basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

NOVEMBER 2021

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1.   This question paper consists of 12 questions.

2.   Answer ALL the questions.

3.   Number the answers correctly according to the numbering system used in this question paper.

4.   Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

5.   Answers only will NOT necessarily be awarded full marks.

6.   You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7.   If necessary, round off answers to TWO decimal places, unless stated otherwise.

8.   Diagrams are NOT necessarily drawn to scale.

9.   An information sheet with formulae is included at the end of the question paper.

10.   Write neatly and legibly.

QUESTION 1

1.1   Solve for x:

  1.1.1 $x^2 - 2x - 24 = 0$ (3)

  1.1.2 $2x^2 - 3x - 3 = 0$ (correct to TWO decimal places)   (3)

  1.1.3 $x^2 + 5x \leq -4$ (4)

  1.1.4 $\sqrt{x + 28} = 2 - x$ (4)

1.2   Solve simultaneously for x and y in:

  $2y = 3 + x$ and $2xy + 7 = x^2 + 4y^2$ (6)

1.3   The roots of an equation are $x = \dfrac{-n \pm \sqrt{n^2 - 4mp}}{2m}$ where m, n and p are positive real numbers. The numbers m, n and p, in that order, form a geometric sequence. Prove that x is a non-real number.   (4)
  [24]

QUESTION 2

Given the geometric series: $x + 90 + 81 + \ldots$

2.1   Calculate the value of x.   (2)

2.2   Show that the sum of the first n terms is $S_n = 1\,000\left(1 - (0,9)^n\right)$.   (2)

2.3   Hence, or otherwise, calculate the sum to infinity.   (2)
  [6]

QUESTION 3

Consider the quadratic number pattern: $-145 ; -122 ; -101 ; \ldots$

3.1	Write down the value of T_4.	(1)

3.2	Show that the general term of this number pattern is $T_n = -n^2 + 26n - 170$.	(3)

3.3	Between which TWO terms of the quadratic number pattern will there be a difference of -121?	(4)

3.4	What value must be added to each term in the number pattern so that the value of the maximum term in the new number pattern formed will be 1?	(3)

[11]

QUESTION 4

Consider the linear pattern: $5 ; 7 ; 9 ; \ldots$

4.1	Determine T_{51}.	(3)

4.2	Calculate the sum of the first 51 terms.	(2)

4.3	Write down the expansion of $\displaystyle\sum_{n=1}^{5000}(2n+3)$. Show only the first 3 terms and the last term of the expansion.	(1)

| 4.4	Hence, or otherwise, calculate $\displaystyle\sum_{n=1}^{5000}(2n+3) + \sum_{n=1}^{4999}(-2n-1)$. ALL working details must be shown.	(4)

[10]

QUESTION 5

Given: $f(x) = \dfrac{-1}{x-3} + 2$

5.1	Write down the equations of the asymptotes of f.	(2)

5.2	Write down the domain of f.	(1)

5.3	Determine the coordinates of the x-intercept of f.	(2)

5.4	Write down the coordinates of the y-intercept of f.	(2)

5.5	Draw the graph of f. Clearly show ALL the asymptotes and intercepts with the axes.	(3)

[10]

QUESTION 6

The graph of $f(x) = \log_4 x$ is drawn below.

