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Integrated unit commitment and natural gas network operational planning under renewable generation uncertainty

Farhad Fallahi, Pouria Maghouli*

Department of Electrical Engineering, Shahed University, Tehran, Iran

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ABSTRACT

In this paper, we study an important variant of integrated unit commitment problem and gas network in which uncertain renewable energy resources, gas and electricity storage, the capability of line-pack of the gas network (gas stored in the pipeline), and Power to Gas (PtG) are taken into account. The motivation of this paper is to enhance the security of power generation and optimize both electrical and gas network operational cost. Uncertainty of renewable sources in a power system will inevitably increase the power system's dependence on traditional gas power plants. As a result, the uncertainty of renewable sources is transferred to the gas network such that the fluctuations of the gas flow would increase. Gas and electricity storage, the capability of line-pack of gas network and the use of PtG technologies help us to manage the effect of renewable sources on electricity and gas network. Here, we formulate a mixed integer non-linear optimization model to present the integrated unit commitment problem. Given the complexity of the underlying problem, we explore the problem structure and propose a novel reformulation consisting of efficient decomposition and convexification techniques. Compared to existing methods whose quality are sensitive to initial solutions, the proposed model is more flexible and offers significant improvements in solution quality and efficiency. The proposed model is tested on a modified IEEE 24-Bus test system and Belgian 20-node gas system.

1. Introduction

In order to achieve future energy security and sustainability, many countries have already begun diversifying their energy portfolio, mainly by focusing on renewable energy resources. High penetration of renewable energy inevitably leads to significant growth in using gas-fired units in power systems [1]. This is due to the fact that gas-fired units are capable of ramping up during peak loads and backing up intermittent renewable generation and contingencies. Given the uncertainty in electricity generated by renewable resources, the uncertainties of renewable generations are transferred to the gas network and the fluctuations of the gas flow are increased. In the short run, this leads to decreasing gas pressure and increasing the risk of venting in the gas network. In such circumstances, gas and electricity storage, the capability of line-pack of gas network and use of PtG technology can alleviate the effects of fluctuations of renewable generations in the operation of gas and electricity network. As a result, studying the correlation between gas network and electricity network becomes important which has been highlighted in recent years [2,3]. The integration of the UC and short-term operation of the natural gas network is a new area in the power system studies [4,5]. Renewable generations such as wind and solar farms impose a significant deal of uncertainty to the power system. In such a situation, the integration of short term operation of the gas system with UC plays a vital role in operating a flexible and reliable power system to guarantee continuity of power supply. We assume that electricity generated by renewable resources can take three states, low, medium and high.

In this study, we address the following key characteristics of the power system with the aforementioned features:

- The study of the direct influence of the natural gas network on the operation of the power system.
- The evaluation of the effectiveness of gas and electricity storage facilities and line-pack on the operation of the power system.
- The impact of uncertain renewable generation on the operation of the power system (See [6;7]).
- Exploring the benefits of PtG for both the natural gas system and power system [8].

Below, we focus on the above characteristics and describe the complexity of the underlying problem.

The UC problem is a well-known optimization problem that creates

* Corresponding author.
E-mail addresses: farhad.fallahi@shahed.ac.ir (F. Fallahi), p.maghouli@shahed.ac.ir (P. Maghouli).

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Nomenclature

Indexes and sets

GN	set of gas nodes, index by *i, j*
LG	set of gas pipelines, index by *lg*
T	scheduling time horizon, index by *t*
TL	set of transmission lines, Index by *le*
N	set of all generation power units, index by *g*
N^{Wd}	set of wind power units
N^{G}	set of Gas-fired power units
N^{H}	set of hydropower units
N^{C}	set of coal power units
B	set of power buses, index by *b* and *m*
U	set of scenarios, index by *u*
S^E, S^G	set of electricity/gas storage facilities, index by *s*
C	set of Compressors, index by c_{ij} (located between node *i* and *j*)
W	set of Gas wells, index by *w*
PG	set of power to gas facilities, index by *pg*
LG_i	set of the gas pipelines connected to node *i*
N_i^G	set of Gas-fired power units connected to node *i*
N_b^G	set of Gas-fired power units connected to bus *b*
N_b^W	set of wind power units connected to bus *b*
N_b^H	set of hydropower units connected to bus *b*
N_b^C	set of coal power units connected to bus *b*
W_i	set of gas wells connected to node *i*
S_b^E, S_i^G	set of Electricity storage facilities connected to bus *b*/ node *i*
PG_b, PG_i	set of PtG facilities connected to bus *b*/ node *i*
TL_b	set of transmission lines connected to bus *b*
k	index of iteration

Parameters

$F_w^{G,Max}$, $F_w^{G,Min}$	maximum/minimum gas output of well *w* [m^3/h]
$F_{ptg}^{G,Max}$	maximum gas production from PtG technology [m^3/h]
C_w^G	gas production cost from gas well *w*[\$/m^3]
C_g^E	electricity generation cost of unit g [\$/MWh]
$C_s^{G,in}$, $C_s^{G,out}$	cost of Injection / Withdrawal to/from gas storage facilities [\$/m^3 h]
$C_s^{E,in}$, $C_s^{E,out}$	cost of Injection / Withdrawal to/from electricity storage facilities [\$/MWh]
$C^{NS,E}$	non-served power cost [\$/MW]
$C^{NS,G}$	non-served gas cost [\$/Sm^3]
$P_g^{E,Max}$, $P_g^{E,Min}$	maximum/minimum power output of unit *g* [MW]
$P_{g,T}^{Max}$	maximum production of hydropower unit due to water limitation over the scheduling time horizon [MWh]
RR_g^{Start}, RR_g^{Stop}	startup/ shutdown ramp rate of unit *g* [MW/h]
RR_g^{Inc}, RR_g^{Dec}	rampup/ rampdown rate of unit *g* [MW/h]
T_g^{Inc}, T_g^{Dec}	rampup/ rampdown time of unit *g* [h]
C_g^{SU}, C_g^{SD}	startup/ shutdown cost of unit *g* [\$/h]
MT_g^{Up}, MT_g^{Down}	minimum uptime/ downtime of unit *g* [h]
T_g^{on}, T_g^{off}	number of the time interval that unit *g* has been on or off before the first time of the scheduling time horizon
G_g^{on}, L_g^{off}	number of time interval unit *g* must be initially on/ off due to its minimum uptime constraint
$R_{g,t}^{Up} R_{g,t}^{Down}$	upward/ downward spinning reserve from unit *g* in time *t* in [MW]
$R_{g,t}^{Reg,Up}$, $R_{g,t}^{Reg,Down}$	upward/ downward regulation spinning reserve from unit *g* in time *t* in [MW]
$intial_g^{Status}$	binary variables indicate the initial status of unit *g*
$L_{b,t}^E$	electricity Load demand at bus *b* in time *t* [MWh]
$B_{b,m}$	admittance of the line connecting bus *b* and *m*
$L_{i,t}^G$	gas Load demand at node *i* in time *t* [Sm^3]
η_g^E	power conversion factor of unit *g* [Sm^3/MWh]
η_{pg}^{PG}	efficiency of PtG facility *pg*
η_s^{in}, η_s^{out}	injection/ withdraw efficiency of storage *s*
M_{Big}	sufficiently large number
Pr_i^{Min}, Pr_i^{Max}	maximum/minimum pressure at node i [kPa]
f_{le}^{Min}, f_{le}^{Max}	maximum/minimum transmission capacity of line *le* [Mw]
$MS_s^{E,Max}$, $MS_s^{G,Max}$	maximum capacity of electricity/gas storage facility *s* [Sm^3] or [MW]
$S_s^{Max,In}$, $S_s^{Max,out}$	maximum injection/ withdraw rate of storage facility s [Sm^3/h] or [MW/h]
Lp_{safe}	the minimum safe level of line-pack [Sm^3]
Γc_{ij}	compressor factor located between node *i and j*
K_{lg}	Weymouth constant
M_{lg}	line-pack constant
H_{Gas}, H_{H_2}	gas/H2 heat rate

Variables

$F_{w,t,u}^G$	gas production from gas well *w* in time *t* for scenario *u* [Sm^3/h]
$F_{ptg,t,u}^G$	the amount of hydrogen production by *ptg* in time *t* for scenario *u* [Sm^3/h]
$P_{g,t,u}^E$	power production of unit g in time *t* for scenario *u* [MWh]
$f_{le,t,u}$	power flow in line *le* in time *t* for scenario *u* [MW]
$S_{s,t,u}^{G,In}$, $S_{s,t,u}^{G,Out}$	injection/ withdraw gas rate of storage *s* in time *t* [Sm^3/h]
$S_{s,t,u}^{E,In}$, $S_{s,t,u}^{E,Out}$	injection/ withdraw electricity rate of storage *s* in time *t* [MW/h]
$MS_{s,t}^G$, $MS_{s,t}^E$	natural gas/ electricity storage level in storage *s* in time *t* [Sm^3] or [MWh]
$Cuw_{g,t,u}$	curtailment wind power of unit *g* in time *t* in for scenario *u* [MWh]
$Loss_{b,t,u}^E$	non-served power at bus b in time *t* for scenario *u* [MWh]
$Loss_{i,t,u}^G$	non-served gas load in node *i* in time *t* for scenario *u* [Sm^3]
$V_{g,t}^E$	binary variables indicate on/off status of power unit *g* in time *t*
$V_{s,t,u}^{St}$	binary variables indicate the injection or the withdraw to/ from storage *s* in time *t* for scenario *u*
$v_{lg,t,u}^i$	binary variables indicate the injection or the withdraw to/ from pipeline *lg* in time *t* for scenario *u* for node *i*
$xv_{lg,t,u}$	binary variables indicate the direction of gas flow rate in pipeline *lg* in time *t* for scenario *u*
$C_{g,t}^{SU}$, $C_{g,t}^{SD}$	startup/ shutdown cost of unit *g* in time *t* [\$]
$\theta_{m,t,u}$	phase angle at bus *m* in time *t* for scenario *u* [rad]
$Lp_{lg,t,u}$	average gas mass (line-pack) in pipeline *lg* in time *t* for scenario *u* [Sm^3]
$Pr_{i,t,u}$	pressure at node *i* in time *t* for scenario *u* [kPa]
$q_{lg,t,u}^{i,in}$, $q_{lg,t,u}^{i,out}$	input and output of natural gas rate of pipeline *lg* in time *t* for scenario *u* [Sm^3/h]
$fg_{lg,t,u}$	natural gas flow rate in pipeline *lg* in time *t* for scenario *u* [Sm^3/h]
$fg_{lg,t,u}^+$, $fg_{lg,t,u}^-$	positive and negative part of pipeline flow rate in pipeline *lg* in time *t* for scenario *u* [Sm^3/h]
$xfg_{lg,t,u}$	the signed square of the pipeline flow rate in pipeline lg in time t for scenario u [Sm^3/h]
$xfg_{lg,t,u}^+$, $xfg_{lg,t,u}^-$	positive and negative part of the signed square of the pipeline flow rate in pipeline *lg* in time *t* for scenario
$NG_{i,t,u}$	the accumulated difference between the withdrawal and injection in node i in time *t* for scenario *u* [Sm^3/h]
$SF_{lg,t,u}^{+,k}$, $SF_{lg,t,u}^{-,k}$	non-negative slack variable for flow rate equation in the sub-problem for iteration *k*
$SB_i^{+,k}$, $SB_i^{-,k}$	non-negative slack variable for gas injection equation in the sub-problem for iteration *k*
$\pi_{lg,t,u}^k$, $\lambda_{i,t,u}$	dual variables corresponding to the conservation of flow and gas injection constraints, respectively

power generators commitment decisions to minimize the cost of serving forecasted net-load in the power market subject to several types of operational constraints on generation resources and transmission lines [9]. In the related literature, the two-stage stochastic programming approach is widely used to present stochastic UC under different random parameters. In these studies, binary decisions related to the status of generators are made before knowing the realization of random parameters in the first-stage decisions, and the dispatch of each generator committed at the first stage will be determined as the second-stage decisions for each scenario (See, e.g., [10–12]). Using scenario-based stochastic programming approaches has two significant challenges: (1) creating scenarios and obtaining their associated probabilities could be a problematic task, especially for practical UC problems, (2) an adequate number of scenarios will normally lead to a large-scale optimization problem whose solution is a formidable task. Alternative approaches which result is much smaller problems are robust optimization [13] and interval model [14].

