PRIMITIVES DE FONCTIONS USUELLES

Dans ce tableau F est une primitive de f.

$f(x)$	$F(x)$
$f[g(x)]\,g'(x)$	$F[g(x)]$
$\dfrac{g'(x)}{g(x)}$	$\ln\lvert g(x)\rvert$
a	ax
$\dfrac{1}{x}$	$\ln\lvert x\rvert$
$\sqrt{x}$	$\dfrac{2}{3}x\sqrt{x}$
$\dfrac{1}{x^n}$	$\dfrac{-1}{(n-1)\,x^{n-1}}$ $\quad (n \neq 1)$
$\ln x$	$x(\ln x - 1)$
$\log_a x$	$x(\log_a x - \log_a e)$
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\tan x$	$-\ln\lvert\cos x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln\cosh x$
$\coth x$	$\ln\lvert\sinh x\rvert$
$\dfrac{1}{(x-a)(x-b)}$	$\dfrac{1}{a-b}\ln\left\lvert\dfrac{x-a}{x-b}\right\rvert$ $\quad (a \neq b)$
$\dfrac{1}{a^2+x^2}$	$\dfrac{1}{a}\arctan\dfrac{x}{a}$ $\quad (a \neq 0)$

$f(x)$	$F(x)$
$f(ax+b)$	$\dfrac{1}{a}F(ax+b)$ $\quad (a \neq 0)$
$[g(x)]^a\,g'(x)$	$\dfrac{[g(x)]^{a+1}}{a+1}$
x^m	$\dfrac{x^{m+1}}{m+1}$ $\quad (m \neq -1)$
$\dfrac{1}{x^2}$	$-\dfrac{1}{x}$
$\dfrac{1}{\sqrt{x}}$	$2\sqrt{x}$
$\dfrac{ax+b}{cx+d}$	$\dfrac{ax+b}{c} - \dfrac{ad-bc}{c^2}\ln\lvert cx+d\rvert$
e^x	e^x
a^x	$\dfrac{a^x}{\ln a}$
$\arcsin x$	$x\arcsin x + \sqrt{1-x^2}$
$\arccos x$	$x\arccos x - \sqrt{1-x^2}$
$\arctan x$	$x\arctan x - \dfrac{1}{2}\ln(1+x^2)$
$\text{arccot}\, x$	$x\,\text{arccot}\, x + \dfrac{1}{2}\ln(1+x^2)$
$\text{arcsinh}\, x$	$x\,\text{arcsinh}\, x - \sqrt{x^2+1}$
$\text{arccosh}\, x$	$x\,\text{arccosh}\, x - \sqrt{x^2-1}$
$\text{arctanh}\, x$	$x\,\text{arctanh}\, x + \dfrac{1}{2}\ln(1-x^2)$
$\text{arccoth}\, x$	$x\,\text{arccoth}\, x + \dfrac{1}{2}\ln(x^2-1)$
$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x-a}{x+a}\right\rvert$
$\dfrac{1}{a^2-x^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{a+x}{a-x}\right\rvert$

Primitives (suite)

$f(x)$	$F(x)$
$\sqrt{x^2-a^2}$	$\dfrac{x}{2}\sqrt{x^2-a^2} - \dfrac{a^2}{2}\ln\lvert x+\sqrt{x^2-a^2}\rvert$
$\sqrt{a^2-x^2}$	$\dfrac{x}{2}\sqrt{a^2-x^2} + \dfrac{a^2}{2}\arcsin\dfrac{x}{a}$
$\sqrt{x^2+a^2}$	$\dfrac{x}{2}\sqrt{x^2+a^2} + \dfrac{a^2}{2}\ln(x+\sqrt{x^2+a^2})$
$\sin^2 x$	$\dfrac{1}{2}(x - \sin x\cos x)$
$\cos^2 x$	$\dfrac{1}{2}(x + \sin x\cos x)$
$\tan^2 x$	$\tan x - x$
$\cot^2 x$	$-\cot x - x$
$\dfrac{1}{1+\sin x}$	$\tan\left(\dfrac{x}{2}-\dfrac{\pi}{4}\right)$
$\dfrac{1}{1+\cos x}$	$\tan\dfrac{x}{2}$
$e^{ax}\sin(bx)$	$\dfrac{e^{ax}}{a^2+b^2}[a\sin(bx) - b\cos(bx)]$
$e^{ax}\cos(bx)$	$\dfrac{e^{ax}}{a^2+b^2}[a\cos(bx) + b\sin(bx)]$
$x\sin(ax)$	$-\dfrac{1}{a}x\cos(ax) + \dfrac{1}{a^2}\sin(ax)$
$x\cos(ax)$	$\dfrac{1}{a}x\sin(ax) + \dfrac{1}{a^2}\cos(ax)$

