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# A Hybrid Fuzzy Regression - SSA Approach for Electricity Consumption Optimisation

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*The critical impact of electricity on the quality of human life and the use of all electrical appliances and equipment is in principle self-evident. Nevertheless, governments cannot afford even productivity to consumption. The countries developed many structures to operate and improve the city and industrial electricity consumption and to manage power generation efficiency. Increase in population and the culture of consumption force the governments to raise the price. However, it is difficult to find ways and means to effectively manage trends in public behaviour and to control the harmful action of the population. This process requires knowing the need and consumption of electricity. The article discusses the primary criterion that influences electricity consumption and uses the singular spectrum analysis based on these factors to predict use. Besides, a fuzzy regression model is represented to optimise function. Results of optimisation show a considerable reduction in comparison with SSA forecasting method, indicating the efficiency of the offered method. Eventually results considerably assume that attention to a way of construction and improvements of the culture of use is a priority of the persons making decisions to reduce radical electric consumption in Iran and to become more optimistic concerning management of an electrical network.*

Keywords: *Singular Spectrum Analysis; Electrical Demand; Electrical Energy Consumption; Fuzzy Regression; Prediction*.

## Introduction

Electrical power is an inevitable part of the sustainable development of countries. Measuring business cycles provides a reference point for assessing macroeconomic theory and policy (Skare & Stjepanovic, 2016). Economic expansions and slowdowns are inherent in modern financial systems and impact energy and resources consumptions. The problems involve a multitude of requirements and uncertain conditions that have to be taken into consideration simultaneously (Hashemkhani Zolfani *et al.*, 2013). Economic, social and ecological components of development have both direct and reverse impacts (Ginevicius *et al.* 2018). An appropriate choice at an early stage of a project is crucial regarding adding value over scope, time and total investment strategic decisions (Saparauskas *et al.*, 2011). Analysis helps to rate the electricity generation technologies and consumption considering their economic, technological, environmental social and political aspects (Medineckiene *et al.*, 2015; Streimikiene *et al.*, 2016). Researchers proposed dozens of different utility aggregation functions for the resolution of similar problems by employing multi-criteria as an aid (Zavadskas *et al.*, 2015c; Bagocius *et al.*, 2014; Zavadskas *et al.*, 2015a; 2015b; 2015c).

Consumption and production are two dynamically changing life-changing sides and depends on dozens of factors (Zavadskas *et al.*, 2009). Consumption as a mean to satisfy ones' needs has been investigated mainly by using economic and psychological approaches (Taujanskaite *et al.*, 2015). The amount of effective used electricity amount indicates the economic prosperity of nations (Tunc *et al.*, 2006). Shiu & Lam (2004) investigated the relationship between electricity consumption and economic growth and found out that the electricity usage increases the amount of GDP. Due to the increase of urbanisation besides the industries EC needs to be monitored and reduced (Pampuri *et al.*, 2016).

Figure 1 shows the rate of electricity consumption (EC) comparison between countries in kWh from 1990 to 2012. Arab countries in the Middle East, like UAE, are consuming electrical power more than Iran. Nonetheless, the pestilent point is that all four considered countries are emphasising to reduce the consumption rate and conspicuously is evident that a declination path in these countries policies is in progress. Iran's strategy is to subsidise electricity for consumers, to buy the produced power from the private sector and provide it to the final consumer as a support act (BEEP, 2011). Besides the industrial usage, the residential sector and buildings are responsible for 33.2 % of Iran's EC. Considering a 41 % growth of this section usage since 1989 and a 7.9 % growth rate of total per capita EC besides of 3.3 % of global concerns us to control and predict Iran's EC, otherwise, by the next decade Iran may face a devastating situation of EC by two times (Pourazarm & Cooray, 2013). A growing trend highlights the need to integrate environmental sustainability into efforts related to new product development, energy consumption, and resources use. It is even operating in different countries and sectors, an essential driver for the proper choice (Jugend *et al*., 2017). However, their short-run economic performance does not correspond to the observed business cycles of other global emerging markets. The business cycles of these countries are longer and more pronounced their recessions. Economic activity in the studied countries is relatively low and volatile, and the trade balance and government purchases have a relatively significant countercyclical character (Szomolanyi *et al*., 2017). To manage the development of regions and thus reduce the social tensions in the country, we need to be able to assess the state of the main components of sustainable development (economic, social and ecological development) at a certain period in time, as well as to determine their interrelationships.

The use of quantitative criteria limits the influence of subjective, non-economic factors on consumption-related resource management and can positively affect its efficiency. Processes and phenomena vary significantly according to the nature of many qualities – physical, chemical, social, and others. The need for the development of socioeconomic systems also arises since they are open; thus, they exposed continuously to the changing environment (Ginevicius *et al*., 2018). Selection of the appropriate management options for a definite machining requirement observed as multi-criteria decision-making (MCDM) problem with different criteria. Chatterjee *et al*. (2017) proposed a novel hybrid method encompassing factor relationship (FARE) (Ginevicius, 2011) and multi-attributive border approximation area comparison (MABAC) methods to select and evaluate feasible alternatives (Pamucar *et al*., 2015).

Typically, the study of market behaviour under the demand and supply law graphically represented by the traditional demand and supply curves embodied in two-dimensional graphs. Meanwhile, multidimensional real-time dynamic economic behaviour (including consumption) is a problem that affects market behaviour. The 2-dimensional and 3-dimensional spaces not able to capture the action of the real industrial world as a whole. It is clear that the real world is continually changing and inside the Mega-Space or Universe, the number of General-Spaces, Sub-Spaces, Micro-Spaces, Nano-Spaces and JI-Spaces. All these spaces move differently and across time. Besides, Ruiz Estrada *et al*. (2016) mentioned the roles of Euclidian geometry and Minkowski's n-dimensional spaces (Einstein, 1961) supporting this proposition. One of the common challenges that exist throughout the World is to compiling, distribution, and sharing of knowledge ready for practice as a critical element for socio-economic development and progress acceleration (Ziemianczyk *et al*., 2017).

