXN], id[MAXN], dfnCnt;
6     int semi[MAXN], idom[MAXN], best[MAXN];
7     vector<int> tree[MAXN]; // tree here
8     void init(int _n){
9         n = _n;
10        for(int i=1; i<=n; ++i)
11            G[i].clear(), rG[i].clear();
12    }
13    void add_edge(int u, int v){
14        G[u].push_back(v);
15        rG[v].push_back(u);
16    }
17    void dfs(int u){
18        id[dfn[u]=++dfnCnt]=u;
19        for(auto v:G[u]) if(!dfn[v])
20            dfs(v), pa[dfn[v]]=dfn[u];
21    }
22    int find(int y, int x){
23        if(y <= x) return y;
24        int tmp = find(pa[y], x);
25        if(semi[best[y]] > semi[best[pa[y]]])
26            best[y] = best[pa[y]];
27        return pa[y] = tmp;
28    }
29    void tarjan(int root){
30        dfnCnt = 0;
31        for(int i=1; i<=n; ++i){
32            dfn[i] = idom[i] = 0;
33            tree[i].clear();
34            best[i] = semi[i] = i;
35        }
36        dfs(root);
37        for(int i=dfnCnt; i>1; --i){
38            int u = id[i];
39            for(auto v:rG[u]) if(v=dfn[v]){
40                find(v, i);
41                semi[i]=min(semi[i], semi[best[v]]);
42            }
43            tree[semi[i]].push_back(i);
44            for(auto v:tree[pa[i]]){
45                find(v, pa[i]);
46                idom[v] = semi[best[v]]==pa[i]
47                    ? pa[i] : best[v];
48            }
49            tree[pa[i]].clear();
50        }
51        for(int i=2; i<=dfnCnt; ++i){
52            if(idom[i] != semi[i])
53                idom[i] = idom[idom[i]];
54            tree[id[idom[i]]].push_back(id[i]);
55        }
56    }
57 } dom;

```

8.2 tnfsb017_2_sat

```

1 #include<bits/stdc++.h>
2 using namespace std;
3 #define MAXN 8001
4 #define MAXN2 MAXN*4
5 #define n(X) ((X)+2*N)
6 vector<int> v[MAXN2], rv[MAXN2], vis_t;
7 int N,M;
8 void addedge(int s,int e){
9     v[s].push_back(e);
10    rv[e].push_back(s);
11 }
12 int scc[MAXN2];
13 bool vis[MAXN2]={false};
14 void dfs(vector<int> *uv,int n,int k=-1){
15     vis[n]=true;
16     for(int i=0;i<uv[n].size();++i)
17         if(!vis[uv[n][i]])
18             dfs(uv,uv[n][i],k);
19     if(uv==v)vis_t.push_back(n);
20     scc[n]=k;
21 }
22 void solve(){
23     for(int i=1;i<=N;++i){
24         if(!vis[i])dfs(v,i);
25         if(!vis[n(i)])dfs(v,n(i));
26     }
27     memset(vis,0,sizeof(vis));
28     int c=0;
29     for(int i=vis_t.size()-1;i>=0;--i)
30         if(!vis[vis_t[i]])
31             dfs(rv,vis_t[i],c++);
32 }
33 int main(){
34     int a,b;
35     scanf("%d%d",&N,&M);
36     for(int i=1;i<=N;++i){
37         // (A or B)&(!A & !B) A^B
38         a=i*2-1;
39         b=i*2;
40         addedge(n(a),b);
41         addedge(n(b),a);
42         addedge(a,n(b));
43         addedge(b,n(a));
44     }
45     while(M--){
46         scanf("%d%d",&a,&b);
47         a = a>0?a*2-1:-a*2;
48         b = b>0?b*2-1:-b*2;
49         // A or B
50         addedge(n(a),b);
51         addedge(n(b),a);
52     }
53     solve();
54     bool check=true;
55     for(int i=1;i<=2*N;++i)
56         if(scc[i]==scc[n(i)])
57             check=false;
58     if(check){
59         printf("%d\n",N);
60         for(int i=1;i<=2*N;i+=2){
61             if(scc[i]>scc[i+2*N]) putchar('+');
62             else putchar('-');
63         }
64     }

```

```

64     puts("");
65 }else puts("0");
66 return 0;
67 }

```

8.3 橋連通分量

```

1 #define N 1005
2 struct edge{
3     int u,v;
4     bool is_bridge;
5     edge(int u=0,int v=0):u(u),v(v),is_bridge
6         (0){}
7 };
8 vector<edge> E;
9 vector<int> G[N]; // 1-base
10 int low[N],vis[N],Time;
11 int bcc_id[N],bridge_cnt,bcc_cnt; // 1-base
12 int st[N],top; // BCC用
13 void add_edge(int u,int v){
14     G[u].push_back(E.size());
15     E.emplace_back(u,v);
16     G[v].push_back(E.size());
17     E.emplace_back(v,u);
18 }
19 void dfs(int u,int re=-1){ // u當前點, re為u連
20     接前一個點的邊
21     int v;
22     low[u]=vis[u]=++Time;
23     st[top++]=u;
24     for(int e:G[u]){
25         v=E[e].v;
26         if(!vis[v]){
27             dfs(v,e^1); // e^1反向邊
28             low[u]=min(low[u],low[v]);
29             if(vis[u]<low[v]){
30                 E[e].is_bridge=E[e^1].is_bridge=1;
31                 ++bridge_cnt;
32             }else if(vis[v]<vis[u]&&e!=re)
33                 low[u]=min(low[u],vis[v]);
34         }
35     }
36     if(vis[u]==low[u]){ // 處理BCC
37         ++bcc_cnt; // 1-base
38         do bcc_id[v=st[--top]]=bcc_cnt; // 每個點
39         所在的BCC
40         while(v!=u);
41     }
42 }
43 void bcc_init(int n){
44     Time=bcc_cnt=bridge_cnt=top=0;
45     E.clear();
46     for(int i=1;i<=n;++i){
47         G[i].clear();
48         vis[i]=bcc_id[i]=0;
49     }

```

8.4 雙連通分量 & 割點

