Análise assintótica

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1 $f_{(n)} = n - 100$; $g_{(n)} = n - 200$

- $f_{(n)} \leq 2 \times g_{(n)}, \forall n \geq 300 \therefore f_{(n)} = O(g_{(n)})$
- $f_{(n)} \geq 1 \times g_{(n)}, \forall n \geq 150 \therefore f_{(n)} = \Omega(g_{(n)})$
- $f_{(n)} = \Theta(g_{(n)}) \rightarrow c_1 = 150, c_2 = 300$
- $\lim_{n\to\infty} \frac{n-100}{n-200} = \lim_{n\to\infty} \frac{1}{1} = 1 \therefore f_{(n)} \neq o(g_{(n)}), f_{(n)} \neq \omega(g_{(n)})$

2 $f_{(n)} = \log n$; $g_{(n)} = (\log n)^2$

- $f_{(n)} \leq 1 \times g_{(n)}, \forall n \geq 1 \therefore f_{(n)} = O(g_{(n)})$
- $\nexists c | f_{(n)} >= c \times g_{(n)} \therefore f_{(n)} \neq \Omega(g_{(n)})$
- $\lim_{n\to\infty} \frac{\log n}{(\log n)^2} = \lim_{n\to\infty} \frac{1}{\log n} = 0 \therefore f_{(n)} = o(g_{(n)}), f_{(n)} \neq \omega(g_{(n)})$

3 $f_{(n)} = \log n$; $g_{(n)} = \log n^2$

- $f_{(n)} \leq 1 \times g_{(n)}, \forall n \geq 1 \therefore f_{(n)} = O(g_{(n)})$
- $f_{(n)} >= \frac{1}{2} \times g_{(n)} \therefore f_{(n)} = \Omega(g_{(n)})$
- $f_{(n)} = \Theta(g_{(n)}) \rightarrow c_1 = \frac{1}{2}, c_2 = 1$
- $\lim_{n\to\infty} \frac{\log n}{\log n^2} = \lim_{n\to\infty} \frac{1}{2} = \frac{1}{2} \therefore f_{(n)} \neq o(g_{(n)}), f_{(n)} \neq \omega(g_{(n)})$

4 $f_{(n)} = 2^n$; $g_{(n)} = 2^{n+1}$

- $f_{(n)} \leq 1 \times g_{(n)}, \forall n \geq 1 \therefore f_{(n)} = O(g_{(n)})$
- $f_{(n)} >= \frac{1}{2} \times g_{(n)} \therefore f_{(n)} = \Omega(g_{(n)})$
- $f_{(n)} = \Theta(g_{(n)}) \rightarrow c_1 = \frac{1}{2}, c_2 = 1$
- $\lim_{n\to\infty} \frac{2^n}{2^{n+1}} = \lim_{n\to\infty} \frac{2^n}{2^n \times 2} = \frac{1}{2} \therefore f_{(n)} \neq o(g_{(n)}), f_{(n)} \neq \omega(g_{(n)})$

5 $f_{(n)} = n!$; $g_{(n)} = 2^n$

- $f_{(n)} \leq 1 \times g_{(n)}, \forall n \geq 4 \therefore f_{(n)} = O(g_{(n)})$
- $\nexists c | f_{(n)} >= c \times g_{(n)} \therefore f_{(n)} \neq \Omega(g_{(n)})$
- para avaliar a notação o e ω é necesário analisar o crescimento individual de cada função, com isso temos:
  $\lim_{n\to\infty} \frac{n!}{2^n} = \infty \therefore f_{(n)} \neq o(g_{(n)}), f_{(n)} = \omega(g_{(n)})$

6 $f_{(n)} = 2n^2 + 5n$; $g_{(n)} = n^2$

- $f_{(n)} \leq 7 \times g_{(n)}, \forall n \geq 1 \therefore f_{(n)} = O(g_{(n)})$
- $f_{(n)} >= 1 \times g_{(n)} \therefore f_{(n)} = \Omega(g_{(n)})$
- $f_{(n)} = \Theta(g_{(n)}) \to c_1 = 1, c_2 = 7$
- $\lim_{n\to\infty} \frac{2n^2+5n}{n^2} = \lim_{n\to\infty} 2 + \frac{5}{n} = 2 \therefore f_{(n)} \neq o(g_{(n)}), f_{(n)} \neq \omega(g_{(n)})$

7 $f_{(n)} = 2n^2 + 5n$; $g_{(n)} = n^3$

- $f_{(n)} \leq 7 \times g_{(n)}, \forall n \geq 1 \therefore f_{(n)} = O(g_{(n)})$
- $\nexists c | f_{(n)} >= c \times g_{(n)} \therefore f_{(n)} \neq \Omega(g_{(n)})$
- $\lim_{n\to\infty} \frac{2n^2+5n}{n^3} = \lim_{n\to\infty} \frac{2+\frac{5}{n}}{n} = 0 \therefore f_{(n)} = o(g_{(n)}), f_{(n)} \neq \omega(g_{(n)})$