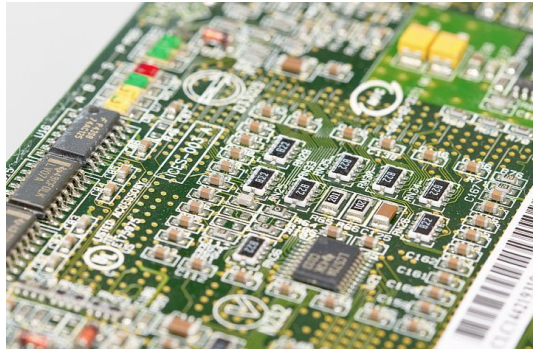
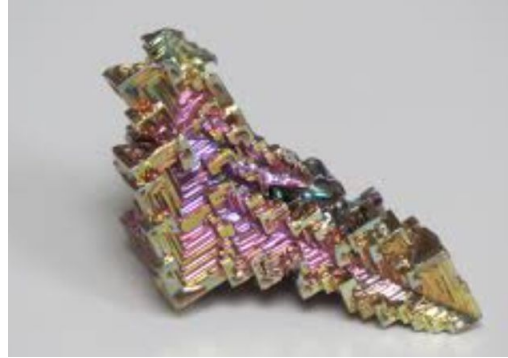


Crystal structure search accelerated by neural networks

Stefaan Hessmann, Niklas W. A. Gebauer, Kristof T. Schütt, Michael Gastegger, Klaus-Robert Müller, Tamio Oguchi, Tomoki Yamashita

Motivation: search for stable structures



Overview

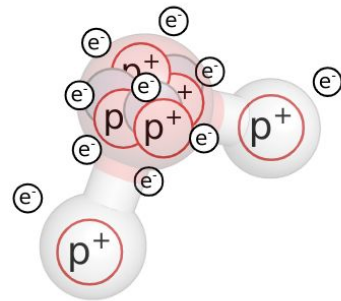
1. Quantum chemistry computations
2. Message passing neural networks
3. Active learning for crystal structure prediction
4. Results

Quantum chemistry computations

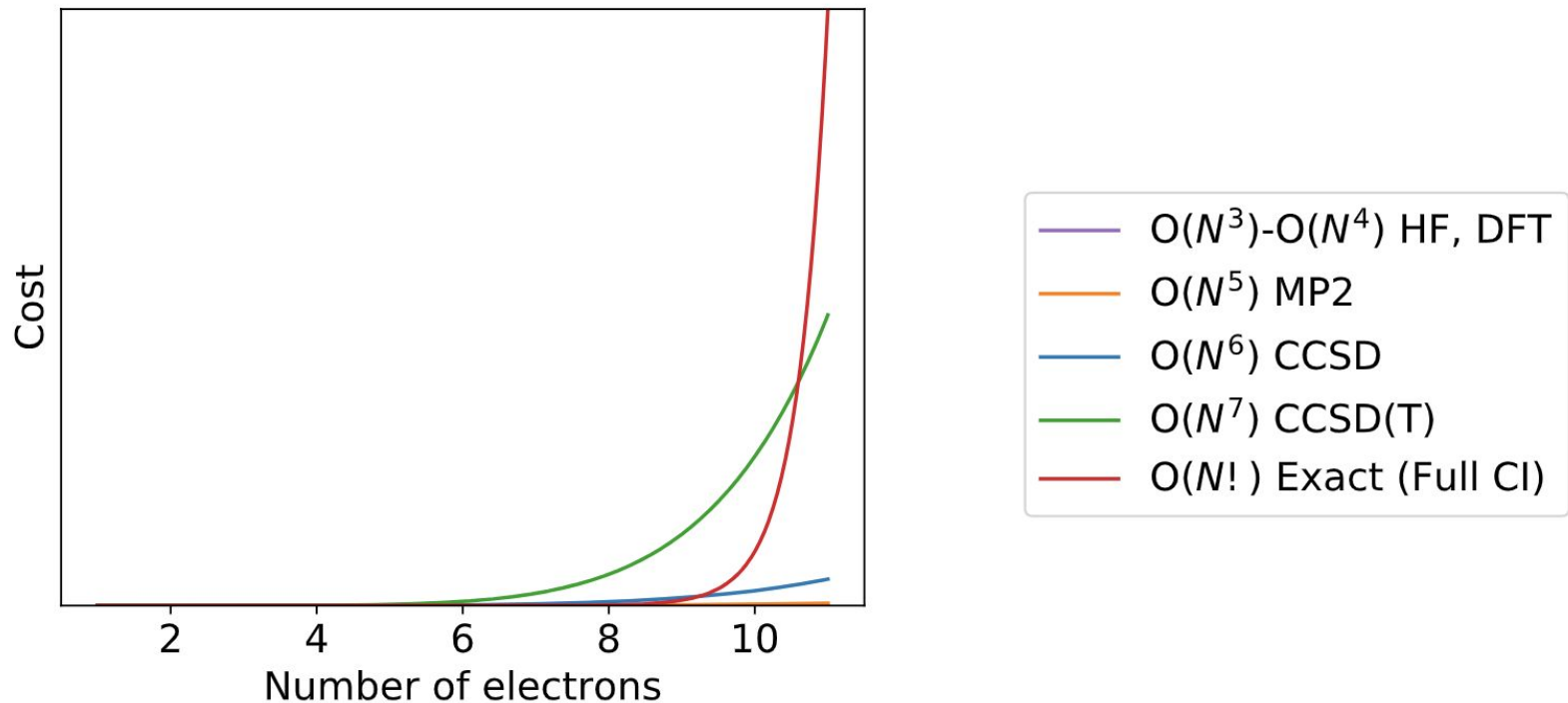
Quantum chemistry computations

$$\hat{H}_{\text{el}}(\bar{\mathbf{R}}, \mathbf{r}, \mathbf{Z}) \Psi_{\text{el}}(\bar{\mathbf{R}}, \mathbf{r}) = E_{\text{el}} \Psi_{\text{el}}(\bar{\mathbf{R}}, \mathbf{r})$$

$$\hat{H}_{\text{el}}(\bar{\mathbf{R}}, \mathbf{r}, \mathbf{Z}) = \underbrace{-\sum_i^N \frac{1}{2} \nabla_i^2}_{\text{electrons}} - \underbrace{\sum_i^N \sum_A^M \frac{Z_A}{|\mathbf{r}_i - \bar{\mathbf{R}}_A|}}_{\text{electrons-nuclei}} + \underbrace{\sum_i^N \sum_{j>i}^N \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|}}_{\text{electrons-electrons}}$$



