Dynamic Portfolio Optimization Across Asset Classes using Multi-variate Volatility Modelling

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1. Introduction

The idea of asset co-movements is important to investors because they provide insight into the underlying relationships between assets and the risk and return characteristics of a portfolio, which can inform investment decisions and help to improve portfolio performance. In particular, portfolio diversification helps to reduce the risk of large losses by spreading investments across a variety of assets and can lead to increased returns by providing exposure to a wider range of economic and market conditions (Brière, Chapelle & Szafarz, 2012).

A large body of literature has investigated the effect of the Global Financial Crisis (GFC) on the relationship across various asset classes (Mensi, Hammoudeh, Nguyen & Kang, 2016; Zhang & Broadstock, 2020). These studies have shown how the GFC has intensified market relations and affected asset allocation and hedging strategies. As such, fluctuations in interdependence across financial markets can greatly affect investors seeking diversification benefits.

Several studies have examined the time-varying correlations between asset classes in South Africa using various techniques. Bouri, Cepni, Gabauer & Gupta (2021), for example, studied the dynamic connectedness across five assets using the TVP-VAR connectedness approach. Bouri *et al.* (2021) shows that total connectedness spiked around the COVID-19 outbreak, which subsequently altered the structure of the network of connectedness and posed a threat to investors' portfolios. Horvath & Poldauf (2012) studied the conditional correlation between sectors of numerous economies, including South Africa, using the BEKK MV-GARCH approach. Duncan & Kabundi (2013) used generalised variance decompositions of a vector autoregressive model to study the time-varying domestic and foreign volatility spillovers and evolving linkages across asset classes in South Africa and global capital

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markets.

The analysis in this paper is divided into two parts. First, I will analyse the time-varying conditional correlations between the different asset classes by using Engle (2002) Dynamic Conditional Correlation (DCC) Model as well as a GO-GARCH model. DCC models are statistical models used in finance and economics to capture the changing correlations between multiple time series. These models allow for changes in the correlations between time series over time, which is important in modelling the behaviour of assets that are dependent on each other (Engle, 2002). DCC models use a multivariate GARCH framework to capture the dynamics of the covariance matrix of the series over time.

The second part of the analysis will focus on portfolio optimization. Portfolio optimization is the process of determining the optimal allocation of assets in a portfolio in order to maximize returns or minimize risk. This typically involves using mathematical models to identify the portfolio that offers the best trade-off between risk and return, taking into account constraints such as investment budgets and risk tolerance (Ledoit & Wolf, 2003). The optimization process takes into account factors such as expected returns, volatilities, and correlations between assets in order to determine the optimal portfolio mix.

2. Methodology

2.1. DCC specification

The method behind DCC models is based on the idea of modeling the covariance matrix of the returns of multiple assets over time. For Engle (2002) DCC model the variance-covariance matrix can be written as [1]:

$$H_t = D_t.R_t.D_t. \tag{2.1}$$

Estimating R_T now requires it to be inverted at each estimated period, and thus a proxy equation is used:

$$\begin{aligned} Q_{ij,t} &= \bar{Q} + a\left(z_{t-1}z'_{t-1} - \bar{Q}\right) + b\left(Q_{ij,t-1} - \bar{Q}\right) \\ &= (1-a-b)\bar{Q} + az_{t-1}z'_{t-1} + b.Q_{ij,t-1} \end{aligned} \tag{2.2}$$

[1] The paper was written using the 'Texevier' package developed by N.F. Katzke (2016)

With non-negative scalars a and b, and with:

- $Q_{ij,t}$ the unconditional (sample) variance estimate between series i and j ,
- $\bar{Q}$ the unconditional matrix of standardized residuals from each univariate pair estimate.

We next estimate R_T as:

$$R_t = diag(Q_t)^{-1/2} Q_t . diag(Q_t)^{-1/2}. \tag{2.3}$$

The dynamic conditional correlation matrix, will therefore have entries in the bivariate framework as follows:

$$R_t = \rho_{ij,t} = \frac{q_{i,j,t}}{\sqrt{q_{ii,t} \cdot q_{jj,t}}} \tag{2.4}$$

2.2. Go-GARCH specification

The methodology behind Go-GARCH models is based on a combination of the GARCH (Generalized Autoregressive Conditional Heteroskedasticity) framework and the concept of orthogonalization. Orthogonalization is used to ensure that the parameters of the model are estimated in an uncorrelated and unbiased manner. This can help to improve the stability and accuracy of the model's predictions.

These orthogonal components are measured by identifying independent and uncorrelated factors that make up the var-covar matrix H_t. The statistical transformations are done as follows:

$$r_t = \mu_t + \varepsilon_t \tag{2.5}$$

$$\varepsilon_t = A.f_t \tag{2.6}$$

with A linking the unobserved uncorrelated components with the observed residual process. Also: f_t represents the unobserved independent factors assigned to each series (factor weights), such that:

$$f_t = H^{1/2}.z_t \tag{2.7}$$

with H_T and z_t as before and:

$$\begin{aligned} E[f_t] = 0; \qquad & E[f_t f_t'] = I_N \\ E[\varepsilon_t] = 0; \qquad & E[\varepsilon_t \varepsilon_t'] = AA' \end{aligned} \tag{2.8}$$

So that the conditional covariance matrix is given by:

$$\Sigma_t = AH_tA' \tag{2.9}$$

3. Data

My objective is to characterise the time-varying correlation estimates to provide insight into the underlying comovement structures of a portfolio of asset classes. The empirical application in this paper aims to diversify a portfolio investing in stocks, property, gold, and Bitcoin. This study collects daily price data on the FTSE/JSE Africa All Share Index, FTSE/JSE Africa Property Index, gold, and Bitcoin [2]. The sample period of this study covers data from the beginning of 2015 to the end of 2019. Considering a time series of prices, the daily asset returns are calculated by taking the log difference of each index series, as:

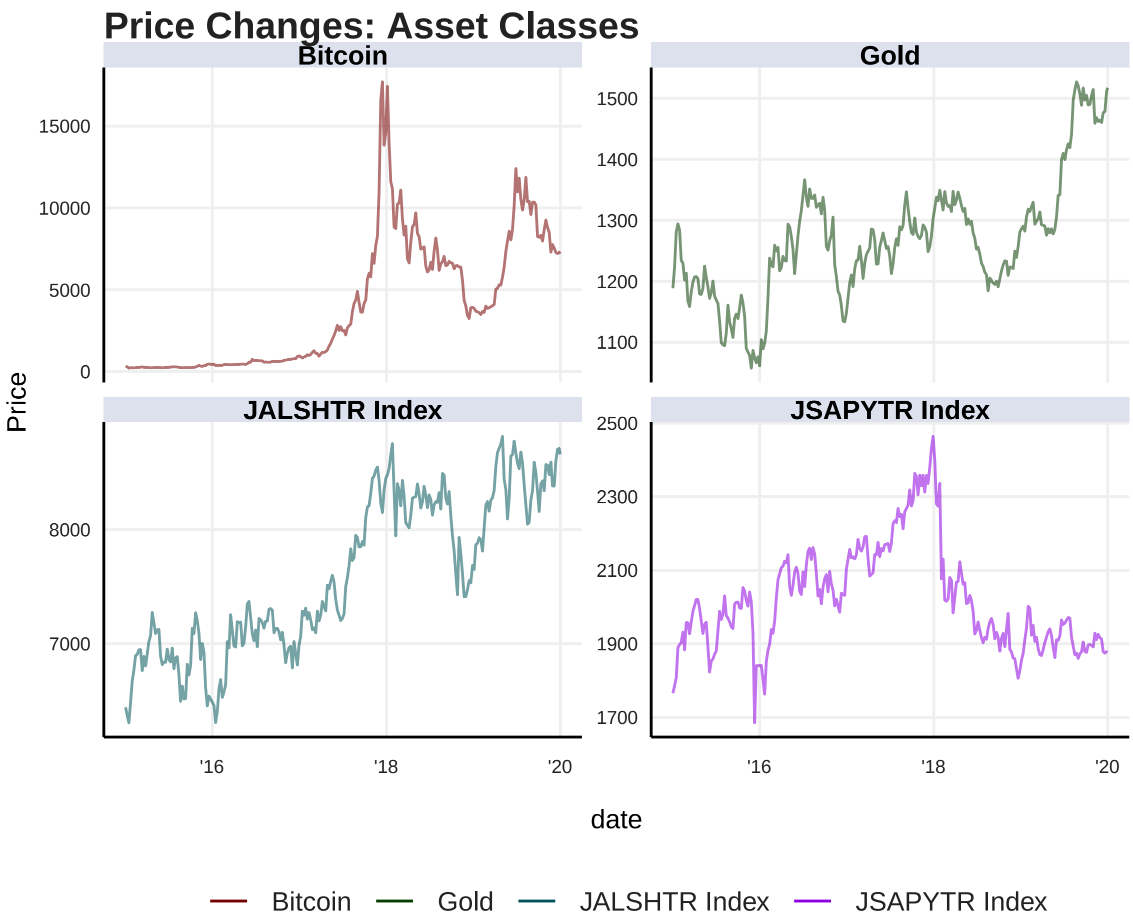
$$r_{i,t} = ln(\frac{P_{i,t}}{P_{i,t-1}}) * 100 \tag{3.1}$$

