You are to write a MATLAB function to implement Romberg integration with an error estimate at the end. Your romberg function should have the form

[int,err,ier]=romberg(f,a,b,tol)

The input variables are

- f–a function handle for the function to be integrated
- a, b– the limits of integration, i.e., you are approximating

$$I = \int_a^b f(x)dx.$$

- *tol*-the error between the last two Romberg approximations. In the notation in the notes this means that

$$|T_k(h) - T_{k-1}(h)| \leq tol.$$

  and $T_k(h)$ is accepted as the integral.

The value of h for your first approximation from the Trapezoid rule should satisfy $h << 1$. I recommend that $h = (b-a)/2^j$ for some appropriate value of j.

The output variables are

- *int*–The approximate integral.
- *err*– The value of $T_k(h) - T_{k-1}(h)$, the error estimate.
- *ier* – You are to have a maximum of 10 levels of extrapolation, i.e., $k = 10$. If you reach 10 levels and $|err| > tol$, then $ier = 1$ (tolerence was not reached), otherwise $ier = 0$ (tolerence was reached).

To design this code you will need to design a function for the composite trapezoid rule, that is, for computing $T_0(h)$. You will also need a function to perform an update, that is, to compute $T_0(h/2)$ from $T_o(h)$ and summing the midpoints. The matlab function **sum** will save you some loops. You should be able to accumulate the approximations with one vector that stores

$$T_0(h/2^k), T_1(h/2^{k-1}), \ldots, T_{k-1}(h/2), T_k(h)$$

at a given point in the Richardson extrapolation process. Again, the maximum for k is 10, and you may take advantage of that in the development on your code.

Test your code on three functions that I will put in the subfolder **Homework Five** on Canvas. They are **pifunc.m**, **logderiv.m**, and **erfderiv.m** and they are all one line functions. For these three functions, approximate the following integrals with $tol = 1e - 14$:

- **pifunc.m**. Compute the integral over the interval $[0, 1]$. You should get a very good approximation of π.
- **logderiv.m**. Compute the integral over the interval $[1, e]$ where $e =$ **exp**(1) is the natural logarithm base. The integral should be 1 (or very close to it).
- **erfderiv.m**. Compute the integral over the intervals $[0, 1]$ and $[0, 3]$. You should get good approximations of **erf**(1) and **erf**(3) from the **erf** function in MATLAB.

Please turn in your codes and the output from these tests on Canvas. Your **romberg** function should be documented to explain how to call it and to explain any unusual code, but you do not need to document every single line.

Note: The easiest way to get an output file is to use the MATLAB **diary** command. Suppose that **romberg_scr** is a MATLAB script for this assignment and you type the following.

$\gg$ diary
$\gg$ romberg_scr
$\gg$ diary off

The output produced appear in a file called diary. If you do this run multiple times, it just adds to the end of the file. Thus, to have the output of one set of MATLAB commands, you should remove the diary file before doing the runnning those commands.