Algebra 1
Exercise sheet 7

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Exercise 4. Let
$$\varphi\colon (f,T) \to A[T] \tag{1}$$
be the inclusion. Then
$$\mathrm{id} \otimes \varphi\colon (f,T) \otimes_{A[T]} (f,T) \to (f,T) \otimes_{A[T]} A[T] \tag{2}$$
is not injective, because
$$(\mathrm{id} \otimes \varphi)(f \otimes T - T \otimes f) = f \otimes T - T \otimes f = fT \otimes 1 - fT \otimes 1 = 0 \tag{3}$$
but $f \otimes T \neq T \otimes f \in (f,T) \otimes_{A[T]} (f,T)$. Module (f,T) is not flat by proposition 1 in lecture notes 12.