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ex. 1

$n = 10$

$X = 0$ "no"

$X = 1$ "sì"

$X \geq 8$ test passato

$i = 2$

a) $P(X=2) = \binom{10}{2}\left(\frac{1}{2}\right)^2\left(1-\frac{1}{2}\right)^8 = \frac{45}{1024} \approx 0{,}0439$

b) $P(X>2) = 1 - P(X \leq 2) = 1 - P(X=2) - P(X=1) - P(X=0)$

$= 1 - \binom{10}{2}\left(\frac{1}{2}\right)^2\left(1-\frac{1}{2}\right)^8 - \binom{10}{1}\left(\frac{1}{2}\right)\left(1-\frac{1}{2}\right)^9 - (1-0{,}5)^{10} \cong 0{,}9453$

c) $P(X \leq 2) = 1 - P(X>2) = 1 - 0{,}9453 = 0{,}0547$

d) $P(X \geq 8) = \binom{10}{8}\left(\frac{1}{2}\right)^8\left(1-\frac{1}{2}\right)^2 + \binom{10}{9}\left(\frac{1}{2}\right)^9\left(1-\frac{1}{2}\right) + \left(\frac{1}{2}\right)^{10} \cdot \binom{10}{10} \cong 0{,}0547$

ex. 2

$n = 5$ $\quad p = \frac{2}{3}$

X = "vive tra 30 anni"

a) $P(X=5) = \binom{5}{5}\left(\frac{2}{3}\right)^5\left(1-\frac{2}{3}\right)^0 = \frac{32}{243} \approx 0{,}1317$

b) $P(X \geq 3) = 1 - P(X<3) = 1 - ~~P(X=3)~~ - P(X=2) - P(X=1)$

$= 1 - ~~\binom{5}{3}\left(\frac{2}{3}\right)^3\left(1-\frac{2}{3}\right)^2~~ - \binom{5}{2}\left(\frac{2}{3}\right)^2\left(1-\frac{2}{3}\right)^3 - \binom{5}{1}\left(\frac{2}{3}\right)\left(1-\frac{2}{3}\right)^4 = 0{,}7901$

$E[X] = 5 \cdot \frac{2}{3} = 3{,}33$

ex. 3

$n = 10$

X = "pezzi difettosi"

$P(X \leq 1)$ lotto accettato

$P(x) = 5\% = 0{,}05$

a) $P(X=2) = \binom{10}{2}(0{,}05)^2(1-0{,}05)^8 \cong 0{,}075$

b) $P(X \leq 2) = \binom{10}{2}(0{,}05)^2(1-0{,}05)^8 + \binom{10}{1}(0{,}05)(1-0{,}05)^9 + (1-0{,}05)^{10} = 0{,}988$

$P(X=2) + P(X=1) + P(X=0)$

c) $P(X \leq 1) = P(X=0) + P(X=1) = (1-0{,}05)^{10} + \binom{10}{1}(0{,}05)(1-0{,}05)^9 \cong 0{,}913862$

d) $E[X] = 10 \cdot 0{,}05 = 0{,}5$

e) $Var(X) = 10 \cdot 0{,}05\,(1-0{,}05) \cong 0{,}475$

$\sigma_X = \sqrt{Var(X)} \cong 0{,}6892$

ex. 4

$n = 1000$ $\quad$ 800 = casa A, 200 = casa X

X = "televisori di dubbia provenienza"

n comprati $= 10$ $\quad P = \frac{200}{1000} = 0{,}2 \quad X \sim B(10; 0{,}2)$

a) $P(X=3) = \binom{10}{3}(0{,}2)^3 \cdot (1-0{,}2)^7 = 0{,}2013$

b) $P(X \leq 2) = \dots$

c) $P(X>4) = 1 - P(X \leq 3) = 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3) \Rightarrow 1 - (P(X=3) + P(X \leq 2))$

d) $E[X] = 10 \cdot 0{,}2 = 2$ (n, p)

ex. 5

$E[X] = 8$

8 tornadi annui

a) $X \sim P(8) \rightarrow$ Poisson.

b) $P(X \leq 5) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) =$

$= \frac{8^0}{1}e^{-8} + \frac{8}{1}e^{-8} + \frac{8^2}{2!}e^{-8} + \dots + \frac{8^5}{5!}e^{-8} \cong 0{,}1912$

c) $P(6 \leq X \leq 9) = P(X=6) + P(X=7) + P(X=8) + P(X=9) \cong 0{,}5254$

d) $E[X] = 6 = n \cdot p = \lambda = \frac{8}{2} = 4 \rightarrow$ perché 8 sono annui

e) $P(X=8 \mid \lambda = 6) = \frac{4^8}{8!}e^{-6} \cong 0{,}0297$

ex. 6

$E[X] = 1 \quad X \sim P(1)$

X = "numero black out"

a) $P(X>1) = 1 - P(X \leq 1) = 1 - P(X=0) - P(X=1) = 1 - e^{-1} - e^{-1} \cong 0{,}265$

b) $P(X=2) = \frac{1^2}{2!}e^{-1} = 0{,}18394$

c) $P(Y \geq 4)$ $\quad Y$ = "black out in 6 mesi"