```

1 #define N 1005
2 vector<int> G[N]; // 1-base
3 vector<int> bcc[N]; // 存每塊雙連通分量的點
4 int low[N],vis[N],Time;
5 int bcc_id[N],bcc_cnt; // 1-base
6 bool is_cut[N]; // 是否為割點
7 int st[N],top;
8 void dfs(int u,int pa=-1){ // u當前點, pa父親
9     int t, child=0;
10    low[u]=vis[u]=++Time;
11    st[top++]=u;
12    for(int v:G[u]){
13        if(!vis[v]){
14            dfs(v,u), ++child;
15            low[u]=min(low[u],low[v]);
16            if(vis[u]<=low[v]){
17                is_cut[u]=1;
18                bcc[++bcc_cnt].clear();
19                do{
20                    bcc_id[t=st[--top]]=bcc_cnt;
21                    bcc[bcc_cnt].push_back(t);
22                }while(t!=v);
23                bcc_id[u]=bcc_cnt;
24                bcc[bcc_cnt].push_back(u);
25            }
26        }else if(vis[v]<vis[u]&&v!=pa) // 反向邊
27            low[u] = min(low[u],vis[v]);
28        // u是dfs樹的根要特判
29        if(pa==-1&&child<2)is_cut[u]=0;
30    }
31 }
32 void bcc_init(int n){
33     Time=bcc_cnt=top=0;
34     for(int i=1;i<=n;++i){
35         G[i].clear();
36         is_cut[i]=vis[i]=bcc_id[i]=0;
37     }
38 }

```

9 Tree_problem

9.1 HeavyLight

```

1 #include<vector>
2 #define MAXN 100005
3 int siz[MAXN],max_son[MAXN],pa[MAXN],dep[
4     MAXN];
5 int link_top[MAXN],link[MAXN],cnt;
6 vector<int> G[MAXN];
7 void find_max_son(int u){
8     siz[u]=1;
9     max_son[u]=-1;
10    for(auto v:G[u]){
11        if(v==pa[u])continue;
12        pa[v]=u;
13        dep[v]=dep[u]+1;
14        find_max_son(v);
15        if(max_son[u]==-1||siz[v]>siz[max_son[u]
16            ])max_son[u]=v;
17        siz[u]+=siz[v];
18    }

```

9.3 link_cut_tree

```

17 }
18 void build_link(int u,int top){
19     link[u]=++cnt;
20     link_top[u]=top;
21     if(max_son[u]==-1)return;
22     build_link(max_son[u],top);
23     for(auto v:G[u]){
24         if(v==max_son[u]||v==pa[u])continue;
25         build_link(v,v);
26     }
27 }
28 int find_lca(int a,int b){
29     //求LCA，可以在過程中對區間進行處理
30     int ta=link_top[a],tb=link_top[b];
31     while(ta!=tb){
32         if(dep[ta]<dep[tb]){
33             swap(ta,tb);
34             swap(a,b);
35         }
36         //這裡可以對a所在的鏈做區間處理
37         //區間為(link[ta],link[a])
38         ta=link_top[a=pa[ta]];
39     }
40     //最後a,b會在同一條鏈，若a!=b還要在進行一次區間處理
41     return dep[a]<dep[b]?a:b;
42 }

```

9.2 LCA

```

1 const int MAXN=100000; // 1-base
2 const int MLG=17; //log2(MAXN)+1;
3 int pa[MLG+2][MAXN+5];
4 int dep[MAXN+5];
5 vector<int> G[MAXN+5];
6 void dfs(int x,int p=0){//dfs(root);
7     pa[0][x]=p;
8     for(int i=0;i<=MLG;++i)
9         pa[i+1][x]=pa[i][pa[i][x]];
10    for(auto &i:G[x]){
11        if(i==p)continue;
12        dep[i]=dep[x]+1;
13        dfs(i,x);
14    }
15 }
16 inline int jump(int x,int d){
17     for(int i=0;i<=MLG;++i)
18         if((d>>i)&1) x=pa[i][x];
19     return x;
20 }
21 inline int find_lca(int a,int b){
22     if(dep[a]>dep[b])swap(a,b);
23     b=jump(b,dep[b]-dep[a]);
24     if(a==b)return a;
25     for(int i=MLG;i>=0;--i){
26         if(pa[i][a]!=pa[i][b]){
27             a=pa[i][a];
28             b=pa[i][b];
29         }
30     }
31     return pa[0][a];
32 }

```

