Validation – Transient Natural Convection for a Circular Cylinder

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1. Introduction

A common study for natural convection has been about a long and hot horizontal cylindrical rod exposed to room temperature air contained in a significantly larger enclosure [1]. There have been various experimental studies conducted to determine convection characteristics with one by Churchill and Chu having summarized the results of a selection of them for both laminar and turbulent cases [2]. Through this they produced a generalized correlation of the Nusselt number to the Rayleigh and Prandtl numbers. Characteristics of the flow that is produced by this due to the diffusion from natural convection can be expected to create a "plume" above the horizontal rod [1,3].

2. Purpose

The purpose of this simulation is to validate these heat transfer correlations through FLUENT using various diameter sizes to test with different Rayleigh numbers. As a secondary verification of this simulation, the flow characteristics are also observed for the plume effect.

3. Problem Description

For this model, an infinitely long cylindrical rod at a temperature of 350 K is placed in a cylindrical enclosure ten times its diameter as seen in Figure 3.1. This enclosure is filled with initially static air at standard atmospheric pressure and a temperature of 290 K and is allowed to reach fully developed flow before measuring the total heat transfer from the cylinder's surface. The three rod diameters and their respective Rayleigh numbers, calculated using Equation 3.1 for a cylinder, used in this simulation are shown in

Table *3.1*.

Table 3.1 Simulated Rayleigh Numbers

Diameter [m]	Rayleigh Number [-]
0.1	4.257e6
0.2	3.406e7
0.3	1.149e8

$$Ra_D = \frac{g\beta(T_s - T_\infty)D^3}{\alpha\nu} \tag{1}$$

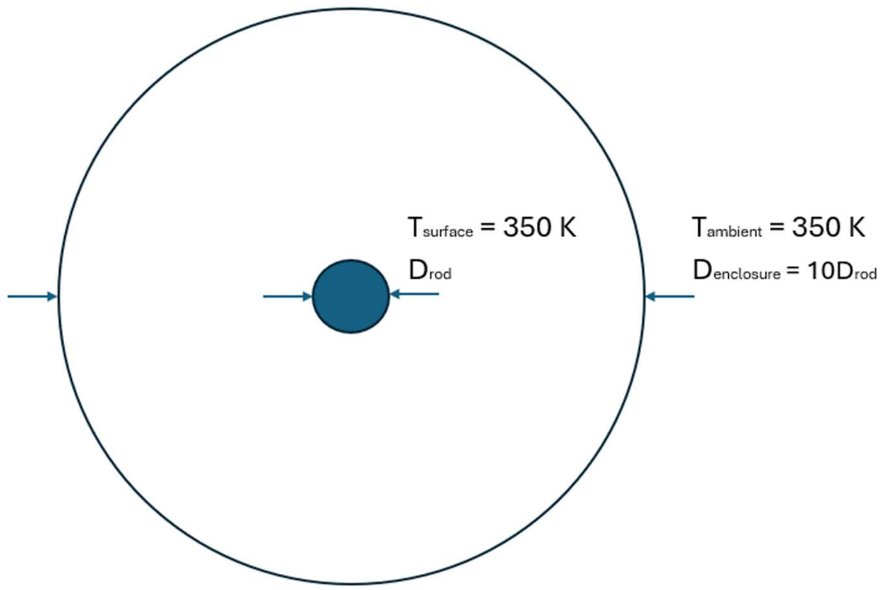

Figure 3.1 Simulation Model Geometry

3.1 Fluid Properties

Using the Engineering Equation Solver (EES), the properties of the air evaluated at a film temperature of 320 K are shown in Table 3.2 and are assumed to be constant with a Boussinesq density. This includes properties required for the simulation as well as those to evaluate the Nusselt number.

