Quantum Computing

Introduction to basic concepts and perspective for lattice field theory

Stefan Kühn

Lattice Practices 2024, 17.09.2024

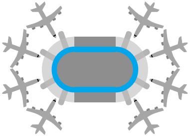
Motivation

Problems for which Quantum Computers might be advantageous

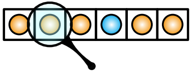
- Factoring

$$70747 = 263 \times 269$$

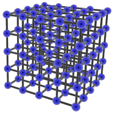
- Optimization problems



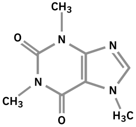
- Searching databases



- Quantum simulation
  - Lattice field theory

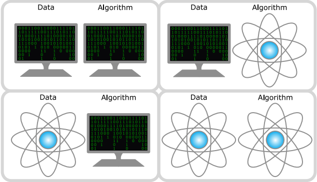


  - Quantum chemistry

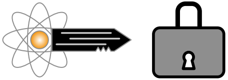


  - Material science
  - ...

- Machine learning



- Cryptography

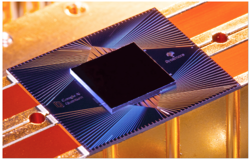


- ...

Motivation

On the verge of the NISQ era

- Noisy intermediate-scale quantum computers with $\mathcal{O}(100)$ qubits are already available
- Noise significantly limits the circuit depths that can be executed reliably, no quantum error correction possible



J. Preskill, Quantum 2, 79 (2018)

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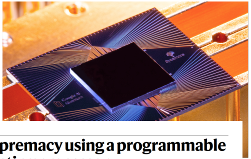

Article

Quantum supremacy using a programmable superconducting processor

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Received: 22 July 2019
Accepted: 20 September 2019
Published online: 23 October 2019

Frank Arute[1], Kunal Arya[1], Ryan Babbush[1], Dave Bacon[1], Joseph C. Bardin[1,2], Rami Barends[1], Rupak Biswas[3], Sergio Boixo[1], Fernando G. S. L. Brandao[1,4], David A. Buell[1], Brian Burkett[1], Yu Chen[1], Zijun Chen[1], Ben Chiaro[5], Roberto Collins[1], William Courtney[1], Andrew Dunsworth[1], Edward Farhi[1], Brooks Foxen[1,5], Austin Fowler[1], Craig Gidney[1], Marissa Giustina[1], Rob Graff[1], Keith Guerin[1], Steve Habegger[1], Matthew P. Harrigan[1], Michael J. Hartmann[1,6], Alan Ho[1], Markus Hoffmann[1], Trent Huang[1], Travis S. Humble[7], Sergei V. Isakov[1], Evan Jeffrey[1], Zhang Jiang[1], Dvir Kafri[1], Kostyantyn Kechedzhi[1], Julian Kelly[1], Paul V. Klimov[1], Sergey Knysh[1], Alexander Korotkov[1,8], Fedor Kostritsa[1], David Landhuis[1], Mike Lindmark[1], Erik Lucero[1], Dmitry Lyakh[9], Salvatore Mandrà[3,10], Jarrod R. McClean[1], Matthew McEwen[5], Anthony Megrant[1], Xiao Mi[1], Kristel Michielsen[11,12], Masoud Mohseni[1], Josh Mutus[1], Ofer Naaman[1], Matthew Neeley[1], Charles Neill[1], Murphy Yuezhen Niu[1], Eric Ostby[1], Andre Petukhov[1], John C. Platt[1], Chris Quintana[1], Eleanor G. Rieffel[3], Pedram Roushan[1], Nicholas C. Rubin[1], Daniel Sank[1], Kevin J. Satzinger[1], Vadim Smelyanskiy[1], Kevin J. Sung[1,13], Matthew D. Trevithick[1], Amit Vainsencher[1], Benjamin Villalonga[1,14], Theodore White[1], Z. Jamie Yao[1], Ping Yeh[1], Adam Zalcman[1], Hartmut Neven[1] & John M. Martinis[1,5]*

J. Preskill, Quantum 2, 79 (2018)
F. Arute et al., Nature 574, 5050 (2019)

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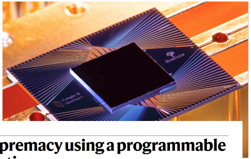

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RESEARCH

QUANTUM COMPUTING

Quantum computational advantage using photons

Han-Sen Zhong[1,2]*, Hui Wang[1,2]*, Yu-Hao Deng[1,2]*, Ming-Cheng Chen[1,2]*, Li-Chao Peng[1,2], Yi-Han Luo[1,2], Jian Qin[1,2], Dian Wu[1,2], Xing Ding[1,2], Yi Hu[1,2], Peng Hu[3], Xiao-Yan Yang[3], Wei-Jun Zhang[3], Hao Li[3], Yuxuan Li[4], Xiao Jiang[1,2], Lin Gan[4], Guangwen Yang[4], Lixing You[3], Zhen Wang[3], Li Li[1,2], Nai-Le Liu[1,2], Chao-Yang Lu[1,2]†, Jian-Wei Pan[1,2]†

Quantum computers promise to perform certain tasks that are believed to be intractable to classical computers. Boson sampling is such a task and is considered a strong candidate to demonstrate the quantum computational advantage. We performed Gaussian boson sampling by sending 50 indistinguishable single-mode squeezed states into a 100-mode ultralow-loss interferometer with full connectivity and random matrix—the whole optical setup is phase-locked—and sampling the output using 100 high-efficiency single-photon detectors. The obtained samples were validated against plausible hypotheses exploiting thermal states, distinguishable photons, and uniform distribution. The photonic quantum computer, *Jiuzhang*, generates up to 76 output photon clicks, which yields an output state-space dimension of 10^{30} and a sampling rate that is faster than using the state-of-the-art simulation strategy and supercomputers by a factor of $\sim 10^{14}$.

F. Arute et al., Nature 574, 5050 (2019)
H.-S. Zhong et al., Science 370, 1460 (2020)

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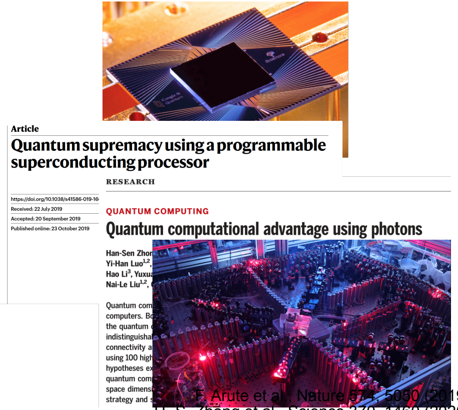

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- For certain simple models NISQ devices results were comparable with state of the art methods

Article

Evidence for the utility of quantum computing before fault tolerance

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Published online: 14 June 2023
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Youngseok Kim[1,6] ✉, Andrew Eddins[2,6] ✉, Sajant Anand[3], Ken Xuan Wei[1], Ewout van den Berg[1], Sami Rosenblatt[1], Hasan Nayfeh[1], Yantao Wu[3,4], Michael Zaletel[3,5], Kristan Temme[1] & Abhinav Kandala[1] ✉

Quantum computing promises to offer substantial speed-ups over its classical counterpart for certain problems. However, the greatest impediment to realizing its full potential is noise that is inherent to these systems. The widely accepted solution to this challenge is the implementation of fault-tolerant quantum circuits, which is out of reach for current processors. Here we report experiments on a noisy 127-qubit processor and demonstrate the measurement of accurate expectation values for circuit volumes at a scale beyond brute-force classical computation. We argue that this represents evidence for the utility of quantum computing in a pre-fault-tolerant era. These experimental results are enabled by advances in the coherence and calibration of a superconducting processor at this scale and the ability to characterize[1] and controllably manipulate noise across such a large device. We establish the accuracy of the measured expectation values by comparing them with the output of exactly verifiable circuits. In the regime of strong entanglement, the quantum computer provides correct results for which leading classical approximations such as pure-state-based 1D (matrix product states, MPS) and 2D (isometric tensor network states, isoTNS) tensor network methods[2,3] break down. These experiments demonstrate a foundational tool for the realization of near-term quantum applications[4,5].

F. Arute et al., Nature 574, 5050 (2019)
H.-S. Zhong et al., Science 370, 1460 (2020)
Y. Kim et al., Nature 618, 500 (2023)

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- Current NISQ devices have already outperformed classical computers for “artificially tailored” tasks
- For certain simple models NISQ devices results were comparable with state of the art methods
- Larger quantum devices in the near future with error correction are announced future

Article

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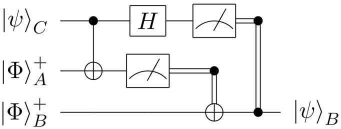
https://www.ibm.com/quantum/technology#scaling-quantum-computing, https://blog.google/technology/ai/unveiling-our-new-quantum-ai-campus/

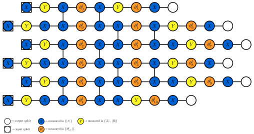
Motivation

Different approaches for Quantum Computing (QC)

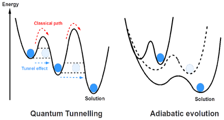
Circuit-based QC



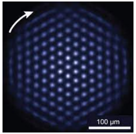
Measurement-based QC



Quantum annealers and adiabatic QC



Quantum Simulators



Outline

The circuit model of Quantum Computing

The variational quantum eigensolver

Real-time dynamics

Summary & Outlook

1.

