M E M O R A N D U M

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ME 318-07

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To: Dr. Hemanth Porumamilla, Professor
Dr. Stephen Kwok-Choon, Postdoctoral Lecturer

Subject: Single and Double Pendulum

Introduction/Procedure

In this lab we analyzed the effect that pendulums connected by a spring have on each other at various initial conditions as seen in Figure 1. We began with measuring values for spring constant, k, mass at the end of the bar, m, and the lengths of the bar and where the springs were attached. We then observed the two pendulums displaced at either 0^o,-5^o, or $+5^o$ to measure their natural frequency, beat frequency, or average frequency of the system. . Our theoretical calculations for each of these cases can be seen in Appendix B. We then simulated the double pendulum system in Matlab to verify our oscilloscope and theoretical results.

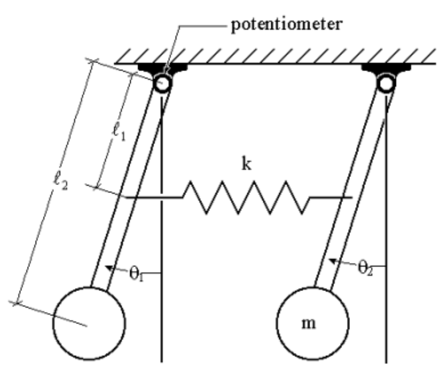


Figure 1. Experimental Double Pendulum Setup - Image from Lab Procedure

Experimental Data

Mass (g)	46.5	
Cycles	Time (s)	Period (s)
10	6.7	0.67
10	6.75	0.675
10	6.76	0.676
10	6.72	0.672
10	6.88	0.688
Average		0.6762
Frequency (Hz)		9.292
Spring Constant (N/m)		4.015

Table 1. Free Oscillation Data to find the Spring Constant

Change in Length (m)	Weight (N)
0.73	0.45126
0.665	0.17658
0.775	0.62784
Spring Constant (N/m)	4.111

Table 2. Static Elongation Data to find the Spring Constant

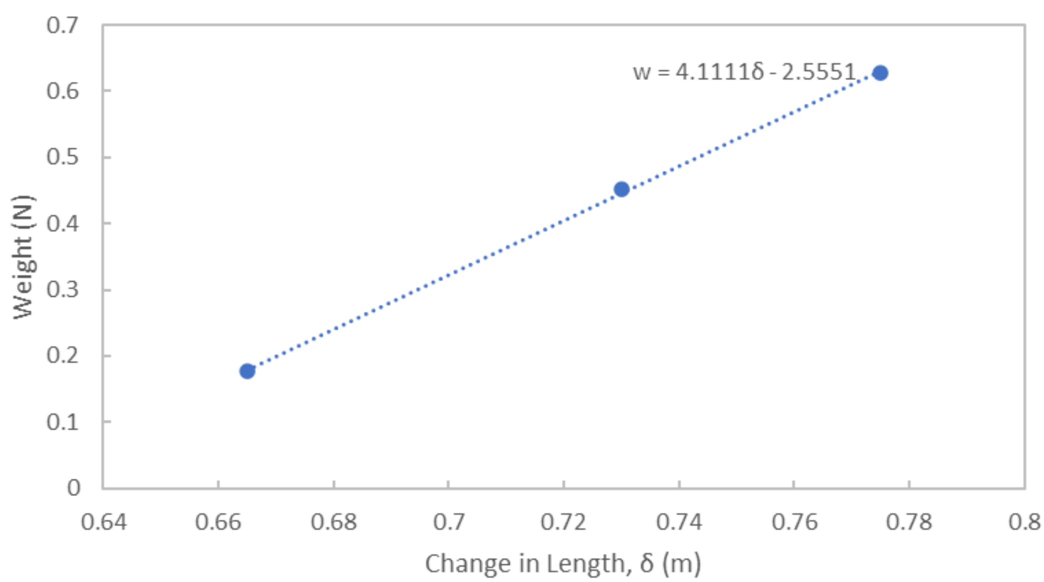
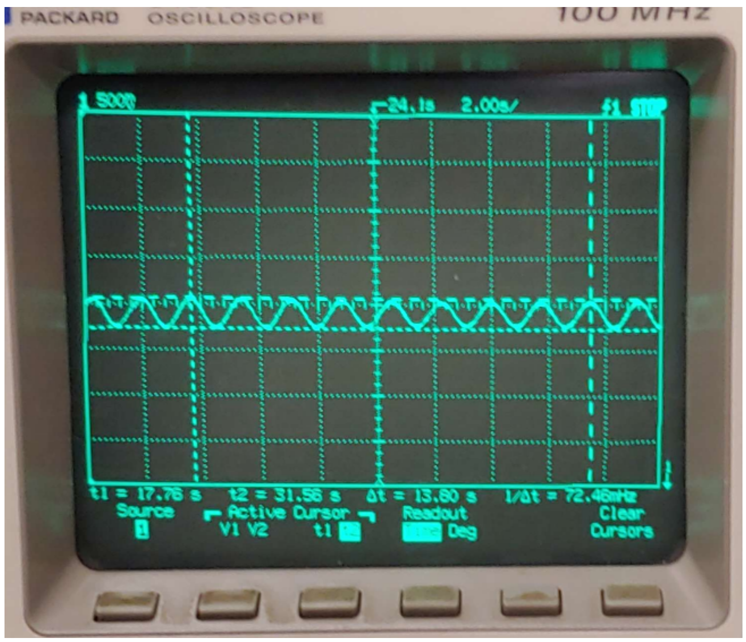


