

icpc 算法模板

Catch-22

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1 数学

1.1 求逆元

注意考虑 x 是 mod 倍数的情况

```

1  ll qpow(ll a, ll b) {
2      ll res = 1;
3      while(b) {
4          if(b & 1) res = res * a % mod;
5          a = a * a % mod;
6          b >>= 1;
7      }
8      return res;
9  }
10
11 ll inv(ll x) { return qpow(x, mod - 2); }
12
13 const int N = 1e6 + 10;
14 // 线性递推求逆元 [1, n] 的所有数关于 p 的逆元
15 int inv[N];
16 void init_inv () {
17     int n, p;
18     cin >> n >> p;
19     inv[0] = 0, inv[1] = 1;
20     for (int i = 2; i <= n; i++)
21         inv[i] = (ll)(p - p / i) * inv[p % i] % p; // 为了保证大于零加了个 p
22     for (int i = 1; i <= n; i++)
23         cout << inv[i] << endl;
24
25     return 0;
26 }

```

1.2 扩展欧几里德算法

bezout 定理: 设 a, b 为正整数, 则关于 x, y 的方程 $ax + by = c$ 有整数解当且仅当 c 是 $\gcd(a, b)$ 的倍数。

返回结果: $ax + by = \gcd(a, b)$ 的一组解 (x, y)

时间复杂度: $\mathcal{O}(n \log n)$

```

1  // 拓欧解线性同余方程  $a \cdot x = b \pmod m$ 
2  #include <bits/stdc++.h>
3  using namespace std;
4  using ll = long long;
5  int a, b, m, n;
6
7  int exgcd(int a, int b, int &x, int &y) {
8      if(b == 0) {
9          x = 1, y = 0;
10         return a;

```

```

11     }
12     int d = exgcd(b, a % b, y, x);
13     y -= a/b * x;
14     return d;
15 }
16
17 int main() {
18     int x, y;
19     cin >> n;
20     while(n -- ) {
21         cin >> a >> b >> m;
22         int d = exgcd(a, m, x, y); // d = gcd(a, m)
23         if(b % d != 0) puts("impossible"); //bezout 定理: 有解的条件, gcd(a, m) | b
24         else printf("%lld\n", (ll)x * (b/d) % m);
25     }
26     return 0;
27 }

```

1.3 筛法

筛质数

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  using ll = long long;
4  const int N = 1e7 + 10;
5  // minp[i] 为 i 的最小素因子 http://oj.daimayuan.top/course/10/problem/733
6  int primes[N], pcnt, minp[N]; // 可用于 Log 级别分解质因数
7  bool vis[N]; //合数 true
8  int n, q;
9  //Linear
10 void get_prime(int n) {
11     for(int i = 2; i <= n; i++) {
12         if(!vis[i]) primes[ ++ pcnt] = i, minp[i] = i;
13         for(int j = 1; j <= pcnt && i * primes[j] <= n; ++ j) {
14             vis[i * primes[j]] = 1;
15             minp[primes[j] * i] = primes[j];
16             if(i % primes[j] == 0) break;
17         }
18     }
19 }
20
21 //about Linear :O(nLogLogn)
22 bool isprime[N];
23 inline void getprime(int n) {
24     for (int i = 2; i <= n; i++) isprime[i] = 1;
25     for (int i = 2; i <= n; i++) {
26         if(isprime[i]) {
27             primes[++pcnt] = i;
28             if((ll)i*i<=n)

```

```

29         for (int j = i * i; j <= n; j+=i){
30             isprime[j] = 0;
31         }
32     }
33 }
34 }

```

筛欧拉函数

```

1  #include <bits/stdc++.h>
2  using namespace std;
3
4  /*phi compute
5  根据给定 n 计算 phi(n) O(aqrt(n))
6  核心公式 phi(n) = n*(1-1/p1)*(1 - 1/p2)*...
7  */
8  int get_phi(int n) {
9      int res = n;
10     for (int i = 2; i <= n / i; i++) {
11         if(n % i == 0) {
12             res = res / i * (i - 1); // res *= (1 - 1/n)
13             while(n % i == 0)    n /= i;
14         }
15     }
16     if(n > 1) res = res / n * (n - 1);
17     return res;
18 }
19
20 using ll = long long;
21 const int N = 1e6 + 10;
22
23 int phi[N], prime[N];
24 bool vis[N]; //合数 true
25
26 void sel_phi(int n) {
27     int cnt = 0;
28     phi[1] = 1;
29     for (int i = 2; i <= n; i++) {
30         if(!vis[i]) {
31             prime[cnt++] = i;
32             phi[i] = i - 1;
33         }
34         for (int j = 0; prime[j] <= n / i; j++) {
35             vis[prime[j] * i] = true;
36             if(i % prime[j] == 0) {
37                 phi[i * prime[j]] = phi[i] * prime[j];
38                 break;
39             }
40             else
41                 phi[prime[j] * i] = phi[i] * (prime[j] - 1);
42         }

```

```

43     }
44 }

```

筛莫比乌斯函数

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  const int N = 50010;
4  int mu[N], p[N]; // p 为素数数组
5  bool flg[N];
6  void init() {
7      int tot = 0; mu[1] = 1;
8      for (int i = 2; i < N; ++i) {
9          if (!flg[i]) {
10             p[++tot] = i;
11             mu[i] = -1;
12         }
13         for (int j = 1; j <= tot && i * p[j] < N; ++j) {
14             flg[i * p[j]] = 1;
15             if (i % p[j] == 0) {
16                 mu[i * p[j]] = 0;
17                 break;
18             }
19             mu[i * p[j]] = -mu[i];
20         }
21     }
22     // 常用 mu 前缀和
23     // for (int i = 1; i <= N; ++i) mu[i] += mu[i - 1];
24 }

```

1.4 组合数

$$1. C_n^m = C_n^{n-m}$$

$$2. C_n^m = C_{n-1}^m + C_{n-1}^{m-1}$$

$$3. C_n^0 + C_n^1 + \cdots + C_n^n = 2^n$$

$$4. lucas: C_n^m \equiv C_{n \bmod p}^{m \bmod p} * C_{n/p}^{m/p}$$

```

1  //求组合数的几种方法
2  //不确定的时候都开 long long
3  #include <bits/stdc++.h>
4  using namespace std;
5  using ll = long long;
6  const int mod = 1e9 + 7, N = 1e6 + 10;
7  //C(a, b) a 上 b 下
8
9  /*1. 依照定义 适用于 a, b 很小的时候 (几十) */
10 int C(ll a, int b) /* a 上 b 下 */{
11     if(a < b) return 0;

```

```

12     int up = 1, down = 1;
13     for (ll i = a; i > a - b; i -- ) up = i % mod * up % mod; //up *= i
14     for (int j = 1; j <= b; j ++ ) down = (ll)j * down % mod; // down *= j
15     return (ll)up * qpow(down, mod - 2) % mod; // (up/down)
16 }
17
18 /*2. 递推 杨辉三角 a, b 在 2000 这个数量级 */
19 //O(N^2) 1e6~1e7
20 void init()
21 {
22     for (int i = 0; i < N; i ++ )
23         for (int j = 0; j <= i; j ++ )
24             if(!j) C[i][j] = 1;
25             else C[i][j] = (C[i - 1][j] + C[i - 1][j - 1]) % mod;
26 }
27
28 //最常用
29 /*3. 预处理 fac[], invfac[]*/
30 /**
31  * //调用:
32  * 1ll * fac[b] * invfac[a] % mod * invfac[b - a] % mod;
33  */
34 // O(N) 1e6 左右 看 N 大小
35 int fac[N], invfac[N];
36 void init() {
37     fac[0] = 1;
38     for (int i = 1; i < N; i ++ ) (ll)fac[i] = fac[i - 1] * i % mod;
39     invfac[N - 1] = qpow(fac[N - 1], mod - 2);
40     for (int i = N - 2; i >= 0; i -- )
41         invfac[i] = (ll)invfac[i + 1] * (i + 1) % mod;
42 }
43
44 /*4. Lucas 定理 当 a, b 的值特别大 如 1e9 以上...1e18 等 */
45 int C(int a, int b) {
46     int res = 1;
47     for (int i = 1, j = a; i <= b; i ++, j -- ) {
48         res = (ll)res * j % p;
49         res = (ll)res * binpow(i, p - 2) % p;
50     }
51     return res;
52 }
53
54 ll lucas(ll a, ll b) { //p 为质 (模) 数
55     if(a < p && b < p) return C(a, b);
56     return (ll)C(a % p, b % p) * lucas(a / p, b / p) % p;
57 }