Table 3.2 Relevant Air Properties at T_{film} = 320 K

Density [kg/m^3]	1.103
Specific Heat [J/kg-K]	1006
Thermal Conductivity [W/m-K]	0.02712
Viscosity [kg/m-s]	1.767e-5
Thermal Expansion Coefficient [K^{-1}]	0.003125
Thermal Diffusivity [m^2/s]	2.444e-5
Prandtl Number [-]	0.7229

3.2 Flow Physics

The modelling method used in this simulation is for laminar natural convection. Studies on natural convection about a cylindrical rod contain correlations for both laminar and turbulent flow with the transition region near a Rayleigh number of 10^{-9}. From Equation 3.1, the Rayleigh number is dependent on the cylinder diameter and temperature difference between the cylinder wall and ambient air. So, to stay within the laminar region and out of the transition region, diameters in *Table 3.1* were chosen to produce Rayleigh numbers between 10^{-6} and 10^{-8}. While the temperature difference could be adjusted as well to avoid adjusting the mesh, this was not an issue because the mesh can be scaled from the meshing software to FLUENT while a larger temperature difference may cause non-constant properties when basing properties around the film temperature.

3.3 Boundary Conditions

This simulation uses three different boundary conditions. To reduce simulation times to convergence, a line of thermal and geometric symmetry along the vertical axis is used. While studies have shown that the plume is not symmetric in most experimental cases, it is found in these simulations that the effects of the plume are not very significant to the results of the thermal system, found in __ [3]. The cylinder is treated as a standard wall with no-slip and a temperature of 350 K. The enclosure is treated as a wall as well, but instead has zero shear and a temperature of 290 K to emulate as if it was open to the environment. This boundary could have also been treated as a pressure-based exit, however, this would increase the simulation time and potentially prevent convergence. As an alternative, experimental studies have suggested that for a higher aspect ratio of rod length to radius, enclosures should be at least ten times in diameter which has been applied in this simulation [4].

4. Grid and Mesh

The grid and meshing used for this simulation is shown in Figure 4.1. This consists of a radially geometric 1 and tangentially uniform grid blocked into the shape of the partial semicircle. This allows there to be an increased mesh density, starting at a spacing of 0.4, near the cylinder wall where most of the important heat transfer and flow will occur and reduces computation time for the regions near the enclosure wall which should ideally be negligible to the overall flow. This grid was then reflected over the vertical line of symmetry to produce

