MAT 137
Tutorial #7– L'Hôpital's Rule
November 21–22, 2016

1. Compute the following limits:

   (a) $\lim_{t\to 0} \dfrac{\tan(3t)}{\ln(1+2t)}$

   (b) $\lim_{h\to 2} \dfrac{h^3-5h^2+3h+6}{h^3-h^2-3h+2}$

   (c) $\lim_{t\to 0} \dfrac{1-\cos(3t)}{t\ln(1+t)}$

   (d) $\lim_{x\to\infty} \left(x^7-5x^3+2\right)e^{-x}$

   (e) $\lim_{x\to 1} \dfrac{(x-1)\sin x}{e^x\cos x}$

   (f) $\lim_{x\to 0} \dfrac{\sqrt{x+1}-1}{x}$

   (g) $\lim_{x\to\infty} x\tan\dfrac{3}{x}$

   (h) $\lim_{x\to 0} \left(\dfrac{1}{x\sin x}-\dfrac{1}{x\tan x}\right)$

   (i) $\lim_{x\to\infty} \dfrac{\sqrt{x^4+5x^3}-x^2}{x}$

   (j) $\lim_{x\to\infty} \dfrac{\sin x}{x}$

2. Compute the following limits:

   (a) $\lim_{x\to 0} (\cos x)^{1/x^2}$

   (b) $\lim_{x\to\infty} x^{\frac{\ln 2}{1+\ln x}}$

   (c) $\lim_{x\to 0^+} x^{\sqrt{x}}$

   (d) $\lim_{x\to 1} (2-x)^{\tan(\pi x/2)}$

A historical question

3. The first appearance in print of L'Hôpital's Rule was in the book *Analyse des Infiniment Petits* published by the Marquis de l'Hôpital in 1696. This was the first calculus *textbook* ever published and the example that the Marquis used in that book to illustrate his rule was to find the limit of the function

$$y = \frac{\sqrt{2a^3x-x^4}-a\sqrt[3]{aax}}{a-\sqrt[4]{ax^3}}$$

   as x approaches a, where $a>0$ is a constant. (At that time it was common to write aa instead of a^2.) Solve this problem.[1]

[1] Problem taken from *Stewart, James: Calculus Early Trascendental*, 7th Ed.