

with  $B$  and  $C$  polynomial  $R$ -algebras. Assume  $B = R[T_i : i \in I]$  and the axiom of choice. Then we have a choice function:

$$F = \{ \psi^{-1}(e(T_i)) \}_{i \in I} \xrightarrow{\Delta} \bigcup F$$

and we can define  $f$  as follows:

$$f(T_i) := \Delta(\psi^{-1}(e(T_i)))$$

This gives the following map:

$$\begin{array}{ccc}
 0 & \xrightarrow{\exists_B} & A \otimes_B \mathcal{R}[B/R] & \sim 0 \\
 \downarrow f_* & & \downarrow f_* & \\
 0 & \xrightarrow{\exists_C} & A \otimes_C \mathcal{R}[C/R] & \sim 0
 \end{array}$$

(2)

$$b \mapsto 1 \otimes db$$

↓

$$f(b) \mapsto 1 \otimes df(b) = 1 \otimes d f(b) =: f_*(1 \otimes db)$$

Hence a map/morphism of complexes.

(ii) We write down the homotopy:

$$\begin{array}{ccccccc} \cdots & \rightarrow & \mathcal{I}_B & \rightarrow & A \otimes_{\mathcal{B}} \Omega^1_{B/R} & \rightarrow & \cdots \\ & \searrow & \mathcal{J}_B^2 & & & & \\ & & f \downarrow f' & \swarrow H & f \downarrow f' & \nearrow \mathcal{J}^1 & \\ \cdots & \rightarrow & \mathcal{I}_C & \rightarrow & A \otimes_{\mathcal{C}} \Omega^1_{C/R} & \rightarrow & \cdots \end{array}$$

Defining  $H(1 \otimes db) := f(b) - f'(b)$  gives the result. We argue that it exists by universal property of Néâller differentials:

$$\begin{array}{ccc} B & \xrightarrow{d} & \Omega^1_{B/R} \\ f-f' \searrow & & \nearrow \mathcal{J}^1_H \\ & \mathcal{I}_C / \mathcal{J}_C^2 & \end{array}$$

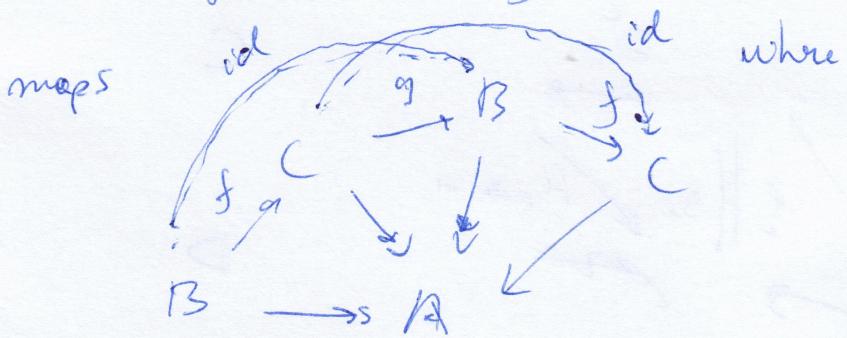
Indeed  $f - f'$  is an  $R$ -derivation of  $B$

into the  $A$ -Module  $\mathbb{Z}/\mathbb{Z}_c^2$ :

$$\begin{aligned} R\text{-linearity: } (f - f')(rb) &:= f(rb) - f'(rb) \\ &= r f(b) - r f'(b) \\ &= r (f - f')(b) \end{aligned}$$

Liebniz rule: ??

(iii) We repeat previous arguments and abstraction



$f \circ g \neq id$ , but we will show that

$h_{f \circ g} \approx h_{id}$ . Indeed the (ii) part

proves it.

④