Figure 2. Static Elongation with Varying Weights to find the Spring Constant, k=w/δ

Results

Frequency	Theoretical	Experimental	Simulated
First Mode f1 (Hz)	0.5718	0.5797	0.5721
Second Mode f2 (Hz)	0.6399	0.6313	0.6242
Beat Frequency (Hz)	0.0341	0.0265	0.0337
Average Frequency (Hz)	0.6059	0.5900	0.6413

Table 3. Theoretical, Experimental, and Simulated Frequencies



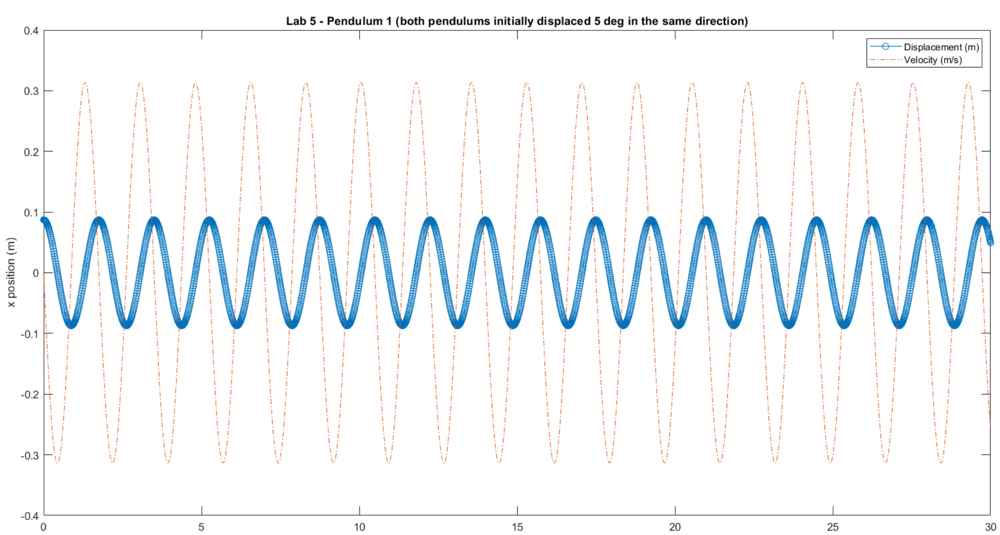


Figure 3. First Mode Response where $\theta_1 = \theta_2 = 5^o$

For the first mode's response, we can approximate it as a free single degree of freedom pendulum performing simple harmonic motion since the spring never changes length and has motion described by just one angle. Simulating this in Matlab (Appendix A) produces a natural frequency of 0.5721 Hz with our oscilloscope reading a similar 0.5797 Hz as well.

