

# basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE/SENIOR SERTIFIKAAT
NATIONAL SENIOR CERTIFICATE/
NASIONALE SENIOR SERTIFIKAAT

GRADE/GRAAD 12

MATHEMATICS P2/WISKUNDE V2

**NOVEMBER 2020** 

MARKING GUIDELINES/NASIENRIGLYNE

MARKS/PUNTE: 150

These marking guidelines consist of 27 pages. *Hierdie nasienriglyne bestaan uit* 27 *bladsye*.

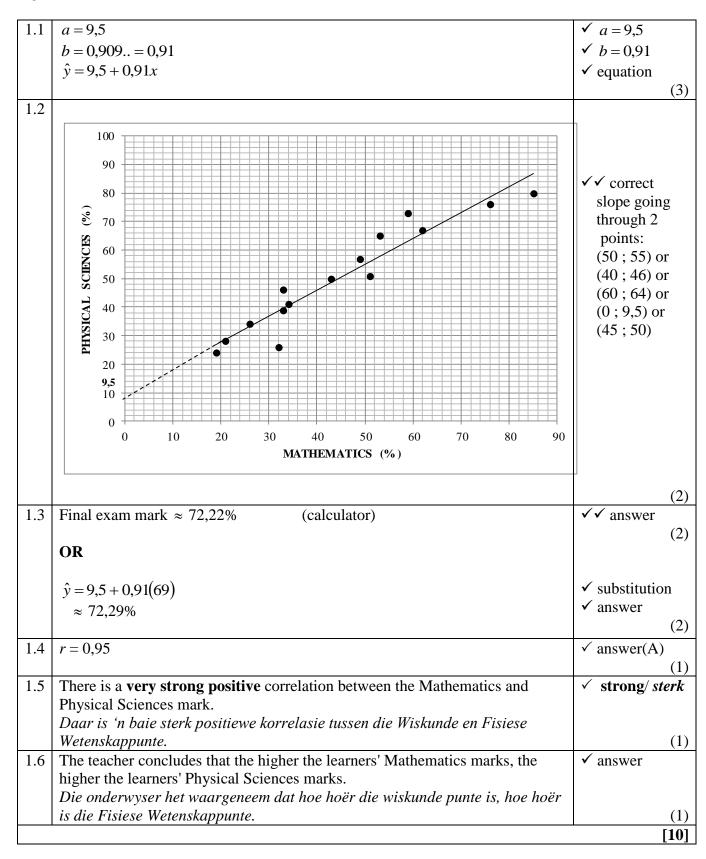
#### **NOTE:**

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

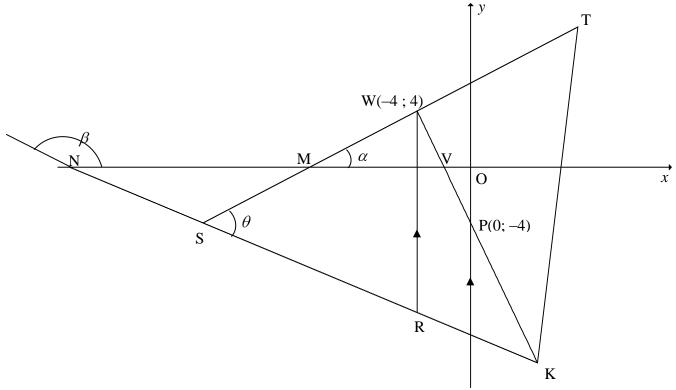
#### **LET WEL:**

- As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.
- As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.
- Volgehoue akkuraatheid word in ALLE aspekte van die memorandum toegepas. Hou op nasien by die tweede berekeningsfout.
- Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.

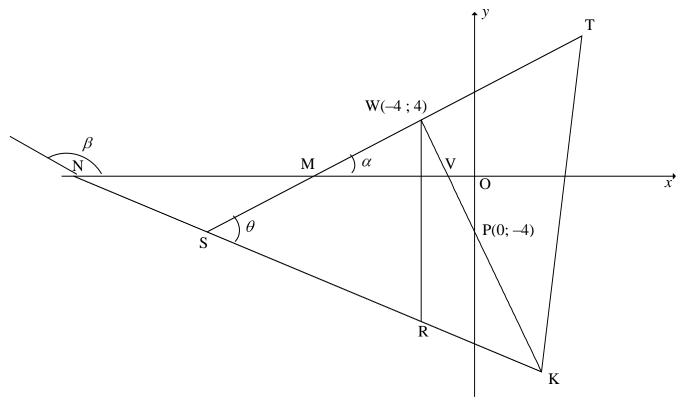
| GEOMETRY |  |  |
|----------|--|--|
| a        | A mark for a correct statement<br>(A statement mark is independent of a reason)                      |  |
| S        | 'n Punt vir 'n korrekte bewering<br>('n Punt vir 'n bewering is onafhanklik van die rede)            |  |
| R        | A mark for the correct reason (A reason mark may only be awarded if the statement is correct)        |  |
| K        | 'n Punt vir 'n korrekte rede<br>('n Punt word slegs vir die rede toegeken as die bewering korrek is) |  |
| S/R      | Award a mark if statement AND reason are both correct  |  |
|          | Ken 'n punt toe as die bewering EN rede beide korrek is  |  |



| 2.1 | July / Julie  |                                   | ✓ answer                  |     |
|-----|---|-----------------------------------|---------------------------|-----|
|     |   |                                   |                           | (1) |
| 2.2 | $\bar{x} = \frac{26941}{12}$  | Angyyan only Evil monks           | ✓ 26 941                  |     |
|     | 12  | Answer only: Full marks           |                           |     |
|     | $= 2\ 245,083 \approx 2\ 245,08$ aircraft land                                  | dings                             | ✓ answer                  | /a\ |
|     |   |                                   |                           | (2) |
| 2.3 | Standard deviation for landings at the K  | King Shaka International airport: | ✓✓ answer                 |     |
|     | $\sigma = 86,30$  |                                   |                           | (2) |
| 2.4 | $(\bar{x} - \sigma; \bar{x} + \sigma) = (2\ 245,08 - 86,30; 2\ 245,08 + 86,30)$ |                                   | $\sqrt{\bar{x}} - \sigma$ |     |
|     | limit = (2 158,78 ; 2 331,38)   |                                   | $\sqrt{x} + \sigma$       |     |
|     | There were 9 months when the aircraft arrivals at the King Shaka                |                                   |                           |     |
|     | International airport were within one sta                                       | andard deviation of the mean.     | ✓ answer                  | (2) |
|     |   |                                   |                           | (3) |
| 2.5 | The standard deviation of the number of landings at the Port Elizabeth          |                                   |                           |     |
|     | Airport will be higher than the standard deviation of the number of             |                                   |                           |     |
|     | arrivals at the King Shaka International Airport <b>OR</b> C.                   |                                   | ✓ answer                  | (4) |
|     |   |                                   |                           | (1) |
|     |   |                                   |                           |     |
|     |   |                                   |                           | [9] |

