Algebraic geometry 1
Exercise sheet 11

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Exercise 1.

1. Since k is algebraically closed, the only irreducible polynomials $f \in k[x, y]$ are of degree 1.

   Hence, we can write

   $$f_r = l_1 \dots l_r,$$

   where $l_i \in k[x, y]$ is of degree 1. From the assumption that f_r is homogenous it follows that the l_i are homogenous.

   Therefore, we can write

   $$Z = V(f_r) = V(l_1 \dots l_r) = \cup_i V(l_i)$$

   and since $V(l_i)$ is a line through the origin, Z can be written as the finite union of lines through the origin.