Q. What about LLG's?

Theorem : $\boldsymbol{L}$ is regular $\Longleftrightarrow\ \exists$ **LLG** $\boldsymbol{G}$ s.t. $\boldsymbol{L} = \boldsymbol{L}(\boldsymbol{G})$.

Proof : Recall: for any language,

$$L^{\mathsf{R}} = \{\boldsymbol{u}^{\mathsf{R}} \mid \boldsymbol{u} \in \boldsymbol{L}\}.$$

And for an **nfa** $\boldsymbol{M}$, define the **reverse nfa** $\boldsymbol{M}^{\mathsf{R}}$ by:

1. **reversing** all **arrows** and,
2. **interchanging start** and **final** states.

(Details on p. 3-12)

Notes:

(1) If $L(M) = L$ then $L(M^{\mathsf{R}}) = L^R$

(2) L is regular $\Longleftrightarrow L^{\mathsf{R}}$ is regular (*)

Now: Given a Σ-LLG $\boldsymbol{G}$ with productions of form:

$$\boldsymbol{A \longrightarrow Bv} \quad \text{and}$$

$$\boldsymbol{A \longrightarrow v}$$

(with $v \in \Sigma^*$)

Define **RLG** $\boldsymbol{G^{\mathsf{R}}}$, by replacing all such productions by:

$$\boldsymbol{A \longrightarrow v^{\mathsf{R}}B} \text{ and}$$

$$\boldsymbol{A \longrightarrow v^{\mathsf{R}}} \text{ (respectively)}$$

Can check: $\boldsymbol{L(G^{\mathsf{R}}) = (L(G))^{\mathsf{R}}}$ (**)

Hence:

L is regular $\Longleftrightarrow$ L^{R} is regular (by (*))

$\Longleftrightarrow$ L^{R} is generated by an **RLG**, G

(by Theorem)

$\Longleftrightarrow$ L is generated by an **LLG**, G^{R} (by (**))