

Notebook - Maratona de Programação

Cabo HDMI, VGA, USB

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1 Math

1.1 Matrix exponentiation

```
Complexity: O(n*n*n log(b)) to raise an nxn matrix to the 9
  power of b.
                                                            11
  Computes powers of matrices efficiently.
1 // TITLE: Matrix exponentiation
2 // COMPLEXITY: O(n*n*n log(b)) to raise an nxn matrix
        to the power of b.
3 // DESCRIPTION: Computes powers of matrices
       efficiently.
5 struct Matrix {
      vector < vi > m;
      int r, c;
      Matrix(vector < vi> mat) {
          m = mat:
           r = mat.size();
                                                            23
12
           c = mat[0].size();
13
14
                                                            2.6
      Matrix(int row, int col, bool ident=false) {
                                                            27
         r = row; c = col;
16
                                                            28
           m = vector < vi > (r, vi(c, 0));
17
                                                            29
           if(ident) {
18
               for(int i = 0; i < min(r, c); i++) {</pre>
19
                   m[i][i] = 1;
20
                                                            31
21
                                                            32
           }
22
                                                            33
2.3
                                                            34
24
                                                            35
      Matrix operator*(const Matrix &o) const {
25
         assert(c == o.r); // garantir que da pra
26
       multiplicar
          vector<vi> res(r, vi(o.c, 0));
                                                            38
           for(int i = 0; i < r; i++) {
               for(int k = 0; k < c; k++) {
3.0
                    for(int j = 0; j < o.c; j++) {</pre>
31
                        res[i][j] = (res[i][j] + m[i][k]*\frac{\pi}{41}
32
      o.m[k][j]) % MOD;
                                                            42
33
                    }
               }
34
                                                            44
           }
35
36
                                                            46
           return Matrix(res);
37
                                                            47
      }
38
                                                            48
39 }:
                                                            49
40
                                                            50
41 Matrix fpow(Matrix b, int e, int n) {
                                                            5.1
       if(e == 0) return Matrix(n, n, true); //
                                                            52
       identidade
                                                            53
       Matrix res = fexp(b, e/2, n);
                                                            54
      res = (res * res);
44
                                                            55
      if(e%2) res = (res * b);
45
                                                            56
46
                                                            57
47
      return res;
                                                            5.8
                                                            5.9
                                                            60
                                                            61
  1.2
       Fast Fourier Transform
                                                            62
                                                            63
```

```
Complexity: O(n \log(n))
 Multiply polynomials quickly
1 // TITLE: Fast Fourier Transform
2 // COMPLEXITY: O(n log(n))
3 // DESCRIPTION: Multiply polynomials quickly
```

```
5 typedef double ld;
 6 typedef long long 11;
 8 struct num{
     ld x, y;
      num() { x = y = 0; }
      num(1d x, 1d y) : x(x), y(y) {}
12 }:
14 inline num operator+(num a, num b) { return num(a.x +
      b.x, a.y + b.y); }
inline num operator-(num a, num b) { return num(a.x -
       b.x, a.y - b.y); }
inline num operator*(num a, num b) { return num(a.x *
       b.x - a.y * b.y, a.x * b.y + a.y * b.x); }
inline num conj(num a) { return num(a.x, -a.y); }
19 int base = 1;
20 vector < num > roots = {{0, 0}, {1, 0}};
21 vector < int > rev = {0, 1};
22 const ld PI = acos(-1);
24 void ensure base(int nbase){
     if(nbase <= base)</pre>
         return;
      rev.resize(1 << nbase);
       for(int i = 0; i < (1 << nbase); i++)</pre>
          rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (
      nbase - 1));
       roots.resize(1 << nbase);</pre>
       while(base < nbase){</pre>
           ld angle = 2*PI / (1 << (base + 1));</pre>
           for(int i = 1 << (base - 1); i < (1 << base);
        i++){
               roots[i << 1] = roots[i];
               ld angle_i = angle * (2 * i + 1 - (1 <<
       base));
               roots[(i << 1) + 1] = num(cos(angle_i),
       sin(angle_i));
           base++;
43 }
45 void fft(vector < num > &a, int n = -1) {
     if(n == -1)
          n = a.size();
       assert((n & (n-1)) == 0);
       int zeros = __builtin_ctz(n);
       ensure_base(zeros);
       int shift = base - zeros;
       for(int i = 0; i < n; i++)</pre>
           if(i < (rev[i] >> shift))
               swap(a[i], a[rev[i] >> shift]);
       for(int k = 1; k < n; k <<= 1)
           for(int i = 0; i < n; i += 2 * k)
               for (int j = 0; j < k; j++) {
                   num z = a[i+j+k] * roots[j+k];
                   a[i+j+k] = a[i+j] - z;
                   a[i+j] = a[i+j] + z;
64 }
66 vector < num > fa, fb;
67 vector<ll> multiply(vector<ll> &a, vector<ll> &b){
     int need = a.size() + b.size() - 1;
68
      int nbase = 0;
69
7.0
      while((1 << nbase) < need) nbase++;</pre>
```

```
ensure_base(nbase);
                                                                       num b2 = (fb[i] - conj(fb[j])) * r4;
72
       int sz = 1 << nbase;</pre>
                                                            138
                                                                        if(i != j){
       if(sz > (int) fa.size())
                                                                            num c1 = (fa[j] + conj(fa[i]));
73
                                                                            num c2 = (fa[j] - conj(fa[i])) * r2;
7.4
           fa.resize(sz);
                                                            140
                                                                            num d1 = (fb[j] + conj(fb[i])) * r3;
       for(int i = 0; i < sz; i++){
                                                                            num d2 = (fb[j] - conj(fb[i])) * r4;
76
                                                            142
            int x = (i < (int) a.size() ? a[i] : 0);</pre>
                                                                            fa[i] = c1 * d1 + c2 * d2 * r5;
                                                            143
            int y = (i < (int) b.size() ? b[i] : 0);</pre>
                                                                            fb[i] = c1 * d2 + c2 * d1;
7.8
                                                            144
           fa[i] = num(x, y);
                                                            145
                                                                        fa[j] = a1 * b1 + a2 * b2 * r5;
80
       fft(fa, sz);
                                                                        fb[j] = a1 * b2 + a2 * b1;
81
                                                            147
       num r(0, -0.25 / sz);
       for(int i = 0; i <= (sz >> 1); i++){
                                                                   fft(fa, sz);
83
                                                            149
           int j = (sz - i) & (sz - 1);
                                                                   fft(fb, sz);
84
           num z = (fa[j] * fa[j] - conj(fa[i] * fa[i]))151
85
                                                                   vector<ll> res(need);
        * r;
                                                                   for(int i=0;i<need;i++){</pre>
            if(i != j) {
                                                                       ll aa = round(fa[i].x);
                                                                       11 bb = round(fb[i].x);
               fa[j] = (fa[i] * fa[i] - conj(fa[j] * fa[154
       j])) * r;
                                                                        ll cc = round(fa[i].y);
           }
                                                                       res[i] = (aa + ((bb % m) << 15) + ((cc % m)
88
           fa[i] = z;
                                                                   << 30)) % m;
89
       fft(fa. sz):
                                                                   return res;
91
                                                            158
       vector<ll> res(need);
                                                            159 }
       for(int i = 0; i < need; i++)</pre>
93
           res[i] = round(fa[i].x);
94
95
                                                                    Graph
96
       return res;
97 }
98
                                                                     Dfs tree
                                                               2.1
99
100 vector<ll> multiply_mod(vector<ll> &a, vector<ll> &b,
                                                               Complexity: O(E + V)
       int m, int eq = 0){
       int need = a.size() + b.size() - 1;
       int nbase = 0;
                                                             1 // TITLE: Dfs tree
       while((1 << nbase) < need) nbase++;</pre>
                                                             2 // COMPLEXITY: O(E + V)
       ensure base(nbase):
104
                                                             3 // DESCRIPION: Create dfs tree from graph
105
       int sz = 1 << nbase;</pre>
       if(sz > (int) fa.size())
106
                                                             5 int desce[mxN], sobe[mxN];
           fa.resize(sz);
                                                             6 int backedges[mxN], vis[mxN];
108
                                                             7 int pai[mxN], h[mxN];
       for(int i=0;i<(int)a.size();i++){</pre>
109
           int x = (a[i] % m + m) % m;
                                                             9 void dfs(int a, int p) {
            fa[i] = num(x & ((1 << 15) - 1), x >> 15);
                                                                   if(vis[a]) return;
                                                            10
112
                                                                   pai[a] = p;
       fill(fa.begin() + a.size(), fa.begin() + sz, num
113
                                                                   h[a] = h[p]+1;
       \{0, 0\}:
                                                                   vis[a] = 1;
                                                             13
       fft(fa, sz);
                                                            14
       if(sz > (int) fb.size())
                                                            15
                                                                   for(auto b : g[a]) {
           fb.resize(sz);
                                                                        if (p == b) continue;
       if(eq)
                                                                        if (vis[b]) continue;
           copy(fa.begin(), fa.begin() + sz, fb.begin())
118
                                                                        dfs(b, a);
                                                                        backedges[a] += backedges[b];
       elsef
119
                                                            20
           for(int i = 0; i < (int) b.size(); i++){</pre>
                                                                   for(auto b : g[a]) {
                                                            21
               int x = (b[i] % m + m) % m;
                                                                       if(h[b] > h[a]+1)
                fb[i] = num(x & ((1 << 15) - 1), x >> 15)
                                                                            desce[a]++;
                                                                        else if (h[b] < h[a]-1)
                                                            24
                                                                            sobe[a]++;
                                                            25
           fill(fb.begin() + b.size(), fb.begin() + sz,
124
                                                            26
       num {0, 0});
                                                                   backedges[a] += sobe[a] - desce[a];
                                                            27
           fft(fb, sz);
                                                            28 }
       ld ratio = 0.25 / sz;
128
       num r2(0, -1);
                                                                     Bellman Ford
       num r3(ratio, 0);
129
       num r4(0, -ratio);
130
                                                               Complexity: O(n * m) | n = |nodes|, m = |edges|
       num r5(0, 1);
131
       for(int i=0;i<=(sz >> 1);i++) {
                                                               Finds shortest paths from a starting node to all nodes of the
           int j = (sz - i) & (sz - 1);
                                                               graph. Detects negative cycles, if they exist.
           num a1 = (fa[i] + conj(fa[j]));
134
           num a2 = (fa[i] - conj(fa[j])) * r2;
                                                             1 // TITLE: Bellman Ford
```

