

Algebraic geometry 1

Exercise sheet 7

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Exercise 1.

1. Since f^* is left adjoint to f_* , we have the following chain of isomorphism

$$\begin{aligned}\mathrm{Hom}_{\mathcal{O}_x}(f_*\tilde{\mathcal{N}}, \widetilde{\mathcal{N}|_A}) &\cong \mathrm{Hom}_A(f_*\tilde{\mathcal{N}}(B), \tilde{\mathcal{N}}|_A(B)) \\ &= \mathrm{Hom}_A(\tilde{\mathcal{N}}(A), \mathcal{N}|_A) \cong \mathrm{Hom}_{\mathcal{O}_x}(\tilde{\mathcal{N}}|_A, \tilde{\mathcal{N}}|_A).\end{aligned}$$

By the Yoneda lemma, this implies that $f_*\tilde{\mathcal{N}} \cong \tilde{\mathcal{N}}|_A$.