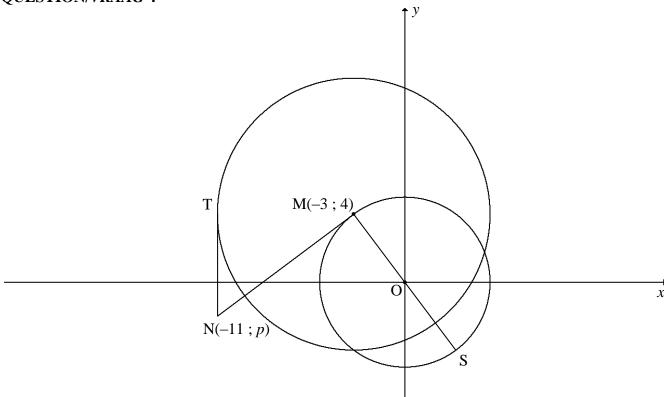


| 3.1 | $m_{\text{WP}} = \frac{4 - (-4)}{-4 - 0} = \frac{8}{-4}$ $m_{\text{WP}} = -2$  | ✓ substitution of W and P $\checkmark m_{WP}$ (2)   |
|-----|--|---|
| 3.2 | $m_{\text{ST}} = \frac{1}{2} \text{ (given)}$ $(m_{\text{WP}})(m_{\text{ST}}) = (-2)(\frac{1}{2})$ $= -1$ $\therefore \text{ST} \perp \text{WP}$   | $(2)$ $(m_{WP})(m_{ST})$ $(m_{WP})(m_{ST}) = -1$ $(2)$  |
| 3.3 | $5y + 2x + 60 = 0$ $\therefore y = -\frac{2}{5}x - 12$ $-\frac{2}{5}x - 12 = \frac{1}{2}x + 6$ $-4x - 120 = 5x + 60$ $9x = -180$ $x = -20$ $\therefore y = -\frac{2}{5}(-20) - 12$ $\therefore y = -4$ $\therefore S(-20; -4)$ | <ul> <li>✓ equating</li> <li>✓ <math>x</math> value</li> <li>✓ substitution</li> <li>✓ <math>y</math> value</li> <li>(4)</li> </ul> |
|     | OR   |   |



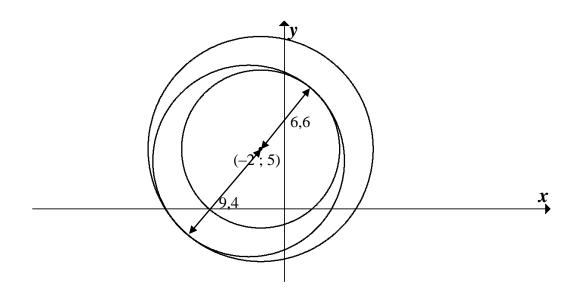
| 3.4 $y = -\frac{2}{5}(-4) - 12  \text{OR}  5y + 2(-4) + 60 = 0$ $y = -\frac{52}{5}$ $\therefore R\left(-4; -\frac{52}{5}\right) \text{ OR } R(-4; -10,4)$ $\therefore WR = 4 - \left(-\frac{52}{5}\right)  \text{OR } WR = \sqrt{(-4 - (-4))^2 + (4 - \left(-\frac{52}{5}\right))^2}  \text{method or subst into distance formula}$ $\therefore WR = \frac{72}{5} \text{ units or } WR = 14\frac{2}{5} \text{ units}$ $OR$ |
|--|
| $\therefore R\left(-4; -\frac{52}{5}\right) \text{ OR } R(-4; -10,4)$ $\therefore WR = 4 - \left(-\frac{52}{5}\right) \text{ OR } WR = \sqrt{(-4 - (-4))^2 + (4 - \left(-\frac{52}{5}\right))^2}$ $\therefore WR = \frac{72}{5} \text{ units or } WR = 14\frac{2}{5} \text{ units}$ $OR$ $(4)$   |
| $\therefore WR = 4 - \left(-\frac{52}{5}\right)  OR \ WR = \sqrt{(-4 - (-4))^2 + (4 - \left(-\frac{52}{5}\right))^2} $ $\therefore WR = \frac{72}{5} \text{ units or } WR = 14\frac{2}{5} \text{ units}$ $OR$ $(4)$  |
| $\therefore WR = 4 - \left(-\frac{32}{5}\right)  OR \ WR = \sqrt{(-4 - (-4))^2 + (4 - \left(-\frac{32}{5}\right))^2}  \text{distance formula}$ $\therefore WR = \frac{72}{5} \text{ units or } WR = 14\frac{2}{5} \text{ units}$ $OR$  |
| OR   |
| OR   |
|  |
| WR = ST - SK   |
| $= \frac{1}{2}x + 6 - \left(-\frac{2}{5}x - 12\right)$ $\checkmark$ substitution   |
| $= \frac{9}{10}x + 18$ $\checkmark$ simplification   |
| $= \frac{9}{10}(-4) + 18$ $\checkmark \text{ subst } x = -4$   |
| = 14,4  units  |
| 2.5  |
| 5  |
| $\beta = 158,19^{\circ}$ (Ref. $\angle = 21,801^{\circ}$ )   |
| MNS = 21,80°   |
| $m_{\rm ST} = \frac{1}{2}$   |
| $N\hat{M}S = 26,56^{\circ}$ $\checkmark$ size of $N\hat{M}S$   |
| $\theta = 21,80^{\circ} + 26,56^{\circ} \text{ [ext } \angle \text{ of } \Delta \text{]}$ $0 = 40,266^{\circ} + 40,270$ $\sqrt{\text{method}}$ $\sqrt{\text{answer}}$  |
| $\theta = 48,366^{\circ} = 48,37^{\circ} $ (5)   |
| 3.6 In $\triangle$ SRW:  |
|  |
| Area $\triangle SRW = \frac{1}{2} (\perp h)(WR)$   |
| $=\frac{1}{2}\left(16\right)\left(\frac{72}{5}\right)$ $\checkmark$ substitution   |
| $= 115,2 \text{ square units} \qquad \qquad \checkmark \text{ area } \Delta$   |
| Area SWRL = 2Area ΔSRW   |
| =2(115,2)  |
| = 230,4square units $\checkmark$ answer (4)  |
| OR (4)   |