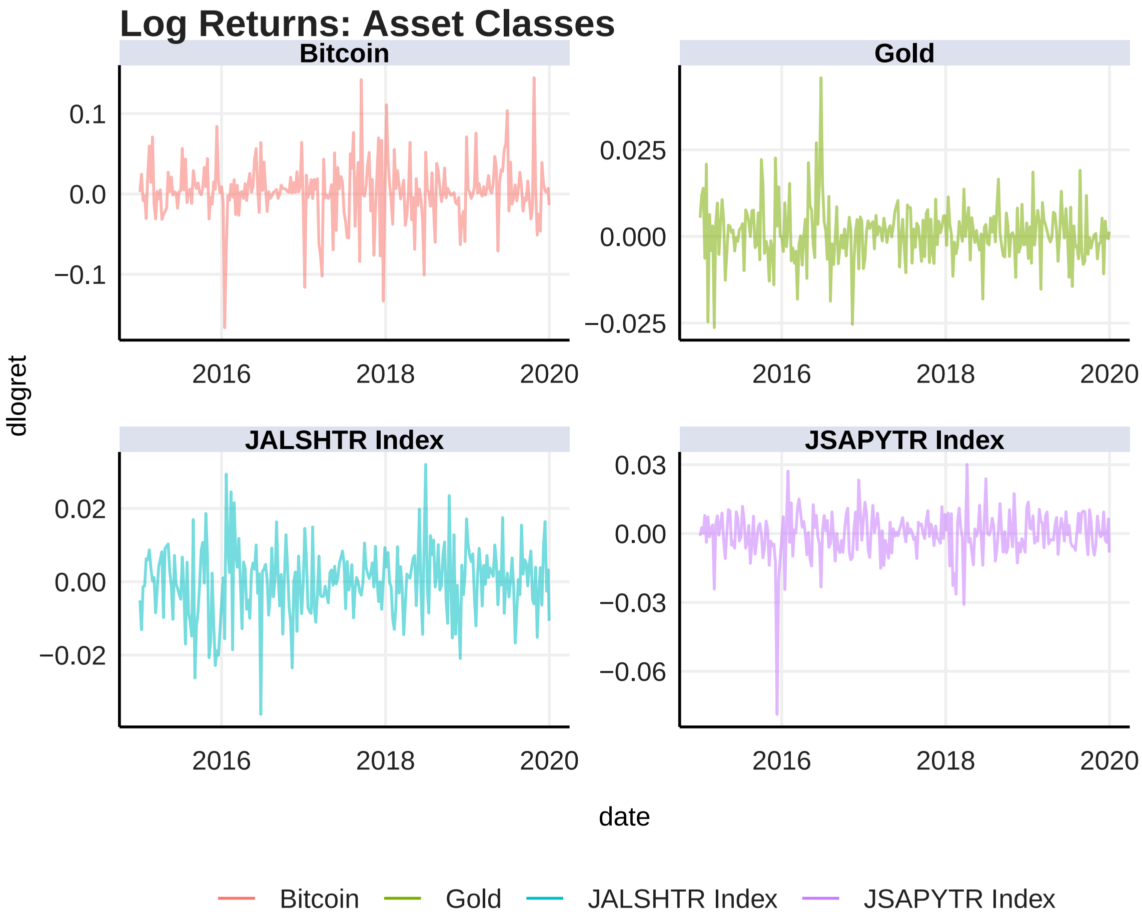
Where $P_{i,t}$ is the closing price of the index, i, at time t. Logarithmic returns are usually preferred by financial econometricians because of their superior properties compared to arithmetic returns.

In order to analyse the characteristics of the data for the return series, I first produce a chart of the price and return series. The two figures below shows the closing prices and returns of the four

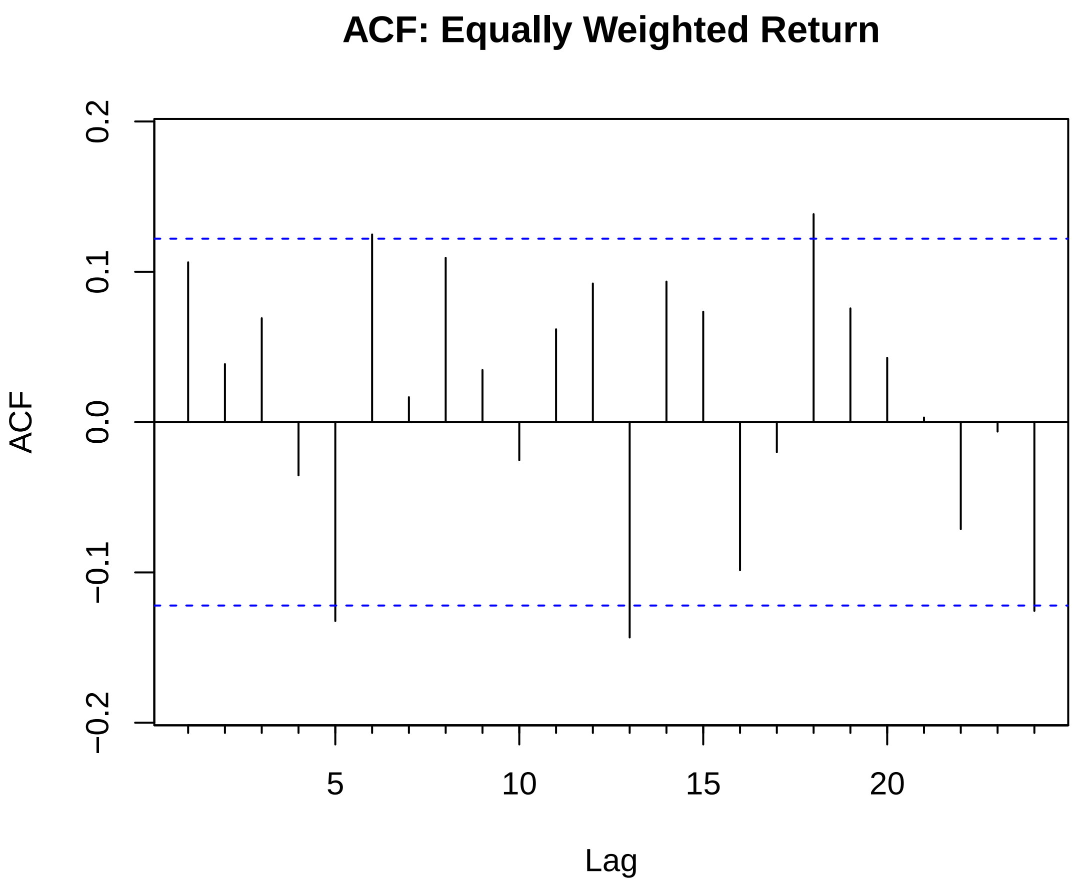
[2] All the data was obtained from the 'fmxdat' package in R

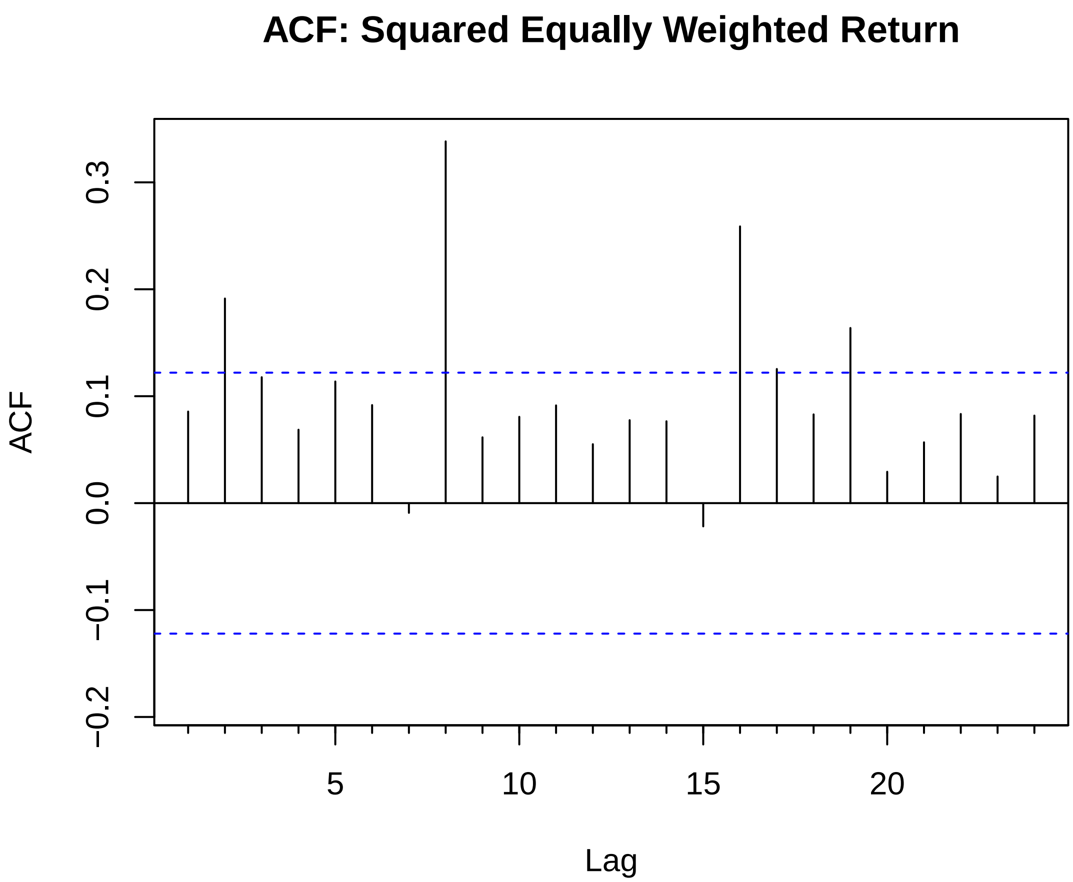
asset classes, respectively. We can see from the second figure that the return series exhibit volatility clustering.

