MEMORANDUM

TO: Dan Castro, Professor, Mechanical Engineering
FROM: Christopher Ng, ME 328 – 15
DATE: 10/13/2022
SUBJECT: Stress Concentrations

In this lab, we examined stress concentrations on beams with different load types and geometry. The concentrated stress regions can be approximated using a factor of the nominal stress near it. This is shown by the relation

$$\sigma_{max} = k\sigma_{nom}$$

Where k is the concentration factor that varies with the geometry of the part and what type of stress is applied. In our case, we examine a beam composed of one wider and one thinner bar of the same thickness with a shoulder fillet at their connection. Our value for k for this geometry can be found using Figure 1 and 2 for an axial load and applied moment, respectively.

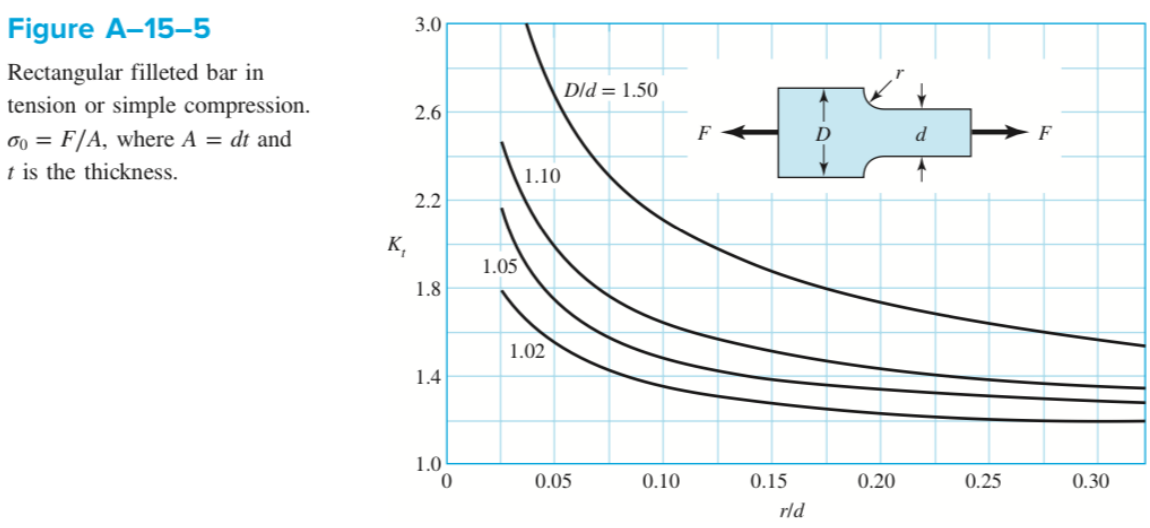


Figure 1. Values for Stress Concentration Factor for Shoulder Fillet Under an Axial Load

Figure A–15–6

Rectangular filleted bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$, $I = td^3/12$, t is the thickness.

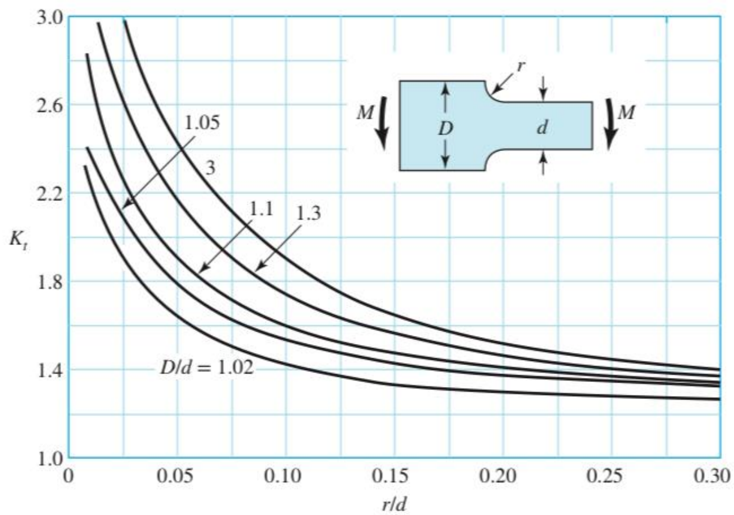
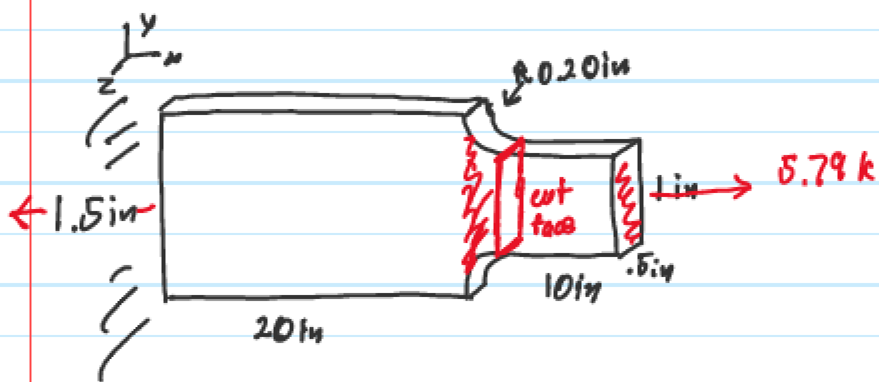