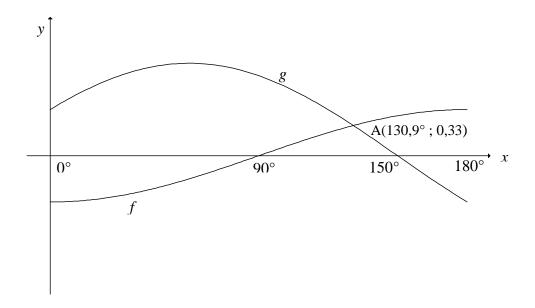
| In ΔSRW:   |  |
|--|--|
| $\perp h = -4 - (-20)$   |  |
| $\perp h = 16$ units   | $\checkmark \perp h$   |
| Area SWRL = $16 \times \frac{72}{5}$<br>= 230,40 square units                        | ✓ ✓ substitution ✓ answer (4)  |
| OR   | $\checkmark$ SW = $8\sqrt{5}$  |
| SW = $\sqrt{(-20+4)^2 + (-4-4)^2} = 8\sqrt{5} = 17,89$                               | $\mathbf{v} \mathbf{S} \mathbf{W} = \mathbf{\delta} \mathbf{V} \mathbf{S}$ |
| $SR = \sqrt{(-20+4)^2 + (-4+10\frac{2}{5})^2} = \frac{16\sqrt{29}}{5} = 17,23$       | $\checkmark SR = \frac{16\sqrt{29}}{5}$                                    |
| Area SWRL = $2 \times$ Area $\Delta$ SRW   |  |
| $=2\left(\frac{1}{2}SW\times SR\sin\theta\right)$                                    |  |
| $=2\left(\frac{1}{2}8\sqrt{5}\times\frac{16\sqrt{29}}{5}\sin 48{,}37^{\circ}\right)$ | ✓substitution  |
| =230,41 square units   | √answer  |
|  | (4)  |
|  | [21]   |



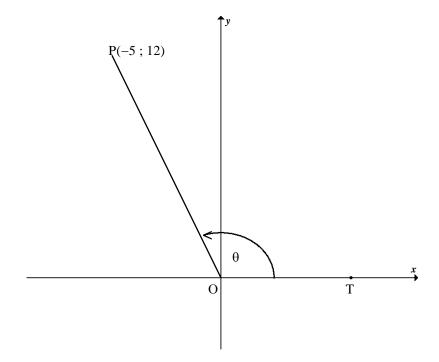
| 4.1 | $x^{2} + y^{2} = r^{2}$ $\therefore r^{2} = (-3)^{2} + (4)^{2} = 25$                            | ✓ substitution                            |
|-----|---|---|
|     | $x^2 + y^2 = 25$  | ✓ answer (2)                              |
| 4.2 | TM $\perp$ TN [tangent $\perp$ radius]<br>T(-11; 4)   |   |
|     | r = -3 - (-11) = 8  | $\checkmark x_T = -11$                    |
|     | $(x+3)^2 + (y-4)^2 = 64$  | ✓ LHS ✓ RHS (3)                           |
| 4.3 | O (0; 0) and M(-3; 4)   | (-)                                       |
|     | $m_{\text{OM}} = \frac{4-0}{-3-0} = -\frac{4}{3}  \text{OR}  \frac{0-4}{0-(-3)} = -\frac{4}{3}$ | $\checkmark m_{\text{OM}} = -\frac{4}{3}$ |
|     | $m_{\text{NM}} = \frac{3}{4}$   | $\checkmark m_{\text{NM}} = \frac{3}{4}$  |
|     | $y-4=\frac{3}{4}(x-(-3))$ <b>OR</b> $y=\frac{3}{4}x+c$  | ✓ substitution of <i>m</i> and M          |
|     | $y-4=\frac{3}{4}x+\frac{9}{4}$ $4=\frac{3}{4}(-3)+c$  |   |
|     | $\therefore y = \frac{3}{4}x + \frac{25}{4} \qquad c = \frac{25}{4}$                            | ✓ equation                                |
|     | $y = \frac{3}{4}x + \frac{25}{4}$   | (4)                                       |

| 4.4 | N(-11; p)   |   |
|-----|---|---|
|     |   |   |
|     | $y = \frac{3}{4}x + \frac{25}{4}$   |   |
|     |   |   |
|     | $p = \frac{3}{4}(-11) + \frac{25}{4}$ OR $\frac{4-p}{-3-(-11)} = \frac{3}{4}$ | $\checkmark$ subst $x = -11$ into eq or   |
|     | - ( )   | gradient                                  |
|     | p=-2  | $\checkmark p=-2$                         |
|     | N( 11 2)  |   |
|     | $\therefore N(-11;-2)$  |   |
|     | -3+r $4+v$  |   |
|     | $\frac{-3+x_S}{2} = 0$ and $\frac{4+y_S}{2} = 0$                              |   |
|     | $\therefore$ S(3;-4)  | $\langle x_S \rangle \langle y_S \rangle$ |
|     |   | 5 75                                      |
|     | $SN = \sqrt{(-11-3)^2 + (-2-(-4))^2}$   |   |
|     | $=10\sqrt{2}$ units or 14,14 units  | ✓ answer (CA)                             |
|     |   | (5)                                       |
| 4.5 | B(-2;5)   |   |
|     | $BM = \sqrt{2}$ units   | $\sqrt{2}$                                |
|     |   |   |
|     | Radius of circle centred at $M = 8$ units                                     |   |
|     | $k = 8 - \sqrt{2}$ or $k = 8 + \sqrt{2}$                                      |   |
|     | $k = 8 - \sqrt{2}$ or $k = 8 + \sqrt{2}$<br>= 6,59 units = 9,41 units         |   |
|     | = 6.6  units $= 6.6  units $ $= 9.4  units $ $= 9.4  units$                   | $\checkmark \checkmark  k = 6,6$          |
|     | 2,1 2,1   | $\checkmark$ $\checkmark$ $k = 9,4$       |
|     |   | (5)                                       |
|     |   | [19]                                      |





