MAT 137
Tutorial #16– Improper Integrals
March 6–7, 2017

1. Determine whether the following integrals are convergent or divergent directly from the definition of improper integral. If they are convergent, calculate their value.

   (a) $\displaystyle\int_{-1}^{\infty} \frac{1}{x^2+1}\,dx$

   (b) $\displaystyle\int_{2}^{\infty} \frac{1}{x+2}\,dx$

   (c) $\displaystyle\int_{0}^{1} \ln x\,dx$

   (d) $\displaystyle\int_{1}^{\infty} \ln x\,dx$

   (e) $\displaystyle\int_{0}^{\infty} \cos x\,dx$

   (f) $\displaystyle\int_{0}^{1} \frac{dx}{x^2}$

   (g) $\displaystyle\int_{0}^{1} \frac{dx}{\sqrt{x}}$

   (h) $\displaystyle\int_{2}^{\infty} \frac{1}{x^2-1}\,dx$

   Hint: For Question 1h, write $\dfrac{1}{x^2-1} = \dfrac{A}{x-1} + \dfrac{B}{x+1}$.

2. For which values of the constant p is the integral $\displaystyle\int_{1}^{\infty} \frac{1}{x^p}\,dx$ convergent?

3. Using the Basic Comparison Test and/or the Limit Comparison Test, determine which ones of the following improper integrals are convergent or divergent. Do not calculate their value.

   (a) $\displaystyle\int_{1}^{\infty} \frac{\sin x + 2\cos x + 10}{x^2}\,dx$

   (b) $\displaystyle\int_{0}^{\infty} \frac{x-7}{x^2+x+5}\,dx$

   (c) $\displaystyle\int_{10}^{\infty} \frac{\sqrt{x-6}}{3x^2+5x+11}$

   (d) $\displaystyle\int_{0}^{\infty} \frac{\arctan x}{x^{1.1}}\,dx$

   (e) $\displaystyle\int_{0}^{1} \frac{\sin x}{x^{4/3}}\,dx$

   (f) $\displaystyle\int_{0}^{\infty} e^{-x^2}\,dx$