

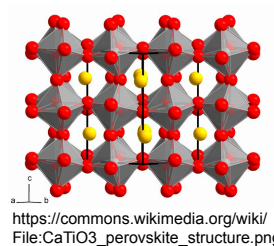
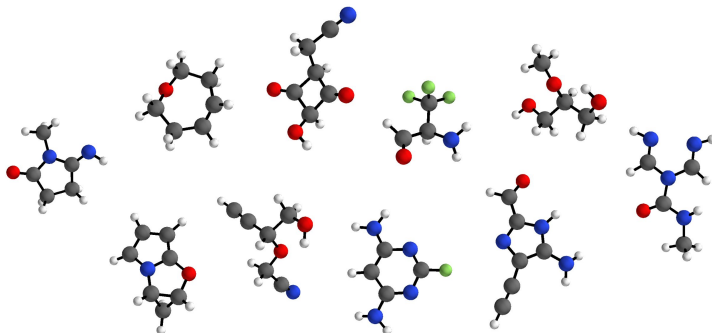
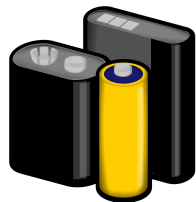
# Inverse design of 3d molecular structures with conditional generative neural networks

Niklas Gebauer

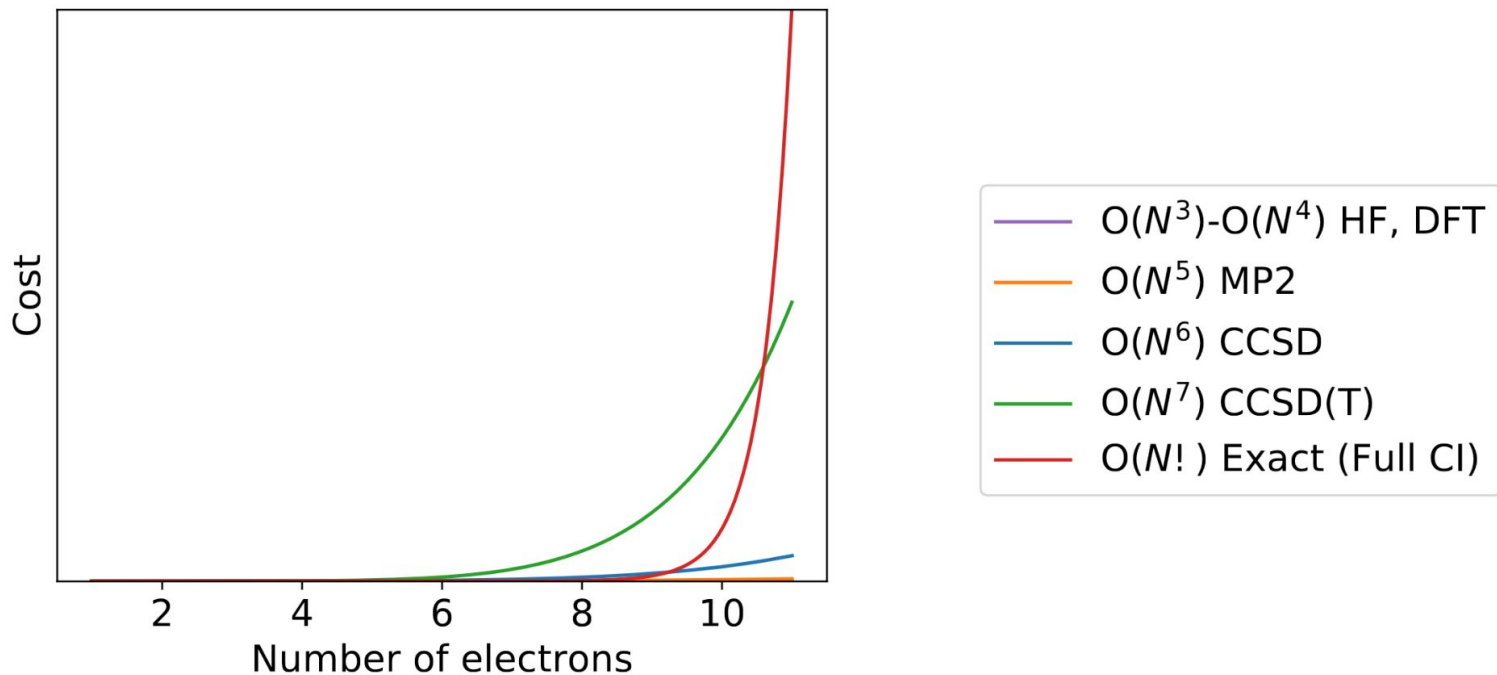
Technische Universität Berlin - Machine Learning Group

27th of February 2024

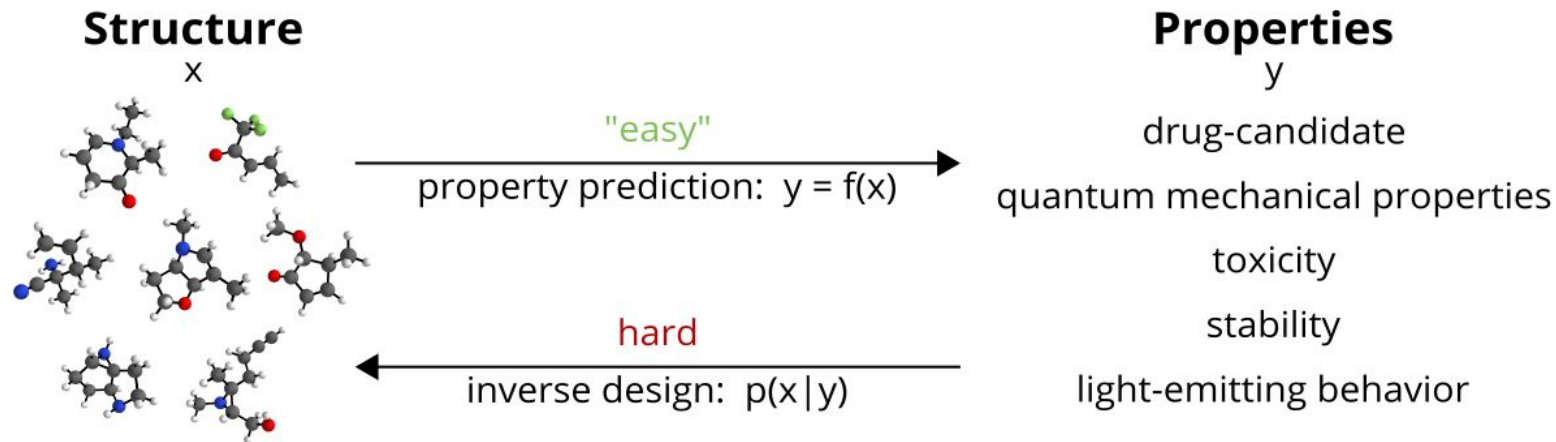
# Quantum chemistry/Physics



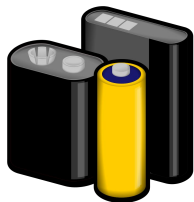
# Computational complexity of QC



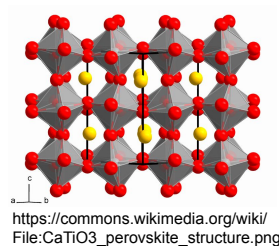
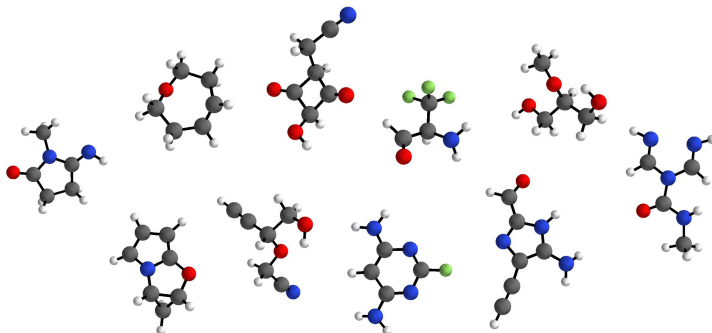
# ML for molecules