```


1.5 容斥原理

S_i 为有限集, $|S|$ 为 S 的大小 (元素个数), 则:

$$|\bigcup_{i=1}^n S_i| = \sum_{i=1}^n |S_i| - \sum_{1 \leq i < j \leq n} |S_i \cap S_j| + \sum_{1 \leq i < j < k \leq n} |S_i \cap S_j \cap S_k| + \cdots + (-1)^{n+1} |S_1 \cap \cdots \cap S_n|$$

```

1 // 容斥原理
2 // 给定素数集合 A(大小为 k), 求 [L, R] 中素数集合的任意元素的倍数的个数
3 // 1<=L<=R<=10^18, 1<=k<=20, 2<=ai<=100
4 #include <bits/stdc++.h>
5 using ll = long long;
6 using namespace std;
7
8 int main() {
9     ll l, r, k, f[25];
10    cin >> l >> r >> k;
11    for (int i = 0; i < k; i++) cin >> f[i];
12
13    ll ans = 0;
14
15    for (int i = 1; i < 1 << k; i++) { // 枚举集合中全部的非空子集
16        ll cnt = 0, a = r, b = l - 1; // cnt 用来表示所取的数的个数
17        for (int j = 0; j < k; j++) {
18            if(i >> j & 1) {
19                cnt++;
20                a /= f[j], b /= f[j];
21            }
22        }
23        if(cnt & 1) ans += (a - b);
24        else ans -= (a - b);
25    }
26    cout << ans << endl;
27    return 0;
28 }

```

1.6 数论分块

考虑和式: $\sum_{i=1}^n f(i) \lfloor \frac{n}{i} \rfloor$, 由于 $\lfloor \frac{n}{i} \rfloor$ 的值成一个块状分布, 故可以一块一块运算。我们先求出 $f(i)$ 的前缀和, 每次以 $[l, r] = [l, \lfloor \frac{n}{\lfloor \frac{n}{l} \rfloor} \rfloor]$ 为一块分块求出贡献累加到结果中。(常配合莫反使用) 常见转换:

- $\lceil \frac{a}{b} \rceil = \lfloor \frac{a-1}{b} \rfloor + 1$
- $a \bmod b = a - \lfloor \frac{a}{b} \rfloor * b$

```

1 // for(int i = st; i <= ed; i++) ans += num/i
2 ll block(ll st, ll ed, ll num) {
3     //sum(num/i i in [st,ed])
4     ll L = 0, res = 0;

```

```

5   ed = min(ed, num);
6   for (ll i = st; i <= ed; i = L + 1) {
7       L = min(ed, num / (num / i)); //该区间的最后一个数
8       res += (L - i + 1) * (num / i); //区间 [i, L] 的 num/i 都是一个值
9       // res += (s(L) - s(i-1)) * (num/i); //s(i) 为 f(i) 前缀和
10  }
11  return res;
12 }

```

1.7 Möbius 反演

μ 为莫比乌斯函数，定义为

$$\mu(x) = \begin{cases} 1 & n = 1 \\ 0 & n \text{ 含有平方因子} \\ (-1)^k & k \text{ 为 } n \text{ 本质不同的质因子个数} \end{cases}$$

性质：

$$\sum_{d|n} \mu(d) = \begin{cases} 1 & n = 1 \\ 0 & n \neq 1 \end{cases}$$

证：设 $n = \prod_{i=1}^k p_i^{c_i}, n' = \prod_{i=1}^k p_i$

那么 $\sum_{d|n} \mu(d) = \sum_{d|n'} \mu(d) = \sum_{i=0}^k C_k^i \cdot (-1)^i = (1 + (-1))^k = 1$

反演：

形式一：

$$f(n) = \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu(d) f\left(\frac{n}{d}\right)$$

证：

$$\sum_{d|n} \mu(d) f\left(\frac{n}{d}\right) = \sum_{d|n} \mu(d) \sum_{k|\frac{n}{d}} g(k) = \sum_{k|n} g(k) \sum_{d|\frac{n}{k}} \mu(d) = g(n)$$

用 $\sum_{d|n} g(d)$ 来替换 $f(\frac{n}{d})$ ，再变换求和顺序。最后一步变换的依据： $\sum_{d|n} \mu(d) = [n=1]$ ，因此在 $\frac{n}{k}=1$ 时第二个和式的值才为 1。此时 $n=k$ ，故原式等价于 $\sum_{k|n} [n=k] \cdot g(k) = g(n)$

形式二：

$$f(n) = \sum_{n|d} g(d) \Leftrightarrow g(n) = \sum_{n|d} \mu\left(\frac{d}{n}\right) f(d)$$

1.8 高斯消元

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  const int N = 110;
4  const double eps = 1e-6;
5  int n;
6  double a[N][N];
7
8  int gauss() {
9      int c, r;

```

```

10     for(c = 0, r = 0; c < n; c++) {
11         int t = r;
12         for(int i = r; i < n; i++)//找到首元素最大
13             if(fabs(a[i][c]) > fabs(a[t][c]))
14                 t = i;
15
16         if(fabs(a[t][c]) < eps) continue;
17
18         for(int i = c; i <= n; i++) swap(a[t][i], a[r][i]);
19         for(int i = n; i >= c; i--) a[r][i] /= a[r][c];
20         for(int i = r + 1; i < n; i++)
21             if(fabs(a[i][c]) > eps)
22                 for(int j = n; j >= c; j--)
23                     a[i][j] -= a[r][j] * a[i][c];
24         r++;
25     }
26     if(r < n) {
27         for(int i = r; i < n; i++)
28             if(fabs(a[i][n]) > eps)
29                 return 2;
30         return 1;
31     }
32
33     for(int i = n - 1; i >= 0; i--)
34         for(int j = i + 1; j < n; j++)
35             a[i][n] -= a[i][j] * a[j][n];
36
37     return 0;//有唯一解
38 }
39
40 int main() {
41     cin >> n ;
42     for(int i = 0; i < n; i++)
43         for(int j = 0; j < n + 1; j++)
44             cin >> a[i][j];
45
46     int t = gauss();
47     if(t == 0)
48         for(int i = 0; i < n; i++) printf("%.2f\n", a[i][n]);
49     else if(t == 1)
50         puts("Infinite group solutions");
51     else puts("No solution");
52
53     return 0;
54 }

```

1.9 Miller Rabin 素数测试

```

1  //loj143 prime test
2  #include <bits/stdc++.h>
3  using namespace std;
4  using ull = unsigned long long;
5  using ll = long long;
6  /* O(sqrt(n))
7  bool is_prime(ll x)
8  {
9      if(x < 2) return false;
10     for(ll i = 2; i <= x / i; ++i)
11         if(x % i == 0) return false;
12     return true;
13 }
14 */
15 //常常是大素数测试，要用到 int128
16 inline ll qmul(ll a, ll b, ll p) { return (ll)((__int128)a * b % p); }
17 ll qpow(ll a, ll b, ll p) {
18     ll res = 1;
19     while(b) {
20         if(b & 1) res = qmul(res, a, p);
21         a = qmul(a, a, p);
22         b >>= 1;
23     }
24     return res;
25 }
26 const int test_time = 8;
27
28 bool mr_test(ll n) {
29     if(n < 3 || n % 2 == 0) return n == 2;
30     ll a = n - 1, b = 0;
31     while(a % 2 == 0) a /= 2, ++b;
32
33     for (int i = 1, j; i <= test_time; ++i) {
34         ll x = rand() % (n - 2) + 2, v = qpow(x, a, n);
35         if(v == 1) continue;
36         for (j = 0; j < b; ++j) {
37             if(v == n - 1) break;
38             v = qmul(v, v, n);
39         }
40         if(j >= b) return 0;
41     }
42     return 1;
43 }
44
45 int main() {
46     srand(time(0));
47     ll x;
48     while(cin >> x) {

```

```

49         if(mr_test(x)) puts("Y");
50         else puts("N");
51     }
52     return 0;
53 }

```

2 数据结构

2.1 (带权) 并查集

```

1  const int N = 1e5 + 10;
2  int fa[N], n, m, d[N];
3
4  int find(int x) {return x == fa[x] ? x : fa[x] = find(fa[x]);}
5  // 对于带权并查集，一般的 find 函数写作：
6  int find(int x) {
7      if(x == fa[x]) return x;
8      int rt = find(fa[x]); //这和下面一行顺序很重要
9      d[x] += d[fa[x]]; //可以改成 d[x] ^= d[fa[x]], 根据权值意义的需要修改
10     return fa[x] = rt;
11 }
12
13 void init() {
14     for (int i = 1; i <= n; i++) fa[i] = i;
15 }

```

2.2 Sparse Table

时间复杂度 $\mathcal{O}(1)$ ，空间复杂度 $\mathcal{O}(n \log n)$

静态区间查询可重复贡献信息，如“区间最值”、“区间按位和”、“区间按位或”、“区间 GCD”

```

1  #include<bits/stdc++.h>
2  using namespace std;
3
4  const int N = 1e5 + 10;
5  int f[N][21], n, m;
6  int a[N];
7  //f[i][j] 表示闭区间 [i, i + 2^j - 1] 的最大值
8
9  void init_st() {
10     // cout << __lg(N) << endl;
11     for (int j = 0; j < 21; j++)
12         for (int i = 1; i + (1 << j) - 1 <= n; i++) //区间长度是 2^j 所以要减一
13             if(!j) f[i][j] = a[i];
14             else
15                 f[i][j] = max(f[i][j - 1], f[i + (1 << j - 1)][j - 1]);
16 }
17
18 int query(int l, int r) {

```

