1º miniteste 2016

1. Calcule, se existir, o valor de $\lim\limits_{x\to+\infty} \frac{\ln(2^x)-x^2}{x^3}$

$$\lim_{x\to+\infty} \frac{\ln(2^x)-x^2}{x^3} = \lim_{x\to+\infty} \frac{\ln(2^x)}{x^3} - \lim_{x\to+\infty} \frac{x^2}{x^3} = \lim_{x\to+\infty} \frac{\frac{2^x ln2}{2^x}}{3x^2} - \lim_{x\to+\infty} \frac{1}{x} =$$

$$\lim_{x\to+\infty} \frac{ln2}{3x^2} - \lim_{x\to+\infty} \frac{1}{x} = 0 - 0 = 0$$

(a) ln2 (b) não existe (c) 0 (d) 1

2. Calcule, se existir, o valor de $\lim\limits_{x\to 0^+} (senx)^{\frac{2}{ln(x)}}$

$$\lim_{x\to 0^+} (senx)^{\frac{2}{\ln(x)}} = e^{ln\left(\lim\limits_{x\to 0^+} (senx)^{\frac{2}{\ln(x)}}\right)} = e^{\lim\limits_{x\to 0^+}\left(\ln(senx)^{\frac{2}{\ln(x)}}\right)} =$$

$$e^{\lim\limits_{x\to 0^+}\left(\frac{2}{\ln(x)}\ln(\text{senx})\right)} = e^{\lim\limits_{x\to 0^+}\left(\frac{\frac{2cosx}{senx}}{\frac{1}{x}}\right)} = e^{\lim\limits_{x\to 0^+}\left(\frac{2xcosx}{senx}\right)} =$$

$$e^{\lim\limits_{x\to 0^+}\left(\frac{2cosx-2xsenx}{cosx}\right)} = e^2$$

(a) 0 (b) 1 (c) e^2 (d) não existe

- **3.** $\boldsymbol{\frac{d}{dx}\left(x\left(ln\left(\sqrt{x}\right)+e^{2x}\right)\right)}$

$$\frac{d}{dx}\left(x(ln(\sqrt{x})+e^{2x})\right)=(ln(\sqrt{x})+e^{2x})+\mathrm{x}\left(\frac{1}{2x}+2e^{2x}\right)=$$

$$ln(\sqrt{x})+e^{2x}(1+2x)+\frac{1}{2}$$

resposta (a)

- 4. $\frac{d}{dx}\left(\frac{\sqrt[3]{1-x^3}}{\sqrt{x}}\right)$

$$\frac{d}{dx}\left(\frac{\sqrt[3]{1-x^3}}{\sqrt{x}}\right)=\frac{\frac{1}{3}(1-x^3)^{-2/3}(-3x^2)\sqrt{x}-\frac{1}{2}x^{-1/2}\sqrt[3]{1-x^3}}{x}$$

$$=\frac{-(x^2)\sqrt{x}}{x\sqrt[3]{(1-x^3)^2}}-\frac{\frac{1}{2\sqrt{x}}}{x}=-\frac{(x^2)\sqrt{x}+\frac{\sqrt{x}}{2x}(1-x^3)}{x\sqrt[3]{(1-x^3)^2}}$$

$$-\frac{\sqrt{x}\left(x^2+\frac{1-x^3}{2x}\right)}{x\sqrt[3]{(1-x^3)^2}}=-\frac{\sqrt{x}\left(\frac{2x^3+1-x^3}{2x}\right)}{x\sqrt[3]{(1-x^3)^2}}$$

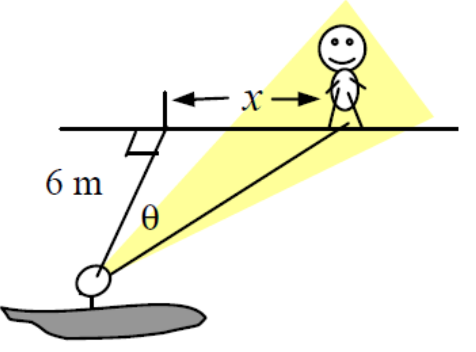
$$=-\frac{x^3+1}{2x\sqrt{x}\sqrt[3]{(1-x^3)^2}}$$

5. Calcule o integral: $\int_{-\sqrt{2}}^{+\sqrt{2}} e^{\frac{x+\sqrt{2}}{\sqrt{2}}}\, dx$

$$\int_{-\sqrt{2}}^{+\sqrt{2}} e^{\frac{x+\sqrt{2}}{\sqrt{2}}}\, dx = (a) \qquad \begin{cases} u = \frac{x+\sqrt{2}}{\sqrt{2}} \quad du = \frac{1}{\sqrt{2}} dx \\ x = -\sqrt{2} \Rightarrow u = 0 \\ x = \sqrt{2} \Rightarrow u = 2 \end{cases}$$

$$(a)\ \int_{-\sqrt{2}}^{+\sqrt{2}} e^{\frac{x+\sqrt{2}}{\sqrt{2}}}\, dx = \sqrt{2}\int_{0}^{2} e^{u}\, du = \sqrt{2}\left[e^{u}\right]_{0}^{2} = \boldsymbol{\sqrt{2}(e^{2}-1)}$$