# Quantum chemistry/Physics



guided exploration of chemical space



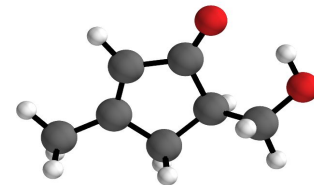
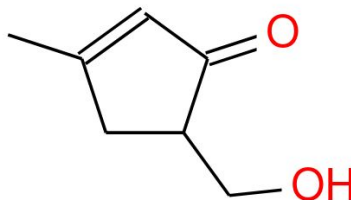
# Molecule representations

*SMILES strings*

[Weininger 1988]

CC1=CC(=O)C(CO)C1

*molecular graph*



*spatial*

atom positions			
$r_1=(x,y,z)$	$r_2=(x,y,z)$	...	$r_{19}=(x,y,z)$
atom types			
$Z_1=O$	$Z_2=C$	...	$Z_{19}=O$

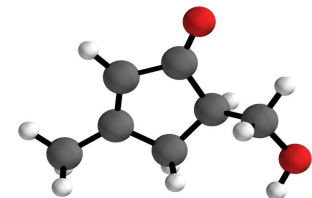
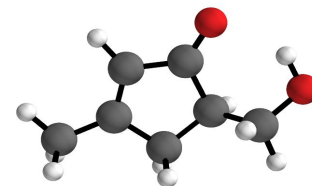
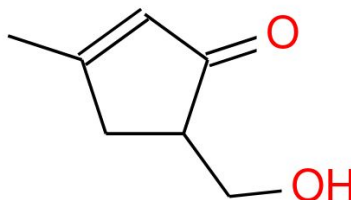
# Molecule representations

*SMILES strings*

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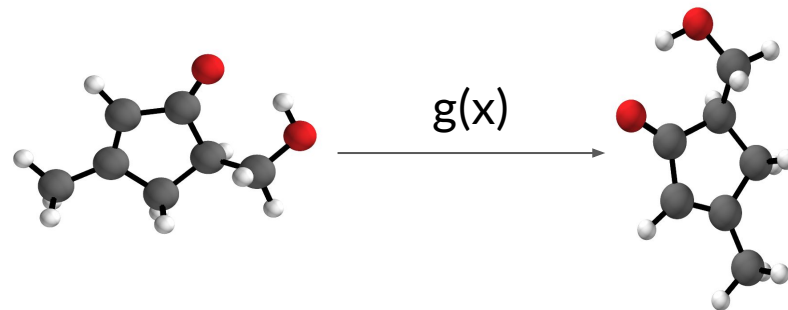


*spatial*

atom positions			
$r_1=(x,y,z)$	$r_2=(x,y,z)$	...	$r_{19}=(x,y,z)$
atom types			
$Z_1=O$	$Z_2=C$	...	$Z_{19}=O$

# Neural network for spatial representation

- invariant networks:
  - $f(g(x)) = f(x)$
  - features based on pairwise distances

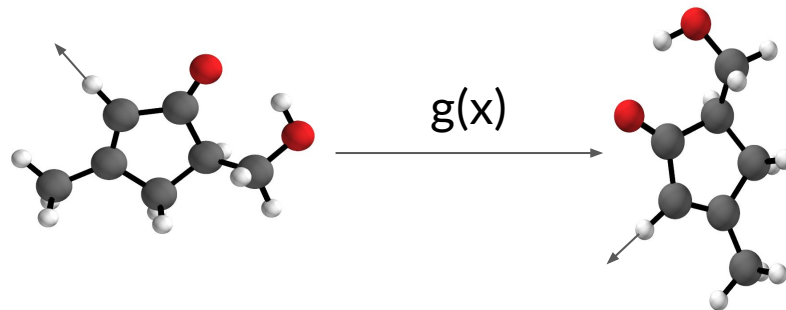




# Neural network for spatial representation

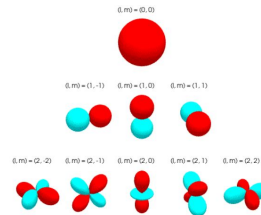
- invariant networks:

- $f(g(x)) = f(x)$
- features based on pairwise distances



- equivariant networks:

- $f(g(x)) = g(f(x))$
- features based on direction vectors
- specific layers required
- spherical harmonics



[https://commons.wikimedia.org/wiki/File:Real\\_Spherical\\_Harmonics\\_Figure\\_Table\\_Complex\\_Radial\\_Magnitude.gif](https://commons.wikimedia.org/wiki/File:Real_Spherical_Harmonics_Figure_Table_Complex_Radial_Magnitude.gif)

# Approaches for 3d molecule generation

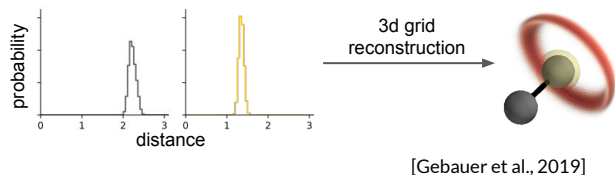
*iterative*

*one-shot*

# Approaches for 3d molecule generation

## *iterative*

reconstruction from pairwise distances

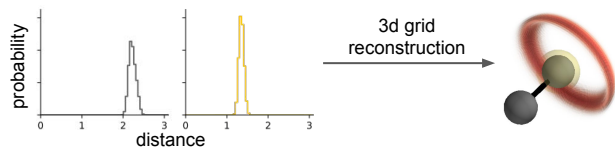


## *one-shot*

# Approaches for 3d molecule generation

## *iterative*

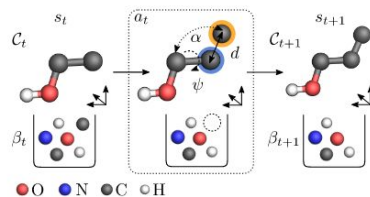
reconstruction from pairwise distances



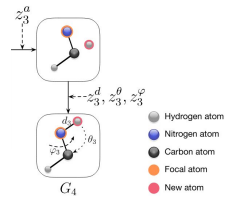
[Gebauer et al., 2019]

## *one-shot*

reconstruction from internal coordinates



[Simm et al., 2020]

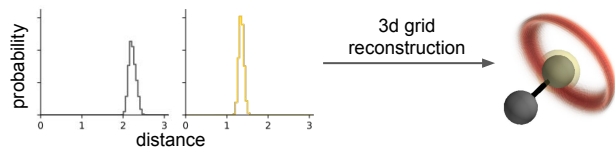


[Luo & Ji, 2022]

# Approaches for 3d molecule generation

*iterative*

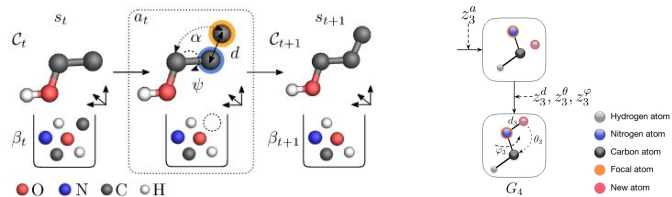
reconstruction from pairwise distances



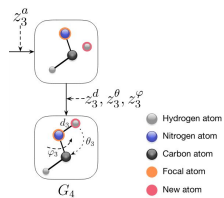
[Gebauer et al., 2019]

*one-shot*

reconstruction from internal coordinates

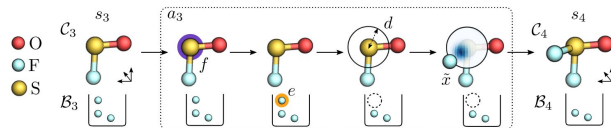


[Simm et al., 2020]



