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Feedback Vertex Set.

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Geometric graph : A geometric graph in a 2D plane is defined by a set of points in the plane called vertices, and a threshold on the distance between the points : there is an edge between two vertices if and only if the Euclidean distance between the two vertices is smaller than this threshold.

Feedback Vertex Set : Given a graph $G = (V, E)$, the minimum Feedback Vertex Set (MinFVS) problem consists in computing a minimum sized subset of vertices $F \subseteq V$ such that the subgraph $G[V \setminus F]$ induced by $V \setminus F$ in G is cycleless, that is, $G[V \setminus F]$ is a forest.

Budgeted Feedback Vertex Set : Given $G = (V, E)$ a graph and B a real number called budget, the Feedback Vertex Set problem of budget B consists in computing a subset of vertices $F \subseteq V$ of size at most B such that the number of connected components with cycle of $G[V \setminus F]$ is minimum.

1 Greedy algorithm

Propose a greedy heuristic method finding a Feedback Vertex Set of small size (which could not be optimal) in a geometric graph. Implement this method in the canvas file. For example, we can estimate that in a geometric graph, the cycles often contain high degree vertices.

2 Naive local search

Heuristics called local search works as follows.

— Start with a non-optimal, yet valid, solution. For instance a greedy result, or even a random result (!) provided it is a valid solution to the problem at hand.

— While we can improve the current solution by applying some RULE, apply it.

This kind of heuristic depends a lot on the RULE to apply. For vertex subset problems like MinFVS, there is a naive rule : if by replacing two vertices of the current solution by a vertex exterior to the current solution, we still have a valid solution, then we perform the replacement. This is because the optimality of the FVS only depends on the number of its vertices, therefore, what the naive rule tries to do is merely to decrease the number of vertices in the FVS by one. We can generalize the naive rule : if by replacing k vertices of the current solution by $k - 1$ external vertices, we still have a valid solution, then we perform the replacement.

Implement this naive heuristic in the canvas file.

3 Optional question : Bafna-Berman-Fujito approximation

A $2-$approximation to MinFVS is an algorithm computing a (valid) solution whose size is between the optimal size and the double of the optimal size Implement the $2-$ approximation described in [Bafna, Berman and Fujito. *SIAM Journal of Discrete Mathema tics*, 1999]. Does this implementation perform better compared to the above naive local search ?

4 Budget consideration

Based on the previous implementations, propose a heuristic for the Feedback Vertex Set problem with a budget $B = 52$.