MAT 137
Tutorial #12– Volumes
January 30–31, 2017

1. Consider the region bounded by the curve $y = 2x - x^2$ and the x-axis. We rotate the region around the x-axis. Calculate the volume of the solid of revolution we obtain this way.

   Answer: $16\pi/15$.

2. Let R be the region bounded by the curves $y = x^2 + 4$ and $y = 6x - x^2$. Calculate the volume of the solid of revolution obtained when we rotate the region R...

   (a) around the x-axis,

   (b) around the y-axis.

   Answers: $13\pi/3$, π.

3. Consider the region in the first quadrant bounded between the x-axis, the y-axis, and the curve $\sqrt{x} + \sqrt{y} = 1$. Calculate the volume of the solid of revolution obtained by rotating this region around the line $x = 3$. Do this in two different ways:

   (a) Do the calculation by integrating with respect to x.

   (b) Do the calculation by integrating with respect to y

   Answer: $14\pi/15$.

Harder questions

Do not try these problems unless you have finished the previous ones.

4. A hemispherical salad bowl has radius R. We place an iron ball with radius r inside the bowl. Assume that $2r \leq R$. Then we pour water into the bowl until the height of the water is h.

   - Assume $0 \leq h \leq 2r$. Calculate the total volume of water.
   - Assume $2r \leq h \leq R$. Calculate the total volume of water.

   Suggestions: You can check your solution! Your two answers should agree when $h = 2r$, and you know what the answer should be when $h = R$.

5. Two cylinders have the same radius r and their axes meet at a right angle. Find the volume of their intersection.

   Answer: $16r^3/3$.