Algebra 1
Exercise sheet 6

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Exercise 1. We note that the map is defined on elementary tensors.

Well defined: Suppose $\sum_{i=1}^{n} a_i \otimes b_i = 0$. Then $(\sum_{i=1}^{n} a_i \cdot b_i)(1 \otimes 1) = 0$.

Injectivity: Take $\sum_{i=1}^{n} a_i \otimes b_i \in \mathfrak{a} \otimes_A \mathfrak{b}$. Suppose $\sum_{i=1}^{n} a_i \cdot b_i = 0$. Then we can simply observe that since $a \otimes b = 1 \otimes (ab)$ we get

$$\sum_{i=1}^{n} a_i \otimes b_i = \sum_{i=1}^{n} 1 \otimes (a_i \cdot b_i) = 1 \otimes (\sum_{i=1}^{n} a_i \cdot b_i) = 1 \otimes 0 = 0$$

which proves injectivity.

Surjectivity: An element $\sum_{i=1}^{n} a_i \cdot b_i \in \mathfrak{a} \cdot \mathfrak{b}$ is the image of an element $\sum_{i=1}^{n} a_i \otimes b_i \in \mathfrak{a} \otimes_A \mathfrak{b}$.