Scholars have presented both linear and non-linear approaches to overcome EC crisis. Kermanshahi (1998) forecasted a long term load of the total power demand amount, Abdel-Aal (2006) investigated electric daily peak loads using abductive networks. Kermanshahi & Iwamiya (2002) used two different artificial neural networks for long-term load forecasting. Buhari & Adamu (2012) forecasted a short term load using an artificial neural network. Scholars have furthermore implemented the important heuristic approaches like a genetic algorithm. Ozturk *et al*. (2004) suggested a generic algorithm notion of estimating EC of residential-commercial sectors, while Canyurt *et al*. (2007) modelled and applied Genetic Algorithm approach to predict the future total energy input values.

Engineers, based upon their ability and experience to design, analyse, and synthesise (Arabzad *et al*., 2015), play a vital role in capital investment decisions (Karaulova & Bashkite, 2016). The proposed model y determines and forecasts electricity consumption in Iran by employing a hybrid approach consisting of Singular Spectrum Analysis (SSA) in conjunction with fuzzy regression methodology to optimise the fitness function. This model enables them to design suitable policies to prevent any possible disaster or ascending trend in power usage. The remainder of the research is organised as follow. Initially SSA methodology and related steps and formulas have been considered; afterwards, the methodology's preliminaries compromising fuzzy concepts are finalised. Consequently, fuzzy regression and its essential foundations are introduced; furthermore, by proposing the optimisation model, the electricity consumption will be controllable. The proposed approach is applied to Iran electricity consumption, and the related objective function is emanated, and by employing a fuzzy optimisation approach, the desired decision variables to achieve minimum electricity consumption are obtained.

## Singular Spectrum Analysis (SSA)

The first principles of SSA emerged in the scene of forecasting methods by Broomhead & King (1986) and Pike *et al*. (1984) works, and since then the non-parametric method is proved to be an accurate method. Hassani (2007) investigated this model and compared it with Box-Jenkins SARIMA models, the ARAR and the Holt-Winter algorithm. One of the critical features of this technique is that the prediction is deployed without prior knowledge of the underlying structure (Ghanati 2016; Zhigljavsky 2011; Chu *et al*., 2014). Many scholars have accepted the uppermost prediction technique. Carniel *et al*. (2006) performed SSA to improve Nakamura spectral ratios. Ghaderi *et al*. (2011) applied SSA to Localize heart sounds in respiratory. As a case in point, an investigation on UK tourism income concluded SSA is outperforming. SARIMA is another proof of this methods validity (Beneki *et al*., 2012). Golyandina & Korobeynikov (2014) described the availing R package for SSA. There is a finite number of SSA-fuzzy researches. Abdollahzade *et al*. (2015) investigated a local linear neuro-fuzzy model and optimised singular spectrum analysis method to forecast nonlinear and chaotic time series. Furthermore, Xiao *et al*. (2014) combined singular spectrum analysis, adaptive-network-based fuzzy inference system and improved particle swarm optimisation to forecast air transport demand.

SSA consists of two parts: Decomposition & Reconstruction (Hassani & Mahmoudvand, 2013). The two choices referred to as window length $L$ and number of eigenvalues $r$, and each of these stages comprises two steps called Embedding, Singular Value Decomposition (SVD) and grouping and diagonal averaging. The rest of methodology has been mainly taken from (Chang *et al*., 2015; Plaza & Lopez, 2017; Hassani, 2007; Hassani *et al*., 2015; Maddirala & Shaik, 2016). The first step presents mapping the original single dimension time series to multi-dimensional lagged vectors. Assuming $u_m[n]$ as an observed data from the $m^{th}$ channel at the $n^{th}$ time step, then the $u[n]$ vector is defined as below:

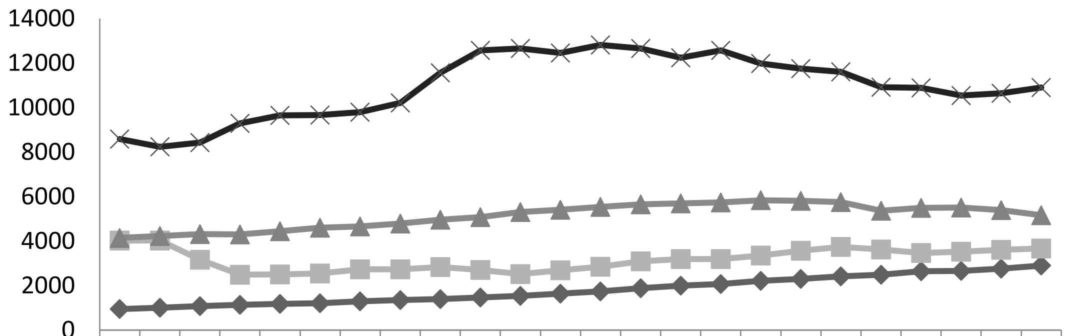
$$u[n] = \begin{bmatrix} u_1[n] \\ u_2[n] \\ \vdots \\ u_M[n] \end{bmatrix}, n = 1 \sim N \tag{1}$$

Considering $N$ as the number of sampling points, the $U$ matrix is embedded with a raw size of $ML$ and column size $k$ as follows:

$$U = \begin{bmatrix} u[1] & u[2] & \cdots & u[k] \\ u[2] & u[3] & \cdots & u[k+1] \\ \vdots & \vdots & \ddots & \vdots \\ u[L] & u[L+1] & \dots & u[N] \end{bmatrix} k = N - L + 1 \; and \; L < k \tag{2}$$

The matrix $U$ is defined as the trajectory matrix being a Henkel matrix formed as $\left(U_{ij}\right)_{i,j=1}^{L,k}$. $u_i$ is the $L$-lagged vectors. It is worth noting here that, the parameter $L$ refers to the window length, this is one of the most critical choices in the process of SSA methodology as the condition of signal decomposition results depends on this factor (García Plaza & Núnez Lopez, 2017). With this fact in mind, remark that choosing the window length too long provides more detailed signal analysis but hinders the identification of trend components, the dynamic elements, and the noise components from the original signal. Although narrowing the amount of the $L$ value simplifies the analysis; nonetheless, there might be a lack of principle component resulting deficient signal decomposition. Maddirala & Shaik (2016) pointed out that criteria $L$ are chosen upon the criteria $M > \frac{f_s}{f}$, where $f_s$ is supposed as a sampling frequency while $f$ is the frequency of the signal of interest. This stage is the embedding stage. This procedure forms $k$ lagged vectors ($k = N - L + 1$) and maps the time series with length $N$ to an $ML$ time-delay series with length $k$ (Bozzo *et al*., 2010).

