LECTURE 3 (SUMMARY)

(1) We recall the definition of constructible sets.
(2) We recall the definition of the constructible topology.
(3) We state Chevalley's theorem. That is, finitely presented morphisms preserve constructible sets.
(4) We use Chevalley's theorem to show all finitely presented flat morphisms are universally open.
(5) We give an example of a flat closed immerion which is not an open immersion (necessarily non-finitely presented).
(6) We show that for any map $f : X \to Y$ of locally Noetherian schemes and any point $x \in X$ we have an inequality
$$\dim(\mathcal{O}_{X_y,x} \geq \dim \mathcal{O}_{X,x} - \dim \mathcal{O}_{Y,f(x)}$$
and that we get equality when the map is flat.
(7) We define what it means to be equidimensional and define morphisms of relative dimension n.
(8) We show that flat maps between irreducible finite type k-schemes X and Y are always of relative dimension $\dim(X) - \dim(Y)$.
(9) We show generic flatness by reducing it to Grothendieck's freeness lemma.
(10) We sketched a proof of Grothendiekc's freeness lemma.

1. Suggested additional reading:

- Section 7.4.4 in Vakil's Foundations of algebraic geometry.