$\lambda = 6 \cdot \lambda_1 = 6 \quad Y \sim P(6)$

$P(Y \geq 4) = 1 - P(Y \leq 3) = 1 - P(Y=0) - P(Y=1) - P(Y=2) - P(Y=3) = 1 - \frac{6^0}{0!}e^{-6} - \frac{6^1}{1!}e^{-6} - \frac{6^2}{2!}e^{-6} - \frac{6^3}{3!}e^{-6} \cong 0{,}8488$

d) Z = "black out in 3 mesi" $\quad \lambda = 3$

$P(Z \leq 3) = P(Z=0) + P(Z=1) + P(Z=2) + P(Z=3) = e^{-3} + 3e^{-3} + \frac{3^2}{2!}e^{-3} + \frac{3^3}{3!}e^{-3} \cong 0{,}64723$

ex. 7

$Var(X) = 300 \quad X \sim P(300)$

X = "chiamate in 1h"

a) $P(X=250) = \frac{300^{250}}{250!}e^{-300} = 0{,}000304$

b) Y = "chiamate in 30 min"

$\lambda = \frac{300}{2} = 150$

$P(Y \geq 160) = 1 - P(Y<160) = 1 - e^{-150}\sum_{i=0}^{159}\frac{150^i}{i!} \cong 0{,}2173$

d) Z = "chiamate al minuto"

$\lambda_Z = \frac{\lambda_X}{60} = \frac{300}{60} = 5$

$E[Z] = \lambda_Z = 5$

$P(Z \leq 2) = P(Z=0) + P(Z=1) + P(Z=2) = e^{-5}\sum_{i=0}^{2}\frac{5^i}{i!} \cong 0{,}1247$

ex. 9

$$f(x) = \begin{cases} 1 & \text{se } 0 \leq x \leq 1 \\ 0 & \text{altrimenti} \end{cases}$$

$\alpha = 0$

$\beta = 1$

a) $\int_\alpha^\beta \frac{1}{\beta - \alpha}\,dx \overset{!}{=} 1 \Rightarrow \frac{1}{\beta-\alpha}\int_\alpha^\beta 1\,dx \overset{!}{=} 1 = 1 = 1$ vero

b) $E[X] = \frac{1}{2} \cong 0{,}5$

c) $Var[X] = \frac{(\beta-\alpha)^2}{12} = \frac{1}{12}$

d) $P(0 < x < 0{,}25) = \frac{b-a}{\beta-\alpha} = 0{,}25$

ex. 10

$X \sim U$

$E[X] = 4{,}5 \quad [a,b] = ?$

$Var[X] = 0{,}75$

a), b) $\begin{cases} \frac{a+b}{2} = 4{,}5 \\ \frac{(b-a)^2}{12} = 0{,}75 \end{cases}$ $\begin{cases} a = 9-b \\ \frac{(b-9+b)^2}{12} = 0{,}75 \end{cases}$ $\begin{cases} a = 9-b \\ \frac{(2b-9)^2}{12} = 0{,}75 \end{cases}$ $\begin{cases} a = 9-b \\ 4b^2 - 36b + 81 = 9 \end{cases} \begin{matrix} 3 \\ 6 \end{matrix}$ $\begin{cases} 9-3 = a \Rightarrow 6 \\ 9-6 = a \Rightarrow 3 \\ b = 3 \\ b = 6 \end{cases}$ $a \overset{!}{<} b$

$a = 3 \quad b = 6$

c) $P(X \geq 5{,}5) = P(5{,}5 < X < 6) = \frac{6-5{,}5}{6-3} = \frac{\frac{1}{2}}{3} = \frac{1}{6} \cong 0{,}1667$

Devo rimanere nell'intervallo quindi uso, perché distribuzione uniforme, il rapporto $\frac{b-a}{\beta-\alpha}$

ex. 12

$E[X] = 1{,}25$ secondi $\quad X \sim N(1{,}25;\ 0{,}46^2)$

$\sigma_X = 0{,}46$ s $\quad N(E[X]; Var(X))$

$\mu = E[X] = 1{,}25 \quad N(\mu; \sigma^2)$

$Var(X) = 0{,}46^2$

X = "tempo di reazione"