 $_2$ // COMPLEXITY: O(n * m) | n = |nodes|, m = |edges|

num b1 = (fb[i] + conj(fb[j])) * r3;

```
_{\rm 3} // <code>DESCRIPTION: Finds shortest paths from a starting</code>
       node to all nodes of the graph. Detects negative
       cycles, if they exist.
5 // a and b vertices, c cost
6 // [{a, b, c}, {a, b, c}]
7 vector < tuple < int , int , int >> edges;
s int N:
void bellman_ford(int x){
       for (int i = 0; i < N; i++){</pre>
11
12
            dist[i] = oo;
13
       dist[x] = 0;
14
1.5
       for (int i = 0; i < N - 1; i++){
16
                                                               10
            for (auto [a, b, c]: edges){
                if (dist[a] == oo) continue;
18
                                                               12
                dist[b] = min(dist[b], dist[a] + w);
                                                               1.3
20
                                                               14
21
                                                               15
22 }
                                                               16
23 // return true if has cycle
                                                               17
24 bool check_negative_cycle(int x){
                                                               18
       for (int i = 0; i < N; i++){
2.5
                                                               19
           dist[i] = oo;
26
                                                               2.0
27
                                                               21
       dist[x] = 0;
28
                                                               22
                                                               23
       for (int i = 0; i < N - 1; i++){
3.0
           for (auto [a, b, c]: edges){
31
                                                               25
                if (dist[a] == oo) continue;
32
                                                               26
                dist[b] = min(dist[b], dist[a] + w);
33
                                                               27
34
           }
                                                               28
       }
35
                                                               3.0
       for (auto [a, b, c]: edges){
3.7
                                                               31
           if (dist[a] == oo) continue;
38
                                                               32
           if (dist[a] + w < dist[b]){</pre>
39
                                                               33
                return true;
40
                                                               34
41
           };
                                                               3.5
       }
42
                                                               36
43
       return false;
                                                               37
44 }
                                                               38
45 ((
                                                               3.9
                                                               40
                                                               41
  2.3
         Floyd Warshall
                                                               42
   Complexity: O(V^*V^*V)
                                                               43
                                                               44
```

Finds shortest distances between all pairs of vertices

```
1 // TITLE: Floyd Warshall
2 // COMPLEXITY: O(V*V*V)
3 // DESCRIPTION: Finds shortest distances between all
                                                                48
       pairs of vertices
                                                                51
5 for(int k=0; k<n; k++) {</pre>
            for(int i = 0; i < n; i++) {</pre>
                                                                5.3
                                                                54
                for(int j=0;j<n;j++) {</pre>
                     graph[i][j]=min(graph[i][j],
                                                                55
                                                                56
                     graph[i][k] + graph[k][j]);
10
                                                                57
                                                                5.8
           }
12
       }
```

2.4 2SAT

Complexity: O(n+m), n = number of variables, $m = number_{64}$ of conjunctions (ands).

Finds an assignment that makes a certain boolean formula true, or determines that such an assignment does not exist.

```
1 // TITLE: 2SAT
2 // COMPLEXITY: O(n+m), n = number of variables, m =
      number of conjunctions (ands).
_{\rm 3} // <code>DESCRIPTION: Finds an assignment that makes a</code>
      certain boolean formula true, or determines that
      such an assignment does not exist.
5 struct twosat {
      vi vis, degin;
6
      stack<int> tout;
      vector < vi> g, gi, con, sccg;
      vi repr, conv;
      int gsize;
      void dfs1(int a) {
          if (vis[a]) return;
           vis[a]=true;
           for(auto& b : g[a]) {
               dfs1(b);
           tout.push(a);
       void dfs2(int a, int orig) {
          if (vis[a]) return;
           vis[a]=true;
           repr[a]=orig;
           sccg[orig].pb(a);
           for(auto& b : gi[a]) {
               if (vis[b]) {
                   if (repr[b] != orig) {
                        con[repr[b]].pb(orig);
                       degin[orig]++;
                   continue;
               dfs2(b, orig);
       // if s1 = 1 and s2 = 1 this adds a \backslash/ b to the
       graph
       void addedge(int a, int s1,
                    int b, int s2) {
           g[2*a+(!s1)].pb(2*b+s2);
           gi[2*b+s2].pb(2*a+(!s1));
           g[2*b+(!s2)].pb(2*a+s1);
46
           gi[2*a+s1].pb(2*b+(!s2));
       twosat(int nvars) {
          gsize=2*nvars;
           g.assign(gsize, vi());
           gi.assign(gsize, vi());
           con.assign(gsize, vi());
           sccg.assign(gsize, vi());
           repr.assign(gsize, -1);
           vis.assign(gsize, 0);
           degin.assign(gsize, 0);
60
      // returns empty vector if the formula is not
      satisfiable.
       vi run() {
          vi vals(gsize/2, -1);
           rep(i,0,gsize) dfs1(i);
```

61

47

```
vis.assign(gsize,0);
66
           while(!tout.empty()) {
               int cur = tout.top();tout.pop();
68
6.9
                if (vis[cur]) continue;
               dfs2(cur,cur);
               conv.pb(cur);
           }
7.3
           rep(i, 0, gsize/2) {
74
               if (repr[2*i] == repr[2*i+1]) {
                   return {};
76
           }
80
            queue < int > q;
           for(auto& v : conv) {
81
                if (degin[v] == 0) q.push(v);
83
           while(!q.empty()) {
8.5
               int cur=q.front(); q.pop();
86
                for(auto guy : sccg[cur]) {
                   int s = guy%2;
88
                    int idx = guy/2;
                    if (vals[idx] != -1) continue;
90
                    if (s) {
91
                        vals[idx] = false;
92
                    } else {
93
                        vals[idx]=true;
9.5
               }
               for (auto& b : con[cur]) {
97
                   if(--degin[b] == 0) q.push(b);
           }
100
           return vals;
103
104 };
```