The circuit model of Quantum Computing

The variational quantum eigensolver

Real-time dynamics

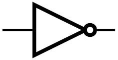
Summary & Outlook

Classical computing

Classical computing

Classical computing

- Goal: compute Boolean functions $f : \mathbb{Z}_2^n \to \mathbb{Z}_2$
- Logic gates



a	$\neg a$
0	1
1	0

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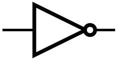

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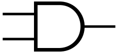

a	b	ab
0	0	0
0	1	0
1	0	0
1	1	1

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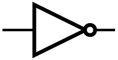

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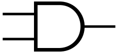

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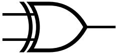

a	b	$a \oplus b$
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0	1	1
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1	1	0

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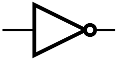

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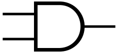

a	b	ab
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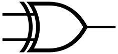

a	b	$a \oplus b$
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- **Universal** gate set: allows for expressing any Boolean function, e.g. $\{\text{NOT}, \text{AND}\}$ are universal

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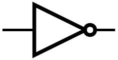

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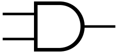

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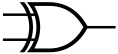

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0	0	0
0	1	1
1	0	1
1	1	0

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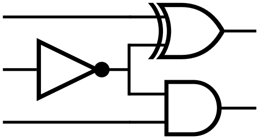
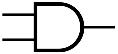
- We can arbitrarily copy bits



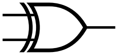
Classical computing

Irreversible vs. reversible gates

- Typically on uses a irreversible gate set, e.g. AND, XOR
- Can we make them reversible?



a	b	ab
0	0	0
0	1	0
1	0	0
1	1	1



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0	0	0
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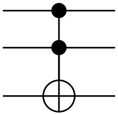
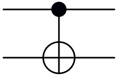


a	b	c	a	b	$c \oplus ab$
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0



a	b	a	$a \oplus b$
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

$\Rightarrow$ It is possible to work with reversible gates

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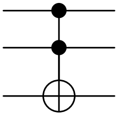
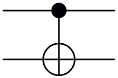


a	b	c	a	b	$c \oplus ab$
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0



a	b	a	$a \oplus b$
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

$\Rightarrow$ It is possible to work with reversible gates

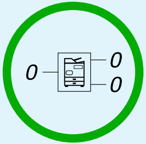
- Toffoli is universal

Classical computing

Classical vs. Quantum computing

Classical computing

- Classical bits are 0 or 1
- Typically irreversible gate set, but reversible is possible
- We can arbitrarily copy bits



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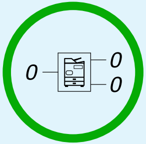

Quantum computing

- Quantum analog of bits?
- Quantum logic gates?
- Copying quantum information?

Quantum bits

The circuit model of Quantum Computing

Quantum bit

- Qubit: two-dimensional quantum system
- Hilbert space $\mathcal{H}$ with basis $\{|0\rangle, |1\rangle\}$, called the **computational basis**
- Contrary to classical bits, it can be in a superposition

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

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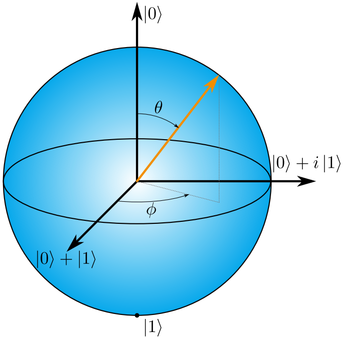
- Hilbert space $\mathcal{H}$ with basis $\{|0\rangle, |1\rangle\}$, called the **computational basis**
- Contrary to classical bits, it can be in a superposition

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

- Angles can be associated to a vector in spherical coordinates

$$\vec{r} = \begin{pmatrix} \sin(\theta)\cos(\phi) \\ \sin(\theta)\sin(\phi) \\ \cos(\theta) \end{pmatrix}$$

$\Rightarrow$ **Bloch-sphere** representation



The circuit model of Quantum Computing

Multiple quantum bits

- n qubits: Hilbert space is the tensor product $\underbrace{\mathcal{H} \otimes \cdots \otimes \mathcal{H}}_{n \text{ times}}$
- Most general state in the computational basis

$$|\psi\rangle = \sum_{i_1,\ldots,i_n=0}^{1} c_{i_1\ldots i_n} |i_1\rangle \otimes \cdots \otimes |i_n\rangle$$

- In the following $\otimes$ often suppressed: $|0\rangle \otimes |0\rangle \rightarrow |0\rangle\,|0\rangle\,, \;|00\rangle$
- Shorthand notation for basis states: $|x\rangle$ where $x \in 0, \ldots, 2^n - 1$

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- Qubits can be **entangled**

The circuit model of Quantum Computing

Extracting information out of qubits: projective measurements

- At the end of the computation we obtain the final wave function in the computational basis

$$|\psi\rangle = \sum_{i=0}^{2^n-1} c_i |i\rangle$$

- Measurement in the computational basis: **project** the wave function **onto one of the basis states of the computational basis**

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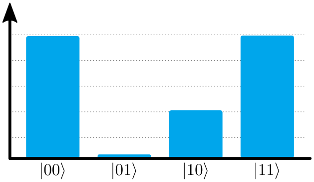
  - Measuring an operator: repeat the experiment and record the **histogram** of the observed basis states $\Rightarrow$ estimate for the $|c_i|^2$



Entanglement

The circuit model of Quantum Computing

Entanglement of bipartite systems

- Consider bipartite systems $\mathcal{H}_A \otimes \mathcal{H}_B$
- A **quantum state** that can be factored as a tensor product of states of its local constituents is called a **separable state** or **product state**

$$|\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$$

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Example

- $|\psi_1\rangle = |0\rangle \otimes |0\rangle + |0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle$

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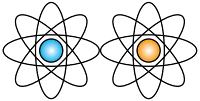
$\Rightarrow$ Entangled state (Bell state)

The circuit model of Quantum Computing

Entanglement of bipartite systems

- Entangled states **cannot be described** by the individual **states of the constituent systems**
- Consider the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$



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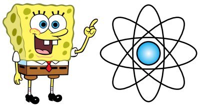

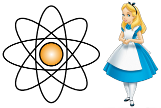

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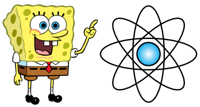

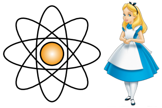

- Bob can measure his qubit

$$p_{\text{Bob}}(0) = 1/2,$$
$$p_{\text{Bob}}(1) = 1/2,$$

$\Rightarrow$ Bob does not obtain information about the state, his outcomes are random

The circuit model of Quantum Computing

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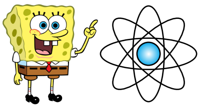

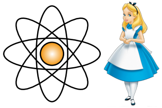

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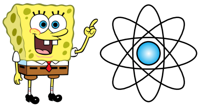

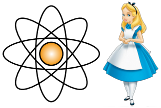

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p_{\text{Bob}}(0) &= 1/2, \qquad & |\psi\rangle &= |00\rangle\,, \qquad & p_{\text{Alice}}(0) &= 1 \\
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\end{aligned}
$$

$\Rightarrow$ Bob does not obtain information about the state, his outcomes are random

- If Alice measures after Bob she obtains the same result as Bob with certainty

$\Rightarrow$ Perfect **correlation** between the measurement outcomes

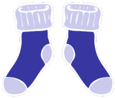
The circuit model of Quantum Computing

Entanglement of bipartite systems

> Let us consider the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle\right)$

$\Rightarrow$ The measurement outcomes are perfectly correlated

> Are these correlations any special?



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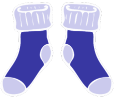

- Outcome is correlated no matter which basis we choose

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle) \quad \Rightarrow \quad |\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle \otimes |+\rangle + |-\rangle \otimes |-\rangle\right)$$

- These type of correlations **do not have a classical counterpart**

Quantum gates

The circuit model of Quantum Computing

Quantum gates

- Quantum mechanics is reversible, $|\psi\rangle$ undergoes unitary evolution under some (time-dependent) Hamiltonian $H(t)$

$$|\psi(t)\rangle = T\,\mathrm{exp}\left(-i\int_0^t ds\,H(s)\right)|\psi_0\rangle$$

- **Quantum gates** are represented by **unitary matrices**

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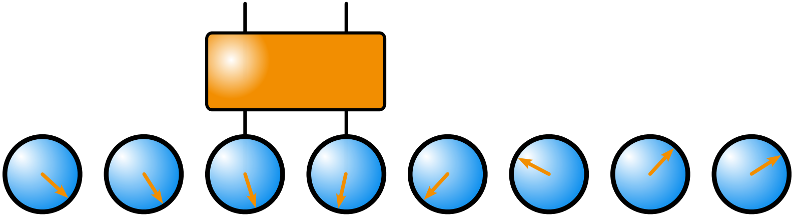
- **Quantum gates** are represented by **unitary matrices**
- Typically gates only act on a few qubits in a nontrivial way



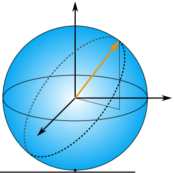
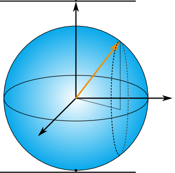
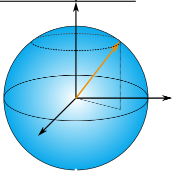
The circuit model of Quantum Computing

Common single-qubit quantum gates

	Circuit symbol	Matrix	Action
Hadamard	$-\boxed{H}-$	$H = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$	$\lvert 0\rangle \to \frac{1}{\sqrt{2}}(\lvert 0\rangle + \lvert 1\rangle)$ $\lvert 1\rangle \to \frac{1}{\sqrt{2}}(\lvert 0\rangle - \lvert 1\rangle)$
X	$-\boxed{X}-$	$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\lvert 0\rangle \to \lvert 1\rangle$ $\lvert 1\rangle \to \lvert 0\rangle$
Y	$-\boxed{Y}-$	$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$	$\lvert 0\rangle \to -i\lvert 1\rangle$ $\lvert 1\rangle \to i\lvert 0\rangle$
Z	$-\boxed{Z}-$	$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\lvert 0\rangle \to \lvert 0\rangle$ $\lvert 1\rangle \to -\lvert 1\rangle$

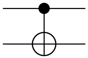
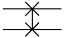
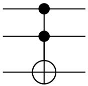
The circuit model of Quantum Computing