1. Um homem anda ao longo de um caminho rectilíneo a uma velocidade de 1.2 m/s. Um holofote localizado no chão a 6 m do caminho focaliza o homem. A que taxa de variação instantânea, $\frac{d\theta}{dt}$, gira o holofote quando o homem está a uma distância de 4.5 m do ponto do caminho mais próximo da luz (x designa genericamente esta distância)?



$$\frac{d\theta}{dt} = ? \qquad \frac{dx}{dt} = \frac{1.2m}{s} \qquad \frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt}$$

$$tg\theta = \frac{x}{6} \Rightarrow x = 6\,tg\theta \Rightarrow \frac{dx}{d\theta} = 6\sec^2\theta$$

$$\frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt} \Rightarrow \quad 1.2 = 6\sec^2\left(\mathrm{arctg}(\frac{4.5}{6})\right)\frac{d\theta}{dt}$$

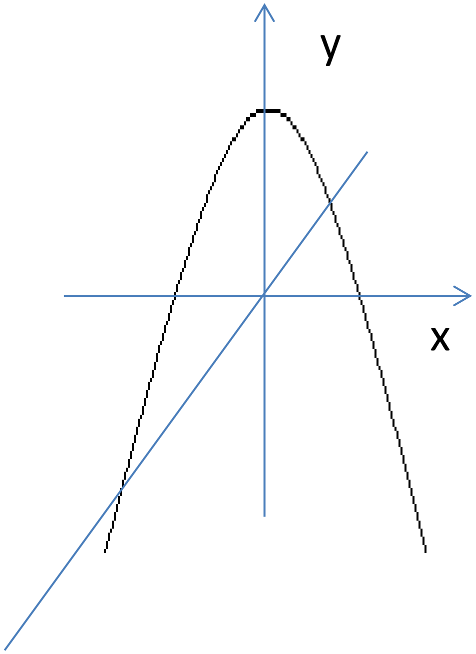
7. $\frac{dy}{dx}$=? para y= arctg x

$$x = tg\ y \qquad\qquad \frac{dx}{dy} = sec^2 y$$

Pelo Teorema da Derivada da Função Inversa:

$$\boldsymbol{\frac{dy}{dx}} = \frac{1}{\frac{dx}{dy}} = \frac{1}{sec^2 y} = \frac{1}{1+tg^2 y} = \boldsymbol{\frac{1}{1+x^2}}$$

8.



$$y = 2 - x^2 = 0 \Rightarrow x = \pm\sqrt{2}$$

$$y = x$$

$$2 - x^2 = x \Rightarrow x^2 + x - 2 = 0 \Rightarrow \mathrm{x} = -2 \text{ ou } \mathrm{x} = 1$$

$$A = \int_{-2}^{1} (2 - x^2 - x)dx = \left[2x - \frac{x^3}{3} - \frac{x^2}{2}\right]_{-2}^{1} =$$

$$\left(2 - \frac{1}{3} - \frac{1}{2}\right) - \left(-4 + \frac{8}{3} - \frac{4}{2}\right) = \frac{12-2-3+24-16+12}{6} = \frac{27}{6}\,(\text{u.a.})$$

$$\mathbf{9}\boldsymbol{a})\ \int \frac{sec^2x}{tgx}dx = \int \frac{1}{u}du = ln|u| + C = ln|tgx| + C$$

$$u = tgx \ \ du = sec^2x$$

$$\mathbf{9}\boldsymbol{b})\ \int \frac{1}{\sqrt{9+x^2}}dx = \int \frac{1}{\sqrt{9+9tg^2\theta}}3sec^2\theta d\theta = \frac{1}{3}\int \frac{1}{\sqrt{sec^2\theta}}3sec^2\theta d\theta = (a)$$

$$\boldsymbol{x = 3tg\theta \quad dx = 3sec^2\theta d\theta \ \Rightarrow \ tg\theta = \frac{x}{3} \ \ e \ \ sec\theta = \sqrt{1+\frac{x^2}{9}}}$$

$$(\mathrm{a}) = \int sec\theta d\theta = \int sec\theta \frac{sec\theta+tg\theta}{sec\theta+tg\theta}d\theta = \int \frac{sec^2\theta+sec\theta tg\theta}{sec\theta+tg\theta}\mathrm{d}\theta =$$

$$\begin{cases} \boldsymbol{u = sec\theta + tg\theta} \\ \boldsymbol{du = sec^2\theta + sec\theta tg\theta} \end{cases}$$

$$= \int \frac{1}{u}\mathrm{du} = ln|u| + C = ln|sec\theta + tg\theta| + C = \boldsymbol{ln\left|\sqrt{1+\frac{x^2}{9}} + \frac{x}{3}\right| + C}$$

9c) $\int xe^{3x}dx$

$$u = x \qquad dv = e^{3x}dx$$

$$du = dx \qquad v = \frac{e^{3x}}{3}$$

$$\int xe^{3x}dx = \frac{1}{3}xe^{3x} - \int \frac{1}{3}e^{3x}\,dx + C =$$

$$= \frac{1}{3}xe^{3x} - \frac{e^{3x}}{9} + C$$