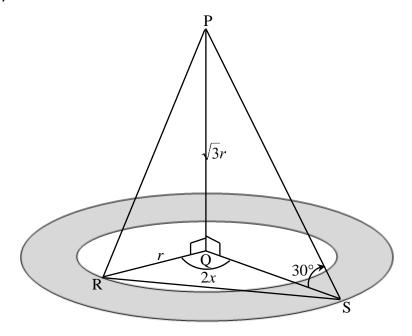
| 5.1   | Period of $g = 360^{\circ}$   | ✓ answer                                |     |
|-------|---|---|-----|
|       |   |   | (1) |
| 5.2   | Amplitude of $f = \frac{1}{2}$  | ✓ answer (A)                            | /1> |
|       |   | (                                       | (1) |
| 5.3   | $f(180^{\circ}) - g(180^{\circ})$   |   |     |
|       | $=\frac{1}{2}-\left(-\frac{1}{2}\right)$  |   |     |
|       | $\begin{bmatrix} -2 & (2) \end{bmatrix}$  | √ 1                                     |     |
|       | = 1   |   | (1) |
| 5.4.1 | $x = 140.9^{\circ}$   | $\checkmark  x = 140.9^{\circ}$         | (1) |
| 3.1.1 | <i>x</i> – 110,5  |   | (1) |
| 5.4.2 | <u></u>   |   | (1) |
| 3.4.2 | V 3 5 11 X + COS X 2 1  |   |     |
|       | $\int \frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x \ge \frac{1}{2}$                   | ✓ dividing by 2                         |     |
|       | $\sin x \cos 30^{\circ} + \cos x \sin 30^{\circ} \ge \frac{1}{2}$                       | ✓ cos 30°; sin 30°                      |     |
|       |   |   |     |
|       | $\sin(x+30^\circ) \ge \frac{1}{2}$  | $\int \sin(x+30^\circ) \ge \frac{1}{2}$ |     |
|       | $\sin(x+30^\circ) = \frac{1}{2}$ at $x = 0^\circ$ or $x = 120^\circ$                    | _                                       |     |
|       | $\therefore x \in [0^{\circ}; 120^{\circ}]$ <b>OR</b> $0^{\circ} \le x \le 120^{\circ}$ | ✓ interval                              |     |
|       | $[\ldots \lambda \in [0], 120]$ <b>ON</b> $[0] \subseteq \lambda \subseteq 120$         |   | (4) |
|       |   |   | [8] |
|       |   | l l                                     | լօյ |



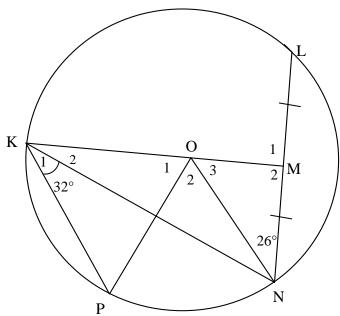
| 6.1.1 | $\tan \theta = -\frac{12}{5}  \text{or}  -2\frac{2}{5}$                                    | ✓ answer (1)   |
|-------|--|--|
| 6.1.2 | $(OP)^2 = (-5)^2 + (12)^2$<br>OP = 13  | ✓ Pythagoras<br>✓ OP                                 |
| (1.2  | $\cos\theta = -\frac{5}{13}$   | ✓ answer (3)   |
| 6.1.3 | $\sin(\theta + 90^\circ) = \frac{b}{6.5}$ $b \qquad P(-5; 12)$                             | $\checkmark \sin(\theta + 90^\circ) = \frac{b}{6.5}$ |
|       | $\cos \theta = \frac{1}{6.5}$  | $\checkmark \cos \theta$                             |
|       | $\begin{vmatrix} -5 \\ 13 = \frac{b}{6.5} \\ b = -\frac{5}{2} \end{vmatrix}$               | $\checkmark \frac{-5}{13} = \frac{b}{6,5}$           |
|       |  | $\checkmark$ value of $b$ (4)                        |
|       | OR $\cos(90^{\circ} + \theta) = \frac{a}{6.5}$ $\sin(30^{\circ} + \theta) = \frac{a}{6.5}$ | $\checkmark \cos(\theta + 90^\circ) = \frac{a}{6.5}$ |
|       | $-\sin\theta = \frac{a}{6.5}$  | $\sqrt{-\sin\theta}$                                 |
|       | $-\frac{12}{13} = \frac{a}{6,5}  \therefore a = -6$  | ✓ value of <i>a</i>                                  |
|       | $b = \sqrt{(6,5)^2 - (-6)^2} = -\frac{5}{2}$   | $\checkmark$ value of $b$ (4)                        |