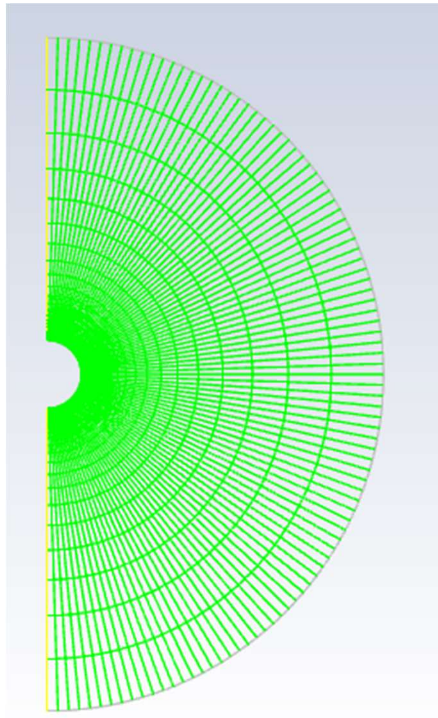

Figure 4.1 30x50 Radially Geometric 1 and Tangentially Uniform Meshing

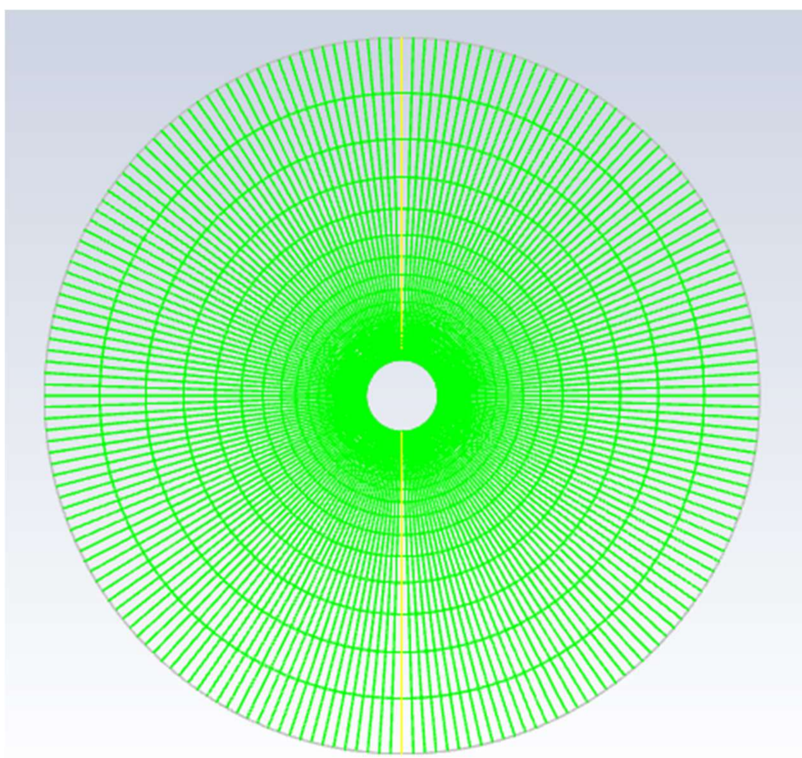

Figure 4.2 Reflected Mesh for 30x50 Radially Geometric 1 and Tangentially Uniform Meshing

4.1 Grid Independence Study

Three different mesh geometries were applied to conduct a grid independence study where the geometric 1 spacing varied from 30, 40, and 50 grids with the initial spacing and uniform spacing remaining 0.4 and 50, respectively. These are all applied to the 0.1 m diameter case and observed at a time of 1 second. Each case's Nusselt numbers are produced as seen in Table 4.1 with the correlation value for this case found to be 18.195. While the simulated values do not appear to approach this correlation value as mesh size increases, it can be observed that the change in Nusselt number is decreasing as the number of mesh sections increases. On top of having very small changes per increase in mesh sections, this demonstrates that the effects of further mesh refinement is not significant enough to justify the increase in computation.

Table 4.1 Mesh Number's Effect on Nusselt Number (Using Geometric 1 Meshing with an Initial 0.4 Spacing and 50 Uniform Meshing Tangentially at a Time of 1 s)

Radial Mesh Sections [-]	Simulated Nusselt Number [-]
30	19.593
40	19.597
50	19.599

5. Case Setup

As described previously in Section 3, three cases are used to verify the combined results provided by Churchill and Chu [1]. Cylinder diameters of 0.1, 0.2, and 0.3 m, are used to produce Rayleigh numbers of magnitudes 6, 7, and 8. An additional three cases are used to observe the effects of heat transfer over time as the flow reaches a fully developed flow are also provided to help interpret the effects of the plume visually compared to numerically.

6. Calculations

As shown in Section 3, the Rayleigh number is used to ensure laminar flows. Since this simulation and model uses air treated as an ideal gas, this allows the thermal expansion coefficient to follow the form shown in Equation 2.

$$\beta = \frac{1}{T_{film}} \tag{2}$$

This Rayleigh number is also used in Churchill and Chu's correlation as described in Equation 3, which can be applied for a laminar region of $10^{-6} < Ra_D < 10^{9}$ and any Prandtl number [1].

$$Nu_D = 0.36 + 0.518\left(\frac{Ra_D}{\left[1+(0.559/Pr)^{9/16}\right]^{16/9}}\right)^{1/4} \tag{3}$$

The values that are measured in this simulation are initially the overall heat transfer rates per unit length from the surface of the cylinder. To produce a Nusselt number to compare to the correlation, Equation 4 is applied for convective heat transfer, then Equation 5 is applied for the Nusselt number correlation over a cylinder.

$$Q = hA(T_{surf} - T_{amb}) \quad (4)$$

$$Nu_D = \frac{hD}{k} \quad (5)$$

7. Results

Sections 7.1 and 7.2 have been conducted for the 30x50 mesh case with Sections 7.1 at 2 seconds and Section 7.2 using a diameter of 0.1 m.

7.1 Diameter and Rayleigh Number Relationship to Nusselt Numbers

As shown in Figures 7.1 through 7.6, simulations of the fluids and heat transfer were conducted through FLUENT and produced velocity vectors to describe the flow around the cylinder due to diffusion along with the temperature contours near the wall of the cylinder and as it follows the plume. While these demonstrate the predicted fluid behavior, what is really evaluated is the Nusselt numbers from the simulations. After applying Equations 4 and 5 to the heat flux results to get simulated Nusselt numbers and Equation 3 for the Nusselt number correlation, the results can be summarized in Table 7.1. As predicted, increases in cylinder diameter increase the Rayleigh number, which in turn increases the Nusselt number both for the simulated and correlation cases. The values produced by the simulation are slightly larger than those by the correlation, but only by a maximum of about 8% error.