Robust UC (RUC) problems in which uncertain parameters are modeled through predefined intervals, called interval-uncertainty, have recently received significant attention (see, e.g., [15–17]). In the RUC with uncertain net-load, upper and lower bounds on the net-load at each optimization interval define the range of uncertainty, and the RUC enforces the feasibility of its schedule over a given uncertainty set and minimizes the dispatch cost under the worst-case realization [18]. The literature review on stochastic and robust formulations of UC problems are presented in [19].

Since the identification of the worst-case might be a difficult task in some problems [20], interval modeling is an alternative way of addressing the issue. Ref. [21] modeled the wind power generation in the UC problem as interval uncertainty, and [22] proposed a UC problem under interval uncertainty related to bus load and wind power generation. An interval gas flow analysis is presented in [23]. Interval UC (IUC) considers only three distinct scenarios including the central forecast (the most probable), the lower scenario (least probable) and upper scenario (most probable) to model the requirements of the ramp of units [24]. The cost of the net dispatch is then minimized in the most probable (the central) forecast while the solution guarantees that any realization of the uncertainty around this central forecast could not change the status of the units [25]. Refs. [24,26] compare scenario-based and interval optimization approaches for UC and similarly [27] for SCUC. Furthermore, regarding the solution algorithm, Benders' decomposition is widely used for solving the RUC J. [15–17] and IUC [22,27] because of the two-stage nature of the problem.

The integration of the UC and short-term operation of the natural gas network is a new area in the power system studies. There are several works in the literature (see, e.g., [4,5,28–35]) which have proposed models to study this problem. Refs. [4] and [5] have combined optimization of natural gas and electricity networks without considering their physical characteristics. Ref. [35] presented a bi-level programming formulation to solve the UC as an MINLP problem and the gas transmission as a successive LP. Ref. [31] developed a combined multi-time gas and electricity network optimization model in which a DC power flow model represents the electricity network and in the gas network, successive times are linked by considering the volume of gas in pipes and important facilities such as compressors and gas storage facilities.

Gas flows are modeled as a function of the pressure gradient and pipe characteristics such as diameter, length and friction coefficient. Thus, the resulting problem which formulates the integration of power and gas networks is typically a non-linear and non-convex problem. The main challenge is that it is not possible to guarantee that a solution for such a non-convex problem is globally optimal. Many studies present optimization methods to ensure feasibility, convergence, and better local optimal solutions for the integrated short-term operation of power and gas networks. Given the complexity of the problem, some works use approximation methods to find near optimal solutions. For instance, [28] approximate non-linear and non-convex functions using piecewise linear functions, and present a mixed-integer linear model. However, their model is not solvable for large-size power systems when the function is approximated using an adequate number of discretization points. Some enhancements for this method include local convexification. The solution quality and efficiency of the resulting method is very sensitive to the initial solutions. In other words, their approximation method depends on the operating interval of gas network nodes. Some other papers focus on specific types of networks (e.g., radial gas network) which simplifies modeling gas and electricity networks [36].

PtG technologies which convert electrical power to hydrogen or synthetic natural gas [37], have led to a significant impact on efficient use of energy resources. PtG connects electricity network and gas network [38] to enhance reliability, enable flexible bi-directional energy conversion between the electricity and gas network, and save surplus power [39]. Given the fact that the gas network consists of non-linear and non-convex constraints, the inclusion of PtG dramatically increases the complexity of the problem in several ways which will be discussed in detail in the next sections.

The decision-making environments under uncertainty can be categorized into two main groups: 1- Decision-making environments with random parameters in which their probability distributions are known for the decision maker, and 2- Decision-making environments with random parameters in which the decision maker has no information about the probability distributions of random parameters [40]. To immune the power system against any possible outcome of uncertain renewable resources, we use the second approach, in other words, we use interval optimization approach which makes conservative (risk averse) decisions upon decision variables (for similar studies [22,23,26]). This approach helps us to find a robust solution against variation of renewable energy source (wind) and also to investigate the level of advantage the network can gain. We include uncertainty of wind using interval optimization via worst cases and forecast values. We analyze the impact of these three scenarios on different decision variables such as line pack and Weymouth equation. For further details on interval optimization, the reader is referred to [26] and [27].

Line-pack is an important feature of the gas network and enables us to store gas for a short time in gas pipelines [1]. Modeling line-pack further increases the flexibility to quick short-term management of fluctuations and time-imbalances in gas production or gas load. The advantage of line-pack is that the constraints involved in modeling are linear. But the disadvantage of this component is that we have to formulate gas network pressure and related equations on an hourly basis.

A significant gap in the related literature is to integrate UC and short-term operation of the natural gas network under uncertainty with reliable optimization models which can be handled by off-the-shelf software. This study addresses this gap by considering interval uncertainty for wind power generation in the power system to foster the body of knowledge in this area.

In this paper, we study the UC problem with two complicating features: the direction of gas flow is not determined in advance and the line pack is also taken into account. As mentioned, there are very few works in the literature to cover these two features. Given the complexity of the resulting problem, for instance, while [41] takes into account line-pack and they assume that direction of flow is predefined. We try to contribute to the literature by proposing an effective modeling and solution method for this complex problem.

We first reformulate the underlying problem using the Benders' decomposition by which we relax some constraints and variables related to the gas network. In other words, the decision on gas flow (value and direction), the status of each unit, gas and electricity storage and PtG variables are made in the first problem (known as the master problem). This decomposition/relaxation moves the simplified form of non-convex constraints to the sub-problem. We call them simplified because the direction of gas flow and its pressure are not determined at the same time. We first decide upon the direction and in the sub-problem based on the fixed directions, the pressure of the gas is determined. It is also worth mentioning that as the relation of line pack and pressure is linear, a lower bound is determined for pressure in the master problem. This decomposition enables us to reformulate a complex problem to an effective formulation while other works in the literature assume that the direction of flow is given in advance. The structure of the master problem paves the way for further decomposition, such that we decompose it into two parts. The first part addresses the standard UC and the second part includes the decisions for each scenario of wind. We consider worst cases which significantly affect the operation factors of the power generation units such as their ramp rate.

Then the pressure of the gas network and line-pack is the secondary decisions and determined from the fixed value of the primary decisions. The master problem includes linear constraints while the sub-problem consists of non-linear constraints (the steady state Weymouth equation). Since the gas flow including value and direction are fixed, the sub-problem is reformulated as a convex quadratic constrained programming.

In summary, the key contributions of this study are as follows:

- We study an integrated UC problem with the gas network, electricity network and uncertainty of renewable generations. We also include line-pack in our model as a short term gas storage which significantly increases the complexity of the underlying problem.
- We develop a novel reformulation technique (a combination of decomposition and convexification) to obtain a tractable model. We decompose the problem into two sections: (1) Master Problem: the amount of gas required for each node and also the electricity load are computed which determine the direction of gas flow in a mixed integer program, and (2) Sub-problem: given the direction of gas flow and the amount of flow determined in the first part, Weymouth equation is written in a second order cone program to minimize error. It is worth mentioning that to enhance solution efficiency the master problem is broken into two parts based where the decisions such as unit statuses are computed as here-and-now decisions and the decisions such as power generated by each unit are determined based on worst case scenarios.
- We test and compare the proposed method with some existing methods and demonstrate our method achieves a significant improvement in the quality and efficiency of the solution.

The rest of the paper is organized as follows. In Section 2, the proposed optimization model of the underlying integrated UC and short-term operation of the natural gas system are formulated. The proposed solution method is described in Section 3. The numerical results are present in Section 4 and the conclusion in Section 5.

2. Problem formulation

Given the notation presented at the beginning of the paper, we formulate the underlying problem as follows. The objective function (1) minimizes the total operation and storage costs for an integrated electricity and gas system which are explained in five terms (*A to E*) within the planning time horizon of 24 h. Term A presents the startup and shutdown cost of generation units as calculated in constraints (7)–(11). Term *B* presents the cost of electricity generation and the cost of production of natural gas from gas wells. Term *C* presents the operational cost of electricity and gas storage systems. In both storage systems, it is assumed that the injection and withdrawal of storage facilities have a different cost. Term *D* presents the cost of not served electricity and natural gas. Term *E* presents the operation cost of PtG facilities to produce gas. The optimization problem is subjected to electricity system constraints and natural gas constraints.

$$
\begin{aligned}
&Min \sum_{t\in T}\sum_{u}(A+B+C+D+E)\\
&A=\sum_{g\in N}(C_{g,t}^{SU}+C_{g,t}^{SD})\\
&B=\sum_{g\in N}(C_g^{E}\times P_{g,t,u}^{E})+\sum_{w\in W}(C_w^{G}\times F_{w,t,u}^{G})\\
&C=\sum_{s\in s^E}(C_s^{E,In}\times S_{s,t,u}^{E,In}+C_s^{E,Out}\times S_{s,t,u}^{E,out})\\
&\quad+\sum_{s\in s^G}(C_s^{G,In}\times S_{s,t,u}^{G,In}+C_s^{G,Out}\times S_{s,t,u}^{G,out})\\
&D=\sum_{b\in B}C^{NS,E}\times Loss_{b,t,u}^{E}+\sum_{i\in GN}C^{NS,G}\times Loss_{i,t,u}^{G}\\
&E=\sum_{pg\in PG}C_{pg}^{PG}P_{pg,t}\\
&\text{Subject to: } Cons1\&Cons2
\end{aligned}
\tag{1}
$$

Below we first define the objective function and its term then the constraints (Cons1 and Cons 2) are explained in detail. Note that the connection between the terms in the objective function is presented in the constraints.

2.1. Objective function

The objective function consists of five terms as follows:

Term A calculates the startup and shutdown costs of generator units.
Term B calculates variable generation cost and gas production cost.
Term C calculates the cost of storing electricity and natural gas.
Term D calculates the load shedding cost for electricity and natural gas.
Term E calculates the cost of PtG.

2.2. Cons1: Electricity system constraints

The DC power flow model is used for the network constraints. The electric power constraints consist of the nodal power balance, the generator capacities, the ramping limits, and the power transmission capacities, which are trivial constraints in a power system.

