Project 2 — Exploring the Fundamentals of Monte-Carlo Simulation

On My Honor as a student, I have neither given nor accepted unauthorized aid on this assignment

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Introduction

"A representative of a high-speed Internet provider calls customers to assess their satisfaction with the service." This representative will call customers up to 4 times before giving up. We define the continuous random variable W as the total number of seconds the representative spends waiting for a customer to answer the phone over the course of the 4 potential calls. W has a minimum of 6 seconds (assuming the customer picks up immediately on the first call) and a maximum of 128 (assuming the customer does not pick up after 4 calls). We are interested in several statistics of W (ie. we want to know how W is distributed, purportedly to learn about the answering behavior of customers). In order to visualize the distribution of W, we use monte-carlo simulation to mimic trials of this experiment.

Part 1 — Formulating a Model: Notation, Equations, and Diagrams

Events

- **D:** probability the customer is dialed | P[D] = 1
- **A:** probability the customer is available | P[A] = 0.5
- **B:** probability the line is busy | P[B] = 0.2
- **O:** probability the line is open | P[O] = 0.3
- **M:** probability the customer is available but misses the call | P[M] = P[A] * P[X > 25]
- **R:** probability the customer is available and receives the call | P[R] = P[A] * P[X ≤ 25]

Random Variables

- **N:** number of calls (out of 4) until the customer picks up
    - N ~Geometric (p = 0.5)
    - $$p_N(n) = \begin{cases} 0.5\,(0.5)^{n-1} & n = 1,2,3,4, \\ 0 & \text{otherwise} \end{cases}$$
    - $$F_N(n) = \begin{cases} 1-(0.5)^n & n = 1,2,3,4, \\ 0 & \text{otherwise} \end{cases}$$
- **X:** number of seconds until an available customer picks up
    - E[X] = 12
    - X ~Exponential ($\lambda = 1/12$)

$$f_X(x) = \begin{cases} \frac{1}{12} e^{-x/12} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$F_X(x) = \begin{cases} 1 - e^{-x/12} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

- $x = -12 \ln[1 - F_X(x)]$
- $P[X \leq 25] = 0.8755 \rightarrow P[R] = 0.4378$
- $P[X > 25] = 0.1245 \rightarrow P[M] = 0.0623$

- **W:** random variable W is defined as the number of seconds it takes, over the course of 4 potential calls, for a customer to pick up
  - W is a function of the two other random variables: N and X | W = f(N, X)

Tree Diagram for Calling Process