```

19     int k = __lg(r - 1 + 1);
20     return max(f[1][k], f[r - (1 << k) + 1][k]);
21 }

```

2.3 01Trie

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  const int N = 1e5 + 10, M = N * 31;
4  int a[N];
5  int son[M][2], idx;
6
7  void insert(int x) {
8      int p = 0;
9      for (int i = 30; i >= 0; --i) {
10         int u = (x >> i) & 1;
11         if (!son[p][u]) son[p][u] = ++idx;
12         p = son[p][u];
13     }
14 }
15 // 集合内和 x 异或的最大值
16 int query(int x) {
17     int p = 0, res = 0;
18     for (int i = 30; i >= 0; --i) {
19         int u = (x >> i) & 1;
20         if (son[p][u ^ 1]) p = son[p][u ^ 1], res |= (1 << i);
21         else p = son[p][u];
22         // 集合内和 x 异或的最小值
23         // if(son[p][u]) p = son[p][u];
24         // else res |= (1 << i), p = son[p][u ^ 1];
25     }
26     return res;
27 }
28
29 int main() {
30     int n, res = 0;
31     cin >> n;
32     for(int i = 0; i < n; i++) cin >> a[i];
33     for(int i = 0; i < n; i++) {
34         insert(a[i]);
35         res = max(res, query(a[i]));
36     }
37     cout << res;
38     return 0;
39 }

```

2.4 树状数组

```

1  const int N = 1e5 + 10;
2  int tr[N], a[N];
3  inline int lowbit(int x) {return x & -x;}
4
5  int query(int x) {
6      int res = 0;
7      for (int i = x; i; i -= lowbit(i)) res += tr[i];
8      return res;
9  }
10
11 void add(int x, int val) {
12     for(int i = x; i <= n; i += lowbit(i))
13         tr[i] += val;
14 }
15
16 //fenwich-tree 写区间修改, 区间查询
17 //记录两个数组 b[i] = a[i] - a[i - 1]; c[i] = i * b[i];
18 #include <bits/stdc++.h>
19 using namespace std;
20 typedef long long ll;
21 const int N = 1e5 + 10;
22 int a[N], b[N];
23 ll t1[N], t2[N]; //维护 b[i], b[i] * i 的前缀和
24 int n, m;
25
26 void add(ll tr[], int x, ll c) {
27     for (int i = x; i <= n; i += lowbit(i))
28         tr[i] += c;
29 }
30
31 ll query(ll tr[], int x) {
32     ll res = 0;
33     for (int i = x; i; i -= lowbit(i))
34         res += tr[i];
35     return res;
36 }
37
38 ll preSum(int x) { return query(t1, x) * (x + 1) - query(t2, x); }
39
40 int main() {
41     scanf("%d%d", &n, &m);
42     for (int i = 1; i <= n; i++) scanf("%d", &a[i]);
43
44     for (int i = 1; i <= n; i++) {
45         int b = a[i] - a[i - 1];
46         add(t1, i, b);
47         add(t2, i, (ll)b * i);
48     }

```

```

49     while(m -- ) {
50         char op[2];
51         int l, r, d;
52         scanf("%s%d%d", op, &l, &r);
53         if(*op == 'Q')
54             printf("%lld\n", preSum(r) - preSum(l - 1));
55         else {
56             scanf("%d", &d);
57             //a[l] += d;
58             add(t1, l, d), add(t2, l, l * d);
59             add(t1, r + 1, -d), add(t2, r + 1, (r + 1) * -d);
60         }
61     }
62     return 0;
63 }

```

2.5 线段树

```

1  //常见维护
2  /**
3   * 区间和, 最值
4   * 维护最大连续字段和 (维护 lmax, rmax, tmax)
5   * 维护区间平方和
6   * 区间修改成之指定数 维护 sum, lazy(指定数值), bool changed;
7   * 区间内开根号: 由于六次根号 1e12 (向下取整) 即得到 1, 所以可以暴力修改
8   * 区间内数字同时乘以一个数 如下:
9   */
10 #include<bits/stdc++.h>
11 using namespace std;
12 using ll = long long;
13 const int N = 1e5 + 10;
14 int n, m, mod;
15 int a[N];
16
17 struct node {
18     int l, r;
19     int sum, add, mul;
20 } t[4 * N];
21
22 void eval(node &t, int add, int mul) {
23     t.sum = ((ll)t.sum * mul + (ll)(t.r - t.l + 1) * add) % mod;
24     t.mul = (ll)t.mul * mul % mod;
25     t.add = ((ll)t.add * mul + add) % mod;
26 }
27
28 void pushup(int p) {
29     t[p].sum = (t[p << 1].sum + t[p << 1 | 1].sum) % mod;
30 }
31

```



```

32 void pushdown(int p) {
33     eval(t[p << 1], t[p].add, t[p].mul);
34     eval(t[p << 1 | 1], t[p].add, t[p].mul);
35
36     t[p].add = 0, t[p].mul = 1;
37 }
38
39 void build(int p, int l, int r) {
40     if(l == r) {
41         t[p] = {l, r, a[l], 0, 1};
42         return;
43     }
44     t[p] = {l, r, 0, 0, 1};
45     int mid = l + r >> 1;
46     build(p << 1, l, mid);
47     build(p << 1 | 1, mid + 1, r);
48     pushup(p);
49 }
50
51 void modify(int p, int l, int r, int add, int mul) {
52     if(t[p].l >= l && t[p].r <= r) eval(t[p], add, mul);
53     else {
54         pushdown(p);
55         int mid = t[p].l + t[p].r >> 1;
56         if(l <= mid) modify(p << 1, l, r, add, mul);
57         if(r > mid) modify(p << 1 | 1, l, r, add, mul);
58         pushup(p);
59     }
60 }
61
62 int query(int p, int l, int r) {
63     if(t[p].l >= l && t[p].r <= r) return t[p].sum;
64
65     pushdown(p);
66     int res = 0;
67     int mid = t[p].l + t[p].r >> 1;
68
69     if(l <= mid) res += query(p << 1, l, r);
70     if(r > mid) res += query(p << 1 | 1, l, r);
71     res %= mod;
72     return res;
73 }
74
75 int main() {
76     scanf("%d%d", &n, &mod);
77     for (int i = 1; i <= n; i++)
78         scanf("%d", &a[i]);
79     build(1, 1, n);
80

```

```

81     scanf("%d", &m);
82     while(m -- ) {
83         int op, l, r, d;
84         scanf("%d%d%d", &op, &l, &r);
85         if(op == 1) {
86             scanf("%d", &d);
87             modify(1, l, r, 0, d);
88         }
89         else if(op == 2) {
90             scanf("%d", &d);
91             modify(1, l, r, d, 1);
92         }
93         else
94             printf("%d\n", query(1, l, r));
95     }
96     return 0;
97 }

```

扫描线: (面积)

```

1  //p1502 线段树扫描线算法
2  #include<bits/stdc++.h>
3  using namespace std;
4  using ll = long long;
5  const ll N = 1e4 + 10;
6  struct L {
7      ll x, y1, y2;
8      ll c;
9      //当左矩形的右边界与右矩形的左边界重合时, 该线上的点应属于能被两个窗户都能看见的状态所以先加
10     bool operator<(const L &rhs) const { return x == rhs.x ? c < rhs.c : x < rhs.x; }
11 } line[2 * N];
12
13 ll n, w, h, m;
14 ll b[2 * N]; //离散化前的 y 轴
15
16 struct node {
17     ll l, r;
18     ll maxv, add;
19 } t[8 * N];
20
21
22 void pushdown(ll p) {
23     node &root = t[p], &nl = t[p << 1], &nr = t[p << 1 | 1];
24     if(root.add) {
25         nl.add += root.add, nl.maxv += root.add;
26         nr.add += root.add, nr.maxv += root.add;
27         root.add = 0;
28     }
29 }
30
31 void pushup(ll p) {

```

```

32     t[p].maxv = max(t[p << 1].maxv, t[p << 1 | 1].maxv);
33 }
34
35 void modify(ll p, ll l, ll r, ll c) {
36     if(t[p].l >= l && t[p].r <= r) {
37         t[p].maxv += c;
38         t[p].add += c;
39         return;
40     }
41     pushdown(p);
42     ll mid = t[p].l + t[p].r >> 1;
43     if(l <= mid) modify(p << 1, l, r, c);
44     if(r > mid) modify(p << 1 | 1, l, r, c);
45     pushup(p);
46
47 }
48
49 void build(ll p, ll l, ll r) {
50     if(l == r) {
51         t[p] = {l, r, 0, 0};
52         return;
53     }
54     t[p].l = l, t[p].r = r;
55     ll mid = l + r >> 1;
56     build(p << 1, l, mid);
57     build(p << 1 | 1, mid + 1, r);
58     //pushup(p); //初始化都是 0 不用 pushup()
59 }
60
61 int main() {
62     ll T;
63     scanf("%lld", &T);
64     while( T -- ) {
65         memset(line, 0, sizeof(line));
66         memset(b, 0, sizeof(b));
67         memset(t, 0, sizeof(t));
68
69         scanf("%lld%lld%lld", &n, &w, &h);
70         for (ll i = 1, j = 0; i <= n; i++) {
71             ll x, y, l;
72             scanf("%lld%lld%lld", &x, &y, &l);
73             line[i] = {x, y, y + h - 1, l};
74             line[i + n] = {x + w - 1, y, y + h - 1, -1};
75             b[ ++ j] = y;
76             b[ ++ j] = y + h - 1;
77         }
78         n <<= 1;
79         sort(b + 1, b + 1 + n);
80         m = unique(b + 1, b + 1 + n) - b - 1; //unique 得到 end() 迭代器