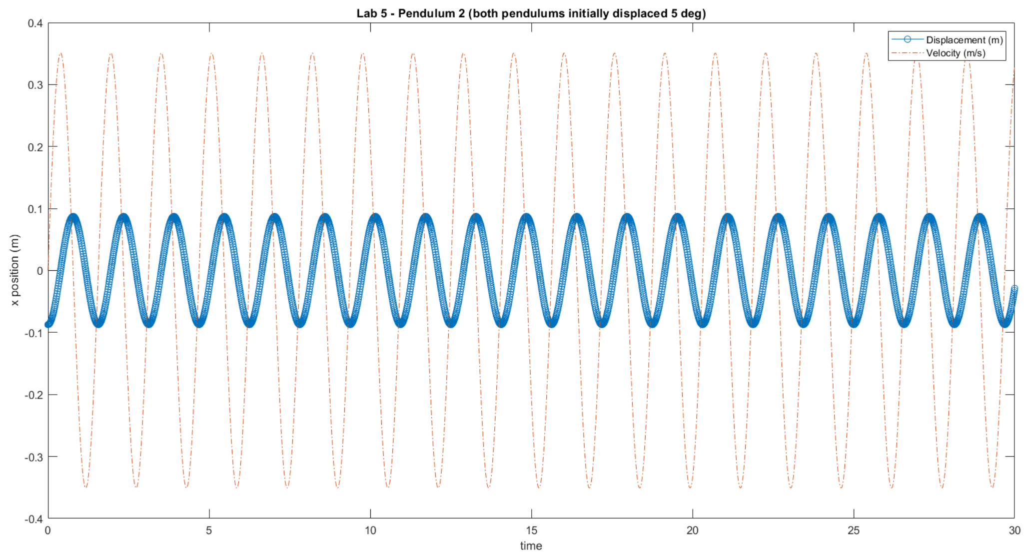


Figure 4. First Mode Response where $\theta_1 = -\theta_2 = 5^o$

For the second mode's response, although we now observe a spring creating a reaction between the two pendulums, we can still approximate the pendulums as a free single degree of freedom pendulum performing simple harmonic motion. This is because the angle displacements will maintain being exactly equal and opposite, so the force of the spring is only dependent on one θ. Simulating this in Matlab (Appendix A) produces a natural frequency of 0.6242 Hz with our oscilloscope again reading a similar 0.6313 Hz.