Table 7.1 Nusselt Numbers for Various Rayleigh Numbers (30x50 Mesh at 2.0 s)

Diameter [m]	Rayleigh Number [-]	Correlation Nusselt Number [-]	Simulated Nusselt Number [-]
0.1	4.257e6	18.195	19.600
0.2	3.406e7	30.354	31.964
0.3	1.149e8	41.014	43.376

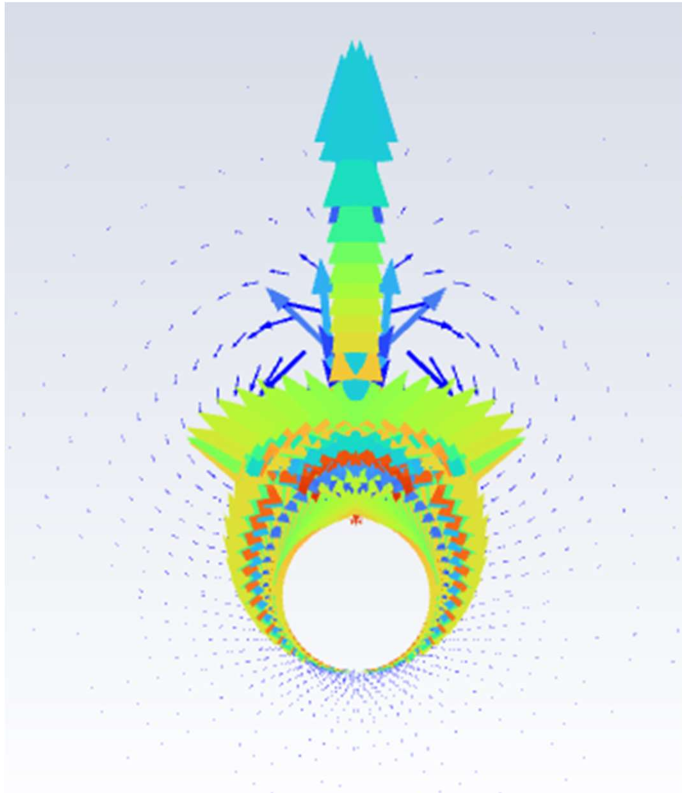

Figure 7.1 Velocity-Temperature Vectors for Diameter of 0.1 m (30x50 Mesh at 2 s)

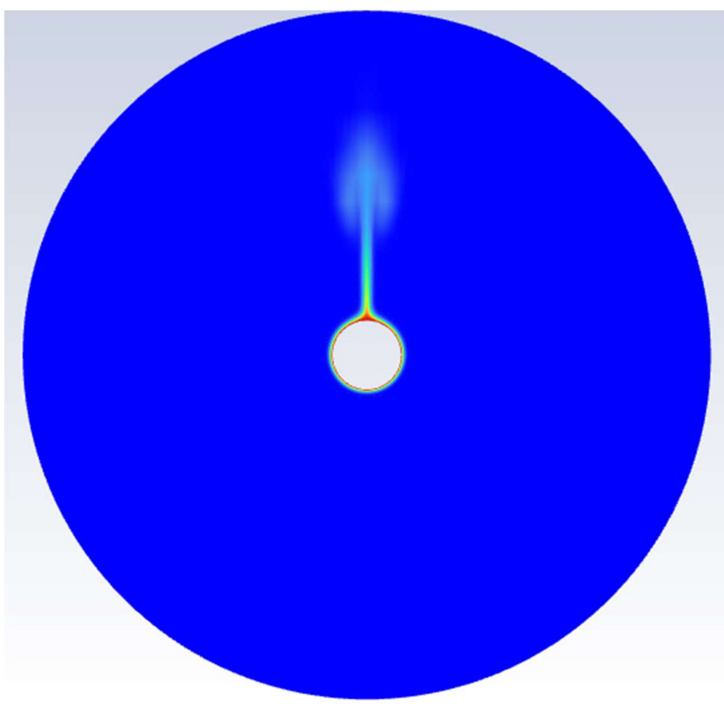

Figure 7.2 Temperature Contour for Diameter of 0.1 m (30x50 Mesh at 2 s)

