Continuous Statistical Distributions

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which may be easier to compute numerically. Note that

$$
\begin{aligned}
f(x;a,b) &= \frac{\Gamma(a+b)}{\Gamma(a)\,\Gamma(b)} x^{a-1}(1-x)^{b-1} I_{(0,1)}(x) \\
F(x;a,b) &= \int_0^x f(y;a,b)\,dy = I(x,a,b) \\
G(\alpha;a,b) &= I^{-1}(\alpha;a,b) \\
M(t) &= \frac{\Gamma(a)\,\Gamma(b)}{\Gamma(a+b)}\, {}_1F_1(a;a+b;t) \\
\mu &= \frac{a}{a+b} \\
\mu_2 &= ab(a+b
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\mu &= \frac{\alpha}{\beta - 1} \quad \beta > 1 \\
\mu_2 &= \frac{\alpha(\alpha+1)}{(\beta-2)(\beta-1)} - \frac{\alpha^2}{(\beta-1)^2} \quad \beta > 2 \\
\gamma_1 &=
\end{aligned}
$$

$$f(x; c, d) = \frac{cd}{x^c}$$

$$M(t) = \Gamma \ \frac{v}{}$$

$$f(x) = 1$$

f

18 Exponentiated Weibull

Two positive shape parameters a and c and $x \in (0, \infty)$

$$\begin{aligned} f(x;a,c) &= ac\left[1-\exp\left(-x^{c}\right)\right]^{a-1}\exp\left(-x^{c}\right)x^{c-1} \\ F(x;a,c) &= \left[1-\exp\left(-x^{c}\right)\right]^{a} \\ G(q;a,c) &= \left[-\log\left(1-q^{1/a}\right)\right]^{1/c} \end{aligned}$$

19 Exponential Power

One positive shape parameter b. Defined for $x \geq 0$.

$$\begin{aligned} f(x;b) &= ebx^{b-1}\exp\left(x^{b}-e^{x^{b}}\right) \\ F(x;b) &= 1-\exp\left(1-e^{x^{b}}\right) \\ G(q;b) &= \log^{1/b}\left[1-\log\left(1-q\right)\right] \end{aligned}$$

20 Fatigue Life (Birnbaum-Sanders)

This distribution's pdf is the average of the inverse-Gaussian ($\mu = 1$) and reciprocal inverse-Gaussian pdf ($\mu = 1$). We follow the notation of JKB here with $\beta = S$. for $x > 0$

$$f(x;c) = \quad x+1$$

$$f(x; c, d) = \frac{c x^{c-1}}{(1 + x^c)^2} I_{(0, \infty)}(x)$$

$$k = \mathrm{erf} \; \frac{c}{}$$

28 Generalized Logistic

Has been used in the analysis of extreme values. Has one shape parameter $c > 0$. And $x > 0$

$$f(x;c) = \quad c \exp(-x)$$

μ

So,

$$\begin{aligned}
\mu_1 &= \frac{1}{c}\left(1 - \Gamma\left(1+c\right)\right) \quad c > -1 \\
\mu_2 &= \frac{1}{c^2}\left(1 - 2\Gamma\left(1+c\right) + \Gamma\left(1+2c\right)\right) \quad c > -\frac{1}{2} \\
\mu_3 &= \frac{1}{c^3}\left(1 - 3\Gamma\left(1+c\right) + 3\Gamma\left(1+2c\right)2\right.
\end{aligned}$$

33 Generalized Half-Logistic

For x $[0, 1/c]$ and $c > 0$ we have

$$f(x;c) = \frac{2(1-cx)^{\frac{1}{c}-1}}{1}$$

$$\mu = \gamma = -\psi_0(1)$$

$$\mu_2 = \frac{\pi^2}{6}$$

γ

$$M(t) = \sqrt{2}$$

$$\mu = \frac{1}{a-1}$$

$$\mu_2 = \frac{1}{(a-2)(a-1)} - \mu^2$$

$$\gamma_1 = \frac{1}{\mu^{3/2}}$$

$$\frac{}{(a- \quad a- \quad a-} - 3\mu\mu_2 - \mu^3$$

46 Johnson SB

Defined for

51 Logistic (Sech-squared)

A special case of the Generalized Logistic distribution with $c = 1$. Defined for $x > 0$

$$f(x) = \quad \exp(-x)$$

$$
\begin{aligned}
\mu &= \mu_1 \\
\mu_2 &= \mu_2
\end{aligned}
$$

56 Noncentral beta*

Defined over x $[0, 1]$ with $a > 0$ and $b > 0$ and c

where U and χ_ν are independent and distributed as a standard normal and chi with ν degrees of freedom. Note $\lambda > 0$ and $\nu > 0$.

$$f(x; \lambda, \nu) = \quad \nu^{\nu/2}\Gamma(\nu+1)$$

$$\mu = 2\sqrt{\frac{2}{\pi}}$$

$$\mu_2 = 3 - 8$$

$$f(x; c) = \frac{c}{(1 + x)}$$

$$\mu = \frac{\pi}{2}$$

$$\mu_2 = \frac{4 - \pi}{2}$$

$$\gamma_1 = 2(\pi - 3)\sqrt{\pi}$$

$$f_{IG}(x;\mu) = \frac{1}{}$$

$$
\begin{aligned}
m_n = m_d = \mu &= 0 \\
\mu_2 &= \frac{\nu}{\nu - 2} \quad \nu > 2 \\
\gamma_1 &= 0 \quad \nu > 3 \\
\gamma_2 &= \frac{6}{\nu - 4} \quad \nu > 4
\end{aligned}
$$

As $\nu \to \infty$, this distribution approaches the standard normal distribution.

77 Symmetric Power*

78 Triangular

One shape parameter $c \in [0, 1]$ giving the distance to the peak as a percentage of the total extent of the non-zero portion. The location parameter is the start of the non-zero portion, and the scale-parameter is the x-length of the non-zero portion. In standard form we have $x \in [0, 1]$.

$$
f(x; c) = \quad 2^x
$$