



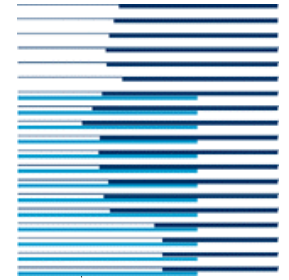
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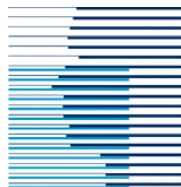
IDSIA

**Istituto Dalle Molle di studi
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Ottimizzazione

Introduction



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Why should I study “Linear Programming” ?

Because you will be able to solve real and practical problems otherwise very difficult

Show me an example of such a problem



Ok! Follow me

Problem: Power Generation



See [problem 12.15](#) for details

- A number of power stations are committed to meeting a given amount of electricity load demand
- There are 3 types of generating unit available: 12 of type 1, 10 of 2 and 5 of 3.
- Each generator has to work between a min and max level.
- Hourly cost of running each generator at min level. Above min: extra hourly cost. To start up a generator involves a cost.
- Robustness: sufficient generators to meet an increase of 15%



- **Which generators should be working in which periods of the day to minimize the total cost?**
- **What tariffs should be charged in each period of the day?**
- **What would be the saving of lowering the 15% reserve output guarantee?**

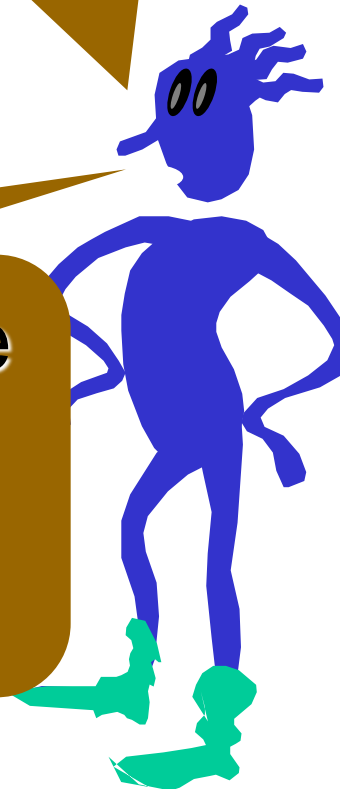




This problem is a real and practical problem. Any idea on how to solve it?

Ok. I do not know. It means that I will not work for power stations!!

The list of problems that can be solved with LP is surprisingly wide! Let me show an other example.



Problem: Factory Planning



See [problem 12.03](#) for details

- An engineering factory makes 7 products on 4 machines.
- Each product yields a profit and requires several processes on different machines to be completed
- There are periods where certain machines will be down for maintenance
- Market limitations on each product each month
- We can store up to 100 of each product at a time at a cost of 0.5 CHF per unit per month. We want a stock of 50 of each type by the end of June.
- The factory works a 6 day week with 2 shifts of 8 hours



- **When and what should the factory make in order to maximize the total profit? Recommend any price increases and the value of acquiring any new machines.**



Problem: Milk Collection



See [problem 12.23](#) for details

- A milk processing company collects milk from 20 farms to the depot for processing
- One tanker lorry: capacity 80k litres
- 11 small farms needs a collection only every other day, the other 9 every day.
- The position of the farms in relation to the depot are given together with their collection requirements.



- **Find the optimal route for the tanker lorry on each day**



Other Typical Applications

- The Petroleum/Chemical Industry
 - Where and how to buy crude oil, how to ship it, which product to produce, blending, etc...
- Manufacturing Industry
 - Product mix, resource allocation, scheduling tasks on machines, etc...
- Transport and Distribution
 - Depot location, routing, network problems, etc...
- Finance
 - Given a sum of money to invest, how to spend it among a portfolio of shares and stocks; setting prices for goods at different times to max revenue; etc...
- Agriculture
 - What to grow where, how to rotate crops, etc...
- Health
 - Resource allocation: doctors, nurses, rooms
- Mining, Manpower Planning, Food, Energy, Defence, etc...