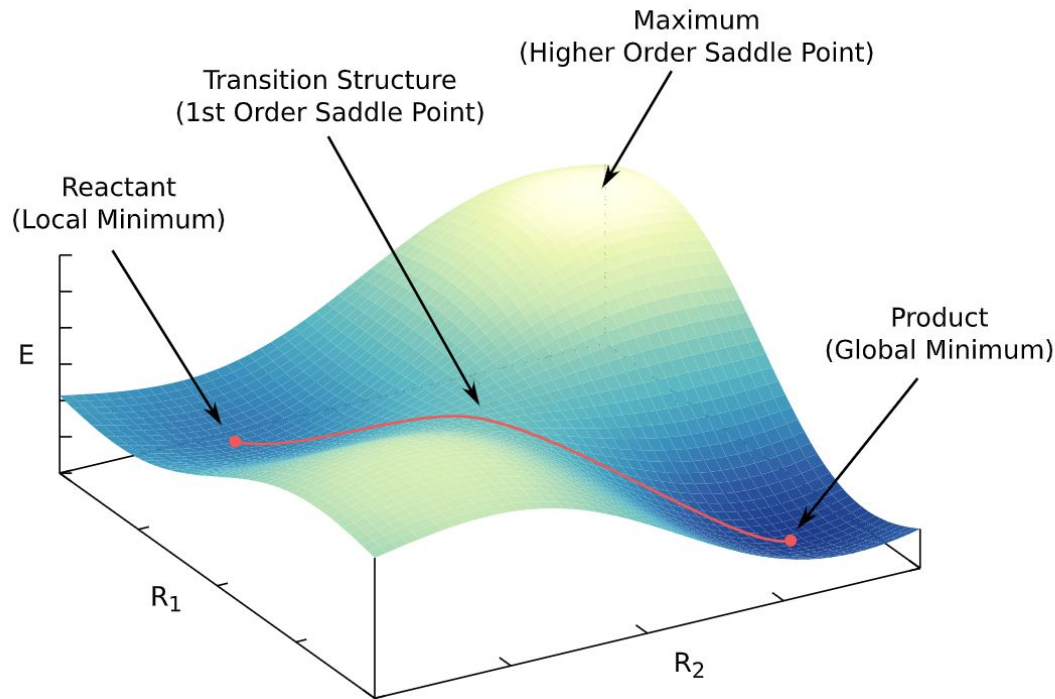
Quantum chemistry computations



Quantum chemistry computations

$$\hat{H}_{\text{el}}(\bar{\mathbf{R}}, \mathbf{r}, \mathbf{Z}) \psi_{\text{el}}(\bar{\mathbf{R}}, \mathbf{r}) = E_{\text{el}} \psi_{\text{el}}(\bar{\mathbf{R}}, \mathbf{r})$$

$$E_{\text{el}} \approx ML(\bar{\mathbf{R}}, \mathbf{Z})$$



Message passing neural networks

Message passing neural networks

Message passing neural network [Gilmer et al 2017]

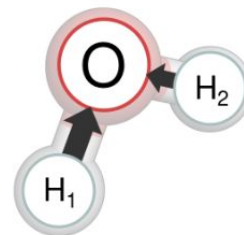
$$\mathbf{m}_i^{t+1} = \sum_{j \in \mathcal{N}(i)} \mathbf{M}_t(\mathbf{s}_i^t, \mathbf{s}_j^t, \|\vec{r}_{ij}\|)$$

$$\mathbf{s}_i^{t+1} = \mathbf{U}_t(\mathbf{s}_i^t, \mathbf{m}_i^{t+1})$$

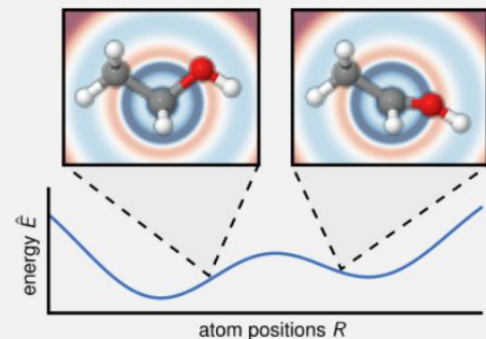
Rotational invariance:

$$\mathbf{M}(\vec{x}) = \mathbf{M}(R \vec{x})$$

for any rotation R .



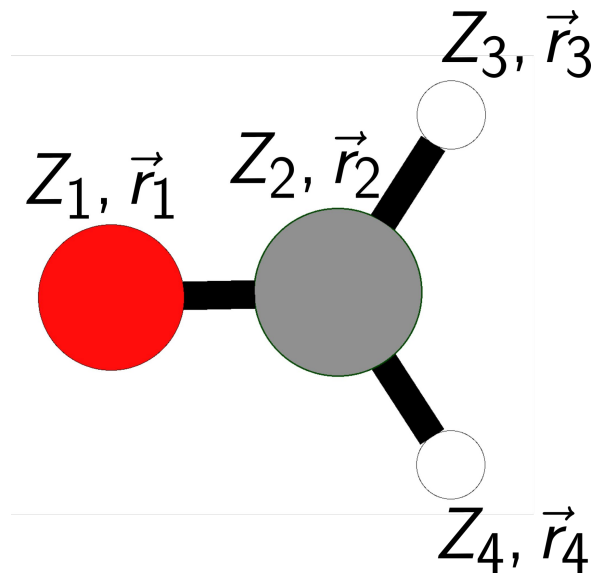
Continuous-filter convolution [Schütt et al 2017]



$$(\mathbf{x} * W)(\mathbf{r}_i) = \sum_{j=1}^{N_{\text{atom}}} \mathbf{x}_j^{(t)} \circ \underbrace{W^{(t)}(\|\mathbf{r}_i - \mathbf{r}_j\|)}_{\text{neural network}}$$

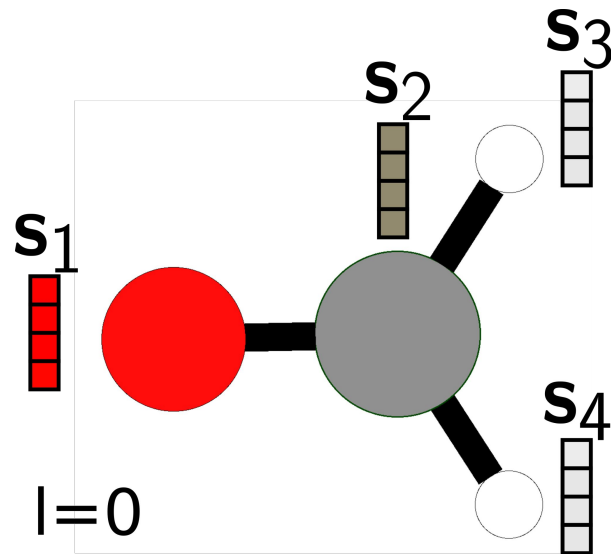
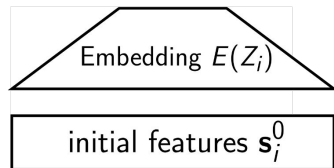
Message passing neural networks

atom types Z_i , positions \vec{r}_i



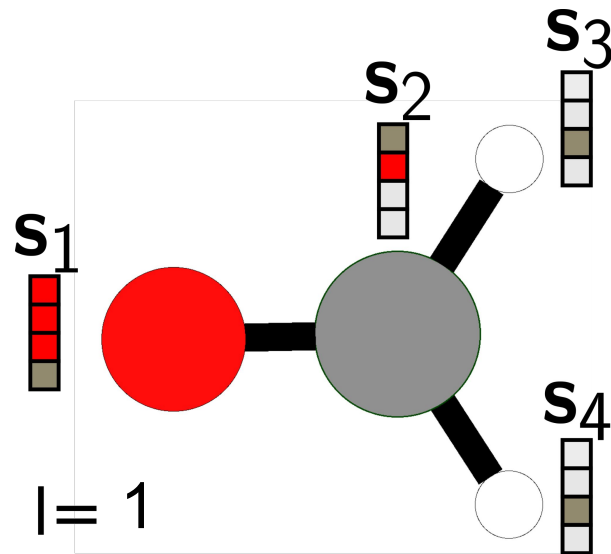
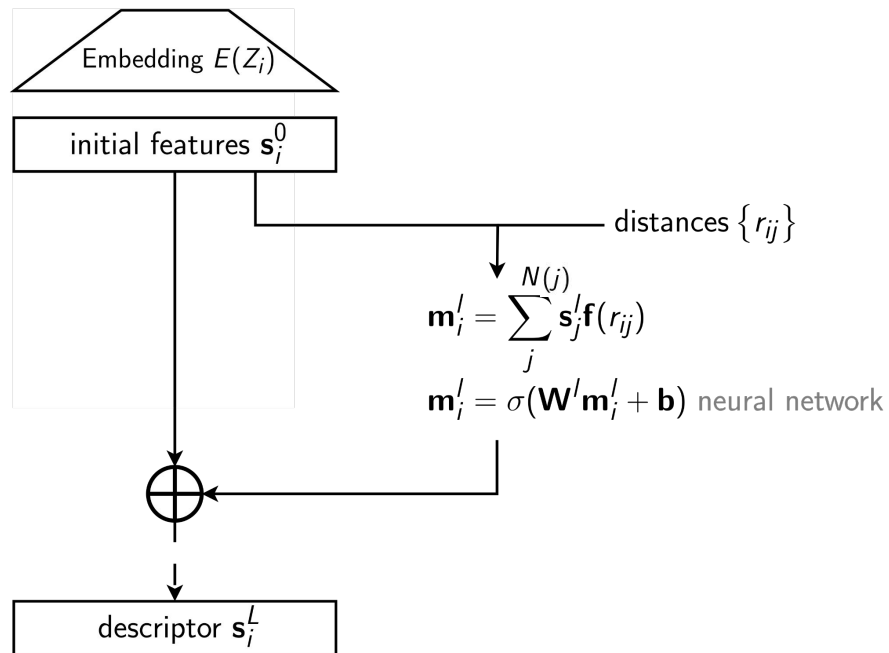
Message passing neural networks

