Integral.

[Vector in field]

$$-\int_C \vec{F}\, d\vec{r} = \int_C P\,dx + Q\,dy + R\,dz$$

$\vec{F} = \langle P(x,y,z), Q(x,y,z), R(x,y,z) \rangle$, $\vec{r} = \langle x, y, z \rangle$

curve in xOy: $f(x,y) = 0$,

$$= \int_a^b \vec{F}[x(t), y(t), z(t)] \cdot \langle x'(t), y'(t), z'(t) \rangle\, dt.$$

$a \le t \le b$. $x = x(t)$, $y = y(t)$, $z = z(t)$

Special: $\int_{C = y(x)/x(y)} P(x,y)\,dx$

$$= \int_a^b P[x, y(x)]\,dx = \int_c^d P[x(y), y]\, x'(y)\,dy.$$

$a \le x \le b$ $\qquad$ $c \le y \le d$

~~$= \int_a^b P[x, y(x)]\, x$~~

[in a function]

$$\int_C f(x,y)\,ds = \int_a^b f[x(t), y(t)] \cdot \sqrt{x'(t)^2 + y'(t)^2}\, dt$$

ds: $dxdy$

curve in xoy: $g(x(t), y(t)) = 0$, $a \le t \le b$

Substitution: $\iint_R f(x,y)\,dxdy = \iint_{R'} f \cdot |J(u,v)|\,du\,dv$,

$J(u,v) = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$, $x = x(u,v)$, $y = y(u,v)$

R' is the new section of uov of R in xoy

Line Integral $\Rightarrow$ Definite Integral

$$\iint_R f(x,y)\,dA = \int_a^b \int_{y_1(x)}^{y_2(x)} f(x,y)\,dy\,dx.$$

$dA = dxdy$

region in xoy: $y_1(x) \le y_{region} \le y_2(x)$, $a \le x_{region} \le b$

can divide into pieces to caculate

$$= \int_c^d \int_{x_1(y)}^{x_2(y)} f(x,y)\,dx\,dy.$$

(often extract constant to get)

$f(x,y) = 1$, $\iint_R dA$ = Area of R.

Green Theorem: $\oint_C \vec{F}\,d\vec{r} = \iint_R \mathrm{Curl}(\vec{F})\,dxdy = \iint_R (N_x - M_y)\,dxdy$

$\vec{F} = \langle M(x,y), N(x,y) \rangle$, R → region of C. in xoy.

Line Integral $\rightleftharpoons$ Double Integral

(closed curve) in plane. $\qquad$ (two variables)

$$-\iint_S \vec{F}\,d\vec{A} = \iint_S P\,dydz + Q\,dzdx + R\,dxdy$$

$\vec{F} = \langle P, Q, R \rangle$, $= \iint_S \vec{F} \hat{n}\, dS$, $\hat{n} = \langle \cos\alpha, \cos\beta, \cos\gamma \rangle$

surface $z = f(x,y)$

$$= \iint_{R_{xy}} \vec{F}[x, y, f(x,y)] \cdot \langle -f_x, -f_y, 1 \rangle\, dxdy$$

region of $f(x,y) = 0$

$$= \iint_R \vec{F}(\vec{r}) \left| \vec{r}_u \times \vec{r}_v \right| dA$$

$\vec{r} = \langle x(u,v), y(u,v), z(u,v) \rangle$

Surface Integral $\Rightarrow$ ~~Double~~ Double Integral

Special: $\iint_S f(x,y,z)\,dS$ → surface $z = z(x,y)$, $x, y \in R_{xy}$.

$$= \iint_{R_{xy}} f(x, y, z(x,y)) \sqrt{1 + z_x^2 + z_y^2}\, dxdy$$

$$\iiint_R f(x,y,z)\,dv = \int_a^b \int_{y_1(x)}^{y_2(x)} \int_{z_1(x,y)}^{z_2(x,y)} f\, dz\,dy\,dx.$$

$dv = dxdydz$

region of a space

$z_1(x,y) \le z_{region} \le z_2(x,y)$

$y_1(x) \le y_{region} \le y_2(x)$

$a \le x_{region} \le b$

$f(x,y,z) = 1$, $\iiint_R dv$ = volume of R.

Divergence Theorem: $\oiint_S \vec{F}\,d\vec{A} = \iiint_R \mathrm{div}(\vec{F})\,dv = \iiint_R P_x + Q_y + R_z = \iiint_R \nabla \cdot \vec{F}\,dv$

Gauss Formula $\Leftrightarrow$ Green theorem in 3D

closed Surface boundary of region R

Surface Integral $\rightleftharpoons$ Triple Integral

(closed surface)

Stoke's Theorem: $\oint_C \vec{F}\,d\vec{r} = \iint_S \mathrm{curl}(\vec{F})\,d\vec{A} = \iint_S \nabla \times \vec{F}\, \hat{n}\, dS = \iint_S \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right) dydz + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right) dzdx + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dxdy$

close curve is boundary of surface S

if $C \in xoy$ → Green theorem.

$$= \iint_S \begin{vmatrix} dydz & dzdx & dxdy \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

Line Integral $\rightleftharpoons$ Surface Integral

(closed line) in space $\qquad$ (three variables)