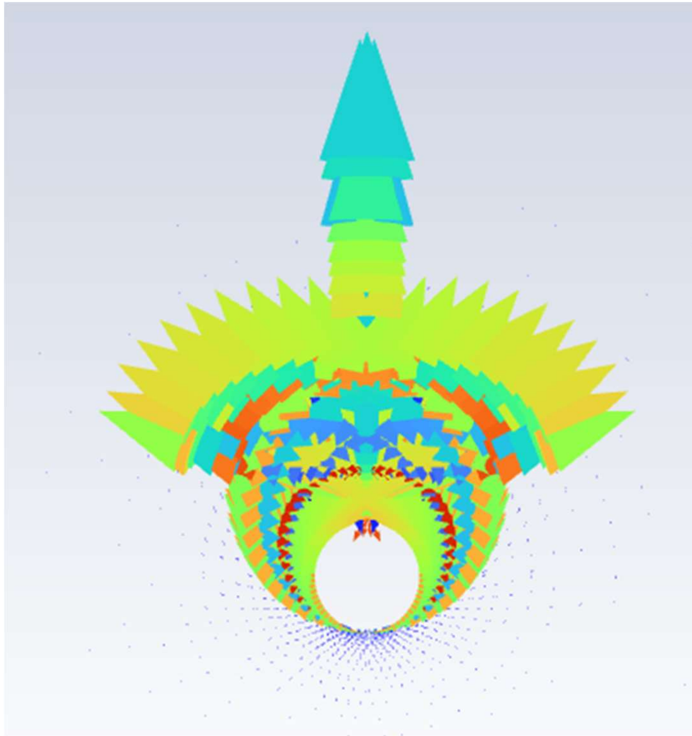

Figure 7.3 Velocity-Temperature Vectors for Diameter of 0.2 m (30x50 Mesh at 2 s)

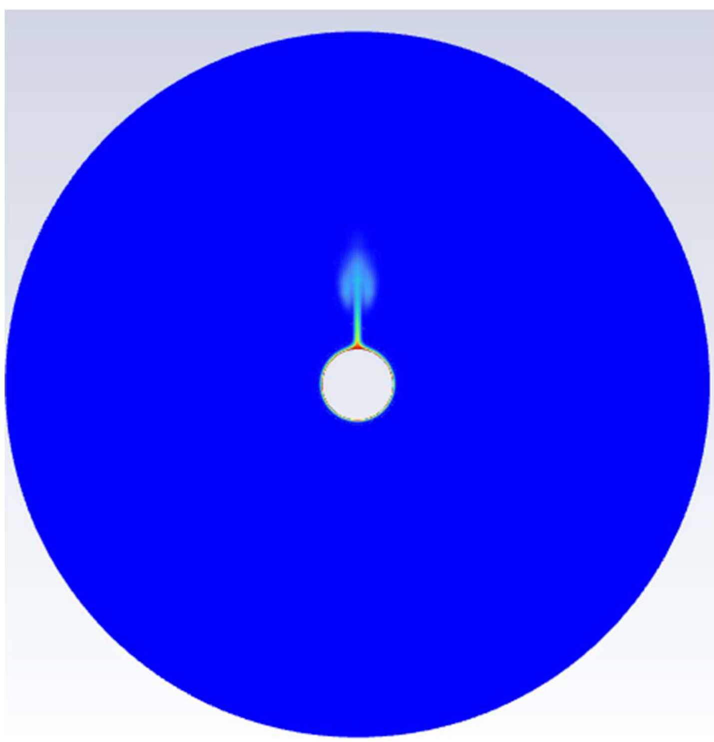

Figure 7.4 Temperature Contour for Diameter of 0.2 m (30x50 Mesh at 2 s)