| 6.2     | $\sin 2x \cdot \cos(-x) + \cos 2x \cdot \sin(360^\circ - x)$                      |  |
|---------|---|--|
|         | $\frac{\sin(180^\circ + x)}{\sin(180^\circ + x)}$                                 |  |
|         | $= \frac{\sin 2x \cos x + \cos 2x(-\sin x)}{\cos x}$                              | $\sqrt{\cos(-x)} = \cos x$                         |
|         | $-\sin x$   | $\checkmark \sin(360^\circ - x) = -\sin x$         |
|         | $=\frac{\sin(2x-x)}{x}$   | $\checkmark \sin(180^\circ + x) = -\sin x$         |
|         | $-\sin x$   |  |
|         | $= \frac{\sin x}{-\sin x}$  | $\checkmark$ numerator = $\sin x$                  |
|         | =-1   | √ answer   |
|         |   | (5)  |
| 6.3     | $6\sin^2 x + 7\cos x - 3 = 0$   |  |
|         | $6(1-\cos^2 x) + 7\cos x - 3 = 0$   | ✓ identity   |
|         | $6 - 6\cos^2 x + 7\cos x - 3 = 0$   |  |
|         |   | ✓ standard form                                    |
|         | $6\cos^2 x - 7\cos x - 3 = 0$   | ✓ factors  |
|         | $(3\cos x + 1)(2\cos x - 3) = 0$  |  |
|         | $\cos x = -\frac{1}{3} \qquad \text{or} \qquad \cos x = \frac{3}{2} (\text{N/A})$ | $\checkmark$ both solutions of $\cos x$            |
|         | $\therefore x = 109,47^{\circ} + k.360^{\circ}; k \in \mathbb{Z} \text{ or }$     | $\sqrt{x} = 109,47^{\circ} \& 250,53^{\circ}$      |
|         | $x = 250,53^{\circ} + k.360^{\circ}; k \in \mathbb{Z}$                            | $\checkmark$ + <i>k</i> .360°; <i>k</i> ∈ <i>Z</i> |
|         |   | (6)  |
| 6.4     | $x + \frac{1}{x} = 3\cos A$   |  |
|         | $(3\cos A)^2 = \left(x + \frac{1}{x}\right)^2$                                    | ✓ squaring both sides                              |
|         | $\begin{pmatrix} x & x \end{pmatrix}$   |  |
|         | $9\cos^2 A = x^2 + \frac{1}{x^2} + 2$   | $\sqrt{9\cos^2 A} = x^2 + \frac{1}{x^2} + 2$       |
|         | $9\cos^2 A = 2 + 2$   |  |
|         | $\cos^2 A = \frac{4}{9}$  | . 4  |
|         | $\cos A = \frac{-9}{9}$   | $\sqrt{\cos^2 A} = \frac{4}{9}$                    |
|         | $\cos 2A = 2\cos^2 A - 1$   | $\sqrt{\cos 2A} = 2\cos^2 A - 1$                   |
|         | $=2\left(\frac{4}{9}\right)-1$  |  |
|         | $= -\frac{1}{2}$  |  |
|         | 9   | ✓ answer   |
|         | OR  | (5)  |
|         |   |  |
| <u></u> |   |  |

| $x^{2} - 2 + \frac{1}{x^{2}} = 0$ $\left(x - \frac{1}{x}\right)^{2} = 0$ $x^{2} = 1$ $x = \pm 1$ $3\cos A = 2  \text{or}  3\cos A = -2$ $\cos A = \frac{2}{3}  \text{or}  \cos A = -\frac{2}{3}$ $\cos 2A = 2\cos^{2} A - 1$ $= 2\left(\pm \frac{2}{3}\right)^{2} - 1$ | $\checkmark x = \pm 1$ $\checkmark \cos A = \frac{2}{3}$ $\checkmark \cos A = -\frac{2}{3}$ |
|--|---|
| $=-\frac{1}{9}$  | $✓ \cos A = -\frac{2}{3}$ $✓ \text{double angle identity}$ $✓ \text{answer}$ (5)            |
|  | [24]  |

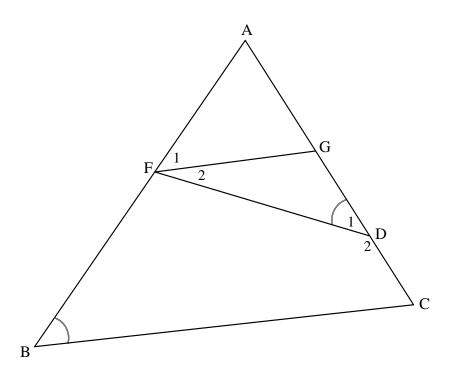


| 7.1 | $\tan 30^{\circ} = \frac{\sqrt{3} r}{\text{QS}}$ <b>OR</b>                         | $\tan 60^\circ = \frac{QS}{\sqrt{3}r}$ | ✓✓ trig ratio  |
|-----|--|--|--|
|     | $QS = \frac{\sqrt{3}r}{\tan 30^{\circ}}$   | $\sqrt{3} = \frac{QS}{\sqrt{3}r}$      | ✓ QS subject   |
|     | $= \frac{\sqrt{3}r}{\frac{1}{\sqrt{3}}}  or  \frac{\sqrt{3}r}{\frac{\sqrt{3}}{3}}$ | QS = 3r                                |  |
|     | =3r  |  | (3)  |
| 7.2 | Area of flower garden = $\pi (3r)^2 - \pi r^2$<br>= $9\pi r^2 - \pi r^2$           |  | ✓ substitution into difference of areas                            |
|     | $=8\pi r^2$  |  | ✓ answer (2)   |
| 7.3 | $RS^{2} = r^{2} + (3r)^{2} - 2(r)(3r)\cos 2x$ $= r^{2} + 9r^{2} - 6r^{2}\cos 2x$   |  | ✓ substitution into cosine rule correctly                          |
|     | $= 10r^2 - 6r^2 \cos 2x$ $= r^2 (10 - 6\cos 2x)$                                   |  | $\checkmark 10r^2 - 6r^2 \cos 2x$ $\checkmark r^2 (10 - 6\cos 2x)$ |
|     | $RS = r\sqrt{10 - 6\cos 2x}$ $RS = r\sqrt{10 - 6\cos 2x}$                          |  | (3)  |
| 7.4 | $RS = 10\sqrt{10 - 6\cos 2(56)}$   |  | ✓substitution  |
|     | = 34,9966<br>≈ 35 m  |  | ✓ answer (2)   |
|     |  |  | [10]   |



