CHAPTER 1

INTRODUCTION

The term “Biomedical Engineering” can refer to any endeavor in which techniques from engineering disciplines are used to solve problems in the life sciences. One such undertaking is the construction of mathematical models of physiological systems and their subsequent analysis. Ideally the insights gained from analyzing these models will lead to a better understanding of the physiological systems they represent.

System identification is a discipline that originated in control engineering; it deals with the construction of mathematical models of dynamic systems using measurements of their inputs and outputs. In control engineering, system identification is used to build a model of the process to be controlled; the process model is then used to construct a controller.

In biomedical engineering, the goal is more often to construct a model that is detailed enough to provide insight into how the system operates. This text deals with system identification methods that are commonly used in biomedical engineering. Since many physiological systems are highly nonlinear, the text will focus on methods for nonlinear systems and their application to physiological systems. This chapter will introduce the concepts of signals, systems, system modeling, and identification. It also provides a brief overview of the system identification problem and introduces some of the notation and terminology to be used in the book. The reader should be acquainted with most of the material covered in this chapter. If not, pedagogical treatments can be found in most undergraduate level signals and systems texts, such as that by Kamen (1990).

1.1 SIGNALS

The concept of a signal seems intuitively clear. Examples would include speech, a television picture, an electrocardiogram, the price of the NASDAQ index, and so on. However, formulating a concise, mathematical description of what constitutes a signal is somewhat involved.

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1.1.1 Domain and Range

In the examples above, two sets of values were required to describe each "signal"; these will be termed the domain and range variables of the signal. Simply put, a signal may be viewed as a function that assigns a value in the range set for each value in the domain set; that is, it represents a mapping from the domain to the range. For example, with speech, the domain is time while the range could be one of a variety of variables: the air pressure near the speaker's mouth, the deflection of the listener's ear drum, or perhaps the voltage produced by a microphone.

This concept can be defined formally by describing a signal, $s(t)$, as a mapping from a domain set, T, which is usually time, to a range set, Y. Thus,

$$s : T \rightarrow Y$$

where $t \in T$ is a member of the domain set, usually time. In continuous time, T is the real line; in discrete time, it is the set of integers. In either case, the value of the signal is in the range set, Y. The range of the signal is given by applying the mapping to the domain set, and is therefore $s(T)$.

The above definition really describes a function. A key point regarding the domain set of a signal is the notion that it is ordered and thus has a direction. Thus, if x_1 and x_2 are members of the domain set, there is some way of stating $x_1 > x_2$, or the reverse. If time is the domain, $t_1 > t_2$ is usually taken to mean that t_1 is later than t_2.

The analysis in this book will focus on signals with one-dimensional domains—usually time. However, most of the ideas can be extended to signals with domains having two dimensions (e.g., X-ray images), three dimensions (e.g., MRI images), or more (e.g., time-varying EEG signals throughout the brain).

1.1.2 Deterministic and Stochastic Signals

A signal is deterministic if its future values can be generated based on a set of known rules and parameters, perhaps expressed as a mathematical equation. For example, the sinusoid

$$y_d(t) = \cos(2\pi f t + \phi)$$

can be predicted exactly, provided that its frequency f and phase ϕ are known. In contrast, if $y_r(k)$ is generated by repeatedly tossing a fair, six-sided die, there is no way to predict the kth value of the output, even if all other output values are known. These represent two extreme cases: $y_d(t)$ is purely deterministic while $y_r(k)$ is completely random, or stochastic.

The die throwing example is an experiment where each repetition of the experiment produces a single *random variable*: the value of the die throw. On the other hand, for a *stochastic process* the result of each experiment will be a signal whose value at each time is a random variable. Just as a single throw of a die produces a single realization of a random variable, a random signal is a single realization of a stochastic process. Each experiment produces a different time signal or realization of the process. Conceptually, the stochastic process is the ensemble of all possible realizations.

In reality, most signals fall between these two extremes. Often, a signal may be deterministic but there may not be enough information to predict it. In these cases, it may be necessary to treat the deterministic signal as if it were a single realization of some underlying stochastic process.

1.1.3 Stationary and Ergodic Signals

The statistical properties of a random variable, such as its mean and variance, are determined by integrating the probability distribution function (PDF) over all possible range values. Thus, if $f(x)$ is the PDF of a random variable x, its mean and variance are given by

$$\mu_x = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$\sigma_x^2 = \int_{-\infty}^{\infty} (x - \mu_x)^2 f(x)\, dx$$

Similar integrals are used to compute higher-order moments. Conceptually, these integrals can be viewed as averages taken over an infinite ensemble of all possible realizations of the random variable, x.

