MAT 137Y: Calculus!
Problem Set D

This problem set contains a few extra problems to help you prepare for Test #4. This is not comprehensive: it only contains problems from some sections that were not included in past problem sets or in past tutorials. You do not need to turn in any of these problems.

1. Find two series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ such that:
   - $\sum_{n=1}^{\infty} a_n$ is divergent,
   - $\sum_{n=1}^{\infty} b_n$ is divergent, and
   - $\sum_{n=1}^{\infty} (a_n + b_n)$ is convergent.

2. Find a sequence $\{b_n\}_{n=1}^{\infty}$ such that:
   - $b_n > 0$ for all $n \geq 1$,
   - $\lim_{n\to\infty} b_n = 0$, and
   - the series $\sum_{n=1}^{\infty} (-1)^{n+1} b_n$ is divergent.

3. Estimate the value of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ with an error less than 0.01.

4. (a) Write the formal definition of the following properties of a sequence:
      i. A sequence is divergent to $-\infty$.
      ii. A sequence is not bounded below.

   (b) Prove the following theorem:

      Theorem: Let $\{a_n\}_{n=1}^{\infty}$ be a sequence.
      - IF the sequence $\{a_n\}_{n=1}^{\infty}$ is decreasing and *not* bounded below,
      - THEN the sequence $\{a_n\}_{n=1}^{\infty}$ is divergent to $-\infty$.

   Do a formal proof directly from the definitions.