2.5 Dominator tree

Complexity: O(E + V)

```
1 // TITLE: Dominator tree
2 // COMPLEXITY: O(E + V)
3 // DESCRIPION: Builds dominator tree
5 vector < int > g[mxN];
6 vector < int > S, gt[mxN], T[mxN];
7 int dsu[mxN], label[mxN];
8 int sdom[mxN], idom[mxN], id[mxN];
9 int dfs_time = 0;
vector < int > bucket[mxN];
vector < int > down[mxN];
14 void prep(int a)
15
16
      S.pb(a);
      id[a] = ++dfs_time;
1.7
      label[a] = sdom[a] = dsu[a] = a;
1.9
      for (auto b: g[a]) {
20
2.1
          if (!id[b]) {
               prep(b);
               down[a].pb(b);
24
           gt[b].pb(a);
25
      }
26
27 }
```

```
28
29 int fnd(int a, int flag = 0)
30 €
31
       if (a == dsu[a]) return a;
       int p = fnd(dsu[a], 1);
32
       int b = label[ dsu[a] ];
33
       if (id [ sdom[b] ] < id[ sdom[ label[a] ] ]) {</pre>
34
           label[a] = b;
3.5
36
37
       dsu[a] = p;
       return (flag ? p: label[a]);
38
39 }
40
41 void build_dominator_tree(int root)
42 {
       prep(root);
43
44
       reverse(all(S));
45
       int w;
       for (int a: S) {
47
           for (int b: gt[a]) {
48
                w = fnd(b);
49
5.0
                if (id[ sdom[w] ] < id[ sdom[a] ]) {</pre>
                    sdom[a] = sdom[w];
5.2
53
           }
54
           gt[a].clear();
           if (a != root) {
55
56
               bucket[ sdom[a] ].pb(a);
5.7
           for (int b: bucket[a]) {
58
                w = fnd(b);
5.9
                if (sdom[w] == sdom[b]) {
60
61
                    idom[b] = sdom[b];
                }
62
63
                    idom[b] = w;
64
65
66
           bucket[a].clear();
67
68
           for (int b: down[a]) {
               dsu[b] = a;
6.9
70
7.1
           down[a].clear();
72
73
       reverse(all(S));
       for (int a: S) {
7.4
           if (a != root) {
               if (idom[a] != sdom[a]) {
7.6
                    idom[a] = idom[ idom[a] ];
78
                T[ idom[a] ].pb(a);
79
80
8.1
82
       S.clear();
83 }
```

2.6 Kth Ancestor

```
Complexity: O(n * log(n))
Preprocess, then find in log n

1 // TITLE: Kth Ancestor

2 // COMPLEXITY: O(n * log(n))

3 // DESCRIPTION: Preprocess, then find in log n

4 const int LOG_N = 30;

6 int get_kth_ancestor(vector<vector<int>> & up, int v, int k)

7 {

8 for (int j = 0; j < LOG_N; j++) {

9 if (k & ((int)1 << j)) {
```

```
v = up[v][i];
                                                             41 }
1.0
           }
       }
12
13
       return v;
                                                                2.8
                                                                     Dkistra
14 }
15
                                                                Complexity: O(E + V \cdot log(v))
16 void solve()
17
       vector < vector < int >> up(n, vector < int > (LOG_N));
18
                                                              1 // TITLE: Dkistra
19
                                                              2 // COMPLEXITY: O(E + V.log(v))
       for (int i = 0; i < n; i++) {</pre>
20
                                                              3 // DESCRIPION: Finds to shortest path from start
21
           up[i][0] = parents[i];
           for (int j = 1; j < LOG_N; j++) {
22
                                                              5 int dist[mxN];
               up[i][j] = up[up[i][j-1]][j-1];
23
                                                              6 bool vis[mxN];
24
                                                              7 vector<pair<int, int>> g[mxN];
       }
25
26
       cout << get_kth_ancestor(up, x, k) << endl;</pre>
                                                              9 void dikstra(int start)
27
                                                             10 ┫
28 }
                                                                    fill(dist, dist + mxN, oo);
                                                                    fill(vis, vis + mxN, 0);
                                                                    priority_queue < pair < int , int >> q;
                                                             13
                                                                    dist[start] = 0;
  2.7
         Topological Sort
                                                             14
                                                                    q.push({0, start});
                                                             15
  Complexity: O(N + M), N: Vertices, M: Arestas
                                                                    while(!q.empty()) {
  Retorna no do grapho em ordem topologica, se a quantidade de
                                                                        auto [d, a] = q.top();
  nos retornada nao for igual a quantidade de nos e impossivel 19
                                                                         q.pop();
                                                                         if (vis[a]) continue;
1 // TITLE: Topological Sort
                                                                         vis[a] = true;
_{2} // COMPLEXITY: O(N + M), N: Vertices, M: Arestas
                                                                         for (auto [b, w]: g[a]) {
                                                             22
3 // DESCRIPTION: Retorna no do grapho em ordem
                                                                             if (dist[a] + w < dist[b]) {</pre>
                                                             23
       topologica, se a quantidade de nos retornada nao
                                                                                  dist[b] = dist[a] + w;
       for igual a quantidade de nos e impossivel
                                                                                  q.push({-dist[b], b});
                                                             2.5
                                                                             }
5 typedef vector<vector<int>> Adj_List;
                                                                         }
                                                             27
6 typedef vector<int> Indegree_List; // How many nodes
                                                                    }
                                                             28
       depend on him
                                                             29 }
  typedef vector<int> Order_List; // The order in
       which the nodes appears
                                                                2.9
                                                                     Dinic Min cost
9 Order_List kahn(Adj_List adj, Indegree_List indegree)
10
                                                                Complexity: O(V^*V^*E), Bipartite is O(\operatorname{sqrt}(V) E)
       queue < int > q;
       // priority_queue < int > q; // If you want in
12
                                                                Gives you the max flow with the min cost
       lexicografic order
       for (int i = 0; i < indegree.size(); i++) {</pre>
                                                              1 // TITLE: Dinic Min cost
13
                                                              2 // COMPLEXITY: O(V*V*E), Bipartite is O(sqrt(V) E)
14
           if (indegree[i] == 0)
                                                              _{\rm 3} // <code>DESCRIPTION:</code> Gives you the <code>max_flow</code> with the min
               q.push(i);
1.5
16
       vector < int > order;
17
                                                              5 // Edge structure
18
       while (not q.empty()) {
                                                              6 struct Edge
19
          auto a = q front();
                                                              7 -{
20
           q.pop();
21
                                                                     int from, to;
                                                                    int flow, capacity;
22
                                                              g
23
           order.push_back(a);
                                                             10
                                                                    int cost;
           for (auto b: adj[a]) {
24
                indegree[b]--;
                                                                    Edge(int from_, int to_, int flow_, int capacity_
25
                                                             12
                if (indegree[b] == 0)
                                                                    , int cost_)
                    q.push(b);
                                                                        : from(from_), to(to_), flow(flow_), capacity
27
                                                             13
           }
                                                                    (capacity_), cost(cost_)
28
       }
29
                                                             14
                                                                    {}
                                                             15 };
       return order;
3.0
31 }
                                                             16
                                                             17 struct Dinic
33 int32_t main()
                                                             18 €
34 {
                                                             19
                                                                    vector < vector < int >> graph;
                                                                    vector < Edge > edges;
35
                                                             20
       Order_List = kahn(adj, indegree);
                                                                    vector < int > dist;
                                                             21
                                                                    vector < bool > inqueue;
       if (Order_List.size() != N) {
3.7
                                                             22
           cout << "IMPOSSIBLE" << endl;</pre>
                                                                    int size;
                                                             23
                                                                    int cost = 0;
39
                                                             24
```