atom types Z_i , positions \vec{r}_i



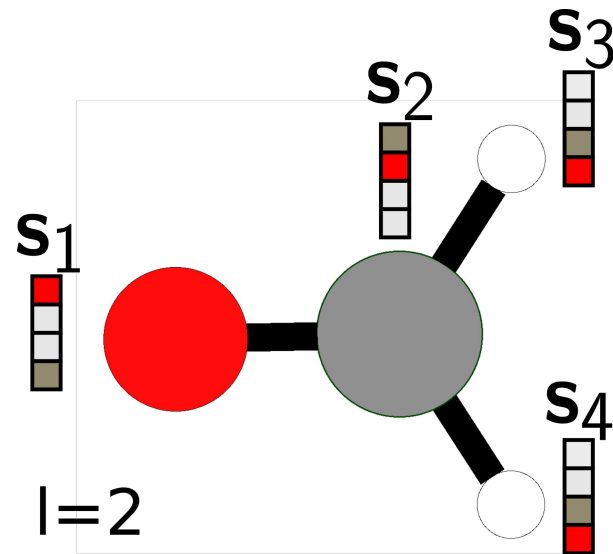
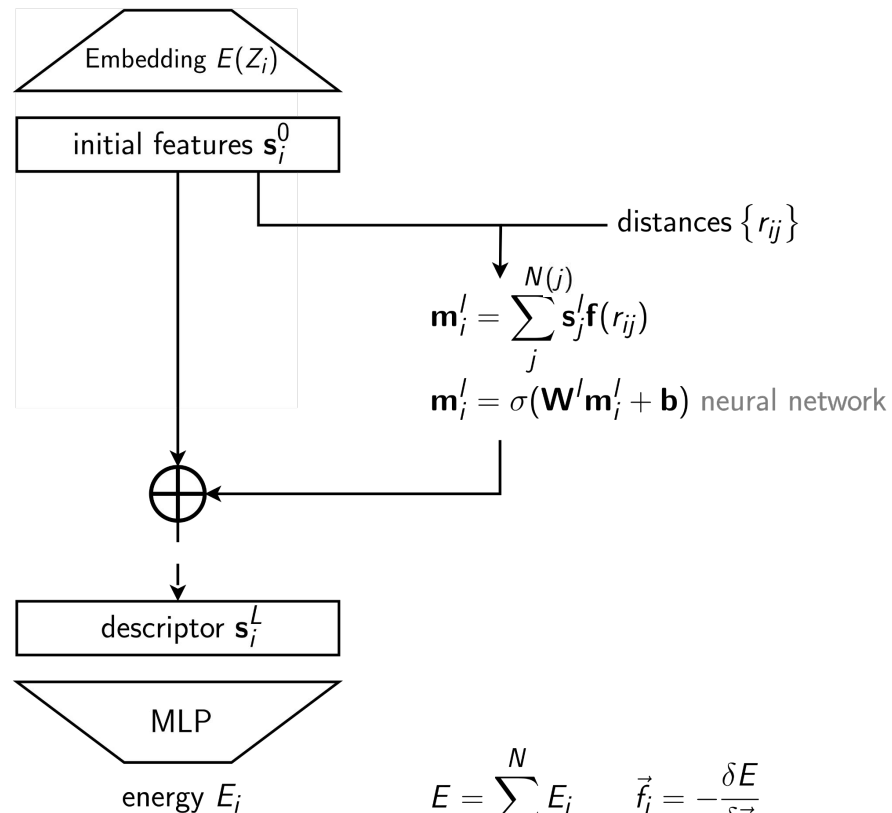
Message passing neural networks

