

Algebraic geometry 1

Exercise sheet 6

Solutions by: Eric Rudolph and David Čadež

18. November 2023

Exercise 1.

1. We have the following commutative diagram

$$\begin{array}{ccc}
 U_i \times_{S_{i,j}} V_j & \xrightarrow{\pi_2} & V_j \\
 \downarrow \pi_1 & & \downarrow \psi \\
 U_i & \xrightarrow{\phi} & S_{i,j} \subset X \times_S Y
 \end{array}$$

Now

$$(\phi \circ \pi_1)^{-1}(S_{i,j}) = U_i \times_{S_{i,j}} V_j =: Z$$

is open as the preimage of an open set under a continuous map. By the second part of exercise sheet 5, this induces a subset (Z, \mathcal{O}_Z) of $X \times_S Y$ as locally ringed spaces.