```

1 struct splay_tree{
2     int ch[2],pa; //子節點跟父母
3     bool rev; //反轉的懶惰標記
4     splay_tree():pa(0),rev(0){ch[0]=ch[1]=0;}
5 };
6 vector<splay_tree> nd;
7 //有的時候用vector會TLE，要注意
8 //這邊以node[0]作為null節點
9 bool isroot(int x){//判斷是否為這棵splay
10    tree的根
11    return nd[nd[x].pa].ch[0]!=x&&nd[nd[x].pa].ch[1]!=x;
12 }
13 void down(int x){//懶惰標記下推
14     if(nd[x].rev){
15         if(nd[x].ch[0])nd[nd[x].ch[0]].rev^=1;
16         if(nd[x].ch[1])nd[nd[x].ch[1]].rev^=1;
17         swap(nd[x].ch[0],nd[x].ch[1]);
18         nd[x].rev=0;
19     }
20 }
21 void push_down(int x){//所有祖先懶惰標記下推
22     if(!isroot(x))push_down(nd[x].pa);
23     down(x);
24 }
25 void up(int x){//將子節點的資訊向上更新
26     void rotate(int x){//旋轉，會自行判斷轉的方向
27         int y=nd[x].pa,z=nd[y].pa,d=(nd[y].ch[1]==x);
28         nd[x].pa=z;
29         if(!isroot(y))nd[z].ch[nd[z].ch[1]==y]=x;
30         nd[y].ch[d]=nd[x].ch[d^1];
31         nd[nd[y].ch[d]].pa=y;
32         nd[y].pa=x,nd[x].ch[d^1]=y;
33         up(y),up(x);
34     }
35 }
36 void splay(int x){//將x伸展到splay tree的根
37     push_down(x);
38     while(!isroot(x)){
39         int y=nd[x].pa;
40         if(!isroot(y)){
41             int z=nd[y].pa;
42             if((nd[z].ch[0]==y)^(nd[y].ch[0]==x))
43                 rotate(y);
44             else rotate(x);
45         }
46         rotate(x);
47     }
48 }
49 int access(int x){
50     int last=0;
51     while(x){
52         splay(x);
53         nd[x].ch[1]=last;
54         up(x);
55         last=x;
56         x=nd[x].pa;
57     }
58     return last; //access後splay tree的根
59 }

```

```

57 void access(int x,bool is=0){//is=0就是一般的access
58     int last=0;
59     while(x){
60         splay(x);
61         if(is&&!nd[x].pa){
62             //printf("%d\n",max(nd[last].ma,nd[nd[x].ch[1]].ma));
63         }
64         nd[x].ch[1]=last;
65         up(x);
66         last=x;
67         x=nd[x].pa;
68     }
69 }
70 void query_edge(int u,int v){
71     access(u);
72     access(v,1);
73 }
74 void make_root(int x){
75     access(x),splay(x);
76     nd[x].rev^=1;
77 }
78 void make_root(int x){
79     nd[access(x)].rev^=1;
80     splay(x);
81 }
82 void cut(int x,int y){
83     make_root(x);
84     access(y);
85     splay(y);
86     nd[y].ch[0]=0;
87     nd[x].pa=0;
88 }
89 void cut_parents(int x){
90     access(x);
91     splay(x);
92     nd[nd[x].ch[0]].pa=0;
93     nd[x].ch[0]=0;
94 }
95 void link(int x,int y){
96     make_root(x);
97     nd[x].pa=y;
98 }
99 int find_root(int x){
100    x=access(x);
101    while(nd[x].ch[0])x=nd[x].ch[0];
102    splay(x);
103    return x;
104 }
105 int query(int u,int v){
106     //傳回uv路徑splay tree的根結點
107     //這種寫法無法求LCA
108     make_root(u);
109     return access(v);
110 }
111 int query_lca(int u,int v){
112     //假設求鏈上點權的總和，sum是子樹的權重和，data是節點的權重
113     access(u);
114     int lca=access(v);
115     splay(u);
116     if(u==lca){
117         //return nd[lca].data+nd[nd[lca].ch[1]].sum
118     }
119 }

```

```

118 }else{
119     //return nd[lca].data+nd[nd[lca].ch[1]].sum+nd[u].sum
120 }
121 }
122 struct EDGE{
123     int a,b,w;
124 }e[10005];
125 int n;
126 vector<pair<int,int>> G[10005];
127 //first表示子節點，second表示邊的編號
128 int pa[10005],edge_node[10005];
129 //pa是父母節點，暫存用的，edge_node是每個編
130 //被存在哪個點裡面的陣列
131 void bfs(int root){
132     //在建構的時候把每個點都設成一個splay tree
133     queue<int> q;
134     for(int i=1;i<=n;++i)pa[i]=0;
135     q.push(root);
136     while(q.size()){
137         int u=q.front();
138         q.pop();
139         for(auto P:G[u]){
140             int v=P.first;
141             if(v!=pa[u]){
142                 pa[v]=u;
143                 nd[v].pa=u;
144                 nd[v].data=e[P.second].w;
145                 edge_node[P.second]=v;
146                 up(v);
147                 q.push(v);
148             }
149         }
150     }
151 }
152 void change(int x,int b){
153     splay(x);
154     //nd[x].data=b;
155     up(x);
156 }

```

9.4 POJ_tree