$f(x)$	$F(x)$
$\dfrac{1}{\sqrt{x^2-a^2}}$	$\ln\lvert x+\sqrt{x^2-a^2}\rvert$
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\arcsin\dfrac{x}{a}$
$\dfrac{1}{\sqrt{x^2+a^2}}$	$\ln(x+\sqrt{x^2+a^2})$
$\dfrac{1}{\sin^2 x}$	$-\cot x$
$\dfrac{1}{\cos^2 x}$	$\tan x$
$\dfrac{1}{\sin x}$	$\ln\left\lvert\tan\dfrac{x}{2}\right\rvert$
$\dfrac{1}{\cos x}$	$\ln\left\lvert\tan\left(\dfrac{x}{2}+\dfrac{\pi}{4}\right)\right\rvert$
$\dfrac{1}{1-\sin x}$	$-\cot\left(\dfrac{x}{2}-\dfrac{\pi}{4}\right)$
$\dfrac{1}{1-\cos x}$	$-\cot\dfrac{x}{2}$
$x\,e^{ax}$	$\left(\dfrac{1}{a}x - \dfrac{1}{a^2}\right)e^{ax}$
$x\ln(ax)$	$\dfrac{x^2}{2}\ln(ax) - \dfrac{x^2}{4}$

Applicazioni del calcolo integrale alla geometria

Si considera un arco di curva di equazione cartesiana $y = f(x)$ con $a \leq x \leq b$.

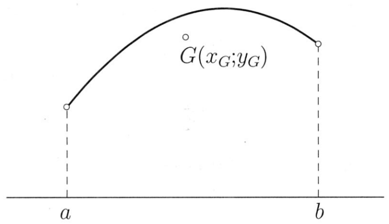
Lunghezza dell'arco

$$l = \int_a^b \sqrt{1 + \left(f'(x)\right)^2}\, dx$$

Baricentro dell'arco

$$x_G = \frac{1}{l} \int_a^b x\sqrt{1 + \left(f'(x)\right)^2}\, dx$$

$$y_G = \frac{1}{l} \int_a^b f(x)\sqrt{1 + \left(f'(x)\right)^2}\, dx$$



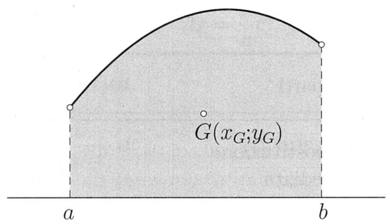
Area della superficie

$$\mathcal{A} = \int_a^b f(x)\, dx \qquad \text{se } f \geq 0$$

Baricentro della superficie

$$x_G = \frac{1}{\mathcal{A}} \int_a^b x f(x)\, dx \qquad \text{se } f \geq 0$$

$$y_G = \frac{1}{2\mathcal{A}} \int_a^b \left(f(x)\right)^2 dx$$



Area della superficie laterale del solido di rotazione

$$\mathcal{A}_{\text{lat}} = 2\pi \int_a^b f(x)\sqrt{1 + \left(f'(x)\right)^2}\, dx \qquad \text{se } f \geq 0$$

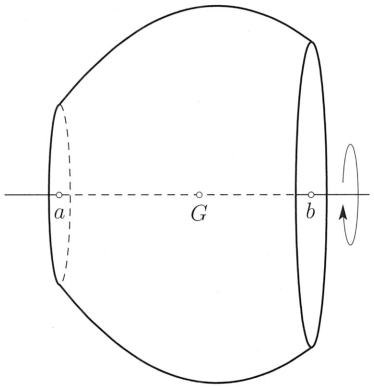
Volume del solido di rotazione

$$\mathcal{V} = \pi \int_a^b \left(f(x)\right)^2 dx$$

Baricentro del solido di rotazione

$$x_G = \frac{\pi}{\mathcal{V}} \int_a^b x\left(f(x)\right)^2 dx$$

$$y_G = z_G = 0$$



Si considera un arco di curva di equazioni parametriche $\begin{cases} x = g(t) \\ y = h(t) \end{cases}$ con $t_1 \leq t \leq t_2$.

