Back to lifting criteria: Proposition Formal Smoothness is local on the source. Theorem (Gruson-Raymand) Let A be a ring and M be an A-module. Let A->B be a faithfully flat map. If Mass is projective B-module, the M is a projective A-modile. Park - projectivity sitisfies frithfilly flyt bscent. - We showed being finite projective sutisfies frithfolly flat dexent.

Let f:x->5 be a morphism of schenes, let X = U U_i open Cover with ui= Spec Ai such that flui: Ui ->) is formally smooth, Fix a diagram and To = Spee By S. I. $T^2 = 0$. By hypothesi's

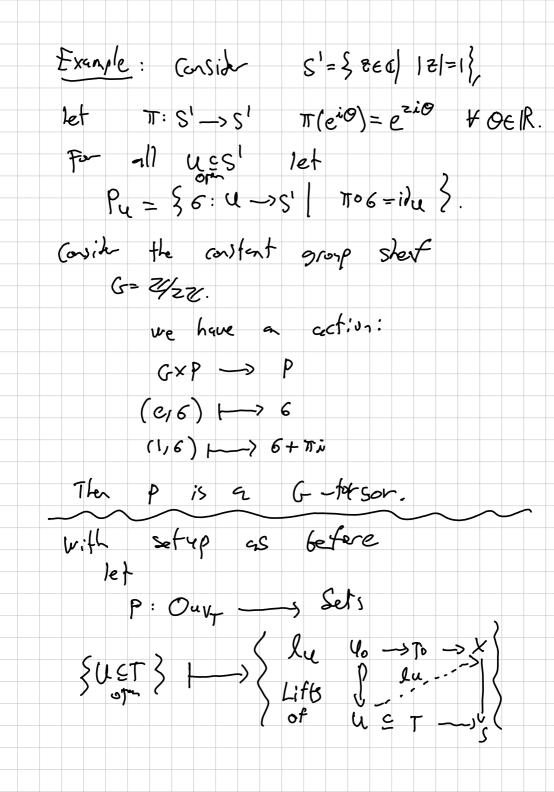
To NUi -> Ui

i [lī. · ·]

rnui - u

hes a lift li Question Do the li glue to a map T -> x ?

Definition Let G be a group. 1) A G-pseudo - torsor is a set P with ection Gxp -> p s.t. + P, Pz & P 3! 96 C with 9-P, =Pz. 2) A G-PSecrat-forse p is a G-tos. - if P + Ø. Let T be a topological spre and G a Sheet of groups 1) A G-pseub-torser is a sheet pover T with an action $G \times P \longrightarrow P$ s.t. If ust open and I pep(u) He map G(u) ->> P(u) 91-39-1 is an isomorphism. e) A pseud-forset p is a forser for all xET Px # Ø.



Claib : P is a

Hom (4. Slxs, I) - torsor. Remark. when x=spec A, S=spec C th: 5 5475 Der (A, I) acts simply fransitively on the set of lifts (see Lecture 6) Key point: This construction plues. Definition: A map of torsus Pr, Pz is a G-equiver and map of sheares. Rot: Any map of G-torses is automitically on isomorphism Detinition: Given a topological space T and a Steef of groups G we let $H'(T,G) = \frac{1}{3}G - \frac{1}{3}e^{-\frac{1}{3}}$

Remark H'(T,G) is a pointed set with distinsuished element being the isomorphism class of the trivial forson. Renarle If sep(T) than the map G -> P indres en ison>(ph.sn ut G-torsus. In other words 9=* in #'(1,6) <=> P(7) ≠ Ø. Functorisdity of torsors? proposition If G, -> G, is a map of 9ron 195, and P, is a Cox P, = Gx P, G is a Gz - torsor.

Prost For XET we con conjute

the Stalk $(P_2)_x = G_{2x} \times P_{1x} \times G_{2x}$. If SE P2(4)
we want to show $G_2(u) \rightarrow \left[P_2(u) \right]$ is an isonorphism, we can check
this map is an isomorphism on
Stalks.

Rook
In general P_2(u) + [G_2(u) x G_1(u)]

Since we have to sheafify. Proposition H'(T, G, xGz) = H'(T, G,) x H'(T, Gz). proof there are evident and ps

Pinz Horizon (Pinz X G, Pinz X Gz) PIXPZ C-1 (PIPZ) since stalks compute with finite

products are shows the constructions are inverse to each other. PNPS: tim H'(T, TTG;) = TT H'(T,G;) Proof If P is a IT G, - torsor the (P; = P x G;) ; et ; s a fam:/y of G; -torsis inlexed 6, jeJ. Given a family of tossis (P;) ic). we let P= TTP;, this is only a
Pseudo-torsor since some ctalks might Le empty. Key point: stalks do not commute with infinik product. Given or diaspam $D = \bigcup_{i=1}^{N_0} X_i$ we set a porse PDEH'(T, Hom(No R'x/s, I)).

and we want to show Po=* in this H'. clain: Us Slx/s is projective. Prosective since li -> 5 is formally Smoth. Since U TONUi -> To 15 a faithfully the coult Tollows from (Gruson - Paynaud). Claim: H'(T, Hom(N, R'xs, I)) = # Let M= U&Slys then Mis a
direct summend of PA sine it is
Projective. The Mom (M, I) is a direct summent of Hon (A, I) = TI This gives as an injection $H'(T, Hom(NT)) \subseteq H'(T, TTL) \subseteq TT H'(T, L)$