B(k ; 2) is a point on f.
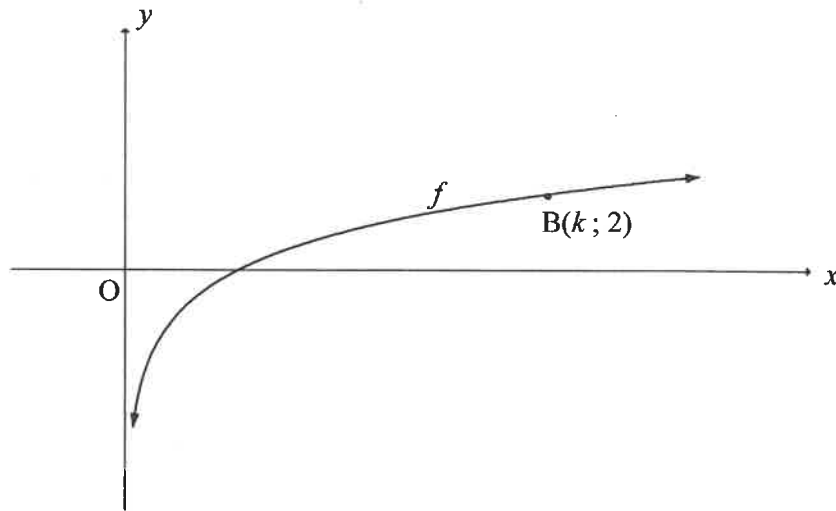
6.1   Calculate the value of k.   (2)

