Grothendieck vanishing: Theorem Let X be a Noetherian topological Space. If dim(x) = d then  $H^{p}(x, f) = 0$  for any sherf of

abelian groups and p > d. RMK Any ringed space (x,0x) there is a forsetf-1 function f: (x, ox) ---> (x, 2) R^F\* =0 Br 131  $H^{i}(x,F) \stackrel{\sim}{=} H^{i}(x,f_{*}F)$ (i.e. Cohomology 15 the same it) trented 95 Ox-module or sheet) of abelian 50075-

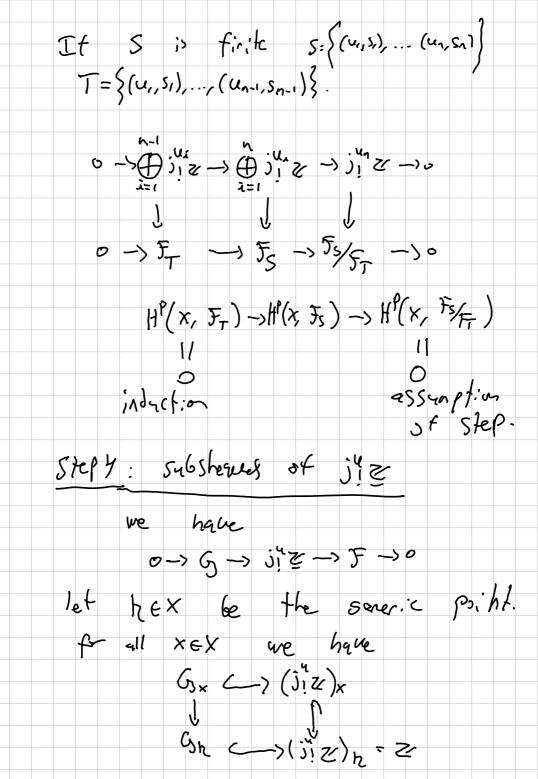
Leng: Let i: 2 -> x be a closed inhersion of topological spres. Giren J a Sheaf of alselian 2100ps on z we have  $H^{p}(z, f) \simeq H^{p}(x, i_{x} f)$ prost ix: Shu(2, Z) -> Shu (x, Z) is an exact functor (check stalks). and sends injectives to 1'-acxclic. Lens If X is an irreducible topological

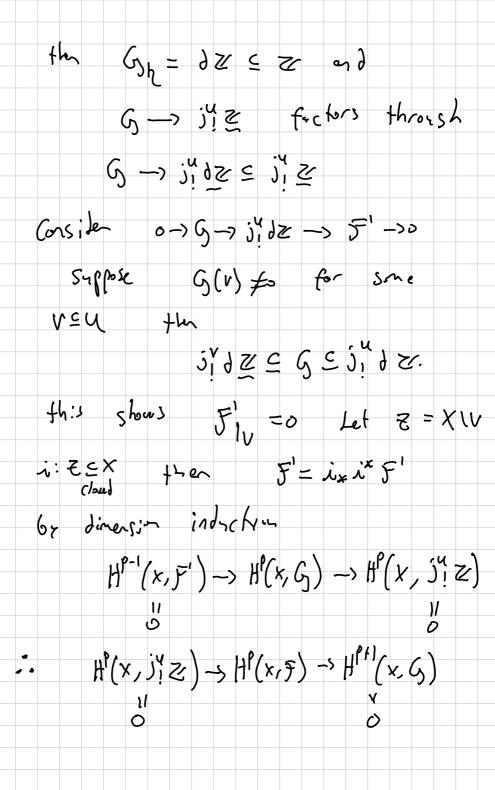
Space and A is an abelian snorth then HP(x, A) =0 for PZ1. Proit Every open USX is cornected so A(u) = A and A is flasque hence r'-acoclic. RMK: This shows H'(x, Z) =0 Por all smooth proper (uvue) V. Mit verted Groffendierk to introduce

sketch of proof of Thin: Step 1 WLOG X is irredycible: It i: 2 SX closed immes: a cont the support of 5, j\* 5 is u ≤ x i.e. j, j\* F = iū, G for some G. Apply this to 2 an irreducible component, then dim(2), dim(ci) < dim(x) and # components to U < # sompount X HP(x, j, j\* 5) -> HP(x, 5) -> HP(x, j\*i\* 5)  $H^{p}(\overline{u},G)$   $H^{p}(\overline{z},i^{*}F)$   $0 \sim 100 \sim 100 \sim 100$ 