atom types Z_i , positions \vec{r}_i



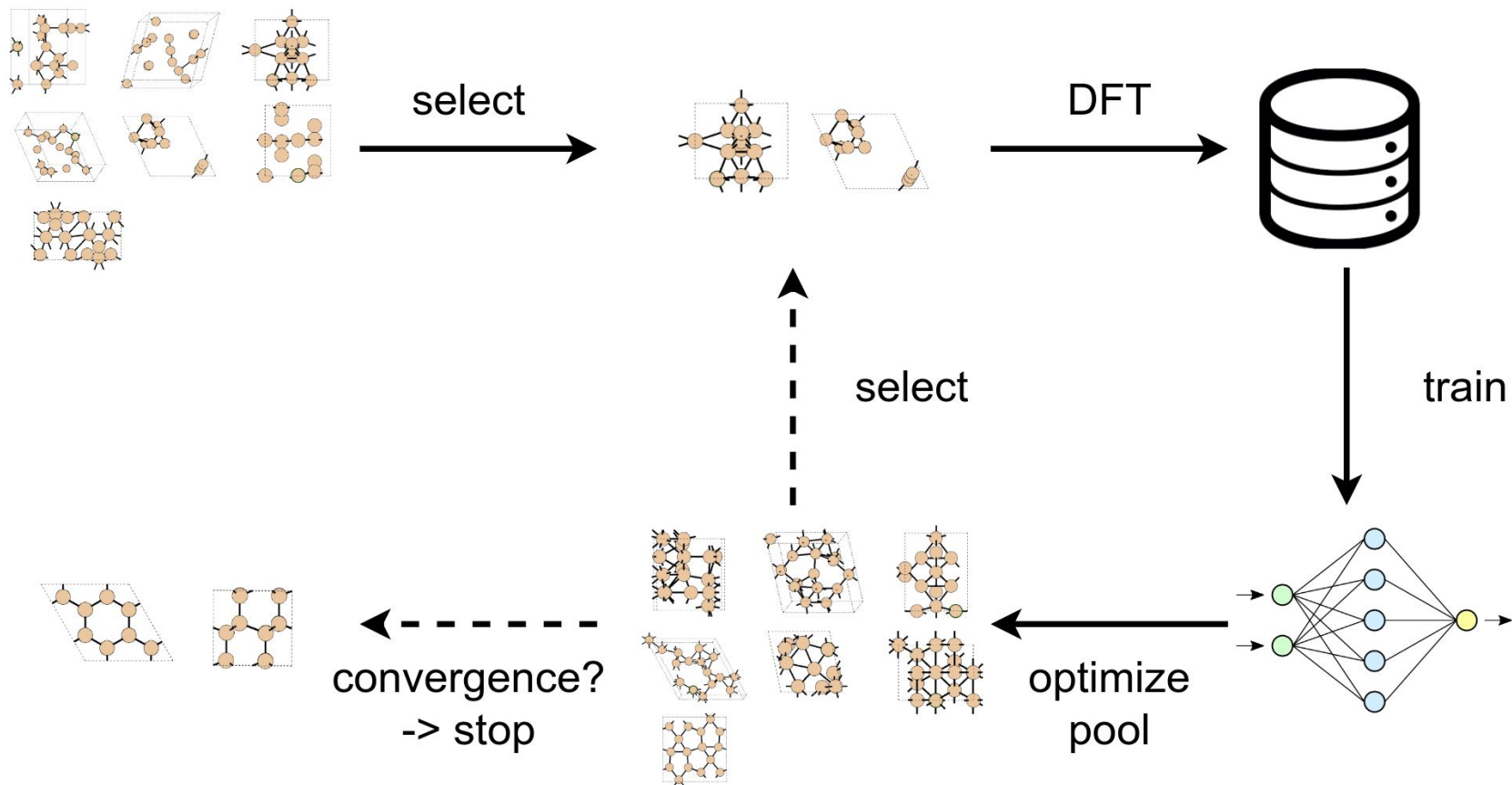
Message passing neural networks

atom types Z_i , positions \vec{r}_i

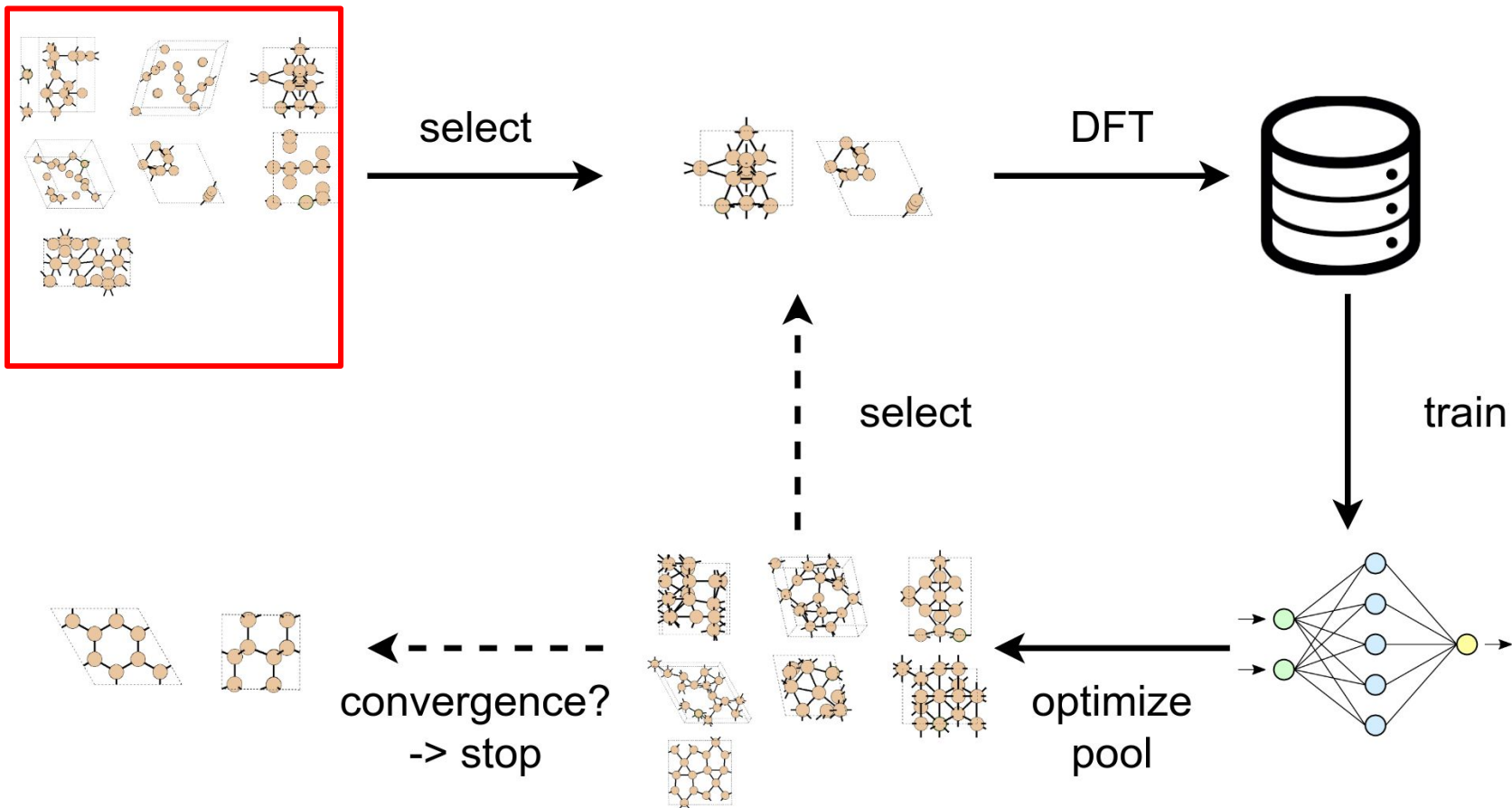


Active learning for crystal structure prediction

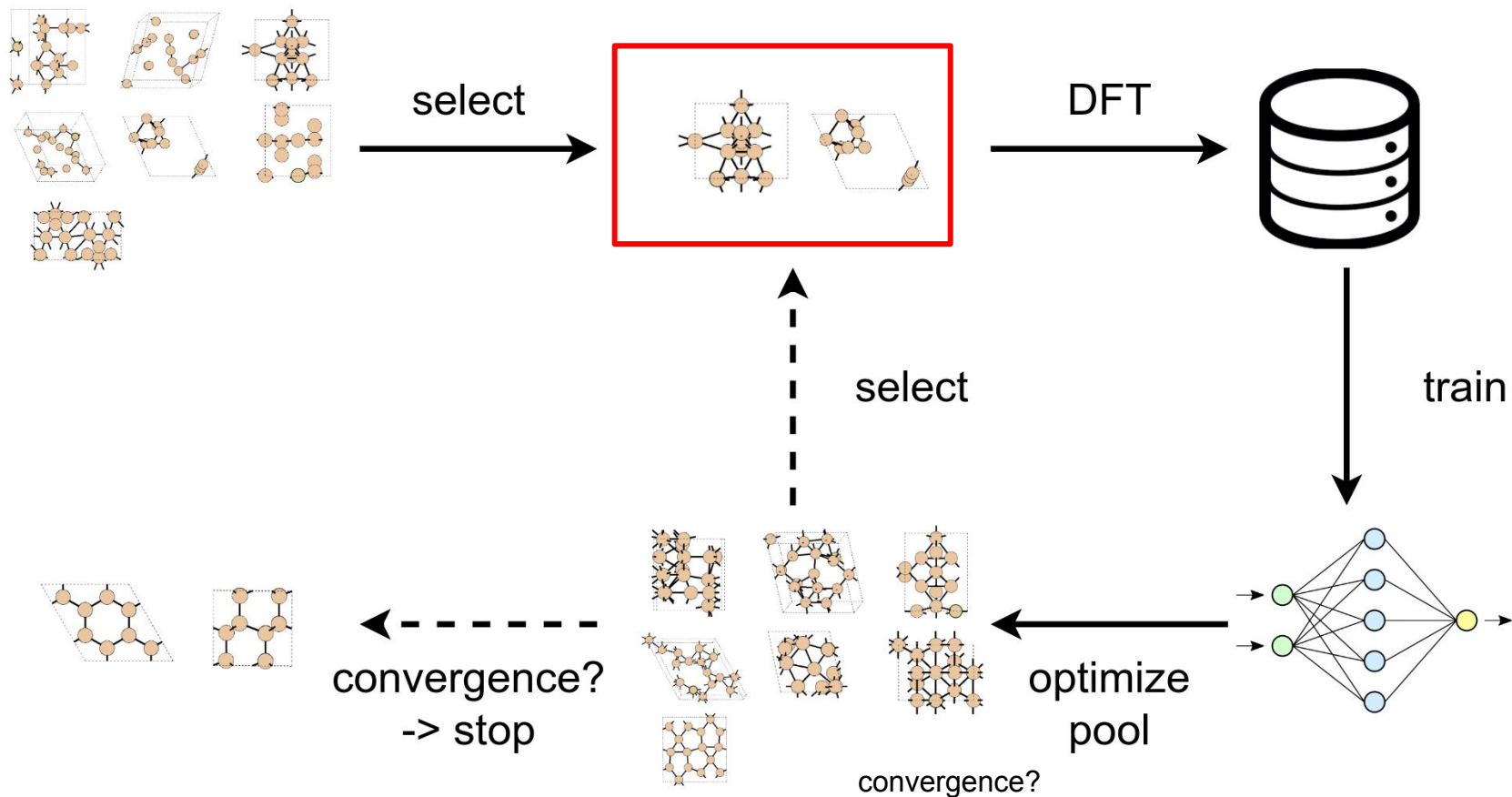
Method overview



Method overview



Method overview



Selecting candidates for DFT evaluation

- scoring function

$$L_{i,\theta} = \overline{E}_{i,\theta} - \omega_F \max(||\overline{F}_{i,\theta}||_2)$$