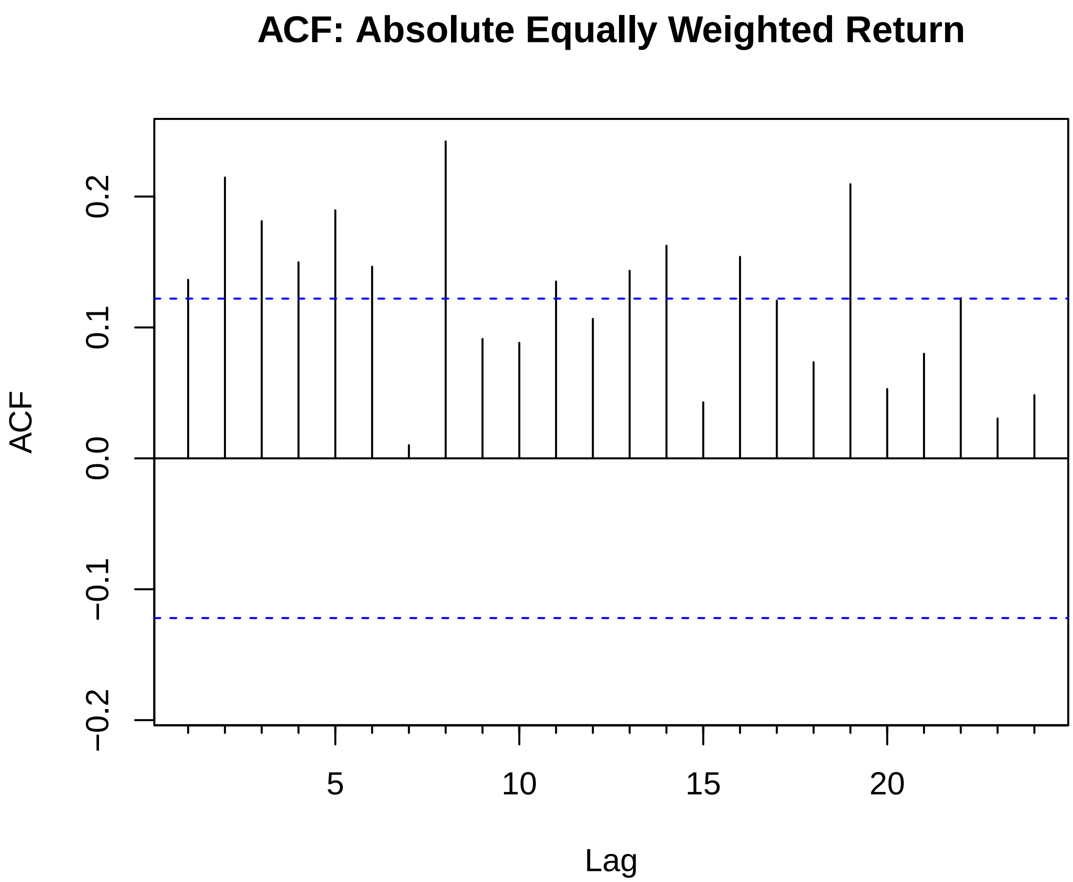




Next, I examine the Autoregressive Conditional Heteroskedasticity (ARCH) effects. To visualize the analysis of autocorrelations in the residuals and squared residuals, I graph the returns, squared returns and the absolute returns for an equally weighted portfolio of the assets using simple returns.


ACF: Equally Weighted Return
ACF
0.2
0.1
0.0
−0.1
−0.2
5
10
15
20
Lag

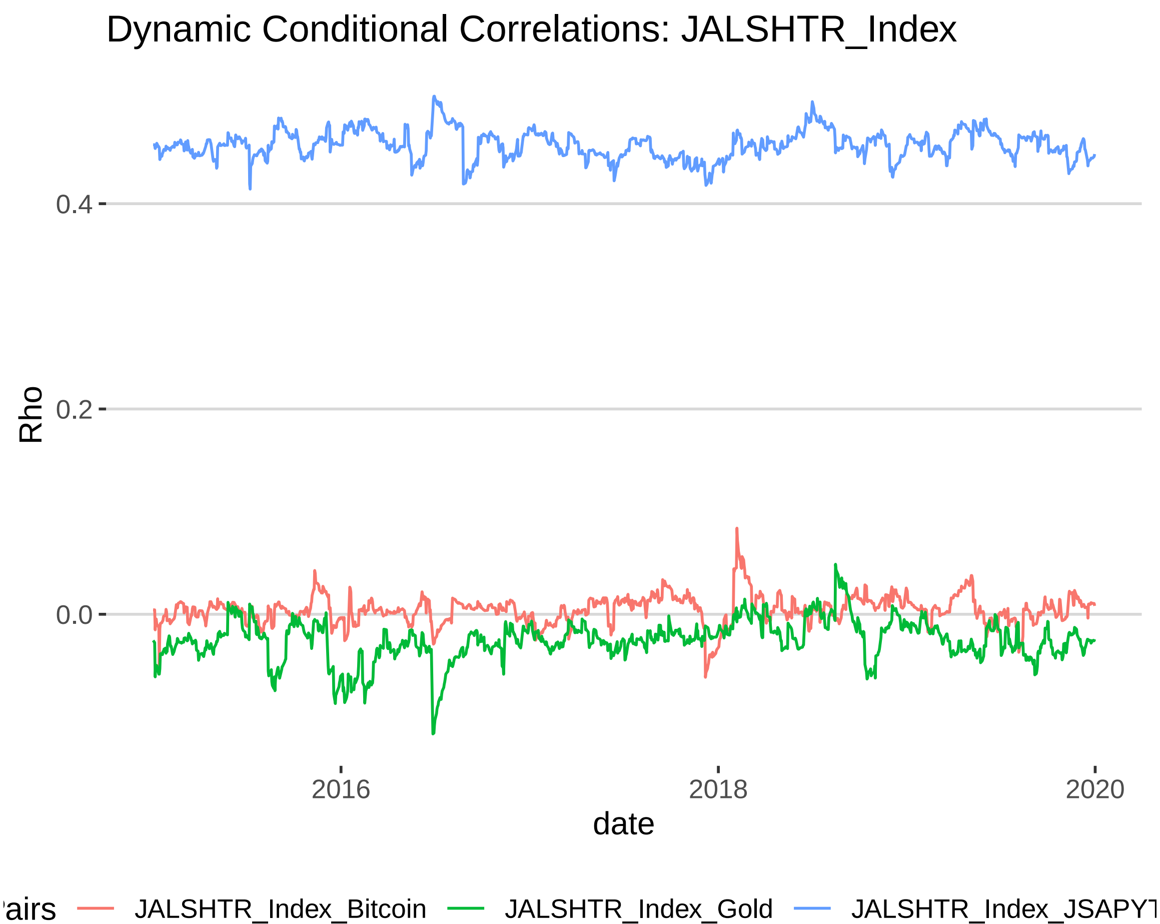

ACF: Squared Equally Weighted Return
ACF
Lag
0.3
0.2
0.1
0.0
−0.1
−0.2
5
10
15
20