```

1 #include<bits/stdc++.h>
2 using namespace std;
3 #define MAXN 10005
4 int n,k;
5 vector<pair<int,int>> g[MAXN];
6 int size[MAXN];
7 bool vis[MAXN];
8 inline void init(){
9     for(int i=0;i<=n;++i){
10         g[i].clear();
11         vis[i]=0;
12     }
13 }
14 void get_dis(vector<int> &dis,int u,int pa,
15             int d){
16     dis.push_back(d);
17     for(size_t i=0;i<g[u].size();++i){
18         int v=g[u][i].first,w=g[u][i].second;
19         if(v!=pa&&vis[v])get_dis(dis,v,u,d+w);
20     }
21 }

```



```

19 }
20 }
21 vector<int> dis;//這東西如果放在函數裡會TLE
22 int cal(int u,int d){
23     dis.clear();
24     get_dis(dis,u,-1,d);
25     sort(dis.begin(),dis.end());
26     int l=0,r=dis.size()-1,res=0;
27     while(l<r){
28         while(l<r&&dis[l]+dis[r]>k)--r;
29         res+=r-(l++);
30     }
31     return res;
32 }
33 pair<int,int> tree_centroid(int u,int pa,
    const int sz){
34     size[u]=1;//找樹重心，second是重心
35     pair<int,int> res(INT_MAX,-1);
36     int ma=0;
37     for(size_t i=0;i<g[u].size();++i){
38         int v=g[u][i].first;
39         if(v==pa||vis[v])continue;
40         res=min(res,tree_centroid(v,u,sz));
41         size[u]+=size[v];
42         ma=max(ma,size[v]);
43     }
44     ma=max(ma,sz-size[u]);
45     return min(res,make_pair(ma,u));
46 }
47 int tree_DC(int u,int sz){
48     int center=tree_centroid(u,-1,sz).second;
49     int ans=cal(center,0);
50     vis[center]=1;
51     for(size_t i=0;i<g[center].size();++i){
52         int v=g[center][i].first,w=g[center][i].second;
53         if(vis[v])continue;
54         ans+=cal(v,w);
55         ans+=tree_DC(v,size[v]);
56     }
57     return ans;
58 }
59 int main(){
60     while(scanf("%d%d",&n,&k),n||k){
61         init();
62         for(int i=1;i<n;++i){
63             int u,v,w;
64             scanf("%d%d%d",&u,&v,&w);
65             g[u].push_back(make_pair(v,w));
66             g[v].push_back(make_pair(u,w));
67         }
68         printf("%d\n",tree_DC(1,n));
69     }
70     return 0;
71 }

```

10 default

10.1 debug

```
1 //volatile
```

```

2 #ifndef DEBUG
3 #define dbg(...) {\
4     fprintf(stderr,"%s - %d : (%s) = ",
5         __PRETTY_FUNCTION__,__LINE__,#
6         __VA_ARGS__);\
7     _DO(__VA_ARGS__); \
8 }
9 template<typename I> void _DO(I&&x){cerr<<x
10     <<endl;}
11 template<typename I,typename...T> void _DO(I
12     &&x,T&&...tail){cerr<<x<<" ";_DO(tail
13     ...);}
14 #else
15 #define dbg(...)
16 #endif

```

10.2 ext

```

1 #include<bits/extc++.h>
2 #include<ext/pd_ds/assoc_container.hpp>
3 #include<ext/pd_ds/tree_policy.hpp>
4 using namespace __gnu_cxx;
5 using namespace __gnu_pbds;
6 template<typename T>
7 using pbds_set = tree<T,null_type,less<T>,
8     rb_tree_tag,
9     tree_order_statistics_node_update>;
10 template<typename T,typename U>
11 using pbds_map = tree<T,U,less<T>,
12     rb_tree_tag,
13     tree_order_statistics_node_update>;
14 using heap=__gnu_pbds::priority_queue<int>;
15 //s.find_by_order(1);//0 base
16 //s.order_of_key(1);

```

10.3 IncStack

```

1 //Magic
2 #pragma GCC optimize "Ofast"
3 //stack resize,change esp to rsp if 64-bit
4 system
5 asm("mov %0,%esp\n" ::"g"(mem+10000000));
6 -WL,--stack,214748364 -trigraphs
7 #pragma comment(linker, "/STACK
8 :1024000000,1024000000")
9 //linux stack resize
10 #include<sys/resource.h>
11 void increase_stack(){
12     const rlim_t ks=64*1024*1024;
13     struct rlimit rl;
14     int res=getrlimit(RLIMIT_STACK,&rl);
15     if(!res&&rl.rlim_cur<ks){
16         rl.rlim_cur=ks;
17         res=setrlimit(RLIMIT_STACK,&rl);
18     }
19 }

```

10.4 input

```

1 inline int read(){
2     int x=0; bool f=0; char c=getchar();
3     while(ch<'0' || '9'<ch)f|=ch=='-',ch=getchar
4     ();
5     while('0'<=ch&&ch<='9')x=x*10-'0'+ch,ch=
6     getchar();
7     return f?-x:x;
8 }
9 // #!/bin/bash
10 // g++ -std=c++11 -O2 -Wall -Wextra -Wno-
11 // unused-result -DDEBUG $1 && ./a.out
12 // -fsanitize=address -fsanitize=undefined
13 // -fsanitize=return

```

11 language

11.1 CNF

