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Homework 9

1)

**Loop Invariant:** s = 1 + 2 + 3 + … + (k – 1)

In other words, s is a summation of k numbers increasing by 1 until reaching a value of (k-1).

**Correctness:**

Initialization

The loop invariant is true before any iterations of the while loop because s = (k-1) = 0 as k at this point is equal to 1.

Maintenance

Assuming the loop invariant is true for any given value for k <= n, it will also be true for any k + 1 <= n as s = 1 + 2 + 3 + … + (k-1) + (k – 1 + 1) or, s = 1 + 2 + 3 + … + (k-1) + k.

Termination

When the loop is about to terminate, s will equal 1 + 2 + 3 + … + n as the while loop runs while k <= n, and k will always reach n exactly since its only ever incremented by 1 and n >= 0.

2)

**Loop Invariant:**

s = 1^2 + 2^2 + 3^2 + … + c ^ 2

In other words, s is a summation of c numbers squared increasing by 1 until reaching a value of (c^2).

**Correctness:**

Initialization

The loop invariant is true before any iterations of the while loop because s = (c^2) = 0 as c at this point is equal to 0.

Maintenance

Assuming the loop invariant is true for any given value for c, it will also be true for any c + 1 as

s = 1^2 + 2^2 + 3^2 + … + c^2 + (c+1)^2 and c + 1 = k. So, as s = s + k \* k, the loop will always terminate before s could surpass 1000 by too much and violate its post condition as a result of c + 1.

Termination

When the loop is about to terminate, s will still equal 1^2 + 2^2 + 3^2 + … + c^2 as the while loop will terminate at some point where s >1000.

3)

**Loop Invariants:**

Outer Loop: m = 2 | x – a | + 1

Inner Loop: s >= 0

**Correctness for Outer Loop:**

Initialization

The loop invariant before any iterations of the while loop gives m = 2 | 10 - 10 | + 1 = 1. On the first pass of the outer loop before reaching the inner loop, m, which began as 0, is incremented by one. So, this is true.

Maintenance

Assuming the loop invariant is true for any given value for x > 0 and a >= 0, it will also be true for any x – 1. Note that a is a static number. So the loop invariant becomes m = 2 | (x-1) – a | + 1.

Termination

When the loop is about to terminate, x will equal 1, as it is only ever decremented by one. So the loop invariant becomes m = 2 | 1 – a | + 1 or m = 2(a-1) + 1.

**Correctness for Inner Loop:**

Initialization

The loop invariant before any iterations of the while loop is true as s initially equals 0, so s >= 0.

Maintenance

Assuming the loop invariant is true before the while loop iterates, it will be true as the while loop iterates as well as s only increments as m decreases.

Termination

When the loop is about to terminate, s will equal the initial value of m.