$Z = \frac{X - 1{,}25}{0{,}46} \sim N(0,1)$

$P(X<1) = P\left(Z < \frac{1-1{,}25}{0{,}46}\right) = \Phi\left(-\frac{0{,}25}{0{,}46}\right) = \Phi(-0{,}54) = 0{,}2946$ ✓

dalla tabella

$P(1<X<2) = -\Phi\left(\frac{1-1{,}25}{0{,}46}\right) + \Phi\left(\frac{2-1{,}25}{0{,}46}\right) = \Phi(-0{,}543) + \Phi(1{,}630) = 0{,}6538$

ex. 13

X = "diametro bulloni"

$P(X<1{,}4) = 25\%$

$P(X>1{,}8) = 10\%$

$E[X] = \mu \quad P\left(\frac{a-\mu}{\sigma}\right)$

$\begin{cases} \frac{1{,}4-\mu}{\sigma} \cong -0{,}67 \\ \frac{1{,}8-\mu}{\sigma} \cong 1{,}28 \end{cases}$

$P\left(\frac{b-\mu}{\sigma}\right)$

?

ex. 14

$E[X] = 64{,}5'' = \mu$

$Var[X] = 5{,}76'' = \sigma^2 \quad X$ = "altezza donne"

$X \sim N(0,1)$

a) $P(X<63) = \Phi\left(\frac{63-64{,}5}{\sqrt{5{,}76}}\right) \cong \Phi(-0{,}63) = 0{,}264$

b) $P(63<X<70) = \Phi\left(\frac{70-64{,}5}{2{,}4}\right) - \Phi\left(\frac{63-64{,}5}{2{,}4}\right) = 0{,}989 - 0{,}2676 \cong 0{,}721$

c) X_1 = h prima donna $\quad X_2$ = h seconda donna

$Y = \frac{X_1+X_2}{2} = \frac{X_1}{2} + \frac{X_2}{2} \rightarrow Y \sim N(64{,}5;\ 2{,}88)$ $\quad \frac{1}{2}2E[X] = E[X]$ $\quad \frac{1}{4}\cdot 2Var(X) = \frac{1}{2}Var(X)$

$Var(X) = \sigma^2$ quindi $\frac{Var(X)}{2} = \left(\frac{1}{2}\sigma\right)^2 = \frac{1}{4}\sigma^2 \cdot 2 = \frac{\sigma^2}{2}$

$P(Y>66) = 1 - P(Y>66) = 1 - \Phi(0{,}88) = 0{,}1894$

ex. 19

$\lambda = \frac{1}{8}$

a) $E[X] = \frac{1}{\lambda} = \frac{1}{\frac{1}{8}} = 8$

b) $Var(X) = \frac{1}{\lambda^2} = \frac{1}{\left(\frac{1}{8}\right)^2} = 16$

c) $P(X=0) = 1 - e^{-\lambda x} = 1 - e^{-\frac{1}{8}0} = 0$

d) $P(X > 5+10 \mid X>5) = P(X \geq 10) = 1 - P(X<10) = 1 - e^{-\frac{10}{8}} \cong 0{,}2865$

ex. 20

$X \sim \mathcal{E}\left(\frac{1}{20}\right)$

$s = 10'000$ km $\quad t = 20'000$ km

X = "T. vita in migliaia di km"

$P(X > 10+20 \mid X>10)$

a) $P(X>30 \mid X>10) = P(X>20) = 1 - (1 - e^{-\frac{1}{20}20}) = 1 - 1 + \frac{1}{e} \cong 0{,}3678$

b) $X \sim U(0,40)$

$$f(x) = \begin{cases} \frac{1}{40} & 0<x<40 \\ 0 & \text{altrimenti} \end{cases}$$

$P(X \geq 30 \mid X \geq 10) = \frac{P(30<X<40)}{P(X>10)} = \frac{\frac{10}{40}}{\frac{30}{40}} = \frac{1}{3} = 0{,}331$

non vale la prop. di assenza di memoria

ex.

$H_0 = 730$ ml $\quad H_1 \neq 730$ ml

$n = 84$ bottiglie

$\bar{X}_n = 726$ ml

$\sigma^2 = 625 \quad \sigma = \sqrt{625} = 25$

$\alpha = 0{,}05$

$T = \frac{726 - 730}{\frac{25}{\sqrt{84}}} = -\frac{36}{25}$

Pvalue $= \left(T < -\frac{36}{25}\right) = 0{,}0749 \rightarrow$ se $> \alpha = 0{,}05$ = rifiuto altrimenti accetto

2 table

ex.

$E[X] = 2$

$Var = 25$

$n = 36$

$S_n = \frac{75 - 2 \cdot 36}{\frac{5}{6}} \quad P(S_n > 75) = 0{,}5398$

$X_n = \frac{4-2}{\frac{5}{6}} \quad P(X_n < 4) = 1 - 0{,}9918 = 0{,}0082$

$P(|X_n - 2| > 3) = 2 \cdot \Phi(3) =$

2 casi del valore assoluto