MAT 137
Tutorial #10– Antiderivatives
January 16–17, 2017

Do not confuse:

- $\int_a^b f(x)dx$ is a number and represents area.
- $\int f(x)dx$ represents the collection of *all* functions whose derivative is $f(x)$.

1. Let's warm up with some easier antiderivatives.

$$\int x^3\,dx \qquad \int \sqrt{x}\,dx \qquad \int \frac{1}{x^3}\,dx \qquad \int \left(x^3 - 2x^2 + 7x - 5\right)\,dx$$
$$\int \sin x\,dx \qquad \int \cos x\,dx \qquad \int e^x\,dx \qquad \int \sec^2 x\,dx$$

2. The poor integral $\int \frac{1}{x}\,dx$ is often misunderstood. Let's get to know her a bit better.

   (a) Calculate the domain and the derivative of $F_1(x) = \ln x$.

   (b) Calculate the domain and the derivative of $F_2(x) = \ln(-x)$.

   (c) Calculate the domain and the derivative of $F_3(x) = \ln|x|$.

   (d) In view of the above, what is $\int \frac{1}{x}\,dx$?

   (e) Calculate the derivative of $F_4(x) = \ln(2x)$. Do we have a problem?

3. Next, try some harder antiderivatives. Remember: the key is often to make an educated guess, try it, and then take it from there.

$$\int (3x+7)^{10}\,dx \qquad \int 3\sin(2x)\,dx \qquad \int 5e^{-2x}\,dx$$
$$\int \frac{2}{(7-6x)^4}\,dx \qquad \int \frac{x^3+2x^2}{x}\,dx \qquad \int \sqrt{x}\,(x+1)\,dx$$
$$\int \frac{2}{3x-1}\,dx \qquad \int \frac{1}{\sqrt[3]{5-2x}}\,dx \qquad \int \tan^2 x\,dx$$

Hint: For the last one, think of the trig identity involving tangents and secants.

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Harder question

4. These antiderivatives are more difficult. We will learn later a more systematic way to approach them, but it is a very good exercise to try to figure them out now without knowing any tricks or formulas, just by (sophisticated) guess and check. You will solidify your understanding in the process.

$$\int x\left(3x^2+1\right)^{100}\,dx \qquad \int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx \qquad \int xe^{-x^2}\,dx$$

$$\int \frac{(\ln x)^3}{x}\,dx \qquad \int \frac{\sin\ln x}{x}\,dx \qquad \int \frac{1}{x\ln x}\,dx$$

Hint: For the first one, study the derivative of a function of the form $F(x) = (3x^2+1)^n$ for some appropriate value of n.
For the second one, what is the derivative of $F(x) = \sin\sqrt{x}$?