2.2.1. Operation constraints of units

Inequities (2) and (3) represent the minimum and maximum capacity limits of the power unit's output by considering up and down and regulation spinning reserve. Also, the constraint (4) limits the summation of the production of hydropower units due to water limitation over the scheduling time horizon.

$$(P_{g,t,u}^{E}+R_{g,t}^{Up}+R_{g,t}^{Reg,Up})\leqslant P_g^{E,Max}\times V_{g,t}^{E}\quad \forall\, g\in N,\ t\in T,\ u\in U \tag{2}$$

$$(P_{g,t,u}^{E}-R_{g,t}^{Down}-R_{g,t}^{Reg,Down})\geqslant P_g^{E,Min}\times V_{g,t}^{E}\quad \forall\, g\in N,\ t\in T,\ u\in U \tag{3}$$

$$\sum_{t\in T}P_{g,t,u}^{E}\leqslant P_{g,T}^{Max}\quad \forall\, g\in N^{H},\ u\in U \tag{4}$$

The IUC approach considers three distinct scenarios: the central forecast (the most probable, (c)), the lower scenario (l) and upper scenario (u), and minimizes the operation cost of the central forecast. Fig. 1 (taken from [22]) shows seven possible inter-hour transitions (the ramping) constraints between two adjacent time intervals (see [22,25]). This statement is not entirely correct. Because the IUC committed the generation units with respect to u_c and production level of units in other scenarios must be determined with respect to these commitment states and other network constraints. So it is not necessary that $P_{g,t,u_l}^{E}>P_{g,t,u_c}^{E}>P_{g,t,u_u}^{E}$ for all units. Suppose that we have two gas-fired power units and in time t the scenario u_c and in time $t+1$ the scenario u_l occurred and $D_{t,u_c}<D_{t+1,u_l}$. We know the total production of units should increase, that means $P_{1,t+1,u_c}^{E}+P_{2,t+1,u_c}^{E}>P_{1,t,u_c}^{E}+P_{2,t,u_c}^{E}$. But it does not mean that the production of all units must increase. For

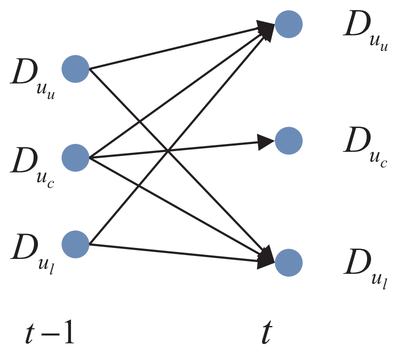


Fig. 1. IUC scenario with transition constraints (D: Net demand) [22].

example, when some constraints (transmission constraint, the minimum and maximum capacity of each unit) are satisfied, one unit may decrease its production, and another unit may increase its production, $P^E_{2,t+1,u_l} > P^E_{2,t,u_c}$ (Fig. 2).

Therefore, all nine inter-hour ramping constraints must be modeled (Fig. 3) and none of them can be removed to ensure that the IUC schedule has sufficient flexibility to handle any uncertainty within given bounds and will not require any changes in the commitment. Eqs. (5) and (6) enforce all possible transition ramping constraints between all scenarios.

$$(P^E_{g,t,u} + R^{Up}_{g,t} + R^{Reg,Up}_{g,t}) - (P^E_{g,t-1,k} - R^{Down}_{g,t-1} - R^{Reg,Down}_{g,t-1}) \leqslant RR^{Inc}_g \times T^{Inc}_g \times V^E_{g,t-1} + RR^{Start}_g \times (V^E_{g,t} - V^E_{g,t-1}) + M_{Big} \times (1 - V^E_{g,t}) \quad \forall\, g \in N,\ t \in T,\ u, k \in U \tag{5}$$

$$(P^E_{g,t-1,k} + R^{Up}_{g,t-1} + R^{Reg,Up}_{g,t-1}) - (P^E_{g,t,u} - R^{Down}_{g,t} - R^{Reg,Down}_{g,t}) \leqslant RR^{Dec}_g \times T^{Dec}_g \times V^E_{g,t} + RR^{Stop}_g \times (V^E_{g,t-1} - V^E_{g,t}) + M_{Big} \times (1 - V^E_{g,t-1}) \quad \forall\, g \in N,\ t \in T,\ u, k \in U \tag{6}$$

2.2.2. Startup and shutdown costs

Eqs. (7)–(11) determine the startup and shutdown cost of each unit.

$$C^{SU}_{g,t} \geqslant C^{SU}_g \times (V^E_{g,t} - V^E_{g,t-1}) \quad \forall\, g \in N,\ t \in \{2, ..,T\} \tag{7}$$

$$C^{SU}_{g,1} \geqslant C^{SU}_g \times (V^E_{g,1} - intial^{Status}_g) \quad \forall\, g \in N \tag{8}$$

$$C^{SD}_{g,t} \geqslant C^{SD}_g \times (V^E_{g,t-1} - V^E_{g,t}) \quad \forall\, g \in N,\ t \in \{2, ..,T\} \tag{9}$$

$$C^{SD}_{g,1} \geqslant C^{SD}_g \times (intial^{Status}_g - V^E_{g,1}) \quad \forall\, g \in N \tag{10}$$

$$C^{SU}_{g,t} \geqslant 0\ ,\ \ C^{SD}_{g,t} \geqslant 0 \quad \forall\, g \in N,\ t \in \{2, ..,T\} \tag{11}$$

2.2.3. Minimum up and down times

Eqs. (12)–(15) are necessary for minimum downtime and uptime requirements for the number of hours that each unit has been on or off before the first time of the scheduling time horizon.

$$\sum_{t=1}^{G_g} V^E_{g,t} = G_g\ ,\ G_g = Max(0,\ \min(T,\ (MT^{Up}_g - T^{on}_g) \times V^E_{g,0})) \tag{12}$$

$$\sum_{t=1}^{L_g} (1 - V^E_{g,t}) = L_g\ , L_g = Max(0,\ \min(T,\ (MT^{Down}_g - T^{off}_g) \times (1 - V^E_{g,0}))) \tag{13}$$

$$\sum_{k=t}^{t+\min(MT^{Up}_g - 1; T-t)} V^E_{g,k} \geqslant \min\left(MT^{Up_g}_g,\ T - t + 1\right) \times (V^E_{g,t} - V^E_{g,t-1}) \quad \forall\, g,\ t = G_g + 1, ..,T \tag{14}$$

$$\sum_{k=t}^{t+\min(MT^{Down}_g - 1; T-t)} (1 - V^E_{g,k}) \geqslant \min(MT^{Down}_g,\ T - t + 1) \times (V^E_{g,t-1} - V^E_{g,t}) \quad \forall\, g,\ t = L_g + 1, ...,T \tag{15}$$

2.2.4. Power balance constraint

The DC power balance constraints are given in Eq. (16). Curtailment of wind power is also allowed (Eq. (17)).

$$\sum_{g \in N^G_b} P^E_{g,t,u} + \sum_{g \in N^H_b} P^E_{g,t,u} + \sum_{g \in N^C_b} P^E_{g,t,u} + \sum_{g \in N^{Wd}_b} (P^E_{g,t,u} - Cuw_{g,t,u}) + \sum_{s \in S^E_b} (S^{E,out}_{s,t,u} - S^{E,in}_{s,t,u}) = L^E_{b,t} - Loss^E_{b,t,u} + \sum_{pg \in PG_b} P^{PG}_{pg,t,u} + \sum_{m \in TL_b} B_{b,m} \times (\theta_{m,t,u} - \theta_{b,t,u}) \quad \forall\ b \in B,\ t \in T,\ u \in U \tag{16}$$

$$0 \leqslant Cuw_{g,t,u} \leqslant P^E_{g,t,u} \quad \forall\ g \in N^{Wd},\ t \in T,\ u \in U \tag{17}$$

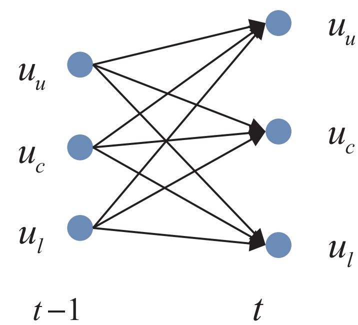


Fig. 3. All transition ramping constraints.

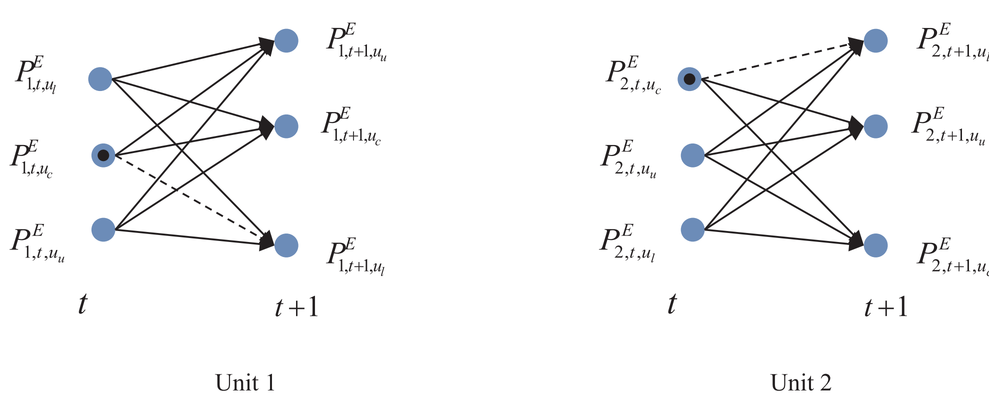


Fig. 2. Illustrative example of the ramping constraints between two adjacent time intervals.