Decomposing the trajectory matrix into a unit-rank elementary matrix $U_i$ using singular value decomposition.




| | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Iran | 944,2 | 1001 | 1077 | 1131 | 1179 | 1206 | 1299 | 1344 | 1388 | 1467 | 1541 | 1635 | 1735 | 1878 | 1999 | 2069 | 2210 | 2295 | 2409 | 2491 | 2642 | 2662 | 2762 | 2899 |
| Lithuania | 4023 | 4023 | 3167 | 2494 | 2499 | 2539 | 2735 | 2728 | 2826 | 2702 | 2517 | 2696 | 2849 | 3088 | 3197 | 3187 | 3353 | 3564 | 3734 | 3622 | 3471 | 3530 | 3608 | 3664 |
| Italy | 4145 | 4225 | 4311 | 4304 | 4439 | 4598 | 4659 | 4791 | 4956 | 5079 | 5300 | 5406 | 5540 | 5653 | 5688 | 5731 | 5833 | 5804 | 5758 | 5368 | 5494 | 5515 | 5398 | 5159 |
| UAE | 8580 | 8236 | 8423 | 9293 | 9650 | 9663 | 9788 | 10217 | 11559 | 12566 | 12653 | 12454 | 12815 | 12660 | 12238 | 12571 | 11985 | 11754 | 11604 | 10911 | 10891 | 10537 | 10636 | 10904 |

**Figure 1.** Electricity Consumption per capita (kWh) (*Source: The World Bank*)

(SVD) is the next step. The decomposition procedure is from the calculated eigenvalues ($\lambda_i$) and eigenvectors ($\theta_i$) of a matrix $P$ of dimension $L \times L$ defined by the expression $P = UU^T$. The trajectory matrix is determined by the sum of elementary matrices $U = \sum_{i=1}^{d} U_i$, where $U_i = \sqrt{\lambda_i}\theta_i V_i^T (i = 1, \dots, d)$,$V_i = U^T\theta_i / \sqrt{\lambda_i}$ and the rank of $U$, $d = max\{i, \lambda_i > 0\}$. The matrices $U_i$ are elementary matrices as they have rank 1. The collection $(\sqrt{\lambda_i}, \theta_i, V_i)$ is called the *i-th* Eigen triple of the matrix $U$, $\sqrt{\lambda_i}$ $(i = 1, \dots, d)$ are the singular values of the matrix $U$ and the set $\left\{\sqrt{\lambda_i}\right\}$ is considered to be the spectrum of the matrix U. by the mean of the equation $A = \lambda_i / \sum_{i=1}^{d} \lambda_i$ the weight of each matrix $U_i$ on the trajectory matrix $U$ from their associated eigenvalues is figured. The graphical designation of the specific weight of the eigenvalues ($A$) in decreasing order of magnitude ($\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_L$) is the singular spectrum.

In toward the second stage of SSA methodology, an individual or grouping analysis is performed to reconstruct the elementary matrices $U_i$ and to do so, we split the $U_i$s into several groups and sum the matrices within each group. Designating $I = \{i_1, \dots, i_p\}$ as a group of indices $i_1, \dots, i_p$, then the matrix $U_I$ which corresponds to the group $I$,is interpreted as $U_I = U_{i1} + \dots + U_{ip}$. The split of the set of indices $J = 1, \dots, d$ into the disjoint subsets $I_1, \dots, I_m$ corresponds to the representation.

$$U = U_{I1} + \cdots + U_{Im}. \tag{3}$$

One of the decisive points in the SSA approach is the selection of the indices $I_m$. García Plaza & Núnez Lopez (2017) point, that the incorrect choice of indices leads to principal components that do not contain prominent processing data of providing redundant data. The nominating procedure of sets $I_1, \dots, I_m$ is denominated as the Eigen triple grouping. The share of the equal eigenvalues $\sum_{i\in I}\lambda_i / \sum_{i=1}^{d}\lambda_i$ measures the benefaction of the component $U_I$ for a given group $I$.

The reconstruction stage maps the estimated trajectory matrix of the signal of interest into a single channel signal. Presume $\hat{a}_{kj}$ as an element positioned in the $k$th row and the $j$th column of the trajectory matrix, then for a single channel motion artefact signal like $\hat{a}(n)$ the mathematical expressions are defined by $f_k = (f_k^1, f_k^2, f_k^3)$, where we calculate elements of the series for $1 \geq n < L$ by $f_k^1$ and for $L \geq n \leq k$ we implement $f_k^2$. Finally, we use $f_k^3$ for $k < n \leq N$ (Eq.4).

$$\hat{a}(n) = \begin{cases} f_k^1 = \dfrac{1}{n}\sum_{k=1}^{n} \hat{a}_{k,n-k+1} \\ f_k^2 = \dfrac{1}{L}\sum_{k=1}^{L} \hat{a}_{k,n-k+1} \\ f_k^3 = \dfrac{1}{N-n-1}\sum_{k=n-k+1}^{N-k+1} \hat{a}_{k,n-k+1} \end{cases} \tag{4}$$

## Fuzzy Set Theory Preliminaries

Fuzzy sets introduced by (Zadeh, 1965) is an efficient and accurate tool in uncertain circumstances (Zimmermann, 1978). Scholars have employed it in many cases (Mahdiraji et al., 2016; Razavi et al., 2015; Razavi *et al.*, 2013). A complete review of Fuzzy sets and its applications can be found in (Buckley & Eslami, 2002; Masulli *et al*., 2007; Wang, 1983); however, a review of basic sets is briefed in the rest of this section.

