(10+10)

1. Number of gas stations on an interstate highway follows a Poisson process with λ = 4 per 100 miles.
   a. What is the probability that you will see 3 gas stations in the next 50 miles?
   b. In the last 40 miles 3 gas stations passed by. What is the probability that you have to drive 30 miles or less for the next gas station?

a)

$$\lambda = \frac{4}{100} \text{ miles} = 0.04$$

$$P(X=3) = \frac{(0.04 \cdot 50)^3}{3!} e^{-(0.04 \cdot 50)}$$

$$= 0.18$$

b) memoryless Process

$$P(x \leq 30) = 1 - e^{-\lambda x}$$

$$P(x \leq 30) = 1 - e^{-(0.04 \cdot 30)}$$

$$P(x \leq) = 0.699$$

(10+10)

2. The lifetime of a sensor can be modeled as a Weibull distribution with lambda 0.26 per year. If the probability that the sensor will last less than 2 years is 0.7,
   a. Find the value of the parameter a?
   b. What is the probability that among ten such sensors at least 2 will last longer than 2 years?

a)

$$e^{-(0.26 \cdot 2)^a} = 0.7$$

$$-(0.26 \cdot 2)^a = \ln(0.7)$$

$$a = 1.58$$

b)

$$P(X \geq 2 \,;\, n=10,\, p=0.7) = 1 - P(X < 3)$$

$$1 - \left(\binom{10}{2} 0.7^2 (0.3)^{10-2} + \binom{10}{1} 0.7^1 (0.3)^{10-1} + \binom{10}{0} 0.7^0 (0.3)^{10-0} \right)$$

$$= 0.998$$

(10)

3. Lifetime of a cell phone part in years follows the lognormal distribution, with a log (to the base 10) adjusted mean of 0.9 and log adjusted variance of 0.4. What is the time to which 60% of the cell phone parts will last?

$$\mu_{\log} = 0.9 \quad \sigma_{\log}^2 = 0.4$$

$$P(X < x) = 0.6$$

$$P(\log(X) < \log(x)) = 0.6$$

$$P\left(\frac{\log(x) - \mu_{\log}}{\sigma_{\log}} < \frac{\log(x) - 0.9}{\sqrt{0.4}} \right)$$

$$Z_{0.6} = \frac{\log(x) - 0.9}{\sqrt{0.4}}$$

$$0.26 = \frac{\log(x) - 0.9}{\sqrt{0.4}}$$

$$x = 11.6$$

(10)

4. X is a random variable with uniform distribution values ranging between 2 and 5. If this student randomly selects 80 numbers from this distribution, what is the probability that the average of these 80 numbers is less than 3.9?

$n = 80$

$\mu = \frac{2+5}{2} = 4.5$

$\sigma^2 = \frac{(5-2)^2}{12} = \frac{3}{4}$ $\qquad \frac{\sigma^2}{n} = 0.0094$

$P(\bar{X} < 3.9)$

$P\left(Z < \frac{3.9 - 4.5}{\sqrt{0.0094}}\right) = -6.18$

$P(Z < -6.18) = 0$

(10+10+10)

5. The probability that a random student primarily uses the google chrome browser is 0.40. The probability that a random student uses internet explorer is 0.35 and that he does not use either of these is 0.25.

a. Among 6 students, what is the probability that 2 students use google chrome and at least 4 use internet explorer as their browsers?

b. What is the probability that I need to ask 4 students to find at least three students who use google chrome as their primary browser?

c. If we randomly survey 150 students, what is the probability at least 70 of them use google chrome as their primary browser?

a)

$P(X_1 = 2, X_2 = 4 ;\ P_1 = 0.4, P_2 = 0.35)$

$= \frac{6!}{2!4!} 0.4^2 0.35^4 = 0.036$

b)

$$\sum_{n=0}^{x=3} \binom{n+4-1}{4-1} (0.4)^4 (1-0.4)^n = 0.29$$

c)

$P(X \geq 70;\ n=150;\ p=0.4)$

$\mu = np = 150 \cdot 0.4 = 60$

$\sigma^2 = 60(0.6) = 36$

$= 1 - P(X < 69.5)$

$= 1 - P\left(Z < \frac{69.5-60}{\sqrt{36}}\right)$

$= 1 - P(Z < 1.58)$

$= 1 - 0.9429 = 0.057$

(10)

6. In a black box, there are 5 red balls, 8 green balls, 7 black balls. You'll win in a bet if out of 4 trials you pick at least one red ball. If you pick without replacement, what is the probability that you will win the bet?

$P(X \geq 1;\ n=20;\ P=0.25)$

$P(X \geq 1) = 1 - P(X < 2)$

$1 - \left(\binom{20}{1} 0.25^1 (1-0.25)^{20-1} + \binom{20}{0} 0.25^0 (1-0.25)^{20-0}\right)$

$= 0.97$