6.2   Determine the values of x for which $-1 \le f(x) \le 2$.   (2)

6.3   Write down the equation of f^{-1}, the inverse of f, in the form $y = \ldots$ (2)

6.4   For which values of x will $x.f^{-1}(x) < 0$?   (2)

                                                                                            [8]
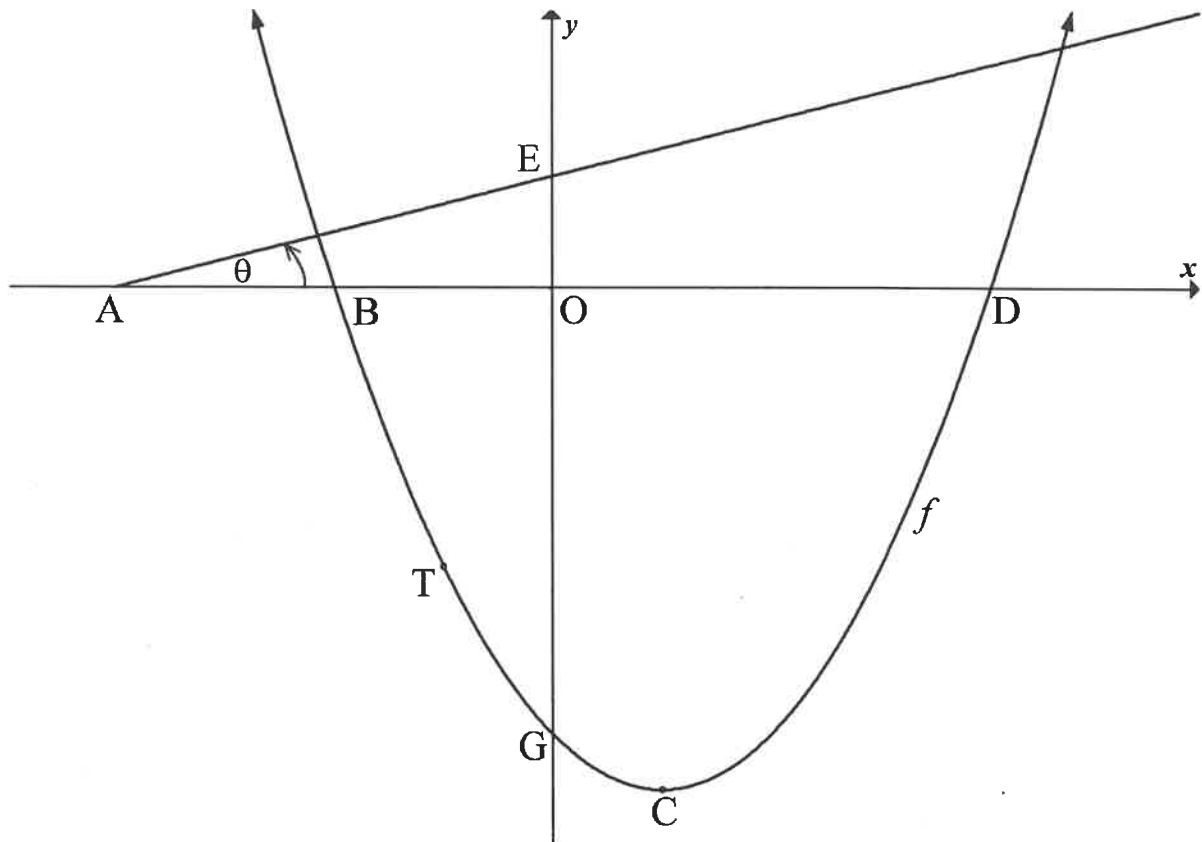
QUESTION 7

The graph of $f(x) = (x+4)(x-6)$ is drawn below.

The parabola cuts the x-axis at B and D and the y-axis at G.

C is the turning point of f.

Line AE has an angle of inclination of θ and cuts the x-axis and y-axis at A and E respectively.

T is a point on f between B and G.

7.1   Write down the coordinates of B and D.   (2)

7.2   Calculate the coordinates of C.   (2)

7.3   Write down the range of f.   (1)

7.4   Given that $\theta = 14{,}04°$ and the tangent to f at T is perpendicular to AE.

        7.4.1   Calculate the gradient of AE, correct to TWO decimal places.   (1)

        7.4.2   Calculate the coordinates of T.   (5)

7.5   A straight line, g, parallel to AE, cuts f at K(–3 ; –9) and R.
        Calculate the x-coordinate of R.   (6)

                                                                                       [17]

QUESTION 8

8.1   A farmer bought a tractor for R980 000. The value of the tractor depreciates annually
        at a rate of 9,2% p.a. on the reducing-balance method. Calculate the book value of
        the tractor after 7 years.   (3)

8.2   How many years will it take for an amount of R75 000 to accrue to R116 253,50 in
        an account earning interest of 6,8% p.a., compounded quarterly?   (4)

8.3   Thabo wanted to save R450 000 as a deposit to buy a house on 30 June 2018.

        8.3.1   He deposited a fixed amount of money at the end of every month into an
                  account earning interest of 8,35% p.a., compounded monthly. His first
                  deposit was made on 31 July 2013 and his 60th deposit on 30 June 2018.
                  Calculate the amount he deposited monthly.   (3)

        8.3.2   Thabo bought a house costing R1 500 000 and used his savings as the
                  deposit. He obtained a home loan for the balance of the purchase price at
                  an interest of 12% p.a., compounded monthly over 25 years. He made
                  his first monthly instalment of R11 058,85 towards the loan on
                  31 July 2018.

                  (a)   What will the balance outstanding on the loan be on 30 June 2039,
                          21 years after the loan was granted?   (3)

                  (a)   Calculate the interest Thabo will have paid over the first
                          21 years of the loan.   (3)
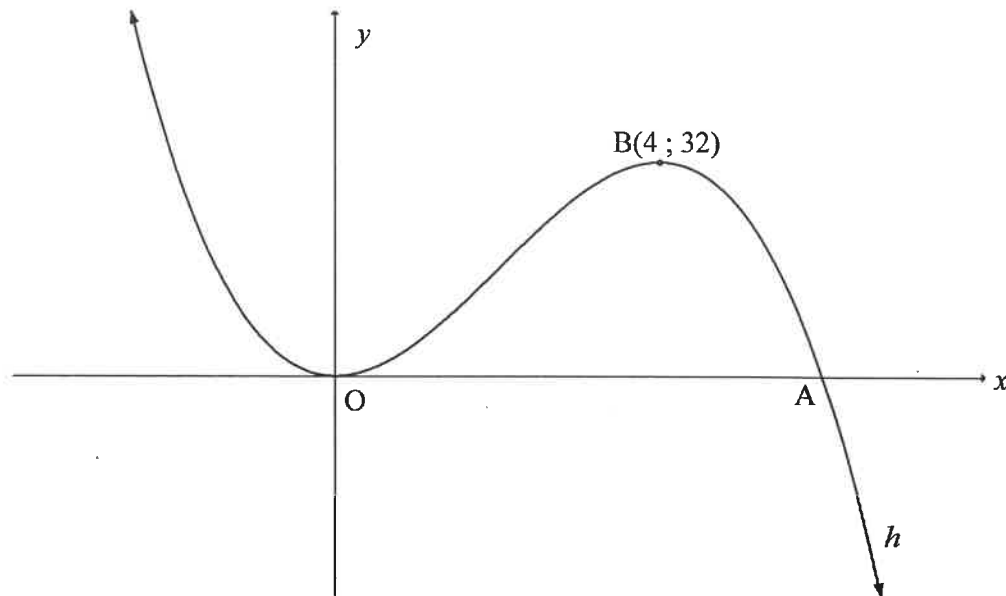   **[16]**