2.5

return 0;

```
Dinic(int n)
                                                                            int b = graph[curr][next[curr]];
26
27
                                                            93
                                                                            auto & edge = edges[b];
           graph.resize(n);
                                                                            auto & rev_edge = edges[b^1];
28
                                                            94
29
           dist.resize(n);
                                                            95
                                                                            int cap = edge.capacity - edge.flow;
30
           inqueue.resize(n);
                                                            96
                                                                            if (cap > 0 && (dist[edge.from] + edge.
           size = n:
31
                                                            97
           edges.clear();
                                                                   cost == dist[edge.to])) {
                                                                                int bottle_neck = dfs(edge.to, sink,
33
                                                                   next, min(flow, cap));
34
      void add_edge(int from, int to, int capacity, int 99
                                                                                if (bottle_neck > 0) {
35
                                                                                     edge.flow += bottle_neck;
       cost)
                                                                                     rev_edge.flow -= bottle_neck;
                                                                                     cost += edge.cost * bottle_neck;
           {\tt edges.emplace\_back(from\,,\,to\,,\,0\,,\,capacity\,,}\\
37
       cost):
                                                                                     return bottle_neck;
38
           graph[from].push_back(edges.size() - 1);
                                                            104
                                                                                }
                                                                            }
40
           edges.emplace_back(to, from, 0, 0, -cost);
                                                                        }
           graph[to].push_back(edges.size() - 1);
                                                                        return 0;
41
42
      }
                                                            108
43
                                                            109
      int get_max_flow(int source, int sink)
                                                                   vector<pair<int, int>> mincut(int source, int
                                                            110
44
                                                                   sink)
45
           int max flow = 0:
                                                                   {
46
           vector<int> next(size);
                                                                        vector<pair<int, int>> cut;
47
           while(spfa(source, sink)) {
                                                                        spfa(source, sink);
48
                                                            113
               next.assign(size, 0);
                                                                        for (auto & e: edges) {
49
                                                            114
               for (int f = dfs(source, sink, next, oo);115
                                                                            if (e.flow == e.capacity && dist[e.from]
        f != 0; f = dfs(source, sink, next, oo)) {
                                                                   != oo && level[e.to] == oo && e.capacity > 0) {
51
                   max_flow += f;
                                                                                cut.emplace_back(e.from, e.to);
               }
52
           }
                                                                        }
                                                            118
           return max_flow;
5.4
                                                            119
                                                                        return cut;
56
                                                           121 };
      bool spfa(int source, int sink)
57
                                                           123 // Example on how to use
           dist.assign(size, oo);
                                                           124 void solve()
59
           inqueue.assign(size, false);
                                                           125
           queue < int > q;
                                                           126
61
           q.push(source);
                                                                   int N = 10;
62
           dist[source] = 0;
                                                            128
           inqueue[source] = true;
                                                                   int source = 8;
64
                                                           129
                                                                   int sink = 9;
                                                           130
66
           while(!q.empty()) {
                                                           131
               int a = q.front();
                                                            132
                                                                   Dinic flow(N);
67
               q.pop();
                                                                   flow.add_edge(8, 0, 4, 0);
68
               inqueue[a] = false;
                                                                   flow.add_edge(8, 1, 3, 0);
69
                                                           134
                                                                   flow.add_edge(8, 2, 2, 0);
               for (int & b: graph[a]) {
                                                                   flow.add_edge(8, 3, 1, 0);
                                                            136
                    auto edge = edges[b];
                                                                   flow.add_edge(0, 6, oo, 3);
                    int cap = edge.capacity - edge.flow; 138
73
                   if (cap > 0 && dist[edge.to] > dist[ 139
                                                                   flow.add_edge(0, 7, oo, 2);
74
       edge.from] + edge.cost) {
                                                                   flow.add_edge(0, 5, oo, 0);
                        dist[edge.to] = dist[edge.from] +141
                                                                   flow.add_edge(1, 4, oo, 0);
        edge.cost;
                                                            142
                        if (not inqueue[edge.to]) {
                                                            143
                            q.push(edge.to);
                                                                   flow.add_edge(4, 9, oo, 0);
                                                            144
                            inqueue[edge.to] = true;
                                                                   flow.add_edge(5, 9, oo, 0);
78
                                                            145
                        }
                                                            146
                                                                   flow.add_edge(6, 9, oo, 0);
                   }
80
                                                            147
                                                                   flow.add_edge(7, 9, oo, 0);
               }
8.1
                                                            148
           }
                                                            149
                                                                   int ans = flow.get_max_flow(source, sink);
82
           return dist[sink] != oo;
                                                                   debug(ans);
                                                                   debug(flow.cost);
                                                            151
84
                                                            152
      int dfs(int curr, int sink, vector<int> & next, 153
86
      int flow)
                                                            154 int32_t main()
      {
                                                            155 {
87
           if (curr == sink) return flow;
                                                           156
                                                                   solve():
88
           int num_edges = graph[curr].size();
                                                           157 }
89
90
           for (; next[curr] < num_edges; next[curr]++)</pre>
91
      {
```

```
2.10 Kosaraju
                                                                    Edge(int from_, int to_, int flow_, int capacity_
  Complexity: O(V+E)
                                                                        : from(from_), to(to_), flow(flow_), capacity
                                                             1.3
                                                                    (capacity_)
  Find the strongly connected components of a graph
                                                             14
1 // TITLE: Kosaraju
                                                            15 }:
2 // COMPLEXITY: O(V+E)
                                                            16
3 // DESCRIPTION: Find the strongly connected
                                                            17 struct Dinic
      components of a graph
                                                            18 {
                                                            19
                                                                    vector < vector < int >> graph;
5 int n,m;
                                                                    vector < Edge > edges;
                                                            20
6 vector < vi> g, gi, scc;
                                                            21
                                                                    vector < int > level;
7 vi vis, order, p;
                                                                   int size:
                                                            22
                                                            23
9 void dfs1(int a) {
                                                                   Dinic(int n)
                                                            24
      if(vis[a]) return;
1.0
                                                            25
       vis[a]=true;
                                                            26
                                                                        graph.resize(n);
       for(auto& b:g[a]) {
                                                                        level.resize(n);
                                                            27
           dfs1(b);
13
                                                                        size = n;
14
                                                                        edges.clear();
                                                            29
       order pb(a);
1.5
                                                            30
16 }
                                                            31
17
                                                            32
                                                                    void add_edge(int from, int to, int capacity)
18 void dfs2(int a, int orig) {
                                                            33
       if (vis[a]) return;
19
                                                                        edges.emplace_back(from, to, 0, capacity);
                                                            3.4
       vis[a]=true;
20
                                                            3.5
                                                                        graph[from].push_back(edges.size() - 1);
21
      p[a]=orig;
                                                            36
22
                                                                        edges.emplace_back(to, from, 0, 0);
                                                            37
23
       for(auto& b:gi[a]) {
                                                            38
                                                                        graph[to].push_back(edges.size() - 1);
           if (vis[b] && p[b] != orig)
24
                                                                   }
                                                            3.9
               scc[p[b]].pb(orig);
25
                                                            40
           dfs2(b,orig);
                                                                    int get_max_flow(int source, int sink)
                                                            4.1
       }
27
                                                            42
28 }
                                                            43
                                                                        int max_flow = 0;
29
                                                                        vector < int > next(size);
                                                            44
30 void solve() {
                                                                        while(bfs(source, sink)) {
                                                             45
       cin >> n >> m;
                                                                            next.assign(size, 0);
                                                            46
32
                                                                            for (int f = dfs(source, sink, next, oo);
                                                            47
       g.assign(n, vi());
                                                                     f != 0; f = dfs(source, sink, next, oo)) {
       gi.assign(n, vi());
                                                                                max_flow += f;
