SOLUZIONI

1) $r: 2x+y+3=0 \qquad \vec{n}=\begin{pmatrix}2\\1\end{pmatrix} \quad \vec{u}=\begin{pmatrix}1\\-2\end{pmatrix}$

$$P=(0;-3)\in r$$

a) $$P_1=\frac{\vec{u}\cdot\vec{u}^T}{\|\vec{u}\|^2}=\begin{pmatrix}\frac{1}{5} & -\frac{2}{5}\\ -\frac{2}{5} & \frac{4}{5}\end{pmatrix}$$

$$\Rightarrow P=\begin{pmatrix}1&0&0\\0&1&-3\\0&0&1\end{pmatrix}\cdot\begin{pmatrix}\frac{1}{5}&-\frac{2}{5}&0\\-\frac{2}{5}&\frac{4}{5}&0\\0&0&1\end{pmatrix}\cdot\begin{pmatrix}1&0&0\\0&1&3\\0&0&1\end{pmatrix}=\begin{pmatrix}\frac{1}{5}&-\frac{2}{5}&-\frac{6}{5}\\-\frac{2}{5}&\frac{4}{5}&-\frac{3}{5}\\0&0&1\end{pmatrix}$$

b) $$Q_1:\ P\cdot\begin{pmatrix}3\\-1\\1\end{pmatrix}=\begin{pmatrix}-\frac{1}{5}\\-\frac{13}{5}\\1\end{pmatrix} \Rightarrow Q_1=\left(-\frac{1}{5};-\frac{13}{5}\right)$$

2) a) $\vec{n}_\alpha=\begin{pmatrix}1\\-1\\1\end{pmatrix} \qquad A=(-1;0;0)\in\alpha$

$$S_1=I-2\frac{\vec{n}_\alpha\cdot\vec{n}_\alpha^T}{\|\vec{n}_\alpha\|^2}=\begin{pmatrix}\frac{1}{3}&\frac{2}{3}&-\frac{2}{3}\\\frac{2}{3}&\frac{1}{3}&\frac{2}{3}\\-\frac{2}{3}&\frac{2}{3}&\frac{1}{3}\end{pmatrix}$$

$$\Rightarrow F=\begin{pmatrix}1&0&0&-1\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\cdot\begin{pmatrix}\frac{1}{3}&\frac{2}{3}&-\frac{2}{3}&0\\\frac{2}{3}&\frac{1}{3}&\frac{2}{3}&0\\-\frac{2}{3}&\frac{2}{3}&\frac{1}{3}&0\\0&0&0&1\end{pmatrix}\cdot\begin{pmatrix}1&0&0&1\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\cdot$$

$$\cdot\begin{pmatrix}1&0&0&2\\0&1&0&1\\0&0&1&-3\\0&0&0&1\end{pmatrix}=\begin{pmatrix}\frac{1}{3}&\frac{2}{3}&-\frac{2}{3}&\frac{8}{3}\\\frac{2}{3}&\frac{1}{3}&\frac{2}{3}&\frac{1}{3}\\-\frac{2}{3}&\frac{2}{3}&\frac{1}{3}&-\frac{7}{3}\\0&0&0&1\end{pmatrix}$$

b) $$A:\ F^{-1}\cdot\begin{pmatrix}1\\0\\1\\1\end{pmatrix}=\begin{pmatrix}-3\\1\\2\\1\end{pmatrix} \Rightarrow A=(-3;1;2)$$

c) $$r_1:\ F\cdot\begin{pmatrix}3-\lambda\\1+\lambda\\2+4\lambda\\1\end{pmatrix}=\begin{pmatrix}3-\frac{7}{3}\lambda\\4+\frac{7}{3}\lambda\\-3+\frac{8}{3}\lambda\\1\end{pmatrix} \Rightarrow r_1:\ \begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}3\\4\\-3\end{pmatrix}+\mu\begin{pmatrix}-7\\7\\8\end{pmatrix}$$

3) a) $$R=\begin{pmatrix}1&0&c_1\\0&1&c_2\\0&0&1\end{pmatrix}\cdot\begin{pmatrix}\frac{1}{2}&-\frac{\sqrt{3}}{2}&0\\\frac{\sqrt{3}}{2}&\frac{1}{2}&0\\0&0&1\end{pmatrix}\cdot\begin{pmatrix}1&0&-c_1\\0&1&-c_2\\0&0&1\end{pmatrix}=$$

$$=\begin{pmatrix}\frac{1}{2}&-\frac{\sqrt{3}}{2}&\frac{c_1-\sqrt{3}c_2}{2}\\\frac{\sqrt{3}}{2}&\frac{1}{2}&\frac{-\sqrt{3}c_1+c_2}{2}\\0&0&1\end{pmatrix}$$

b) $$R\cdot\begin{pmatrix}3\\-1\\1\end{pmatrix}\overset{!}{=}\begin{pmatrix}0\\5\\1\end{pmatrix}\iff\begin{cases}\frac{1}{2}\left(c_1+\sqrt{3}c_2+\sqrt{3}+3\right)=0\\\frac{1}{2}\left(-\sqrt{3}c_1+c_2+3\sqrt{3}-1\right)=5\end{cases}$$

$$\begin{cases}c_1=-\sqrt{3}c_2-\sqrt{3}-3=\frac{3}{2}-3\sqrt{3}\\c_2=2-\frac{3\sqrt{3}}{2}\end{cases}$$

$$\Rightarrow\boxed{C=\left(\frac{3}{2}-3\sqrt{3};\ 2-\frac{3}{2}\sqrt{3}\right)}$$

4) $$\vec{n}_\alpha=\begin{pmatrix}1\\2\\-3\end{pmatrix}\qquad A=(4;0;0)\in\alpha$$

a) $$P_1=I-\frac{\vec{n}_\alpha\cdot\vec{n}_\alpha^{\,T}}{\|\vec{n}_\alpha\|^2}=\begin{pmatrix}\frac{13}{14}&-\frac{1}{7}&\frac{3}{14}\\-\frac{1}{7}&\frac{5}{7}&\frac{3}{7}\\\frac{3}{14}&\frac{3}{7}&\frac{5}{14}\end{pmatrix}$$

$$\Rightarrow P=\begin{pmatrix}1&0&0&4\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\cdot\begin{pmatrix}\frac{13}{14}&-\frac{1}{7}&\frac{3}{14}&0\\-\frac{1}{7}&\frac{5}{7}&\frac{3}{7}&0\\\frac{3}{14}&\frac{3}{7}&\frac{5}{14}&0\\0&0&0&1\end{pmatrix}\cdot\begin{pmatrix}1&0&0&-4\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}=$$

$$=\begin{pmatrix}\frac{13}{14}&-\frac{1}{7}&\frac{3}{14}&\frac{2}{7}\\-\frac{1}{7}&\frac{5}{7}&\frac{3}{7}&\frac{4}{7}\\\frac{3}{14}&\frac{3}{7}&\frac{5}{14}&-\frac{6}{7}\\0&0&0&1\end{pmatrix}$$

b) $$P_1:\ P\cdot\begin{pmatrix}3\\1\\2\\1\end{pmatrix}=\begin{pmatrix}47/14\\12/7\\13/14\\1\end{pmatrix}\Rightarrow P_1=\left(\frac{47}{14};\frac{12}{7};\frac{13}{14}\right)$$

c) $$Q\in q:\ \begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}2\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\-3\end{pmatrix}$$