```

1 #define MAXN 55
2 struct CNF{
3     int s,x,y;//s->xy | s->x, if y==1
4     int cost;
5     CNF(){}
6     CNF(int s,int x,int y,int c):s(s),x(x),y(y)
7     ,cost(c){}
8 };
9 map<char,int> rule;//每個字元對應到的規則，
10 小寫字母為終端字符
11 vector<CNF> cnf;
12 void init(){
13     state=0;
14     rule.clear();
15     cnf.clear();
16 }
17 void add_to_cnf(char s,const string &p,int
18     cost){
19     //加入一個s -> <p>的文法，代價為cost
20     if(rule.find(s)==rule.end())rule[s]=state
21     ++;
22     for(auto c:p)if(rule.find(c)==rule.end())
23     rule[c]=state++;
24     if(p.size()==1){
25         cnf.push_back(CNF(rule[s],rule[p[0]],-1,
26             cost));
27     }else{
28         int left=rule[s];
29         int sz=p.size();
30         for(int i=0;i<sz-2;++i){
31             cnf.push_back(CNF(left,rule[p[i]],
32                 state,0));
33             left=state++;
34         }
35         cnf.push_back(CNF(left,rule[p[sz-2]],
36             rule[p[sz-1]],cost));
37     }
38 }
39 }
40 }

```

```

32 vector<long long> dp[MAXN][MAXN];
33 vector<bool> neg_INF[MAXN][MAXN];//如果花費
34 是負的可能會有無限小的情形
35 void relax(int l,int r,const CNF &c,long
36     long cost,bool neg_c=0){
37     if(!neg_INF[l][r][c.s]&&(neg_INF[l][r][c.x
38     ][cost<dp[l][r][c.s]])){
39         if(neg_c|neg_INF[l][r][c.x]){
40             dp[l][r][c.s]=0;
41             neg_INF[l][r][c.s]=true;
42         }else dp[l][r][c.s]=cost;
43     }
44 }
45 void bellman(int l,int r,int n){
46     for(int k=1;k<=state;++k)
47     for(auto c:cnf)
48     if(c.y==1)relax(l,r,c,dp[l][r][c.x]+c
49     .cost,k==n);
50 }
51 void cyk(const vector<int> &tok){
52     for(int i=0;i<(int)tok.size();++i){
53         for(int j=0;j<(int)tok.size();++j){
54             dp[i][j]=vector<long long>(state+1,
55                 INT_MAX);
56             neg_INF[i][j]=vector<bool>(state+1,
57                 false);
58         }
59         dp[i][i][tok[i]]=0;
60         bellman(i,i,tok.size());
61     }
62     for(int r=1;r<(int)tok.size();++r){
63         for(int l=r-1;l>=0;--l){
64             for(int k=1;k<r;++k)
65             for(auto c:cnf)
66             if(~c.y)relax(l,r,c,dp[l][k][c.x]+
67                 dp[k+1][r][c.y]+c.cost);
68             bellman(l,r,tok.size());
69         }
70     }
71 }

```

12 other

12.1 WhatDay

```

1 int whatday(int y,int m,int d){
2     if(m<2)m+=12,--y;
3     if(y<1752||y==1752&&m<9||y==1752&&m==9&&d
4     <3)
5         return (d+2*m+3*(m+1)/5+y+y/4+y/100+y/400)
6         %7;
7 }

```

12.2 上下最大正方形

```

1 void solve(int n,int a[],int b[]){// 1-base
2   int ans=0;
3   deque<int>da,db;
4   for(int l=1,r=1;r<n;++r){
5     while(da.size()&&a[da.back()]>=a[r]){
6       da.pop_back();
7     }
8     da.push_back(r);
9     while(db.size()&&b[db.back()]>=b[r]){
10      db.pop_back();
11    }
12    db.push_back(r);
13    for(int d=a[da.front()]+b[db.front()];r-
14      1+l>d;++l){
15      if(da.front()==l)da.pop_front();
16      if(db.front()==l)db.pop_front();
17      if(da.size()&&db.size()){
18        d=a[da.front()]+b[db.front()];
19      }
20    }
21    ans=max(ans,r-l+1);
22  }
23  printf("%d\n",ans);
}

```

12.3 最大矩形

