Algebraic geometry 1
Exercise sheet 3

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Exercise 1.

1. Let X be a finite set that is irreducible with respect to some topology $\mathcal{F}$ on X. Then we get $\mid \mathcal{F} \mid < \infty$ and since finite unions of closed sets are closed again, we get that

$$X' := \bigcup_{\substack{U \subsetneq X \, \text{closed}}} U$$

is closed in X. Since X is by assumption irreducible, $X \neq X'$, so we can pick $x_0 \in X \backslash X'$, which is by construction generic. The second part is then an immediate consequence of part 2 of Hochster's Theorem. As a finite set, X is quasicompact and as a basis $\mathcal{B}$ just take all of the open sets. It remains to show that X is sober.

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