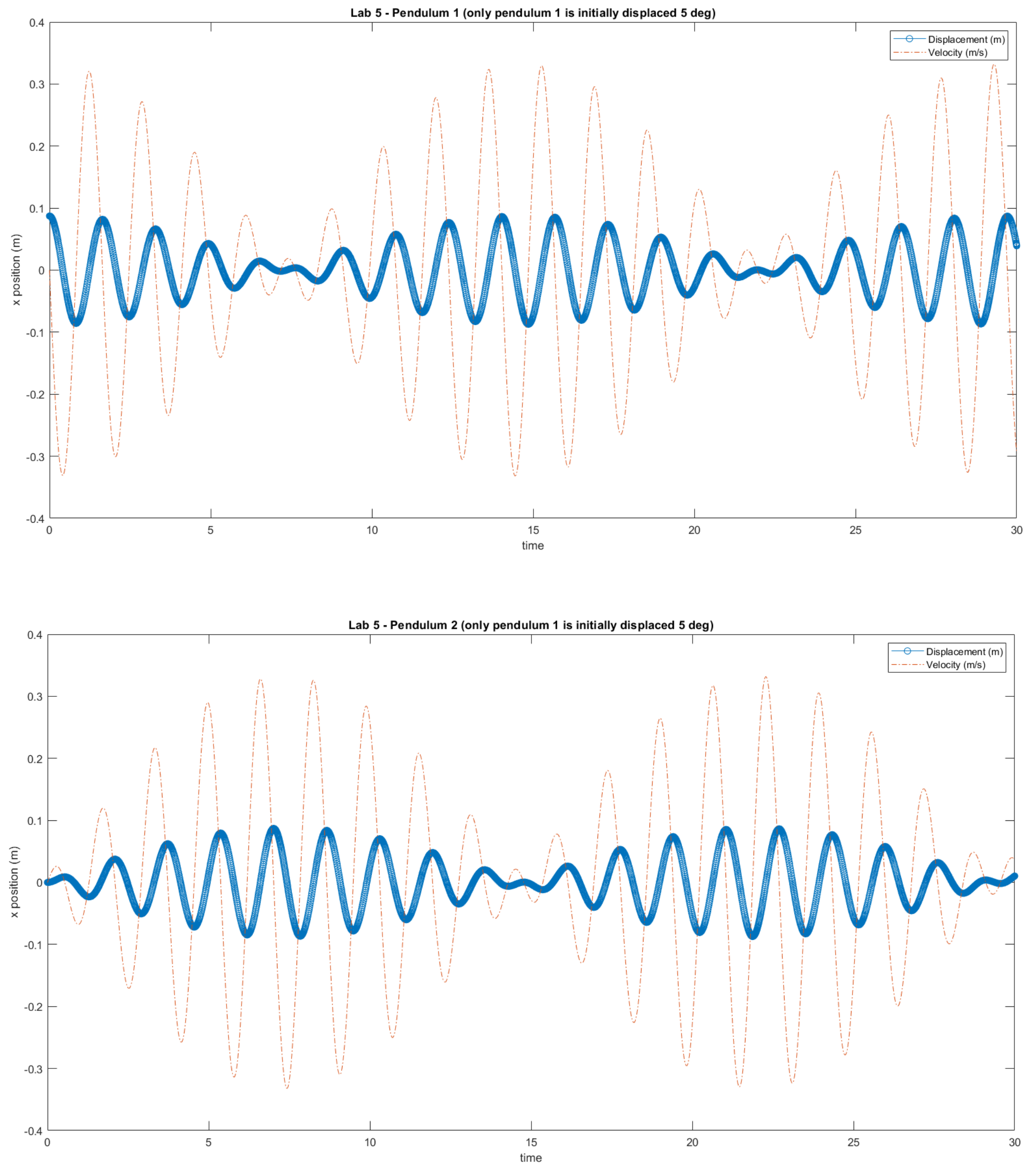


Figure 5. Arbitrary θ_1 and θ_2 Response

This third response that we tested contains two different initial values for θ_1 and θ_2, and because the spring is now dependent on two varying values, each pendulum system now has two degrees of freedom as they observe a forcing function on one another. Compared to the previous models, we can't say that this is the third mode since our values for θ could be any arbitrary values and can have infinite natural frequencies. As we observe in our pendulums' responses, there's two frequencies; a beat, or enveloping, frequency, and the average frequency that fills in the beat envelope measured with Matlab to be 0.0337 Hz and 0.6413 Hz, respectively. We observe this specific harmonic excitation due to the two pendulums having similar natural frequencies.

Error Analysis

	%error	
Frequency	Th and Exp	Sim and Exp
First Mode f1 (Hz)	1.38%	1.33%
Second Mode f2 (Hz)	1.34%	1.14%
Beat Frequency (Hz)	22.17%	21.36%
Average Frequency (Hz)	2.62%	8.00%

Table 4. %Error between Theoretical and Simulated compared to Experimental Frequencies

As we can see in Table 4, our models for the theoretical and simulated calculations were fairly accurate to what occurs in the real world. The only discrepancy we found was with the beat frequency which could have been caused by not exact initial conditions. By varying our spring constant ±5%, we found, using our Matlab simulation, only a less than two percent difference in beat frequency variation, so the error from our experimental spring constant is negligible. We could also look at our assumptions for error. We assumed small angle approximation, a massless bar and spring, and no friction. We only used 5^{o}, so small angle approximation is still true, but since the bars aren't massless, nor can we avoid friction, we'll have some small variation in our experimental frequency.