Common single-qubit rotations

$R_x(\theta)$	$-\boxed{R_x(\theta)}-$	$R_x(\theta) = \exp\left(-i\frac{\theta}{2}X\right)$	
$R_y(\theta)$	$-\boxed{R_y(\theta)}-$	$R_y(\theta) = \exp\left(-i\frac{\theta}{2}Y\right)$	
$R_z(\theta)$	$-\boxed{R_z(\theta)}-$	$R_z(\theta) = \exp\left(-i\frac{\theta}{2}Z\right)$	

The circuit model of Quantum Computing

Common multi-qubit quantum gates

CNOT		$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	$\|00\rangle \to \|00\rangle$ $\|01\rangle \to \|01\rangle$ $\|10\rangle \to \|11\rangle$ $\|11\rangle \to \|10\rangle$
SWAP gate		$\text{SWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	$\|00\rangle \to \|00\rangle$ $\|01\rangle \to \|10\rangle$ $\|10\rangle \to \|01\rangle$ $\|11\rangle \to \|11\rangle$
Toffoli gate		$\begin{pmatrix} \mathbb{1}_{6\times 6} & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$	CCNOT

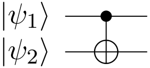
The circuit model of Quantum Computing

Quantum gates

- Gate operations allow for replicating classical computation on a quantum computer
- The Hadamard gate can **create superpositions** out of a single basis state

$$|0\rangle \rightarrow |+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

- The CNOT gate can **create entanglement**



$$|\psi_1\rangle \otimes |\psi_2\rangle \rightarrow |\phi_{12}\rangle \neq |\phi_1\rangle \otimes |\phi_2\rangle$$

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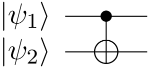


$$|\psi_1\rangle \otimes |\psi_2\rangle \to |\phi_{12}\rangle \neq |\phi_1\rangle \otimes |\phi_2\rangle$$

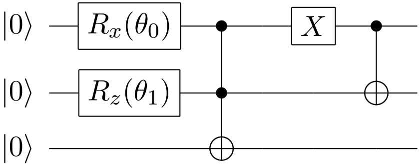
- Since quantum mechanics is linear, we can apply gates to superpositions of basis states

$$\begin{aligned}\text{CNOT}\big(\alpha\,|00\rangle + \beta\,|01\rangle + \gamma\,|10\rangle + \delta\,|11\rangle\big)\\ =\alpha\,|00\rangle + \beta\,|01\rangle + \gamma\,|11\rangle + \delta\,|10\rangle\end{aligned}$$

The circuit model of Quantum Computing

Quantum circuits

> Combining multiple gates we can build **quantum circuits**



The circuit model of Quantum Computing

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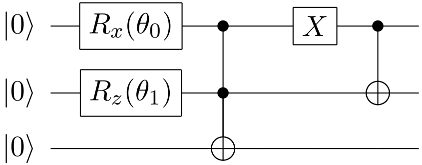

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The circuit model of Quantum Computing

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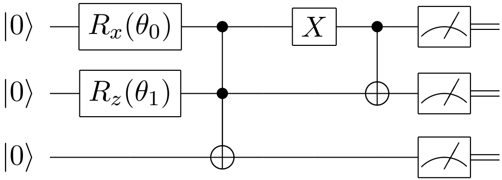

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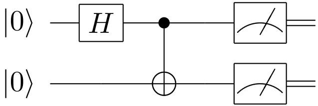
The circuit model of Quantum Computing

Example: preparing a Bell state

- Simple circuit preparing an entangled state (Bell state)



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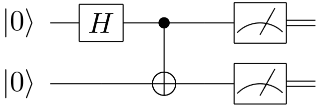

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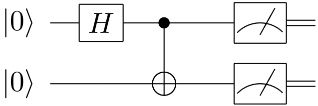

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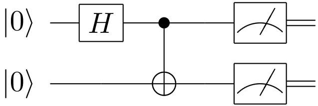

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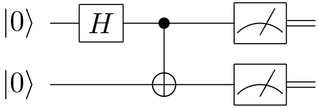

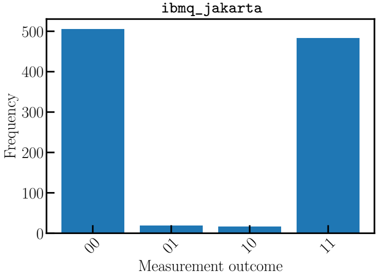

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The circuit model of Quantum Computing

Universal gate set

A set of gates is **universal** if, by composing gates from it, one can express **any unitary transformation** on any number of qubits.

- Since the n-qubit unitaries form an uncountable infinite set $U(2^n)$, this requires an infinite number of gates
- Example: $\{\text{CNOT}, R_x(\theta), R_y(\theta), R_z(\theta)\}$, $\theta \in [0, 2\pi)$

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Approximate universal gate set

A set of gates is **universal** if, by composing gates from it, one can **approximate any unitary transformation** on any number of qubits to **any desired precision**.

- Examples: $\{\text{CNOT}, R_y(\pi/4), R_z(\pi/2)\}$, $\{\text{CCNOT}, H, R_z(\pi/2)\}$

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A set of gates is **universal** if, by composing gates from it, one can express **any unitary transformation** on any number of qubits.

- Since the n-qubit unitaries form an uncountable infinite set $U(2^n)$, this requires an infinite number of gates
- Example: $\{\text{CNOT}, R_x(\theta), R_y(\theta), R_z(\theta)\}$, $\theta \in [0, 2\pi)$

Approximate universal gate set

A set of gates is **universal** if, by composing gates from it, one can **approximate any unitary transformation** on any number of qubits to **any desired precision**.

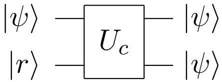
- Examples: $\{\text{CNOT}, R_y(\pi/4), R_z(\pi/2)\}$, $\{\text{CCNOT}, H, R_z(\pi/2)\}$
- Approximation can be done efficiently (Solovay-Kitaev theorem, $\mathcal{O}(\text{polylog}(1/\varepsilon))$)

Copying quantum states

The circuit model of Quantum Computing

Can we copy quantum states?

- Classically: we can arbitrarily copy bits
- Quantum version: we want a unitary U_c which allows for



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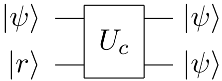

- In particular we have have to be able to copy basis states

$$U_c\,|0\rangle \otimes |r\rangle \rightarrow |0\rangle \otimes |0\rangle$$
$$U_c\,|1\rangle \otimes |r\rangle \rightarrow |1\rangle \otimes |1\rangle$$

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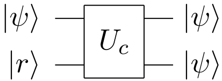

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$$U_c\,|0\rangle \otimes |r\rangle \to |0\rangle \otimes |0\rangle$$
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- Since $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

$$\begin{aligned} U_c\,|\psi\rangle \otimes |r\rangle &= U_c\big(\alpha\,|0\rangle + \beta\,|1\rangle\big) \otimes |r\rangle \\ &= U_c\big(\alpha\,|0\rangle \otimes |r\rangle + \beta\,|1\rangle \otimes |r\rangle\big) \\ &= \alpha\,|0\rangle \otimes |0\rangle + \beta\,|1\rangle \otimes |1\rangle \\ &\neq |\psi\rangle \otimes |\psi\rangle \end{aligned}$$

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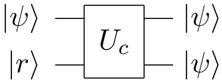

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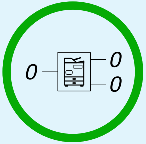
No cloning theorem

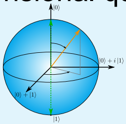
A quantum state cannot be copied with perfect fidelity.

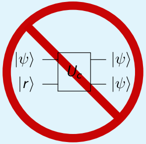
The circuit model of Quantum Computing

Summary: Classical vs. Quantum Computing

Classical computing	Quantum computing
> Classical bits are 0 or 1 > Typically irreversible gate set, but reversible is possible > We can arbitrarily copy bits	> Qubit: two-dimensional quantum system ⇒ Can be in superposition > Unitary evolution, reversible > No cloning theorem







The circuit model of Quantum Computing

Why is quantum computing more powerful?

- The Hilbert space of n qubits is the tensor product $\underbrace{\mathcal{H} \otimes \cdots \otimes \mathcal{H}}_{n \text{ times}}$

  $\Rightarrow$ Dimension 2^n, **number of basis states grows exponentially**

The circuit model of Quantum Computing

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- We can build **superpositions** of basis states and apply unitary gates to them

$$|0\rangle + |1\rangle \; -\!\!\boxed{U}\!\!- \; U|0\rangle + U|1\rangle$$

$\Rightarrow$ **“Quantum parallelism”**

The circuit model of Quantum Computing

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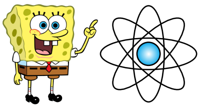
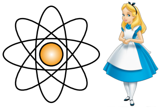
$\Rightarrow$ **“Quantum parallelism”**

- Multiple **qubits can be entangled**



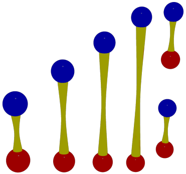


$\Rightarrow$ Correlations that have no classical analog

The circuit model of Quantum Computing

Why quantum computing for lattice field theories?

> Conventional **Monte Carlo methods** rely on a Wick rotation resulting in a theory in **Euclidean time**
$\Rightarrow$ No access to real-time dynamics



M. Troyer, U.-J. Wiese, Phys. Rev. Lett. 94, 170201 (2005)

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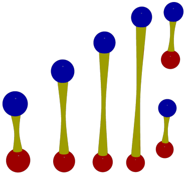
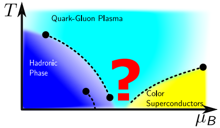


- **Monte Carlo approach** suffers from the **sign problem** in certain parameter regimes even for static problems
  $\Rightarrow$ Not solvable in its full generality



M. Troyer, U.-J. Wiese, Phys. Rev. Lett. 94, 170201 (2005)

The circuit model of Quantum Computing

How to use quantum computing for lattice field theories?