A presumed fuzzy number like $D$ can be delineated as an interval$[q_l, q_u]$. The apparent fact is that $q_l$ is the lower boundary of $D$ and $q_u$, the upper. Respectively $\bar{q}_\alpha^L$ and $\bar{q}_\alpha^U$ are outlined as (Wang 2015):

$$\bar{q}_\alpha^L = \inf(z)_{\mu_{\bar{q}}(z)\geq\alpha} \tag{5}$$

$$\bar{q}_\alpha^U = \sup(z)_{\mu_{\bar{q}}(z)\geq\alpha} \tag{6}$$

Here for any $q \in (0,1]$, $F(\mathbb{R})$ is the set of fuzzy numbers and $[\,\bar{q}_\alpha^L(0),\ \bar{q}_\alpha^U(0)] = q_0$.

For any fuzzy number like $\bar{q}$, (Heilpern 1992) defines the expected interval $EI(\bar{q})$ and expected value $EV(\bar{q})$ as below:

$$EI(\bar{q}) = [E_*(\bar{q}), E^*(\bar{q})] = \left[\int_0^1 \bar{q}_\alpha^L(\beta)d\beta, \int_0^1 q_\alpha^U(\beta)d\beta\right] \tag{7}$$

$$EV(\bar{q}) = \frac{1}{2}(E_*(\bar{q}) + E^*(\bar{q})) \tag{8}$$

ambiguity $Amb(\bar{q})$, value $Val(\bar{q})$, width $w(\bar{q})$, left-hand ambiguity $Amb_L(\bar{q})$, right-hand ambiguity $Amb_U(\bar{q})$ were investigated by (Delgado *et al.*, 1998; Grzegorzewski 1998). Grzegorzewski (1998) defined one of the eminent matrices in fuzzy sets, the Euclidean distance as:

$$d^2(\bar{q},\bar{p}) = \int_0^1 (\,\bar{q}_\alpha^L(\beta) - \bar{p}_\alpha^L(\beta))^2 d\beta + \int_0^1 (\,\bar{q}_\alpha^U(\beta) - \bar{p}_\alpha^U(\beta))^2 d\beta \tag{9}$$

Literature review suggests that scholars implement triangular and trapezoidal sets to investigate fuzzy numbers, wherein this research considers trapezoidal numbers; therefore, by reviewing trapezoidal fuzzy numbers (henceforth TrFN), main operations and illustrating its schematic view in Figure 2, the section is finalised.

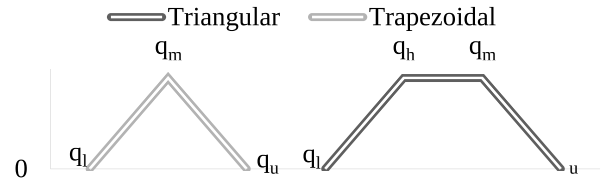



**Figure 2.** Schematic View of Triangular and Trapezoidal
*Source: Own*

Chen & Chen (2007) and Kumar & Gupta (2011) designate the arithmetic operations on two TrFN $\bar{q} = (q_l, q_h, q_m, q_u)$ and $\bar{p} = (p_l, p_h, p_m, p_u)$ as follow:

$$\lambda \geq 0, \lambda\bar{q} = (\lambda q_l, \lambda q_h, \lambda q_m, \lambda q_u) \tag{10}$$

$$\lambda \leq 0, \lambda\bar{q} = (\lambda q_u, \lambda q_m, \lambda q_h, \lambda q_l) \tag{11}$$

$$\bar{q} + \bar{p} = (q_l + p_l, q_h + p_h, q_m + p_m, q_u + p_u) \tag{12}$$

$$\bar{q} - \bar{p} = (q_l - p_u, q_h - p_m, q_m - p_h, q_u - p_l) \tag{13}$$

## Fuzzy Linear Regression

Regression analysis is applied to study the relation among an affected variable, called the dependent variable, with a set of affecting variables, called independent variables. The linear regression model can be displayed as:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + \beta_k X_{i,k} + \varepsilon_i \tag{14}$$

For $i = 1,2,\dots,n$, where $Y = [Y_1, Y_2, \dots, Y_n]^t$ is the vector of dependent variables and $X = [X_1, X_2, \dots, X_n]$ is the matrix of independent variables with $X_j = [X_{1j}, X_{2j}, \dots, X_{nj}]^t, j = 1,2,\dots,k$ as the vector of $j$th independent variable. Also, $\varepsilon_i$ are independent normal variables with $E(\varepsilon_i) = 0$ and $Var(\varepsilon_i) = \sigma^2$. The least square estimation of coefficients vector $\beta = [\beta_0, \beta_1, \beta_2, \dots, \beta_k]^t$ is obtained as

$$\beta = (X^t X)^{-1} X^t Y \tag{15}$$

where

$$X = \begin{bmatrix} 1 x_{11} \cdots x_{1k} \\ 1 x_{21} \cdots x_{2k} \\ \vdots \ \vdots \ \ddots \ \vdots \\ 1 x_{n1} \cdots x_{nk} \end{bmatrix}$$

In ordinal regression, both dependent and independent variables are crisp. However, in a world of uncertain events, usually, the information is unknown and are determined vaguely. Therefore, fuzzy regression analysis is proposed to examine the problem of the relationship among several variables in a fuzzy environment. Tanaka *et al.* (1982) initiated the topic of fuzzy linear regression. The issue is later extended and studied by scholars. Based on Ubale & Sananse (2015) investigation on fuzzy regression application and literature review, Chen *et al.* (2004)

Arulchinnappan & Rajendran (2011), Abdullah & Zamri (2012), Pushpa & Vasuki (2013), Shafi & Rusiman (2015), Muzzioli *et al.* (2015), and Chen *et al.* (2016), are among the most illustrious recent jobs on this topic.