```

1 LL max_rectangle(vector<int> s){
2   stack<pair<int,int> > st;
3   st.push(make_pair(-1,0));
4   s.push_back(0);
5   LL ans=0;
6   for(size_t i=0;i<s.size();++i){
7     int h=s[i];
8     pair<int,int> now=make_pair(h,i);
9     while(h<st.top().first){
10      now=st.top();
11      st.pop();
12      ans=max(ans,(LL)(i-now.second)*now.first);
13    }
14    if(h>st.top().first){
15      st.push(make_pair(h,now.second));
16    }
17  }
18  return ans;
19 }

```

13 zformula

13.1 formula

13.1.1 Pick 公式

給定頂點坐標均是整點的簡單多邊形，面積 = 內部格點數 + 邊上格點數/2-1

13.1.2 圖論

- 對於平面圖 $\cdot F = E - V + C + 1 \cdot C$ 是連通分量數
- 對於平面圖 $\cdot E \leq 3V - 6$
- 對於連通圖 G ，最大獨立點集的大小設為 $I(G)$ ，最大匹配大小設為 $M(G)$ ，最小點覆蓋設為 $C_v(G)$ ，最小邊覆蓋設為 $C_e(G)$ ，對於任意連通圖：

- $I(G) + C_v(G) = |V|$
- $M(G) + C_e(G) = |V|$

- 對於連通二分圖：

- $I(G) = C_v(G)$
- $M(G) = C_e(G)$

- 最大權閉合圖：

- $C(u, v) = \infty, (u, v) \in E$
- $C(S, v) = W_v, W_v > 0$
- $C(v, T) = -W_v, W_v < 0$
- $ans = \sum_{W_v > 0} W_v - flow(S, T)$

- 最大密度子圖：

- 求 $max\left(\frac{W_e + W_v}{|V|}\right), e \in E', v \in V'$
- $U = \sum_{v \in V} 2W_v + \sum_{e \in E} W_e$
- $C(u, v) = W_{(u,v)}, (u, v) \in E$ ，雙向邊
- $C(S, v) = U, v \in V$
- $D_u = \sum_{(u,v) \in E} W_{(u,v)}$
- $C(v, T) = U + 2g - D_v - 2W_v, v \in V$
- 二分搜 g ：
 $l = 0, r = U, eps = 1/n^2$
 if $((U \times |V| - flow(S, T))/2 > 0)$ $l = mid$
 else $r = mid$
- $ans = min_cut(S, T)$
- $|E| = 0$ 要特殊判斷

- 弦圖：

- 點數大於 3 的環都要有一條弦
- 完美消除序列從後往前依次給每個點染色，給每個點染上可以染的最小顏色
- 最大團大小 = 色數
- 最大獨立集：完美消除序列從前往後能選就選
- 最小團覆蓋：最大獨立集的點和他延伸的邊構成
- 區間圖是弦圖
- 區間圖的完美消除序列：將區間按造又端點由小到大排序
- 區間圖染色：用線段樹做

13.1.3 dinic 特殊圖複雜度

- 單位流： $O\left(\min\left(V^{3/2}, E^{1/2}\right)E\right)$
- 二分圖： $O\left(V^{1/2}E\right)$

13.1.4 0-1 分數規劃

$x_i \in \{0, 1\} \cdot x_i$ 可能會有其他限制，求 $max\left(\frac{\sum B_i x_i}{\sum C_i x_i}\right)$

- $D(i, g) = B_i - g \times C_i$
- $f(g) = \sum D(i, g) x_i$
- $f(g) = 0$ 時 g 為最佳解， $f(g) < 0$ 沒有意義
- 因為 $f(g)$ 單調可以二分搜 g
- 或用 Dinkelbach 通常比較快