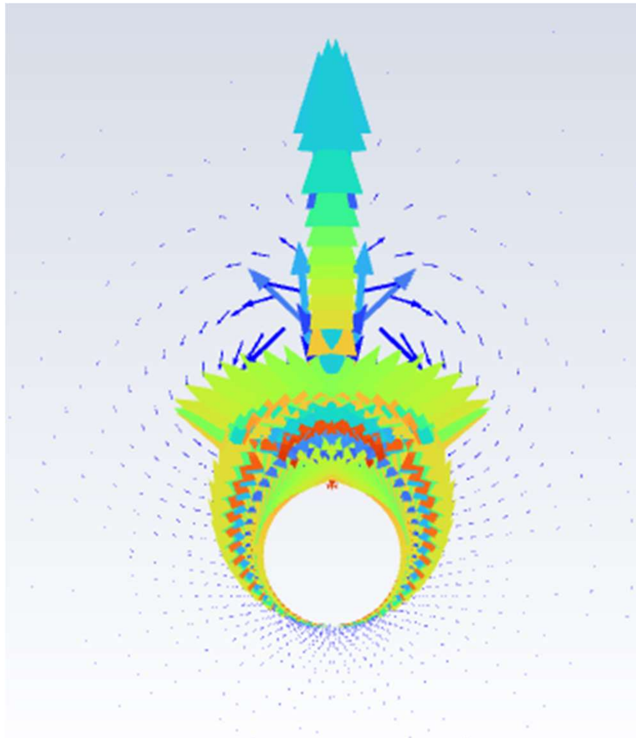

Figure 7.5 Velocity-Temperature Vectors for Diameter of 0.3 m (30x50 Mesh at 2 s)

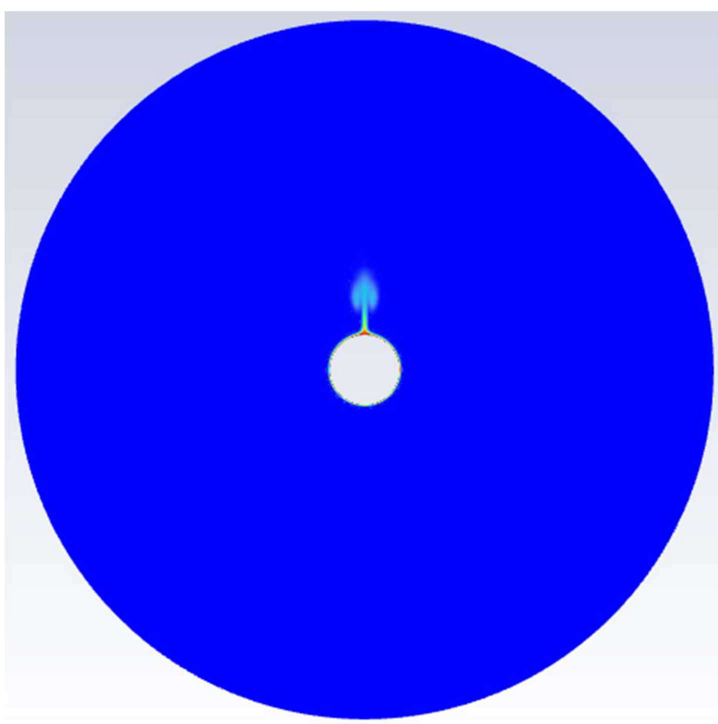

Figure 7.6 Temperature Contour for Diameter of 0.3 m (30x50 Mesh at 2 s)

7.2 Time and Plume Effects on Heat Transfer

Another notable detail with this flow that has been mentioned is the plume that is produced due to the buoyancy of less dense air, making air exit from the top of the cylinder. As seen in Figure 7.7, the total heat transfer rate over time plateaus to around 19.6, which can also be seen in Table 7.2. Although in Figures 7.8, 7.9, and 7.10, there is clearly an effect on the fluid behavior of the plume, these values imply that the mechanics of the plume itself does not have a large impact on the heat transfer occurring at the surface of the cylinder.

