LECTURE 2 (SUMMARY)

(1) We recall the definition of flat modules.
(2) We mention Lazard's theorem.
(3) We recall flat A-algebras and mention main examples: polynomial algebras, localizations, completions of Noetherian rings.
(4) We recall that flatness is a property that can be verified on stalks.
(5) We define f-flatness for a quasicoherent sheaf $\mathcal{M} \in \mathrm{Qcoh}(X)$ and a map of schemes $f : X \to Y$, from the stalks perspective.
(6) We see that flatness is stable under composition, stable under basechange, local on target and local on source, since this can be checked on stalks.
(7) We reinterpret finitely presented flat quasicoherent sheaves as locally free modules of locally constant rank.
(8) We define the degree of a finite flat map.
(9) We recall the definition of faithfully flat modules.
(10) We show that flat map of rings $A \to B$ is faithfully flat if and only if $\operatorname{Spec} B \to \operatorname{Spec} A$ is surjective.
(11) We show that flat maps are generalizing.
(12) We have a short discussion of Chevalley's theorem for constructible sets, and use this theorem to prove that finitely presented flat maps are universally open.

1. Suggested additional reading:

- For flatness in the context of commutative algebra, Görtz–Wedhorn Algebraic Geometry I: Schemes (§ B.4, B.5, B.6, B.8).
- For a proof that the I-adic completion of a Noetherian ring is flat (https://stacks.math.columbia.edu/tag/06LD).
- For a proof of Lazard's theorem (https://stacks.math.columbia.edu/tag/058G)
- For a proof of Chevalley's theorem (https://stacks.math.columbia.edu/tag/00FE and https://stacks.math.columbia.edu/tag/054H).