Figure 2. Values for Stress Concentration Factor for Shoulder Fillet Under an Applied Moment

For our beam, we calculated the predicted values for the maximum concentrated stress of 20.27kpsi for an axial load and 30.06kpsi for an applied moment (Figure 3 and 4). When calculating these values, we first predicted where the stress regions would occur. Since we can think of the stress concentration as a “flow” of stress lines throughout the beam, we base our analysis at the fillet where one end of an element might have stress pointing at an angle instead of straight through. We can then use our geometric values and find k using the plots on Figure 1 and 2. Once we calculate the nominal stress at the fillet’s cross-section, we use k to find the maximum stresses.

To verify if these values are close to those of a real model of this beam, we use FEA modeling to examine the stress concentrations. Our results are seen in Figure 5 and 6 with maximum stress values of 19.69kpsi and 28.21. These are relatively close to those that we hand calculated and are also at the fillet as we predicted, showing that our approximation for maximum stress using k is reasonable.



$$\sigma_{nom} = \frac{P}{A}$$

$$= \frac{5.79\,k}{(1in)(.5in)} = 11.68\ k/in^2$$

$$\frac{r}{d} = \frac{0.20in}{1in} = 0.20$$

$$\frac{D}{d} = \frac{1.5in}{1.0in} = 1.5$$

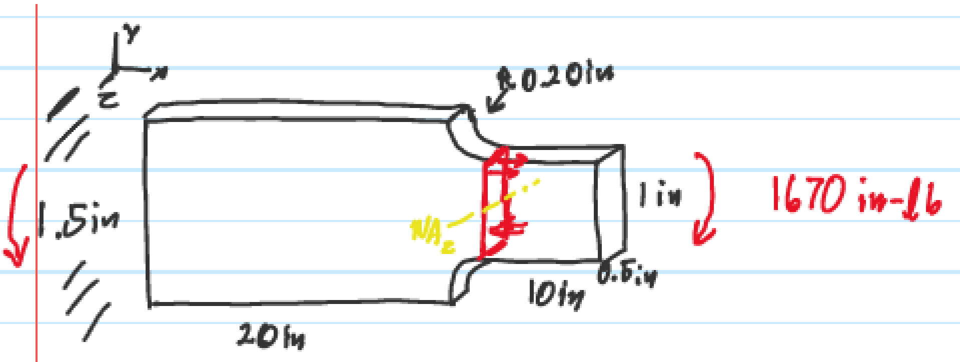
$$k_t = 1.75$$

$$\sigma_{max} = \sigma_{nom}\, k_t$$
$$= 11.58\ k/in^2 \cdot 1.75$$
$$= \underline{20.265\ k/in^2}$$

plot

$$\sigma_{max} = 19.69\ ksi$$

Figure 3. Hand Calculations for Theoretical Maximum Stress Due to an Axial Load



$$\sigma_{nom} = \frac{Mc}{I_z}$$

$$= \frac{(1670\ in\text{-}lb)(0.5\ in)}{\frac{1}{12}(0.5\ in)(1\ in)^3} = 20040\ lb/in^2$$

$$= 20.04\ k/in^2$$

$$\frac{r}{d} = \frac{0.20\ in}{1\ in} = 0.20$$

$$\frac{D}{d} = \frac{1.5\ in}{1.0\ in} = 1.50$$

$$k_t = 1.50$$

$$\sigma_{max} = \sigma_{nom}\, k_t$$

$$= 20.04\ k/in^2\ 1.50$$

$$= \underline{30.06\ k/in^2}$$

plot

$$\sigma_{max} = 28.21\ k/in^2$$

Figure 4. Hand Calculations for Theoretical Maximum Stress Due to an Applied Moment