```

```

81     sort(line + 1, line + 1 + n);
82
83     for (ll i = 1; i <= n; i++) {
84         line[i].y1 = lower_bound(b + 1, b + m + 1, line[i].y1) - b - 1;
85         line[i].y2 = lower_bound(b + 1, b + m + 1, line[i].y2) - b - 1;
86     }
87     build(1, 1, m - 1);
88
89     ll res = 0;
90     for (ll i = 1; i <= n; i++) {
91         modify(1, line[i].y1, line[i].y2, line[i].c);
92         res = max(res, t[1].maxv);
93     }
94     printf("%d\n", res);
95 }
96 return 0;
97 }

```

2.6 可持久化线段树

```

1 //Luogu 3824 kth-number
2 #include <bits/stdc++.h>
3 using namespace std;
4 const int N = 2e5 + 10, M = (N << 2) + 17 * N;
5
6 struct node {
7     int l, r;
8     int cnt;
9 } t[M];
10 int idx, a[N];
11 vector<int> num;
12 int find(int x) { return lower_bound(num.begin(), num.end(), x) - num.begin(); }
13
14 int insert(int now, int l, int r, int x) {
15     int p = ++ idx;
16     t[p] = t[now];
17     if (l == r) {
18         t[p].cnt ++ ;
19         return p;
20     }
21     int mid = l + r >> 1;
22     if(x <= mid) t[p].l = insert(t[now].l, l, mid, x);
23     else t[p].r = insert(t[now].r, mid + 1, r, x);
24     t[p].cnt = t[t[p].l].cnt + t[t[p].r].cnt;
25
26     return p;
27 }
28
29 int build(int l, int r) {

```

```

30     int p = ++ idx;
31     if (l == r) return p;
32     int mid = l + r >> 1;
33     t[p].l = build(l, mid), t[p].r = build(mid + 1, r);
34     return p;
35 }
36
37 int query(int x, int y, int l, int r, int k) {
38     if(l == r) return l;
39     int cnt = t[t[y].l].cnt - t[t[x].l].cnt;
40     int mid = l + r >> 1;
41     if(k <= cnt) return query(t[x].l, t[y].l, l, mid, k);
42     else return query(t[x].r, t[y].r, mid + 1, r, k - cnt);
43 }
44
45 int n, m, root[N];
46
47 int main() {
48     scanf("%d%d", &n, &m);
49     for (int i = 1; i <= n; i ++ ) {
50         scanf("%d", &a[i]);
51         num.push_back(a[i]);
52     }
53
54     sort(num.begin(), num.end());
55     num.erase(unique(num.begin(), num.end()), num.end());
56
57     root[0] = build(0, num.size() - 1);
58
59     for (int i = 1; i <= n; i ++ )
60         root[i] = insert(root[i - 1], 0, num.size() - 1, find(a[i]));
61     while (m -- ) {
62         int l, r, k;
63         scanf("%d%d%d", &l, &r, &k);
64         printf("%d\n", num[query(root[l - 1], root[r], 0, num.size() - 1, k)]);
65     }
66
67     return 0;
68 }

```

2.7 线段树合并

```

1 int merge(int p, int q, int l, int r) {
2     if(!p || !q) return p + q;
3     if(l == r) {
4         //维护信息, 一般是 t[p].val += t[q].val 等
5         // t[p].val.first += t[q].val.first;
6         return p;
7     }

```

```

8     int mid = l + r >> 1;
9     t[p].l = merge(t[p].l, t[q].l, l, mid);
10    t[p].r = merge(t[p].r, t[q].r, mid + 1, r);
11    // pushup();
12    // t[p].val = max(t[t[p].l].val, t[t[p].r].val);
13    return p;
14 }

```

2.8 树链剖分

```

1  #include<bits/stdc++.h>
2  #define pb push_back
3  using namespace std;
4
5  typedef long long ll;
6  const int N = 1e5 + 10;
7
8  struct node {
9      int l, r;
10     ll add, sum;
11 } t[N << 2];
12 int n, m, w[N], nw[N];
13 vector<int> G[N];
14
15 int dep[N], top[N], son[N], dfn[N], sz[N], fa[N], cnt;
16 ////////////////线段树部分////////////////////
17
18 void pushdown(int p) {
19     auto &rt = t[p], &nl = t[p << 1], &nr = t[p << 1 | 1];
20     if(rt.add) {
21         nl.add += rt.add, nl.sum += (ll)(nl.r - nl.l + 1) * rt.add;
22         nr.add += rt.add, nr.sum += (ll)(nr.r - nr.l + 1) * rt.add;
23         rt.add = 0;
24     }
25 }
26
27 void pushup(int p) { t[p].sum = t[p << 1].sum + t[p << 1 | 1].sum; }
28
29 void build(int p, int l, int r) {
30     t[p] = {l, r, 0, nw[l]};
31     if(l == r) return;
32
33     int mid = l + r >> 1;
34     build(p << 1, l, mid);
35     build(p << 1 | 1, mid + 1, r);
36     pushup(p);
37 }
38
39 ll query(int p, int l, int r) {

```

```

40     if(t[p].l >= 1 && t[p].r <= r) return t[p].sum;
41
42     pushdown(p);
43     int mid = t[p].l + t[p].r >> 1;
44     ll res = 0;
45     if(1 <= mid) res += query(p << 1, 1, r);
46     if(r > mid) res += query(p << 1 | 1, 1, r);
47     //pushup(p);
48     return res;
49 }
50
51 void modify(int p, int l, int r, int k) {
52     if(t[p].l >= 1 && t[p].r <= r) {
53         t[p].sum += (t[p].r - t[p].l + 1) * k;
54         t[p].add += k;
55         return;
56     }
57
58     pushdown(p);
59     int mid = t[p].l + t[p].r >> 1;
60     if(1 <= mid) modify(p << 1, l, r, k);
61     if(r > mid) modify(p << 1 | 1, l, r, k);
62     pushup(p);
63 }
64
65 //////////////////////////////////////////////////////////////////树剖部分////////////////////////////////////
66 //第一次 dfs 维护 sz, 重儿子, dep[], fa[]
67 void dfs1(int u, int fath) {
68     sz[u] = 1, dep[u] = dep[fath] + 1, fa[u] = fath;
69     for(int v:G[u]) {
70         if(v == fath) continue;
71         dfs1(v, u);
72         sz[u] += sz[v];
73         if(sz[son[u]] < sz[v]) son[u] = v;
74     }
75 }
76 //第二次 dfs, 维护 dfs 序,
77 void dfs2(int u, int tp) {
78     dfn[u] = ++cnt, nw[cnt] = w[u], top[u] = tp;
79     if(!son[u]) return;
80     dfs2(son[u], tp); //递归重儿子
81     //维护轻儿子信息
82     for(int v:G[u]) {
83         if(v == fa[u] || v == son[u]) continue;
84         dfs2(v, v);
85     }
86 }
87
88 void modify_path(int u, int v, int k) {

```

```

89     while(top[u] != top[v]) {
90         if(dep[top[u]] < dep[top[v]]) swap(u, v);
91         modify(1, dfn[top[u]], dfn[u], k);
92         u = fa[top[u]];
93     }
94     if(dep[u] < dep[v]) swap(u, v);
95     modify(1, dfn[v], dfn[u], k);
96 }
97
98 void modify_tree(int u, int k) {
99     modify(1, dfn[u], dfn[u] + sz[u] - 1, k);
100 }
101
102 ll query_tree(int u) {
103     return query(1, dfn[u], dfn[u] + sz[u] - 1);
104 }
105
106 ll query_path(int u, int v) {
107     ll res = 0;
108     while(top[u] != top[v]) {
109         if(dep[top[u]] < dep[top[v]]) swap(u, v);
110         res += query(1, dfn[top[u]], dfn[u]);
111         u = fa[top[u]];
112     }
113     if(dep[u] < dep[v]) swap(u, v);
114     res += query(1, dfn[v], dfn[u]);
115     return res;
116 }
117
118 ///////////////////////////////////////////////////////////////////
119 int main() {
120
121     scanf("%d", &n);
122     for(int i = 1; i <= n; i++) scanf("%d", &w[i]);
123     for(int i = 1; i < n; i++) {
124         int u, v; scanf("%d%d", &u, &v);
125         G[u].pb(v), G[v].pb(u);
126     }
127     dfs1(1, 0);
128     dfs2(1, 1);
129
130     build(1, 1, n);
131
132     scanf("%d", &m);
133     while(m--) {
134         int op, u, v, k;
135         scanf("%d%d", &op, &u);
136         if(op == 1) {
137             scanf("%d%d", &v, &k);

```



```

138     modify_path(u, v, k);
139 }
140 else if(op == 2) {
141     scanf("%d", &k);
142     modify_tree(u, k);
143 }
144 else if(op == 3) {
145     scanf("%d", &v);
146     printf("%lld\n", query_path(u, v));
147 }
148 else
149     printf("%lld\n", query_tree(u));
150 }
151 return 0;
152 }

