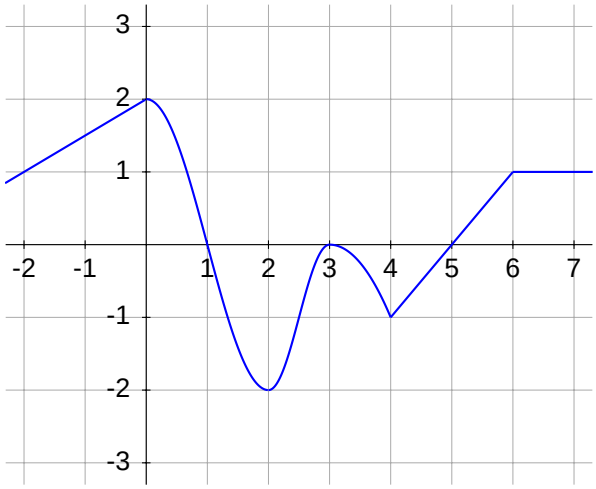
MAT 137Y: Calculus!
Problem Set C

This problem set contains a few extra problems to help you prepare for Test #3. This is not comprehensive: it only contains problems from some sections that were not included in past problem sets or in past tutorials. You do not need to turn in any of these problems.

1. Below is the graph of the function f:



We define a new function by $F(x) = \int_0^x f(t)dt$. Answer the following questions about F and justify your answers.

   (a) Is $F(6)$ positive or negative?

   (b) At which point or points does F have a local minimum?

   (c) Where is the function F concave up?

   (d) On the domain $[-1, 5]$, at which point or points does F have an inflection point?

   (e) On the domain $[-1, 5]$, at which point or points does F have an global minimum?

   (f) Find two values of x such that $F(x) = 0$.

2. Calculate the following:

   (a) $\int_2^7 \sqrt{2+x}\, dx$, (b) $\int_0^2 e^{3t}\, dt$, (c) $\int_{-1}^2 (tx^2 + 2)\, dx$.

3. Read Theorems 4.2.3 and 4.2.4 on section 4.2 of the book. Prove that there is a constant C such that

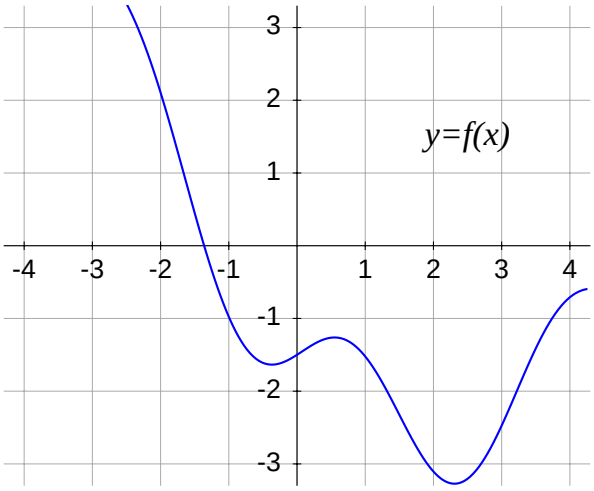
$$\arcsin \frac{1-x}{1+x} + 2 \arctan \sqrt{x} = C$$

for all x in a certain domain. What is the largest domain on which this identity is true? What is the value of the constant C?

4. Let g be a function that has all of its derivatives everywhere. We define a new function G by $G(x) = \displaystyle\int_0^x g(t)\, dt$. Calculate:

$$\lim_{x \to 0} \frac{\int_0^{G(x)} \arctan(s + 2s^2)\, ds}{x^2}$$

5. Here is the graph of the function f:



We define a new function g by the equation

$$g(x) = \int_1^x \left[f(t)^2 - t^2 \right] dt$$

At which point or points does the function g have a local maximum or a local minimum?

Hint: The solution includes three points.

6. Let f, u, and v be functions which are continuous and differentiable everywhere. We define a new function F by

$$F(x) = \int_{v(x)}^{u(x)} f(t)\, dt$$

Find a formula for $F'(x)$ in terms of f, u, v, and their derivatives.