ECE 707: Assignment 1

1. Find out if the sets of vectors below are linearly independent.

   (a)
   $$\left\{ \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 4 \\ -4 \\ 14 \end{pmatrix} \right\} \tag{1}$$

   (b)
   $$\left\{ \begin{pmatrix} 1 \\ 7 \\ 7 \end{pmatrix}, \begin{pmatrix} 2 \\ 7 \\ 7 \end{pmatrix}, \begin{pmatrix} 3 \\ 7 \\ 7 \end{pmatrix} \right\} \tag{2}$$

   (c)
   $$\left\{ \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\} \tag{3}$$

2. Use Gauss' elimination to solve the following sets of equations.

   (a)
   $$\begin{aligned} 2x + 3y &= 13 && (4) \\ x - y &= -1 && (5) \end{aligned}$$

   (a)
   $$\begin{aligned} x + -z &= 0 && (6) \\ 3x + y &= 1 && (7) \\ -x + y + z &= 4 && (8) \end{aligned}$$

3. Find out if the following sets of equations have unique, many or no solutions.

   (a)
   $$\begin{aligned} 2x + 2y &= 5 && (9) \\ x - 4y &= 0 && (10) \end{aligned}$$

(b)

$$\begin{aligned} -x - y &= 1 && (11)\\ -3x - 3y &= 2 && (12) \end{aligned}$$

4. What conditions do b_1, b_2, b_3 and b_4 have to satisfy in order for the following equations to have a solution?

   (a)

$$\begin{aligned} x - 3y &= b_1 && (13)\\ 3x + y &= b_2 && (14)\\ x + 7y &= b_3 && (15)\\ 2x + 4y &= b_4 && (16) \end{aligned}$$

   (b)

$$\begin{aligned} x + 2y + 3z &= b_1 && (17)\\ 2x + 5y + 3z &= b_2 && (18)\\ x + 8z &= b_3 && (19) \end{aligned}$$

5. For which rational numbers a does the following system have (i) no solutions (ii) exactly one solution (iii) infinitely many solutions?

$$\begin{aligned} x + 2y - 3z &= 4 && (20)\\ 3x - y + 5z &= 2 && (21)\\ 4x + y + (a^2 - 14)z &= a + 2 && (22) \end{aligned}$$

6. The points (0, 0), (1, 0), (2, -1), (3, 4) and (4, 8) are required to lie on a parabola $y = a + bx + cx^2$. Find a least squares solution for a, b and c. Also prove that no parabola passes through these points.

7. Let $A = \begin{bmatrix} a & b & c \\ 1 & 1 & 1 \end{bmatrix}$. Find conditions on a, b and c such that (i) $\text{rank}(A) = 1$; (ii) $\text{rank}(A) = 2$.

8. Determine the rational values of t for which B^{-1} exists if

$$B = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \\ 2 & 4 & 7 & 2t+6 \\ 2 & 2 & 6-t & t \end{bmatrix} \qquad (23)$$

9. Find the ranks of the matrices below.

   (a) $\begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 2 \\ 1 & 0 & 3 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & -1 & 2 \\ 3 & -3 & 6 \\ -2 & 2 & 4 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 3 & 2 \\ 5 & 1 & 1 \\ 6 & 4 & 3 \end{bmatrix}$

10. A submarine is moving at a speed of 10m/s heading West and 5m/s heading North starting from (5000m, 35000m). The motion of another submarine, which is tracking the first one, consists of two velocity legs starting from the origin: 1) in the Northwest direction for 450s and 2) in the Northeast direction for the next 450s with a constant speed of 7.1m/s. The second submarine measures the angle to the first one at regular intervals of 30s. The total observation period is 900s. The state of each submarine is represented by its position and velocity.

    Write the equations to describe the states of the submarines over time. Write an equation to describe the measurements. Use MATLAB to plot the trajectories and measurements over time.

Notes:

1. Due: September 25, 2019 (late penalty: 10%/day and no credit after one week.)
2. Work on your own.

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