Combinatorial optimization
Exercise sheet 4

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Exercise 4.1. By greedily and heuristically looking at the graph we find the following matching $M = \{\{4,15\},\{1,5\},\{2,6\},\{7,8\},\{3,10\},\{9,12\},\{11,16\}\}$. It has size 7, so we can try to show $\nu(G) = 7$. We can do that by finding a set X such that $q_G(X) = |X| + 2$. Again heuristically looking at the graph yields $X = \{1,16,8\}$, for which the graph $G \setminus X$ contains 5 odd components, namely $\{5\}$, $\{13\}$, $\{11\}$, $\{2,6,7\}$, $\{3,9,10,12,14\}$. So $\max_{X\subseteq V(G)}(q_G(X)-|X|) \geq 2$. But also, using Berge-Tutte formula and $\nu(G) \geq 7$, we have $\max_{X\subseteq V(G)}(q_G(X) - |X|) \leq 2$. So $\nu(G) = 7$ and M is maximum.