34
                                                            48
35
      scc.assign(n, vi());
                                                             49
      vis.assign(n, 0);
36
                                                                        }
                                                            5.0
      p.assign(n, 0);
37
                                                            51
                                                                        return max_flow;
       rep(i, 0, m) {
                                                            52
           int a,b;cin>>a>>b;a--;b--;
39
                                                            53
           g[a].pb(b);
40
                                                            54
                                                                   bool bfs(int source, int sink)
41
           gi[b].pb(a);
                                                            5.5
42
                                                            56
                                                                        level.assign(size, -1);
43
                                                                        queue < int > q;
                                                            5.7
      rep(i,0,n)dfs1(i);
44
                                                                        q.push(source);
45
       vis.assign(n,0);
                                                                        level[source] = 0;
       for(int i=n-1; i>=0;i--) dfs2(order[i],order[i]);60
46
47
                                                                        while(!q.empty()) {
       vis.assign(n,0);
                                                                            int a = q.front();
                                                            62
49 }
                                                                            q.pop();
                                                            64
                                                                            for (int & b: graph[a]) {
                                                                                 auto edge = edges[b];
  2.11 Dinic
                                                                                 int cap = edge.capacity - edge.flow;
                                                            67
                                                                                 if (cap > 0 && level[edge.to] == -1)
                                                            68
  Complexity: O(V^*V^*E), Bipartite is O(\operatorname{sqrt}(V) E)
                                                                   {
  Dinic
                                                            6.9
                                                                                     level[edge.to] = level[a] + 1;
                                                                                     q.push(edge.to);
1 // TITLE: Dinic
2 // COMPLEXITY: O(V*V*E), Bipartite is O(sqrt(V) E)
                                                            7.1
3 // DESCRIPTION: Dinic
                                                                            }
                                                                        }
                                                            73
                                                            74
                                                                        return level[sink] != -1;
5 const int oo = 0x3f3f3f3f3f3f3f3f3f;
                                                            75
6 // Edge structure
                                                            76
7 struct Edge
                                                                    int dfs(int curr, int sink, vector < int > & next,
8 -
                                                                   int flow)
       int from, to;
      int flow, capacity;
10
                                                            7.9
                                                                        if (curr == sink) return flow;
```

```
int num_edges = graph[curr].size();
                                                                       if (edge.from == sink || edge.to == sink)
80
                                                           145
                                                                    continue;
81
            for (; next[curr] < num_edges; next[curr]++) 146</pre>
                                                                       if (edge.flow == 0) continue; // Is not
82
                                                                    participant
                int b = graph[curr][next[curr]];
                auto & edge = edges[b];
                                                                        cout << edge.from + 1 << " " << edge.to -n +
84
                                                            148
                auto & rev_edge = edges[b^1];
                                                                    1 << endl;
86
                                                            149
                int cap = edge.capacity - edge.flow;
                                                            150
87
                if (cap > 0 && (level[curr] + 1 == level[
       edge.to])) {
                    int bottle_neck = dfs(edge.to, sink,
       next, min(flow, cap));
                                                               3
                                                                    Segtree
                    if (bottle_neck > 0) {
                         edge.flow += bottle_neck;
91
                                                                      Standard SegTree
                         rev_edge.flow -= bottle_neck;
                                                               3.1
92
                         return bottle_neck;
                    }
                                                               Complexity: O(log(n)) query and update
94
                }
                                                               Sum segment tree with point update.
           }
96
           return 0;
97
                                                             1 // TITLE: Standard SegTree
98
                                                             2 // COMPLEXITY: O(log(n)) query and update
99
                                                             _{\rm 3} // <code>DESCRIPTION:</code> Sum segment tree with point update.
       vector<pair<int, int>> mincut(int source, int
       sink)
                                                             5 using type = int;
            vector<pair<int, int>> cut;
102
                                                             7 type iden = 0;
           bfs(source, sink);
                                                             8 vector < type > seg;
           for (auto & e: edges) {
104
                                                             9 int segsize;
                if (e.flow == e.capacity && level[e.from] 10
105
        != -1 && level[e.to] == -1 && e.capacity > 0) { _{11} type func(type a, type b)
                    cut.emplace_back(e.from, e.to);
                                                             12 {
                                                             13
                                                                    return a + b;
            }
                                                             14 }
            return cut;
109
110
                                                             16 // query do intervalo [1, r)
111 }:
                                                             17 type query(int 1, int r, int no = 0, int lx = 0, int
112
                                                                    rx = segsize)
113 // Example on how to use
                                                             18 €
114 void solve()
                                                                    // 1 lx rx r
                                                             19
115 {
                                                                    if (r <= lx or rx <= 1)</pre>
                                                             20
116
       int n, m;
                                                                        return iden;
                                                             21
       cin >> n >> m;
117
                                                                    if (1 <= lx and rx <= r)</pre>
                                                             22
       int N = n + m + 2;
118
                                                                        return seg[no];
                                                             23
119
                                                             24
       int source = N - 2;
120
                                                                    int mid = lx + (rx - lx) / 2;
                                                             25
       int sink = N - 1:
                                                             26
                                                                    return func(query(1, r, 2 * no + 1, lx, mid),
                                                                                 query(1, r, 2 * no + 2, mid, rx));
                                                             2.7
       Dinic flow(N);
                                                            28 }
124
                                                            29
       for (int i = 0; i < n; i++) {</pre>
125
                                                             30 void update(int dest, type val, int no = 0, int lx =
           int q; cin >> q;
126
                                                                    0, int rx = segsize)
            while(q--) {
127
                                                             31
                int b; cin >> b;
128
                                                                    if (dest < lx or dest >= rx)
                                                             32
                flow.add_edge(i, n + b - 1, 1);
129
                                                             3.3
                                                                        return;
           }
130
                                                             34
                                                                    if (rx - 1x == 1)
                                                             35
       for (int i =0; i < n; i++) {</pre>
                                                                        seg[no] = val;
                                                             36
           flow.add_edge(source, i, 1);
                                                             37
                                                                        return:
134
                                                             38
       for (int i =0; i < m; i++) {</pre>
135
                                                             39
           flow.add_edge(i + n, sink, 1);
136
                                                                    int mid = lx + (rx - lx) / 2;
                                                             40
                                                                    update(dest, val, 2 * no + 1, lx, mid);
                                                             41
138
                                                                    update(dest, val, 2 * no + 2, mid, rx);
       cout << m - flow.get_max_flow(source, sink) <<</pre>
                                                                    seg[no] = func(seg[2 * no + 1], seg[2 * no + 2]);
                                                             43
       endl:
                                                             44 }
140
                                                             45
       // Getting participant edges
141
                                                             46 signed main()
       for (auto & edge: flow.edges) {
142
                                                             47 {
            if (edge.capacity == 0) continue; // This
143
                                                                    ios_base::sync_with_stdio(0);
                                                             48
       means is a reverse edge
                                                                    cin.tie(0);
           if (edge.from == source || edge.to == source) 50
144
                                                                    cout.tie(0);
        continue;
                                                                    int n;
                                                             5.1
```

```
43 int query(int 1, int r, node *no) {
      cin >> n:
52
53
      segsize = n;
                                                           44
                                                                  if (r <= no->1x or no->rx <= 1) return 0;
                                                                  if (1 <= no->lx and no->rx <= r) return no->val;
      if (__builtin_popcount(n) != 1)
5.4
                                                           4.5
                                                           46
           segsize = 1 + (int)log2(segsize);
                                                           47
                                                                  return query(1,r,no->1) + query(1,r,no->r);
           segsize = 1 << segsize;</pre>
                                                           48 }
57
      seg.assign(2 * segsize - 1, iden);
59
60
                                                              3.3
                                                                    Set and update lazy seg
      rep(i, 0, n)
61
62
63
           int x;
                                                              Sum segtree with set and update
           cin >> x;
64
           update(i, x);
65
                                                            1 // TITLE: Set and update lazy seg
66
67 }
                                                            5 vector <int> lazy, opvec;
                                                            6 vector < int > seg;
        Persistent sum segment tree
                                                            8 constexpr int SET = 30;
  Complexity: O(\log(n)) query and update, O(k \log(n)) memory, constexpr int ADD = 31;
```

