Example 2: Give a PDA which accepts

$$L = \{a^n b^n \mid n \geq 0\}$$

Define $\boldsymbol{M}$:

$$Q = \{s,\, q,\, f$$
$$\Sigma = \{a,\, b\}$$
$$\Gamma = \{\perp,\, A\}$$ (stack items '$\boldsymbol{A}$', count '$\boldsymbol{a}$'s)
$$F = \{f\}$$

$\boldsymbol{\delta}$:

(1) $(s,\, a,\, \perp) \longrightarrow (s,\, A\perp)$

(2) $(s,\, a,\, A) \longrightarrow (s,\, A\,A)$ — **building up** stack of '$\boldsymbol{A}$'s

(3) $(s,\, b,\, A) \longrightarrow (q,\, \lambda)$ — change state for '$\boldsymbol{b}$'s; **remove** '$\boldsymbol{A}$' from stack

(4) $(q,\, a,\, \perp/A) \longrightarrow \ldots ?$ — <u>**OMIT! word has form $\boldsymbol{a^n b^m a \ldots}$**</u>

(5) $(q,\, b,\, A) \longrightarrow (q,\, \lambda)$ — **breaking down** stack of '$\boldsymbol{A}$'s

(6) $(q,\, b,\, \perp) \longrightarrow \ldots ?$ — <u>**OMIT! Too many '$\boldsymbol{b}$'s**</u>

(7) $(q,\, \lambda,\, \perp) \longrightarrow (f,\, \lambda)$

(8) $(q,\, \lambda,\, A) \longrightarrow \ldots ?$ — <u>**OMIT! Too many '$\boldsymbol{a}$'s**</u>

<u>Note</u>:

$Q.$ Does this work for the empty word?

$A.$ <u>No. Must add: (9) $\boldsymbol{(s, \lambda, \perp) \longrightarrow (f, \lambda)}$</u>

Example 3: Give a PDA which accepts

$$L = \mathbf{WN}_{[\,]}$$

(see p. 5-3)

Define M :

$$Q = \{\, s,\ f \,\}$$
$$\Sigma = \{\, \mathbf{[}\,,\ \mathbf{]} \,\}$$
$$\Gamma = \{\, \perp,\ \mathbf{[} \,\}$$
$$F = \{\, f \,\}$$

δ :

(1) $(\, s,\ \mathbf{[}\,,\ \perp \,) \longrightarrow (\, s,\ \mathbf{[}\perp \,)$

(2) $(\, s,\ \mathbf{[},\ \mathbf{[} \,) \longrightarrow (s,\ \mathbf{[\,[} \,)$ **building up** stack of ‘ **[** ’s

(3) $(\, s,\ \mathbf{]}\,,\ \mathbf{[} \,) \longrightarrow (\, s,\ \lambda \,)$ **cancelling** ‘ **[** ’ on stack with ‘ **]** ’

(4) $(s,\ \lambda,\ \perp \,) \longrightarrow (\, f,\ \lambda \,)$

<u>Notes</u>:

(1) At any stage, stack of ‘ **[** ’s represents the **excess** of ‘ **[** ’ so far

(2) What about $(\, s,\ \mathbf{]},\ \perp \,) \ \longrightarrow\ \ldots$? <u>Bad! Too many ‘]’</u>

(3) What about $(\, s,\ \lambda,\ \mathbf{[} \,) \ \longrightarrow\ \ldots$? <u>Bad! Too few ‘]’</u>

(4) Does this work for the empty word? <u>Yes, see (4)</u>

Example: Take $w = \mathbf{[\,[\,]\,[\,[\,]\,]\,]}$

Configuration sequence:

$$
\begin{array}{ll}
\quad (s,\ \mathbf{[\,[\,]\,[\,[\,]\,]\,]}, \perp) & \\
\to (s,\ \mathbf{[\,]\,[\,[\,]\,]\,]},\ \mathbf{[}\perp) & \text{by (1)} \\
\to (s,\ \mathbf{]\,[\,[\,]\,]\,]},\ \mathbf{[\,[}\perp) & \text{by (2)} \\
\to (s,\ \mathbf{[\,[\,]\,]\,]},\ \mathbf{[}\perp) & \text{by (3)} \\
\to (s,\ \mathbf{[\,]\,]\,]},\ \mathbf{[\,[}\perp) & \text{by (2)} \\
\to (s,\ \mathbf{]\,]\,]},\ \mathbf{[\,[\,[}\perp) & \text{by (2)} \\
\to (s,\ \mathbf{]\,]},\ \mathbf{[\,[}\perp) & \text{by (3)} \\
\to (s,\ \mathbf{]},\ \mathbf{[}\perp) & \text{by (3)} \\
\to (s, \lambda, \perp) & \text{by (3)} \\
\to (f, \lambda, \lambda) & \text{by (4)}
\end{array}
$$

Note:

Could have started with (4):

$\to (f, w, \lambda)$

-Fail! (Need $(f,\ \lambda,\ \lambda)$)

So M is **nondeterministic** – can **choose** transition!

Note:

There are **2 possible definitions** for acceptance of a string, w, by a PDA, $M = (Q,\ \Sigma,\ \Gamma,\ \delta,\ S,\ \perp,\ F)$

(1) By **final state**:

$$(s, w, \perp) \overset{*}{\underset{M}{\vdash}} (f, \lambda, \alpha) \qquad\qquad \text{where } f \in F$$

(2) By **empty stack**:

$$(s, w, \perp) \overset{*}{\underset{M}{\vdash}} (q, \lambda, \lambda) \qquad\qquad \text{for any } q$$

We have used (1) (see p. 7-7) (following Linz).

Both definitions are **equivalent**,

i.e. we can **transform** a PDA which accepts a language, L, by one method to a PDA which accepts it by the other method.

[see Linz, end of Sec. 7.2 (6ed, p.191, Ex. 19, or 5ed, p. 185, Ex. 17), or Kozen, Lecture E]

In most common examples (e.g. our Examples 1, 2, 3), these two definitions are clearly equivalent.