2.2.5. Electricity transmission constraint

The maximum line flow limit is denoted by Inequality (18).

$$-f_{le}^{Max} \leqslant f_{le,t,u} = B_{b,m} \times (\theta_{m,t,u} - \theta_{b,t,u}) \leqslant f_{le}^{Max} \quad \forall \ le \in TL, t \in T, u \in U \tag{18}$$

2.2.6. Electricity storage constraint

Eqs. (19)–(23) enforce the limitation of the maximum capacity of electricity storage facilities and the limitation of injection and withdrawal rate from gas storage facilities.

$$MS_{s,t}^{E} \leqslant MS_{s}^{E,Max} \quad \forall \ s \in S^E, t \in T \tag{19}$$

$$MS_{s,t.u}^{E} = MS_{s,t-1,u}^{E} + S_{s,t,u}^{E,in} \times \eta_s^{in} - S_{s,t,u}^{E,out}/\eta_s^{out} \quad \forall \ s \in S^E, t \in T, u \in U \tag{20}$$

$$S_{s,t,u}^{E,in} \leqslant V_{s,t,u}^{St} \times S_s^{Max,In} \ , S_{s,t,u}^{E,out} \leqslant (1 - V_{s,t,u}^{St}) \times S_s^{Max,out} \quad \forall \ s \in S^E, t \in T, u \in U \tag{21}$$

$$MS_{s,0}^{E} \leqslant MS_{s,T}^{E} \quad \forall \ s \in S^E \tag{22}$$

$$S_{s,t,u}^{E,in} \geqslant 0, S_{s,t,u}^{E,out} \geqslant 0 \quad \forall \ s \in S^E, t \in T, u \in U \tag{23}$$

2.3. Cons2: Natural gas system constraints

In this section, the constraints of the natural gas system are presented. The gas network includes sources, pipelines, compressors, storage facilities and gas loads. By controlling the pressures, the stored gas in the pipeline (line-pack) can increase or decrease as the withdrawal and injection rates from each other, so we need to precisely model gas flow. Many papers model gas flow in one direction while very few studies model bi-directional gas flow [28,42]. In this paper, the gas flow in pipes is modeled bi-directional. We can consider the case when the direction of natural gas flow in the pipeline changes due to a sudden change in gas consumption of gas-fired power units, which occurs to compensate the changes in the production of renewable power plants. To model bi-directional gas flow, two non-negative variables, $q_{lg,t,u}^{i,out}$ and $q_{lg,t,u}^{i,in}$ are defined which indicates the input and output gas flow of the pipeline from the node i as shown in Fig. 4. Using a binary variable $v_{lg,t,u}^{i}$, Eqs. (24) and (25) guarantee that on each side of the pipeline, there is either input or output flow rate.

$$q_{lg,t,u}^{i,out} \leqslant M_{big} \times (1 - v_{lg,t,u}^{i}) \quad \forall \ lg \in LG, i \in lg, t \in T, u \in U \tag{24}$$

$$q_{lg,t,u}^{i,in} < M_{big} \times v_{lg,t,u}^{i} \quad \forall \ lg \in LG, i \in lg, t \in T, u \in U \tag{25}$$

Equality (26) is the relationship between gas flow rate and variables which models the input and output flow on either side of each pipe.

$$fg_{lg,t,u} = 0.5 \times \left(q_{lg,t,u}^{i,in} - q_{lg,t,u}^{i,out} + q_{lg,t,u}^{j,out} - q_{lg,t,u}^{j,in}\right) \quad \forall \ lg \in LG, (ij) \in lg, t \in T, u \in U \tag{26}$$

2.3.1. Gas production constraint

Gas production from each well is constrained by minimum and maximum levels modeled by Inequality (27).

$$F_{w}^{G,Min} \leqslant F_{w,t,u}^{G} \leqslant F_{w}^{G,Max} \quad \forall \ w \in W, t \in T, u \in U \tag{27}$$

2.3.2. Gas flow constraints

Eqs. (28) and (29) state the steady state Weymouth equation which are the most common and widely used equations for modeling gas flow rate.

$$fg_{lg,t,u} = sgn(Pr_{i,t,u}, Pr_{j,t,u}) \times K_{lg} \times \sqrt{|Pr_{i,t,u}^2 - Pr_{j,t,u}^2|} \quad \forall \ lg \in LG, (ij) \in lg, t \in T, u \in U \tag{28}$$

$$sgn(Pr_{i,t,u}, Pr_{j,t,u}) = \begin{cases} +1 Pr_{i,t,u} \geqslant Pr_{j,t,u} \\ -1 Pr_{i,t,u} < Pr_{j,t,u} \end{cases} \tag{29}$$

where K_{lg} is constant and depends on the characteristics of the gas pipeline. The pressure difference between nodes i and j determines the direction of the gas flow which is modeled by sign function. These constraints are non-linear and non-convex [42].

To ensure the safety of the gas network, the pressure on each node should be within a certain limit. The minimum and maximum pressures allowed at each node are present in Inequality (30).

$$Pr_i^{Min} \leqslant Pr_{i,t,u} \leqslant Pr_i^{Max} \quad \forall \ i \in GN, t \in T, u \in U \tag{30}$$

2.3.3. Gas storage constraints

Inequalities (31)–(35) enforce the limitation of the maximum capacity of gas storage facilities and the limitation of injection and withdrawal rate from gas storage facilities.

$$MS_{s,t}^{G} \leqslant MS_{s}^{G,Max} \quad \forall \ s \in S^G, t \in T \tag{31}$$

$$MS_{s,t,u}^{G} = MS_{s,t-1,u}^{G} + S_{s,t,u}^{G,in} \times \eta_s^{in} - S_{s,t,u}^{G,out}/\eta_s^{out} \quad \forall \ s \in S^G, t \in T, u \in U \tag{32}$$

$$S_{s,t,u}^{G,in} \leqslant V_{s,t,u}^{St} \times S_s^{Max,In} \ , \ S_{s,t,u}^{G,out} \leqslant (1 - V_{s,t,u}^{St}) \times S_s^{Max,out} \quad \forall \ s \in S^G, t \in T, u \in U \tag{33}$$

$$MS_{s,0}^{G} \leqslant MS_{s,T}^{G} \quad \forall \ s \in S^G \tag{34}$$

$$S_{s,t,u}^{G,in} \geqslant 0, S_{s,t,u}^{G,out} \geqslant 0 \quad \forall \ s \in S^G, t \in T, u \in U \tag{35}$$

2.3.4. PtG constraints

PtG technology can convert the surplus power (especially not absorbed power generated by wind) to hydrogen which can be injected into the gas system. Eqs. (36) and (37) model the amount of hydrogen produced using this technology [8].

$$F_{pg,t,u}^{G} = \eta_{pg}^{PG} P_{pg,t,u}^{PG} / H_{H_2} \quad \forall \ pg \in PG, t \in T, u \in U \tag{36}$$

$$F_{pg,t,u}^{G} \leqslant F_{pg}^{G,Max} \quad \forall \ pg \in PG, t \in T, u \in U \tag{37}$$

2.3.5. Gas balance constraints

The gas balance at each node can be expressed by Equality (38):

$$\sum_{w \in W_i} F_{w,t,u}^{G} + \sum_{lg \in LG_i} (q_{lg,t,u}^{i,out} - q_{lg,t,u}^{i,in}) + \sum_{s \in S_i^G} (S_{s,t,u}^{G,out} - S_{s,t,u}^{G,in}) + \sum_{pg \in PG_i} F_{pg,t,u}^{G} = L_{i,t}^{G} - Loss_{i,t,u}^{G} + \sum_{g \in N_i^G} P_{g,t,u}^{E}/(\eta_g^E \times H_{Gas}) \quad \forall \ i \in GN, t \in T, u \in U \tag{38}$$

Also, the accumulated difference between the withdrawal and injection in node i can be calculated with Equality (39):

$$NG_{i,t,u} = \sum_{lg \in LG_i} \left(q_{lg,t,u}^{i,out} - q_{lg,t,u}^{i,in}\right) \quad \forall \ i \in GN, t \in T, u \in U \tag{39}$$

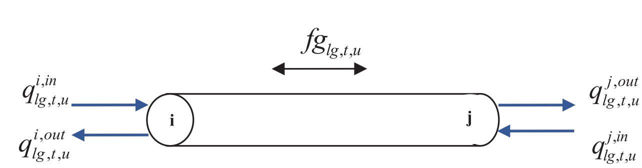


Fig. 4. Gas flow diagram.

2.3.6. Line-pack constraints

Line-packing as a short-term gas storage capability provides flexibility to handle and cover rapid short-term fluctuations and temporary imbalances in gas production or gas loads. By controlling the pressures, the stored gas in the pipeline (line-pack) can increase or decrease by the accumulated difference between the withdrawal and injection gas flow rates. Eqs. (40) and (41) model the line-pack [43]:

$$Lp_{lg,t,u} = Lp_{lg,t-1,u} + \left(q^{i,in}_{lg,t,u} - q^{i,out}_{lg,t,u}\right) + \left(q^{j,in}_{lg,t,u} - q^{j,out}_{lg,t,u}\right) \quad \forall \; lg \in LG, (ij) \in lg, t \in T, u \in U \tag{40}$$

$$Lp_{lg,t,u} = M_{lg} \times \widetilde{P}r_{lg,t,u} \quad \forall \; lg \in LG, t \in T, u \in U \tag{41}$$

where M_{lg} is constant and depends on the characteristics of the gas pipeline, and $\widetilde{P}r_{lg,t,u} = (Pr_{j,t,u} + Pr_{i,t,u})/2$ is the average pressure in each pipeline.

In order to ensure that the gas stored in the gas pipes (line-pack) is suitable for the next scheduling time horizon, Inequality (42) enforces the minimum safe level of line-pack for the end of the time horizon ($t = T$).

$$Lp_{safe} \leqslant \sum_{lg \in LG} Lp_{lg,T,u} \quad \forall \; u \in U \tag{42}$$

2.3.7. Compressor constraint

A compressor is a mechanical device that increases the pressure of a gas by reducing its volume and can be modeled using Inequality (43). The power consumption with compressors is ignored in this paper.

$$pr_{i,t,u} \leqslant pr_{j,t,u} \leqslant \Gamma c_{ij} \times pr_{i,t,u} \quad \forall \; c_{ij} \in C, t \in T, u \in U \tag{43}$$

3. Proposed solution method

As discussed above, this problem is a mixed integer optimization problem with quadratic terms (Weymouth equation) in the constraints (MIQCP). To solve this problem, we suggest a novel solution approach based on a cut generation scheme using Benders' cuts, a model to solve the gas flow equation. The Benders' decomposition decomposes the problem into a master problem and a sub-problem. The master problem aims to find the status of generation units and gas flow rates, and then, the sub-problem checks the feasibility of the solution of the master problem and computes nodal gas pressure. The overall flowchart of the proposed method is depicted in Fig. 5.