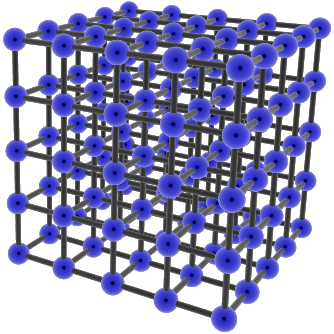
- Typically **quantized Hamiltonian formulation** of the problem
- Hamiltonian formulation can also be tackled classical methods
  - Strong coupling expansions
  - Gaussian states
  - Tensor Networks
  - ...
- These methods (partially) allow for overcoming the limitations of Monte Carlo methods but have other limitations
- **Quantum computers** are believed to be able to **efficiently tackle** problems inaccessible with Monte Carlo approaches

T. Banks, L. Susskind, J. Kogut, Phys. Rev. D 13, 1043 (1976)
T. Banks, S. Raby, L. Susskind, J. Kogut, D. R. T. Jones, P. N. Scharbach, D. K. Sinclair, Phys. Rev. D 15, 1111 (1977)
C. J.Hamer, J. Kogut, L. Susskind, Phys. Rev. D 19, 3091 (1979)
P. Sala, T. Shi, S. Kühn, M. C. Bañuls,E. Demler, J. I. Cirac, Phys. Rev. D 98, 034505 (2018)
M.C. Bañuls, K. Cichy, J. I. Cirac, K. Jansen, S. Kühn, PoS(LATTICE 2018), 022 (2019)

The circuit model of Quantum Computing

Challenges

- Continuous (gauge) symmetries lead to **infinite dimensional Hilbert spaces**, but qubits have two-dimensional Hilbert spaces
- Dealing with **fermionic degrees of freedom**
- Finding the ground state of a given Hamiltonian
- Preparing “interesting states”
- Maintaining **gauge invariance** in a noisy device
- ...



2.

The circuit model of Quantum Computing

The variational quantum eigensolver

Real-time dynamics

Summary & Outlook

The variational quantum eigensolver

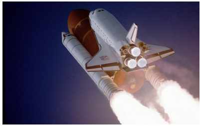
Current NISQ devices

- Small or intermediate scale
- Considerable amount of noise
- Only shallow circuits can be executed faithfully/no error correction
- Quantum advantage demonstrated

The variational quantum eigensolver

Current NISQ devices	Solving "useful" problems
> Small or intermediate scale > Considerable amount of noise > Only shallow circuits can be executed faithfully/no error correction > Quantum advantage demonstrated	> Large number of qubits > Deep circuits > Quantum error correction necessary > So far only proof of principle demonstrations

The variational quantum eigensolver



Current NISQ devices

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Solving “useful” problems

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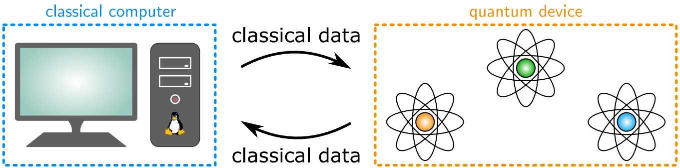
- Deep circuits
- Quantum error correction necessary
- So far only proof of principle demonstrations

How can we utilize existing quantum hardware in a beneficial way?

The variational quantum eigensolver

Hybrid quantum-classical algorithms

- Combine classical and quantum devices
- Rely on classical computing where possible
- Use the quantum device as a coprocessor
  - Tackle the classically hard/intractable part of the problem
  - Feed the classical data obtained from a measurement back to the classical computer



The variational quantum eigensolver

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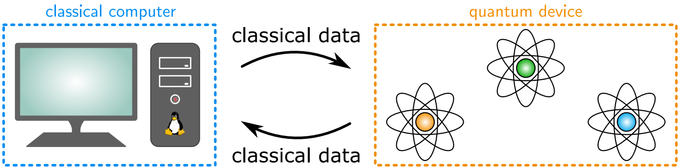


Hope: even modest quantum hardware can yield advantages

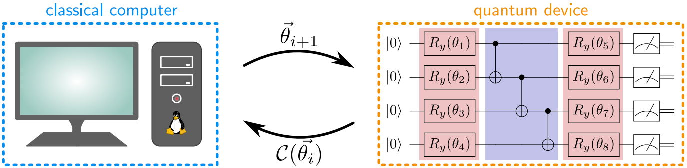
The variational quantum eigensolver

Hybrid quantum-classical algorithms

- Focus on optimization problems trying to minimize a cost function $\mathcal{C}(\vec{\theta})$

$$\min_{\vec{\theta}} \mathcal{C}(\vec{\theta}) = \langle \psi(\vec{\theta}) | H | \psi(\vec{\theta}) \rangle, \qquad \vec{\theta} = \mathbb{R}^n$$

- Solve them iteratively using a parametric ansatz
  - Quantum coprocessor: prepare the **variational ansatz** $|\psi(\vec{\theta}_i)\rangle$ and evaluate $\mathcal{C}(\vec{\theta}_i)$
  - Classical computer: given $\mathcal{C}(\vec{\theta}_i)$, find optimized $\vec{\theta}_{i+1}$



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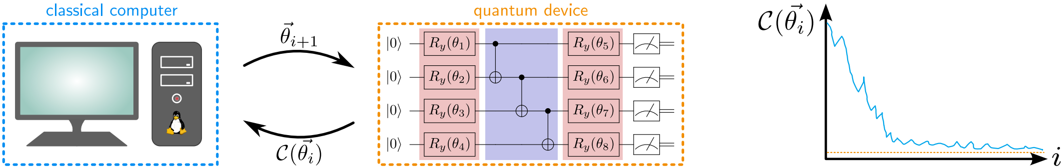

$\Rightarrow$ Run **feedback loop** between the classical computer and the quantum device until convergence

The variational quantum eigensolver

Variational Quantum Eigensolver (VQE) for computing ground states

- Principle can be used to find ground states of (quantum) Hamiltonians H
- Use the energy expectation value as a cost function

$$\mathcal{C}(\vec{\theta}) = \langle \psi(\vec{\theta}) | H | \psi(\vec{\theta}) \rangle$$

- $|\psi(\vec{\theta})\rangle$ ansatz realized by a parametric quantum circuit
- If $|\psi(\vec{\theta})\rangle$ is expressive enough the ground state of H corresponds to the global minimum of of $\mathcal{C}(\vec{\theta})$

Peruzzo et al., Nat. Commun. 5, 1 (2014)
J. R. McClean et al., New J. Phys. 18, 023023 (2016)

The variational quantum eigensolver

Variational Quantum Eigensolver (VQE)

- For Hamiltonians H that are the sum of a polynomial number of terms the cost function can be measured efficiently on a quantum device, for example

$$H = \sum_{i=1}^{N} h_{i,i+1} \qquad \Leftrightarrow \qquad \mathcal{C}(\theta) = \sum_{i} \langle \psi(\vec{\theta}) | h_{i,i+1} | \psi(\vec{\theta}) \rangle$$

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- Example: the Ising model

$$H_{\text{Ising}} = \sum_{i=1}^{N-1} Z_i Z_{i+1} + h \sum_{i=1}^{N} X_i$$

- There exist generalizations to obtain excited states

G. Xu, Y. B. Guo, X. Li, K. Wang, Z. Fan, Z. S. Zhou, H. J. Liao, T. Xiang, Phys. Rev. A 107, 052423 (2023)
O. Higgot, D.v Wang, S. Brieley, Quantum 3, 156 (2019)
T. Jones, S. Endo, S. McArdle, X. Yuan, S. C. Benjamin, Phys. Rev. A 99, 062304 (2019)
K. M. Nakanishi, K. Mitarai, K. Fujii, Phys. Rev. Research 1, 033062 (2019)

The variational quantum eigensolver

Example: Schwinger model

- Lagrangian of the theory: QED in 1+1 dimensions

$$\mathcal{L} = \int \mathrm{d}t\,\mathrm{d}x \left(\underbrace{i\overline{\Psi}\partial\!\!\!/\Psi}_{\text{Kinetic term}} - \underbrace{g\overline{\Psi}A\!\!\!/\Psi}_{\text{Coupling to gauge field}} - \underbrace{m\overline{\Psi}\Psi}_{\text{Mass}} - \underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{Electromagnetic term}} \right)$$

- Matter fields: $\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$

J. Schwinger, Phys. Rev. 128, 2425 (1962)

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- Matter fields: $\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$
- Hamiltonian formulation in temporal gauge $(A^0 = 0)$

$$H = \int \mathrm{d}x \left(\underbrace{-i\bar{\Psi}\gamma^1\partial_1\Psi}_{\text{Kinetic term}} + \underbrace{\bar{\Psi}\gamma^1 g A_1 \Psi}_{\text{Coupling to gauge field}} + \underbrace{m\bar{\Psi}\Psi}_{\text{Mass}} + \underbrace{\frac{1}{2}E^2}_{\text{Electric energy}} \right)$$

- Additional constraint on the physical states: Gauss law

$$\partial_1 E = g\bar{\Psi}\gamma^0\Psi$$

J. Schwinger, Phys. Rev. 128, 2425 (1962)

The variational quantum eigensolver

Example: Schwinger model

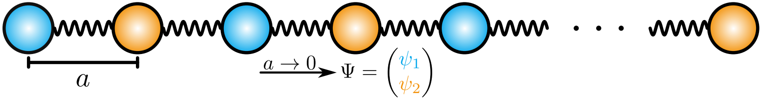
> Lattice formulation with Kogut-Susskind staggered fermions

$$H = -\frac{i}{2a}\sum_{n=1}^{N-1}\left(\phi_n^\dagger e^{i\theta(n)} \phi_{n+1} - \text{h.c.}\right) + \sum_{n=1}^{N}(-1)^n m\, \phi_n^\dagger \phi_n + \frac{ag^2}{2}\sum_{n=1}^{N-1} L_n^2$$

Kinetic part + Coupling to gauge field

Staggered mass term

Electric energy



J. Kogut, L. Susskind, Phys. Rev. D 11, 395 (1975)

The variational quantum eigensolver

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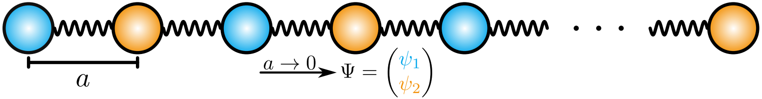
> Gauss Law: $L_n - L_{n-1} = Q_n = \phi_n^\dagger \phi_n - \frac{1}{2}\left(1 - (-1)^n\right)$