Appendix A: MATLAB Code for System Response for Arbitrary Initial Angles

```
% lab 5 - Vibrations
% Christopher Ng, Roman Bradley

m = 0.625; % kg
g = 9.81;    % m/s^2
l1 = .38;      % m
l2 = .76;      % m
k = 4.07 ;    % N/m

c1 = (m*g*l2+k*l1^2)/(m*l2^2);
c2 = (k*l1^2)/(m*l2^2);
c3 = (m*g*l2+k*l1^2)/(m*l2^2);
c4 = (k*l1^2)/(m*l2^2);

tspan = [0, 30];
y0 = [0,0,0,0];

opts = odeset('RelTol',1e-6,'AbsTol',1e-9);
[t,y] = ode45(@(t,y) odefcn(t,y,c1,c2,c3,c4), tspan, y0,opts);
figure(1);
plot(t,y(:,1),'-o',t,y(:,2),'-.')
xlabel('time');
ylabel('x position (m)');
title('Lab 5 - Pendulum 1 (initially displaced _ deg)');
legend('Displacement (m)','Velocity (m/s)');
figure(2);
plot(t,y(:,3),'-o',t,y(:,4),'-.')
xlabel('time');
ylabel('x position (m)');
title('Lab 5 - Pendulum 2 (initially displaced _ deg)');
legend('Displacement (m)','Velocity (m/s)');

function dydt = odefcn(t,y,c1,c2,c3,c4)
  dydt = zeros(4,1);
  dydt(1) = y(2);
  dydt(2) = -c1*y(1)+c2*y(3);
  dydt(3) = y(4);
  dydt(4) = -c3*y(3)+c4*y(1);
End
```

Appendix B: Prelab Hand Calculations for Theoretical Frequencies

Prelab 5
Single and Double Pendulum

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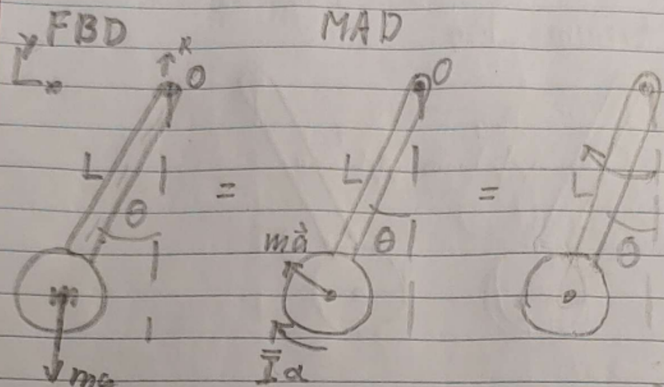
ME 318-07
10/31/22

2a) Single Pendulum



$I_o\alpha = I_o\ddot{\theta}$

$I_o = mL^2$

for small θ
$\sin\theta = \theta$

$\Sigma M_o \Rightarrow mgL\sin\theta = -I\ddot{\theta}$

$mgL\theta = -mL^2\ddot{\theta}$

EOM

$L\ddot{\theta} + g\theta = 0$

b) $\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{g}{L}}$

$f_n = \omega_n \left(\frac{1\,\text{cycle}}{2\pi\,\text{rad}}\right)$

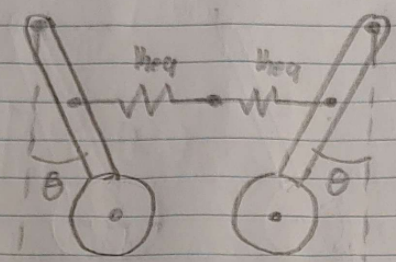
$f_{n_1} = \sqrt{\frac{g}{L}}\left(\frac{1\,\text{cycle}}{2\pi\,\text{rad}}\right)$

$g = 9.81\,m/s^2$
$L = 76cm = 0.76m$

$f_{n_1} = \sqrt{\frac{9.81\,m/s^2}{0.76m}}\left(\frac{1\,\text{cycle}}{2\pi\,\text{rad}}\right)$
$= 0.5718\ Hz$

3b) Double Pendulum

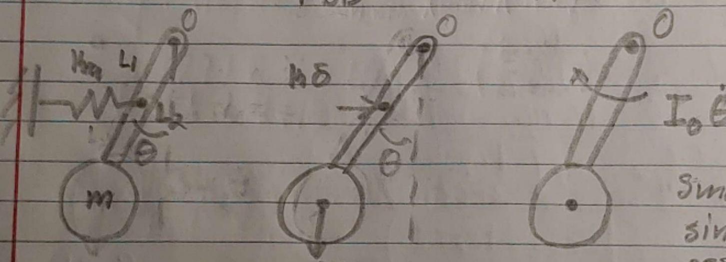
to determine k_{eq}



$k = \frac{k_{eq}\cdot k_{eq}}{k_{eq}+k_{eq}} = \frac{1}{2}k_{eq}$

$\underline{k_{eq} = 2k}$

FBD MAD



Small θ approx
$\sin\theta \approx \theta$
$\cos\theta \approx 1$

$\Sigma M_O \Rightarrow mgL_2\sin\theta + 2kL_1^2\sin\theta\cos\theta = -I_O\ddot{\theta}$