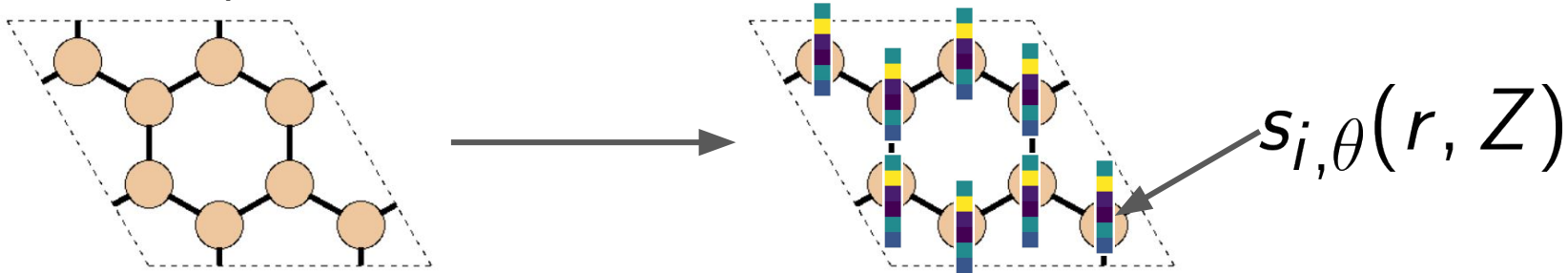
- next candidates

$$\{\arg \min_x (L_\theta(x))\}_n$$

- problem: similar structures yield similar scores
- solution: clustering

Global representation for clustering

- use local representations

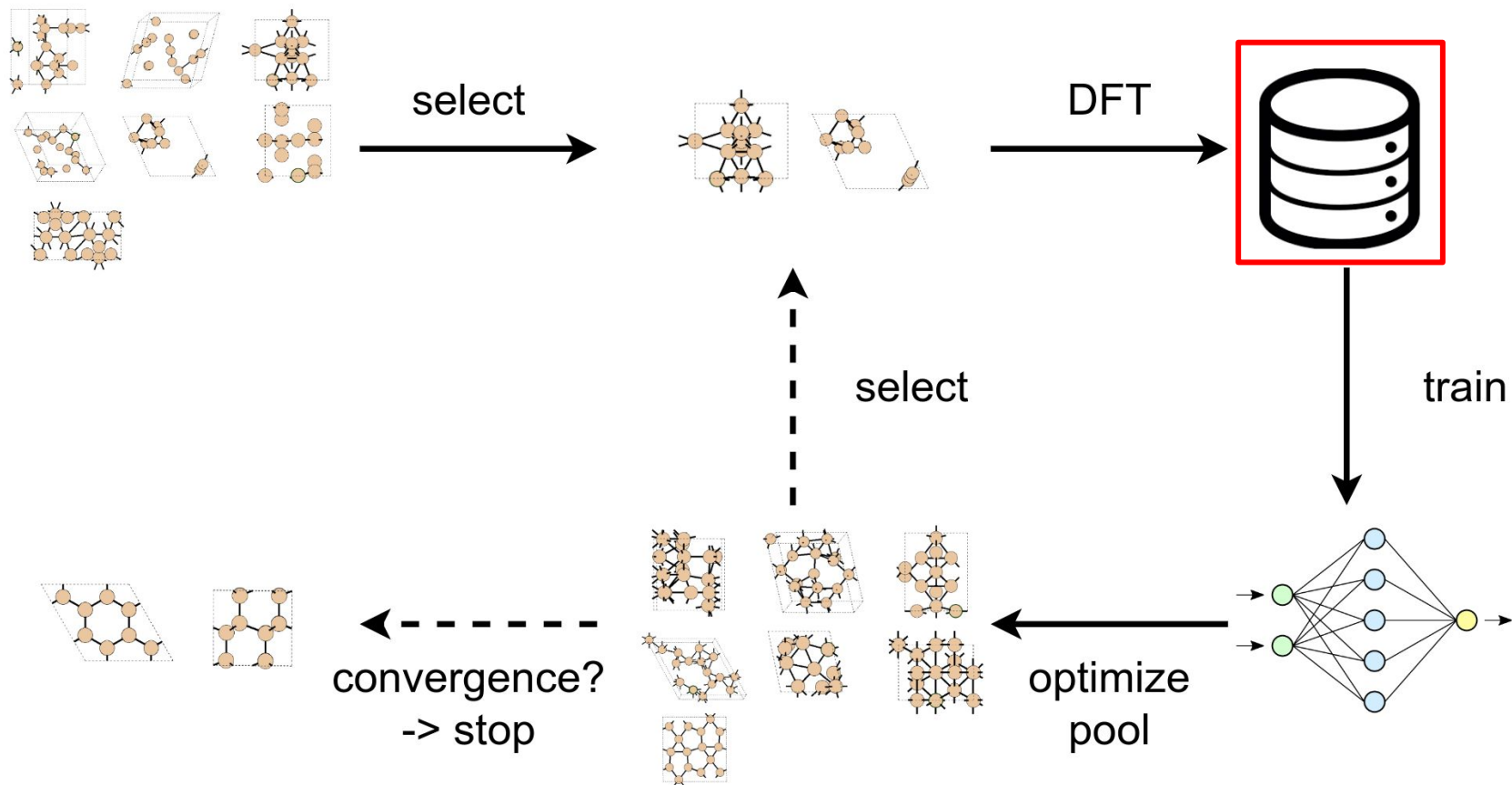


- pooling introduces invariance against permutation

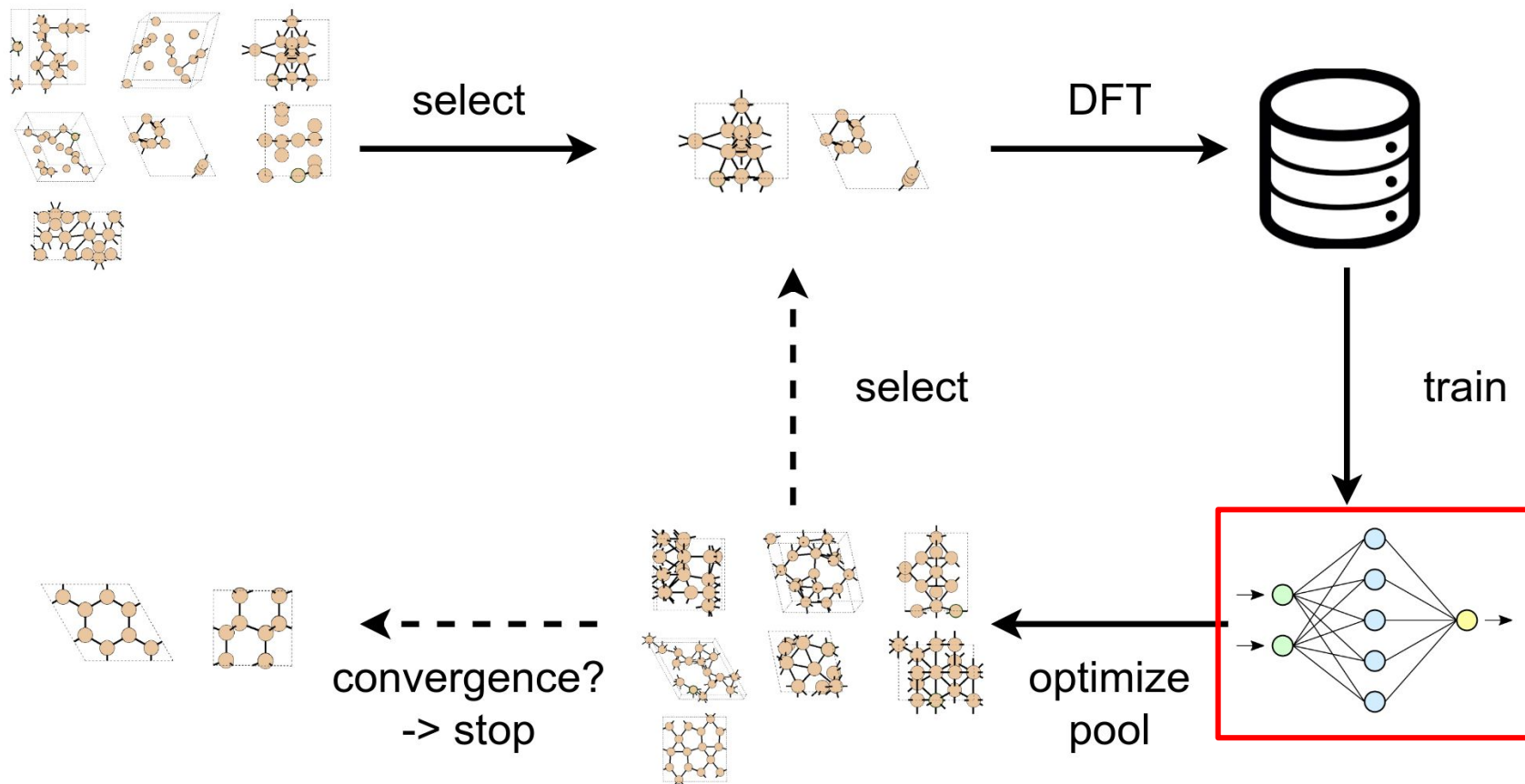
$$\bar{s}_{\theta} = \sum_{i=0}^n s_{i,\theta}(r, Z)$$

