

OR Exercise 1: Linear Programming Modeling

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1 Warm up: Using the LP Solver

LPSolve IDE gives a very intuitive way to model an LP as shown in the example below. We will use the CPLEX format. This type of input format is selected in the menu "view". The inequalities $\{\leq, \geq\}$ are replaced by $\{<=, >=\}$ and a variable x_1 is denoted x_1 in the CPLEX model language. An important thing to notice is that default bound of a variable x_i in the CPLEX model language is $0 \leq x_i \leq \infty$. If variable x_i has another bound for example $-\infty \leq x_i \leq \infty$ then the row "-inf <= x_i <= +inf" has to be added in the "BOUNDS" section of the model. For more information see the help and search for "CPLEX LP file format".

1.1 Example

A Linear Program

maximize/minimize: $x_1 + 2x_2 + 3x_3$
subject to: $x_1 + x_2 \geq 0$
 $x_3 \leq 1$;
 $x_1 + x_2 + x_3 = 1$
 $x_1, x_2, x_3 \geq 0$

CPLEX Model of the LP

Maximize/Minimize
 $x_1 + 2x_2 + 3x_3$
Subject To
 $x_1 + x_2 >= 0$
 $x_1 + x_2 + x_3 = 1$
Bounds
 $x_3 \leq 1$
End

- (a) Try to solve a linear program using the LPSolve IDE. Example of suitable linear programs can be found above and in the slides (page 9 and 11).

2 Exporting Swiss Chocolate and Clocks

Your task is to load a container with clocks and chocolate bars. The container will then be shipped from Switzerland to USA where the clocks and chocolate bars will be sold. For each shipped clock and chocolate bar you will profit 4 CHF and 1 CHF, respectively. However, each clock has weight 1 kg and volume $400cm^3$ whereas each chocolate bar has weight 200 g and volume $150cm^3$. The volume of the container is $500000cm^3$ and the weight of the load cannot exceed 1000kg. Model the problem of packing the container to maximize profit as a linear program and solve it using LPSolve IDE.

3 Resource Management

A food producing company produces two different kinds of cereals. Both types include nuts, oat flakes, and raisins. One unit of cereal A consists of two units of nuts, four units oat flakes, and one unit of raisins. The cereal B has ingredient ratio three to one to one. The cereals (per unit) are sold for five and four CHF, respectively. The company has stored 12000 units of nuts, 16000 units of oat flakes and 4300 units of raisins.

- (a) Formulate and solve a linear program maximizing the companies profit.

4 Mortgage Portfolio Optimization

The optimality of a portfolio depends heavily on the model used for defining risk and other aspects of financial instruments. Here is a particularly simple model that is amenable to linear programming techniques.

Consider a mortgage team with 100,000,000 to finance various investments. There are five categories of loans, each with an associated return and risk (1-10, 1 best):

Loan/investment	Return(%)	Risk
First Mortgage	9	3
Second Mortgage	12	6
Personal Loans	15	8
Commercial Loans	8	2
Government Securities	6	1

Any uninvested money goes into saving account with no risk and 3% return. The goal for the mortgage team is to allocate the money to the categories so as to:

1. Maximize the average return per dollar,
2. Have an average risk of no more than 5 (all averages and fractions taken over the invested money (not over the saving account)),
3. Invest at least 20% in commercial loans,

4. The amount in second mortgage and personal loans combined should be no higher than the amount in first mortgage.

Questions:

- (a) Write out the entire linear programming model formulation for this problem.
- (b) Use LPSolve to solve this problem.

5 Personal Scheduling

The Metropolis Subway System (MSS) has obtained the contract to operate the subway in the Greater Metropolis Metropolitan Area. The Traffic Planning Department has estimated that at least the following number of staff is needed each four-hour interval throughout a standard 24-hour period:

Time Period	Staff
06-10	700
10-14	200
14-18	800
18-22	300
22-02	100
02-06	100

All staff works in 8-hour shifts. There are six possible shifts that starts on the hour in the beginning of each 4-hour period in the table. All staff has the same hourly salary, except that the hourly salary is 50% higher between 22.00 and 6.00.

- (a) Find how many employees that are needed for each of the six shifts in order to minimize the salary costs for MSS, while at the same time satisfying the staffing requirements.
- (b) The staffing company SUPERMAN offers external staff at a more flexible schedule, with 4-hour shift starting on the hour in the beginning of each 4-hour period in the table. The hourly cost for using SUPERMAN's staff is a flat (that is, the same at all hours) 75% higher than the daytime cost for MSS's own staff. Management wants to know if there is any benefit of using SUPERMAN. Find out the optimal mix of MSS and SUPERMAN personnel in order to minimize the salary costs for MSS, while at the same time satisfying the staffing requirements. Is there any financial benefit compared to (a)?