[Luo & Ji, 2022]

equivariant prediction of direction (spherical harmonics)

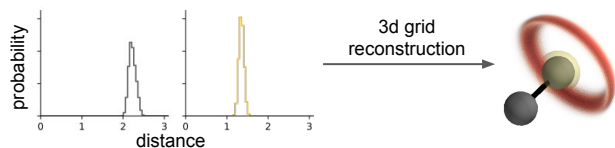


[Simm et al., 2021]

# Approaches for 3d molecule generation

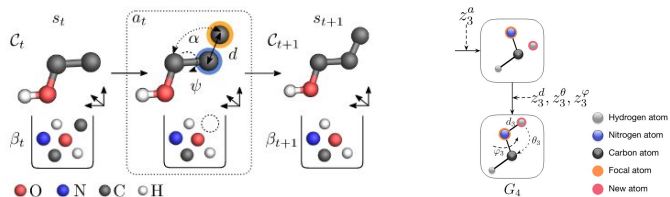
## *iterative*

reconstruction from pairwise distances



[Gebauer et al., 2019]

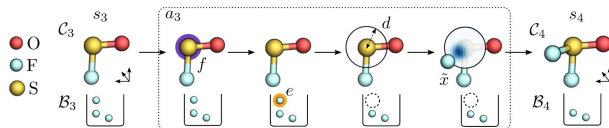
reconstruction from internal coordinates



[Simm et al., 2020]

[Luo & Ji, 2022]

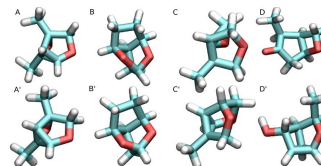
equivariant prediction of direction (spherical harmonics)



[Simm et al., 2021]

## *one-shot*

generating valid euclidean distance matrices

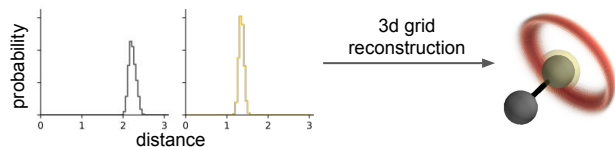


[Hoffmann & Noé, 2019]

# Approaches for 3d molecule generation

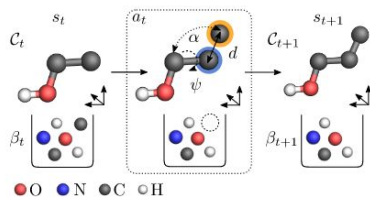
## iterative

reconstruction from pairwise distances

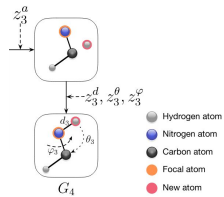


[Gebauer et al., 2019]

reconstruction from internal coordinates

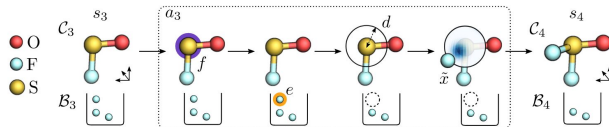


[Simm et al., 2020]



[Luo & Ji, 2022]

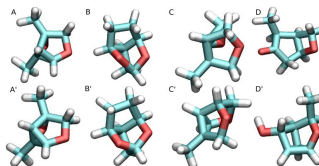
equivariant prediction of direction (spherical harmonics)



[Simm et al., 2021]

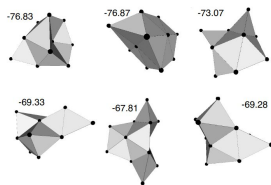
## one-shot

generating valid euclidean distance matrices



[Hoffmann & Noé, 2019]

equivariant normalizing flow

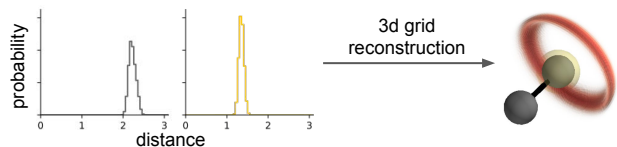


[Köhler et al., 2020]

# Approaches for 3d molecule generation

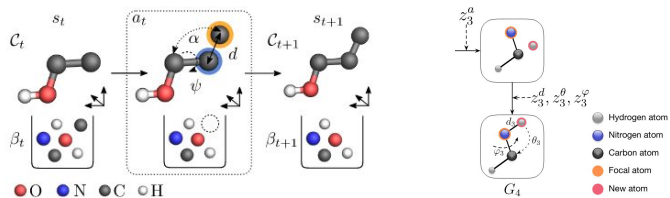
## iterative

reconstruction from pairwise distances

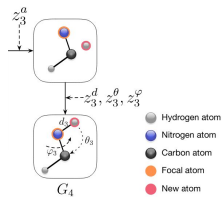


[Gebauer et al., 2019]

reconstruction from internal coordinates

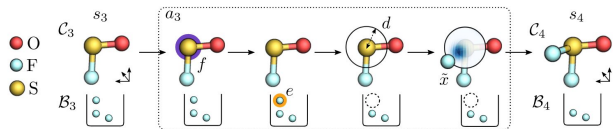


[Simm et al., 2020]



[Luo & Ji, 2022]

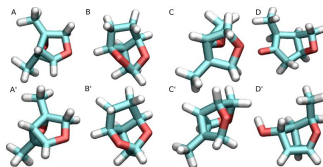
equivariant prediction of direction (spherical harmonics)



[Simm et al., 2021]

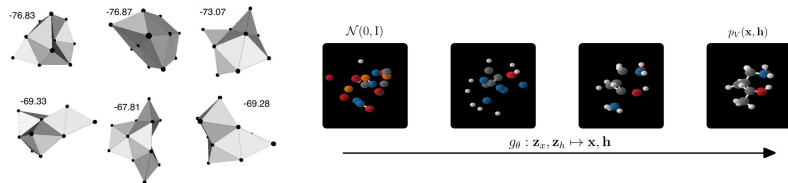
## one-shot

generating valid euclidean distance matrices



[Hoffmann & Noé, 2019]

equivariant normalizing flow



[Köhler et al., 2020]

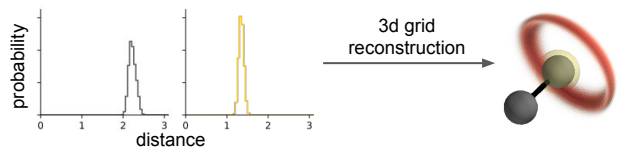
[Satorras et al., 2021]



# Approaches for 3d molecule generation

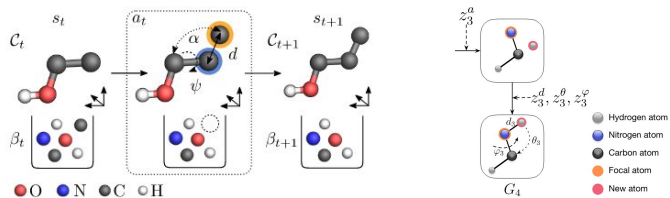
## iterative

reconstruction from pairwise distances



[Gebauer et al., 2019]

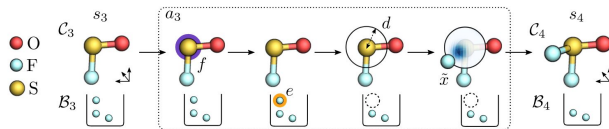
reconstruction from internal coordinates



[Simm et al., 2020]

[Luo & Ji, 2022]

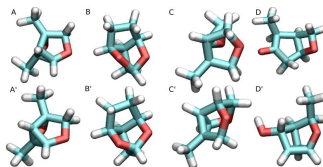
equivariant prediction of direction (spherical harmonics)



[Simm et al., 2021]

