MAT 137
Tutorial #2– The definition of limit
October 3–4, 2016

Let f be a function. Let $a, L \in \mathbb{R}$. Assume that f is defined on some open interval around a, except maybe at a. As you know, the definition of the statement $\lim_{x \to a} f(x) = L$ is

For every $\varepsilon > 0$, there exists $\delta > 0$ such that $\quad 0 < |x-a| < \delta \implies |f(x) - L| < \varepsilon$.

Below is a list of eight other statements.

a. For every $\varepsilon > 0$, there exists $\delta > 0$ such that $\quad |x-a| < \delta \implies |f(x) - L| < \varepsilon$.

b. For every $\varepsilon > 0$, there exists $\delta > 0$ such that $\quad 0 < |x-a| < \delta \implies 0 < |f(x) - L| < \varepsilon$.

c. For every $\varepsilon \geq 0$, there exists $\delta > 0$ such that $\quad 0 < |x-a| < \delta \implies |f(x) - L| < \varepsilon$.

d. For every $\varepsilon > 0$, there exists $\delta \geq 0$ such that $\quad 0 < |x-a| < \delta \implies |f(x) - L| < \varepsilon$.

e. For every $\varepsilon > 0$, there exists $\delta > 0$ such that $\quad 0 < |x-a| < \delta \implies |f(x) - L| \leq \varepsilon$.

f. For every $\delta > 0$, there exists $\varepsilon > 0$ such that $\quad 0 < |x-a| < \delta \implies |f(x) - L| < \varepsilon$.

g. For every $\delta > 0$, there exists $\varepsilon > 0$ such that $\quad 0 < |x-a| < \varepsilon \implies |f(x) - L| < \delta$.

h. There exists $\delta > 0$ such that for every $\varepsilon > 0$, $\quad 0 < |x-a| < \delta \implies |f(x) - L| < \varepsilon$.

Match each of the eight statements above to one of the following (there may be repeats):

1. Every function satisfies this statement.
2. There isn't any function which satisfies this statement.
3. This statement is equivalent to the definition of limit.
4. This statement means that $\lim_{x \to a} f(x) = L$ and that, in addition, $f(a) = L$.
5. This statement means that $\lim_{x \to a} f(x) = L$ and that, in addition, f does not take the value L anywhere on some interval centered at a, except maybe at a.
6. This statement is equivalent to saying that f must be constantly equal to L on an interval centered at a, except maybe at a.
7. This statement means that f is bounded on every interval centered at a.