Nondeterministic Finite Acceptors

Why use **nondeterministic** algorithms or automata?

They are sometimes **simpler** than their deterministic counterparts

Famous open problem: $P \stackrel{?}{=} NP$

Nondeterministic finite acceptors (**nfa's**) are also often **simpler** than the corresponding dfa.

An **nfa** can have $>$ 1 possible next state (or none!) from a given state with a given input symbol.

An nfa, $\boldsymbol{A}$, **accepts** a word, $\boldsymbol{w}$, if there is **at least** 1 path through $\boldsymbol{A}$ with input $\boldsymbol{w}$ from the start state to some final state.

Definition: An **nfa** is a quintuple [cf. dfa's, p.2-2]

$$M = (Q,\ \Sigma,\ \delta,\ q_0,\ F)$$

where Q, Σ, q_0, F are as for **dfa's**, but

$$\delta\ :\ Q \times (\Sigma \cup \{\lambda\}) \longrightarrow \mathcal{P}(Q)$$

Note:

There are 3 major differences with dfa's:

(1) Range of δ is the **powerset** of Q.

(2) Edge of transition graph can be labeled

$$q_1 \xrightarrow{\lambda} q_2, \quad (q_1, q_2 \in Q)$$

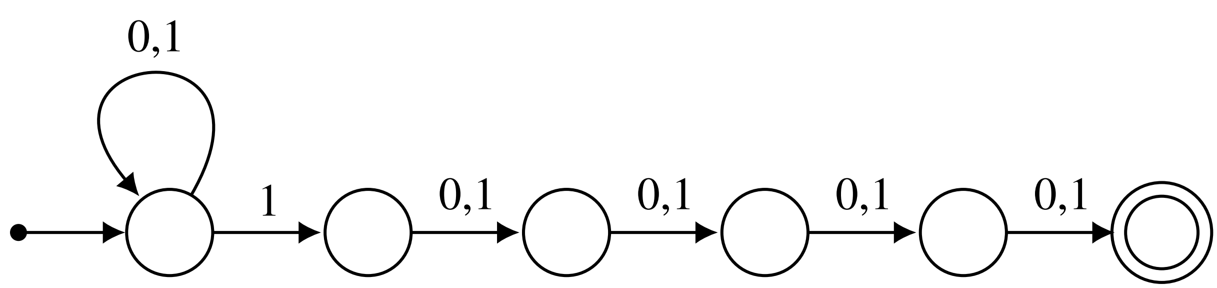
i.e. M can change state without reading a symbol.

(3) $\delta(q, a) = \varnothing$ is possible.

Example 1: Let $\Sigma = \{0, 1\}$,

$L = \{u \in \Sigma^* \mid$ **the 5th symbol from the right is 1**$\}$.

Here is an **nfa** N for A:



When N sees a 1 at the start state, it must "guess" whether to loop around again or go right.

If $u \in L$, then there is **some** path for u from q_0 to F.

If $u \notin L$, there is no such path.

<u>Exercises:</u> [Linz § 2.1: Exercise 4.(a, b, c)]

Construct **dfa's** that accept the following languages:

4.a) All strings with exactly one 'a' (Hwk!)

4.b) All strings with at least two 'a''s (cf. p. 2-8, problem (5))

4.c) All strings with no more than two 'a''s

The Subset Construction

We will see: for every **nfa**, N, which accepts a language, L, there is a **dfa**, M, (usually more complicated) which accepts the same language, i.e. $L(M) = L(N)$.

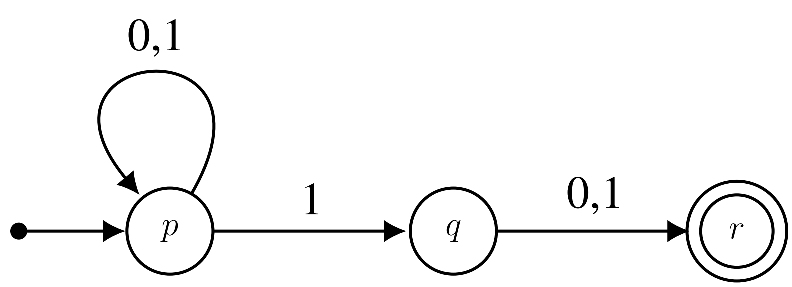
The General Idea:

Each state of M is a **set of states** of N: the set of **all states** of N which can be reached by a particular word.