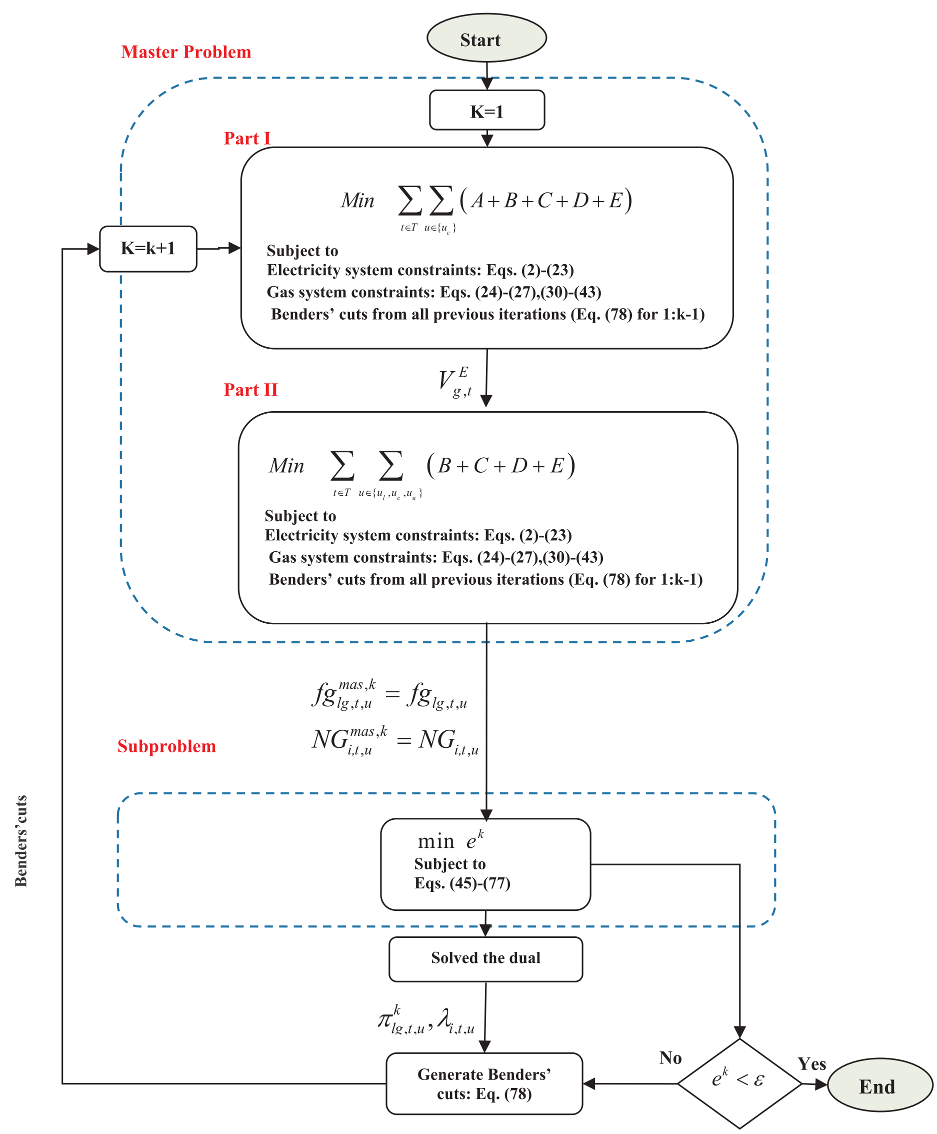


Fig. 5. Process of Benders' decomposition for solving the problem.

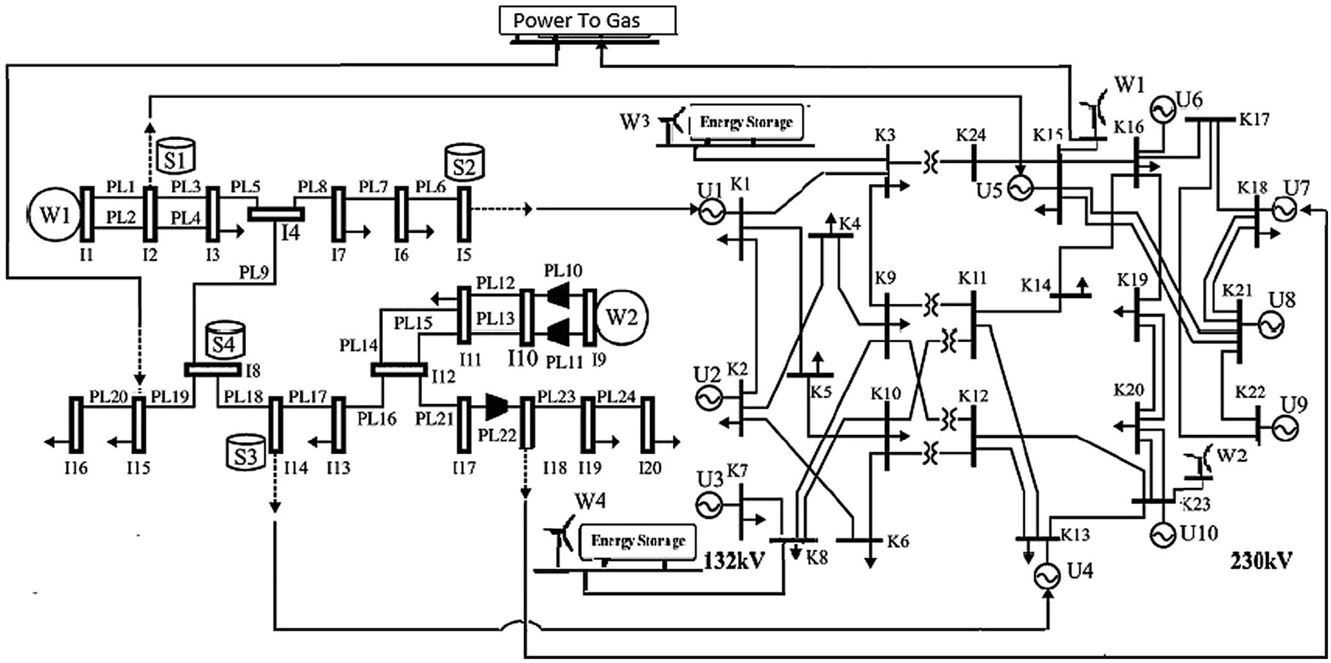


Fig. 6. Modified integrated power and gas system [28].

Table 1
The modified IEEE 24-bus power system and Belgian 20-node gas system.

Modified IEEE 24-bus		Belgian 20-node gas system	
Buses	24	Nodes	20
Branches	38	Pipelines	21
Loads	17	Non-electrical loads	9
Gas-fired power units	4	Wells	2
Coal power units	3	Compressors	3
Hydropower units	3	Natural gas storages	4
Wind sites	4	PtG	1
Storages	2		

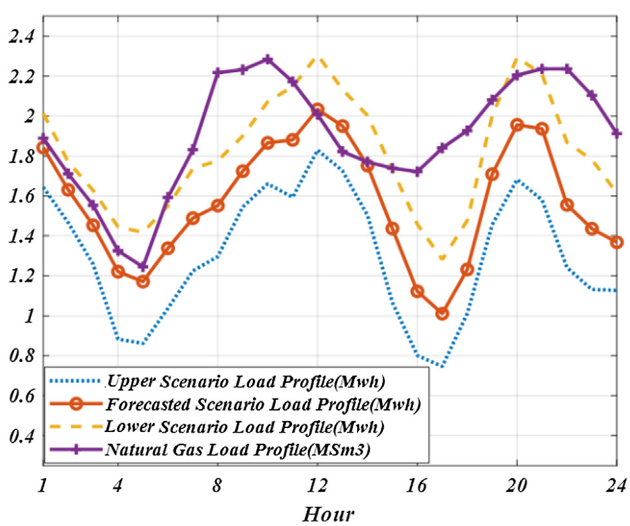


Fig. 7. The nonelectrical natural gas and electrical net-load profiles in three scenarios.

3.1. Master problem

The master problem consists of two parts: the status of generation units (Part I) and gas flow rates (Part II).

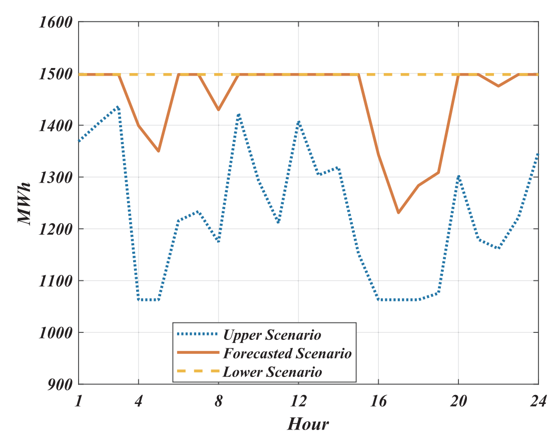


Fig. 8. Total production of gas-fired units in Case I.

3.1.1. Master problem: Part I

Part I solves the unit commitment problem under expected wind scenario (central scenario, u_c). The objective function of part one consists of all *A-E* cost terms which is formulated in Equality (1). The constraints of this part are all electricity system constraints which are expressed in Eqs. (2)–(23), all gas system constraints which are expressed in Eqs. (24)–(43) except the gas flow constraints (Eqs. (28) and (29)) and also includes additional Benders' cuts, which are derived from the sub-problem. Please note that the objective function in Part I is only solved for the central scenario, u_c. The purpose of solving this part is to obtain the status of generation units.

3.1.2. Master problem: Part II

The objective function of part II is the same as the objective function of part I except that the startup and shutdown cost of generation units (term A) is omitted. Thus the objective function consists of *B to G* cost terms. The constraints of this part are also the same as part I. Another important difference between Part I and Part II is that the objective function in part II is solved for all scenarios, $u \in \{u_l, u_c, u_u\}$, and the status of generation units is getting from part I. The purpose of solving this part is to find out nodal gas pressures. In each iteration, the results

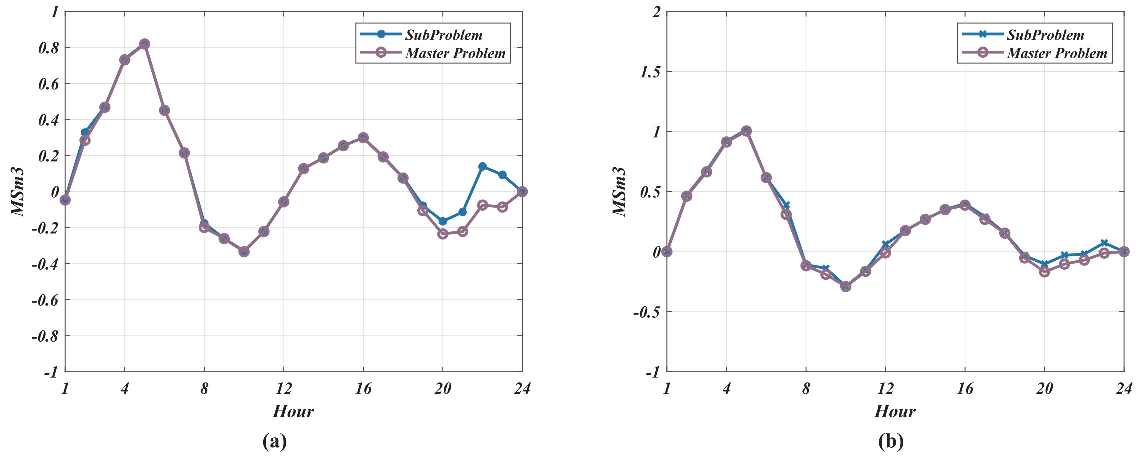


Fig. 9. The net withdrawal and injection gas flow rates in pipelines (a) case II (b) case III.

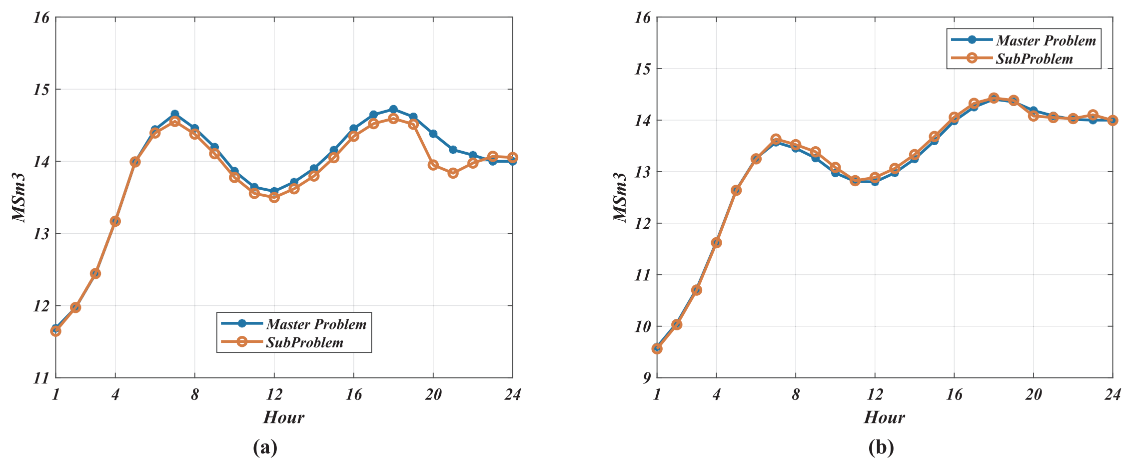


Fig. 10. Total line-pack level for the central forecast (a) case II (b) case III.