$mgL_2\theta + 2kL_1^2\theta = -mL_2^2\ddot{\theta}$

$mL_2^2\ddot{\theta} + (mgL_2 + 2kL_1^2)\theta = 0$

c) $\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{mgL_2 + 2kL_1^2}{mL_2^2}}$

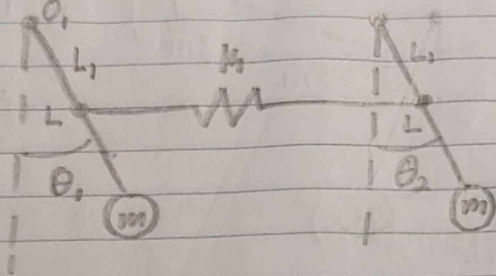
$f_n = \sqrt{\frac{mgL_2 + 2kL_1^2}{mL_2^2}}\left(\frac{1\text{ cycle}}{2\pi\text{ rad}}\right)$

$m = 625\text{g} = 0.625\text{kg}$
$k = 4.07\text{ N/m}$
$L_1 = 38\text{cm} = 0.38\text{m}$
$L_2 = 76\text{cm} = 0.76\text{m}$

$f_{n_2} = \sqrt{\frac{(0.625\text{kg})(9.81\text{m/s}^2)(0.76\text{m}) + 2(4.07\text{N/m})(0.38\text{m})^2}{(0.625\text{kg})(0.76\text{m})^2}}\left(\frac{1\text{ cycle}}{2\pi\text{ rad}}\right)$

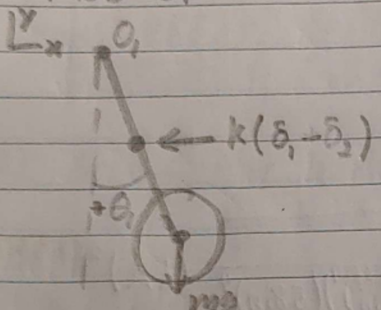
$= 0.6399\text{ Hz}$

5a) Computer Simulation

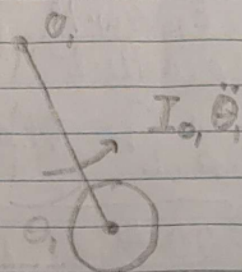


$\theta_1 > \theta_2 > 0$

FBD θ_1



MAD θ_1



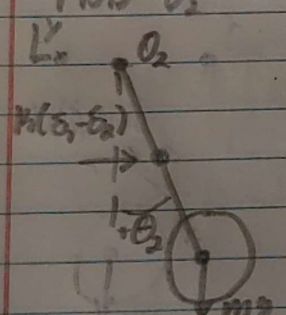
$\Sigma M_{O_1} \Rightarrow mgL\sin\theta_1 + k(\delta_1 - \delta_2)L_1\cos\theta_1 = -I_O\ddot{\theta}_1$

$mgL\sin\theta_1 + k(L_1\sin\theta_1 - L_1\sin\theta_2)L_1\cos\theta = -I_O\ddot{\theta}_1$

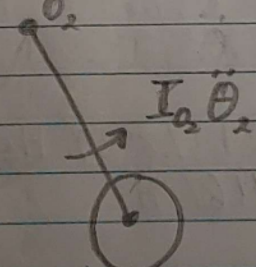
$mgL\theta_1 + kL_1^2(\theta_1 - \theta_2) = -mL^2\ddot{\theta}_1$

$mL^2\ddot{\theta}_1 + (mgL + kL_1^2)\theta_1 - kL_1^2\theta_2 = 0$

FBD θ_2



MAD



$\Sigma M_{O_2} \Rightarrow mgL\sin\theta_2 + k(\delta_1 - \delta_2)L_1\cos\theta_2 = -I_O\ddot{\theta}_2$

$mgL\theta_2 + kL_1^2(\theta_1 - \theta_2) = -mL^2\ddot{\theta}_2$

$mL^2\ddot{\theta}_2 + (mgL - kL_1^2)\theta_2 + kL_1^2\theta_1 = 0$