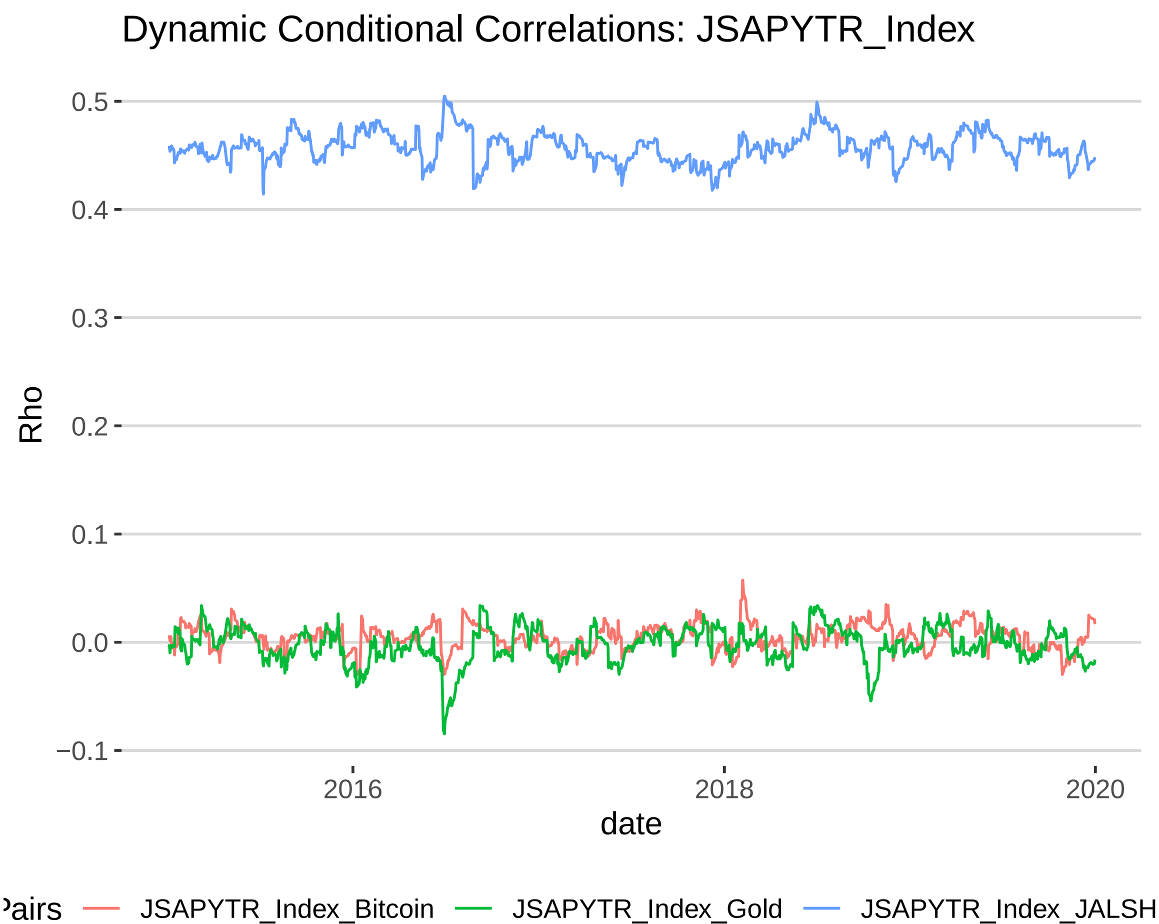


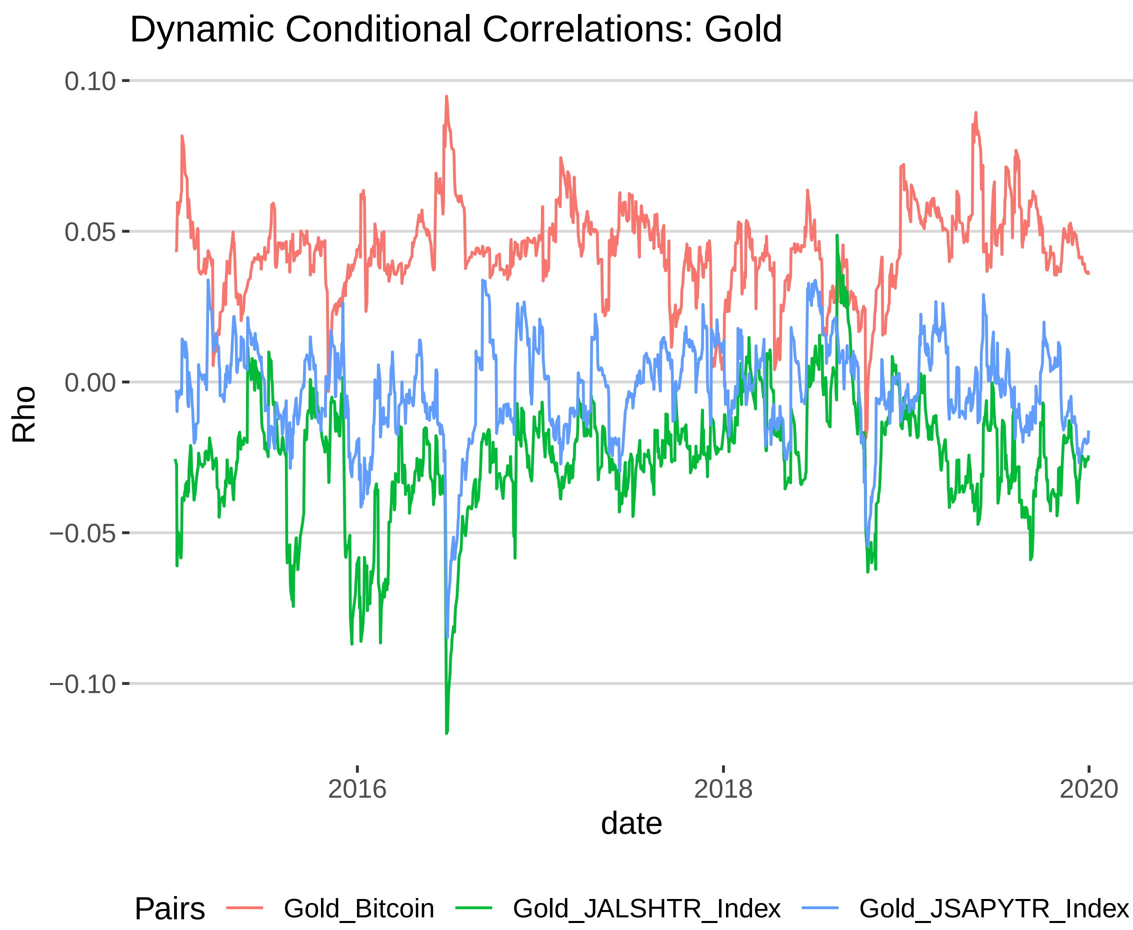
The above figure displays a pattern of squared residuals, which indicates strong conditional heteroskedasticity. In addition, I formally test for ARCH effects using a Ljung–Box test which rejected the null of no ARCH effects - hence I need to control for the remaining conditional heteroskedasticity in the returns series.

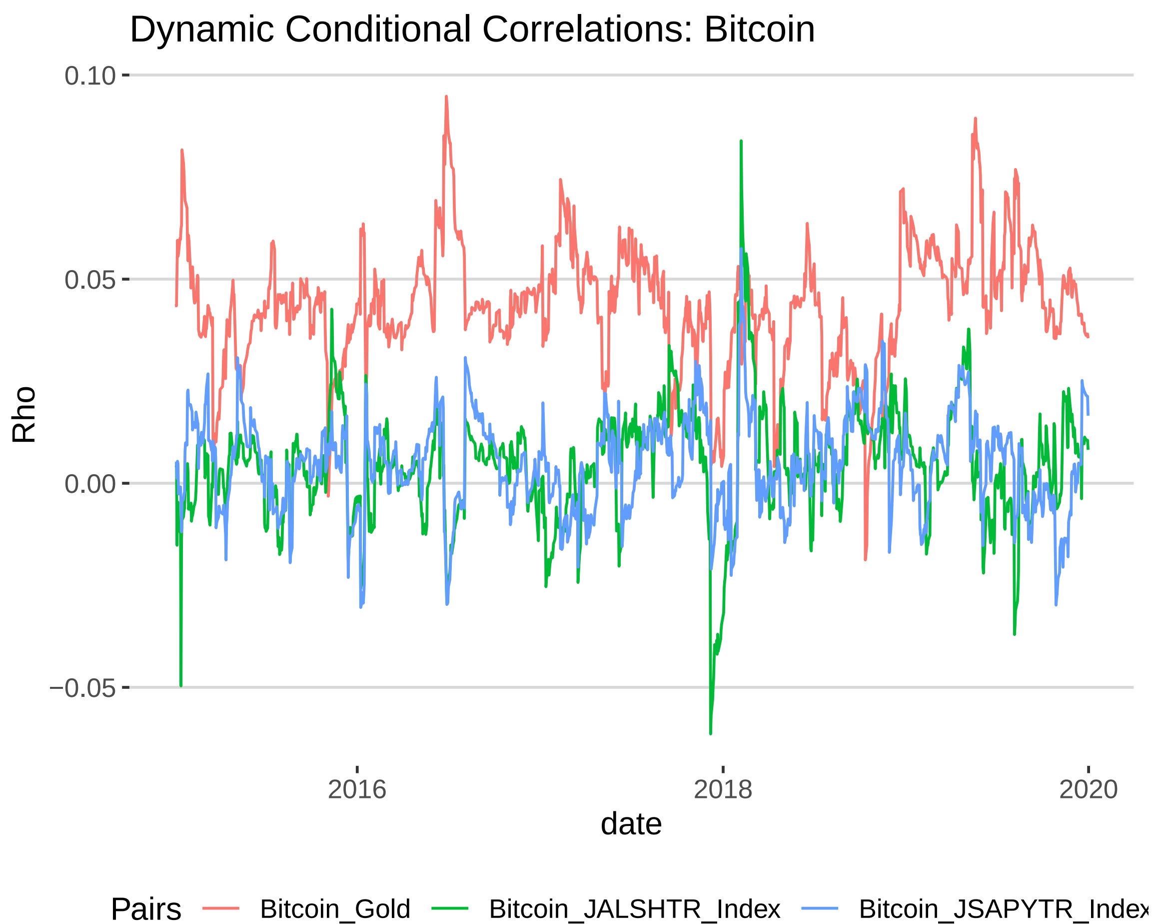
4. DCC model

Before fitting the most appropriate univariate GARCH specifications to each series, I check and clean the data for any missing values and outliers. Thereafter, I specify a GARCH model for each time series, which is used as the conditional covariance structure in the DCC model. Finally, I compute the dynamic correlations between each pair of assets.