Table 7.2 Heat Transfer Variation Over Time (30x50 Mesh and 0.1 m Diameter)

Time [s]	Q [W]	Simulated Nusselt Number [-]
1.0	49.0434	19.593
1.5	49.0851	19.601
2.0	49.0825	19.600

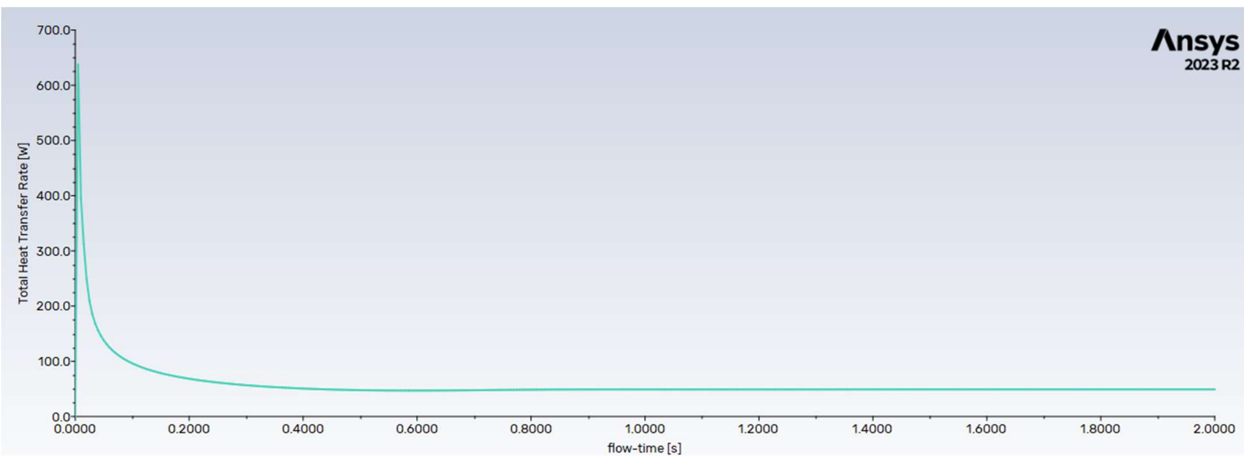

Figure 7.7 Total Heat Transfer Rate Over Time to Steady State (30x50 Mesh)

