

# Elliptic curves and their moduli spaces

## Exercise sheet 6

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### Problem 2.

1. By definition of degree on line bundles we have  $\deg(\mathcal{L}^{\otimes n}) = n \deg(\mathcal{L})$ .  
Assume  $\mathcal{L}$  is ample and  $\mathcal{L}^{\otimes n}$  defines an immersion into projective space  $\mathbb{P}^n$ .
2. Pick  $x \in C$  a closed point. Define a divisor  $D = x$ . Let  $\mathcal{L}$  be the line bundle associated to  $D$ . Since  $\deg(\mathcal{L}) = 1$ , the line bundle  $\mathcal{L}$  is ample and thus defines an immersion  $C \rightarrow \mathbb{P}^n$  for some  $n \geq 1$ .