1)

$$T(n) \begin{cases} O(1), & \forall n = 1 \\ 4T(n/2) + O(n), & \forall n > 1 \end{cases}$$

$$T(n) = 4\left(4T\left(\frac{n}{2^2}\right) + \Theta(n)\right) + \Theta(n)$$

$$T(n) = 4^2 T\left(\frac{n}{2^2}\right) + 4\Theta\left(\frac{n}{2}\right) + \Theta(n)$$

$$T(n) = 4^2\left(4T\left(\frac{n}{2^3}\right) + \Theta\left(\frac{n}{2^2}\right)\right) + 4\Theta\left(\frac{n}{2}\right) + \Theta(n)$$

$$T(n) = 4^3 T\left(\frac{n}{2^3}\right) + 4^2\Theta\left(\frac{n}{2^2}\right) + 4\Theta\left(\frac{n}{2}\right) + \Theta(n)$$

$$T(n) = 4^k T\left(\frac{n}{2^k}\right) + \sum_{i=0}^{k-1} 4^i \Theta\left(\frac{n}{2^i}\right)$$

$$\frac{n}{2^k} = 1 \quad \therefore \quad n = 2^k \iff k = \log_2(n)$$

$$T(n) = 4^{\log_2(n)} O(1) + d2^{\log_2(n)+1} n) - \Theta(n)$$

$$T(n) = O(n^{\log_2 4}) + \Theta(3n) - \Theta(n) = \Theta(n^2)$$

2. $T(n) \begin{cases} O(1), & \forall\, n = 1 \\ 3\,T(n/2) + O(n) \end{cases}$

$$T(n) = 3\left(3\,T\left(\frac{n}{2^2}\right) + O\left(\frac{n}{2}\right)\right) + O(n)$$

$$T(n) = 3^2\,T\left(\frac{n}{2^2}\right) + 3^1\,O\left(\frac{n}{2}\right) + O(n)$$

$$T(n) = 3^2\left(3\,T\left(\frac{n}{2^3}\right) + O\left(\frac{n}{2^2}\right)\right) + 3\,O\left(\frac{n}{2}\right) + O(n)$$

$$= 3^3\,T\left(\frac{n}{2^3}\right) + 3^2\,O\left(\frac{n}{2^2}\right) + 3\,O\left(\frac{n}{2}\right) + O(n)$$

$$= 3^k\,T\left(\frac{n}{2^k}\right) + \sum_{i=0}^{k-1} 3^i\,O\left(\frac{n}{2^i}\right)$$

$$= 3^k\,T\left(\frac{n}{2^k}\right) + 3\left(\frac{3}{2}\right)^k O(n) - 2\,O(n)$$

$$= 3^{\log_2 n}\,O(1) + 3\left(\frac{3}{2}\right)^{\log_2 n} O(n) - 2\,O(n)$$

$$= O\left(n^{\log_2 3}\right) + 3\,O\left(n^{\log_2 \frac{3}{2}}\right) - 2\,O(n)$$

$$= O\left(n^{\log_2 3}\right) + O\left(3n^{\log_2 3 - 1}\right) - O(2n)$$

$$= O(n)$$