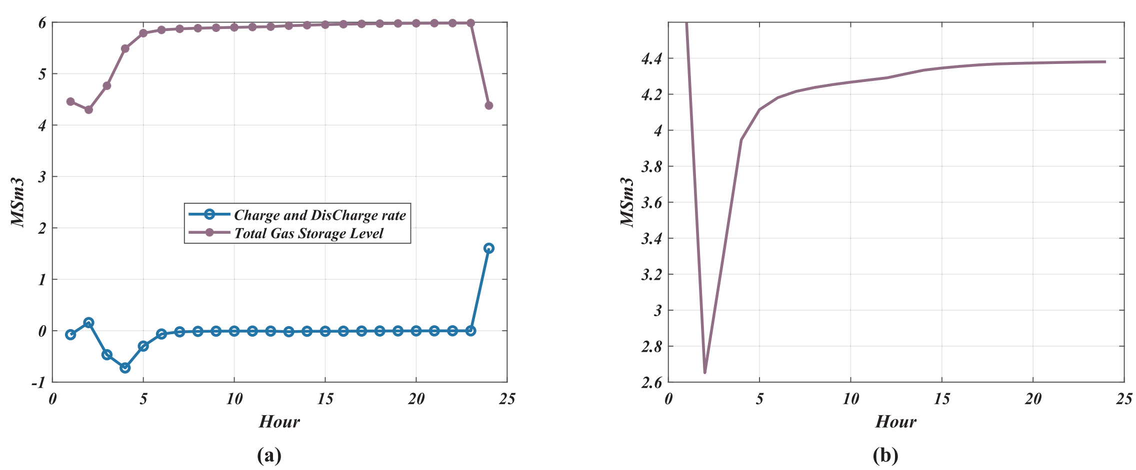


Fig. 11. Total gas storage level in gas storage facilities for the central forecast (a) case II (b) case III.

of the master problem including the status of generation units and gas flow rate will be transferred to the sub-problem for feasibility check and computing nodal gas pressure.

3.2. Sub-problem

Once the master problem calculates the status of generation units and gas flow rates, the sub-problem minimizes the error to find out the nodal gas pressures with respect to constraints (44)–(77). In the sub-problem, we model all constraints that have variables of gas flow rate, fg, and nodal pressure, Pr.

The objective function (44) minimizes the deviations between gas flow rates, gas injection to each node determined by the master problem and the sub-problem and the errors of Weymouth equations. These deviations are modeled by Eqs. (45)–(57).

Please note that the steady state Weymouth equation is initially nonlinear and non-convex. In the literature, two types of relaxation/approximation are commonly used to deal with such a case. First, a

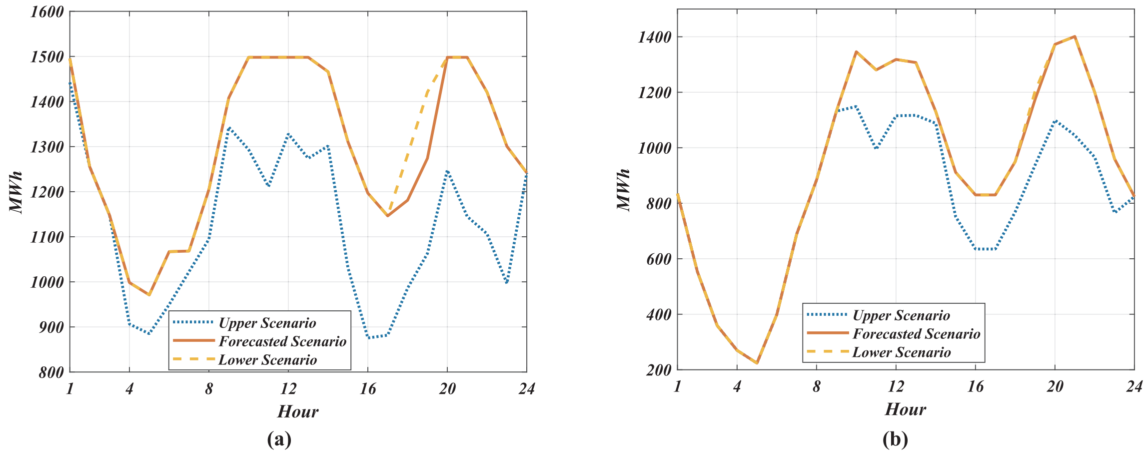


Fig. 12. Total production of gas-fired power units (a) case II (b) case III.

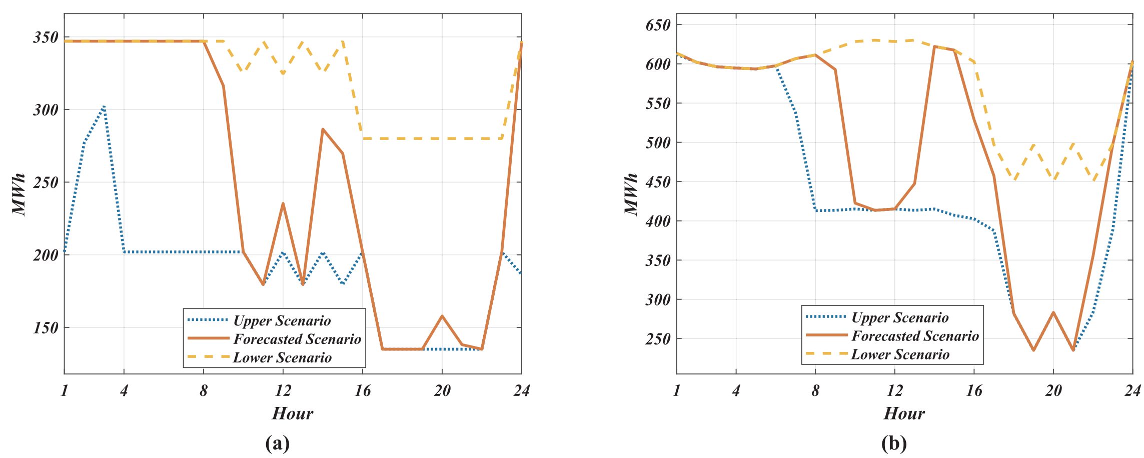


Fig. 13. Total production of coal power units (a) case II (b) case III.

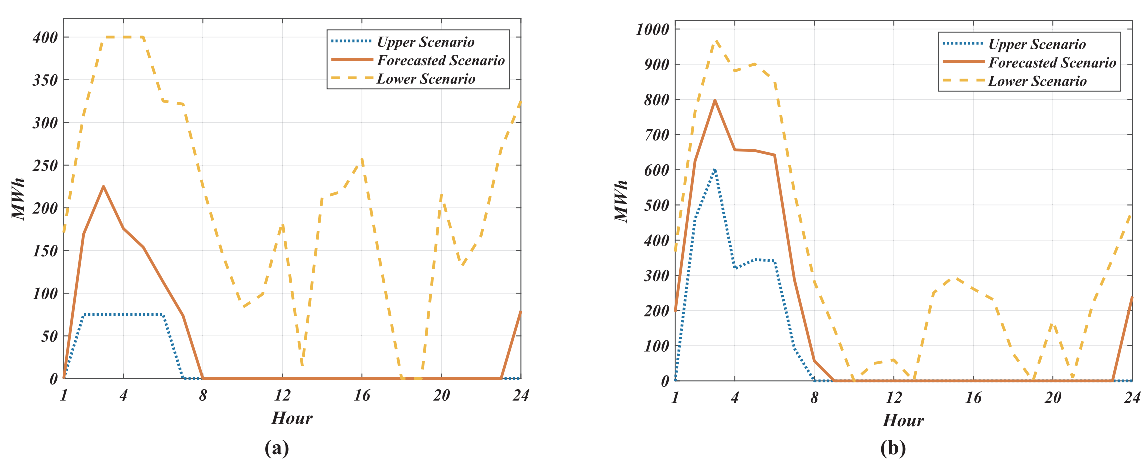
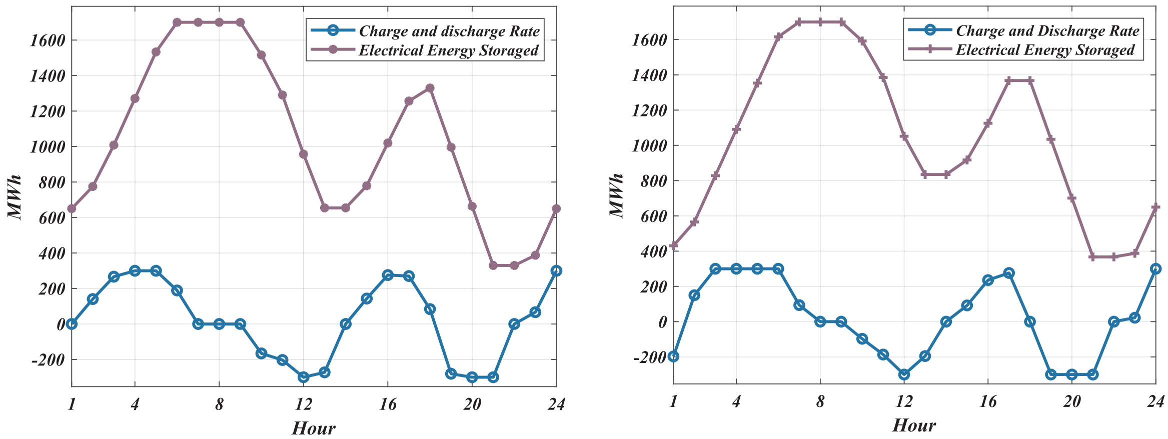


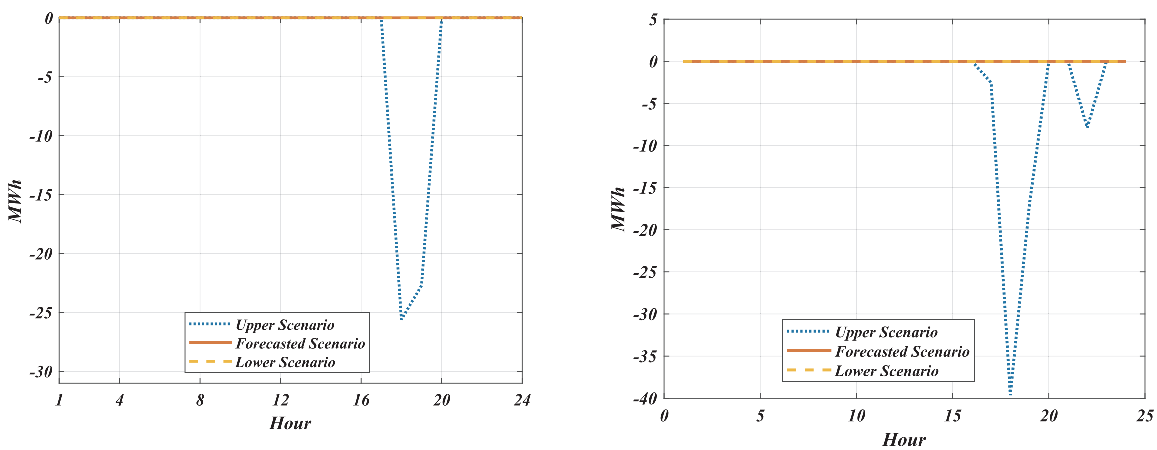
Fig. 14. Total production of hydropower units (a) case II (b) case III.

piecewise linear approximation is applied for the nonlinear term which involves introducing binary variables for each segment. The quality of this type of approximation highly depends on the number of segments. The better approximation, the more binary variables. That is, the approximation may result in a huge problem with too many integer variables [28]. Second, McCormick relaxation is also commonly used. While it is easy to apply this relaxation, it is known to be weak [44]. We propose a relaxation scheme which locally linearizes the nonlinear term based on the solution obtained in the master problem. If the slack variables are not set to zero in the optimal solution of the sub-problem, it means that the flow and balance constraints of gas must be modified. We derive a set of cuts based on the current solution of the subproblem and add it to the master problem. The advantages of the proposed scheme are that no additional variables are needed and also the main objective function remains intact instead it is enforced in the sub-problem.