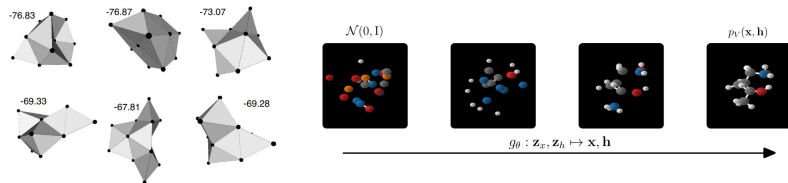
## one-shot

generating valid euclidean distance matrices



[Hoffmann & Noé, 2019]

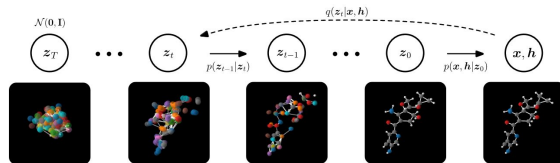
equivariant normalizing flow



[Köhler et al., 2020]

[Satorras et al., 2021]

equivariant diffusion



[Hoogeboom et al., 2022]

# Autoregressive approach

$$p(\mathbf{r}_1, \dots, \mathbf{r}_n, Z_1, \dots, Z_n) = \prod_{i=0}^{n-1} p(\mathbf{r}_{i+1}, Z_{i+1} | \mathbf{r}_1, \dots, \mathbf{r}_i, Z_1, \dots, Z_i)$$

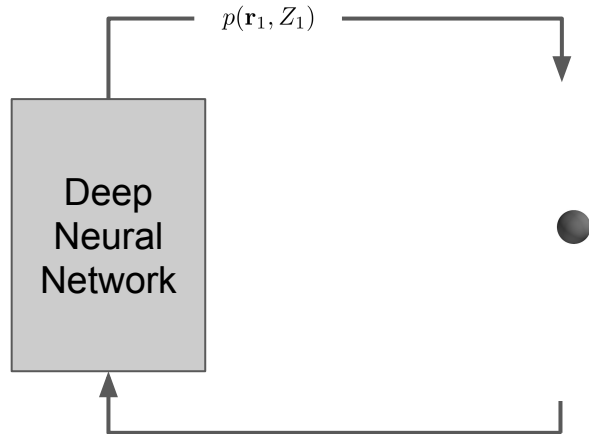


Deep  
Neural  
Network

G-SchNet  
[Gebauer et al., 2019]

# Autoregressive approach

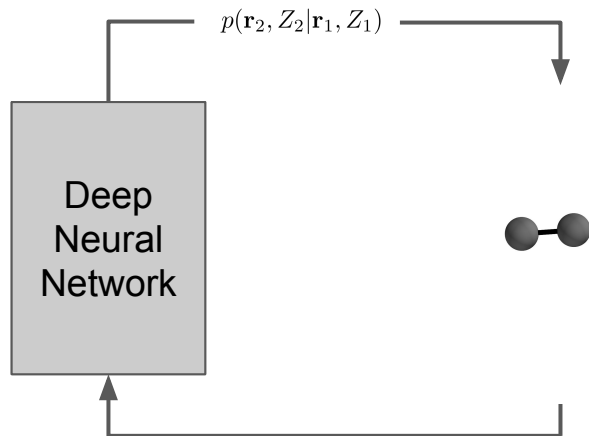
$$p(\mathbf{r}_1, \dots, \mathbf{r}_n, Z_1, \dots, Z_n) = \prod_{i=0}^{n-1} p(\mathbf{r}_{i+1}, Z_{i+1} | \mathbf{r}_1, \dots, \mathbf{r}_i, Z_1, \dots, Z_i)$$



G-SchNet  
[Gebauer et al., 2019]

# Autoregressive approach

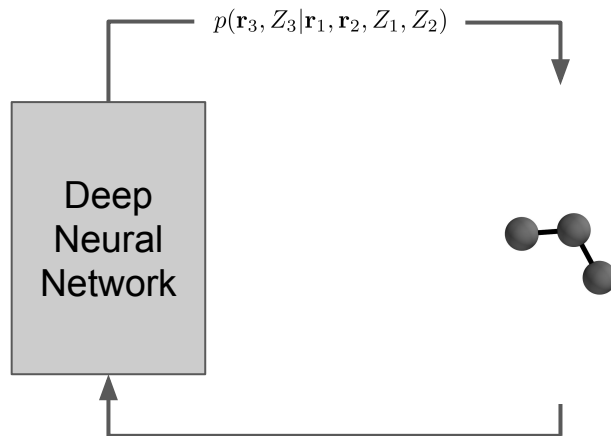
$$p(\mathbf{r}_1, \dots, \mathbf{r}_n, Z_1, \dots, Z_n) = \prod_{i=0}^{n-1} p(\mathbf{r}_{i+1}, Z_{i+1} | \mathbf{r}_1, \dots, \mathbf{r}_i, Z_1, \dots, Z_i)$$



G-SchNet  
[Gebauer et al., 2019]

# Autoregressive approach

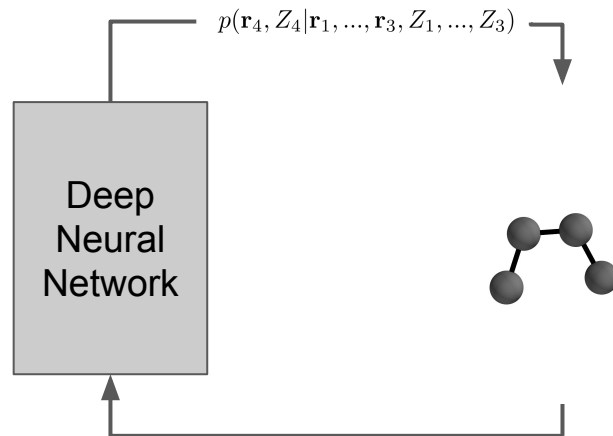
$$p(\mathbf{r}_1, \dots, \mathbf{r}_n, Z_1, \dots, Z_n) = \prod_{i=0}^{n-1} p(\mathbf{r}_{i+1}, Z_{i+1} | \mathbf{r}_1, \dots, \mathbf{r}_i, Z_1, \dots, Z_i)$$



G-SchNet  
[Gebauer et al., 2019]

# Autoregressive approach

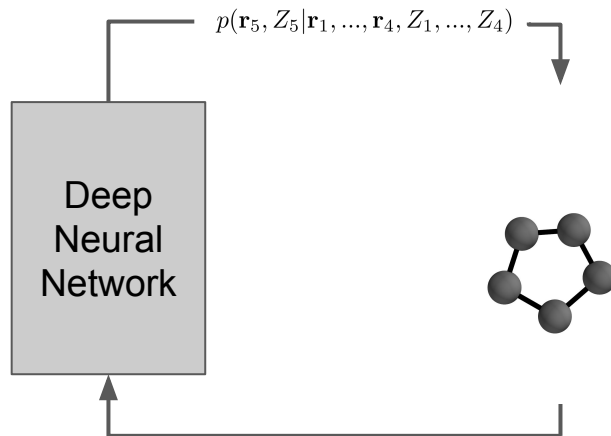
$$p(\mathbf{r}_1, \dots, \mathbf{r}_n, Z_1, \dots, Z_n) = \prod_{i=0}^{n-1} p(\mathbf{r}_{i+1}, Z_{i+1} | \mathbf{r}_1, \dots, \mathbf{r}_i, Z_1, \dots, Z_i)$$



G-SchNet  
[Gebauer et al., 2019]