QUESTION 9

9.1   Determine $f'(x)$ from first principles if it is given that $f(x) = 2x^2 - 3x$.   (5)

9.2   Determine:

        9.2.1 $\dfrac{dy}{dx}$ if $y = 4x^5 - 6x^4 + 3x$ (3)

        9.2.2 $D_x\left[-\dfrac{\sqrt[3]{x}}{2} + \left(\dfrac{1}{3x}\right)^2\right]$ (4)

                                                                                                                       [12]

QUESTION 10

The graph of $h(x) = ax^3 + bx^2$ is drawn.
The graph has turning points at the origin, O(0 ; 0) and B(4 ; 32).
A is an x-intercept of h.

10.1   Show that $a = -1$ and $b = 6$.   (5)

10.2   Calculate the coordinates of A.   (3)

10.3   Write down the values of x for which h is:

      10.3.1   Increasing   (2)

      10.3.2   Concave down   (2)

10.4   For which values of k will $-(x-1)^3 + 6(x-1)^2 - k = 0$ have one negative and
      two distinct positive roots?   (3)
                                                                                         [15]

QUESTION 11

After travelling a distance of 20 km from home, a person suddenly remembers that he did not close a tap in his garden. He decides to turn around immediately and return home to close the tap.

The cost of the water, at the rate at which water is flowing out of the tap, is R1,60 per hour.

The cost of petrol is $\left(1,2+\dfrac{x}{4000}\right)$ rands per km, where x is the average speed in km/h.

Calculate the average speed at which the person must travel home to keep his cost as low as possible.   [7]

QUESTION 12

12.1   A and B are independent events. It is further given that:

P(A and B) = 0,3 and P(only B) = 0,2

    12.1.1   Are A and B mutually exclusive? Motivate your answer.   (1)

    12.1.2   Determine:

        (a)   P(only A)   (4)

        (b)   P(not A or not B)   (2)

12.2   A teacher has 5 different poetry books, 4 different dramas and 3 different novels. She must arrange these 12 books from left to right on a shelf.

    12.2.1   Write down the probability that a novel will be the first book placed on the shelf.   (1)

    12.2.2   Calculate the number of different ways these 12 books can be placed on the shelf if any book can be placed in any position.   (2)

    12.2.3   Calculate the probability that a poetry book is placed in the first position, the three novels are placed next to each other and a drama is placed in the last position.   (4)
                                                                                                                          [14]

                                                                                              TOTAL: **150**

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad\qquad A = P(1 - ni) \qquad\qquad A = P(1 - i)^n \qquad\qquad A = P(1 + i)^n$$

$$T_n = a + (n-1)d \qquad\qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \qquad\qquad S_n = \frac{a(r^n - 1)}{r - 1} \; ; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \; ; \; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad\qquad P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad\qquad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad\qquad y - y_1 = m(x - x_1) \qquad\qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad\qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \triangle ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad\qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad\qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad\qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad\qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad\qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad\qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$