hw02

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5.3

$$\hat{\theta} = \int_0^{0.5} e^{-x} dx = 0.5 E e^{-U(0,0.5)}$$

$$\theta^* = \int_0^{0.5} e^{-x} dx = F(x <= 0.5)$$

其中 F 为 $exp(1)$ 的 cdf

```
int5.3_1 <- function(N=1e4){
  x <- runif(N, 0, 0.5)
  y <- runif(N, 0, 2)
  return(sum(as_vector(map2(x, y, ~ .y < exp(-.x)))) / N)
}

int5.3_2 <- function(N=1e4){
  x <- runif(N, 0, 0.5)
  return(mean(exp(-x))/2)
}

int5.3_3 <- function(N=1e4){
  x <- rexp(N)
  return(sum(as_vector(x %>% map(~.<0.5))) / N)
```

```
}
int5.3_1()#theta_hat
```

```
## [1] 0.3979
```

```
int5.3_2()#theta_hat
```

```
## [1] 0.39394
```

```
int5.3_3()#theta*
```

```
## [1] 0.391
```

$\theta^* > \hat{\theta}$

5.6

$$Cov(e^U, e^{1-U}) = Ee^1 - (Ee^U)^2 = e - (e-1)^2 = -e^2 + 3e - 1$$

$$Var(e^U + e^{1-U}) = Var(e^U) + Var(e^{1-U}) + 2Cov(e^U, e^{1-U}) = -3e^2 + 10e - 5$$

$$Var\left(\frac{e^U + e^{1-U}}{2}\right) = \frac{-3e^2 + 10e - 5}{4}$$

$$Var(e^U) = \frac{-e^2 + 4e - 3}{2}$$

$$1 - \frac{Var\left(\frac{e^U + e^{1-U}}{2}\right)}{Var(e^U)} \approx 0.98$$

5.7

```r
N <- 1e4
int_5.7_anti <- function(N=1e4){
  U <- runif(N)
  ans <- (exp(U) + exp(1-U)) / 2
  return(list(mean=mean(ans), var=var(ans)))
}

int_5.7_mc <- function(N=1e4){
  U <- runif(N)
  ans <- exp(U)
  return(list(mean=mean(ans), var=var(ans)))
}



1 - int_5.7_anti(N)$var / int_5.7_mc(N)$var
```

```
## [1] 0.9840088
```

5.11

$$Var(\hat{\theta}_c)(c) = Var\left(c\hat{\theta}_1 + (1-c)\hat{\theta}_2\right) = c^2 Var(\hat{\theta}_1) + (1-c)^2\hat{\theta}_2 + 2Cov(\hat{\theta}_1, \hat{\theta}_2)$$

$$Var(\hat{\theta}_c)(c) \geqslant 2Cov(\hat{\theta}_1, \hat{\theta}_2) + \frac{Var(\hat{\theta}_1)Var(\hat{\theta}_2)}{Var(\hat{\theta}_1) + Var(\hat{\theta}_2)}$$

when

$$c = \frac{Var(\hat{\theta}_2)}{Var(\hat{\theta}_1) + Var(\hat{\theta}_2)}$$

5.13

$$f_0(x) = \int_1^{+\infty} \frac{x^2}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$

$$f_1(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-0.5)^2}{2}}$$

$$f_2(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-1)^2}{2}}$$

```
N <- 1e5

int_5.13_f <- function(N, mu=0){
  x <- rnorm(N, mu)
  y <- x^2 * exp((mu^2-2*mu*x)/2) * (x > 1)
  return(list(value=mean(y), var=var(y)))
}

int_5.13_f(N, 0) %>% str()#f0
```

```
## List of 2
##  $ value: num 0.395
##  $ var  : num 1.26
```

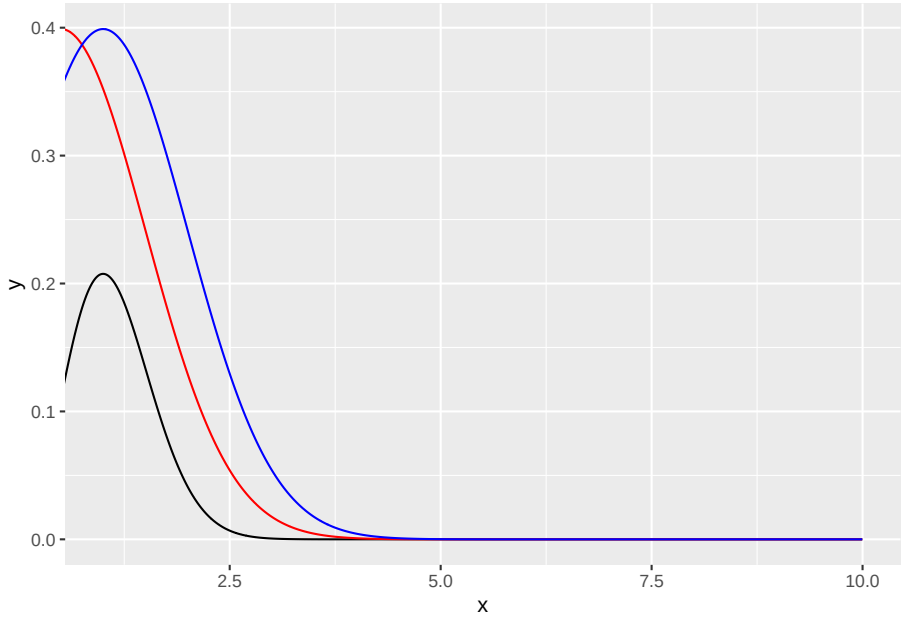
```
int_5.13_f(N, 0.5) %>% str()#f1
```

```
## List of 2
##  $ value: num 0.399
##  $ var  : num 0.416
```

```
int_5.13_f(N, 1) %>% str()#f2
```

```
## List of 2
##  $ value: num 0.402
##  $ var  : num 0.165
```

$Var(f_1) > Var(f_2)$, because $E(\frac{f_0^2}{f_1^2}) < E(\frac{f_0^2}{f_2^2})$



5.15

```
int_5.15 <- function(N, interval=1){
  mu <- numeric(interval)
  se <- numeric(interval)
  x <- runif(N %/% interval, 0,  1  %/% N)
  g <- function(x){exp(-x - log(1+x^2)) * (x > 0) * (x < 1)}
  for (i in 1:interval){
    x <- runif(N %/% interval, (i-1) * 1  / interval, i / interval)
    mu[i] <- mean(g(x))
    se[i] <- sd(g(x))
  }
  return(list(mu=mean(mu), se=sum(se)))
}

int_5.15(1e5, 1) %>% str()
```

```
## List of 2
##  $ mu: num 0.525
##  $ se: num 0.245
```

```
int_5.15(1e5, 1e2) %>% str()
```

```
## List of 2
##  $ mu: num 0.525
##  $ se: num 0.235
```