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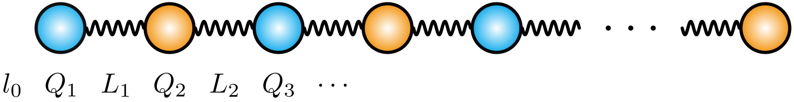
Kinetic part + Coupling to gauge field — Staggered mass term — Electric energy

- Gauss Law: $L_n - L_{n-1} = Q_n = \phi_n^\dagger \phi_n - \frac{1}{2}\left(1 - (-1)^n\right)$
- Use OBC and Gauss Law to integrate out the gauge field



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$l_0 \quad Q_1 \quad L_1 + Q_2 = L_2 \quad Q_3 \quad \cdots$

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$l_0{+}Q_1 \quad + \quad Q_2{=}L_2 \quad Q_3 \quad \cdots$

J. Kogut, L. Susskind, Phys. Rev. D 11, 395 (1975)

The variational quantum eigensolver

Example: Schwinger model at $l_0 = 0$

- Residual gauge transformation allows for fully removing the gauge fields

$$H' = -\frac{i}{2a}\sum_{n=1}^{N-1}\left(\phi_n^\dagger \phi_{n+1} - \text{h.c.}\right) + \sum_{n=1}^{N}(-1)^n m \phi_n^\dagger \phi_n + \frac{ag^2}{2}\sum_{n=1}^{N-1}\left(\boxed{l_0} + \sum_{k=1}^{n} Q_k\right)^2$$

Topological term $l_0 = \theta/2\pi$

C. J. Hamer, Z. Weihong, Zheng J. Oitmaa, Phys. Rev. D 56, 55 (1997)

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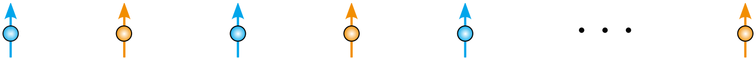
Topological term $l_0 = \theta/2\pi$

- Hamiltonian can be translated to qubits using a **Jordan-Wigner** transformation

$$\phi_n = \prod_{l<n}(iZ_l)\sigma_n^-, \qquad \sigma^\pm = (X + iY)$$

- Resulting spin Hamiltonian

$$H' = \frac{1}{2a}\sum_{n=1}^{N-1}\left(\sigma_n^+ \sigma_{n+1}^- + \text{h.c.}\right) + \frac{1}{2}\sum_{n=1}^{N}(-1)^n m(Z_n + \mathbb{1}) + \frac{ag^2}{2}\sum_{n=1}^{N-1}\left(l_0 + \sum_{k=1}^{n} Q_k\right)^2$$



C. J. Hamer, Z. Weihong, Zheng J. Oitmaa, Phys. Rev. D 56, 55 (1997)

The variational quantum eigensolver

Example: Schwinger model at $l_0 = 0$

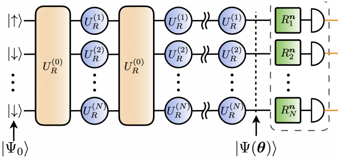
> Resource Hamiltonians used for gate operations $U = \exp(-i\theta H_R^k)$

$$H_R^0 = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} J_{ij}(\sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+) + B \sum_{i=1}^{N} Z_i, \qquad H_R^j = \frac{\Delta_0}{2} Z_j$$

> Resource Hamiltonians respect charge conservation

> Restrict the parameters of the single-qubit gates as $\theta^j = -\theta^{N+1-j}$ to enforce CP symmetry

> For large systems the bulk should approximately be translation invariant



$\Rightarrow$ Restrict parameters in bulk to the same value

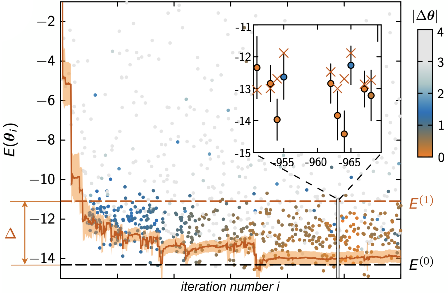
> Initial wave function compatible with the symmetries: Neel state $|\psi_0\rangle = |\uparrow\downarrow \cdots \uparrow\downarrow\rangle$

C. Kokail et al., Nature 569, 355 (2019)

The variational quantum eigensolver

Example: Schwinger model at $l_0 = 0$

- Results on a trapped ion quantum computer with 20 qubits formed by $^{40}\text{Ca}^+$ ions
- Simulation involves 6 layers and 15 parameters



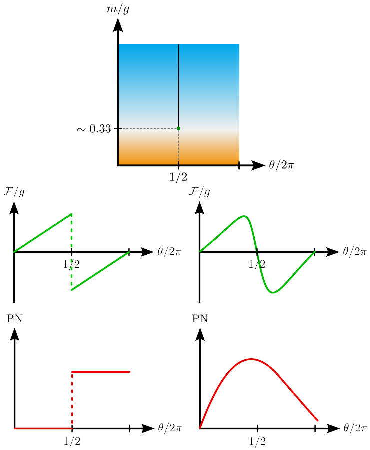
$\Rightarrow$ Ground state can be determined with good accuracy

C. Kokail et al., Nature 569, 355 (2019)

The variational quantum eigensolver

Example: Schwinger model with topological term

- Topological term: sign problem for MC methods
- Phase structure: known from analytical arguments and Tensor-Network calculations
- Regime $m/g \gg 1$:
  - $l_0 < 1/2$: no particles produced, dynamical electric field matches background field
  - $l_0 > 1/2$: production of a particle-antiparticle pair resulting in a nonvanishing dynamical electric field
- Regime $m/g \ll 1$:
  - Complete screening of the electric field
- First-order quantum phase transition above a certain critical mass $m_c/g \approx 0.33$

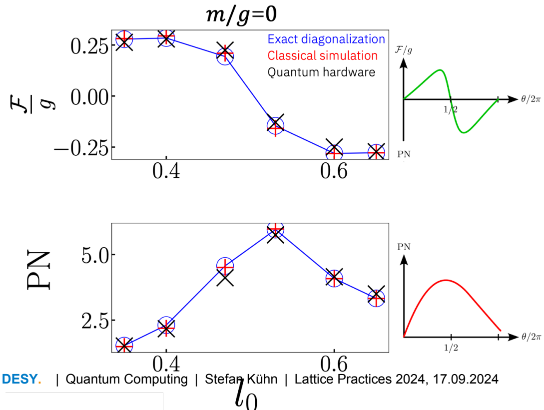


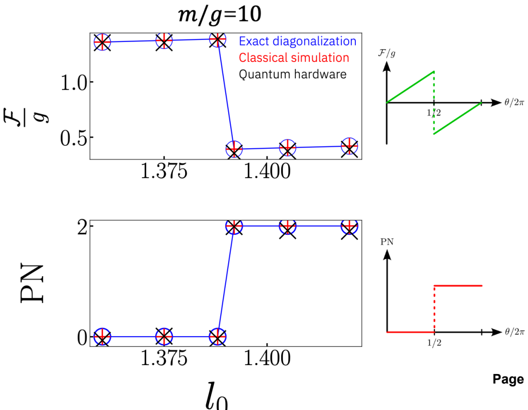
T. Angelides, P. Naredi, A. Crippa, K. Jansen, S. Kühn, I. Tavernelli, D. S.. Wang, arXiv:2312.12831

The variational quantum eigensolver

Example: Schwinger model with topological term

- Focus on $N = 12$ and both mass regimes $m/g \ll 1$ and $m/g \gg 1$
- Results from QC with Pauli twirling, dynamical decoupling and zero-noise extrapolation

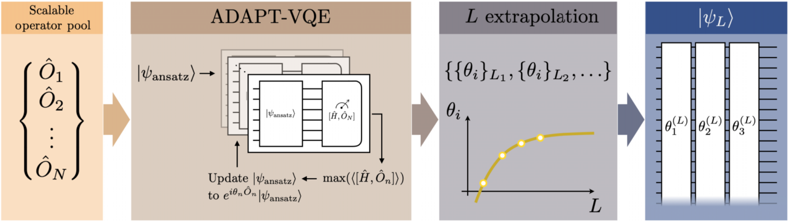




The variational quantum eigensolver

Example: Schwinger model at large scales

- Scaling up a naive VQE approach does typically not work (later more)
  - Idea: physics should be governed at length scale of the correlation length ξ , correlations decay exponentially for $r \gg \xi$
  - State preparation circuits need to have structure only for qubits spatially separated by $r \leq \xi$
- Because of translational invariance: ground state for an arbitrarily large lattice can be prepared by repetition of these circuits



R. C. Farrell, M. Illa, A. N. Ciavarella, M. J. Savage, PRX Quantum 5, 020315

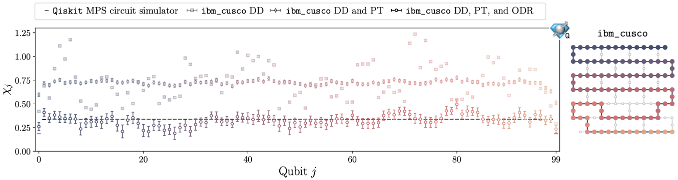
The variational quantum eigensolver

Example: Schwinger model at large scales

- Tune the variational parameters by simulating the VQE classically for up to 28 qubits
- Using the extrapolation go to larger systems
- Chiral condensate

$$\chi = \frac{1}{2L}\sum_{j=0} 2L-1\langle(-1)^j Z_j + 1\rangle = \frac{1}{2L}\sum_{j=0} 2L-1\chi_j$$

for 100 qubits



R. C. Farrell, M. Illa, A. N. Ciavarella, M. J. Savage, PRX Quantum 5, 020315

The variational quantum eigensolver

Barren plateaus

- Optimizing the parameters using a classical algorithms turns out to be challenging
- For a wide class of parametrized circuits the **probability to have a non-vanishing gradient** along any direction **vanishes exponentially** with the number of qubits

