

But they do get pretty big...



There Is No Largest Prime Number



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Prime Numbers

There Is No Largest Prime Number

The proof uses *reductio ad absurdum*.

Theorem

There is no largest prime number.

Proof.

1. Suppose p were the largest prime number.

2. Consider $q = p + 1$.
Since p is the largest prime, q is either prime or composite.
If q is prime, then we have found a larger prime than p , a contradiction.
If q is composite, then it is divisible by some prime number r .
Since r is a divisor of q , $r \neq p$.
But $q + 1$ is greater than 1, thus divisible by some prime number not in the first p numbers.



There Is No Largest Prime Number

The proof uses *reductio ad absurdum*.

Theorem

There is no largest prime number.

Proof.

1. Suppose p were the largest prime number.
2. Let q be the product of the first p numbers.
3. Then $q + 1$ is greater than 1.
4. But $q + 1$ is greater than 1, thus divisible by some prime number not in the first p numbers. □

There Is No Largest Prime Number

The proof uses *reductio ad absurdum*.

Theorem

There is no largest prime number.

Proof.

1. Suppose p were the largest prime number.
2. Let q be the product of the first p numbers.
3. Then $q + 1$ is not divisible by any of them.
4. But $q + 1$ is greater than 1, thus divisible by some prime number not in the first p numbers.



The Proof

Implications Of Our Theorem

Counting Primes

No matter which prime you have, you can find a bigger one.

How We Count Primes

3 is prime, but so is 7 and 7 is bigger than 3.

Warning!

Thinking you have the biggest prime can lead to poor results:

- ▶ Embarrassment
- ▶ Wrong results
- ▶ Broken encryption

Next steps

- ▶ Look for some big prime numbers.
- ▶ Then look for a bigger one.