MAT 137
Tutorial #17– Infinite series
March 13–14, 2017

1. **Geometric series.** You have learned that

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \text{ if } |x| < 1$$

and the series is divergent if $|x| \geq 1$. Calculate the following infinite sums:

(a) $\displaystyle\sum_{n=0}^{\infty} (\ln 2)^n$ (b) $\displaystyle\sum_{n=0}^{\infty} (\ln 3)^n$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{e^{2n+3}}$ (d) $\displaystyle\sum_{n=m}^{\infty} x^n$

(e) $\displaystyle\frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^4} + \frac{1}{5^5} + \frac{1}{5^7} + \frac{1}{5^8} + \frac{1}{5^{10}} + \frac{1}{5^{11}} + \dots$

(f) $\displaystyle\frac{1}{2^{0.5}} + \frac{1}{2} + \frac{1}{2^{1.5}} - \frac{1}{2^2} + \frac{1}{2^{2.5}} + \frac{1}{2^3} + \frac{1}{2^{3.5}} - \frac{1}{2^4} + \frac{1}{2^{4.5}} + \frac{1}{2^5} + \frac{1}{2^{5.5}} - \frac{1}{2^6} + \dots$

2. **Telescopic series.** Calculate the value of the following infinite sums. In all cases, you can start by finding a formula for the N-th partial sum, and then taking the limit. For the last two, think of partial-fraction decomposition.

(a) $\displaystyle\sum_{n=0}^{\infty} [\arctan n - \arctan(n+1)]$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+3n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \left[\ln \frac{n}{n+1}\right]$ (d) $\displaystyle\sum_{n=3}^{\infty} \frac{n+2}{n^3-n}$

Hint: For Question 2c, write $\displaystyle\frac{1}{n^2+3n} = \frac{A}{n} + \frac{B}{n+3}$. Do something similar for Question 2d.

3. **Infinite decimal expansions.** We can interpret any finite decimal expansion as a finite sum. For example:

$$2.13096 = 2 + \frac{1}{10} + \frac{3}{10^2} + \frac{0}{10^3} + \frac{9}{10^4} + \frac{6}{10^5}$$

Similarly, we can interpret any infinite decimal expansion as an infinite series.

Interpret the following numbers as series, and add up the series to calculate their value as fractions:

(a) $0.99999\dots$ (b) $0.11111\dots$ (c) $0.252525\dots$ (d) $0.3121212\dots$