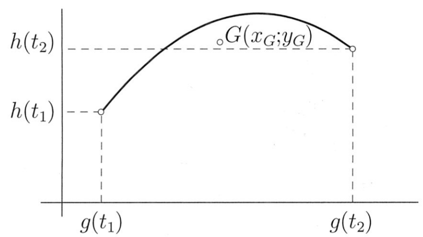
Lunghezza dell'arco

$$l = \int_{t_1}^{t_2} \sqrt{\left(g'(t)\right)^2 + \left(h'(t)\right)^2}\, dt$$

Baricentro dell'arco

$$x_G = \frac{1}{l} \int_{t_1}^{t_2} g(t)\sqrt{\left(g'(t)\right)^2 + \left(h'(t)\right)^2}\, dt$$

$$y_G = \frac{1}{l} \int_{t_1}^{t_2} h(t)\sqrt{\left(g'(t)\right)^2 + \left(h'(t)\right)^2}\, dt$$



Per le formule seguenti si suppone che g sia monotona crescente e che h sia positiva in $[t_1; t_2]$.

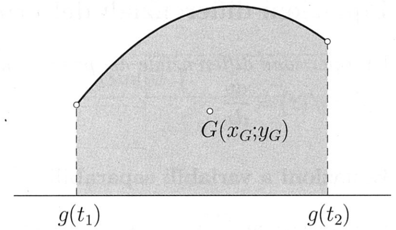
Area della superficie

$$\mathcal{A} = \int_{t_1}^{t_2} h(t)g'(t)\, dt$$

Baricentro della superficie

$$x_G = \frac{1}{\mathcal{A}} \int_{t_1}^{t_2} g(t)h(t)g'(t)\, dt$$

$$y_G = \frac{1}{2\mathcal{A}} \int_{t_1}^{t_2} \left(h(t)\right)^2 g'(t)\, dt$$



Area della superficie laterale del solido di rotazione

$$\mathcal{A}_{\text{lat}} = 2\pi \int_{t_1}^{t_2} h(t)\sqrt{\left(g'(t)\right)^2 + \left(h'(t)\right)^2}\, dt$$

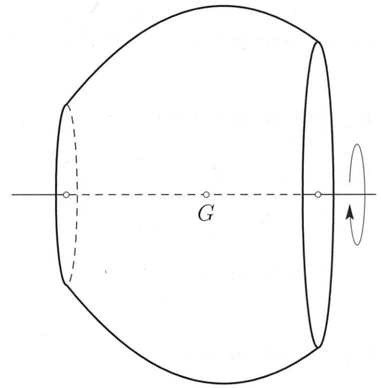
Volume del solido di rotazione

$$\mathcal{V} = \pi \int_{t_1}^{t_2} \left(h(t)\right)^2 g'(t)\, dt$$

Baricentro del solido di rotazione

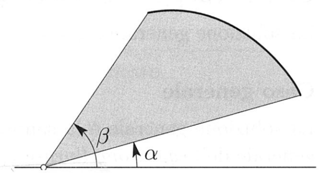
$$x_G = \frac{\pi}{\mathcal{V}} \int_{t_1}^{t_2} g(t)\left(h(t)\right)^2 g'(t)\, dt$$

$$y_G = z_G = 0$$



Si considera un arco di curva di equazione polare $r = f(\varphi)$ con $\alpha \leq \varphi \leq \beta$.

Lunghezza dell'arco	$l = \int_\alpha^\beta \sqrt{\left(f(\varphi)\right)^2 + \left(f'(\varphi)\right)^2}\, d\varphi$
Area della superficie	$\mathcal{A} = \frac{1}{2} \int_\alpha^\beta \left(f(\varphi)\right)^2 d\varphi$



Baricentro di una figura piana racchiusa tra due funzioni

$$x_G = \frac{1}{A} \int_a^b x\left[f(x) - g(x)\right] dx\,,$$

$$y_G = \frac{1}{2A} \int_a^b \left[f(x) + g(x)\right] \cdot \left[f(x) - g(x)\right] dx$$

dove $A = \int_a^b \left[f(x) - g(x)\right] dx$