The problem considered in this paper is to estimate a function of electricity consumption with fuzzy regression; therefore, the Wu (2003) proposed method is applied to determine the corresponding function. In this method, an α-level set $\left(\tilde{\tilde{\beta}}_j\right)_\alpha$ is computed for each regression coefficient as follow:

$$\left(\tilde{\tilde{\beta}}_j\right)_\alpha = \left[\left(\tilde{\tilde{\beta}}_j\right)_\alpha^L, \left(\tilde{\tilde{\beta}}_j\right)_\alpha^U\right] \quad (16)$$

To this end, Wu (2003) proposed that first, the α-level sets of dependent and independent variables are computed. Supposing these sets as $\left[\left(\tilde{X}\right)_\alpha^L, \left(\tilde{X}\right)_\alpha^U\right]$ and $\left[\left(\tilde{Y}\right)_\alpha^L, \left(\tilde{Y}\right)_\alpha^U\right]$, two regression functions are fitted on $\left[\left(\tilde{X}\right)_\alpha^L, \left(\tilde{Y}\right)_\alpha^L\right]$ with $B_1$ coefficients vector and $\left[\left(\tilde{X}\right)_\alpha^U, \left(\tilde{Y}\right)_\alpha^U\right]$ with $B_2$ coefficients vector. Afterwards, the interval regression coefficient at confidence level α is defined as:

$$\left(\tilde{\tilde{\beta}}_j\right)_\alpha = \left[\left(\tilde{\tilde{\beta}}_j\right)_\alpha^L, \left(\tilde{\tilde{\beta}}_j\right)_\alpha^U\right] = [min(B_1, B_2), max(B_1, B_2)] \quad (17)$$

## Proposed Structure

The proposed model is hinged on an SSA/Fuzzy-regression approach to discuss and optimise the electric power consumption in Iran. At the first phase experts suggested primary criterions as the main concern to be addressed in our research, and by the further interviews with economic and energy experts, besides analysing demographics of Iran, mentioned criterions were reduced to principal objects to be examined. Relative and discord data of those criterions in the second phase were gathered and applied by the SSA approach to reduce noise and forecast the next 15 years' behaviour. Consequently, Fuzzy-regression method was scheduled to determine the power consumption function based on independent variables. Eventually, the objective function subjected to governmental policies is optimised. Figure 3 demonstrates the main steps of our proposed approach.

## Iran's Electricity Consumption Evaluation

***Phase 1: Preparation.*** The expert's suggestions gathered the critical criterions in Power consumption. Table 1 illustrates the primal data list, comprising criteria's description (input variables), quantitative or qualitative type of variables and positive or negative influence of each input variable on electricity consumption. Further interviews with economic and energy experts in conjunction with analysing demographics of Iran in brainstorming sessions, lead us to a narrower chose of criterions and five targets were picked from Table 1. Table 2 demonstrates the final criteria (input variables) choice. The discord data in the above table and related data of electrical consumption in Iran present the main objects to implement in Fuzzy-regression optimisation approach.

Table 1

**Initial Criteria (Initial Input Variables)**

| No. | Criteria description | Type | Effect |
|---|---|---|---|
| 1 | Home appliance technology level | Quantitative | + |
| 2 | Population growth rate | Quantitative | - |
| 3 | Industrial development state | Quantitative | - |
| 4 | Urbanization growth rate | Quant./ Qual. | - |
| 5 | Alternative energy sources substitution option | Qualitative | + |
| 6 | Power consumption price | Quantitative | + |
| 7 | Electricity price share from the household expense | Quantitative | + |
| 8 | Iran's population structure | Qualitative | + |
| 9 | Modern construction implementation | Qualitative | + |
| 10 | Energy Productivity in Iran | Quantitative | + |
| 11 | Proper timing and supplement usage | Qualitative | + |

(*Source: Own*)

Thus, related data were gathered from official databases encompassing Statistical Centre of Iran, Iran Ministry of energy, National Iranian Productivity Organization and Central Bank of Iran, uncovering electrical consumption rate in the last 20 years (presented in Table 3).

Table 2

**Final Criteria Selection**

| No. | Criteria description | Type | Effect |
|---|---|---|---|
| 1 | Power consumption price | Quantitative | + |
| 2 | Population growth rate | Quantitative | - |
| 3 | Modern construction implementation | Qualitative | + |
| 4 | Energy Productivity in Iran | Quantitative | + |
| 5 | Proper timing and supplement usage | Qualitative | + |

(*Source: Own*)

Table 3

**Iranian Electrical Consumption in the Last 20 Years**

| Year | Consumption/Million KWh | Year | Consumption/Million KWh |
|---|---|---|---|
| 1996 | 65854 | 2006 | 132897 |
| 1997 | 69671 | 2007 | 144598 |
| 1998 | 73358 | 2008 | 152330 |
| 1999 | 77646 | 2009 | 161445 |
| 2000 | 84656 | 2010 | 168438 |
| 2001 | 90336 | 2011 | 184182 |
| 2002 | 97171 | 2012 | 183905 |
| 2003 | 105076 | 2013 | 194148 |
| 2004 | 114625 | 2014 | 203088 |
| 2005 | 124466 | 2015 | 219653 |

(*Source: Mentioned Databases*)

The population growth rate, energy productivity criteria and power costs are denoted in Table 4. Data for 2016 and 2017 are not yet prepared at the time of developing the research. These two years were forecasted; however, considering the publication time, results for 2016 and 2017 were deleted by the authors.

Table 4

**Total Population, Energy Productivity, Cost**

| Year | Population Growth rate ( Million) [1] | Power cost (IRR/KWh) [2] | Energy Productivity criteria (in billions of euros of GDP per exajoule on energy consumed) [3] |
|---|---|---|---|
| 1996 | 61306632 | 6 | 118 |
| 1997 | 62426086 | 6 | 128 |
| 1998 | 63616065 | 8 | 106 |
| 1999 | 64780362 | 8 | 113 |
| 2000 | 65850062 | 9 | 109 |
| 2001 | 66812736 | 10 | 108 |
| 2002 | 67696677 | 10 | 88 |
| 2003 | 68522074 | 10 | 75 |
| 2004 | 69321953 | 12 | 123 |
| 2005 | 70122115 | 14 | 75 |
| 2006 | 70923164 | 14 | 93 |
| 2007 | 71720859 | 16 | 111 |
| 2008 | 72530693 | 18 | 117 |
| 2009 | 73370982 | 18 | 116 |
| 2010 | 74253373 | 36 | 91 |
| 2011 | 75184322 | 36 | 100 |
| 2012 | 76156975 | 36 | 129 |
| 2013 | 77152445 | 49 | 96 |
| 2014 | 78143644 | 50 | 103 |
| 2015 | 79109272 | 53 | 117 |

(*Source [1]: World Bank, Source [2]: Ministry of Energy, Source [3]: National Iranian Productivity Organization*)

***Phase 2: SSA Implementation***. The window length of time series is set as $L = 10$, and the variables were grouped based on their dispersion and eigenvalues. Afterwards, the time series was reconstructed and employed to forecast the next 15 years. Table 5 demonstrates the reconstructed time series. Moreover, Table 6 illuminates the forecast of electricity consumption criterions between 2016 and 2030. It is worth noting here that SSA implementation, reconstruction of time series, noise reduction of time series and SSA forecasts of the response variable and all input variables were calculated by Caterpillar SSA software.