Complexity: $O(\log(n))$ query and update, $O(k \log(n))$ memory, n = number of elements, k = number of operations Sum segment tree which preserves its history.

1 // TITLE: Persistent sum segment tree

```
_2 // COMPLEXITY: O(log(n)) query and update, O(k log(n) _{14}
      ) memory, n = number of elements, k = number of
      operations
3 // DESCRIPTION: Sum segment tree which preserves its 17
      history.
5 int segsize;
                                                            20
                                                            21
7 struct node {
      int val;
                                                            23
       int lx, rx;
                                                            24
      node *1=0, *r=0;
1.0
                                                            25
11
                                                            26
      node() {}
      node(int val, int lx, int rx, node *1, node *r) : 28
1.3
       val(val), lx(lx),rx(rx),l(l),r(r) {}
15 }:
                                                            3.0
16
                                                            3.1
17 node* build(vi& arr, int lx=0, int rx=segsize) {
                                                            3.2
      if (rx - lx == 1) {
18
                                                            33
           if (lx < (int)arr.size()) {</pre>
19
               return new node(arr[lx], lx, rx, 0, 0);
20
                                                            3.5
21
                                                            36
                                                            3.7
           return new node(0,lx,rx,0,0);
23
                                                            38
                                                            39
25
                                                            40
      int mid = (1x+rx)/2;
       auto nol = build(arr, lx, mid);
27
                                                            42
28
       auto nor = build(arr, mid, rx);
                                                            43
       return new node(nol->val + nor->val, lx, rx, nol, 44
       nor):
                                                            45
30 }
31
                                                            47
32 node* update(int idx, int val, node *no) {
                                                            48
       if (idx < no->lx or idx >= no->rx) return no;
33
                                                            49
       if (no->rx - no->lx == 1) {
3.4
                                                            50
           return new node(val+no->val, no->lx, no->rx, 51
      no->1, no->r);
                                                            53
37
38
       auto nol = update(idx, val, no->1);
       auto nor = update(idx, val, no->r);
39
       return new node(nol->val + nor->val, no->lx, no-> 56
40
       rx, nol, nor);
41 }
                                                            5.8
42
```

```
Complexity: O(\log(n)) query and update
2 // COMPLEXITY: O(log(n)) query and update
3 // DESCRIPTION: Sum segtree with set and update
11 int segsize;
void propagate(int no, int lx, int rx) {
      if (lazy[no] == -1) return;
      if (rx-1x == 1) {
          if(opvec[no] == SET) seg[no] = lazy[no];
           else seg[no] += lazy[no];
          lazy[no]=-1;
           opvec[no] = -1;
           return;
      if(opvec[no] == SET) {
          seg[no] = (rx-lx) * lazy[no];
           lazy[2*no+1] = lazy[no];
          lazy[2*no+2] = lazy[no];
           opvec[2*no+1] = SET;
           opvec[2*no+2] = SET;
          lazy[no] = -1;
           opvec[no] = -1;
           return:
      seg[no] += (rx-lx) * lazy[no];
      if (lazy[2*no+1] == -1) {
          lazy[2*no+1] = 0;
           opvec[2*no+1] = ADD;
      if (lazy[2*no+2] == -1) {
          lazy[2*no+2] = 0;
           opvec[2*no+2] = ADD;
      lazy[2*no+1] += lazy[no];
      lazy[2*no+2] += lazy[no];
      lazy[no] = -1;
      opvec[no] = -1;
52 }
54 void update(int 1, int r, int val, int op, int no=0,
      int lx=0, int rx=segsize) {
      propagate(no, lx, rx);
      if (r <= lx or l >= rx) return;
      if (1x >= 1 \text{ and } rx <= r) {
          lazy[no] = val;
           opvec[no] = op;
59
```

```
propagate(no, lx, rx);
60
61
62
                                                            11
63
       int mid = (rx+lx)/2;
       update(1, r, val, op, 2*no+1, lx, mid);
65
                                                            14
       update(1, r, val, op, 2*no+2, mid, rx);
       seg[no] = seg[2*no+1]+seg[2*no+2];
67
                                                            16
68 }
69
70 int query(int 1, int r, int no=0, int lx=0, int rx=
                                                            19
       segsize) {
       propagate(no, lx, rx);
       if (r <= lx or l >= rx) return 0;
                                                            22 void update(int 1, int r, int val,int no=0, int lx=0,
72
       if (lx >= l and rx <= r) return seg[no];</pre>
7.3
74
75
       int mid = (rx+lx)/2;
       return
7.6
                                                            2.5
          query(1,r,2*no+1,lx,mid) +
           query(1,r,2*no+2, mid, rx);
78
                                                            27
79 }
                                                            28
                                                            29
                                                            3.0
```

Binary Indexed Tree

Complexity: $O(\log(n))$ query and update Range sum queries with point update. One-indexed.

```
1 // TITLE: Binary Indexed Tree
2 // COMPLEXITY: O(log(n)) query and update
3 // DESCRIPTION: Range sum queries with point update.
      One-indexed.
5 struct BIT{
      #define lowbit(x) ( x & -x )
      int n:
      vi b;
10
      BIT( int n ) : n(n) , b(n+1, 0){};
      BIT( vi &c ){
          n = c.size(), b = c;
          for( int i = 1 , fa = i + lowbit(i) ; i <= n 49
13
          i ++ , fa = i + lowbit(i) )
14
              if( fa <= n ) b[fa] += b[i];</pre>
15
      void add( int i , int y ){
          for( ; i <= n ; i += lowbit(i) ) b[i] += y;</pre>
19
20
      int calc( int i ){
21
          int sum = 0;
22
           for( ; i ; i -= lowbit(i) ) sum += b[i];
          return sum;
24
25
26 };
```

3.5Lazy SegTree

Complexity: $O(\log(n))$ query and update Sum segment tree with range sum update.

```
1 // TITLE: Lazy SegTree
2 // COMPLEXITY: O(log(n)) query and update
_{\rm 3} // <code>DESCRIPTION:</code> Sum segment tree with range sum
      update.
4 vector < int > seg, lazy;
5 int segsize;
_{7} // change 0s to -1s if update is
8 // set instead of add. also,
```

Set

9 // remove the +=s

if(rx-lx>1) {

lazy[no]=0;

int rx=segsize) { // 1 r 1x rx

lazy[no]=val;

int mid=1x+(rx-1x)/2;

prop(no,lx,rx);

prop(no, lx, rx);

return;

segsize) {

48 signed main() {

int n; cin>>n;

segsize=n:

prop(no,lx,rx);

int mid=1x+(rx-1x)/2:

12

15

17

18

20 }

21

23

24

31

3.2 33

34

35 36 }

3.7

41

43

44

45

47

50

51

52

5.4

55

56

57

58

59

60

61

62 }

46 }

void prop(int no, int lx, int rx) { if (lazy[no] == 0) return;

seg[no]+=(rx-lx)*lazy[no];

lazy[2*no+1] += lazy[no];

lazy[2*no+2] += lazy[no];

if (r <= lx or rx <= l) return;

update(1,r,val,2*no+1,lx,mid);

update(1,r,val,2*no+2,mid,rx);

seg[no] = seg[2*no+1] + seg[2*no+2];

if (r <= lx or rx <= 1) return 0;

return query(1,r,2*no+1, lx, mid)+

if(__builtin_popcount(n) != 1) {

segsize = 1<<segsize;</pre>

seg.assign(2*segsize-1, 0);

lazy.assign(2*segsize-1, 0);

// update is set instead of add

// use -1 instead of 0 if

segsize = 1 + (int) log2(segsize);

38 int query(int 1, int r, int no=0, int 1x=0, int rx=

if (1 <= lx and rx <= r) return seg[no];</pre>

query(1,r,2*no+2,mid,rx);

ios_base::sync_with_stdio(0);cin.tie(0);cout.tie

if (1 <= lx and rx <= r) {</pre>

4.1Set

Complexity: Insertion Log(n) Keeps elements sorted, remove duplicates, upper bound, lower bound, find, count $_{1}$ // TITLE: Set

```
2 // COMPLEXITY: Insertion Log(n)
3 // Description: Keeps elements sorted, remove
     duplicates, upper_bound, lower_bound, find, count
```

```
5 int main() {
    set < int > set1;
                            // O(log(n))
    set1.insert(1);
    set1.erase(1);
                           // O(log(n))
9
10
    set1.upper_bound(1); // O(log(n))
    set1.lower_bound(1); // O(log(n))
12
    set1.find(1);
                            // O(log(n))
13
    set1.count(1);
                           // O(log(n))
14
15
                           // 0(1)
16
    set1.size();
                           // 0(1)
17
    set1.empty();
18
                           // 0(1)
    set1.clear()
19
20
    return 0:
21 }
```

4.2 Multiset

4.3 Ordered Set

Complexity: log n

Worst set with adtional operations

```
1 // TITLE: Ordered Set
2 // COMPLEXITY: log n
_{\rm 3} // DESCRIPTION: Worst set with adtional operations
6 #include <bits/extc++.h>
\label{eq:constraints} \mbox{\tt 7 using namespace $\_\_gnu\_pbds$; // or $pb\_ds$;}
 8 template < typename T, typename B = null_type >
9 using ordered_set = tree<T, B, less<T>, rb_tree_tag,
       tree_order_statistics_node_update>;
1.0
11 int32_t main() {
       ordered_set < int > oset;
12
1.3
       oset.insert(5);
       oset.insert(1):
1.5
       oset.insert(2);
16
       // o_set = {1, 2, 5}
       5 == *(oset.find_by_order(2)); // Like an array
18
       2 == oset.order_of_key(4); // How many elements
19
       are strictly less than 4
20 }
```

5 Misc

5.1 Template

1 // TITLE: Template

2 // COMPLEXITY: O(1)

```
Complexity: O(1)
Standard template for competitions
```

```
3 // DESCRIPTION: Standard template for competitions
5 #include <bits/stdc++.h>
7 #define int long long
8 #define endl '\n'
9 #define pb push_back
10 #define eb emplace_back
# define all(x) (x).begin(), (x).end()
12 #define rep(i, a, b) for(int i=(int)(a);i < (int)(b);</pre>
       i++)
13 #define debug(var) cout << #var << ": " << var <<
       endl
14 #define pii pair<int, int>
15 #define vi vector <int>
17 int MAX = 2e5;
19 int oo = 0 x3f3f3f3f3f3f3f3f3f;
21 using namespace std;
22
24 {
2.5
26 }
27
28 signed main()
29 {
       ios_base::sync_with_stdio(0);cin.tie(0);cout.tie
30
       (0);
       int t=1;
3.1
32
       // cin>>t;
33
       while(t -- ) solve();
34 }
```