$\Rightarrow$ **Barren plateaus**, gradient-based optimizers will fail

J. R. McClean, S. Boixo, V. N. Smelyanskiy, R. Babbush, H. Neven, Nat Commun **9**, 4812 (2018)

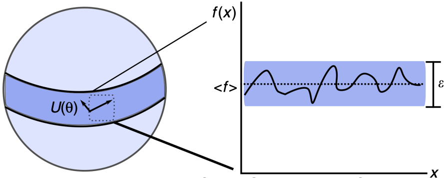
The variational quantum eigensolver

Barren plateaus

- Optimizing the parameters using a classical algorithms turns out to be challenging
- For a wide class of parametrized circuits the **probability to have a non-vanishing gradient** along any direction **vanishes exponentially** with the number of qubits

⇒ **Barren plateaus**, gradient-based optimizers will fail

- Mathematical reason: concentration of measure, sufficiently smooth function is concentrated in an exponentially small region around the mean

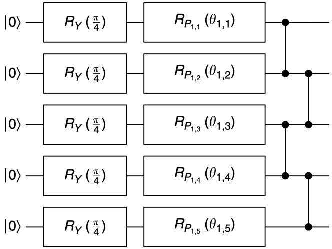


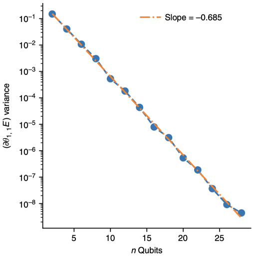
J. R. McClean, S. Boixo, V. N. Smelyanskiy, R. Babbush, H. Neven, Nat Commun **9**, 4812 (2018)

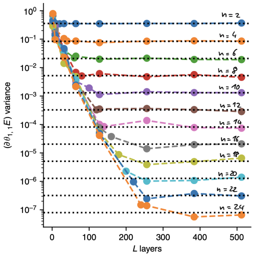
The variational quantum eigensolver

Barren plateaus

- This happens for sufficiently random circuits (match the Haar distribution up to the second moment)
- In practice this phenomena is already observed for relative simple ansatz circuits







J. R. McClean, S. Boixo, V. N. Smelyanskiy, R. Babbush, H. Neven, Nat Commun **9**, 4812 (2018)

The variational quantum eigensolver

Barren plateaus

- It was shown that not only does the gradient vanish, but also the **cost function** has **exponentially narrow minima**
- Not only the gradients vanish exponentially, but also the variance of the cost function itself

⇒ Switching to a **gradient free optimization algorithm** does **not avoid the problem**

- So far it seems that all provably trainable circuits are efficiently classical simulateable

M. Cerezo et al., arXiv:2312.09121
A. Arrasmith, M. Cerezo, P. Czarnik, L. Cincio, P. J. Coles, Quantum **5**, 558 (2022)
A. Arrasmith, Z. Holmes, M. Cerezo, P. J. Coles, Quantum Science and Technology **7**, 045015 (2022)
M. Larocca, S. Thanasilp, S. Wang, K. Sharma, J. Biamonte, P. J. Coles, L. Cincio, J. R. McClean, Z. Holmes, M. Cerezo, arXiv:2405.00781

The variational quantum eigensolver

VQE

- Hybrid quantum-classical algorithm for computing ground states/low-lying excitations
- Efficient as long as H has only a polynomial number of terms
- Hamiltonian exists only as a measurement
- Great freedom choosing the circuit implementing $|\psi(\vec{\theta})\rangle$
- Problem of barren plateaus
- Unclear if one can solve anything efficiently that cannot be simulated classically

3.

The circuit model of Quantum Computing

The variational quantum eigensolver

Real-time dynamics

Summary & Outlook

Real-time dynamics

Time evolution on quantum computer

- Evolution of the wave function $|\psi_0\rangle$ under the Hamiltonian

$$|\psi(t)\rangle = \exp\left(-iHt\right)|\psi_0\rangle$$

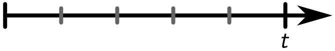
- In general $\exp\left(-iHt\right)$ cannot be efficiently implemented on a quantum computer

Real-time dynamics

Time evolution on quantum computer

- Evolution of the wave function $|\psi_0\rangle$ under the Hamiltonian

$$|\psi(t)\rangle = \exp\left(-iHt\right)|\psi_0\rangle = \left[\exp\left(-iH\frac{t}{n}\right)\right]^n|\psi_0\rangle$$



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Real-time dynamics

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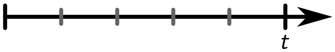

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- Physically relevant Hamiltonians are typically the sum of local terms $H = \sum_k h_k$

Real-time dynamics

Time evolution on quantum computer

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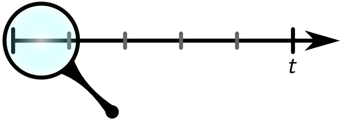


- In general $\exp\left(-iHt\right)$ cannot be efficiently implemented on a quantum computer
- Physically relevant Hamiltonians are typically the sum of local terms $H = \sum_k h_k$
- Using a Suszuki-Trotter decomposition with $\Delta t = t/n$ we can approximate

$$\exp\left(-iH\Delta t\right) \approx \prod_k \exp\left(-ih_k\Delta t\right) + \mathcal{O}\left((\Delta t)^2\right)$$

- $\exp\left(-ih_k\Delta t\right)$ is often simple enough to be efficiently implementable

Real-time dynamics

Example: The Ising model

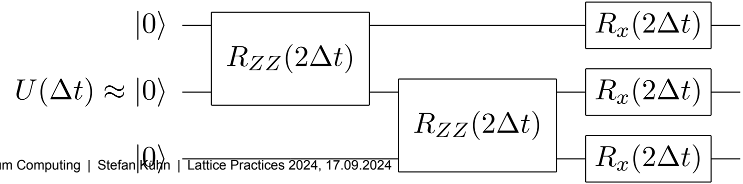
> The Ising Hamiltonian with open boundary conditions for n qubits

$$H = \sum_{k=1}^{n-1} Z_k Z_{k+1} + h \sum_{k=1}^{n} X_k$$

> Trotterizing the time evolution operator for one step, we can find

$$U(\Delta t) = \text{exp}(-iH\Delta t) \approx \prod_{k=1}^{n-1} \text{exp}(-iZ_k Z_{k+1}\Delta t) \prod_{k=1}^{n} \text{exp}(-iX_k \Delta t)$$

> Quantum circuit for one time step (for 3 qubits)



Real-time dynamics

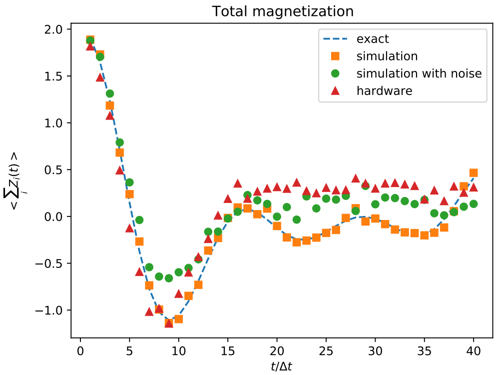
Example: The Ising model

- Look at the Ising Hamiltonian for $N = 4$ and $h = 1.5$
- Simulate the out-of-equilibrium dynamics following the quench

$$|\psi(t = n\Delta t)\rangle = U(\Delta t)^n \, |0010\rangle$$

- Measure the total magnetization $\langle \sum_i Z_i \rangle$ as a function of time

Results on `ibmq_toronto`



Real-time dynamics

Scattering in the Thirring model

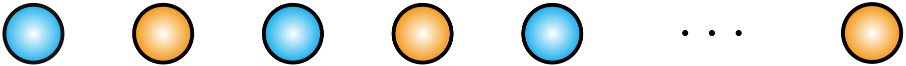
- Exactly solvable fermionic model in one spatial dimension
- Toy model sharing some similarities with QCD
- Lattice formulation with staggered fermions assuming periodic boundary conditions

$$H = \frac{i}{2a}\sum_{n=0}^{N-1}\left(\xi_{n+1}^{\dagger}\xi_{n}^{\dagger} - \xi_{n}^{\dagger}\xi_{n+1}\right) + m\sum_{n=0}^{N-1}(-1)^{n}\xi_{n}^{\dagger}\xi_{n} + \frac{g}{a}\sum_{n=0}^{N-1}\xi_{n}^{\dagger}\xi_{n}\xi_{n+1}^{\dagger}\xi_{n+1}$$

Kinetic term

Staggered mass term

Four-fermion interaction



Y. Chai, A. Crippa, Karl Jansen, SK, V. R. Pascuzzi, F. Tacchino, I. Tavernelli, arXiv:2312.02272

Real-time dynamics

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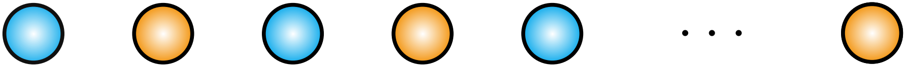
- Lattice formulation with staggered fermions assuming periodic boundary conditions

$$H = \frac{i}{2a}\sum_{n=0}^{N-1}\left(\underbrace{\boxed{\xi_{n+1}^\dagger \xi_n^\dagger - \xi_n^\dagger \xi_{n+1}}}_{\text{Kinetic term}}\right) + m\sum_{n=0}^{N-1}\underbrace{\boxed{(-1)^n \xi_n^\dagger \xi_n}}_{\text{Staggered mass term}} + \frac{g}{a}\sum_{n=0}^{N-1}\underbrace{\boxed{\xi_n^\dagger \xi_n \xi_{n+1}^\dagger \xi_{n+1}}}_{\text{Four-fermion interaction}}$$