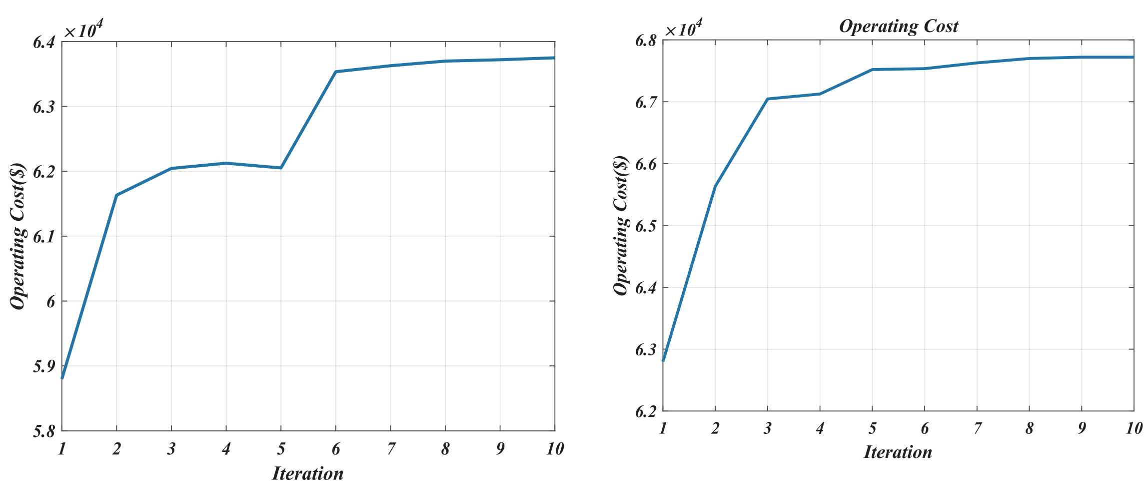
(a) (b)

Fig. 15. Total electrical storage level for the central forecast (a) case II (b) case III.



(a) (b)

Fig. 16. Consumption input of PtG.



(a) (b)

Fig. 17. Total Cost of Operation.

$$\min\ e^k = \sum_{t\in T}\sum_{i}(SB_i^{+,k} + SB_i^{-,k}) + \sum_{t\in T}\sum_{lg\in LG}\sum_{u\in U}(SF_{lg,t,u}^{+,k} + SF_{lg,t,u}^{-,k}) + M_{big}\times\sum_{t\in T}\sum_{lg\in LG}\sum_{u\in U}(xb_{lg,t,u}^{k} + xc_{lg,t,u}^{k}) \tag{44}$$

Subject to

$$fg_{lg,t,u}^{sub} = SF_{lg,t,u}^{+,k} - SF_{lg,t,u}^{-,k} + fg_{lg,t,u}^{mas,k} \qquad : \pi_{lg,t,u}^{k} \quad \forall\ lg\in LG,\ t\in T,\ u\in U \tag{45}$$

$$NG_{i,t,u}^{sub} = SB_i^{g+,k} - SB_i^{g-,k} + NG_{i,t,u}^{master,k} \qquad : \lambda_{i,t,u} \quad \forall\ i\in GN,\ t\in T,\ u\in U \tag{46}$$

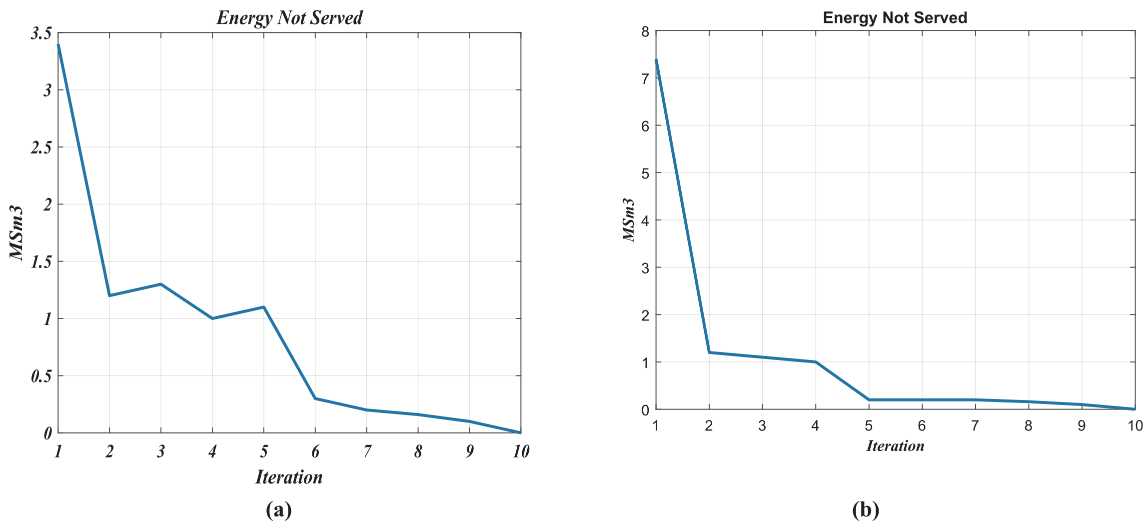


Fig. 18. Total cost of the sub-problem (energy not served).

The direction of the gas flow rate, which has been achieved in the master problem is modeled by Eqs. (47)–(50). Also, Eqs. (51)–(54) determines the direction of the gas flow rate in the sub-problem.

$$fg_{lg,t,u}^{mas,k} = fg_{lg,t,u}^{mas,+} - fg_{lg,t,u}^{mas,-} \quad \forall\, lg \in LG,\, t \in T,\, u \in U \tag{47}$$

$$fg_{lg,t,u}^{mas,+} \leqslant M_{big} \times xv_{lg,t,u}^{mas} \quad \forall\, lg \in LG,\, t \in T,\, u \in U \tag{48}$$

$$fg_{lg,t,u}^{mas,-} \leqslant M_{big} \times (1 - xv_{lg,t,u}^{mas}) \quad \forall\, lg \in LG,\, t \in T,\, u \in U \tag{49}$$

$$fg_{lg,t,u}^{mas,+} \geqslant 0,\, fg_{lg,t,u}^{mas,-} \geqslant 0 \quad \forall\, lg \in LG,\, t \in T,\, u \in U \tag{50}$$

$$fg_{lg,t,u}^{sub} = fg_{lg,t,u}^{sub,+} - fg_{lg,t,u}^{sub,-} \quad \forall\, lg \in LG,\, t \in T,\, u \in U \tag{51}$$

$$fg_{lg,t,u}^{sub,+} \leqslant M_{big} \times xv_{lg,t,u}^{sub} \quad \forall\, lg \in LG,\, t \in T,\, u \in U \tag{52}$$

$$fg_{lg,t,u}^{sub,-} \leqslant M_{big} \times (1 - xv_{lg,t,u}^{sub}) \quad \forall\, lg \in LG,\, t \in T,\, u \in U \tag{53}$$

$$fg_{lg,t,u}^{sub,+} \geqslant 0,\, fg_{lg,t,u}^{sub,-} \geqslant 0 \quad \forall\, lg \in LG,\, t \in T,\, u \in U \tag{54}$$

In the master problem, the gas flow rate equations (Eqs. (28) and (29)) is not modeled and the gas flow rates are achieved only by Eq. (26). The gas flow rate equations (Eqs. (28) and (29)) is non-linear and non-convex, these equations can be written as Eq. (55).

$$\begin{aligned} & fg_{lg,t,u}\, |fg_{lg,t,u}| \\ & \quad = K_{lg}^2 \times (Pr_{i,t,u}^2 - Pr_{j,t,u}^2) \quad \forall\, lg \in LG,\, (ij) \in lg,\, t \in T,\, u \in U \end{aligned} \tag{55}$$

By replacing Pr^2 with xpr and $fg\,|fg|$ with xfg, Eq. (55) can be simplified as Eq. (58). The steady state Weymouth equation can be rewritten as two inequality: one is already a convex constraint, i.e., $y \geqslant x^2$, while the other one is not convex i.e., $y \leqslant x^2$. To convexify this term, we approximate it around the initial solution obtained from the master problem i.e., we rewrite the non-convex constraint as $error \geqslant y - x \times x'$ (Eqs. (56) and (57)) where the error (xb and xc) is a positive variable which is penalized in the objective function and x' (fg^{mas}) is the initial solution value computed in the master problem. If the error is not set to zero, another constraint which connects the master problem and sub-problem will be violated. In this case, using a cut plane scheme known as feasibility cuts which ensure the feasibility of the gas network is derived and imposed to the master problem. As can be observed, the non-convex problem is reformulated by a linear program. So the steady state Weymouth equation is replaced by Eqs. (56)–(65).