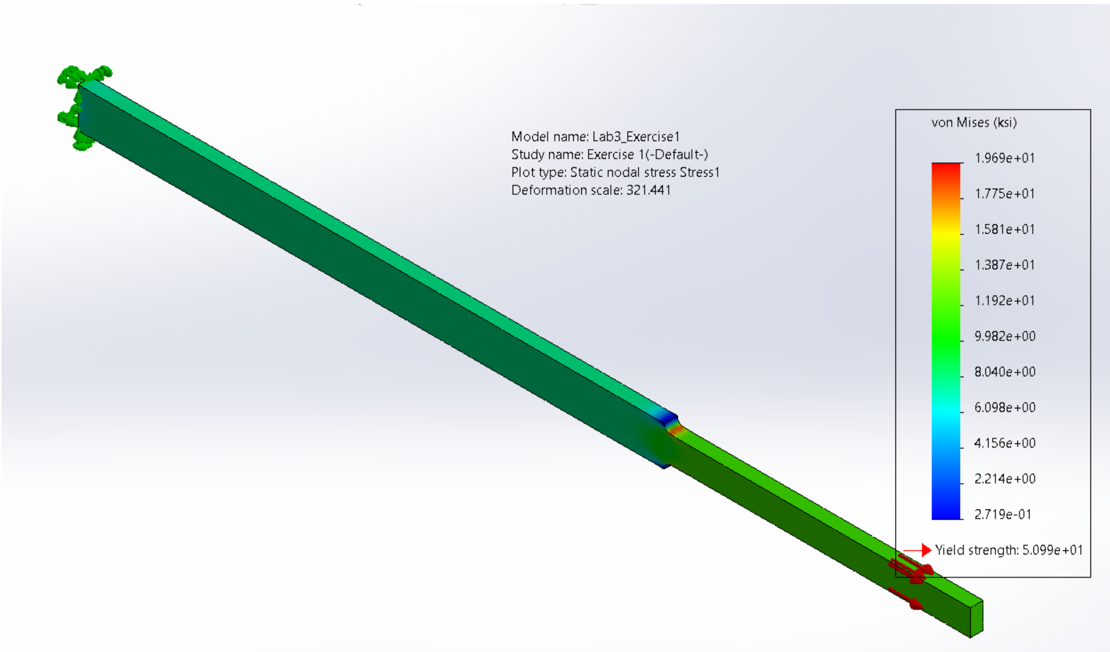


Figure 5. Von Mises Stress Plot for Maximum Stress Due to an Axial Load

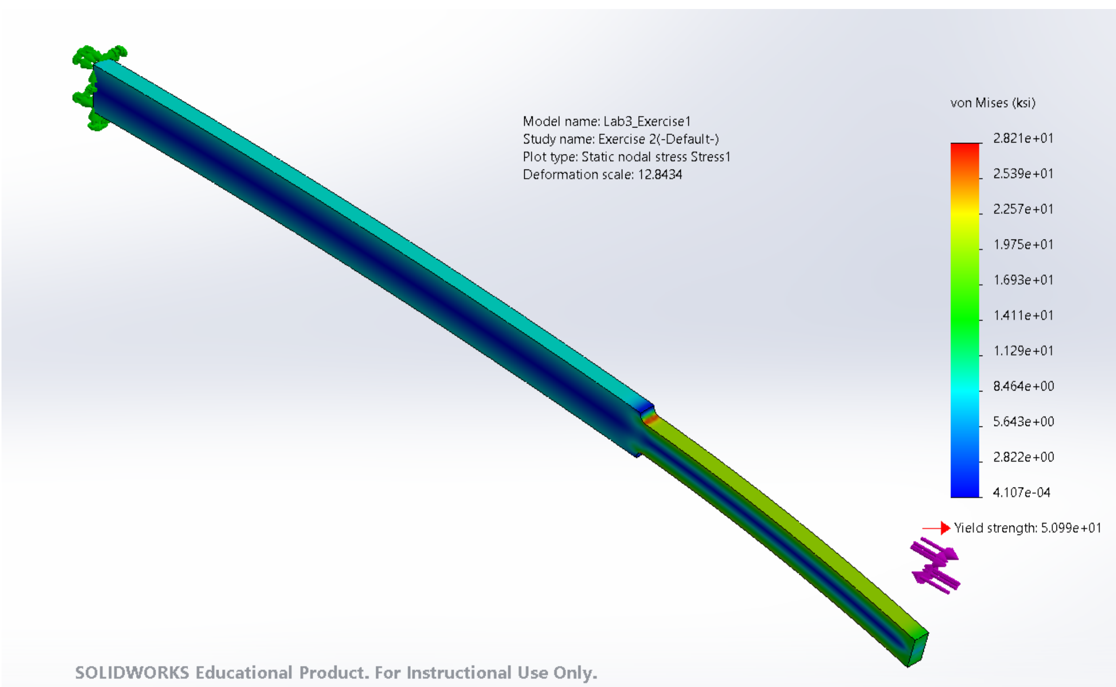


Figure 6. Von Mises Stress Plot for Maximum Stress Due to an Applied Moment

The next step of this lab was experimenting with different types of geometry to attempt to decrease the maximum stress of the beam with the same applied moment as before. As we can see in Figure 2, there are a few geometric factors that contribute to decreasing k, which will in turn decrease the maximum stress. The most prominent feature that could be optimized is r/d, the ratio of the fillet radius to smaller width. By increasing the radius of the fillet, we can imagine the stress concentration "flow" to become smoother and less dense over the corner. Another factor is D/d, the ratio of larger to smaller widths. This is obvious since as D/d approaches 1, it will become a constant width beam