```

2.9 左偏树

支持操作（以维护最小值为例）：

1. 找到最小值 $\mathcal{O}(1)$
2. 删除最小值 $\mathcal{O}(\log n)$
3. 插入一个值 $\mathcal{O}(\log n)$
4. 合并两个堆 $\mathcal{O}(\log n)$

```

1  #include <bits/stdc++.h>
2  #define endl '\n'
3  using namespace std;
4  const int N = 2e5 + 10;
5  int val[N], lson[N], rson[N], dis[N];
6  int fa[N], idx, n;
7  int find(int x) { return x == fa[x] ? x : fa[x] = find(fa[x]); }
8
9  bool cmp(int x, int y) { return val[x] == val[y] ? x < y : val[x] < val[y]; }
10
11 int merge(int x, int y) {
12     if(!x || !y) return x + y;
13     if(cmp(y, x)) swap(x, y);
14     rson[x] = merge(rson[x], y);
15     if(dis[rson[x]] > dis[lson[x]]) swap(lson[x], rson[x]);
16     dis[x] = dis[rson[x]] + 1;
17     return x;
18 }
19
20 int main() {
21     ios::sync_with_stdio(false), cin.tie(0);
22     cin >> n;
23     val[0] = 2e9;

```

```

24     while(n --) {
25         int op, x, y;  cin >> op;
26         if(op == 1) {
27             cin >> x;
28             val[++idx] = x;
29             fa[idx] = idx;
30             dis[idx] = 1;
31         }
32         else if(op == 2) {
33             cin >> x >> y;
34             x = find(x), y = find(y);
35             if(x != y) {
36                 if(cmp(y, x)) swap(x, y); //x 为较小的
37                 fa[y] = x;
38                 merge(x, y);
39             }
40         }
41         else if(op == 3) {
42             cin >> x;
43             cout << val[find(x)] << endl;
44         }
45         else { // 删除 x 所在堆的最小值
46             cin >> x; x = find(x);
47             if(cmp(rson[x], lson[x])) swap(lson[x], rson[x]);
48             fa[x] = lson[x], fa[lson[x]] = lson[x];
49             merge(lson[x], rson[x]);
50         }
51     }
52     return 0;
53 }

```

3 图论

3.1 spfa

```

1  #include <bits/stdc++.h>
2  #define pb push_back
3  using namespace std;
4
5  const int N = 1e5 + 10, inf = 0x3f3f3f3f;
6  struct node{int v, w;};
7  vector<node> G[N];
8  int dis[N], n, m;
9  bool inq[N];
10
11 void spfa() {
12     memset(dis, 0x3f, sizeof dis);
13     dis[1] = 0;
14     inq[1] = 1;

```

```

15     queue<int> q;
16     q.push(1);
17     while(q.size()) {
18         int u = q.front(); q.pop();
19         inq[u] = 0;
20         for(auto [v, w]:G[u]) {
21             if(dis[v] > w + dis[u]) {
22                 dis[v] = dis[u] + w;
23                 if(!inq[v])
24                     q.push(v), inq[v] = true;
25             }
26         }
27     }
28 }
29
30 int main() {
31     cin >> n >> m;
32     while(m -- ) {
33         int u, v, w;
34         cin >> u >> v >> w;
35         G[u].pb({v, w});
36     }
37     spfa();
38     if(dis[n] == inf)    cout << "impossible";
39     else                cout << dis[n];
40     return 0;
41 }

```

3.2 dijkstra

稀疏图 dijkstra:

```

1  //acwing 849
2  #include <bits/stdc++.h>
3  using namespace std;
4  const int N = 510, inf = 0x3f3f3f3f;
5  int dis[N], G[N][N], n, m;
6  bool vis[N];
7
8  void dij() {
9      memset(dis, 0x3f, sizeof dis);
10     dis[1] = 0;
11     for (int j = 0; j < n; j++) {
12         int minv = inf, pos = -1;
13         for(int i = 1; i <= n; i++)
14             if (!vis[i] && minv > dis[i])
15                 minv = dis[i], pos = i;
16
17         if(pos == -1) break;
18         vis[pos] = 1;

```

```

19         for (int i = 1; i <= n; i++)
20             if(!vis[i] && dis[pos] + G[pos][i] < dis[i])
21                 dis[i] = dis[pos] + G[pos][i];
22     }
23 }
24
25 int main() {
26     cin >> n >> m;
27     scanf("%d %d", &n, &m);
28     memset(G, 0x3f, sizeof(G));
29     while(m--) {
30         int u, v, w; scanf("%d %d %d", &u, &v, &w);
31         G[u][v] = min(G[u][v], w);
32     }
33
34     dij();
35
36     cout << (dis[n] == inf ? -1 : dis[n]);
37     return 0;
38 }

```

稠密图 dijkstra:

```

1  #include <bits/stdc++.h>
2  #define pb push_back
3  #define fi first
4  #define se second
5  using namespace std;
6
7  using P = pair<int, int>;
8  const int N = 151000, inf = 0x3f3f3f3f;
9  struct node{int v, w;};
10 vector<node> G[N];
11 int dis[N], n, m;
12 bool vis[N];
13
14 void dij() {
15     memset(dis, 0x3f, sizeof dis);
16     priority_queue<P, vector<P>, greater<P>> q;
17     q.push({0, 1});
18     while(q.size()) {
19         auto t = q.top(); q.pop();
20         int u = t.se, d = t.fi;
21         if(vis[u]) continue;
22         vis[u] = true;
23         for(auto [v, w] : G[u]) {
24             if(dis[v] > d + w) {
25                 dis[v] = d + w;
26                 q.push({dis[v], v});
27             }
28         }
29     }
30 }

```

```

29     }
30 }
31
32 int main() {
33     ios::sync_with_stdio(false);
34     cin >> n >> m;
35     while(m -- ) {
36         int u, v, w; cin >> u >> v >> w;
37         G[u].pb({v, w});
38     }
39     dij();
40     cout << (dis[n] == inf ? -1 : dis[n]);
41     return 0;
42 }

```

3.3 最小生成树

```

1  // kruskal
2  const int N = 1e5 + 10;
3  struct edge {
4      int u, v, w;
5      bool operator<(const edge &rhs) const { return w < rhs.w; }
6  } edges[N];
7
8  int fa[N], n, m;
9  int find(int x) { return x == fa[x] ? x : fa[x] = find(fa[x]); }
10
11 int kruskal() {
12     cin >> n >> m;
13     int u, v, w, ans = 0;
14     for (int i = 1; i <= m; i++) {
15         cin >> u >> v >> w;
16         edges[i] = {u, v, w};
17     }
18     sort(edges + 1, edges + 1 + m);
19     for (int i = 1; i <= n; i++) fa[i] = i;
20     for (int i = 1; i <= m; i++) {
21         auto [u, v, w] = edges[i];
22         u = find(u), v = find(v);
23         if(u == v) continue;
24         fa[u] = v;
25         ans += w;
26     }
27     return ans;
28 }
29
30 //prim
31 const int N = 510, inf = 0x3f3f3f3f;
32 int G[N][N], dis[N];

```

```

33 int n, m;
34 bool vis[N];
35
36 int prim() {
37     int res = 0;
38     memset(dis, 0x3f, sizeof dis);
39     dis[1] = 0;    //随便选一点进入 mst 集合
40     for(int j = 0; j < n; j++) {
41         int minv = inf, pos = -1;
42         for(int i = 1; i <= n; i++)
43             if(!vis[i] && dis[i] < minv)
44                 pos = i, minv = dis[i];
45
46         if(pos == -1) return inf;
47         vis[pos] = true;
48         res += dis[pos];
49
50         for(int i = 1; i <= n; i++)
51             if(!vis[i] && dis[i] > G[pos][i])
52                 dis[i] = G[pos][i];
53     }
54     return res;
55 }

```

另外, 对于完全图的 *MST* 问题, 可以考虑使用 *Boruvka* 算法。我们要在 $n \log n$ 或 $n \log^2 n$ 时间内求出每个连通块最小的连接的边, 而这个边权一般可通过点权以一定方式求出。通常不用直接写出, 运用该思想求解。

3.4 kruskal 重构树

```

1 //kruskal 重构树
2
3 //性质:
4 //两个点之间的所有简单路径上最大边权的最小值
5 // = 最小生成树上两个点之间的简单路径上的最大值
6 // = Kruskal 重构树上两点之间的 LCA 的权值。
7 //Loj136
8 #include <bits/stdc++.h>
9 #define pb push_back
10 using namespace std;
11
12 const int N = 1010 << 1, M = 3e5 + 10;
13 int n, m, k, val[N]; // kruskal 重构树的点权
14 int idx; //重构树的节点数
15
16 struct Edge{
17     int u, v, w;
18     bool operator<(const Edge &rhs) const { return w < rhs.w; }
19 }edges[M];
20