Dynamic Conditional Correlations: JALSHTR_Index
Rho
0.4
0.2
0.0
2016
2018
2020
date
airs
JALSHTR_Index_Bitcoin
JALSHTR_Index_Gold
JALSHTR_Index_JSAPY


Dynamic Conditional Correlations: JSAPYTR_Index
0.5
0.4
0.3
0.2
0.1
0.0
−0.1
Rho
2016
2018
2020
date
Pairs
JSAPYTR_Index_Bitcoin
JSAPYTR_Index_Gold
JSAPYTR_Index_JALSH


Dynamic Conditional Correlations: Gold
0.10
0.05
0.00
−0.05
−0.10
Rho
2016
2018
2020
date
Pairs
Gold_Bitcoin
Gold_JALSHTR_Index
Gold_JSAPYTR_Index

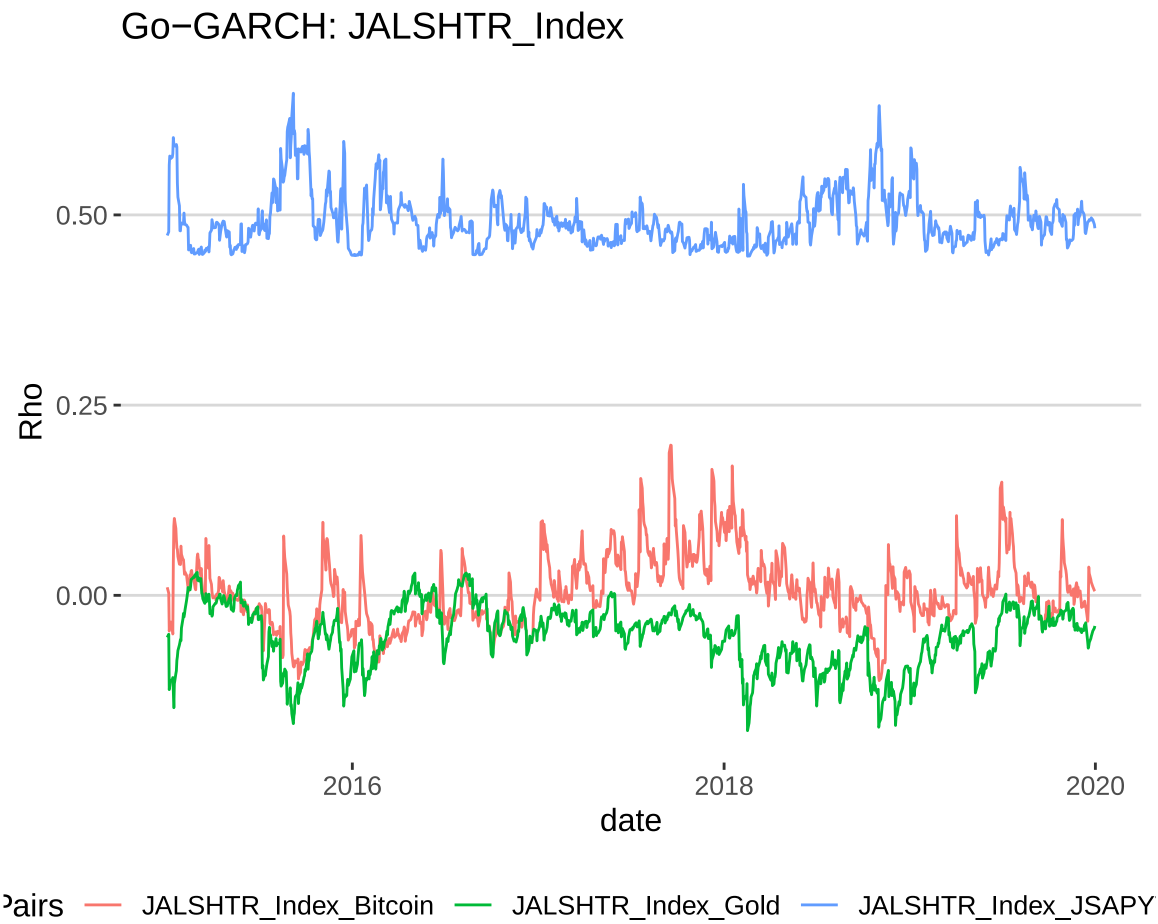


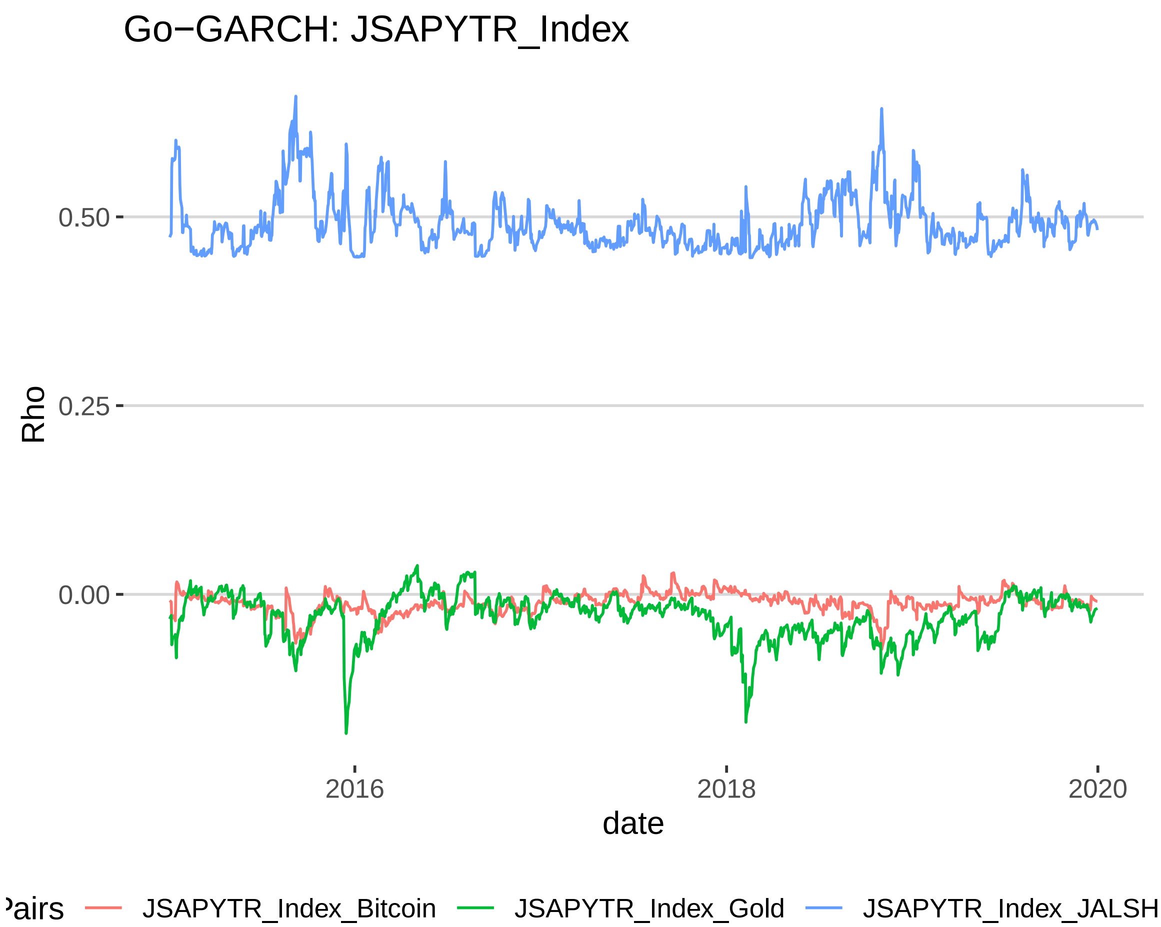
The figures above illustrate the evolution of correlation processes over time. Some asset pairs show correlations that are very volatile and some that flow from positive to negative, or from negative to positive. It indicates that the diversification potential changes over time.

5. Go-GARCH

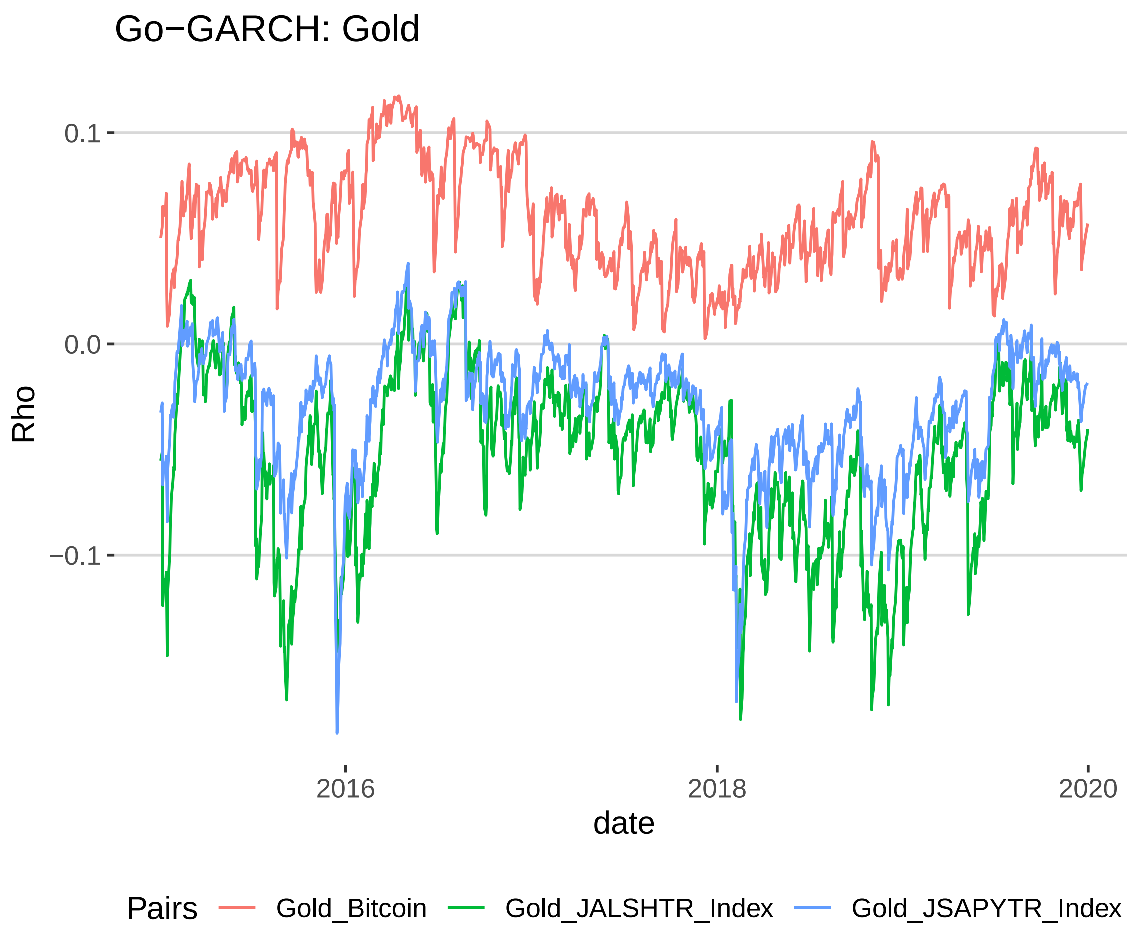
Generalized Orthogonal GARCH (GO-GARCH) models are used to model and forecast volatility in financial time series data. GO-GARCH models are an extension of the popular GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models and combine features of both ARCH (Autoregressive Conditional Heteroskedasticity) and GARCH models. GO-GARCH models are characterized by using orthogonal (uncorrelated) innovations in the GARCH process, which results in a reduced number of parameters compared to traditional GARCH models (Boswijk & Weide, 2006). This makes the GO-GARCH models computationally more efficient and easier to estimate. Additionally, the use of orthogonal innovations can lead to better forecasting performance, especially in situations where the underlying volatility process is highly non-linear.