| 8.1.1(a) | $\hat{O}_2 = 64^{\circ}$ [ $\angle$ at centre = 2 × $\angle$ at circumference/   | ✓ S ✓ R    |     |
|----------|--|------------|-----|
|          | $Middelpts \angle = 2 \times \angle omtreks \angle$  |            | (2) |
| 8.1.1(b) | $\hat{M}_2 = 90^{\circ}$ [Line from centre to midpt of chord/lyn v midpt   | ✓ S ✓ R    |     |
|          | na midpt v koord]  |            |     |
|          | $\hat{KON} = 90^{\circ} + 26^{\circ} = 116^{\circ} \text{ [ext } \angle \text{ of } \triangle/buite \angle van \triangle]$ | ✓ S        |     |
|          | $\hat{O}_1 = 116^{\circ} - 64^{\circ} = 52^{\circ}$  | ✓ answer   |     |
|          | OR   |            | (4) |
|          | $\hat{M}_2 = 90^{\circ}$ [Line from centre to midpt of chord/lyn v midpt   | ✓ S ✓ R    |     |
|          | na midpt v koord]  |            |     |
|          | $\hat{O}_3 = 64^{\circ}$ [sum of $\angle$ s in $\Delta$ ]  | ✓ S        |     |
|          | $\hat{O}_1 = 52^{\circ}$ [ $\angle$ s on straight line/op 'n reguitlyn]  | ✓ answer   |     |
|          |  |            | (4) |
| 8.1.2    | $\hat{P}KO + \hat{P} = 128^{\circ} \text{ [sum of } \angle \text{s in } \Delta/som \angle e  van \Delta]$                  |            |     |
|          | $\hat{PKO} = \hat{P}$ [\( \setminus \text{ opp} = \text{sides} \setminus \( e \text{ teenoor} = sye \)]                    | ✓ S        |     |
|          | =64°   | ✓ S        |     |
|          | $\therefore \hat{\mathbf{K}}_2 = 32^{\circ} or \ \hat{\mathbf{K}}_2 = \hat{\mathbf{K}}_1$                                  | ✓ S        | (2) |
|          | ∴ KN bisects/halveer OKP   |            | (3) |
|          | OR   |            |     |
|          | $\hat{K}_2 = \hat{KNO} \ [\angle s \ opp = sides/\angle e \ teenoor = sye]$  | √ S        |     |
|          | $\hat{K}_2 + K\hat{N}O = 64^{\circ} [\text{sum of } \angle \text{s in } \Delta/\text{som } \angle \text{e van } \Delta]$   | $\sqrt{S}$ |     |
|          | $\therefore \hat{\mathbf{K}}_2 = 32^{\circ} or \ \hat{\mathbf{K}}_2 = \hat{\mathbf{K}}_1$                                  | √ S        |     |
|          | ∴ KN bisects/halveer OKP   |            | (3) |

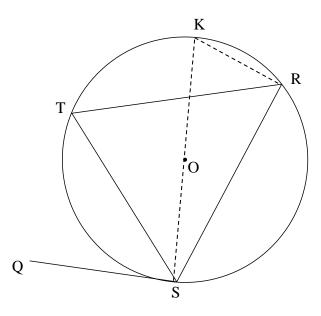
8.2



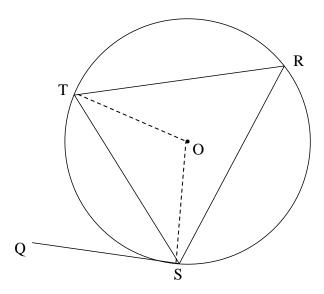
| 8.2.1 | $\hat{\mathbf{F}}_1 = \hat{\mathbf{D}}_1$ [tan chord theorem/raaklyn koordst]                  | ✓ S ✓ R                          |
|-------|--|----------------------------------|
|       | $\hat{\mathbf{D}}_1 = \hat{\mathbf{B}}$ [Given/Gegee]  |                                  |
|       | $\therefore \hat{\mathbf{F}}_1 = \hat{\mathbf{B}}$   | $\checkmark \hat{F}_1 = \hat{B}$ |
|       | $\therefore$ FG    BC [corresp $\angle$ s =/Ooreenkomstige $\angle$ e =]                       | √ R                              |
|       |  | (4)                              |
| 8.2.2 | $\frac{GC}{AC} = \frac{FB}{AB}$ [line    one side of $\Delta / lyn / leen sy v \Delta$ ]       | ✓ S ✓ R                          |
|       | $\frac{x+9}{2x-6} = \frac{5}{7}$   | ✓ substitution                   |
|       | 7x + 63 = 10x - 30   |                                  |
|       | 3x = 93 $x = 31$   | ✓ answer                         |
|       | $\lambda = 31$   | (4)                              |
|       | OR   |                                  |
|       | AG = 2x - 6 - (x+9) = x - 15   |                                  |
|       | $\frac{AG}{GC} = \frac{AF}{FB}$ [line    one side of $\Delta / lyn // een \ sy \ v \ \Delta$ ] | ✓ S ✓ R                          |
|       | $\frac{x-15}{x+9} = \frac{2}{5}$   | ✓ substitution                   |
|       | 5x - 75 = 2x + 18  |                                  |
|       | 3x = 93  | ✓ answer                         |
|       | x = 31   | (4)                              |
|       | OR   |                                  |

| $\frac{AF}{AB} = \frac{AG}{AC}  [\text{line }    \text{ one side of } \Delta / \text{lyn}    \text{ een sy } v\Delta ]$ | ✓ S ✓ R        |
|---|----------------|
| $\frac{2}{7} = \frac{x-15}{2x-6}$   | ✓ substitution |
| 7x - 105 = 4x - 12  |                |
| 3x = 93   | ✓ answer       |
| x = 31  | (4)            |
|   |                |
|   | [17]           |

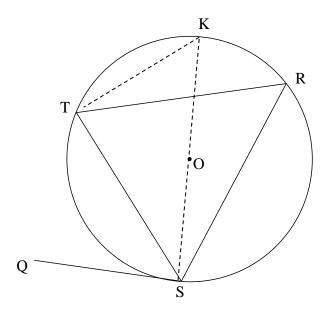
9.1



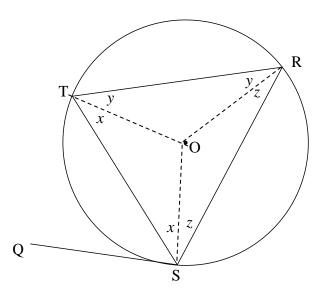
| 9.1 |   | iameter KS and draw KR iddellyn KS en verbind KR | ✓ construction |
|-----|---|--|----------------|
|     | $Q\hat{S}T = 90^{\circ} - T\hat{S}K$            | [radius $\perp$ tangent/raaklyn]                 | ✓ S/R          |
|     | $\hat{SRK} = 90^{\circ}$                        | $[\angle \text{ in semi circle}/halfsirkel}]$    | ✓ S/R          |
|     | $\therefore \hat{SRT} = 90^{\circ} - \hat{KRT}$ |  | ✓ S            |
|     | TŜK = TRK                                       | [∠s same segment/∠e dieselfde segment]           | ✓ S/R          |
|     | $\therefore \hat{QST} = \hat{R}$                | 0 1  | (5)            |