- Use as a prototype to study scattering between wave packets

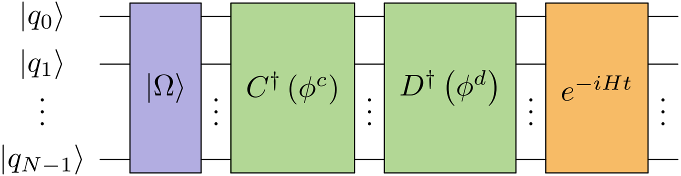
Y. Chai, A. Crippa, Karl Jansen, SK, V. R. Pascuzzi, F. Tacchino, I. Tavernelli, arXiv:2312.02272

Real-time dynamics

Scattering in the Thirring model

- Simulating real-time dynamics of scattering

$$|\psi(t)\rangle = \exp(-iHt)\,|\psi(0)\rangle = \underbrace{\exp(-iHt)}_{\text{Time evolution}}\,\underbrace{D^\dagger(\phi^d)C^\dagger(\phi^d)}_{\text{Creating wave packets}}\,\underbrace{|\Omega\rangle}_{\text{Ground state}}$$



- Problem: creating Gaussian wave packets is in general not a unitary operations that can be directly executed on a quantum computer

Y. Chai, A. Crippa, Karl Jansen, SK, V. R. Pascuzzi, F. Tacchino, I. Tavernelli, arXiv:2312.02272

Real-time dynamics

Scattering in the Thirring model

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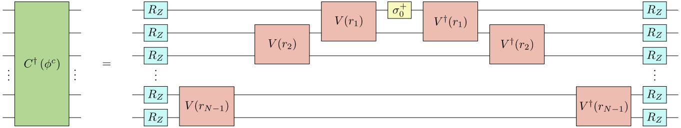
$$|\psi(t)\rangle = \exp(-iHt)\,|\psi(0)\rangle = \exp(-iHt)\; D^\dagger(\phi^d) C^\dagger(\phi^d)\; |\Omega\rangle$$

Time evolution

Creating wave packets

Ground state



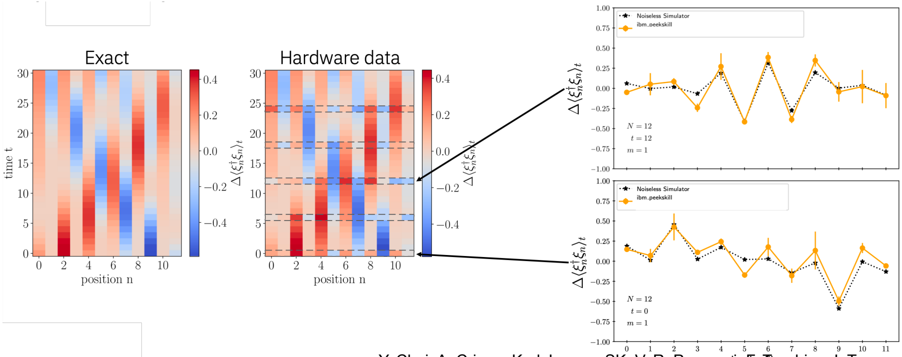
- Problem: creating Gaussian wave packets is in general not a unitary operations that can be directly executed on a quantum computer
- Observables can be decomposed using **Givens rotation** which allows for efficiently measuring observables

Y. Chai, A. Crippa, Karl Jansen, SK, V. R. Pascuzzi, F. Tacchino, I. Tavernelli, arXiv:2312.02272

Real-time dynamics

Scattering in the Thirring model

- Focus on the noninteracting case $g = 0$
- Results for $N = 12$, $m = 1.0$ on `ibm_peekskill` after Pauli twirling, dynamical decoupling and zero-noise extrapolation

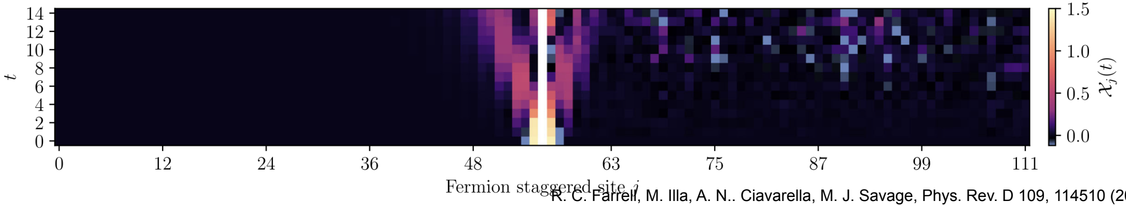


Y. Chai, A. Crippa, Karl Jansen, SK, V. R. Pascuzzi, F. Tacchino, I. Tavernelli, arXiv:2312.02272

Real-time dynamics

Schwinger Model dynamics at large scales

- Use the ADAPT VQE procedure to prepare the ground state just as previously
- Use a variant of the approach to prepare a hadron wave packet on top
- Evolve the hadron wave packet in time using a Trotter decomposition of the Hamiltonian
- Long-range interactions due to integrating out the gauge fields are truncated
- Results for 112 qubits on `ibm_torino`



R. C. Farrell, M. Illa, A. N.. Ciavarella, M. J. Savage, Phys. Rev. D 109, 114510 (2024)

4.

The circuit model of Quantum Computing

The variational quantum eigensolver

Real-time dynamics

Summary & Outlook

Summary

Quantum computing for lattice field theory

- Currently and in the near future only NISQ devices available
- These are most likely not able to overcome the big open problems in lattice field theory
- First steps can already be taken now to learn the technology and develop new methods suited for quantum devices
- Quantum computing for lattice field theories is still in its infancy, further developments are needed
  - Quantum hardware
  - Formulations and techniques suitable for quantum computing
  - Algorithms

Outlook

Outlook

- There are techniques for systematically designing ansatz circuits for VQE

S. Sim, P. D. Johnson, A. Aspuru-Guzik. Adv. Quantum Technol. 2, 1900070 (2019)
M. Bataille, arXiv:2009.13247
L. Funcke, T. Hartung, K. Jansen, S. Kühn, P. Stornati, Quantum 5, 422 (2021)

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- There are techniques for mitigating errors designed for current noisy intermediate-scale devices

S. Endo, S. C. Benjamin, Y. Li, Phys. Rev. X , 8 , 031027 (2018)
M. R. Geller, Quantum Science and Technology, 5(3):03LT01, (2020)
L. Funcke, T. Hartung, K. Jansen, S. Kühn, P. Stornati, X. Wang, arXiv:2007.03663

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S. Sim, P. D. Johnson, A. Aspuru-Guzik. Adv. Quantum Technol. 2, 1900070 (2019)
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L. Funcke, T. Hartung, K. Jansen, S. Kühn, P. Stornati, Quantum 5, 422 (2021)

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M. R. Geller, Quantum Science and Technology, 5(3):03LT01, (2020)
L. Funcke, T. Hartung, K. Jansen, S. Kühn, P. Stornati, X. Wang, arXiv:2007.03663

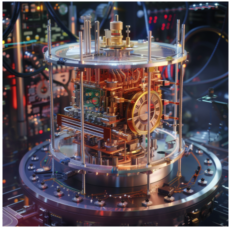
- IBM offers free access to small scale quantum computers
  - These can programmed with a Python frameworks called Qiskit
  - There is a great amount of tutorials and introductory material available

`https://qiskit.org/`
`hhttps://www.ibm.com/quantum`

Thank you!

Some references

- L. Funcke, T. Hartung, K. Jansen, S. Kühn, Review on Quantum Computing for Lattice Field Theory PoS (LATTICE2022), 228 (2023)
- M.C. Bañuls, K. Cichy, Review on novel methods for lattice gauge theories Rep. Prog. Phys. 83, 024401 (2022)
- M.C. Bañuls et al., Simulating lattice gauge theories within quantum technologies, The European Physical Journal D 74, 8 (2023)



Contact

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Appendix B. Projective measurements

Measuring observables

- Given an observable O we want to compute $\langle\psi| O |\psi\rangle$
- State can only be measured in the computational basis

$$\langle\psi| O |\psi\rangle = \langle\psi| U^\dagger U O U^\dagger U |\psi\rangle = \left\langle\psi'\right| U O U^\dagger \left|\psi'\right\rangle = \left\langle\psi'\right| D \left|\psi'\right\rangle$$

Appendix B. Projective measurements

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- Choose U such that $D = UOU^\dagger = \text{diag}(\lambda_0, \ldots, \lambda_{2^N-1}) = \sum_x \lambda_x |x\rangle\langle x|$ in the computational basis
- Expand $|\psi'\rangle$ in the computational basis: $|\psi'\rangle = \sum_x c'_x |x\rangle$

$$\langle\psi| O |\psi\rangle = \langle\psi'| D |\psi'\rangle = \sum_{x=0}^{2^N-1} \lambda_x \langle\psi' \underbrace{|x\rangle\langle x|}_{P_x} \psi'\rangle = \sum_{x=0}^{2^N-1} \lambda_x \langle\psi'|P_x|\psi'\rangle = \sum_{x=0}^{2^N-1} |c'_x|^2 \lambda_x$$

Appendix B. Projective measurements

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- Choose U such that $D = UOU^\dagger = \text{diag}(\lambda_0, \ldots, \lambda_{2^N-1}) = \sum_x \lambda_x |x\rangle\langle x|$ in the computational basis
- Expand $|\psi'\rangle$ in the computational basis: $|\psi'\rangle = \sum_x c'_x |x\rangle$

$$\langle\psi|\, O\, |\psi\rangle = \langle\psi'|\, D\, |\psi'\rangle = \sum_{x=0}^{2^N-1} \lambda_x \langle\psi'\, \underbrace{|x\rangle\langle x|}_{P_x}\, \psi'\rangle = \sum_{x=0}^{2^N-1} \lambda_x \langle\psi'|P_x|\psi'\rangle = \sum_{x=0}^{2^N-1} |c'_x|^2 \lambda_x$$

- U is often called **post rotation**
- Instead of $|\psi\rangle$ we prepare $|\psi'\rangle$ and measure the probability distribution $|c'_x|^2$