$$xb_{lg,t,u} \geqslant xfg_{lg,t,u}^{sub,+} - fg_{lg,t,u}^{mas,+} \times fg_{lg,t,u}^{sub,+} \quad \forall\, lg \in LG,\, t \in T,\, u \in U \tag{56}$$

$$xc_{lg,t,u} \geqslant xfg_{lg,t,u}^{sub,-} - fg_{lg,t,u}^{mas,-} \times fg_{lg,t,u}^{sub,-} \quad \forall\, lg \in LG,\, t \in T,\, u \in U \tag{57}$$

$$xfg_{lg,t,u}^{sub} = {K_{lg}}^2 \times (xpr_{i,t,u} - xpr_{j,t,u}) \quad \forall\, lg \in LG,\, (ij) \in lg,\, t \in T,\, u \in U \tag{58}$$

$$xfg_{lg,t,u}^{sub} = xfg_{lg,t,u}^{sub,+} - xfg_{lg,t,u}^{sub,-} \quad \forall\, lg \in LG,\, t \in T,\, u \in U \tag{59}$$

$$xfg_{lg,t,u}^{sub,+} \leqslant M_{big} \times xv_{lg,t,u}^{sub} \quad \forall\, lg \in LG,\, t \in T,\, u \in U \tag{60}$$

$$xfg_{lg,t,u}^{sub,-} \leqslant M_{big} \times (1 - xv_{lg,t,u}^{sub}) \quad \forall\, lg \in LG,\, t \in T,\, u \in U \tag{61}$$

$$xfg_{lg,t,u}^{sub,+} \geqslant 0,\, xfg_{lg,t,u}^{sub,-} \geqslant 0 \quad \forall\, lg \in LG,\, t \in T,\, u \in U \tag{62}$$

$$(pr_{i,t,u})^2 \leqslant xpr_{i,t,u} \quad \forall\, i \in GN,\, t \in T,\, u \in U \tag{63}$$

$$(fg_{lg,t,u}^{sub,+})^2 \leqslant xfg_{lg,t,u}^{sub,+} \quad \forall\, lg \in LG,\, t \in T,\, u \in U \tag{64}$$

$$(fg_{lg,t,u}^{sub,-})^2 \leqslant xfg_{lg,t,u}^{sub,-} \quad \forall\, lg \in LG,\, t \in T,\, u \in U \tag{65}$$

The relationship between gas flow rate and variables which models the input and output flow on either side of each pipe, the compressor constraints, the line-pack constraints, and the minimum and maximum pressure constraints are written again in Eqs. (66)–(69), Eqs. (70) and (71), Eqs. (72) and (73), and Eqs. (74)–(77), respectively, to be used in the sub-problem. Also Eq. (71) and Eq. (73) are used for the new variable xpr.

$$\begin{aligned} & fg_{lg,t,u}^{sub} = 0.5 \times \left(q_{lg,t,u}^{i,in} - q_{lg,t,u}^{i,out} + q_{lg,t,u}^{j,out} - q_{lg,t,u}^{j,in} \right) \quad \forall \\ & \qquad lg \in LG,\, (ij) \in lg,\, t \in T,\, u \in U \end{aligned} \tag{66}$$

$$q_{lg,t,u}^{i,out} \leqslant M_{big} \times (1 - v_{lg,t,u}^{i}) \quad \forall\, lg \in LG,\, i \in lg,\, t \in T,\, u \in U \tag{67}$$

$$q_{lg,t,u}^{i,in} < M_{big} \times v_{lg,t,u}^{i} \quad \forall\, lg \in LG,\, i \in lg,\, t \in T,\, u \in U \tag{68}$$

$$NG_{i,t,u}^{sub} = \sum_{lg \in LG_i} \left(q_{lg,t,u}^{i,out} - q_{lg,t,u}^{i,in} \right) \quad \forall\, i \in lg,\, t \in T,\, u \in U \tag{69}$$

$$pr_i^{Min} \leqslant pr_{i,t,u} \leqslant pr_i^{Max} \quad \forall\, i \in GN,\, t \in T,\, u \in U \tag{70}$$

$$(pr_i^{Min})^2 \leqslant xpr_{i,t,u} \leqslant (pr_i^{Max})^2 \quad \forall\, i \in GN,\, t \in T,\, u \in U \tag{71}$$

$$pr_{i,t,u} \leqslant pr_{j,t,u} \leqslant \Gamma c_{ij} \times pr_{i,t,u} \quad \forall\, c_{ij} \in C,\, t \in T,\, u \in U \tag{72}$$

$$xpr_{i,t,u} \leqslant xpr_{j,t,u} \leqslant (\Gamma c_{ij})^2 \times xpr_{i,t,u} \quad \forall\, c_{ij} \in C,\, t \in T,\, u \in U \tag{73}$$

$$Lp_{lg,t,u} = Lp_{lg,t-1,u} + (q^{i,in}_{lg,t,u} - q^{i,out}_{lg,t,u}) + (q^{j,in}_{lg,t,u} - q^{j,out}_{lg,t,u}) \quad \forall lg \in LG, (ij) \in lg, t \in T, u \in U \tag{74}$$

$$Lp_{lg,t,u} = M_{lg} \times pr_{g,t,u} \quad \forall lg \in LG, t \in T, u \in U \tag{75}$$

$$pr_{lg,t,u} = (pr_{j,t,u} + pr_{i,t,u})/2 \quad \forall lg \in LG, (ij) \in lg, t \in T, u \in U \tag{76}$$

$$Lp_{safe} <= \sum_{lg \in LG} Lp_{lg,T,u} \quad \forall u \in U \tag{77}$$

Once the sub-problem is solved, if the optimal objective value e^k is greater than the predefined threshold ε, the feasibility Benders' cuts represented in Eq. (78) are generated from dual values and added to the master problem.

$$\begin{aligned}&\sum_{t\in T}\sum_{lg\in LG}\sum_{u\in U}(\pi^k_{lg,t,u} \times (fg^{mas,k}_{lg,t,u} - fg^{mas,k-1}_{lg,t,u}))\\&+\sum_{t\in T}\sum_{lg\in LG}\sum_{u\in U}(SF^{+,k}_{lg,t,u} + SF^{-,k}_{lg,t,u})\\&+\sum_{t\in T}\sum_{i\in GN}(\lambda^k_{i,t,u} \times (NG^{mas,k}_{i,t,u} - NG^{mas,k-1}_{i,t,u}))\\&+\sum_{t\in T}\sum_{i\in GN}(SB^{+,k}_i + SB^{-,k}_i) <= 0\end{aligned} \tag{78}$$

4. Case study

In this paper, one PtG technology, four wind power units, and two electricity storage facilities are added in the modified IEEE 24-bus power system and Belgian 20-node gas presented in [33] and [28]. The details of both systems are shown in Fig. 6 and Table 1.

Wind power generation in wind units are molded in three scenarios as follows:

(1) The central forecast: the most probable forecast from wind power plants.
(2) The lower scenario: The scenario for the least possible electric energy generated by wind power units.
(3) The upper scenario: The scenario for the maximum possible electric energy generated by wind power units.
(4) According to these scenarios, the net-load profile (the total electrical demand in the power system minus renewable generation) and also the non-electrical natural gas load profile are presented in Fig. 7. We perform our computations on a 64-bit PC with 3.8-GHz CPU and 8-GB RAM in MATLAB and in the GUROBI environment.

Case I: In this case, the constraints of the gas network are not taken into account except the constraints of the capacity of gas resources. The cost of producing natural gas from wells is considered as the only cost of the gas network in the objective function. The results show that the gas-fired units generate electricity at their maximum level in the center and the lower scenario while they follow wind generation units in the upper scenario (see Fig. 8).

Case II: In this case, all constraints of both electricity and gas system are taken into account. We set the initial line-pack of the grid is 11.7 million m^3 at the beginning of the day while we assume that Lp_{safe} is set to 14 million m^3.

The efficiency of the proposed method*:* It is worth mentioning that we implemented the standard model presented by [28] for Case II where the piecewise linear function approximating the non-linear gas constraints included only 40 pieces. The resulting problem contains +56,000 binary variables. We were not able to obtain even a bound for this instance after 1 h. As we will discuss in detail, the proposed method was able to solve all test problems.

Case III: same as Case II, but in this case, all constraints of both electricity and gas system are assumed. However, the initial line-pack of the grid is set to 9.6 m^3 at the beginning of the day and the network must be programmed with the minimum line-pack. The operation cannot use the sources of the line-pack for the first hours of the day because of a contingency during the day before.

According to the load curve (Fig. 7), the morning peak load of power and that of the gas network (non-electrical) occur with a two-hour difference i.e., in 10 and 12 o'clock, respectively. Also, the second (night) peak times for power and gas network occur at 20 and 22 o'clock, respectively. Consequently, the gas network must have the capability to supply the gas required for both the gas network and gas-fired units.

With respect to the constraints of the gas network, in cases II, the gas network tries to inject the gas into pipelines (Fig. 9(a)) to increase line-pack level (Fig. 10(a)) and to increase gas storage level in storage facilities (Fig. 11(a)) at hours 1–7 to ensure that it can provide the gas for morning peak at 10 a.m. and to increase line-pack level at hours 12–18 for night peak time. In case III, because of the low level of the line-pack at the beginning of the day, the level of the line-pack must be increased in order to be able to transfer gas through the network during peak hours. For this purpose, extracted gas from wells and gas storages are injected into the gas network, which results in another reduction in the gas storage levels as shown in Fig. 11(b). Also, gas supply for gas-fired power units is reduced as shown in Fig. 12(b).

As shown by Fig. 12, in both cases, total production of gas-fired power units is at their minimum level at 5 a.m. But in case III, total production of gas-fired power units in this hour is lower than the case II and reaches 200 MWh. In addition, despite the low cost of the gas-fired power units and high flexibility with respect to coal plants, it is expected that total production of gas-fired power units will be at its maximum level in both central and lower scenarios as in the case I. But the constraints of the gas network limit the production of gas-fired power units. Given the peak of the natural gas profile at 10 a.m., the gas-fired power units cannot support the center and upper load scenarios, i.e., the usage of gas-fired power units with maximum level is not possible. This implies that the flexibility of gas fired units will be limited by the gas network. Due to the limitation of capacity and the high cost of production, the hydropower units are dispatched only in the first hours of the day (Fig. 14). As shown in Figs. 13 and 14, coal fuel and hydropower units compensate the wind generation fluctuations and help gas-fired units at the first hours of the day under study.

In both cases, as shown in Fig. 15, at the first hours of the day, electricity storage facilities are charged and then, at the electricity profile's peak times, inject electricity to the network. Further, the storages have been charged between 13 and 17 p.m. and discharged during the night peak. Also, as can be seen in Fig. 16, PtGs transform the surplus energy of wind generations to gas network during hours 17–19 in the upper scenario.

In Fig. 17 the convergence of sub-problem is illustrated. As shown in Fig. 18 the algorithm converges in 10 iterations in which the slack variables related to the problem feasibility are zero and the percentage error of problem linearization is less than 0.01%.

5. Conclusion

In this paper, we studied an important variant of unit commitment problem in which uncertain renewable energy resources, gas and electricity storage and Power to Gas (PtG) are taken into account in order to enhance the security of power generation and optimize its operational cost considering. Due to the complexity of the resulting problem, non-linear constraints of the gas network are not usually integrated into the electricity network and power operation problem. Ignoring the gas network in a power system with high renewable energy penetration significantly increases operation costs. Gas and electricity storage and Power to Gas were formulated to improve the reliability of power system and minimize the undesirable impact of uncertainty of renewable energy resources on short term power generation. The resulting problem belongs to a class of complex problems known as non-linear non-convex mixed integer problems. We explored the structure of the problem and applied decomposition and convexification techniques to reformulate and solve it. Interval Optimization which is a conservative, but practical approach is used to invoke the uncertainty of renewable energy generation. Using the aforementioned integrated problem, we found a **better** near optimal values for key decision variables (including gas pressure in the network, line-pack level, the power generated by the gas fired unit plan, the level of gas and electricity storage and PtG) which was not achievable using existing methods for relatively large problems.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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