$$\bar{s}_{\theta} \in \mathbb{R}^f$$

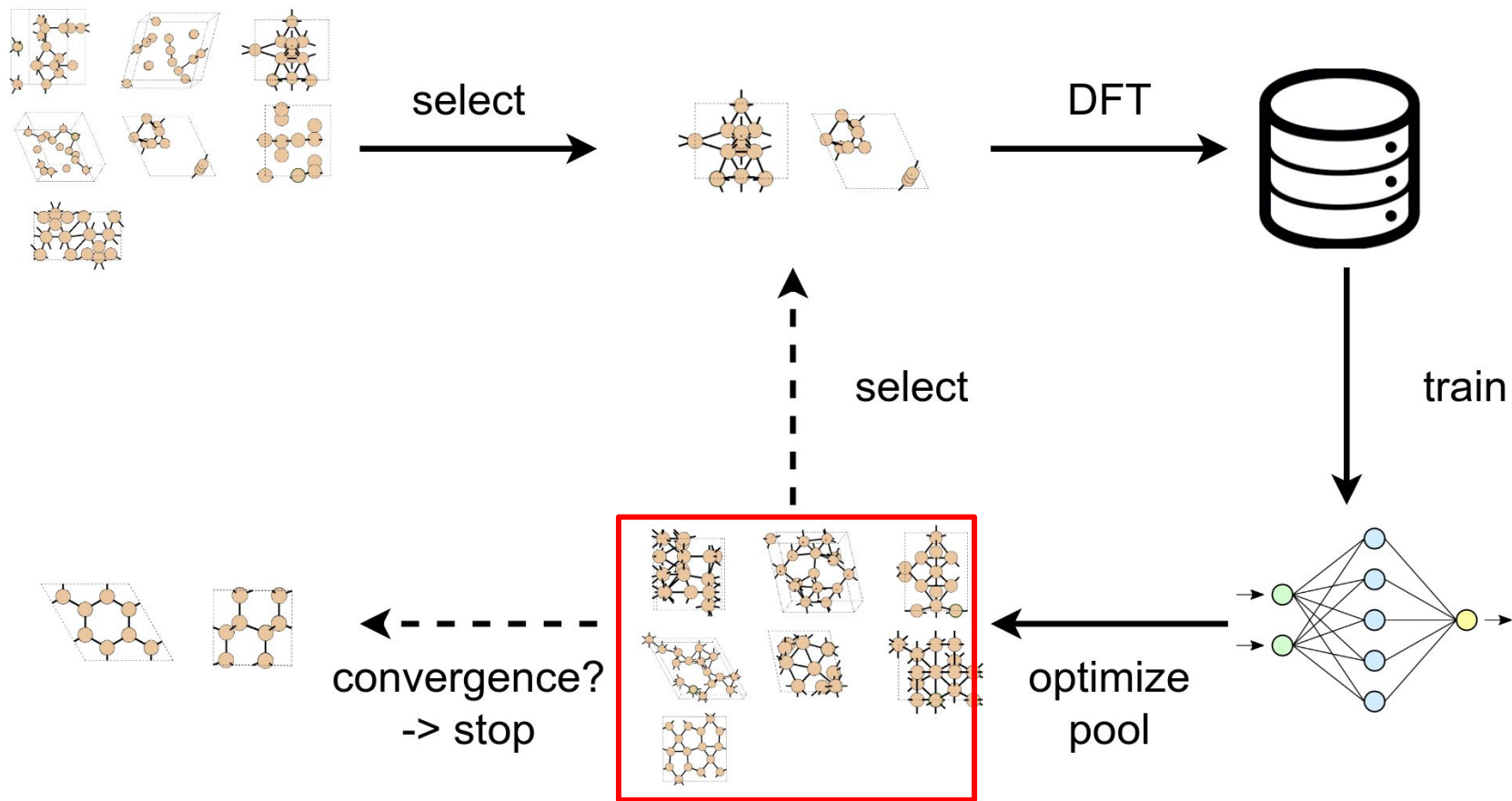
Method overview



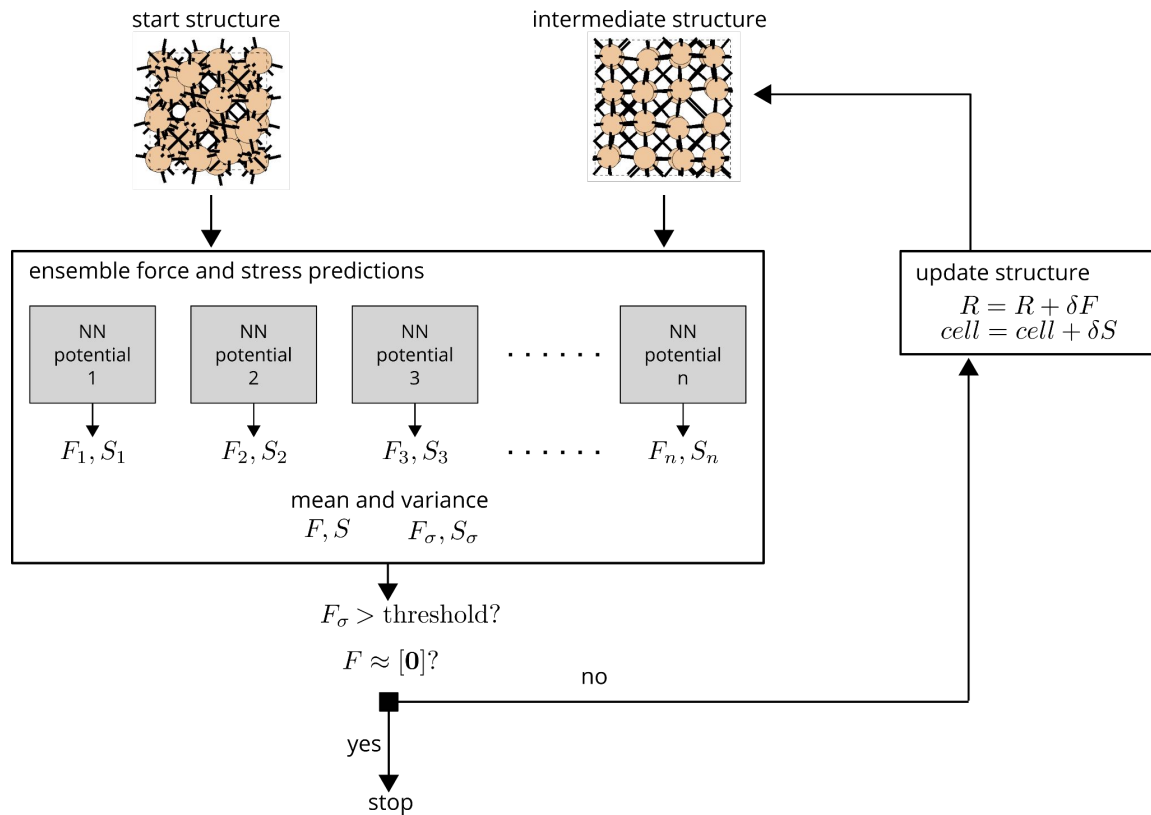
Method overview



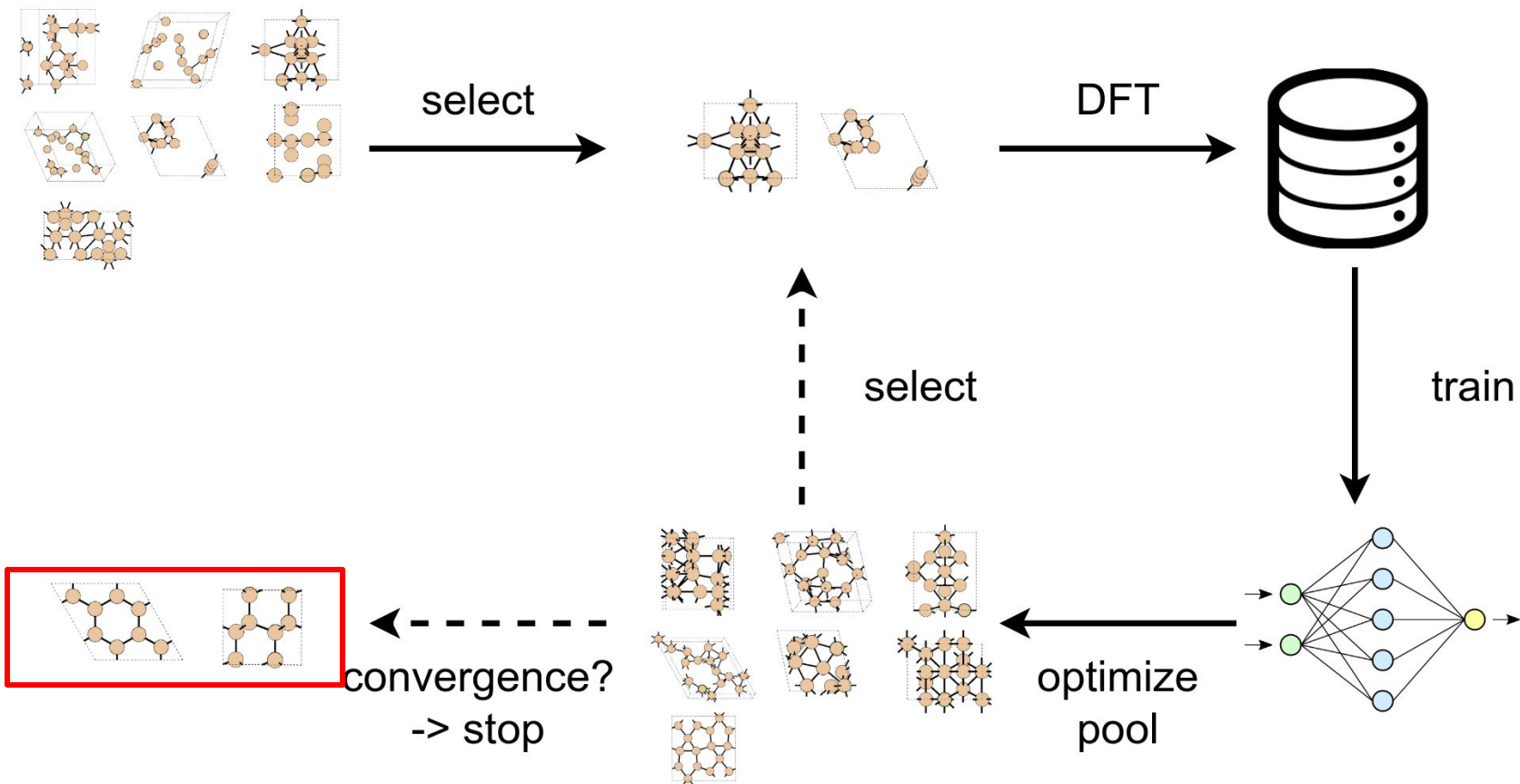
Method overview



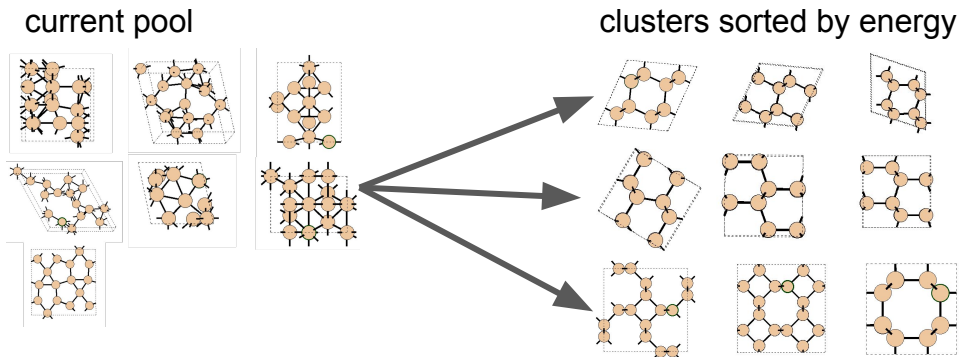
Optimization of candidate pool



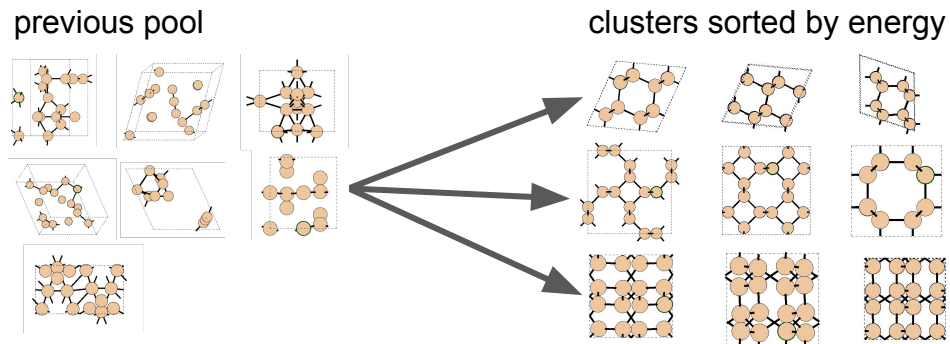
