Algebra 1
Exercise sheet 5

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Exercise 1.

Define $\varphi\colon A \to \bigoplus_{i=1}^{n} A/a_i$.

Lets show that it is injective. Pick $a \in A$ with $\varphi(a) = 0$. Then $a \in a_j$ for every $j = 1, \ldots, n$. Since intersection of these ideals is trivial, we get $a = 0$.

So A is isomorphic to the image $\varphi(A)$. The image is a submodule of the module $\bigoplus_{i=1}^{n} A/a_i$. The direct sum $\bigoplus_{i=1}^{n} A/a_i$ is noetherian because it is the sum of neotherian modules. And the submodule of a noetherian module is obviously also noetherian.