Linear Programming

- Significance.
 - Quintessential tool for optimal allocation of scarce resources, among a number of competing activities.
 - Powerful model generalizes many classic problems:
 - shortest path, max flow, multicommodity flow, MST, matching, 2-person zero sum games
 - Ranked among most important scientific advances of 20th century.
 - accounts for a major proportion of all scientific computation
 - Helps find "good" solutions to NP-hard optimization problems.
 - optimal solutions (branch-and-cut)
 - provably good solutions (randomized rounding)



Case study: AntIngegaglia (1)

- **AntOptima** is a Swiss company based in Lugano which develops innovative optimisation methodologies to increase the efficiency of productive and logistic processes.
- **Number 1 Logistics Group** is Italian logistic broker leader in grocery with 400.000.000 of necks managed, 2.500.000 delivers for about 2.100.000 tons, 250.000.000 of km traveled in a year, 2.600 vehicles and 110.000 customers served.



Case study: AntIngegaglia (2)

AntIngegaglia chooses in a few minutes for each tour the best available vehicle based on objectives and constraints depending on the company.

Objectives:

- Minimization of costs
- Minimization of full/empty vehicle movements
- Minimization of empty kilometers for the next day
- Attainability of annual target kilometers for any vehicle
- Maximization of the feasibility of tours
- Automated optimization of the fleet engagement according to the tariff schemas of various transport operators



Case study: AntIngegria (3)

Constraints:

- Own and external vehicles
- Availability and geographical position of all vehicles
- Compatibility between tour and vehicle (weight, capacity, pallet, ...)
- Tours including week-end and festive days
- Intermodal tours
- The usage preference of vehicle
- The choice preference between a vehicle and a certain tour and/or a particular customer
- The possibility to use the same vehicle for several tours on the same day
- Different costs per transport operator, means of transportation, type of vehicle and per geographical zone



Case study: AntIngegaglia (4)

Problem dimension:

- 800 available trucks
- 500 tours

Manual solution:

- 5 people for 4 hours
each

Automatic solution:

- a model with more than
300.000 binary variables
- 4 minutes of elaboration
time



LP =



Wow!! I really want to know the techniques for solving these problems!