with no stress concentrations. While it doesn't make sense to change the widths of the beam segments, we can change the fillet and modify it to have a smoother transition from the widths D to d. As seen in Figure 7, we make this transition using a modified fillet radius of 5in and find a maximum stress of about 26.90kpsi at the fillet. With a 3in fillet radius, we find the maximum stress to be about 28.05kpsi, also showing a trend of smoother transitions having less maximum stress.

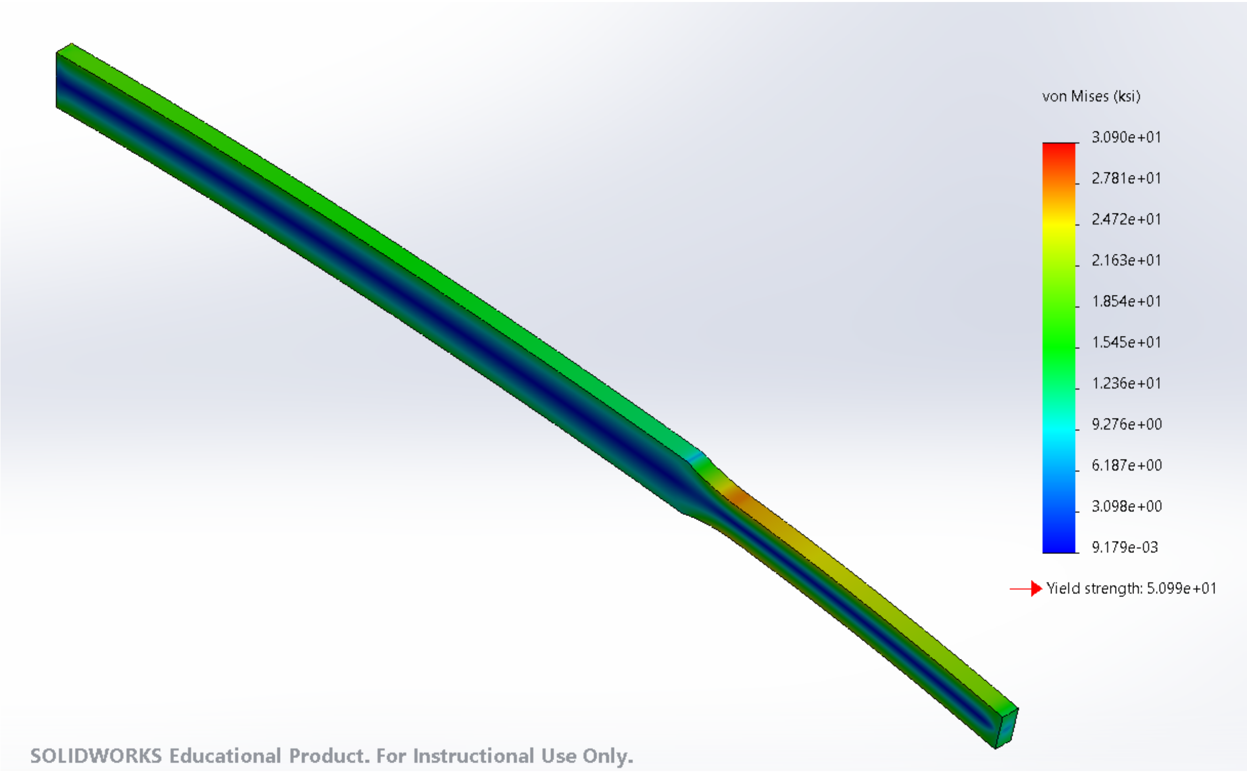


Figure 7. Von Mises Stress Plot for Maximum Stress Due to an Applied Moment with 5in Modified Fillet

We also tried to make the transition of stress concentration smoother using holes throughout the beam with different radii (Figure 8). As seen in Table 1, there doesn't appear to be a relationship with hole size and maximum stress, nor does it decrease the maximum stress from the initial solid beam.