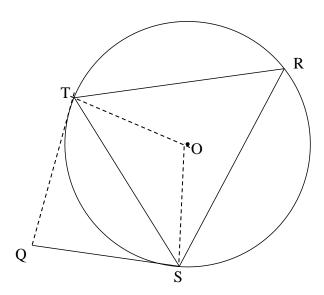
| 9.1 | Construction: Draw rad                               | ii OS and OT   | ✓ construction |
|-----|--|--|----------------|
|     | Konstruksie: Trek radii                              | OS en OT   |                |
|     | $\hat{QST} = 90^{\circ} - \hat{OST}$                 | [radius $\perp$ tangent/raaklyn]                                       | ✓ S/R          |
|     | $\hat{OST} = \hat{STO}$                              | $[\angle s \text{ opp} = \text{sides}/\angle e \text{ teenoor} = sye]$ | ✓ S/R          |
|     | $\therefore \hat{SOT} = 180^{\circ} - 2\hat{OST}$    | $[\angle s \text{ of } \Delta / \angle e  van \Delta]$                 | ✓ S            |
|     | $\hat{\mathbf{R}} = 90^{\circ} - \hat{\mathbf{OST}}$ | $[\angle \text{ at centre} = 2 \times \angle \text{ circumf}/$         | ✓ S/R          |
|     |  | $midpts \angle = 2 \times omtreks \angle$                              |                |
|     | $\therefore \hat{QST} = \hat{R}$                     |  | (5)            |
|     |  |  | (5)            |



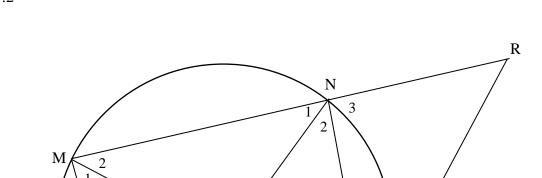
| 9.1 | Construction: Draw diameter KS and join K to T.   | ✓ construction |
|-----|---|----------------|
|     | Konstruksie: Trek middellyn KS en verbind K tot $T$ K   |                |
|     | $\hat{QST} = 90^{\circ} - \hat{TSK}$ [radius $\perp$ tangent/raaklyn]                                 | ✓ S/R          |
|     | $\hat{STK} = 90^{\circ}$ [ $\angle$ in semi circle/halfsirkel]  | ✓ S/R          |
|     | $\therefore \hat{\mathbf{K}} = 90^{\circ} - T\hat{\mathbf{S}}\mathbf{K}$                              | ✓ S            |
|     | $\therefore \hat{QST} = \hat{K}$  |                |
|     | but $\hat{R} = \hat{K}$ [\( \setminus \text{same segment} \) \( \setminus \text{dieselfde segment} \) | ✓ S/R          |
|     | $\therefore \hat{QST} = \hat{R}$  |                |
|     | 45.1  | (5)            |



| 9.1 | Construction: Draw radii OT, OR and OS  | ✓ construction |
|-----|---|----------------|
|     | Konstruksie: Trek radiuse OT, OR en OS  |                |
|     | $\hat{OST} = \hat{OTS}$ [ $\angle s \text{ opp} = \text{radii}/\angle e \text{ teenoor} = \text{radiuse}$ ] | ✓ S/R          |
|     | Also: $\hat{OTR} = \hat{ORT}$ and $\hat{ORS} = \hat{OSR}$   |                |
|     | $2x + 2y + 2z = 180^{\circ} \ [\angle s \text{ of } \Delta]$  |                |
|     | $x + y + z = 90^{\circ}$  |                |
|     | $y + z = 90^{\circ} - x$  | ✓ S            |
|     | $\hat{OSQ} = 90^{\circ}$ [radius $\perp$ tangent/raaklyn]   | ✓ S/R          |
|     | $\therefore \hat{TSQ} = 90^{\circ} - x$   |                |
|     | $\therefore \hat{TSQ} = y + z = \hat{R}$  | ✓ S            |
|     | $ 13Q = y + \zeta = K $   | (5)            |
|     |   | (5)            |

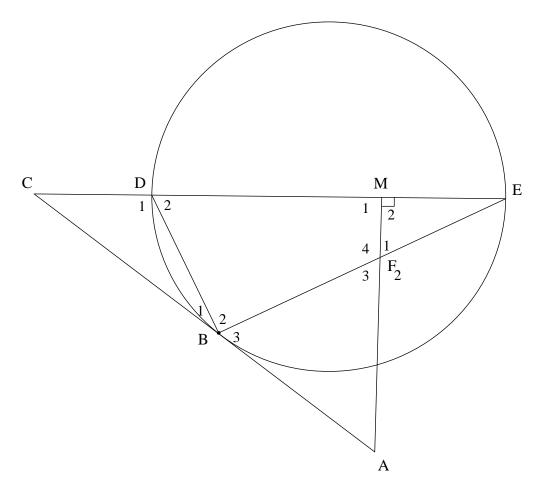


| 9.1 | Construction: Draw radii OT and OS, tangent QT  | ✓ construction |
|-----|---|----------------|
|     | Konstruksie: Trek radiuse OT en OS, raaklyn QT $\hat{OSQ} = 90^{\circ}$ [radius $\perp$ tangent/raaklyn]                              | ✓ S/R          |
|     | $\therefore T\hat{S}Q = 90^{\circ} - T\hat{S}O$   |                |
|     | $\therefore \hat{TSO} = \hat{STO} \ [\angle s \ opp = radii/\angle e \ teenoor = radiuse]$  | ✓ S            |
|     | $\hat{TOS} = 180^{\circ} - 2\hat{TSO} \ [\angle s \text{ of } \Delta]$  | ✓ S            |
|     | $\hat{R} = 90^{\circ} - \hat{TSO}$ [ $\angle$ at centre = $2 \times \angle$ circumf/<br>midpts $\angle = 2 \times$ omtreks $\angle$ ] |                |
|     | $\therefore \hat{TSQ} = \hat{R}$  | ✓ S/R          |
|     |   | (5)            |