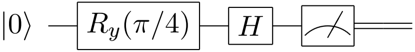
Appendix B. Projective measurements

Example

- State $|\psi\rangle = R_y(\pi/4)\,|0\rangle$
- Observable we want to measure $O = X$

$$D = UOU^\dagger = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \; X \; \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$
$$= HXH = Z$$

- Circuit to prepare and measure
$|\psi'\rangle = U\,|\psi\rangle = HR_y(\pi/4)\,|0\rangle$



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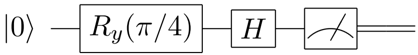

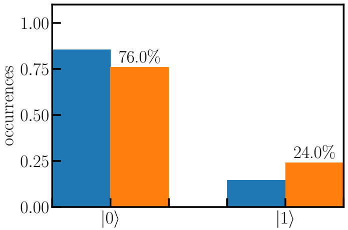

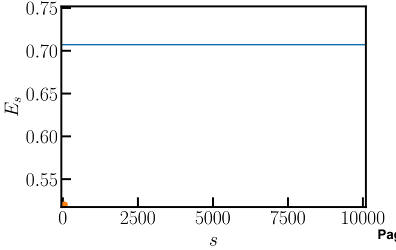

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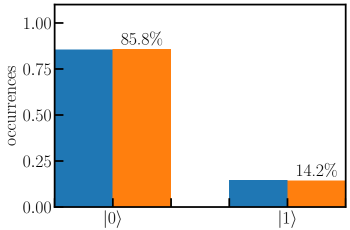
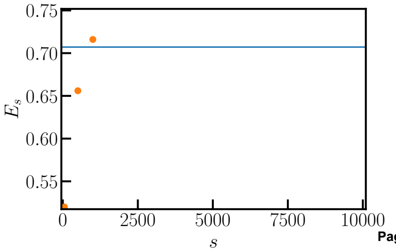
$$|0\rangle \; — \boxed{R_y(\pi/4)} — \boxed{H} — \boxed{\text{measure}} =$$





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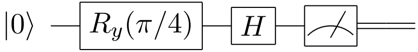

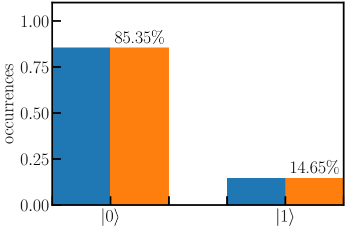

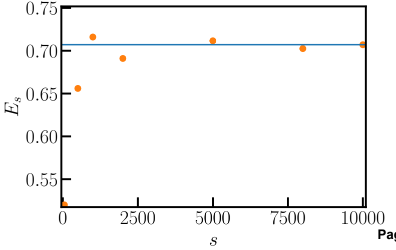

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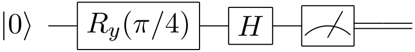


$\Rightarrow$ Error is $\propto 1/\sqrt{s} \to 0$ for $s \to \infty$

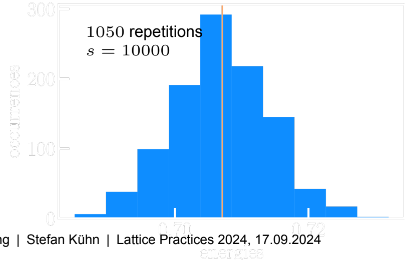
Appendix B. Projective measurements

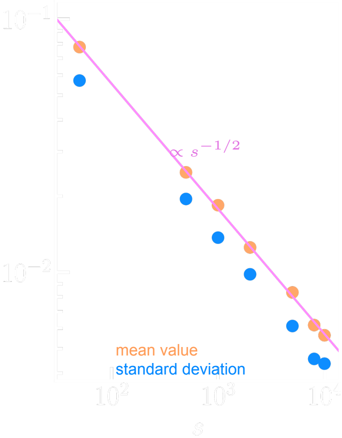
Example

- Repeating the measurement a number of times for fixed s yields a histogram with peak around $E_0 = \langle\psi| X |\psi\rangle$
- The mean and standard deviation of the error

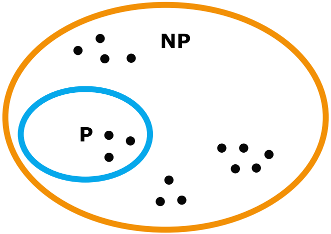
$$\left|\langle\psi| D |\psi\rangle_{\text{measured}} - \langle\psi| D |\psi\rangle_{\text{exact}}\right|$$

decay as a power law in s





Appendix C. Complexity theory



- P: decision problems solvable by a deterministic Turing machine in **polynomial time**
- NP: decision problems solvable by a non-deterministic Turing machine in polynomial time
  - **Solution can be checked** on a deterministic Turing machine in **polynomial time**

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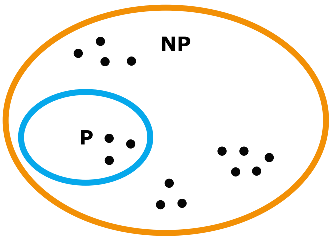

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- Since an exponential function grows asymptotically faster than any polynomial **problems in P** are considered the **"easy" ones**, and the **problems in NP** are considered **the "hard" ones**

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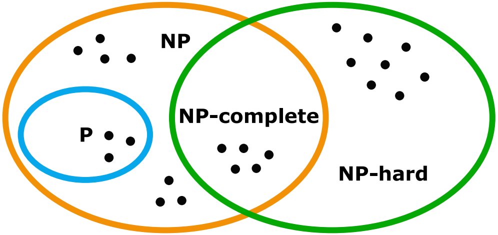

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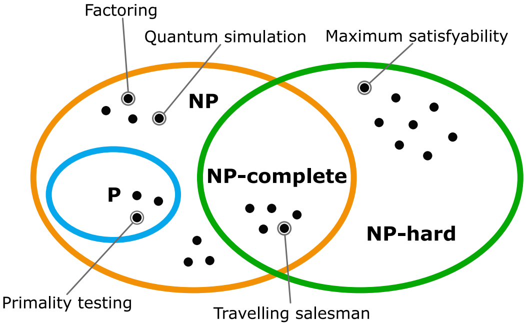

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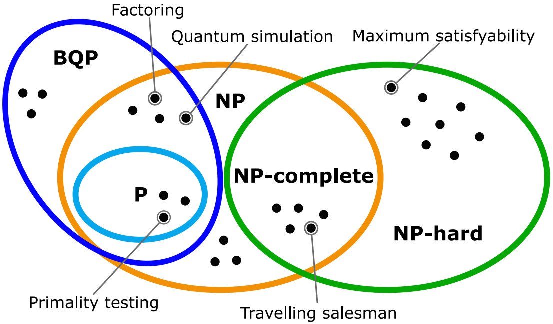

BQP (bounded-error quantum polynomial time):

- Decision problems solvable by a quantum computer in **polynomial time** with error probability less than $1/3$
- Quantum equivalent to P, “easy problems”

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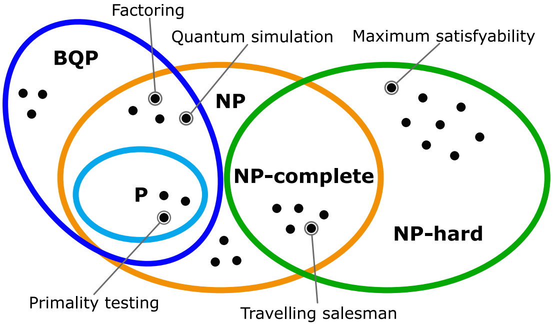

BQP (bounded-error quantum polynomial time):

- Decision problems solvable by a quantum computer in **polynomial time** with error probability less than $1/3$
- Quantum equivalent to P, "easy problems"

Take home message

- **Exponential speedup** on a quantum computer only for **very specific problems**
- **No exponential speedup** for **NP-complete problems**!

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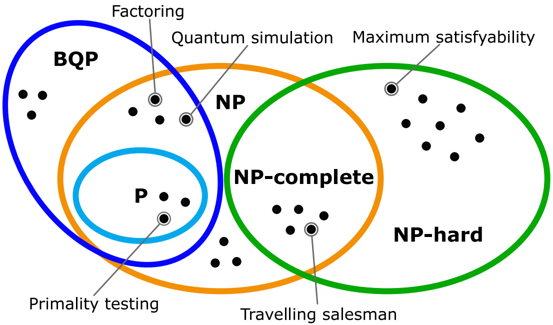

> Not strictly proven, proving P $\neq$ NP is one of the millennium problems

https://www.claymath.org/millennium-problems/

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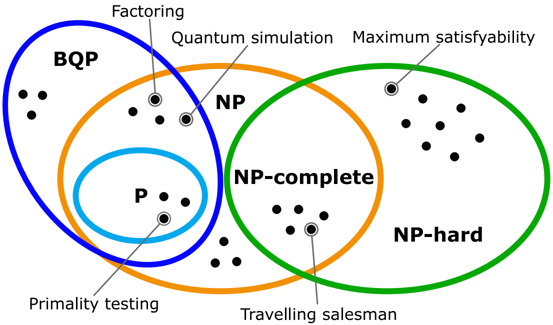

- Not strictly proven, proving P $\neq$ NP is one of the millennium problems
- If P $=$ NP then quantum computers would not allow for an exponential speedup
- Empirically, nobody has found a polynomial time algorithm for (all instances of) a problem in NP

https://www.claymath.org/millennium-problems/

Appendix C. Complexity theory

The Church-Turing thesis

All **physically reasonable models of computation** have the **same set** of **computable functions**.

$\Rightarrow$ Quantum computers cannot compute functions that are uncomputable on a classical computer

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The Church-Turing thesis

All **physically reasonable models of computation** have the **same set** of **computable functions**.

⇒ Quantum computers cannot compute functions that are uncomputable on a classical computer

The extended Church-Turing thesis

All **physically reasonable models of computation differ** in complexity by **at most polynomial factors**.

⇒ Extended Church-Turing thesis would no long hold if **quantum supremacy** is demonstrated

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Article

Quantum supremacy using a programmable superconducting processor

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RESEARCH

QUANTUM COMPUTING

Quantum computational advantage using photons

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