Step 2 Dinonsional induction: and x is imedicible If d:m(k)=0 then x= 3\*3, Shv(x, 2) = A6 ad M(x,-) is en equivilence. Let j: UEX any open with complement z=xvu i: ZEX. we have SES: 0 -> J! Z -> Z -> i\*Z -> 0 ad  $din(z) \leq d-1$ HP-1(2,1,2)->HP(x,5,2)->HP(x,4) induction. (Flisque) Intrition: The Siuzy as u ranges over open subsets "generate" Shu(x, 21)

5tcp 3: From several to 1scally severaked. by one selfion Let B denote the set of local section i.e. B= {(u,s) / uex 6pm and se r'(u, Flu) We have A 5!2 -> 5 (us)EB is surjective. Cive seB FSEF the insue of *let*  $(\alpha^{i})\in \mathbb{Z}$   $(\alpha^{i})\in \mathbb{Z}$   $(\alpha^{i})\in \mathbb{Z}$   $(\alpha^{i})\in \mathbb{Z}$   $(\alpha^{i})\in \mathbb{Z}$   $(\alpha^{i})\in \mathbb{Z}$   $(\alpha^{i})\in \mathbb{Z}$ then  $\overline{S} = \frac{\text{Colim}}{8 \leq B} \overline{S}$ s 5 finite and  $H^{p}(x, f) = colim_{scB} H^{p}(x, fs)$ Hone work. Stink





Rfi preserve QCoh; Proposition Let f:x->x be a gc73 map of schenes and let FEQCh(x) Then Rifx F & Qcoh(y) and for all oper affine VEY with U=f-1(V) we have  $R^{i}f_{x}F(v)=H^{*}(u, F).$ Prest Rufy 5 is the shefifiction of VI > Har (F-YV), Jiv). so the formula holds (Rify 5) V = Rify (5/2-40). V= Spec B ad are mant to MLOG Show Rif\* 5 = Hi (x, F).

Let g E B it 5-17:ces to show  $H^{\alpha}(x,F)[5] = H^{\alpha}(f^{-\alpha}(0(5)),F).$ Case 1 X is separated: write X = Dui affire cove. Hi(x, 5) is comprised by Eech complex c'(suish, 5). with  $C^{K}(\{u_{i}\}_{i=1}^{h}, F) = \bigoplus_{J \in \{u_{i}=n\}} F(u_{J})$  |J| = KMorrower, each  $U_{\overline{J}}$  is affine,  $f^{-1}(D(5)) = U_{\overline{J}}D_{U_{1}}(5)$  and  $J_{\overline{J}}=1$  $\int Du_{i}(9) = Du_{j}(9)$ is again affine.

Hi(f-1(D(5)), f) is compiled by Each (smplex c ({ Du, (9) }, 5)

Note that  $C'\left(\frac{1}{3}D_{u_{\lambda}}(5)\frac{1}{5},\frac{1}{5}\right)=C'\left(\frac{1}{3}u_{\lambda}^{2},\frac{1}{5}\right)$ Since localization is exact, it Commertes with cohomology  $\beta^{*}(f'(p(s)), f)$   $\beta^{*}(x, f) \begin{bmatrix} \frac{1}{3} \end{bmatrix}$ (ase Z, sever) X: we only assume x is gras over Y-slee B. We have finite (over by affine)

X = () Ui and the Us are grasicompact separated schenes but not necessarilly affine, in perticular HP(us,5) might not Vanish for p=1.

$$C'(3Du_{i}(9))^{n}, H^{i}(-, F)) and$$

67 the separated case this myp induces an isomorphism.

$$C^{\bullet}(\S Du_{i}(9)\S_{i=1}^{h}, H^{i}(-, F))$$

$$= C^{\bullet}(\S U_{i}\S_{i=1}^{h}, H^{i}(-, F)) \setminus \begin{bmatrix} \frac{1}{9} \end{bmatrix}$$

Let 
$$N = \{u_i\}_{i=1}^n$$
 and  $N = \{u_i\}_{i=1}^n$  we have functions

$$F_x : Shv_{(x,0x)-nod} \longrightarrow PSh_x$$

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$$F_{(x,0x)-nod} \longrightarrow PSh_x$$

$$F_{(x,0x)-no$$

We set morphism of convergent Grothulieck speetal sequences  $H_{2}^{p,q} = H_{u}^{p}(X, H^{q}(u_{3}, F)) = H^{p,q}(x, F)$ V E 2 = Ho (f'D(s), H (f'D(s) \ U S, F) => H (f'D(s), F) B, the separated (see  $V = \frac{1}{3}$   $V = \frac{1}{2}$ since each us is separated. Since localization is exact this Shows uppg [ ] ~ V pg # z=r=0. By construction HP+4(x, F)[=] ]-> HP+4(f'(D(>)), F) is compatible with filtation and isomorphism on smiled pieces

since it is at Exp.