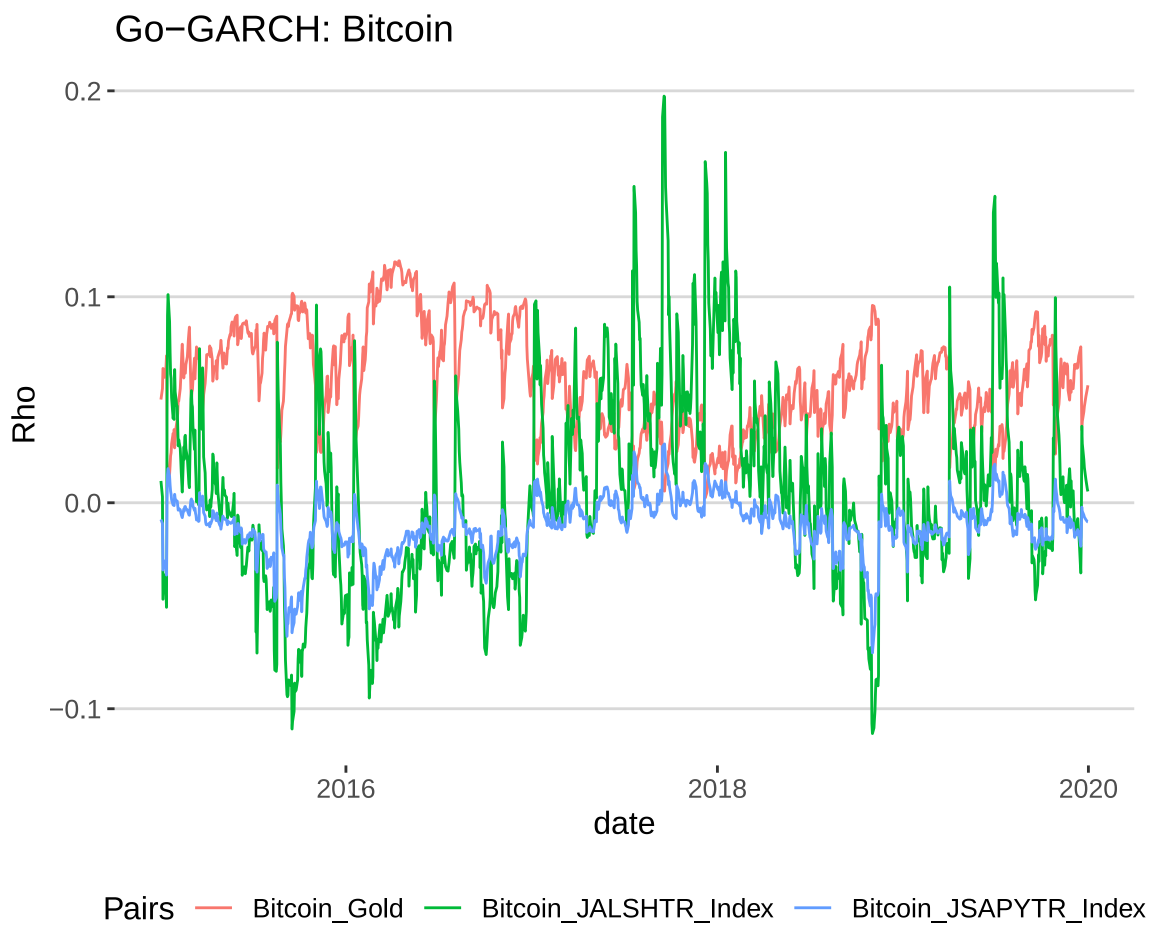
The figure below depict the time-varying correlations between the asset classes using a GO-GARCH model.





Go−GARCH: JSAPYTR_Index
Rho
0.50
0.25
0.00
2016
2018
2020
date
Pairs
JSAPYTR_Index_Bitcoin
JSAPYTR_Index_Gold
JSAPYTR_Index_JALSH


Go−GARCH: Gold
0.1
0.0
−0.1
Rho
2016
2018
2020
date
Pairs
Gold_Bitcoin
Gold_JALSHTR_Index
Gold_JSAPYTR_Index

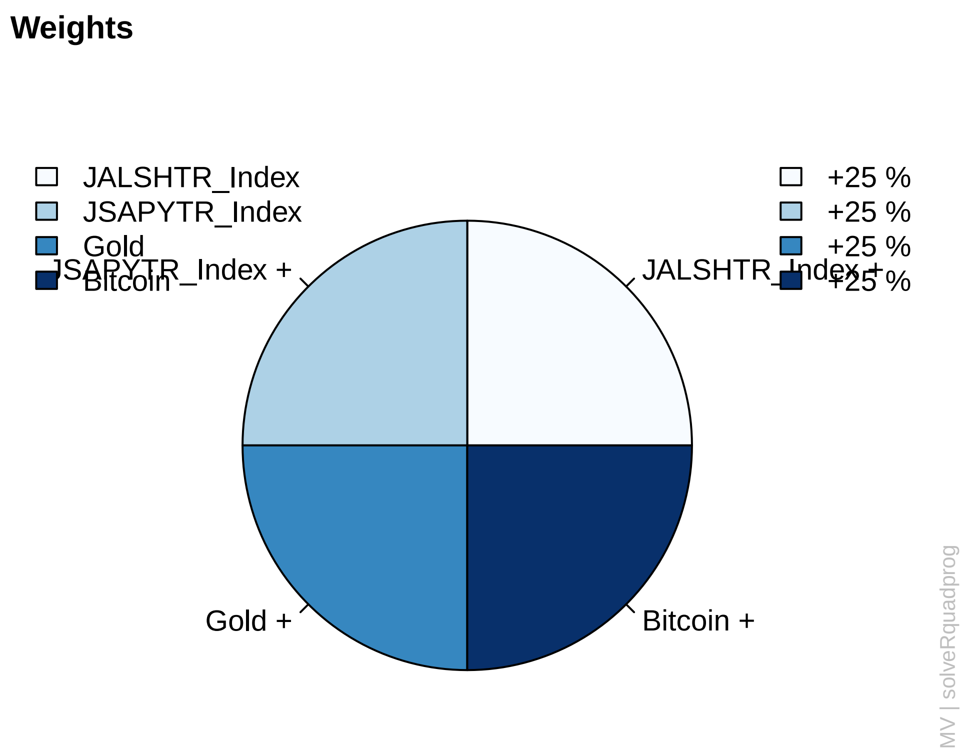


The most noticeable difference between the GO-GARCH correlations and the DCC correlations is the range in which they vary. Similar to Boswijk & Weide (2006) I find that the GO-GARCH and DCC correlation patterns are similar, however, from my analyse I do not find that the GO-GARCH model behaves like a smoothed version of the DCC model. Furthermore, the difference between these two models should be interpreted with caution since this property may be favourable to some and not others.