6 String

6.1 String hash

Complexity: O(n) preprocessing, O(1) query Computes the hash of arbitrary substrings of a given string s.

```
1 // TITLE: String hash
2 // COMPLEXITY: O(n) preprocessing, O(1) query
_{\rm 3} // DESCRIPTION: Computes the hash of arbitrary
      substrings of a given string s.
4 int m1, m2;
5 int n; string s;
7 struct Hash {
     const int P = 31;
      int n; string s;
      vector < int > h, hi, p, p2, h2, hi2;
10
      Hash() {}
11
      Hash(string s):
      s(s), n(s.size()), h(n), hi(n), p(n), h2(n), hi2(
1.3
      n), p2(n) {
          for (int i=0;i<n;i++) p[i] = (i ? P*p[i-1]:1)
14
       % m1;
```

```
for (int i=0;i<n;i++) p2[i] = (i ? P*p2[i</pre>
1.5
       -1]:1) % m2;
                                                                    int n=s.size();
                                                             24
                                                                    vi p(n);
           for (int i=0;i<n;i++)</pre>
                                                                    vi c(n);
               h[i] = (s[i] + (i ? h[i-1]:0) * P) % m1; 26
           for (int i=0;i<n;i++)</pre>
                                                                        vector<pair<char, int>> a(n);
19
               h2[i] = (s[i] + (i ? h2[i-1]:0) * P) % m2_28
                                                                        rep(i,0,n) a[i]={s[i],i};
                                                                        sort(all(a)):
                                                             29
           for (int i=n-1; i>=0; i--)
                                                                        rep(i,0,n) p[i]=a[i].second;
               hi[i] = (s[i] + (i+1 < n ? hi[i+1]:0) * P)
                                                             32
                                                                        c[p[0]]=0;
24
           for (int i=n-1; i>=0; i--)
                                                             34
                                                                        rep(i,1,n) {
               hi2[i] = (s[i] + (i+1 < n ? hi2[i+1]:0) * P_{35}
                                                                             if(s[p[i]] == s[p[i-1]]) {
       ) % m2;
                                                             36
                                                                                 c[p[i]]=c[p[i-1]];
       }
                                                             37
       int gethash(int 1, int r) {
                                                                             else c[p[i]]=c[p[i-1]]+1;
           int hash = (h[r] - (1 ? h[l-1]*p[r-l+1]%m1 :
                                                                        }
                                                            3.9
           int hash2 = (h2[r] - (1 ? h2[1-1]*p2[r-1+1]% 41
29
      m2 : 0));
                                                                    for(int k=0; (1<<k) < n; k++) {</pre>
                                                             42
           hash = hash < 0 ? hash + m1 : hash;
                                                                        rep(i, 0, n)
                                                             43
                                                                             p[i] = (p[i] - (1 << k) + n) % n;
           hash2 = hash2 < 0 ? hash2 + m2 : hash2;
3.1
                                                             44
           return (hash << 30) ^ hash2;</pre>
                                                             45
       }
                                                                        countingsort(p,c);
33
                                                             46
       int gethashi(int 1, int r) {
34
           int hash = (hi[1] - (r+1 < n ? hi[r+1]*p[r-1 48])
                                                                        vi nc(n);
                                                                        nc[p[0]]=0;
       +1] % m1 : 0));
           int hash2 = (hi2[1] - (r+1 < n ? hi2[r+1]*p2[50])
                                                                        rep(i,1,n) {
                                                                             pii prev = \{c[p[i-1]], c[(p[i-1]+(1<<k))\%\}
      r-l+1] % m2 : 0));
                                                             5.1
           hash = hash < 0 ? hash + m1 : hash;
                                                                    n]};
           hash2 = hash2 < 0 ? hash2 + m2 : hash2;
                                                                             pii cur = {c[p[i]], c[(p[i]+(1<<k))%n]};
38
           return (hash << 30) ^ hash2;</pre>
39
40
       }
                                                                             if (prev == cur)
                                                                                 nc[p[i]]=nc[p[i-1]];
41 }:
                                                             54
                                                                             else nc[p[i]]=nc[p[i-1]]+1;
42
                                                                        }
43 void solve()
                                                             56
44 {
                                                             57
                                                                        c=nc;
                                                                    }
       srand(time(0));
                                                             58
45
       m1 = rand()/10 + 1e9;
46
                                                             59
47
       m2 = rand()/10 + 1e9;
                                                             60
                                                                    return p;
       Hash hasher(s);
                                                             61 }
48
49 }
```

6.2 Suffix Array

s += "\$";

21

Complexity: O(n log(n)), contains big constant (around 25). Computes a sorted array of the suffixes of a string.

```
1 // TITLE: Suffix Array
_{2} // COMPLEXITY: O(n log(n)), contains big constant (
       around 25).
_{\rm 3} // DESCRIPTION: Computes a sorted array of the
       suffixes of a string.
5 void countingsort(vi& p, vi& c) {
      int n=p.size();
       vi count(n,0);
       rep(i,0,n) count[c[i]]++;
10
       vi psum(n); psum[0]=0;
      rep(i,1,n) psum[i]=psum[i-1]+count[i-1];
12
13
       vi ans(n);
14
       rep(i,0,n)
15
           ans[psum[c[p[i]]]++]=p[i];
16
       p = ans;
17
18 }
20 vi sfa(string s) {
```

6.3 Z function

```
Complexity: O(n)
  z[i] = largest m such that s[0..m] = s[i..i+m]
1 // TITLE: Z function
2 // COMPLEXITY: O(n)
_3 // DESCRIPTION: z[i] = largest m such that s[0..m]=s[
      i..i+m]
4
5 vector<int> Z(string s) {
      int n = s.size();
      vector < int > z(n);
      int x = 0, y = 0;
       for (int i = 1; i < n; i++) {
9
           z[i] = max(0, min(z[i - x], y - i + 1));
10
           while (i + z[i] < n and s[z[i]] == s[i + z[i]
      ]]) {
               x = i; y = i + z[i]; z[i]++;
1.3
      }
14
15
       return z;
16 }
```

Geometry

Point structure

```
Complexity: Does not apply
  Basic 2d point functionality
1 // TITLE: Point structure
2 // COMPLEXITY: Does not apply
_{\rm 3} // <code>DESCRIPTION:</code> Basic 2d point functionality
_{5} // Point/vector structure definition and sorting
7 #define T int
8 \text{ float EPS} = 1e-6;
9 bool eq(T a, T b){ return abs(a-b)<=EPS; }</pre>
11 struct point{
12
      Тх, у;
      point(t x=0, t y=0): x(x), y(y){}
13
       point operator+(const point &o) const{ return {x
1.5
       + o.x, y + o.y; }
       point operator -(const point &o) const{ return \{x\}
16
       - o.x, y + o.y}; }
       point operator*(T k) const{ return {x*k, y*k}; }
       point operator/(T k) const{ return {x/k, y/k}; }
       T operator*(const point &o) const{ return x*o.x +
       T operator^(const point &o) const{ return x*o.y -
20
       y*o.x; }
       bool operator < (const point &o) const{ return (eq(
      x, o.x) ? y < o.y : x < o.x); }
      bool operator == (const point &o) const{ return eq(
22
      x, o.x) and eq(y, o.t); }
       friend ostream & operator << (ostream & os, point p) { ^{8}
24
           return os << "(" << p.x << "," << p.y << ")";
26
                                                              12
29 int ret[2][2] = {{3, 2}, {4, 1}};
                                                              14
30 inline int quad(point p){
                                                              15
      return ret[p.x >= 0][p.y >= 0];
31
                                                              16
32 }
                                                              17
3.3
                                                              18
34 bool comp(point a, point b){
      int qa = quad(a), qb = quad(b);
return (qa == qb ? (a ^ b) > 0 : qa < qb);</pre>
35
                                                              2.0
36
                                                              21
37 }
                                                              22
```