```

1 binary_search(){
2   while(r-l>eps){
3     g=(l+r)/2;
4     for(i:所有元素)D[i]=B[i]-g*C[i];//D(i,g)
5     找出一組合法x[i]使f(g)最大;
6     if(f(g)>0) l=g;
7     else r=g;
8   }
9   Ans = r;
10 }
11 Dinkelbach(){
12   g=任意狀態(通常設為0);
13   do{
14     Ans=g;
15     for(i:所有元素)D[i]=B[i]-g*C[i];//D(i,g)
16     找出一組合法x[i]使f(g)最大;
17     p=0,q=0;
18     for(i:所有元素)
19       if(x[i])p+=B[i],q+=C[i];
20     g=p/q;//更新解，注意q=0的情況
21   }while(abs(Ans-g)>EPS);
22   return Ans;
23 }

```

13.1.5 學長公式

- $\sum_{d|n} \phi(n) = n$
- $g(n) = \sum_{d|n} f(d) \Rightarrow f(n) = \sum_{d|n} \mu(d) \times g(n/d)$
- Harmonic series $H_n = \ln(n) + \gamma + 1/(2n) - 1/(12n^2) + 1/(120n^4)$
- $\gamma = 0.57721566490153286060651209008240243104215$
- 格雷碼： $n \oplus (n >> 1)$
- $SG(A + B) = SG(A) \oplus SG(B)$
- 選轉矩陣 $M(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

13.1.6 基本數論

- $\sum_{d|n} \mu(n) = [n == 1]$
- $g(m) = \sum_{d|m} f(d) \Leftrightarrow f(m) = \sum_{d|m} \mu(d) \times g(m/d)$
- $\sum_{i=1}^n \sum_{j=1}^m \text{互質數量} = \sum \mu(d) \left\lfloor \frac{n}{d} \right\rfloor \left\lfloor \frac{m}{d} \right\rfloor$
- $\sum_{i=1}^n \sum_{j=1}^n lcm(i, j) = n \sum_{d|n} d \times \phi(d)$

13.1.7 排組公式

- k 卡特蘭 $\frac{C_n^{kn}}{n(k-1)+1} \cdot C_m^n = \frac{n!}{m!(n-m)!}$
- $H(n, m) \cong x_1 + x_2 + \dots + x_n = k, num = C_k^{n+k-1}$
- Stirling number of 2^{nd} , n 人分 k 組方法數目
 - $S(0, 0) = S(n, n) = 1$
 - $S(n, 0) = 0$
 - $S(n, k) = kS(n-1, k) + S(n-1, k-1)$
- Bell number, n 人分任意多組方法數目
 - $B_0 = 1$
 - $B_n = \sum_{i=0}^n S(n, i)$
 - $B_{n+1} = \sum_{k=0}^n C_k^n B_k$
 - $B_{p+n} \equiv B_n + B_{n+1} \pmod{p}$, p is prime
 - $B_{p^m+n} \equiv mB_n + B_{n+1} \pmod{p}$, p is prime
 - From $B_0 : 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975$
- Derangement, 錯排，沒有人在自己位置上
 - $D_n = n!(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \dots + (-1)^n \frac{1}{n!})$
 - $D_n = (n-1)(D_{n-1} + D_{n-2}), D_0 = 1, D_1 = 0$
 - From $D_0 : 1, 0, 1, 2, 9, 44, 265, 1854, 14833, 133496$

- Binomial Equality

- $\sum_k \binom{r}{m+k} \binom{s}{n-k} = \binom{r+s}{m+n}$
- $\sum_k \binom{r}{m+k} \binom{s}{n+k} = \binom{l+s}{l-m+n}$
- $\sum_k \binom{r}{m+k} \binom{s+k}{n} (-1)^k = (-1)^{l+m} \binom{s-m}{n-l}$
- $\sum_{k \leq l} \binom{l-k}{m} \binom{s}{n-k} (-1)^k = (-1)^{l+m} \binom{s-m-1}{l-n-m}$
- $\sum_{0 \leq k \leq l} \binom{l-k}{m} \binom{q+k}{n} = \binom{l+q+1}{m+n+1}$
- $\binom{r}{k} = (-1)^k \binom{r-k}{k}$
- $\binom{r}{m} \binom{m}{k} = \binom{r}{k} \binom{r-k}{m-k}$
- $\sum_{k \leq n} \binom{r+k}{k} = \binom{r+n+1}{n}$
- $\sum_{0 \leq k \leq n} \binom{k}{m} = \binom{n+1}{m+1}$
- $\sum_{k \leq m} \binom{m+r}{k} x^k y^k = \sum_{k \leq m} \binom{-r}{k} (-x)^k (x+y)^{m-k} =$

13.1.8 冪次，冪次和

- $a^{b\%P} = a^{b\% \varphi(P) + \varphi(P)}, b \geq \varphi(P)$
- $1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$
- $1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}$
- $1^5 + 2^5 + 3^5 + \dots + n^5 = \frac{n^6}{6} + \frac{n^5}{2} + \frac{5n^4}{12} - \frac{n^2}{12}$
- $0^k + 1^k + \dots + n^k = P(k), P(k) = \frac{(n+1)^{k+1} - \sum_{i=0}^k C_i^{k+1} P(i)}{k+1}, P(0) = n+1$
- $\sum_{k=0}^{m-1} k^n = \frac{1}{n+1} \sum_{k=0}^m C_k^{n+1} B_k m^{n+1-k}$
- $\sum_{j=0}^m C_j^{m+1} B_j = 0, B_0 = 1$
- 除了 $B_1 = -1/2$ ，剩下的奇數項都是 0
- $B_2 = 1/6, B_4 = -1/30, B_6 = 1/42, B_8 = -1/30, B_{10} = 5/66, B_{12} = -691/2730, B_{14} = 7/6, B_{16} = -3617/510, B_{18} = 43867/798, B_{20} = -174611/330,$

13.1.9 Burnside's lemma

- $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$
- $X^g = t^{c(g)}$
- G 表示有幾種轉法, X^g 表示在那種轉法下, 有幾種是會保持對稱的, t 是顏色數, $c(g)$ 是循環節不動的面數。
- 正立方體塗三顏色, 轉 0 有 3^6 個元素不變, 轉 90 有 6 種, 每種有 3^3 不變, 180 有 3×3^4 , 120(角) 有 8×3^2 , 180(邊) 有 6×3^3 , 全部 $\frac{1}{54}(3^6 + 6 \times 3^3 + 3 \times 3^4 + 8 \times 3^2 + 6 \times 3^3) = \frac{24}{54}$

13.1.10 Count on a tree

- Rooted tree: $s_{n+1} = \frac{1}{n} \sum_{i=1}^n (i \times a_i \times \sum_{j=1}^{\lfloor n/i \rfloor} a_{n+1-i \times j})$
- Unrooted tree:
 - Odd: $a_n - \sum_{i=1}^{n/2} a_i a_{n-i}$
 - Even: $Odd + \frac{1}{2} a_{n/2} (a_{n/2} + 1)$
- Spanning Tree
 - 完全圖 $n^n - 2$
 - 一般圖 (Kirchhoff's theorem) $M[i][i] = \text{degree}(V_i), M[i][j] = -1, \text{if have } E(i, j), 0 \text{ if no edge. delete any one row and col in } A, \text{ans} = \det(A)$

13.2 java

13.2.1 文件操作

```
1 import java.io.*;
2 import java.util.*;
3 import java.math.*;
4 import java.text.*;
5
6 public class Main{
7
8     public static void main(String args[]){
9         throws FileNotFoundException,
10         IOException
11         Scanner sc = new Scanner(new FileReader(
12             "a.in"));
13         PrintWriter pw = new PrintWriter(new
14             FileWriter("a.out"));
15         int n,m;
16         n=sc.nextInt();//读入下一个INT
17         m=sc.nextInt();
18
19         for(ci=1; ci<=c; ++ci){
20             pw.println("Case #"+ci+": easy for
21                 output");
22         }
23
24         pw.close();//关闭流并释放, 这个很重要,
25             否则是没有输出的
26         sc.close();//关闭流并释放
```

```
21 | }
22 | }
```

13.2.2 优先队列

```
1 PriorityQueue queue = new PriorityQueue( 1,
2     new Comparator(){
3     public int compare( Point a, Point b ){
4         if( a.x < b.x || a.x == b.x && a.y < b.y )
5             return -1;
6         else if( a.x == b.x && a.y == b.y )
7             return 0;
8         else return 1;
9     });
```

13.2.3 Map