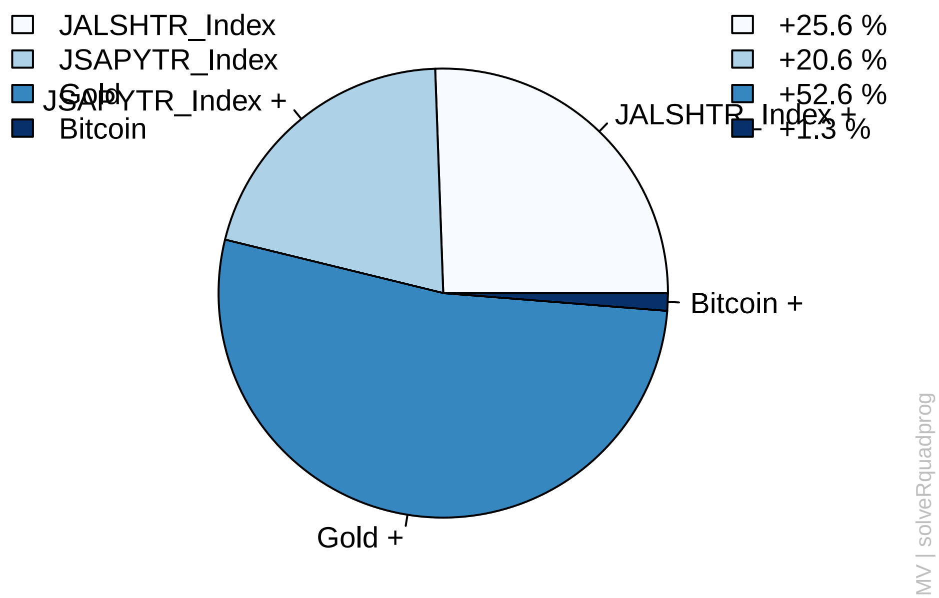
6. Portfolio optimization

In the final section of this paper, I use the data on equities, property, gold and Bitcoin to optimize three portfolios with a linear constraint of long only. In particular, I construct an equal weight portfolio, a global minimum variance portfolio, and a tangency portfolio. The equal weight portfolio gives equal weights of 25 percent to each of the assets in the portfolio. The global minimum portfolio refers to a portfolio that has the minimum possible risk or variance among all possible portfolios constructed from the set of assets. In other words, it is the portfolio with the lowest possible risk among all portfolios with a given expected return. Finally, the tangency portfolio is a portfolio that lies on the efficient frontier and is the portfolio with the highest Sharpe ratio. The Sharpe ratio is a measure of the risk-adjusted return of an investment, defined as the excess return over the risk-free rate divided by the standard deviation of the investment's returns (Sharpe, 1966).

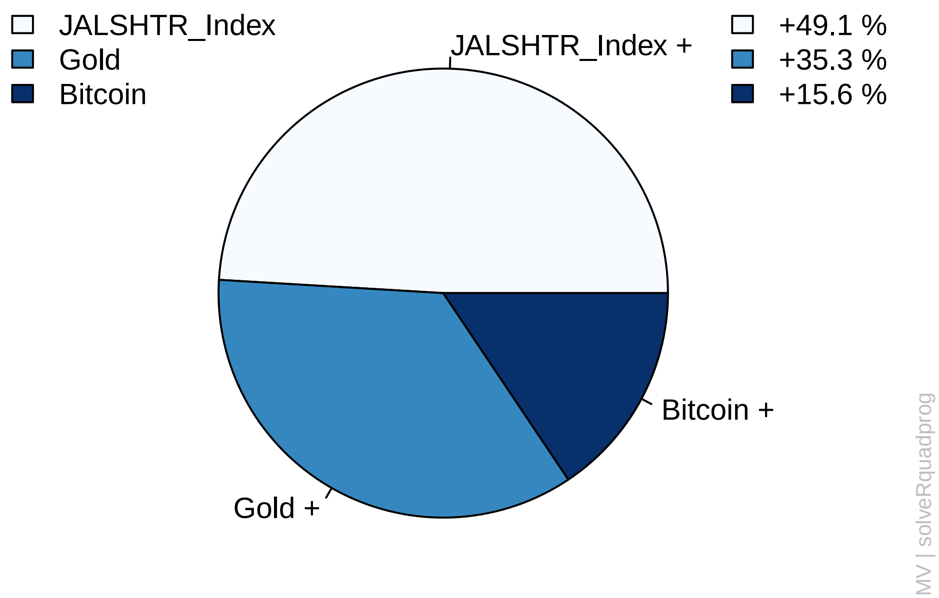
The pie charts below illustrate the optimal weighting as defined by the equally weighted, global minimum, and tangency portfolios', respectively. As expected, we see that Bitcoin receives a very small weighting (1.3%) in the global minimum portfolio since it is deemed high risk.

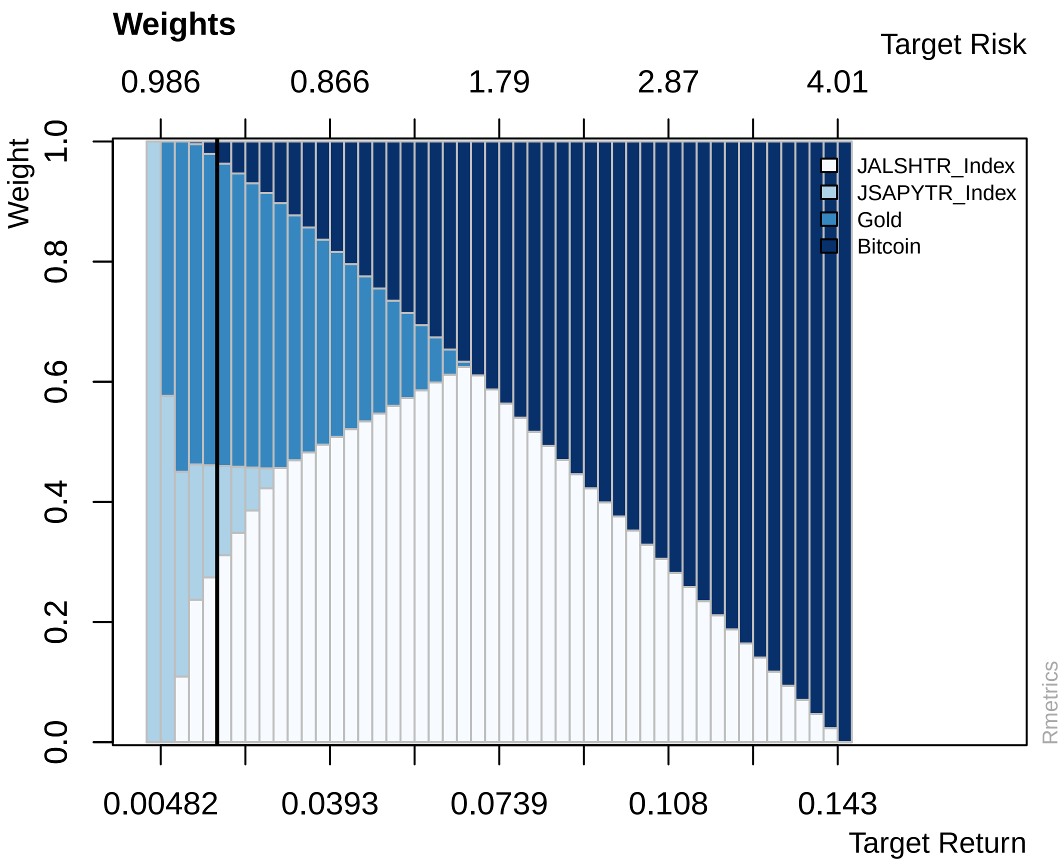


Weights


JALSHTR_Index
JSAPYTR_Index
Gold
Bitcoin
+25.6 %
+20.6 %
+52.6 %
+1.3 %
JSAPYTR_Index +
JALSHTR_Index +
Bitcoin +
Gold +
MV | solveRquadprog

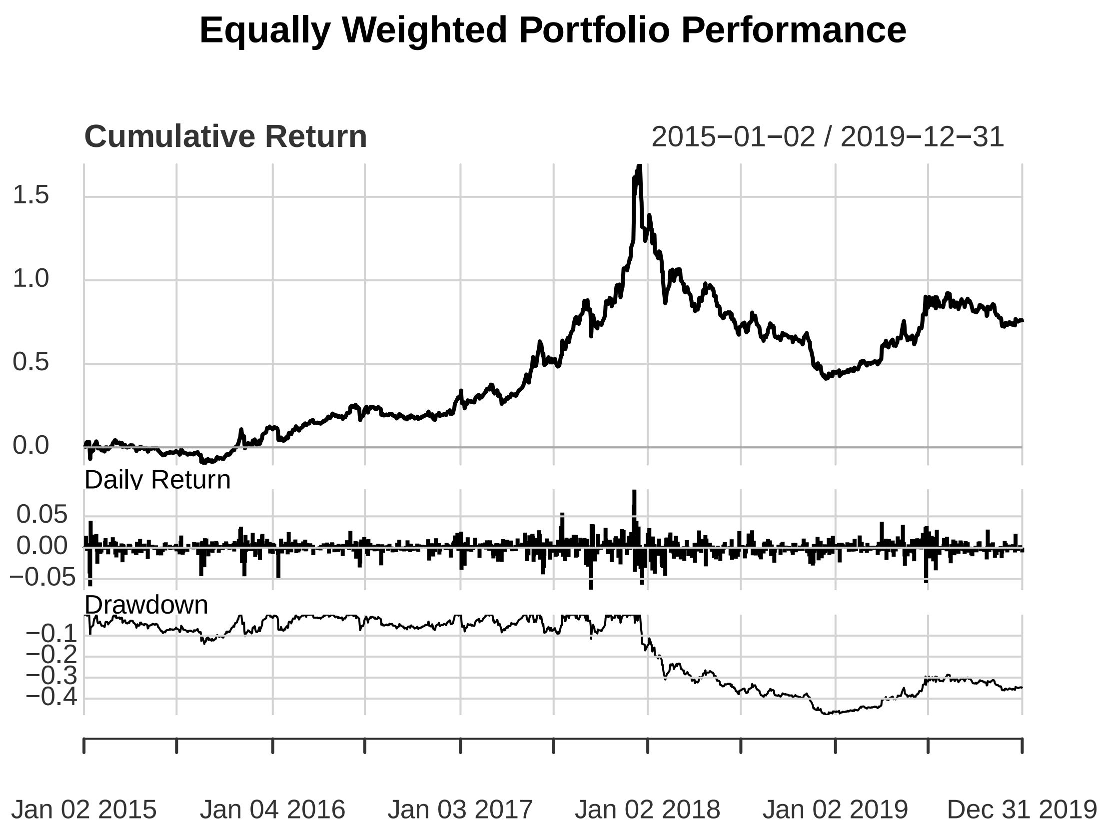
Weights


JALSHTR_Index
Gold
Bitcoin
JALSHTR_Index +
+49.1 %
+35.3 %
+15.6 %
Bitcoin +
Gold +
MV | solveRquadprog