# Autoregressive approach

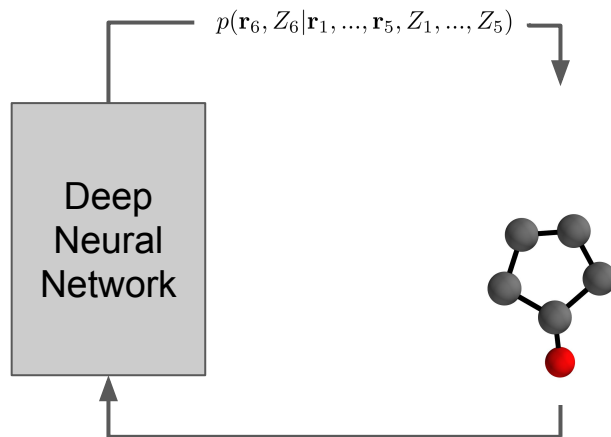
$$p(\mathbf{r}_1, \dots, \mathbf{r}_n, Z_1, \dots, Z_n) = \prod_{i=0}^{n-1} p(\mathbf{r}_{i+1}, Z_{i+1} | \mathbf{r}_1, \dots, \mathbf{r}_i, Z_1, \dots, Z_i)$$



G-SchNet  
[Gebauer et al., 2019]

# Autoregressive approach

$$p(\mathbf{r}_1, \dots, \mathbf{r}_n, Z_1, \dots, Z_n) = \prod_{i=0}^{n-1} p(\mathbf{r}_{i+1}, Z_{i+1} | \mathbf{r}_1, \dots, \mathbf{r}_i, Z_1, \dots, Z_i)$$

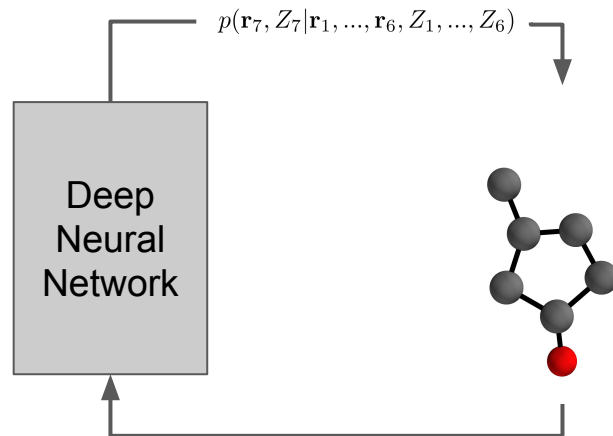


G-SchNet  
[Gebauer et al., 2019]



# Autoregressive approach

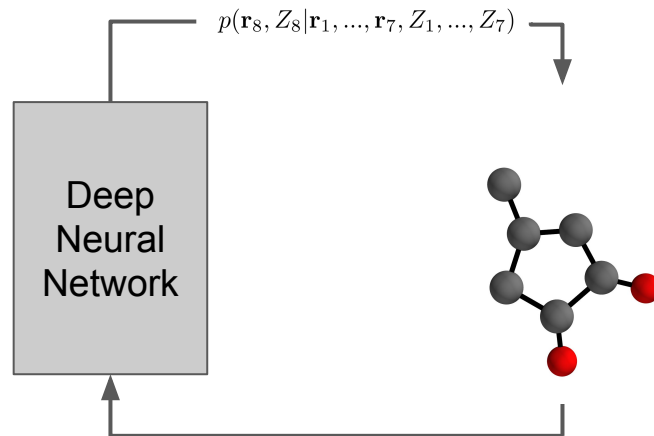
$$p(\mathbf{r}_1, \dots, \mathbf{r}_n, Z_1, \dots, Z_n) = \prod_{i=0}^{n-1} p(\mathbf{r}_{i+1}, Z_{i+1} | \mathbf{r}_1, \dots, \mathbf{r}_i, Z_1, \dots, Z_i)$$



G-SchNet  
[Gebauer et al., 2019]

# Autoregressive approach

$$p(\mathbf{r}_1, \dots, \mathbf{r}_n, Z_1, \dots, Z_n) = \prod_{i=0}^{n-1} p(\mathbf{r}_{i+1}, Z_{i+1} | \mathbf{r}_1, \dots, \mathbf{r}_i, Z_1, \dots, Z_i)$$

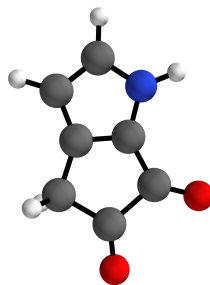


G-SchNet  
[Gebauer et al., 2019]

# Autoregressive approach

$$p(\mathbf{r}_1, \dots, \mathbf{r}_n, Z_1, \dots, Z_n) = \prod_{i=0}^{n-1} p(\mathbf{r}_{i+1}, Z_{i+1} | \mathbf{r}_1, \dots, \mathbf{r}_i, Z_1, \dots, Z_i)$$

Deep  
Neural  
Network



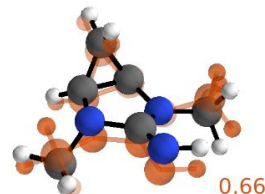
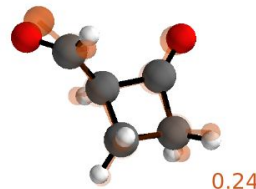
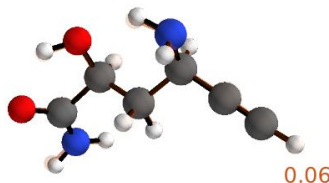
G-SchNet  
[Gebauer et al., 2019]

# Autoregressive approach

$$p(\mathbf{r}_1, \dots, \mathbf{r}_n, Z_1, \dots, Z_n) = \prod_{i=0}^{n-1} p(\mathbf{r}_{i+1}, Z_{i+1} | \mathbf{r}_1, \dots, \mathbf{r}_i, Z_1, \dots, Z_i)$$

Deep  
Neural  
Network

novel



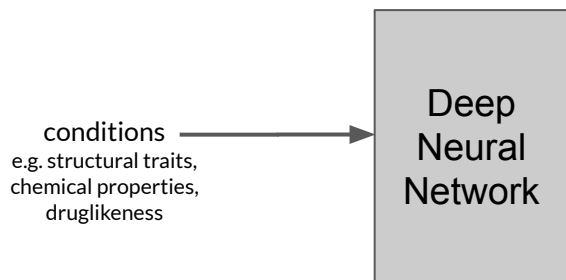
- sampling of valid, stable small organic molecules
- captures characteristics of the training data

## G-SchNet

[Gebauer et al., 2019]

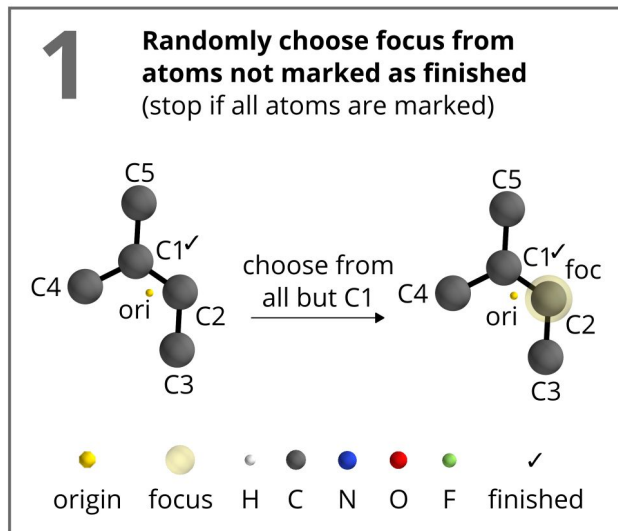
# Autoregressive approach - conditional

$$p(\mathbf{r}_1, \dots, \mathbf{r}_n, Z_1, \dots, Z_n | \lambda_1, \dots, \lambda_k) = \prod_{i=0}^{n-1} p(\mathbf{r}_{i+1}, Z_{i+1} | \mathbf{r}_1, \dots, \mathbf{r}_i, Z_1, \dots, Z_i, \lambda_1, \dots, \lambda_k)$$