```

```

21 vector<int> G[N];
22
23 int p[N];
24 int find(int x) { return x == p[x] ? x : p[x] = find(p[x]); }
25
26 int dep[N], fa[N][21];
27
28 void bfs(int s) {
29     dep[0] = 0, dep[s] = 1;
30     queue<int> q;
31     q.push(s);
32     while(q.size()) {
33         int u = q.front(); q.pop();
34         for(int v:G[u]) {
35             if(dep[v] > dep[u] + 1) {
36                 dep[v] = dep[u] + 1;
37                 q.push(v);
38                 fa[v][0] = u;
39                 for (int i = 1; i <= 20; i++)
40                     fa[v][i] = fa[fa[v][i - 1]][i - 1];
41             }
42         }
43     }
44 }
45
46 int lca(int a, int b) {
47     if(dep[a] < dep[b]) swap(a, b);
48     for (int k = 20; k >= 0; k--)
49         if(dep[fa[a][k]] >= dep[b])
50             a = fa[a][k];
51     if(a == b) return a;
52     for (int k = 20; k >= 0; k--)
53         if(fa[a][k] != fa[b][k])
54             a = fa[a][k], b = fa[b][k];
55     return fa[a][0];
56 }
57
58 void build() {
59     idx = n;
60     int cnt = 0;
61     for (int i = 1; i <= m; i++) {
62         int u = edges[i].u, v = edges[i].v, w = edges[i].w;
63         int fu = find(u), fv = find(v);
64         if(fu != fv) {
65             val[++idx] = w;
66             G[idx].pb(fu), G[idx].pb(fv);
67             G[fu].pb(idx), G[fv].pb(idx);
68             p[fu] = p[fv] = idx;
69             cnt++;

```

```

70     }
71     if(cnt >= n - 1) break;
72 }
73 }
74
75 int main() {
76     scanf("%d %d %d", &n, &m, &k);
77     for (int i = 1; i <= m; i++) {
78         int u, v, w; scanf("%d %d %d", &u, &v, &w);
79         edges[i] = {u, v, w};
80     }
81     sort(edges + 1, edges + m + 1);
82     for (int i = 1; i <= (n << 1); i++) p[i] = i;
83
84     build(); // kruskal 重构树
85
86     memset(dep, 0x3f, sizeof dep);
87     bfs(idk); //bfs 的根节点一定要是重构树的最高点
88
89     while(k -- ) {
90         int s, t;
91         scanf("%d %d", &s, &t);
92         if(find(s) != find(t)) puts("-1");
93         else
94             printf("%d\n", val[lca(s, t)]);
95     }
96     return 0;
97 }

```

3.5 二分图匹配

二分图匹配的模型有两个要素：

1. 节点能分成独立的两个集合，每个集合内部有 0 条边
2. 每个节点只能与 1 条匹配边相连

二分图最小覆盖模型特点是：每条边有 2 个端点，二者至少选择一个。

könig 定理：二分图最小点覆盖包含的点数等于二分图最大匹配数包含的边数。

图的最大独立集：点集 S 中任意两点之间都没有边相连。其大小等于 $n - \text{最大匹配数}$ 。（ n 是二分图总点数）

```

1  /* 染色法判断二分图
2  bool vis[N];
3  int col[N], flag = 1, n, m;
4  void dfs(int u, int t) {
5      if (vis[u]) {
6          if (col[u] != t) flag = 0;
7          return;
8      }

```



```

9     vis[u] = 1; col[u] = t;
10    for (int v : g[u]) {
11        dfs(v, t ^ 1);
12    }
13 }
14 bool isbit() { // 是否为二分图
15     for (int u = 1; u <= n; u++) {
16         if (!vis[u]) dfs(u, 0);
17     }
18     return flag;
19 }
20 */
21 int G[N][M]; // 左半部 n, 右半部 m
22 int n, m, p[M], vis[M];
23 bool match(int u) {
24     for (int i = 1; i <= m; i++) {
25         if (G[u][i] && !vis[i]) {
26             vis[i] = true;
27             if (p[i] == 0 || match(p[i])) {
28                 p[i] = u; return true;
29             }
30         }
31     }
32     return false;
33 }
34 int main() {
35     /* 建图 */
36     int res = 0;
37     for (int i = 1; i <= n; i++) {
38         memset(vis, 0, sizeof vis);
39         if (match(i)) res++;
40     }
41     return 0;
42 }

```

3.6 强连通分量缩点

时间复杂度 $O(m + n)$, 反向枚举 `scc_cnt` 即是新图拓扑序。

```

1  #include <bits/stdc++.h>
2  #define pb push_back
3  using namespace std;
4
5  const int N = 1e4 + 10;
6  vector<int> G[N], G2[N];
7  stack<int> s;
8  int n, m, tim, scc_cnt;
9  int w[N], dfn[N], low[N], id[N];
10 int dist[N], ind[N], W[N];
11 bool ins[N];

```

```

12
13 void tarjan(int u) {
14     low[u] = dfn[u] = ++tim;
15     s.push(u); ins[u] = true;
16     for(int v:G[u]) {
17         if(!dfn[v]) {
18             tarjan(v);
19             low[u] = min(low[v], low[u]);
20         }
21         else if(ins[v])
22             low[u] = min(low[u], dfn[v]);
23     }
24     if(low[u] == dfn[u]) {
25         int y; ++scc_cnt;
26         do {
27             y = s.top(); s.pop();
28             ins[y] = false;
29             id[y] = scc_cnt;
30             W[scc_cnt] += w[y];
31         } while (y != u);
32     }
33 }
34
35 int sol() {
36     queue<int> q;
37     for (int i = 1; i <= scc_cnt; i++)
38         if(!ind[i]) {
39             q.push(i);
40             dist[i] = W[i];
41         }
42
43     while(q.size()) {
44         //cout << "cnt = " << ++cnt << endl;
45         int u = q.front(); q.pop();
46         for (int v:G2[u]) {
47             ↪ //当有重边时, dist[v] 被更新的值始终不变, 即 dist[v] = dist[u] + w[v]; 所以不会影响
48             dist[v] = max(dist[v], dist[u] + W[v]);
49             if(--ind[v] == 0)
50                 q.push(v);
51         }
52     }
53
54     int ans = 0;
55     for (int i = 1; i <= scc_cnt; i++)
56         ans = max(ans, dist[i]);
57     return ans;
58 }
59

```

```

60
61 int main() {
62     ios::sync_with_stdio(false), cin.tie(0);
63     cin >> n >> m;
64     for (int i = 1; i <= n; i++) cin >> w[i];
65     while(m--) {
66         int u, v;
67         cin >> u >> v;
68         G[u].pb(v);
69     }
70     for (int i = 1; i <= n; i++)
71         if(!dfn[i])
72             tarjan(i);
73     //缩点
74     for (int u = 1; u <= n; ++u) {
75         for(int v : G[u]) {
76             if(id[v] != id[u]) {
77                 G2[id[u]].pb(id[v]);
78                 ind[id[v]]++;
79                 //printf("ind[%d] = %d\n", id[v], ind[id[v]]);
80             }
81         }
82     }
83     // debug
84     // for (int i = 1; i <= scc_cnt; i++)
85     //     printf("ind[%d] = %d\n", i, ind[i]);
86     // for (int i = 1; i <= scc_cnt; i++)
87     // {
88     //     printf("%d->", i);
89     //     for (int v:G2[i])
90     //         printf("%d ", v);
91     //     puts("");
92     // }
93     printf("%d\n", sol());
94     return 0;
95 }
96

```

3.7 lca

```

1  /*
2  求 lca: 1. 倍增 2. 树剖 3.tarjan 离线
3
4  lca 用处
5  1. 树上两点之间的距离 (多维护一个 dist 数组,  $dis[u] + dis[v] - 2 * dis[lca(u, v)]$ )
6  2. 树上两条路径是否相交 (如果两条路径相交, 那么一定有一条路径的 LCA 在另一条路径上)
7  */
8  //acwing1171 树上距离
9  #include <bits/stdc++.h>

```

```

10  #define pb push_back
11  #define endl '\n'
12  using namespace std;
13  const int N = 1e4 + 10;
14
15  struct node{int v, w;};
16  vector<node> G[N];
17  int fa[N][19], dep[N], dis[N];
18  int n, m;
19
20  void bfs(int s) {
21      memset(dep, 0x3f, sizeof dep);
22      dep[0] = 0, dep[s] = 1;
23      dis[s] = 0;
24      queue<int> q; q.push(s);
25      while(q.size()) {
26          int u = q.front(); q.pop();
27          for(auto [v, w] : G[u]) {
28              if(dep[v] > dep[u] + 1) {
29                  dis[v] = dis[u] + w;
30                  dep[v] = dep[u] + 1;
31                  fa[v][0] = u;
32                  q.push(v);
33                  for(int i = 1; i < 19; ++i)
34                      fa[v][i] = fa[fa[v][i - 1]][i - 1];
35              }
36          }
37      }
38  }
39
40  int lca(int a, int b) {
41      if(dep[a] < dep[b]) swap(a, b);
42      for(int k = 18; k >= 0; k--)
43          if(dep[fa[a][k]] >= dep[b])
44              a = fa[a][k];
45      if(a == b) return a;
46
47      for(int k = 18; k >= 0; --k)
48          if(fa[a][k] != fa[b][k])
49              a = fa[a][k], b = fa[b][k];
50      return fa[a][0];
51  }
52
53  int main() {
54      ios::sync_with_stdio(false), cin.tie(0);
55      cin >> n >> m;
56      for(int i = 1; i < n; i++) {
57          int u, v, w; cin >> u >> v >> w;
58          G[u].pb({v, w}), G[v].pb({u, w});