***Phase 3: Estimation.*** The fuzzy regression method of Wu (2003) is applied to estimate the amount of energy consumption. The below function is calculated from the observed data using this method, and the results presented in Table 7.

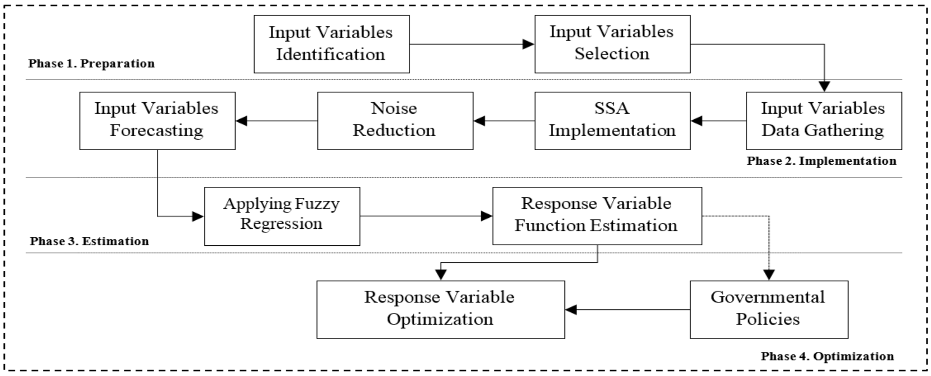



**Figure 3.** Proposed Approach

(*Source: Own*)

$$[\underline{C}_t, \overline{C}_t] = [-398718, -398235] + 771Pr_t + 198Pro_t + 0.00708Po_t - 142CM_t - 2272UC_t \quad (18)$$

Response variable prediction, input variables prediction and qualitative variables are required to estimate electricity consumption, *r*. Furthermore, to consider uncertain information and fuzzy logic, based on qualitative forecasting method (specifically Delphi method) the future behaviour of construction and usage culture of electricity power in Iran were emanated. The results are presented upon linguistic terms, observable in Table 7.

***Phase 4: Optimisation***. The fitted model is an approximation of electricity consumption in Iran. This model can be used to predict consumption as a function of independent variables. However, to propose an optimisation model of whether to improve the consumption of Iran, an optimisation model is formulated in this section. The proposed optimisation model seeks to find the way of decreasing yearly 5 % of electricity consumption in Iran in a period between 2017 to 2020.

In fact, $\left[\underline{C}^*_{t+1}, \overline{C}^*_{t+1}\right] \in 0.95\left[\underline{C}_t, \overline{C}_t\right]$. Ishibuichi & Tanaka (1990) suggest that $\left[\underline{A}, \overline{A}\right] \leq \left[\underline{B}, \overline{B}\right]$ if $\overline{A} \leq \overline{B}$ and $\left(\underline{A} + \overline{A}\right) \leq \left(\underline{B} + \overline{B}\right)$. Following Ishibuichi & Tanaka (1990), the inequality is transformed into two equivalent inequalities:

Table 5

**The Reconstructed Time Series**

| Year | Price | Productivity | Consumption | Population |
|---|---|---|---|---|
| 2005 | 13.6 | 99.05 | 121996.81 | 69975504 |
| 2006 | 14.76 | 97.83 | 130169.63 | 70855181.3 |
| 2007 | 18.05 | 103.14 | 139295.83 | 71758274.2 |
| 2008 | 20.01 | 103.44 | 148794.35 | 72651258.2 |
| 2009 | 22.39 | 105.31 | 158665.69 | 73537926.9 |
| 2010 | 30.5 | 97.49 | 168843.49 | 74424126.6 |
| 2011 | 34.2 | 105.20 | 179414.61 | 75316196.9 |
| 2012 | 36.97 | 115.77 | 189851.74 | 76219381.3 |
| 2013 | 47.91 | 100.97 | 200700.45 | 77137065.5 |
| 2014 | 51.8 | 107.25 | 211941.98 | 78070467.7 |
| 2015 | 50.6 | 121.24 | 224175.53 | 79018774.3 |

(*Source: Own; Software Caterpillar SSA*)

Table 6

**Forecasted Criterions in the Period of 2016-2030**

| Year | Price | Productivity | Consumption | Population |
|---|---|---|---|---|
| 2018 | 83.03 | 115.7 | 270756.46 | 81996472.8 |
| 2019 | 104.12 | 115.34 | 287392.10 | 82995083.4 |
| 2020 | 130.27 | 115.87 | 305010.31 | 84006902.4 |
| 2021 | 136.11 | 114.32 | 323687.35 | 85032749.9 |
| 2022 | 158.16 | 120.64 | 343565.60 | 86072956.1 |
| 2023 | 197.10 | 120.02 | 364746.89 | 87127512.6 |
| 2024 | 211.62 | 115.92 | 387347.57 | 88196275 |
| 2025 | 238.3 | 123.1 | 411437.13 | 89279173.6 |
| 2026 | 299.38 | 122.53 | 436820.72 | 90370511.1 |
| 2027 | 336.26 | 121.17 | 463762.90 | 91475363.1 |
| 2028 | 373.12 | 124.41 | 492367.12 | 92594126 |
| 2029 | 461.27 | 124.96 | 522742.56 | 93727040.5 |
| 2030 | 531.48 | 126.40 | 555003.34 | 94874219.8 |

(*Source: Own; Software Caterpillar SSA*)

