

Algebraic geometry 1

Exercise sheet 3

Solutions by: Eric Rudolph and David Čadež

3. November 2023

Exercise 1.

1. Let X be a finite set that is irreducible with respect to some topology \mathbb{F} on X . Then we get $|\mathbb{F}| < \infty$ and since finite unions of closed sets are closed again we get that

$$X' := \bigcup_{U \subsetneq X \text{ closed}}$$

is closed in X . Since X is by assumption irreducible, $X \neq X'$, so we can pick $x_0 \in X \setminus X'$, which is by construction generic. The second part is then an immediate consequence of part 2 of Hochster's Theorem.