Method overview



Selecting structures for DFT relaxation

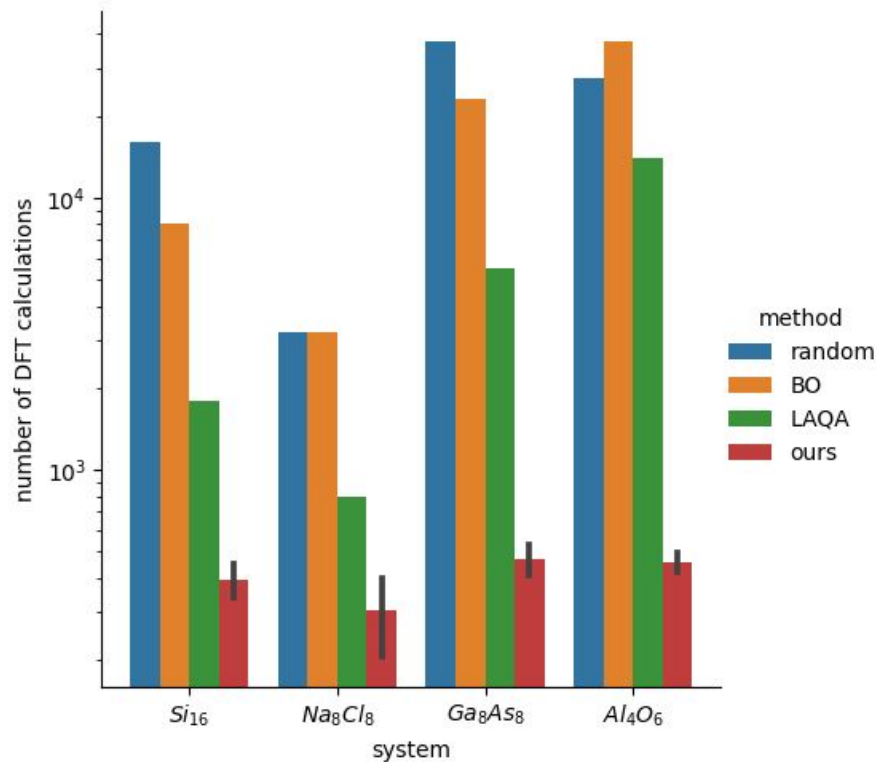


stop if top-k clusters are equal



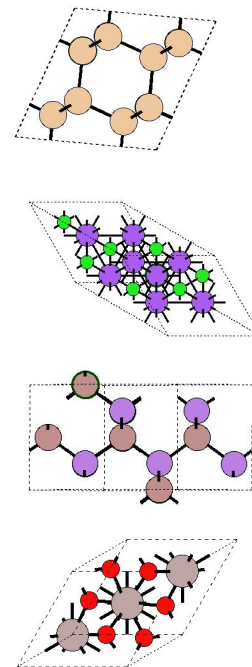
Results

Results

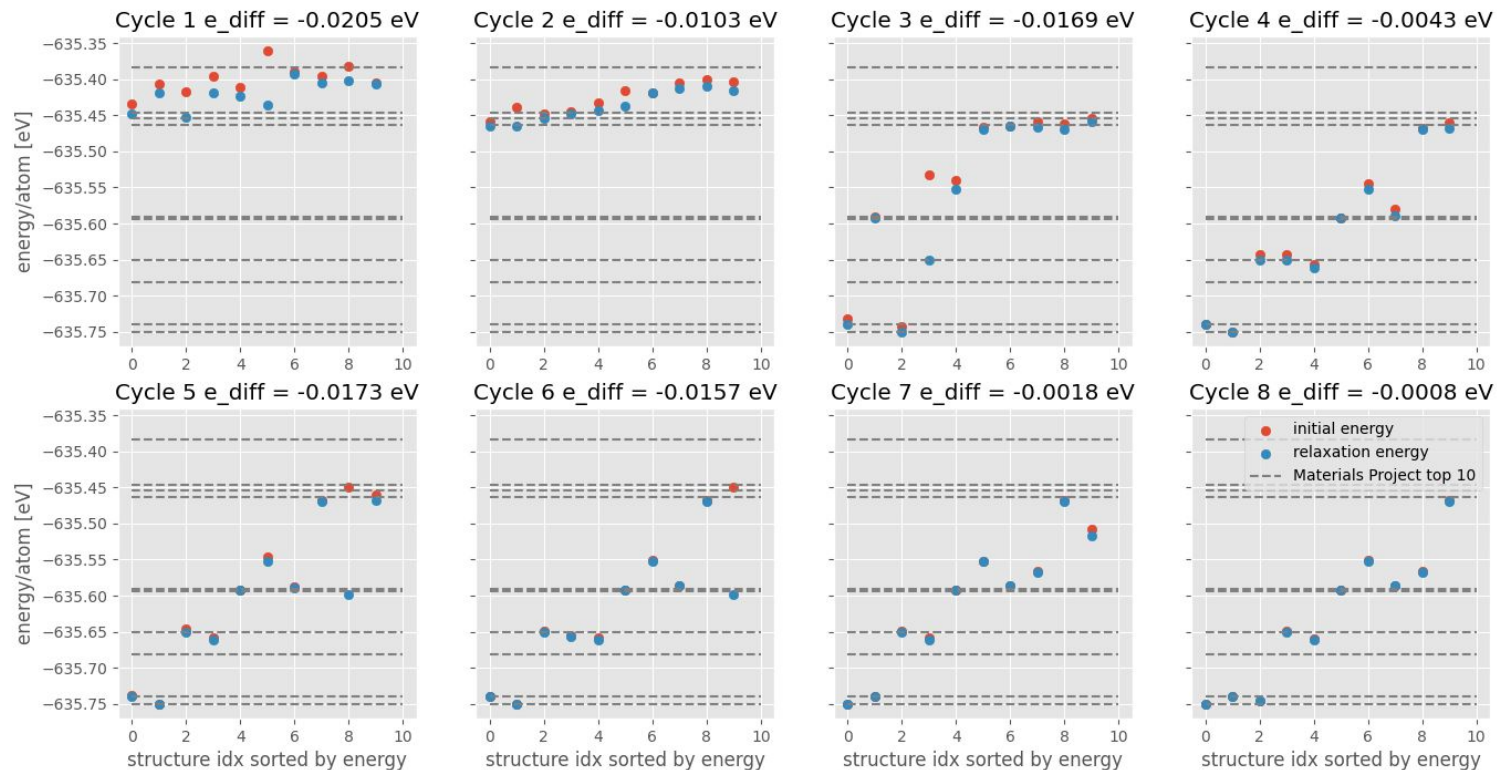


target structures:

- Si: MP-149
- NaCl: MP-22862
- GaAs: MP-2534
- AlO: MP-1143

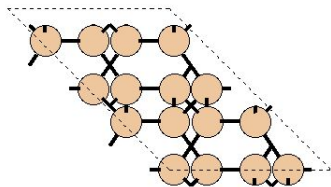
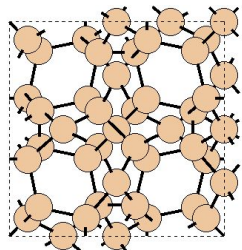
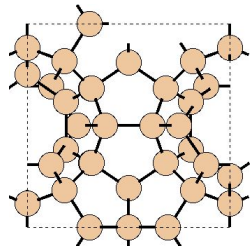


Results: multiple minima



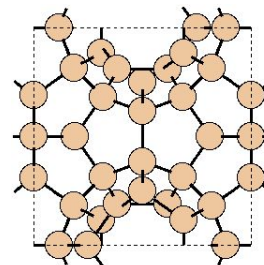
Results: transferability Si16 to Si46

top 3 minima:

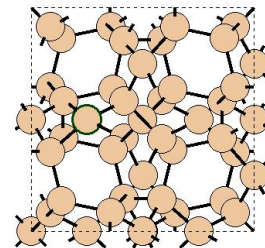


target structures:

- MP-971662



- MP-971661



Thank you!

Questions?