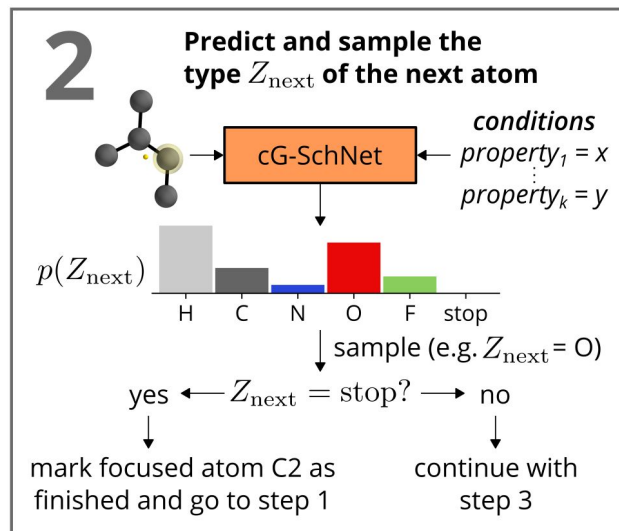
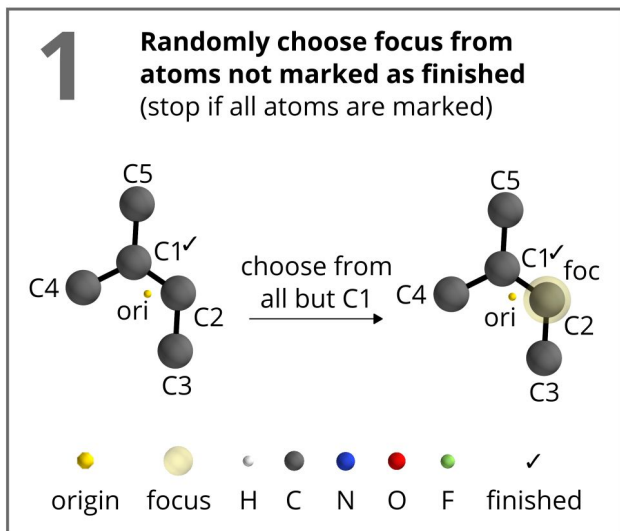


**cG-SchNet**  
[Gebauer et al., 2022]

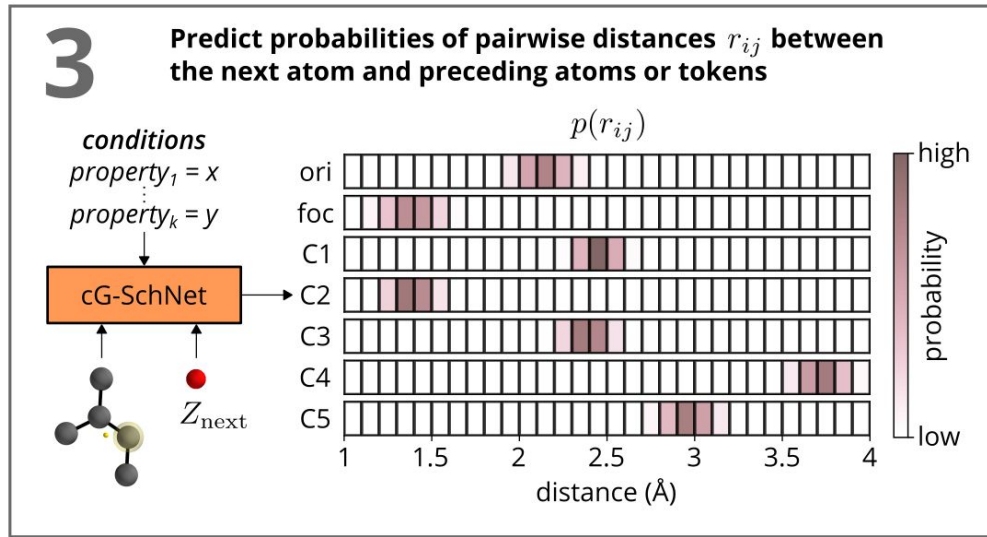
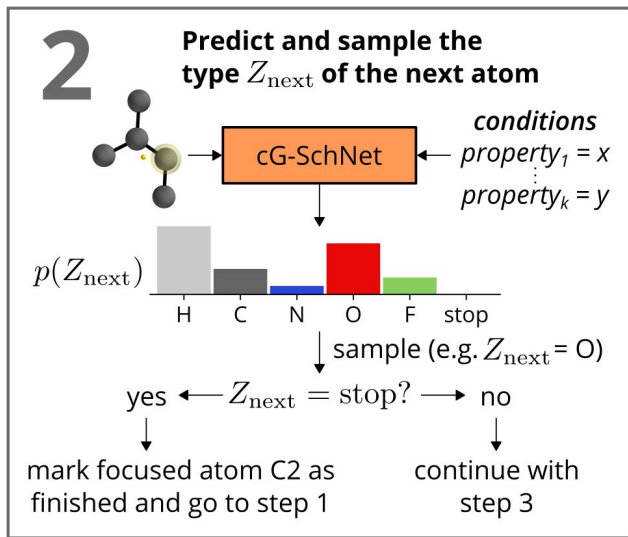
# Atom placement loop with cG-SchNet



# Atom placement loop with cG-SchNet



# Atom placement loop with cG-SchNet

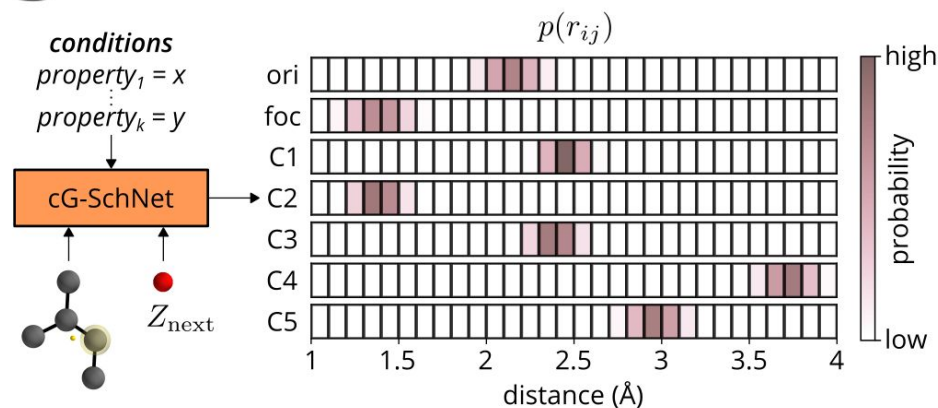




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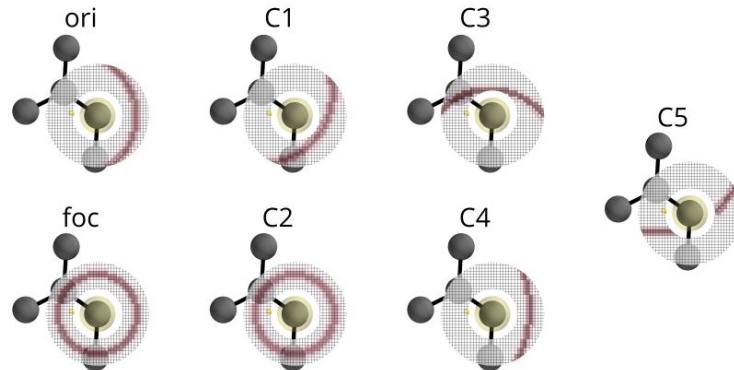
3

Predict probabilities of pairwise distances  $r_{ij}$  between the next atom and preceding atoms or tokens



