Exercício 3 - Análise Assintótica

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▼ 1 - $f(n) = n - 100;\ g(n) = n - 200$

$$\begin{gathered} f(n) <= 2 \times g(n)\ \forall n >= 300 \\ f(n) >= 1 \times g(n)\ \forall n >= 150 \\ f(n) = \Theta(g(n)) \quad \forall n >= 300 \end{gathered}$$

$$\begin{gathered} \lim_{n \to \infty} \frac{n-100}{n-200} = \lim_{n \to \infty} \frac{1}{1} = 1 \therefore \\ f(n) \neq o(g(n)),\ f(n) \neq \omega(g(n)) \end{gathered}$$

▼ 2 - $f(n) = \log(n);\ g(n) = (\log(n))^2$

$$\begin{gathered} f(n) <= 1 \times g(n)\ \forall n >= 1 \\ \nexists\ c\ | f(n) >= c \times g(n) \therefore \\ f(n) = O(g(n))\ \forall n >= 1 \end{gathered}$$

$$\begin{gathered} \lim_{n \to \infty} \frac{\log(n)}{(\log(n))^2} = \lim_{n \to \infty} \frac{1}{\log(n)} = 0 \therefore \\ f(n) = o(g(n)),\ f(n) \neq \omega(g(n)) \end{gathered}$$

▼ 3 - $f(n) = \log(n);\ g(n) = \log(n^2)$

$$\begin{gathered} f(n) <= 1 \times g(n)\ \forall n >= 0 \\ f(n) >= \frac{1}{2} \times g(n)\ \forall n >= 0 \therefore \\ f(n) = \Theta(g(n))\ \forall n >= 0 \end{gathered}$$

$$\begin{gathered} \lim_{n \to \infty} \frac{\log(n)}{\log(n^2)} = \lim_{n \to \infty} \frac{1}{2} = \frac{1}{2} \therefore \\ f(n) \neq o(g(n)),\ f(n) \neq \omega(g(n)) \end{gathered}$$

▼ 4 - $f(n) = 2^n;\ g(n) = 2^{n+1}$

$$f(n) <= 1 \times g(n)\ \forall n >= 0$$
$$f(n) >= \frac{1}{2} \times g(n)\ \forall n >= 0 \therefore$$
$$f(n) = \Theta(g(n))\ \forall n >= 0$$

$$\lim_{n\to\infty} \frac{2^n}{2^{n+1}} = \lim_{n\to\infty} \frac{1}{2} = \frac{1}{2} \therefore$$
$$f(n) \neq o(g(n)),\ f(n) \neq \omega(g(n))$$

▼ 5 - $f(n) = n!;\ g(n) = 2^n$

$$f(n) >= g(n)\ \forall n >= 4$$
$$\nexists\ c\ |f(n) <= c \times g(n) \therefore$$
$$f(n) = \Omega(g(n))\ \forall n >= 4$$

- Observando o crescimento individual de cada função, observamos que:

$$\lim_{n\to\infty} \frac{n!}{2^n} = \infty \therefore$$
$$f(n) \neq o(g(n)),\ f(n) = \omega(g(n))$$

▼ 6 - $f(n) = 2n^2 + 5n;\ g(n) = n^2$

$$f(n) <= 7 \times g(n)\ \forall n >= 0$$
$$f(n) >= 1 \times g(n)\ \forall n >= 0 \therefore$$
$$f(n) = \Theta(g(n))\ \forall n >= 0$$

$$\lim_{n\to\infty} \frac{2n^2 + 5n}{n^2} = \lim_{n\to\infty} 2 + \frac{5}{n} = 2 \therefore$$
$$f(n) \neq o(g(n)),\ f(n) \neq \omega(g(n))$$

▼ 7 - $f(n) = 2n^2 + 5n;\ g(n) = n^3$

$$f(n) <= 7 \times g(n)\ \forall n >= 0$$
$$\nexists\ c\ | f(n) >= c \times g(n) \therefore$$
$$f(n) = O(g(n))\ \forall n >= 0$$

$$\lim_{n \to \infty} \frac{2n^2 + 5n}{n^3} = \lim_{n \to \infty} \frac{2 + \frac{5}{n}}{n} = 0 \therefore$$
$$f(n) = o(g(n)),\ f(n) \neq \omega(g(n))$$