Cache Ellsworth

Linear Constrained Optimization Exercises

Exercise 8.1

See the jupyter notebook in the same folder.

Exercise 8.2

See the jupyter notebook in the same folder.

Exercise 8.3

The linear optimization problem in standard form which would maximize Kenny's profit on these two toys.

$$\begin{aligned} \text{maximize} \quad & 4s + 3d \\ \text{subject to} \quad & 15s + 10d \leq 1800 \\ & 2s + 2d \leq 350 \\ & d \leq 200 \\ & s, d \geq 0 \end{aligned}$$

Exercise 8.4

The linear optimization problem.

$$\begin{aligned} \text{minimize} \quad & 2x_{AB} + 5x_{AD} + 2x_{BD} + 5x_{BC} + 7x_{BE} + 9x_{BF} + 4x_{DE} + 3x_{EF} + 2x_{CF} \\ \text{subject to} \quad & (x_{AB} + x_{AD}) - 0 = 10 \\ & (x_{BC} + x_{BF} + x_{BE} + x_{BE}) - (x_{AB}) = 1 \\ & (x_{CF}) - (x_{BC}) = -2 \\ & (x_{DE}) - (x_{AD}) = -3 \\ & (x_{EF}) - (x_{BE} + x_{DE}) = 4 \\ & 0 - (x_{CF} + x_{BF} + x_{EF}) = -10 \end{aligned}$$

Exercise 8.5

i) The dictionary process is as follows

$$\begin{aligned} \zeta &= \quad 3x_1 + x_2 \\ \hline w_1 &= 15 - x_1 - 3x_2 \\ w_2 &= 18 - 2x_1 - 3x_2 \\ w_3 &= 4 - x_1 + x_2 \end{aligned}$$

where $x_1 = 0, x_2 = 0, w_1 = 15, w_2 = 18, w_3 = 4$

$$\begin{aligned} \zeta &= \quad 12 - 3w_3 + 4x_2 \\ \hline w_1 &= 11 - w_3 - 5x_2 \\ w_2 &= 10 + 2w_3 - 5x_2 \\ x_1 &= 4 - w_3 + x_2 \end{aligned}$$

where $x_1 = 4, x_2 = 0, w_1 = 11, w_2 = 10, w_3 = 0$

$$\begin{aligned} \zeta &= \quad 20 - \frac{7}{5}w_3 - \frac{4}{5}w_2 \\ \hline w_1 &= 1 - w_3 + w_2 \\ x_2 &= 2 + \frac{2}{5}w_3 - \frac{w_2}{5} \\ x_1 &= 6 - \frac{3}{5}w_3 - \frac{w_2}{5} \end{aligned}$$

where $x_1 = 6, x_2 = 2, w_1 = 1, w_2 = 0, w_3 = 0$
This shows that the optimal point is $(6, 2)$ and the optimum value is 20. These agree with the same answers I got in Exercise 8.2. ii) The dictionary process is as follows

$$\begin{aligned} \zeta &= \quad 4x + 6y \\ \hline w_1 &= 11 + x - y \\ w_2 &= 27 - x - y \\ w_3 &= 90 - 2x - 5y \end{aligned}$$

where $x = 0, y = 0, w_1 = 11, w_2 = 27, w_3 = 90$

$$\begin{aligned} \zeta &= \quad 108 - 4w_2 + 2y \\ \hline x &= 27 - w_2 - y \\ w_1 &= 38 - w_2 - 2y \\ w_2 &= 27 - x - y \\ w_3 &= 36 + 2w_2 - 3y \end{aligned}$$

where $x = 27, y = 0, w_1 = 38, w_2 = 0, w_3 = 68$

$$\begin{array}{rl}
\zeta = & 132 - \frac{8}{3}w_2 - \frac{2}{3}w_3 \\
\hline
x = & 15 - \frac{5}{3}w_2 + \frac{w_3}{3} \\
y = & 12 + \frac{2w_2}{3} - \frac{w_3}{3} \\
w_1 = & 14 - \frac{7}{3}w_2 + \frac{2}{3}w_3
\end{array}$$

where $x = 15, y = 12, w_1 = 14, w_2 = 0, w_3 = 0$
This shows that the optimal point is $(15, 12)$ and the optimum value is 132. These agree with the same answers I got in Exercise 8.2.