Exame de Época Normal / First Exam (2017/01/19)

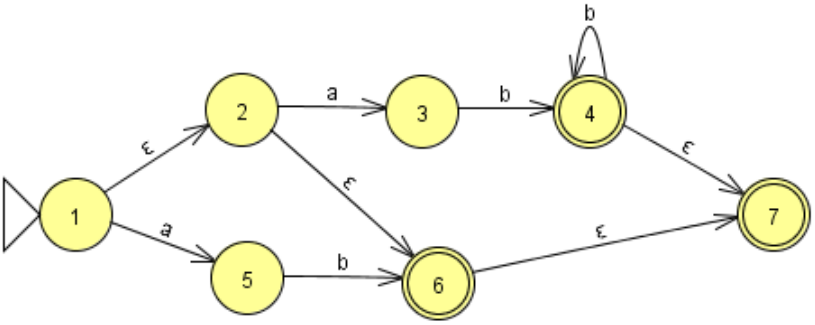
Duration: 2h30 **Version A**

No consultation is allowed, other than the supplied document.
No electronic means are allowed (computer, cellphone, …).
Fraud attempts lead to the annulment of the exam for all participants.

Answer each group in separate sheets!
Write your full name and exam version in all sheets!

Group I: [4.5 Points] Finite Automata and Regular Expressions

Consider the following ε-NFA.

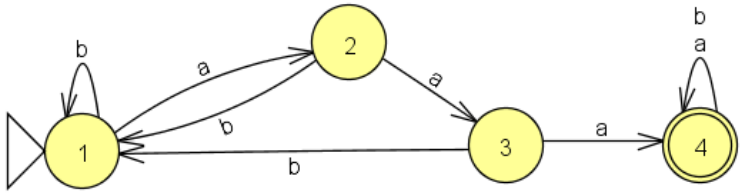


a) Show the ε-closure for each of the states of the ε-NFA.

b) Obtain the equivalent DFA to the ε-NFA. Show the transition table and the state diagram of the DFA.

c) Minimize the obtained DFA. Show the table of distinguishable states, and the state diagram for the minimized DFA.

Consider the DFA below.



d) Obtain a regular expression for the language defined by the DFA using the state elimination method and considering the ordering of elimination 2→3 (i.e., first eliminate state 2 and then state 3). Show all the intermediate steps.

e) Show the expressions for the terms $R_{12}^{(0)}$, $R_{11}^{(0)}$, $R_{22}^{(0)}$, $R_{23}^{(1)}$, $R_{12}^{(1)}$ and $R_{14}^{(1)}$ obtained by the path construction method to convert the DFA in a regular expression.

Consider the existence of a DFA of a regular language L:

f) Suppose that we intend to implement L1 = L:/{a} which must accept all the strings accepted by L, but without the a's that can exist, e.g., if L={abcaa, bb, ab, ccaaaa} then L1={bc, bb, b, cc}. Indicate a method that can be applied to the DFA of L to obtain a DFA that implements L1= L:/{a}.

The insertion of grades in SIGARRA can be done with the use of a file formatted with the grades of the students for a given course:

g) Obtain a regular expression to validate the format of the information for a student, which consists of the student number, name, and grade, separated by ':', as shown in the examples below. Use the symbols 'M' to represent an uppercase letter, 'm' for a lowercase letter (both include letters with accentuated), 'D' for a digit, 'E' for a space. Indicate the alphabet used in the regular expression and in case you use an extra symbol indicate its meaning. The student number must have 9 digits, in which the first 4 digits form a number between 1970 and 2019. The grade information can be RFC, RFF, or a number between 0 and 20, corresponding to the grade obtained. Examples:

197560430 : Dionísio Adalberto da Silva Côrte-Real : RFC
200540075 : Leonilde Maria do Ouro dos Anjos e Ramos da Árvore : 12
201550336 : Marcílio Osvaldo de Espírito-Santo das Almas e Santos : 5

Group II: [2 Pts] Properties of Regular Languages

Prove for each of the following languages if it is a regular language or not. If it is, show a finite automaton that represents the language.

a) L = {xwywz | x,y,z ∈ {a, b}* and w ∈ {c, d}*}.

b) L = {xwywz | x,w,y, z ∈ {a, b}*}.

Group III: [4.5 Pts] Context-Free Grammars (CFG) and Push-Down Automata (PDA)

S → aAS | a
A → SbA | SS | ba

Consider the CFG G on the left, in which S is the initial variable.

a) Show an analysis tree and a leftmost derivation for the string: aabbaa.

b) Is the CFG G ambiguous? Justify. If it is ambiguous, modify the grammar in order it is non-ambiguous.

c) Suppose that we intend to represent the language L given by G with a CFG in which the analysis trees are binary trees. Show a CFG for L.

d) Indicate a PDA accepting by final state to recognize the language of the strings in the alphabet {x, y, z} in which the total number of y's is twice of the sum of x's and z's.

e) The previous PDA is deterministic or not? Justify your answer.

f) Indicate the sequence of instantaneous descriptions to process the string: yxyzyy.

Group IV: [4 Pts] Turing Machine

We intend a Turing machine to perform the operation 4×a+1, in which "a" is a natural number represented in binary. The input in the tape uses the format: "f(", followed by the binary number and ")=", and the result must be placed immediately after the symbol "=" of the input string (the input string must be maintained).

a) Describe a strategy for a deterministic Turing Machine that implements this operations.

b) Draw the respective Turing Machine.

c) Show the sequence of instantaneous descriptions when the input in the tape is: f(1010)=.

Group V: [5 Pts] Statements about Languages (V/F: 20%; justification: 80%))

Indicate, justifying succinctly, whether each of the following statements is True or False.

a) A language L defined by a CFG can never be a regular language.

b) For a CFG G there exists always a non-deterministic PDA which defines L(G).

c) All regular expressions can be implemented with PDAs.

d) If a CFG is ambiguous, then the PDA for that CFG, and obtained by the algorithm to convert CFG→PDA, is non-deterministic.

e) A language L is a CFL if and only if exists a CFG G non-ambiguous such that L=L(G).

f) In order to prove that a CFG G is ambiguous we have to find a string w ∈ L(G) for which there exist at least two distinct syntactic trees.

g) When we eliminate the ambiguity of grammars that represent languages of arithmetic expressions we obtain a grammar that satisfies the priority of the arithmetic operators.

h) If the tape of a Turing Machine is finite then the languages recognized by that machine can be recognized by a DFA.