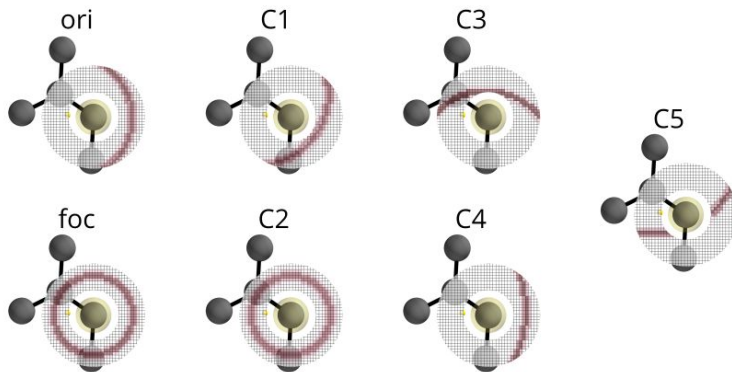
4

Look up the probabilities of pairwise distances between grid positions and preceding atoms or tokens in the output of the network from step 3



# Atom placement loop with cG-SchNet

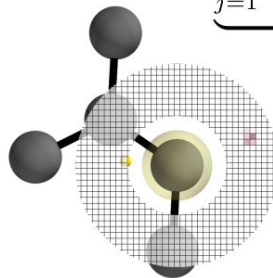
- 4** Look up the probabilities of pairwise distances between grid positions and preceding atoms or tokens in the output of the network from step 3



- 5** Multiply probabilities at each position to reconstruct a distribution for the position  $\mathbf{r}_{\text{next}}$  of the next atom

$$p(\mathbf{r}_{\text{next}}) = \frac{1}{\alpha} \prod_{j=1}^{i-1} p(r_{ij} = \|\mathbf{r}_j - \mathbf{r}_{\text{next}}\|_2)$$

$\omega(\mathbf{r}_{\text{next}})$



where  $\alpha$  normalizes  
w.r.t. all grid positions:

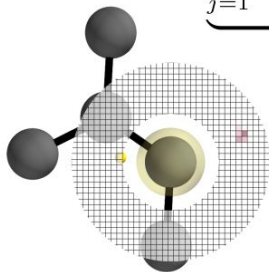
$$\alpha = \sum_{\mathbf{r} \in \text{grid}} \omega(\mathbf{r})$$

# Atom placement loop with cG-SchNet

**5** Multiply probabilities at each position to reconstruct a distribution for the position  $\mathbf{r}_{\text{next}}$  of the next atom

$$p(\mathbf{r}_{\text{next}}) = \frac{1}{\alpha} \prod_{j=1}^{i-1} p(r_{ij} = \|\mathbf{r}_j - \mathbf{r}_{\text{next}}\|_2)$$

$\underbrace{\hspace{10em}}_{\omega(\mathbf{r}_{\text{next}})}$

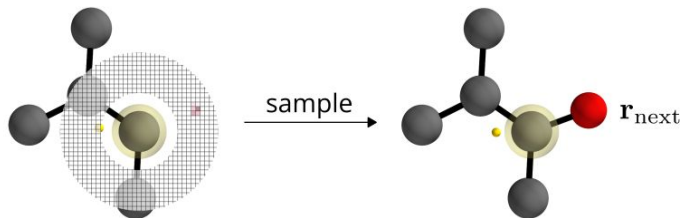


where  $\alpha$  normalizes  
w.r.t. all grid positions:

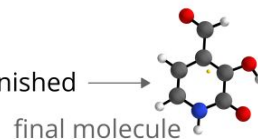
$$\alpha = \sum_{\mathbf{r} \in \text{grid}} \omega(\mathbf{r})$$

**6**

Sample position  $\mathbf{r}_{\text{next}}$   
of the next atom

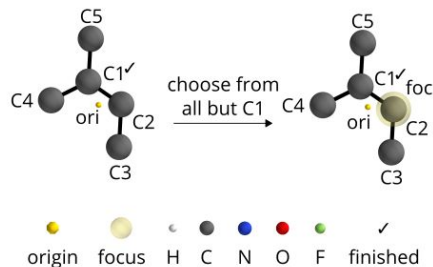


go to step 1 and repeat until finished

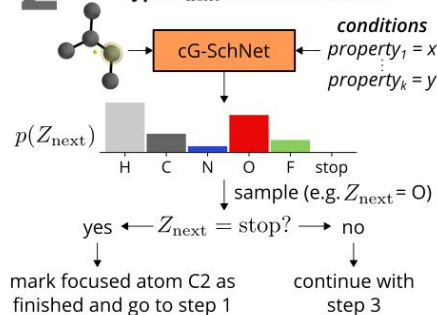


# Atom placement loop with cG-SchNet

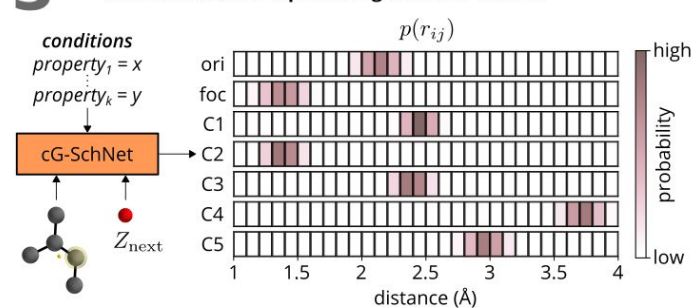
**1** Randomly choose focus from atoms not marked as finished (stop if all atoms are marked)



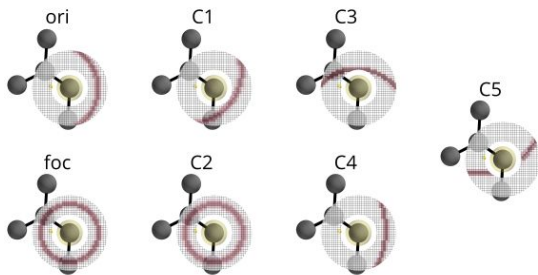
**2** Predict and sample the type  $Z_{\text{next}}$  of the next atom



**3** Predict probabilities of pairwise distances  $r_{ij}$  between the next atom and preceding atoms or tokens



**4** Look up the probabilities of pairwise distances between grid positions and preceding atoms or tokens in the output of the network from step 3



**5** Multiply probabilities at each position to reconstruct a distribution for the position  $\mathbf{r}_{\text{next}}$  of the next atom

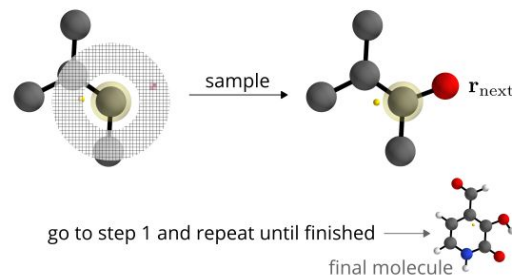
$$p(\mathbf{r}_{\text{next}}) = \frac{1}{\alpha} \prod_{j=1}^{i-1} p(r_{ij} = \|\mathbf{r}_j - \mathbf{r}_{\text{next}}\|_2)$$

$$\omega(\mathbf{r}_{\text{next}})$$

where  $\alpha$  normalizes w.r.t. all grid positions:

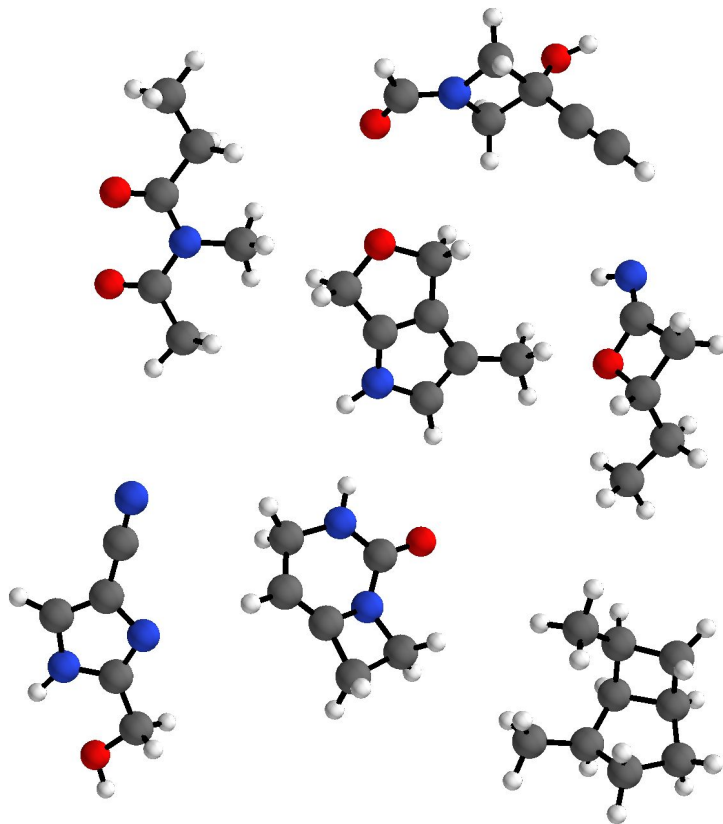
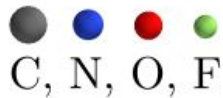
$$\alpha = \sum_{\mathbf{r} \in \text{grid}} \omega(\mathbf{r})$$

**6** Sample position  $\mathbf{r}_{\text{next}}$  of the next atom

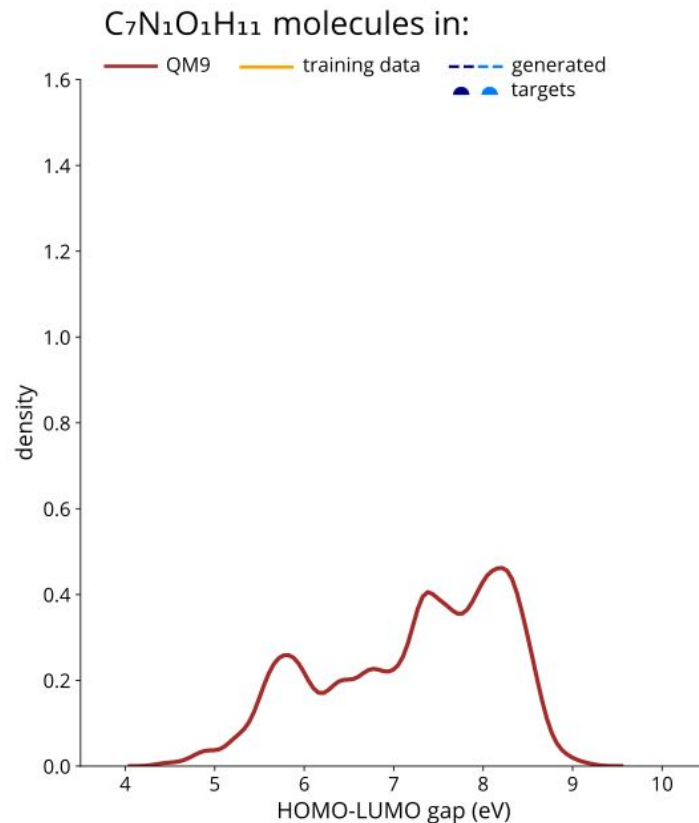


# QM9

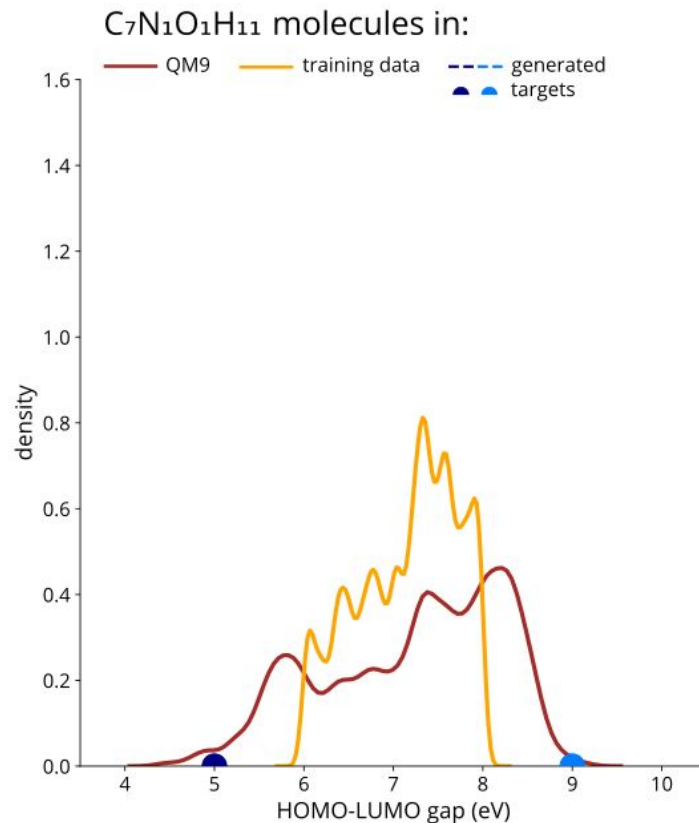
- ~130k stable molecules
- up to 9 heavy atoms
- 55k for training



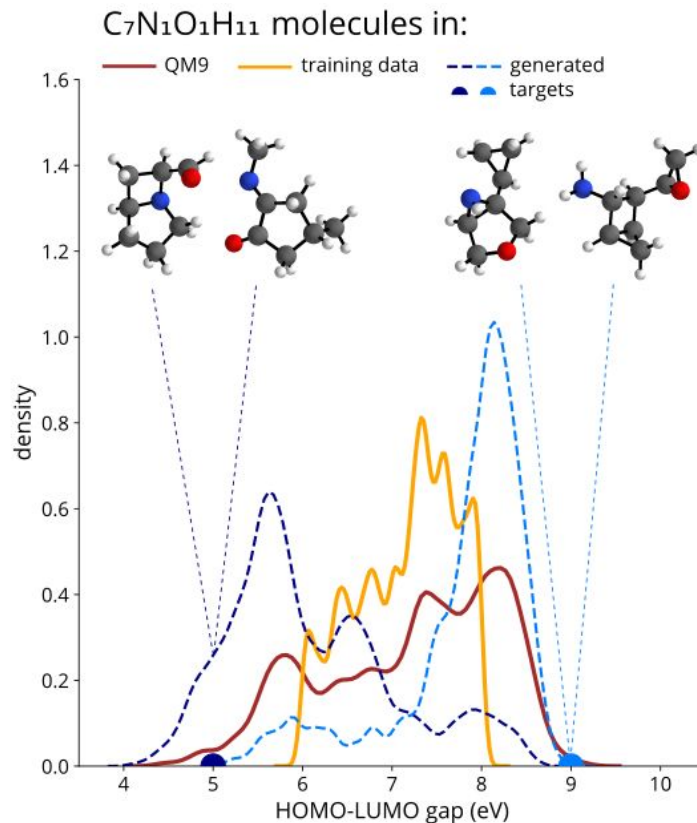
# Results: Generalization across compositions



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# Results: Generalization across compositions





# Challenges

- no real-world success story yet
- limited data availability
  - benchmark data sets have no practical applications
  - more interesting data owned by companies
  - data costly to compute
- mostly applied to:
  - small structures
  - non-periodic systems
- targeting many properties is difficult

# Conclusion

- diverse applications require guided exploration of chemical space
  - cG-SchNet for the conditional generation of 3d molecules
    - deep neural network respecting local and global symmetries
    - generates stable molecules capturing statistics of the training data
    - generalizes to unseen conditions to sample target-dependent candidates
    - allows for data-efficient, guided exploration of small organic compounds
  - generative models for 3d structures are gaining traction
    - ignoring spatial component is a limiting factor
    - data availability is comparatively bad
    - models are still limited
- increase focus on targeted exploration (conditioning/biasing)
- application to crystal structures and larger molecules

# Thank you!

Questions?

# Literature

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