7.2 Lattice Points

```
Complexity: N
  Points with integer coordinate
1 // TITLE: Lattice Points
2 // COMPLEXITY: N
3 // DESCRIPTION: Points with integer coordinate
5 // Picks theorem
_6 // A = area
7 // i = points_inside
8 // b = points in boundary including vertices
9 // A = i + b/2 - 1
11 void solve()
12 {
    int n; cin >> n;
   vector < Point > points(n);
14
  for (int i = 0; i < n; i++) {
```

```
points[i].read();
16
17
18
19
    // Calculatting points on boundary
    int B = 0;
    for (int i =0; i < n; i++) {</pre>
21
      int j = (i + 1) % n;
      Point p = points[j] - points[i];
2.3
      B += \_\_gcd(abs(p.x), abs(p.y)); // Unsafe for 0
24
    }
    // Calculating Area
26
    int a2 = 0;
    for (int i = 0; i < n; i++) {
     int j = (i + 1) % n;
      a2 += points[i] * points[j];
3.0
31
32
    a2 = abs(a2);
    // Picks theorem
3.3
   int I = (a2 - B + 2)/2;
    cout << I << " " << B << endl;
```

7.3 Convex Hull

Complexity: N Gives you the convex hull of a set of points

```
1 // TITLE: Convex Hull
2 // COMPLEXITY: N
 _{\rm 3} // <code>DESCRIPTION:</code> Gives you the convex hull of a set of
       points
 6 struct Point
     int x, y;
     void read()
       cin >> x >> y;
     Point operator - (const Point & b) const
       Point p;
       p.x = x - b.x;
       p.y = y - b.y;
       return p;
     void operator -= (const Point & b)
     {
       x = b.x;
       y -= b.y;
     int operator* (const Point & b) const
       return x * b.y - b.x * y;
     bool operator < (const Point & b) const
       return make_pair(x, y) < make_pair(b.x, b.y);</pre>
39 }:
41 int triangle (const Point & a, const Point & b, const
       Point & c)
42 {
     return (b - a) * (c - a);
```

43

24

25

26

27

29

31

32

3.4

36

37

```
44 }
                                                            40 bool intersect(const Point & p1, const Point & p2,
                                                                  const Point & p3, const Point & p4) {
46 vector < Point > convex_hull(vector < Point > points)
                                                                bool ans = true;
47 - €
                                                                int s1 = triangle(p1, p2, p3);
                                                            42
    vector < Point > hull;
                                                                int s2 = triangle(p1, p2, p4);
    sort(all(points));
49
                                                            44
                                                                if (s1 == 0 && s2 == 0) {
                                                                  int a_min_x = min(p1.x, p2.x);
    for (int z = 0; z < 2; z++) {
5.1
                                                            46
                                                                  int a_max_x = max(p1.x, p2.x);
      int s = hull.size();
52
                                                            47
       for (int i = 0; i < points.size(); i++) {</pre>
                                                                  int a_min_y = min(p1.y, p2.y);
                                                            48
                                                                  int a_max_y = max(p1.y, p2.y);
           while(hull.size() >= s + 2) {
54
                                                            49
               auto a = hull.end()[-2];
                                                            50
               auto b = hull.end()[-1];
                                                                  int b_min_x = min(p3.x, p4.x);
56
                                                            5.1
               if (triangle(a, b, points[i]) <= 0) {</pre>
                                                                  int b_max_x = max(p3.x, p4.x);
                                                            52
58
                   break;
                                                           5.3
                                                                  int b_min_y = min(p3.y, p4.y);
                                                                  int b_max_y = max(p3.y, p4.y);
59
                                                            54
60
               hull.pop_back();
                                                            55
                                                                  if (a_min_x > b_max_x || a_min_y > b_max_y) {
                                                                    ans = false;
6.1
                                                           56
           hull.push_back(points[i]);
                                                           57
                                                                  if (b_min_x > a_max_x || b_min_y > a_max_y) {
63
                                                           5.8
      hull.pop_back();
                                                           59
                                                                    ans = false;
64
65
       reverse(all(points));
                                                           60
                                                           6.1
                                                                  return ans;
66
    return hull;
                                                                }
                                                           62
                                                                int s3 = triangle(p3, p4, p1);
                                                           63
                                                           64
                                                                int s4 = triangle(p3, p4, p2);
                                                           65
                                                                if ((s1 < 0) && (s2 < 0)) ans = false;
         Line Intersegment
                                                           66
                                                                if ((s1 > 0) \&\& (s2 > 0)) ans = false;
                                                            67
                                                                if ((s3 < 0) \&\& (s4 < 0)) ans = false;
  Complexity: O(1)
                                                            68
                                                                if ((s3 > 0) \&\& (s4 > 0)) ans = false;
                                                            69
  Check if two half segments intersect with which other
                                                           7.0
                                                                return ans;
                                                            71 }
1 // TITLE: Line Intersegment
2 // COMPLEXITY: O(1)
3 // DESCRIPTION: Check if two half segments intersect
      with which other
                                                                   Algorithms
5 struct Point
6 {
                                                              8.1
                                                                    Sparse table
    int x, y;
                                                              Complexity: O(n \log(n)) preprocessing, O(1) query
    void read()
1.0
                                                              Computes the minimum of a half open interval.
      cin >> x >> y;
12
                                                            1 // TITLE: Sparse table
                                                            2 // COMPLEXITY: O(n log(n)) preprocessing, O(1) query
13
                                                            3 // DESCRIPTION: Computes the minimum of a half open
14
    Point operator - (const Point & b) const
                                                                  interval.
1.5
16
      Point p;
      p.x = x - b.x;
17
                                                            5 struct sptable {
      p.y = y - b.y;
                                                                  vector < vi> table;
18
                                                            6
19
      return p;
                                                                  int ilog(int x) {
20
                                                                       return (__builtin_clzll(111) -
21
    void operator -= (const Point & b)
                                                                   __builtin_clzll(x));
22
23
    {
                                                            10
      x -= b \cdot x;
24
      y -= b.y;
                                                                   sptable(vi& vals) {
25
                                                            12
                                                                       int n = vals.size();
27
                                                           14
                                                                       int ln= ilog(n)+1;
    int operator* (const Point & b) const
                                                                       table.assign(ln, vi(n));
28
                                                            15
29
                                                            16
      return x * b.y - b.x * y;
                                                                       rep(i,0,n) table[0][i]=vals[i];
30
                                                            17
31
                                                            18
                                                                       rep(k, 1, ln) {
32
                                                            1.9
                                                                           rep(i,0,n) {
33 }:
                                                                               table[k][i] = min(table[k-1][i],
34
35 int triangle(const Point & a, const Point & b, const 22
                                                                               table [k-1] [min(i + (1<<(k-1)), n-1)])
      Point & c)
36 {
                                                                           }
                                                            23
     return (b - a) * (c - a);
                                                                       }
                                                            24
```

2.5

26

}

38 }

```
// returns minimum of vals in range [a, b)
                                                         2.0
2.7
28
      int getmin(int a, int b) {
                                                         21
                                                                    curr.push_back(s[i]);
          int k = ilog(b-a);
29
                                                         22
          return min(table[k][a], table[k][b-(1<<k)]); 23
                                                                add(curr);
30
31
                                                                return ans;
32 };
                                                         25 }
                                                         27 vector<int> parse_vector_int(string & s)
                                                         28 {
      Parser
                                                         29
                                                                vector < int > nums;
                                                                for (string x: split_string(s)) {
                                                         30
                                                         31
                                                                    nums.push_back(stoi(x));
       Parsing Functions
                                                         32
                                                                return nums;
                                                         33
  Complexity:
                                                         34 }
                                                         35
                                                         36 vector<float> parse_vector_float(string & s)
1 // TITLE: Parsing Functions
                                                         37 {
                                                                vector < float > nums;
3 vector<string> split_string(const string & s, const
                                                                for (string x: split_string(s)) {
                                                         39
      string & sep = " ") {
                                                                    nums.push_back(stof(x));
                                                          40
      int w = sep.size();
                                                          41
      vector<string> ans;
                                                         42
                                                                return nums;
      string curr;
                                                         43 }
                                                         44
      auto add = [&](string a) {
                                                         45 void solve()
          if (a.size() > 0) {
                                                         46 {
9
              ans.push_back(a);
                                                                cin.ignore();
                                                         47
10
11
                                                         48
                                                                string s;
                                                                getline(cin, s);
     };
12
                                                         49
13
                                                         50
     for (int i = 0; i + w < s.size(); i++) {
                                                                auto nums = parse_vector_float(s);
14
                                                         51
          if (s.substr(i, w) == sep) {
                                                         52
                                                                for (auto x: nums) {
15
              i += w-1;
                                                         53
                                                                    cout << x << endl;</pre>
              add(curr);
17
                                                         54
```

55 }

curr.clear();

continue;