Example 2: $\Sigma = \{0, 1\}$. [Kozen, p. 29]

$L = \{u \in \Sigma^* \mid$ **the 2[nd] symbol from the right is 1**$\}$

L is accepted by **nfa**, $N =$



Here is the (**nondeterministic**) **transition table** for $\boldsymbol{N}$: [See p. 2-3]

	0	1
$\longrightarrow p$	$\underline{\boldsymbol{p}}$	$\underline{\boldsymbol{p, q}}$
q	$\underline{\boldsymbol{r}}$	$\underline{\boldsymbol{r}}$
F : r	$\underline{-}$	$\underline{-}$

From this we can make a **deterministic transition table** for an **equivalent** dfa, $\boldsymbol{M}$.

Its **states** are all possible **sets of states** of $\boldsymbol{N}$:

	0	1
$\varnothing$	$\underline{\boldsymbol{\varnothing}}$	$\underline{\boldsymbol{\varnothing}}$
$\longrightarrow$ $\{p\}$	$\underline{\boldsymbol{\{p\}}}$	$\underline{\boldsymbol{\{p, q\}}}$
$\{q\}$	$\underline{\boldsymbol{\{r\}}}$	$\underline{\boldsymbol{\{r\}}}$
F : $\{r\}$	$\underline{\boldsymbol{\{-\}}}$	$\underline{\boldsymbol{\{-\}}}$
$\{p, q\}$	$\underline{\boldsymbol{\{p, r\}}}$	$\underline{\boldsymbol{\{p, q, r\}}}$
F : $\{p, r\}$	$\underline{\boldsymbol{\{p\}}}$	$\underline{\boldsymbol{\{p, q\}}}$
F : $\{q, r\}$	$\underline{\boldsymbol{\{r\}}}$	$\underline{\boldsymbol{\{r\}}}$
F : $\{p, q, r\}$	$\underline{\boldsymbol{\{p, r\}}}$	$\underline{\boldsymbol{\{p, q, r\}}}$

Note:

- The **start state** of M is the set containing the **start state** of N, i.e. $\{p\}$.
- The **final states** of M are those states that **contain any of the final states** of N
- Some states of M are **inaccessible** from the start state, $\{p\}$: $\{q, r\}$, $\{q\}$, $\{r\}$, $\varnothing$.

  These can be left out of the table.

Note:

The dfa, M, formed by the subset construction from N in this way is more **complex** than N:

If N has k states, then M has 2^k states.

Example: Let $\Sigma = \{a\}$, [Kozen, p. 30]

$L = \{x \in \{a\}^* \mid |x| \textbf{ is divisible by 3 or 5}\}$

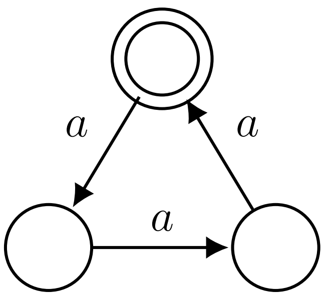
Find a **dfa** and **nfa** for L.

Let $L_1 = \{u \in \Sigma^* \mid |u| \textbf{ is divisible by 3}\}$,

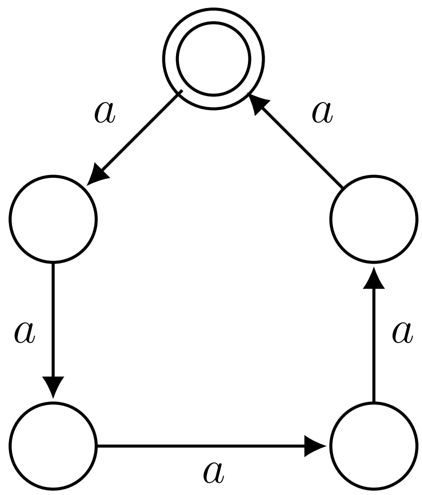
and $L_2 = \{u \in \Sigma^* \mid |u| \textbf{ is divisible by 5}\}$.

Then have **dfa**'s for $\boldsymbol{L_1}$ and $\boldsymbol{L_2}$:

$\boldsymbol{M_1}$:

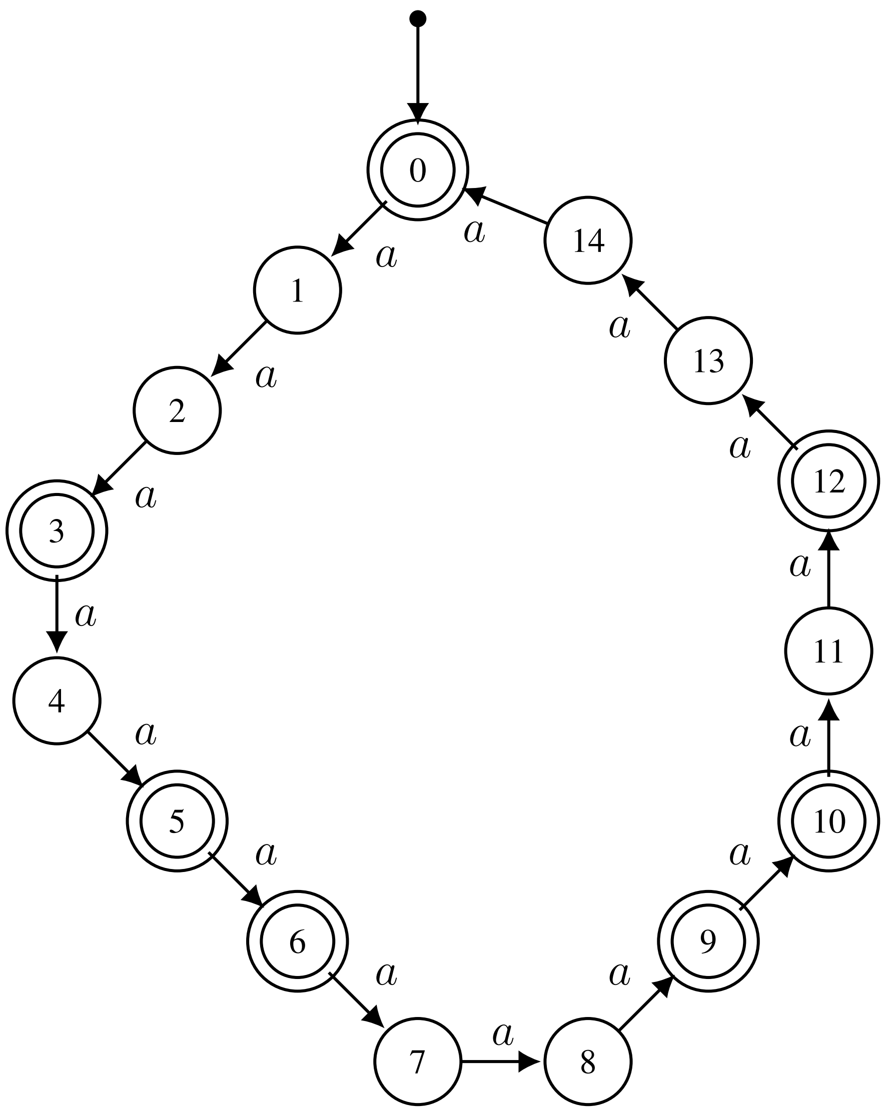


$\boldsymbol{M_2}$:



Then an **nfa** for $L = L_1 \cup L_2$ is...

Here is a **dfa** for $\boldsymbol{L}$:



15 states!

Which are the **final states**?

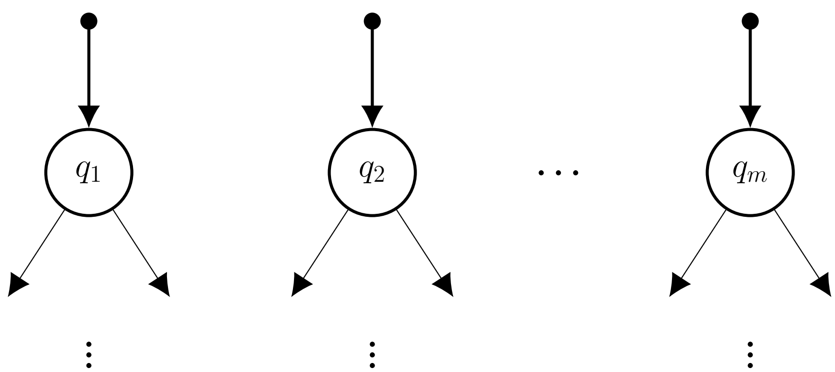
$\underline{\mathbf{0, 3, 6, 9, 12, 5, 10}}$

Note:

Suppose we **change** the **definition** of **nfa** to allow > 1 start state.

$Q.$ Would that make a difference to the power of nfa's?

$A.$ No! Consider an **nfa** with $\boldsymbol{m > 1}$ start states:



We can **transform** this to an **equivalent** nfa with 1 new start state $\boldsymbol{q_0}$, and $\boldsymbol{\lambda}$-edges from $\boldsymbol{q_0}$ to $\boldsymbol{q_1}, \ldots, \boldsymbol{q_m}$, which are no longer start states.

[See Linz Sec. 2.2, Exercises: 19 (6ed), or 18 (5ed).]