```
1 Map map = new HashMap();
2 map.put("sa","dd");
3 String str = map.get("sa").toString();
4
5 for(Object obj : map.keySet()){
6     Object value = map.get(obj );
7 }
```

13.2.4 sort

```
1 static class cmp implements Comparator{
2     public int compare(Object o1,Object o2){
3         BigInteger b1=(BigInteger)o1;
4         BigInteger b2=(BigInteger)o2;
5         return b1.compareTo(b2);
6     }
7 }
8 public static void main(String[] args)
9     throws IOException{
10     Scanner cin = new Scanner(System.in);
11     int n;
12     n=cin.nextInt();
13     BigInteger[] seg = new BigInteger[n];
14     for (int i=0;i<n;i++)
15         seg[i]=cin.nextBigInteger();
16     Arrays.sort(seg,new cmp());
```

14

14.1 ganadoQuote

```
1 ¡Allí está!
2 ¡Un forastero!
3 ¡Agarrenlo!
4 ¡Os voy a romper a pedazos!
5 ¡Cógelo!
6 ¡Te voy a hacer picadillo!
7 ¡Te voy a matar!
8 ¡Míralo, está herido!
9 ¡Sos cerdo!
10 ¿Dónde estás?
11 ¡Detrás de tí, imbécil!
12 ¡No dejes que se escape!
13 ¡Basta, hijo de puta!
14 Lord Saddler...
15
16 ¡Mátalo!
17 ¡Allí está!
18 Morir es vivir.
19 ¡Síííí, ¡Quiero matar!
20 Muere, muere, muere....
21 Cerebros,cerebros,cerebros...
22 Cógedlo, cógedlo, cógedlo...
23 Lord Saddler...
24 Dieciséis.
25
26 ¡Va por él!
27 ¡Muérete!
28 ¡Cógelo!
29 ¡Te voy a matar!
30 ¡Bloqueale el paso!
31 ¡Te cogí!
32 ¡No dejes que se escape!
33
34 ¿Qué carajo estás haciendo aquí? ¡Lárgate,
35     cabrón!
36 Hay un rumor de que hay un extranjero entre
37     nosotros.
38 Nuestro jefe se encargará de la rata.
39 Su "Las Plagas" es mucho mejor que la
40     nuestra.
41 Tienes razón, es un hombre.
42 Usa los músculos.
43 Se vuelve loco!
44 ¡Hey, acá!
45 ¡Por aquí!
46 ¡El Gigante!
47 ¡Del Lago!
48 ¡Cógelo!
49 ¡Cógenlo!
50 ¡Allí!
51 ¡Rápido!
52 ¡Empieza a rezar!
53 ¡Mátenlos!
54 ¡Te voy a romper en pedazos!
55 ¡La campana!
56 Ya es hora de rezar.
57 Tenemos que irnos.
58 ¡Maldita sea, mierda!
59 ¡Ya es hora de aplastar!
60 ¡Mierda!
61 ¡Puedes correr, pero no te puedes esconder!
62 ¡Sos cerdo!
63 ¡Está en la trampa!
64 ¡Ah, que madre!
65 ¡Vámonos!
```

```
63 ¡Ándale!
64 ¡Cabrón!
65 ¡Coño!
66 ¡Agárrenlo!
67 Cógerlo, Cógerlo...
68 ¡Allí está, máta!
69 ¡No dejas que se escape de la isla vivo!
70 ¡Hasta luego!
71 ¡Rápido, es un intruso!
```

14.2

```
1 /*****
2 L'Internationale,
3     Sera le genre humain.
4
5
6
7
8
9
10
11
12
13
14
15 \./
16 *****/
17 Вставай, проклятем заклеимённый,
18 Весь мир голодных и рабов!
19 Кипит наш разум возмущённый
20 И в смертный бой вести готов.
21 Весь мир насилия мы разрушим
22 До основания, а затем
23 Мы наш, мы новый мир построим, —
24 Кто был ничем, тот станет всем.
25
26 Chorus
27 Это есть наш последний
28 И решительный бой;
29 С Интернационалом
30 Воспрянет род людской!
31
32 Никто не даст нам избавленья:
33 Ни бог, ни царь и не герой!
34 Добьёмся мы освобожденья
35 Своею собственной рукой.
36 Чтоб свергнуть гнёт рукой умелой,
37 Отвоевать своё добро, —
38 Вдуйте горн и куйте смело,
39 Пока железо горячо!
40
41 Chorus
42
43 Довольно кровь сосать, вампиры,
44 Тьмой, налогом, нищетой!
45 У вас — вся власть, все блага мира,
46 А наше право — звук пустой !
47 Мы жизнь построим по-иному —
48 И вот наш лозунг боевой:
49 Вся власть народу трудовому!
50 А дармоедов всех долой!
51
```

```

88 //
89 //
90 //
91 //
92 //
93 //
94 //
95 //
96 //
97 //
98 //
99 //
100 //
101 //

```

神獸保佑 永無BUG

68	//	##	#####
69	//	##	##
70	//	##	##
71	//	##	##
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85	//		
86	//	元首保佑 永無BUG	

ACM ICPC TEAM REFERENCE - ANGRY CROW TAKES FLIGHT!

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