MAT 137
Tutorial #3– Limit computations
October 17–18, 2016

Note: L'Hôpitals' Rule is neither required, nor recommended, for any of these limits.

1. Calculate the following limits:

   (a) $\lim\limits_{x\to 3} \dfrac{x^2-9}{x-3}$

   (b) $\lim\limits_{x\to 1} \left(x^2+2^x\right)$

   (c) $\lim\limits_{h\to 2} \dfrac{h^3-5h^2+3h+6}{h^3-h^2-3h+2}$

   (d) $\lim\limits_{x\to 3} \dfrac{2-x}{x-3}$

   (e) $\lim\limits_{x\to 3} \dfrac{2-x}{(x-3)^2}$

   (f) $\lim\limits_{x\to 0} \dfrac{\sqrt{x+1}-1}{x}$

   (g) $\lim\limits_{x\to 1} \dfrac{\sin x}{x}$

   (h) $\lim\limits_{t\to 0} \dfrac{t}{\sin(2t)}$

   (i) $\lim\limits_{x\to 0} \dfrac{\sin(2x)}{\sin(3x)}$

   (j) $\lim\limits_{z\to 0} \dfrac{\sin(2z^2)}{\cos(3z)\ \sin^2(5z)}$

   (k) $\lim\limits_{x\to 3} \dfrac{\tan(x-3)}{2x-6}$

   (l) $\lim\limits_{x\to 0} \dfrac{2e^x}{\sin(2e^x)}$

   (m) $\lim\limits_{t\to 0} \dfrac{1-\cos(3t)}{t^2}$

   (n) $\lim\limits_{y\to 1} \dfrac{\sqrt{y+4}-\sqrt{4y+1}}{\sqrt{y}-1}$

2. Calculate the following limits:

   (a) $\lim\limits_{x\to 0} \dfrac{\sin\left(1-\cos x\right)}{x\tan(\pi x)}$

   (b) $\lim\limits_{u\to 2} \dfrac{1}{2-u}\left(\sqrt{\dfrac{u+2}{u-1}}-2\right)$

   (c) $\lim\limits_{x\to 0^+} e^{1/x}$

   (d) $\lim\limits_{x\to 0^-} e^{1/x}$

3. *[Challenge question]*

   Construct a function f with domain $\mathbb{R}$ such that $\lim\limits_{x\to a} f(x)$ exists at exactly two values of a, and it does not exist for any other value.