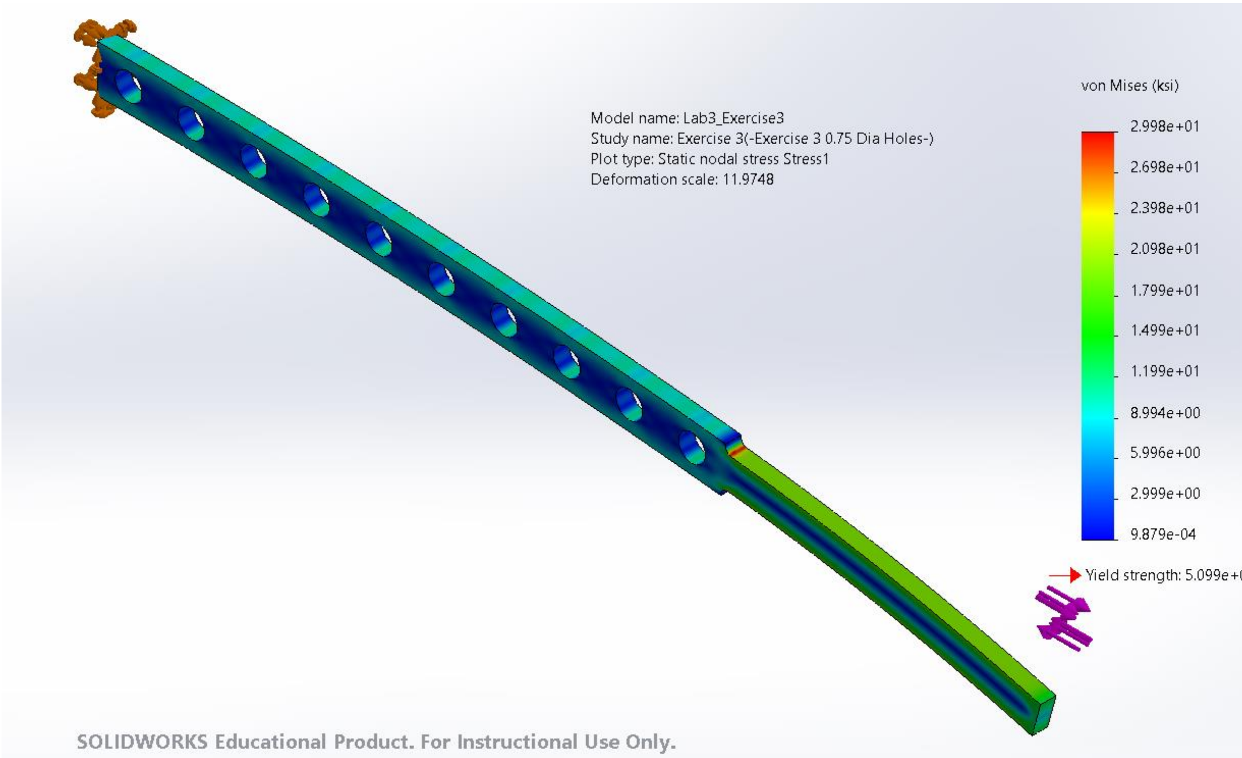


Figure 8. Von Mises Stress Plot for Maximum Stress Due to an Applied Moment with 0.5in Dia Holes

Beam Feature	Load Type	Maximum Stress (kpsi)
Solid – 0.20in Radius Fillet	Axial Load	19.69
Solid – 0.20in Radius Fillet	Moment	28.21
Solid – 3.00in Modified Radius	Moment	28.05
Solid – 5.00in Modified Radius	Moment	26.90
Holes – 0.50in Dia Holes	Moment	29.98
Holes – 0.60in Dia Holes	Moment	29.53
Holes – 0.75in Dia Holes	Moment	29.80

Table 1. Experimental Maximum Stresses for Each Beam Feature and Load Type

Through this lab, we experimented with different geometries and loads on a beam to determine both the accuracy of our theoretical maximum stress concentration and to find out what geometry we can change to decrease the maximum stress at the fillet. We found that our theoretical maximum was close to that of our FEA modeling, and also demonstrated how changing the radius of the fillet in a way that the stress concentration is smoother will decrease the maximum stress in the beam. For a beam consisting of two different widths, to decrease the stress concentration, we suggest that the connection between the two is as smooth as possible, specifically as it reaches the smaller width, and avoids sharp corners.