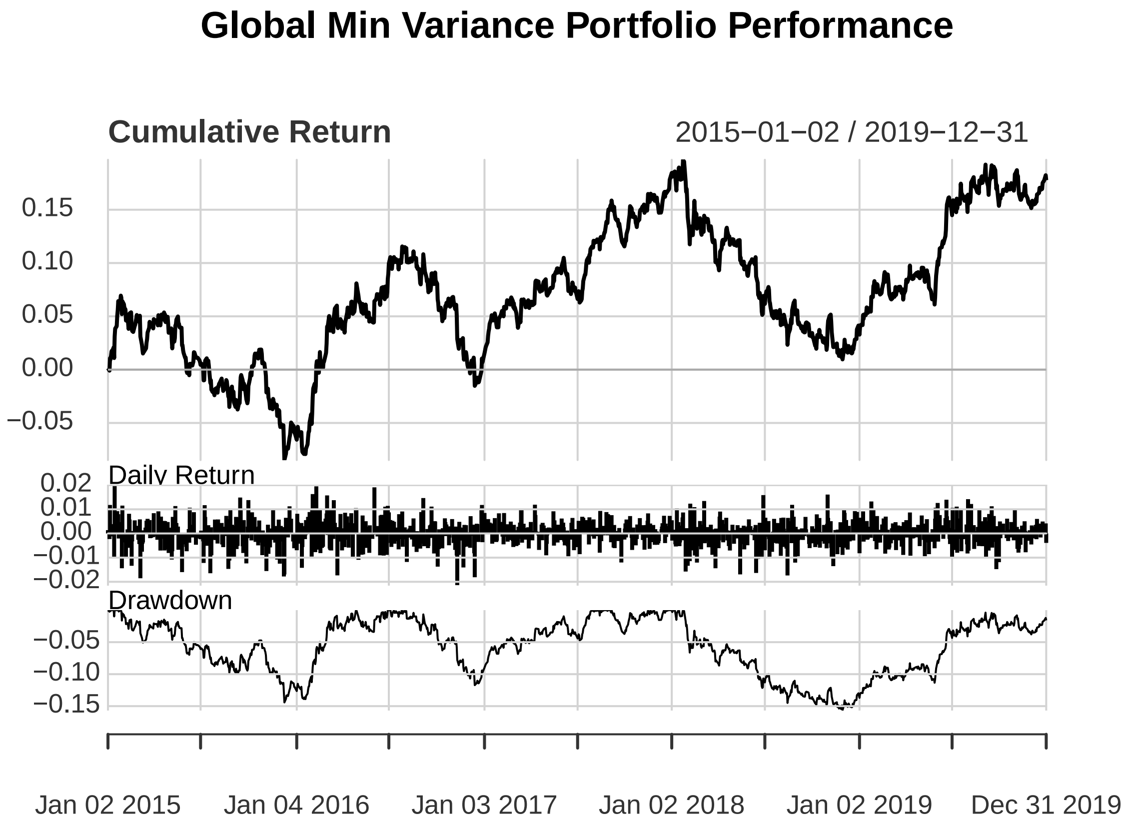


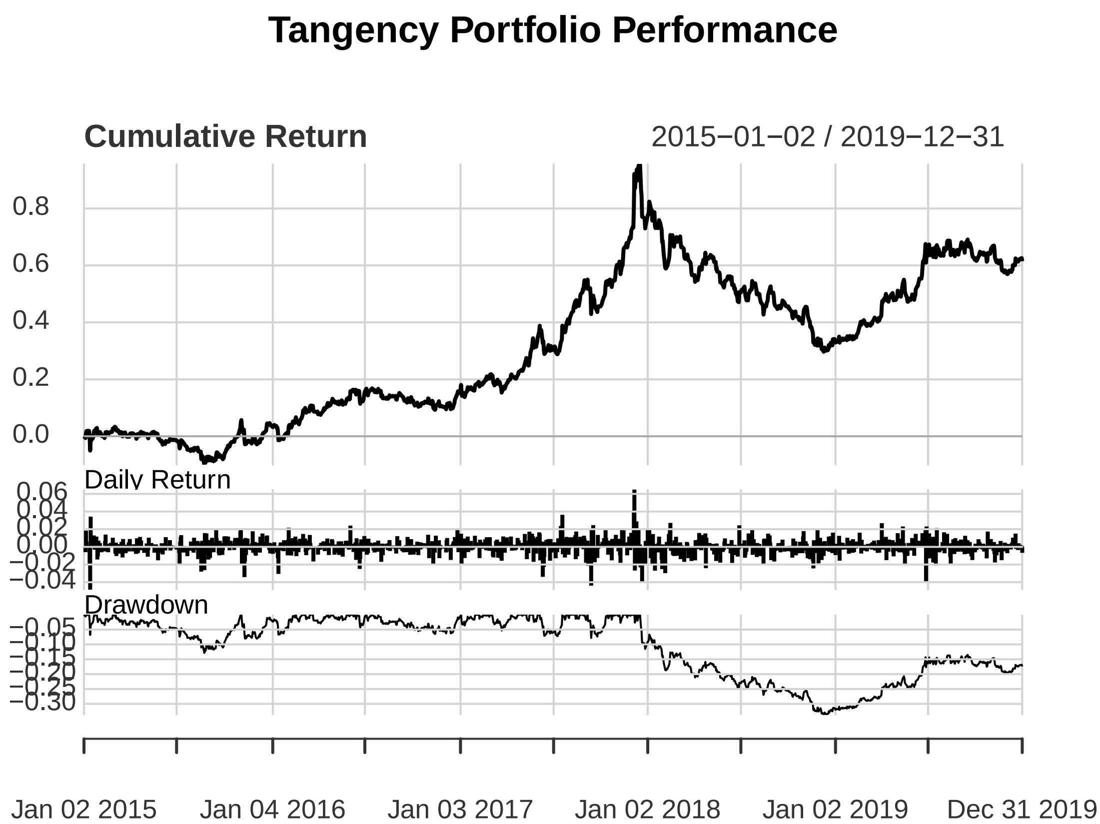
The figure above is another interesting chart that displays the weights on the different assets, the risk, and the return along the frontier. The black line through the chart indicates the minimum variance portfolio.

Next, I present the performance summary charts which sums up the information we need in analysing all the portfolios' performance over time. The charts depict each portfolio's cumulative return, daily return, and drawdown. Each portfolio is rebalanced every quarter.



Equally Weighted Portfolio Performance
Cumulative Return
2015−01−02 / 2019−12−31
1.5
1.0
0.5
0.0
Daily Return
0.05
0.00
−0.05
Drawdown
−0.1
−0.2
−0.3
−0.4
Jan 02 2015
Jan 04 2016
Jan 03 2017
Jan 02 2018
Jan 02 2019
Dec 31 2019


Global Min Variance Portfolio Performance
Cumulative Return
2015−01−02 / 2019−12−31
0.15
0.10
0.05
0.00
−0.05
Daily Return
0.02
0.01
0.00
−0.01
−0.02
Drawdown
−0.05
−0.10
−0.15
Jan 02 2015
Jan 04 2016
Jan 03 2017
Jan 02 2018
Jan 02 2019
Dec 31 2019



Finally, the table displays the target returns and risks associated with each portfolio.

Portfolio	Mean Return	Cov	CVaR	VaR
Equally Weighted	0.0464	1.1288	2.7860	1.8735
Global Minimum Variance	0.0149	0.5473	1.2432	0.9095
Tangency	0.0383	0.8425	1.9634	1.3444

It is surprising that the expected returns of the equally weighted portfolio are higher than the tangency portfolio. However, from the figure displaying the weights along the frontier we see that as the amount of risk increases the weights assigned to the property index decreases. Thus, it could be that the tangency portfolio deems property stocks to risky relative to its return series, hence assigning it a weight if zero.

Conditional Value at Risk (CVaR) is a risk measure that quantifies the expected loss for a portfolio or investment for a given confidence level. It provides a more complete picture of the risk associated with an investment by focusing on the tail risk and providing information about the potential losses in the worst-case scenarios. It is evident that the equally weighted portfolio has the highest expected loss followed by the tangency portfolio and the global minimum portfolio. By considering both the mean return and the CVaR, it seems that the risk and return trade-off for an equally weighted portfolio is quite high. As such, the tangency portfolio appears to perform the best, even though its mean return is slightly lower.

7. Conclusion

In conclusion, this study on asset class correlations and portfolio optimization has shown the importance of considering the relationships between assets when constructing a portfolio. By analysing the covariance between assets, investors can better understand the risk associated with the interactions between the assets in their portfolio and make informed decisions about the risk and return trade-off.

The study also highlighted the potential benefits of portfolio optimization in terms of improving risk-adjusted returns and reducing portfolio risk. By using optimization techniques, investors can identify portfolios that lie on the efficient frontier and achieve the highest possible return for a given level of risk, or the lowest possible risk for a given level of return.

Furthermore, the study demonstrated the significance of regularly monitoring and adjusting portfolios in response to changes in the market and in the correlations between assets. By keeping up-to-date with market developments and making necessary changes, investors can maintain a well-diversified portfolio that is better suited to their investment goals.

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