Table 7

**Energy Consumption Estimation and Qualitative Input Variables Estimation**

| Year | Consumption | Construction Methods | Usage Culture |
|---|---|---|---|
| 2018 | [269658.4, 271959.5] | L | L |
| 2019 | [289776.6, 292129.8] | L | L |
| 2020 | [310919.7, 313337.3] | M | L |
| 2021 | [321969, 324470.5] | M | M |
| 2022 | [343028.3, 345585.3] | NH | M |
| 2023 | [373535.7, 376181.2] | NH | M |
| 2024 | [388949.4, 391635.6] | H | M |
| 2025 | [413816.2, 416597.6] | H | NH |
| 2026 | [456995.6, 459910.1] | H | NH |
| 2027 | [485630.2, 488630.5] | VH | NH |
| 2028 | [515822.2, 518938.3] | VH | H |
| 2029 | [574657.6, 577961.4] | VH | H |
| 2030 | [622703.6, 626160.3] | VVH | H |

$$\begin{aligned} -398235 + 771Pr_{t+1} + 198Pro_{t+1} &+ 0.00708Po_{t+1} - 142CM_{t+1} - 2272UC_{t+1} \\ &\leq -378323.25 + 732.45Pr_t + 188.1Pro_t + 0.006726Po_t - 134.9CM_t - 2158.4UC_t \\ -398477 + 771Pr_{t+1} + 162Pro_{t+1} &+ 0.00708Po_{t+1} - 142CM_{t+1} - 2272UC_{t+1} \\ &\leq -378553 + 732.45Pr_t + 188.1Pro_t + 0.006726Po_t - 134.9CM_t - 2158.4UC_t \end{aligned} \tag{19}$$

Defining the deviation variables $d_1^-, d_1^+, d_2^-, d_2^+$; the goal programming based consumption optimisation model is formulated:

$$\begin{aligned} &Min\ d_1^+ + d_2^+ \text{ S.T.} \\ &-398235 + 771Pr_{t+1} + 198Pro_{t+1} + 0.00708Po_{t+1} - 142CM_{t+1} - 2272UC_{t+1} + d_1^- - d_1^+ \\ &\quad = -378323.25 + 732.45Pr_t + 188.1Pro_t + 0.006726Po_t - 134.9CM_t - 2158.4UC_t \\ &-398477 + 771Pr_{t+1} + 162Pro_{t+1} + 0.00708Po_{t+1} - 142CM_{t+1} - 2272UC_{t+1} + d_2^- - d_2^+ \\ &\quad = -378553 + 732.45Pr_t + 188.1Pro_t + 0.006726Po_t - 134.9CM_t - 2158.4UC_t \end{aligned} \tag{20}$$

(i) $CM \in \{VL, L, ML, M, MH, H, VH\}$
(ii) $UC \in \{VL, L, ML, M, MH, H, VH\}$
(iii) $Po_{t+1} \geq (1 + gr)Po_t$
(iv) $Pr_{t+1} \geq (1 + i_t)Pr_t, Pro_{t+1} \geq Pro_{t+1}$

The objective function of Eq. (20) seeks to minimise undesirable deviations from the considered goals of decreasing the amount of consumption to a value between 75 % - 95 % of the current consumption. Constraints (i) and (ii) expressed the possible variation in two qualitative variables of construction method and usage culture. Eventually, constraint (iii) illustrated that the population of period $t + 1$ could not be lower than the population at period *t* when $gr$ is the growth rate of population from 1996 to 2015. Constraint (iv) illustrated that price at year $t + 1$ is at least as large as the price at a period *t* plus the expected inflation rate of the year that is illustrated by $i_t$. To solve the above model, first, two variables of CM and UC are substituted with the following equivalents:

$$CM = y_1 + 2y_2 + 3y_3 + 4y_4 + 5y_5 + 6y_6 + 7y_7$$
$$UC = y_8 + 2y_9 + 3y_{10} + 4y_{11} + 5y_{12} + 6y_{13} + 7y_{14}$$

Where, $y_1, y_8 \approx VL$, $y_2, y_9 \approx L$, $y_3, y_{10} \approx ML$, $y_4, y_{11} \approx M$, $y_5, y_{12} \approx MH$, $y_6, y_{13} \approx H$, $y_7, y_{14} \approx VH$.

Substituting above equivalents into the model (20), the following integer goal programming model is obtained:

$$\begin{aligned} &Min\ d_1^+ + d_2^+ \text{ S.T.} \\ &-398235 + 771Pr_{t+1} + 198Pro_{t+1} + 0.00708Po_{t+1} - 142(y_{1,t+1} + 2y_{2,t+1} + 3y_{3,t+1} + 4y_{4,t+1} + 5y_{5,t+1} + 6y_{6,t+1} + 7y_{7,t+1}) \\ &\quad - 2272(y_{8,t+1} + 2y_{9,t+1} + 3y_{10,t+1} + 4y_{11,t+1} + 5y_{12,t+1} + 6y_{13,t+1} + 7y_{14,t+1}) + d_1^- - d_1^+ \\ &\quad = -378323.25 + 732.45Pr_t + 188.1Pro_t + 0.006726Po_t - 134.9(y_{1,t} + 2y_{2,t} + 3y_{3,t} + 4y_{4,t} + 5y_{5,t} + 6y_{6,t} + 7y_{7,t}) \\ &\quad - 2158.4(y_{8,t} + 2y_{9,t} + 3y_{10,t} + 4y_{11,t} + 5y_{12,t} + 6y_{13,t} + 7y_{14,t}) \\ &-398477 + 771Pr_{t+1} + 162Pro_{t+1} + 0.00708Po_{t+1} - 142(y_{1,t+1} + 2y_{2,t+1} + 3y_{3,t+1} + 4y_{4,t+1} + 5y_{5,t+1} + 6y_{6,t+1} + 7y_{7,t+1}) \\ &\quad - 2272(y_{8,t+1} + 2y_{9,t+1} + 3y_{10,t+1} + 4y_{11,t+1} + 5y_{12,t+1} + 6y_{13,t+1} + 7y_{14,t+1}) + d_2^- - d_2^+ \\ &\quad = -378553 + 732.45Pr_t + 188.1Pro_t + 0.006726Po_t - 134.9(y_{1,t} + 2y_{2,t} + 3y_{3,t} + 4y_{4,t} + 5y_{5,t} + 6y_{6,t} + 7y_{7,t}) \\ &\quad - 2158.4(y_{8,t} + 2y_{9,t} + 3y_{10,t} + 4y_{11,t} + 5y_{12,t} + 6y_{13,t} + 7y_{14,t}) \\ &CM_t \leq CM_{t+1} \leq CM_t + 1 \\ &UC_t \leq UC_{t+1} \leq UC_t + 1 \\ &Po_{t+1} \geq (1 + gr)Po_t \\ &Pr_{t+1} \geq (1 + i_t)Pr_t, Pro_{t+1} \geq Pro_{t+1} \end{aligned} \tag{21}$$