```

```

59     }
60     bfs(1);
61     while(m -- ) {
62         int u, v; cin >> u >> v;
63         int anc = lca(u, v);
64         cout << dis[u] + dis[v] - 2 * dis[anc] << endl;
65     }
66     return 0;
67 }

```

3.8 基环树

基环树的性质：点数等于边数；度数是点数两倍。一般题目中出现“从一个点到另一个点建一条边”，“N 个点通过恰好 N 条双向道路连接起来，不存在任何两条道路连接了相同的两个点”等类似信息可以判定该图是基环树森林。以下是求基环树（森林）直径（和）代码

```

1  //基环树森林求直径和最大
2  #include <bits/stdc++.h>
3  #define endl '\n'
4  #define pb push_back
5  using ll = long long;
6  using namespace std;
7  const int N = 1e6 + 10, M = N << 1;
8  int h[N], e[M], w[M], ne[M], idx;
9  ll s[N], sum[M], d[M]; //环上的前缀和数组，破环成链后两倍的前缀和
10 bool ins[N], vis[N];
11 int n, cir[M], ed[M], cnt; //cnt 环的个数
12 int fa[N], fw[N]; //父节点，反向权值
13 int q[M];
14 ll ans;
15
16 void add(int a, int b, int c) {
17     e[idx] = b, ne[idx] = h[a], w[idx] = c, h[a] = idx++;
18 }
19
20 //深搜 + 栈 找环
21 void dfs(int u, int from) {
22     vis[u] = ins[u] = true;
23     for (int i = h[u]; ~i; i = ne[i]) {
24         //如果是反向边则跳过，必须用边来判断，这样才能确定是通过反向变回到父节点
25         if (i == (from ^ 1)) continue;
26         int v = e[i];
27         fa[v] = u, fw[v] = w[i];
28         if (!vis[v]) dfs(v, i);
29         else if (ins[v]) {
30             cnt++;
31             ed[cnt] = ed[cnt - 1];
32             ll tot = w[i];
33             for (int k = u; k != v; k = fa[k]) {
34                 s[k] = tot;

```

```

35         tot += fw[k];
36         cir[++ ed[cnt]] = k;
37     }
38     s[v] = tot, cir[++ ed[cnt]] = v;
39 }
40 }
41 ins[u] = false;
42 }
43
44 // 求以 u 为根节点的子树的最大深度
45 ll dfs_d(int u) {
46     vis[u] = true;
47     ll d0 = 0, d1 = 0; //最大距离, 次大距离
48     for (int i = h[u]; ~i; i = ne[i]) {
49         int v = e[i];
50         if (vis[v]) continue;
51         ll d = dfs_d(v) + w[i];
52         if (d >= d0) d1 = d0, d0 = d;
53         else if (d > d1) d1 = d;
54     }
55     ans = max(ans, d1 + d0);
56     return d0;
57 }
58
59 int main() {
60     ios::sync_with_stdio(false), cin.tie(0);
61     cin >> n;
62     memset(h, -1, sizeof h);
63     for (int u = 1; u <= n; u++) {
64         int v; ll w; cin >> v >> w;
65         add(u, v, w), add(v, u, w);
66     }
67
68     for (int i = 1; i <= n; i++)
69         if (!vis[i])
70             dfs(i, -1);
71
72     memset(vis, 0, sizeof vis);
73     for (int i = 1; i <= n; i++) vis[cir[i]] = 1; //标记环上所有点
74
75     ll res = 0;
76     for (int i = 1; i <= cnt; i++) {
77         ans = 0; // 当前基环树的直径
78         int sz = 0; // 当前基环树的环的大小
79         for (int j = ed[i - 1] + 1; j <= ed[i]; j++) {
80             int k = cir[j];
81             d[sz] = dfs_d(k); // 求以当前点为根的子树的最大深度
82             sum[sz] = s[k];
83             sz++;

```

```

84     }
85     // 破环成链, 前缀和数组和 d[] 数组延长一倍
86     for (int j = 0; j < sz; j++)
87         d[sz + j] = d[j], sum[sz + j] = sum[j] + sum[sz - 1];
88
89     // 做一遍滑动窗口, 比较依据是 d[k] - sum[k]
90     int hh = 0, tt = -1;
91     for (int j = 0; j < sz * 2; j++) {
92         while (hh <= tt && q[hh] <= j - sz) hh++;
93         if (hh <= tt) ans = max(ans, d[j] + sum[j] + d[q[hh]] - sum[q[hh]]);
94         while (hh <= tt && d[j] - sum[j] >= d[q[tt]] - sum[q[tt]]) tt--;
95         q[++tt] = j;
96     }
97     res += ans;
98 }
99 cout << res << endl;
100 return 0;
101 }

```

3.9 dinic

```

1  #include <bits/stdc++.h>
2  #define pb push_back
3  using namespace std;
4  using ll = long long;
5  const int N = 1e4 + 10;
6  const ll inf = 0x3f3f3f3f3f3f3f3f;
7  int n, m, s, t, dep[N];
8  struct node {int v, cap, rev;};
9  vector<node> G[N];
10
11 bool bfs() {
12     queue<int> q;
13     q.push(s);
14     memset(dep, -1, sizeof dep);
15     dep[s] = 0;
16     while (q.size()) {
17         int u = q.front(); q.pop();
18         for(auto [v, cap, rev] : G[u])
19             if(dep[v] == -1 && cap)
20                 dep[v] = dep[u] + 1, q.push(v);
21     }
22     return dep[t] != -1;
23 }
24
25 ll dfs(int u, ll lim) {
26     if(u == t || lim == 0) return lim;
27     ll tot_flow = 0;
28     for(auto& [v, cap, rev] : G[u]) {

```

```

29         if(dep[v] == dep[u] + 1 && cap > 0) {
30             ll d = dfs(v, min(lim, (ll)cap));
31             cap -= d, G[v][rev].cap += d;
32             lim -= d, tot_flow += d;
33             if(lim == 0) return tot_flow;
34         }
35     }
36     if(lim != 0) dep[u] = -1;
37     return tot_flow;
38 }
39
40 ll dinic() {
41     ll max_flow = 0;
42     while(bfs())
43         max_flow += dfs(s, inf);
44     return max_flow;
45 }
46
47 int main() {
48     scanf("%d%d%d", &n, &m, &s, &t);
49     while(m --) {
50         int u, v, cap; scanf("%d%d", &u, &v, &cap);
51         G[u].pb({v, cap, G[v].size()});
52         G[v].pb({u, 0, G[u].size() - 1});
53     }
54     printf("%lld\n", dinic());
55     return 0;
56 }

```

4 动态规划

4.1 数位 dp

4.2 换根 dp

换根 dp 一般时间复杂度为 $\mathcal{O}(n)$ ，需要对树处理得到大规模答案，如对每个点得到一个答案。

```

1 // 求树上 对某个点来说包含他的连通点集个数
2 #include <bits/stdc++.h>
3 #define pb push_back
4 #define endl '\n'
5 using ll = long long;
6 using namespace std;
7 const int N = 1e6 + 10, mod = 1e9 + 7;
8
9 ll f[N], ans[N], n;
10 vector<int> G[N];
11
12 ll qpow(ll a, ll b) {

```



```

13     ll res = 1;
14     while(b) {
15         if(b & 1) res = res * a % mod;
16         a = a * a % mod;
17         b >>= 1;
18     }
19     return res;
20 }
21
22 void dfs(int u, int fa) {
23     f[u] = 1;
24     for (auto v:G[u]) {
25         if(v == fa) continue;
26         dfs(v, u);
27         f[u] = f[u] * (f[v] + 1) % mod;
28     }
29 }
30
31 /*
32 考虑换根,  $ans[u]$  记为以  $u$  为根, 和整棵树其他点能形成的所有子树数量。(即最终答案)
33 换根方程:  $ans[v] = (ans[u] / (f[v] + 1) + 1) * f[v]$ 
34 解释:  $u$  点答案除以  $v$  点贡献  $(f[v] + 1)$  为与  $v$  无关的  $u$  点答案,  $+1$  后为其余点对  $v$  点贡献, 再乘上  $f[v]$ 
35
36 有一个很坑的地方, 就是  $(f[v] + 1)$  求逆元可能得到  $0$  ( $f[v]$  可能为  $mod - 1$ ), 这时相当于除以  $0$ , 出错
37 当逆元  $inv$  为  $0$  时,  $ans[u]$  实际是由在树形  $dp$  的时候求出的  $f[u]$ , 而  $f[u]$  又等于 (他所有儿子  $f$  的值  $+1$ ) 的乘积。
38 所以  $ans[u] / (f[v] + 1)$  又可以变成  $u$  的其他儿子的乘积:  $u$  除  $v$  外的其他儿子记  $brother$ 。
39  $(f[brother\_1] + 1) * (f[brother\_2] + 1) * \dots$  他的所有兄弟的值乘积。
40 */
41
42 void dp(int u, int fa) {
43     for (int v:G[u]) {
44         if(v == fa) continue;
45         ll inv = qpow(f[v] + 1, mod - 2);
46         if(inv) ans[v] = (ans[u] * inv % mod + 1) % mod * f[v] % mod;
47         else {
48             ll t = 1;
49             for (auto other:G[u]) {
50                 if(other == v || other == fa) continue;
51                 t = t * (f[other] + 1) % mod;
52             }
53             ans[v] = (t + 1) * f[v] % mod;
54         }
55         dp(v, u);
56     }
57 }
58
59 int main() {
60     cin >> n;
61     for (int i = 1; i < n; i++) {