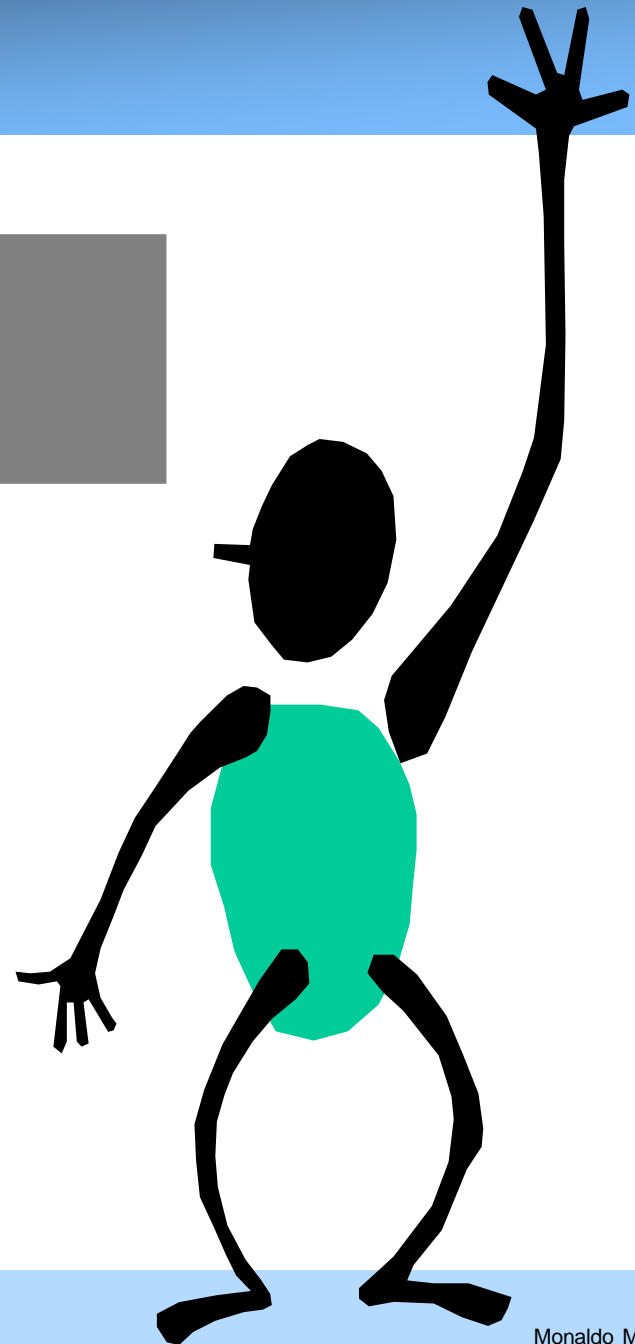
Ok! Follow me

Please feel free
to ask questions!

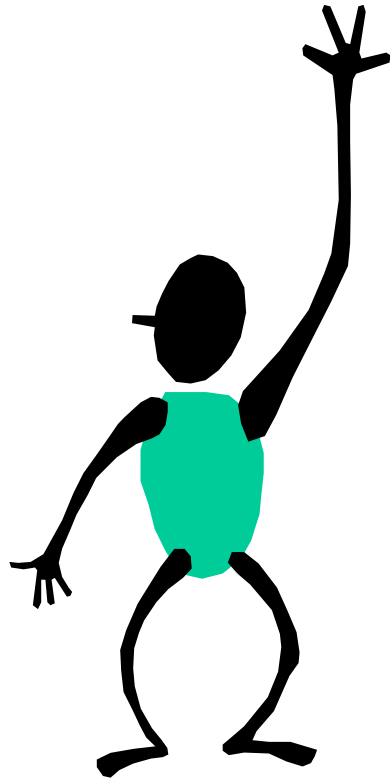
Help me know what people
are not understanding

Slow down the slides

We do have a lot of material

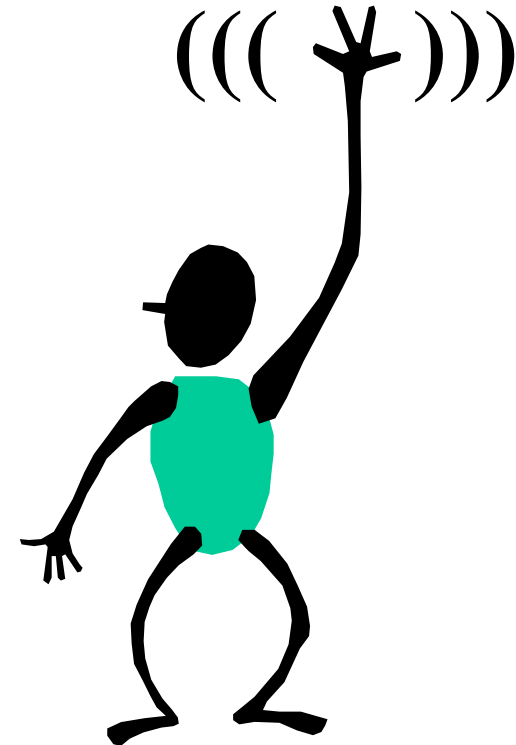


Stationary Hand



You have a question or comment of any nature

Wiggling Hand



You have something relevant to say to what is being spoken now



Course Outline

1. Introduction to Linear Programming (LP) and Integer Linear Programming (ILP)
2. Model Building:
 - a) Techniques for LP
 - b) Techniques for ILP
3. Solving LP: The Simplex Method
4. Network flow problems
5. Solving ILP: The Branch & Bound Method