Solving the above model using Lingo package, the optimal solution can be obtained. Now, consider the case when $t = 2016$. At this year, it is aimed that the consumption of 2016 is about 95 % of the use of 2015, i.e. $0.95 \times 555003.34 = 527253.2$. Furthermore, $CM_{2015} =$

6, $UC_{2015} = 4$, $Pr_{2016}$ is predicted to be nearly 66.74, $Po_{2016}$ is predicted to be at least 80032947.1. Solving the above model, the optimal consumption of the year 2016 eventuates as $y^*_{2,2016} = y^*_{9,2016} = 1$, i.e. constructions method should be kept at high-level, while the usage culture should be improved to a very high level. Regarding other factors, the population is proposed to keep at 80032950, the price at 66.74, and productivity improved to 121.24. With this case in mind, the consumption is estimated at the interval [526770.1, 527495.1]. The model is constructed and solved for years from 2017 to 2020 employing a similar manner. The results are presented in Table 8. The acquired data by optimisation are compared to the estimation methodology (34), the results are briefed in Table 9.

Table 8

**Optimising the Consumption of Electricity in Iran from 2016 to 2020**

| Year | Price | Productivity | Population | Construction methods | Usage culture | Consumption |
|---|---|---|---|---|---|---|
| 2016 | 66.7 | 121.2 | 80032950 | Low | Low | [526770.1, 527495.1] |
| 2017 | 80.5 | 121.2 | 81009700 | Moderately low | Moderately low | [500637.3, 501017.9] |
| 2018 | 83.0 | 121.2 | 81996473 | Medium | Medium | [475545.2, 475967] |
| 2019 | 104.1 | 121.2 | 82995080 | Moderately high | Moderately high | [451685.7, 452210.3] |
| 2020 | 130.3 | 1875.7 | 84006900 | High | High | [429116.8, 429592.6] |

Table 9

**Comparing Methods for Consumption of Electricity in Iran from 2016 to 2020**

| Year | Estimation | Optimisation |
|---|---|---|
| 2016 | [575149, 602896] | [526770.1, 527495.1] |
| 2017 | [590236.2, 622527.2] | [500637.3, 501017.9] |
| 2018 | [596824.3, 633759.3] | [475545.2, 475967] |
| 2019 | [620254.8, 657089.8] | [451685.7, 452210.3] |
| 2020 | [648559.1, 685394.1] | [429116.8, 429592.6] |

As it can be seen from Table 8, if decision makers try to improve construction method and usage culture in a stepwise manner, i.e. construction method is developed from very low in 2015 to $low \rightarrow moderately\ low \rightarrow medium \rightarrow moderately\ high \rightarrow high$ in next years, respectively, and usage culture is enhanced from very low similar to construction method, and the growth in price, population, and productivity is controlled on the basis of the predicted values; thus, the consumption will be diminished from its forecasted values. If the estimated value at time period *t* is shown by $Es_t$ and the optimised method by $Op_t$, then the grey possibility degree (Li *et al.*, 2007), that shows the degree of dominance of a grey number to another one, then it can be concluded that the optimised values are lower than estimated values with a 100 % possibility.

Figure 4 indicates the interval plot of estimated (reported by $E2016 - E2020$) against optimised (indicated by $O2016 - O2020$) values of consumption. It is clear that the mean and range of usage is decreased based on the optimised scenario.

## Conclusion

Scheduling a specific approach to optimise the electricity consumption rate due to the scarce resources are considered in this research. In conjunction with the limitations of data gathering, the scarce annual datasheets and the native character of forecasting procedures causing an error, a novel SSA-fuzzy regression model was designed to minimise the possible negligence. Furthermore, a goal programming approach was developed and employed to solve the fuzzy-based mathematical model. Applying singular spectrum analysis for reconstructing time series, proposing a fuzzy regression approach for energy consumption estimation and designing a goal programming interval based model to optimise EC are the main highlights of this research. Based on this novel approach, policy-making in many infrastructural areas was proposed to organise power consumption in Iran. The case of Iran was chosen due to the lack of a monitoring framework and the large electrical consumption rate in recent years.

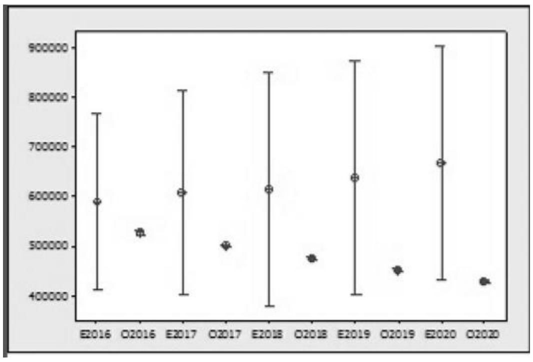



**Figure 4.** Interval Plot of Estimated Against Optimised Consumption

By investigating the relevant data to EC in the last two decades to forecast the EC for the next five years, an incremental pattern has resulted. A useful optimising model using fuzzy regression is presented to control the consumption.

Improvements in construction method and usage culture in a stepwise manner are found to be the solution for the next five years. Researchers can investigate cases from other countries or use a fuzzy singular spectrum analysis to forecast these factors in an uncertain situation. Moreover, researchers can implement the proper and rarely used SSA-fuzzy regression framework to optimise other controversial variables. The results of this research are beneficial for governmental authorities in policymaking. Based on the findings emanated from the proposed optimisation method, the optimal amount of each dependent criteria to decrease nearly 30 % of consumption rate are estimated; hence, policymaking is significantly facilitated.

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