```

```

62         int u, v; cin >> u >> v;
63         G[u].pb(v), G[v].pb(u);
64     }
65     dfs(1, 0);
66     ans[1] = f[1];
67     dp(1, 0);
68
69     for (int i = 1; i <= n; i++) cout << ans[i] << endl;
70     return 0;
71 }

```

5 字符串

5.1 KMP

```

1  //poj2406
2  #include <bits/stdc++.h>
3  using namespace std;
4  const int N = 1e6 + 10;
5  char s[N];
6  int nxt[N], n;
7  /*
8  //区间 l->r 的 kmp
9      nxt[l] = 0;
10     for (int i = l + 1; i <= r; i++) {
11         int j = nxt[i - 1];
12         while(j && s[i] != s[l + j]) j = nxt[l + j - 1];
13         if(s[i] == s[l + j]) j++;
14         nxt[i] = j;
15     }
16 */
17 void get_nxt() {
18     nxt[1] = 0;
19     for (int i = 2, j = 0; i <= n; i++) {
20         while(j && s[i] != s[j + 1]) j = nxt[j];
21         if(s[i] == s[j + 1]) j++;
22         nxt[i] = j;
23     }
24 }
25
26 int main() {
27     while(~scanf("%s", s + 1)) {
28         if(s[1] == '.') break;
29         n = strlen(s + 1);
30         get_nxt();
31         int period = n - nxt[n];
32         if(n % period == 0) printf("%d\n", n / period);
33         else puts("1");
34     }

```

```

35     return 0;
36 }

```

5.2 字符串 Hash

```

1  using ull = unsigned long long;
2  const int N = 1e5 + 10;
3  char s[N];
4  int n;
5
6  namespace Hash
7  { //字符串 s 定义在全局, 且下标从 1 开始
8      ull h[N], p[N], ht[N];
9      const int base = 131;
10
11     void build() {
12         p[0] = 1; //注意 n 是 s 的长度
13         for (int i = 1; i <= n; i++) {
14             p[i] = p[i - 1] * base;
15             h[i] = h[i - 1] * base + s[i] - '0';
16         }
17         // 对于另一个字符串 t, 需要 char t[N], ht[N], m=|t|
18         // for (int i = 1; i <= m; i++)
19         //     ht[i] = ht[i - 1] * base + t[i] - '0';
20     }
21     ull get(int l, int r) {
22         // if(r < l || l > n) return 0; //根据题目需要处理边界情况
23         return h[r] - h[l - 1] * p[r - l + 1]; }
24 };

```

5.3 Trie

```

1  #include <bits/stdc++.h>
2  using namespace std;
3
4  const int N = 1e5 + 10;
5  char str[N];
6  int son[N][26], cnt[N], idx;
7
8  void insert(char *str) {
9      int p = 0;
10     for (int i = 0; str[i]; i++) {
11         int u = str[i] - 'a';
12         if(!son[p][u]) son[p][u] = ++idx;
13         p = son[p][u];
14     }
15     ++cnt[p];
16 }
17

```

```

18 int query(char *str) {
19     int p = 0;
20     for (int i = 0; str[i]; ++i) {
21         int u = str[i] - 'a';
22         if(!son[p][u]) return 0;
23         p = son[p][u];
24     }
25     return cnt[p];
26 }

```

5.4 AC 自动机

```

1 //Luogu3808
2 #include <bits/stdc++.h>
3 using namespace std;
4
5 const int N = 1e6 + 10;
6 int n;
7 char s[N];
8
9 namespace ac
10 {
11
12     int tr[N][26], fail[N], idx;
13     queue<int> q;
14     int cnt[N];
15
16     void insert(char* s) {
17         int p = 0;
18         for (int i = 1; s[i]; ++i) {
19             int u = s[i] - 'a';
20             if(!tr[p][u]) tr[p][u] = ++idx;
21             p = tr[p][u];
22         }
23         ++cnt[p];
24     }
25
26     void build() {
27         for (int i = 0; i < 26; ++i)
28             if(tr[0][i]) q.push(tr[0][i]);
29
30         while(q.size()) {
31             int u = q.front(); q.pop();
32             for (int i = 0; i < 26; ++i) {
33                 if(tr[u][i])
34                     fail[tr[u][i]] = tr[fail[u]][i], q.push(tr[u][i]);
35                 ↪ //原本这个 tr[fail[u]][i] 可能不存在 (为 0)

```

↪ // 但是下一步 else 做了一个优化 (类似

```

36         else
37             tr[u][i] = tr[fail[u]][i];
38     }
39 }
40 }
41
42 int query(char *s) {
43     int u = 0, res = 0;
44     for (int i = 1; s[i]; ++i) {
45         u = tr[u][s[i] - 'a'];
46         for (int j = u; j && cnt[j] != -1; j = fail[j])
47             res += cnt[j], cnt[j] = -1;
48     }
49     return res;
50 }
51
52 }
53
54 int main() {
55     scanf("%d", &n);
56     for (int i = 1; i <= n; i++) {
57         scanf("%s", s + 1);
58         ac::insert(s);
59     }
60     ac::build();
61     scanf("%s", s + 1);
62     printf("%d\n", ac::query(s));
63     return 0;
64 }

```

5.5 SA

5.6 Manacher

```

1  #include <bits/stdc++.h>
2  using namespace std;
3
4  const int N = 22000010;
5  char s[N], a[N];
6  int p[N], n;
7
8  void init() {
9     int k = 0;
10    s[k++] = '$', s[k++] = '#';
11    for (int i = 0; i < n; i++) s[k++] = a[i], s[k++] = '#';
12    s[k++] = '^';
13    n = k;
14 }
15
16 void manacher() {

```

```

17     int mr = 0, mid;
18     for (int i = 1; i < n; i++) {
19         if(i < mr) p[i] = min(p[mid * 2 - i], mr - i);
20         else p[i] = 1;
21         while(s[i - p[i]] == s[i + p[i]]) p[i]++;
22         if(i + p[i] > mr) {
23             mr = i + p[i];
24             mid = i;
25         }
26     }
27 }
28
29 int main() {
30     scanf("%s", a);
31     n = strlen(a);
32     init();
33     manacher();
34     int res = 0;
35     for (int i = 0; i < n; i++) res = max(res, p[i]);
36     cout << res - 1;
37     return 0;
38 }

```

6 其他

6.1 glibc 内置函数

```

1 // Returns the number of 1-bits in x.
2 int __builtin_popcount(unsigned int x);
3
4 // Returns the number of trailing 0 (undefined when x == 0)
5 int __builtin_ctz(unsigned int x);
6
7 // Returns log_2(x)
8 int __lg(int x);
9
10 int __gcd(int x, int y);

```

6.2 __int128 读写

```

1 inline __int128 read(){
2     __int128 x = 0, f = 1;
3     char ch = getchar();
4     while (ch < '0' || ch > '9') { if(ch == '-') f = -1; ch = getchar(); }
5     while (ch >= '0' && ch <= '9') { x = x * 10 + ch - '0'; ch = getchar(); }
6     return x * f;
7 }
8
9 inline void print(__int128 x) {

```

```

10     if(x < 0) { putchar('-'); x = -x; }
11     if(x > 9) print(x / 10);
12     putchar(x % 10 + '0');
13 }

```

6.3 单调栈

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  const int N = 100010;
4  //单调栈，记录每个数左边比他小（大）的第一个数（也可以记录其下标）
5  int stk[N], tt, a[N];
6
7  int main() {
8      ios::sync_with_stdio(false), cin.tie(0), cout.tie(0);
9      int n; cin >> n;
10     for (int i = 1; i <= n; i++) cin >> a[i];
11
12     for (int i = 1; i <= n; i++) {
13         while(tt && stk[tt] >= a[i]) tt--;
14         if(tt) cout << stk[tt] << ' ';
15         else cout << -1 << ' ';
16         stk[++tt] = a[i];
17     }
18     return 0;
19 }

```

6.4 单调队列

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  const int N = 1e6 + 10;
4  int a[N], q[N],n, k;
5  //滑动窗口
6  int main() {
7      cin >> n >> k;
8      for(int i = 0; i < n; i++) cin >> a[i];
9      int hh = 0, tt = -1;
10     for(int i = 0; i < n; i++) {
11         //判断队头是否已经划出窗口
12         if( hh <= tt && i - k + 1 > q[hh]) hh++;
13         while(hh <= tt && /* 后面改成要维护的最小值 */a[q[tt]] >= a[i]) tt -- ;//求区间最小
14         q[ ++ tt ] = i;
15         if(i >= k-1) printf("%d ",a[q[hh]]);
16
17     }
18     return 0;
19 }

```