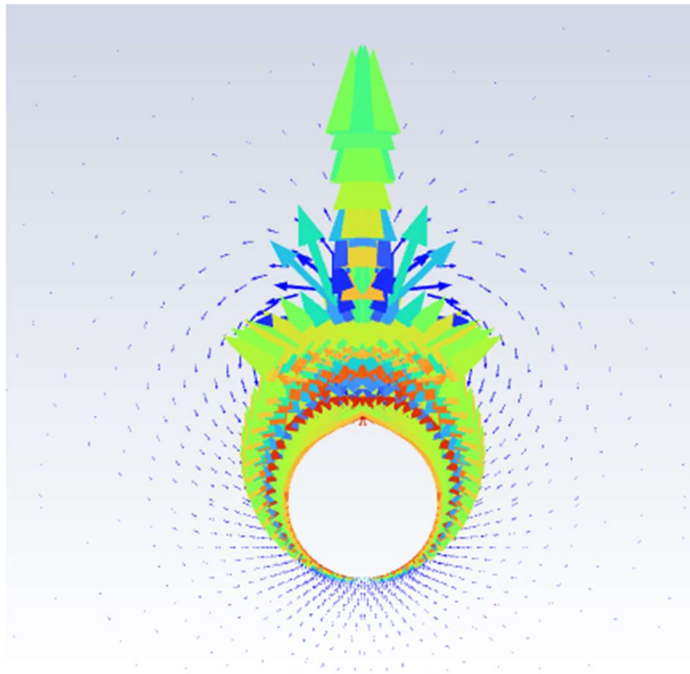

Figure 7.8 Velocity Vectors at a Time of 1 Seconds

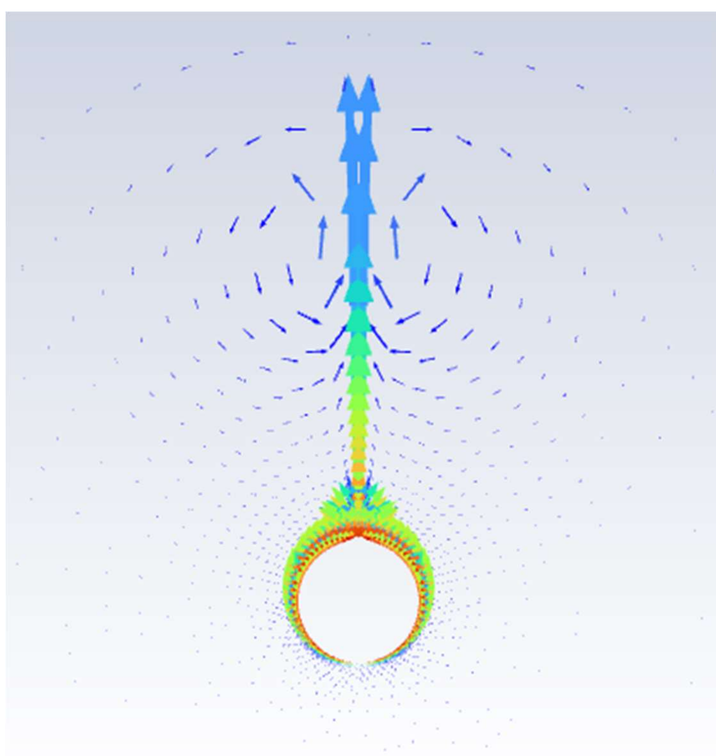

Figure 7.9 Velocity Vectors at a Time of 1.5 Seconds

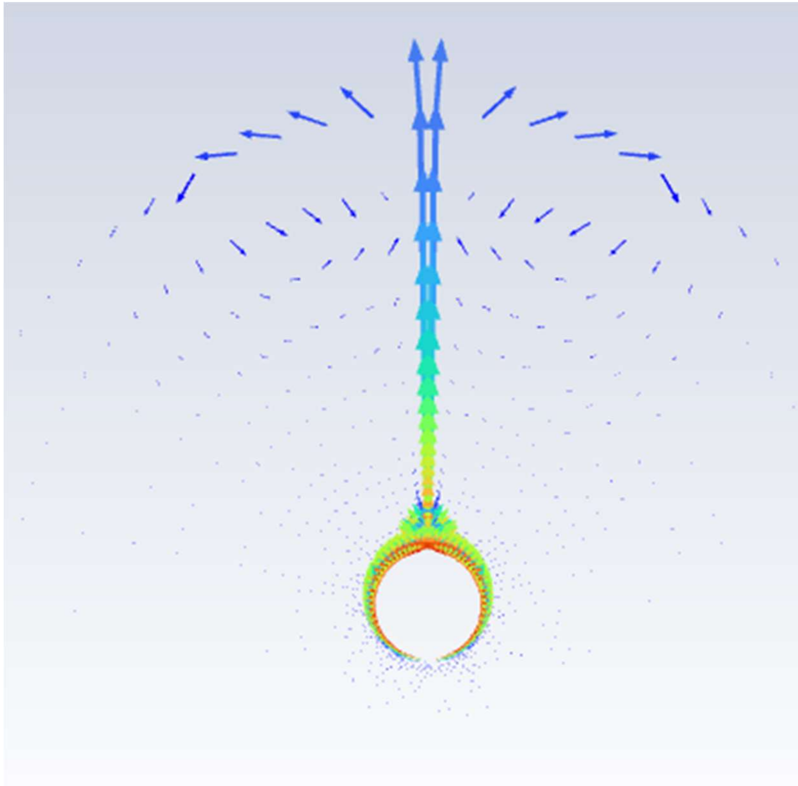

Figure 7.10 Velocity Vectors at a Time of 2 Seconds

8. Conclusions

From this simulation, this study was successful in reproducing the common studied example of natural convection over a cylinder. The error in simulation and correlation values turned out to only be a maximum of about 8% error, which could be explained by various modelling settings, ideal environments, or density approximations with diffusion.

9. References

1. Churchill, S and Chu, H, Correlating equations for laminar and turbulent free convection from a horizontal cylinder, *International journal of heat and mass transfer*, 1049-1053, 1975.
2. Morgan, V, The Overall Convective Heat Transfer from Smooth Circular Cylinders, Advances in heat transfer, 11, 199-264.
3. Tessier, F, Analysis of the Buoyant Plume Above a Heated Horizontal Cylinder in a Free Convection Flow.