S

| 9.2.1(a) | $\hat{\mathbf{N}}_2 = x$  | [alt $\angle$ s; PR    NQ/verw. $\angle$ e; PR    NQ]  | ✓ S ✓ R                    | (2)  |
|----------|---|--|----------------------------|------|
| 9.2.1(b) | $\hat{Q}_2 = x$ OR  | [tan chord theorem/raaklyn koordstelling]  | ✓ S ✓ R                    | (2)  |
|          | $\mathbf{M}_2 = x$ $\hat{\mathbf{Q}}_2 = x$                     | [tan chord theorem/raaklyn koordstelling] [∠s in same segment/∠e in dieselfde segm]  | ✓ S/R<br>✓ S/R             | (2)  |
| 9.2.2    | $\hat{P}_3 = \hat{Q}_2$ $SQ = PS$ $\frac{MN}{M} = \frac{MS}{M}$ | [QN    PR; Prop Th]  [given]  [ $\angle$ s in same segment/ $\angle$ e in dieselfde segm]  [ $= x$ ]  [sides opp = $\angle$ /sye teenoor = $\angle$ e] | ✓ S ✓ R  ✓ S  ✓ S ✓ R  ✓ R | (6)  |
|          | NR SQ   |  |                            | [15] |



| 10.1.1 | $D\hat{B}E = 90^{\circ}$ [ $\angle$ in semi-circle/ $\angle$ in halfsirkel]  | ✓S ✓R |     |
|--------|--|-------|-----|
|        | ∴ DMA = 90° [AM ⊥ DE]<br>∴ FBDM is a cyclic quadrilateral/koordevh<br>[converse opp ∠s cyclic quad/omgek teenoorst ∠e kvh] | ✓ R   | (3) |
|        | OR   |       |     |
|        | $\hat{DBE} = 90^{\circ}$ [ $\angle$ in semi-circle/ $\angle$ in halfsirkel]<br>$\hat{M}_2 = \hat{DBE} = 90^{\circ}$        | ✓S ✓R |     |
|        | ∴ FBDM is a cyclic quadrilateral/koordevh [converse ext∠ of cyclic quad/omgek buite∠van kvh]                               | ✓ R   | (3) |

| 10.1.2 | $\hat{B}_3 = \hat{D}_2$ [tangent chord th/raaklyn koordst]   | ✓ S ✓ R   |       |
|--------|--|---|-------|
|        | $\hat{\mathbf{F}}_1 = \hat{\mathbf{D}}_2$ [ext $\angle$ cyc quad/buite $\angle$ koordevh]                                    | ✓ S ✓ R   |       |
|        | $\therefore \hat{\mathbf{B}}_3 = \hat{\mathbf{F}}_1$   |   | (4)   |
|        |  |   | ( - ) |
|        | OR<br>â â â  | (   |       |
|        | $\hat{\mathbf{B}}_1 = \hat{\mathbf{E}} = x$ [tangent chord th/raaklyn koordst]   | $ \begin{array}{c} \checkmark \text{ S } \checkmark \text{ R} \\ \checkmark \hat{F}_1 = 90^\circ - x \\ = \hat{D}_2 \end{array} $ |       |
|        | $\hat{F}_1 = 90^\circ - x \left[ \angle \text{ sum in } \Delta / \angle \text{ van } \Delta \right]$                         | $\checkmark P_1 = 90^\circ - x$   |       |
|        | $\hat{D}_2 = 90^\circ - x \left[ \angle \text{ sum in } \Delta / \angle \text{ van } \Delta \right]$                         | $= D_2$   |       |
|        | $\therefore \hat{\mathbf{F}}_1 = \mathbf{D}_2$   |   |       |
|        | $\hat{\mathbf{B}}_3 = \hat{\mathbf{D}}_2$ [tangent chord th/raaklyn koordst]   | ✓ R   |       |
|        | $\therefore \hat{\mathbf{B}}_3 = \hat{\mathbf{F}}_1$   |   | (4)   |
|        | OR   |   |       |
|        | $\hat{\mathbf{B}}_1 = \hat{\mathbf{E}} = x$ [tangent chord th/raaklyn koordst]   | ✓ S ✓ R   |       |
|        | $\hat{B}_3 = 90^{\circ} - x$ [straight line/reguitlyn]   | ✓ S   |       |
|        | $\hat{F}_1 = 90^{\circ} - x \text{ [sum of } \angle \text{s } \Delta / \text{som van } \angle \text{e van } \Delta \text{]}$ | ✓ S   |       |
|        | $\therefore \hat{\mathbf{B}}_3 = \hat{\mathbf{F}}_1$   | V 5   | (4)   |
| 10.1.3 | In ΔCDB and ΔCBE   |   |       |
|        | $\hat{\mathbf{C}} = \hat{\mathbf{C}}$ [common $\angle/gemeenskaplike \angle$ ]   | ✓ S   |       |
|        | $\hat{CBD} = \hat{CEB}$ [tangent chord th/raaklyn koordst]   | ✓ S/R   |       |
|        | $\hat{CDB} = \hat{CBE} \ [\angle \text{ sum in } \Delta / \angle \text{ van } \Delta]$                                       | ✓ R   |       |
|        | ΔCDB     ΔCBE  |   | (3)   |
|        | OR   |   |       |
|        | In $\triangle CDB$ and $\triangle CBE$   |   |       |
|        | $\hat{CBD} = \hat{CEB}$ [tangent chord th/raaklyn koordst]   | ✓ S/R   |       |
|        | $\hat{C} = \hat{C}$ [common $\angle/gemeenskaplike \angle$ ]   | ✓ S/K ✓ S   |       |
|        | $\triangle CDB \parallel \triangle CBE [\angle, \angle, \angle]$   | ✓ R   |       |
|        |  |   | (3)   |
| 10.2.1 | $\frac{BC}{DG} = \frac{DC}{DG}$ [    $\Delta s$ ]  |   |       |
|        | EC BC  | ✓ ratio   |       |
|        | $BC^2 = EC \times DC$  |   |       |
|        | $=8\times2$  | ✓ substitution  |       |
|        | = 16 $BC = 4$  | ✓answer   |       |
|        | DC - 4   |   | (3)   |

| 10.2.2 | $\frac{BC}{EC} = \frac{DB}{BE} \qquad [\parallel \Delta s]$ |                                  |
|--------|---|----------------------------------|
|        | $\frac{DB}{BE} = \frac{4}{8} = \frac{1}{2}$                 |                                  |
|        | BE = 2DB  | ✓ BE = 2DB                       |
|        | $DB^2 + BE^2 = DE^2$ [Pyth theorem]                         | ✓ substitution into              |
|        | $DB^2 + (2DB)^2 = 36$                                       | Pyth theorem                     |
|        | $5DB^2 = 36$  |                                  |
|        | $DB^2 = \frac{36}{5}$                                       | $\checkmark DB^2 = \frac{36}{5}$ |
|        | $DB = \frac{6}{\sqrt{5}} = 2,68 \text{ units}$              | ✓answer                          |
|        | ,,,   | (4)                              |
|        |   | [17]                             |

TOTAL/TOTAAL: 150