The value of a random signal at a point in time, considered as a random variable, will have a PDF, $f(x, t)$, that depends on the time, t. Thus, any statistic obtained by integrating over the PDF will be a function of time. Alternately, the integrals used to compute the statistics can be viewed as averages taken over an infinite ensemble of realizations of the stochastic process, at a particular point in time. If the PDF, and hence statistics, of a stochastic process is independent of time, then the process is said to be *stationary*.

For many practical applications, only a single realization of a stochastic process will be available; therefore, averaging must be done over time rather than over an ensemble of realizations. Thus, the mean of a stochastic process would be estimated as

$$\hat{\mu}_x = \frac{1}{T} \int_0^T x(t)\, dt$$

Many stochastic process are *ergodic*, meaning that the ensemble and time averages are equal.

1.2 SYSTEMS AND MODELS

Figure 1.1 shows a block diagram of a system in which the "black box," $\mathbf{N}$, transforms the input signal, $u(t)$, into the output $y(t)$. This will be written as

$$y(t) = \mathbf{N}(u(t)) \tag{1.1}$$

to indicate that when the input $u(t)$ is applied to the system $\mathbf{N}$, the output $y(t)$ results. Note that the domain of the signals need not be time, as shown here. For example, if the system operates on images, the input and output domains could be two- or three-dimensional spatial coordinates.

This book will focus mainly on single-input single-output (SISO) systems whose domain is time. Thus $u(t)$ and $y(t)$ will be single-valued functions of t. For multiple-input

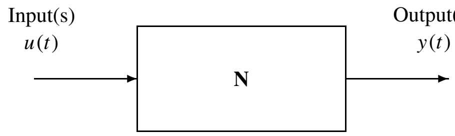

Figure 1.1 Block diagram of a "black box" system, which transforms the input(s) $u(t)$, into the output(s), $y(t)$. The mathematical description of the transformation is represented by the operator **N**.

multiple-output (MIMO) systems, Figure 1.1, equation (1.1), and most of the development to follow will not change; the input and output simply become vector-valued functions of their domains. For example, a multidimensional input signal may be written as a time-dependent vector,

$$\mathbf{u}(t) = \left[u_1(t)\ u_2(t)\ \ldots\ u_n(t)\right] \tag{1.2}$$

1.2.1 Model Structure and Parameters

Using **M** to indicate a mathematical model of the physical system, **N**, the model output can be written as

$$\hat{y}(t) = \mathbf{M}(u(t)) \tag{1.3}$$

where the caret, or "hat," indicates that $\hat{y}(t)$ is an estimate of the system output, $y(t)$.

In general, a model will depend on a set of parameter parameters contained in the *parameter vector* $\boldsymbol{\theta}$. For example, if the model, $\mathbf{M}(\boldsymbol{\theta})$, was a third-degree polynomial,

$$\begin{aligned}\hat{y}(\boldsymbol{\theta}, t) &= \mathbf{M}(\boldsymbol{\theta}, u(t)) \\ &= c^{(0)} + c^{(1)}u(t) + c^{(2)}u^2(t) + c^{(3)}u^3(t)\end{aligned} \tag{1.4}$$

the parameter vector, $\boldsymbol{\theta}$, would contain the polynomial coefficients,

$$\boldsymbol{\theta} = \left[c^{(0)}\ c^{(1)}\ c^{(2)}\ c^{(3)}\right]^T$$

Note that in equation (1.4) the dependence of the output, $\hat{y}(\boldsymbol{\theta}, t)$, on the parameter vector, $\boldsymbol{\theta}$, is shown explicitly.

Models are often classified as being either *parametric* or *nonparametric*. A parametric model generally has relatively few parameters that often have direct physical interpretations. The polynomial in equation (1.4) is an example of a parametric model. The model structure comprises the constant, linear, quadratic and third-degree terms; the parameters are the coefficients associated with each term. Thus each parameter is related to a particular behavior of the system; for example, the parameter $c^{(2)}$ defines how the output varies with the square of the input.

In contrast, a nonparametric model is described by a curve or surface defined by its values at a collection of points in its domain, as illustrated in Figure 1.2. Thus, a set of samples of the curve defined by equation (1.4) would be a nonparametric model of the same system. Here, the model structure would contain the domain values, and the "parameters" would be the corresponding range values. Thus, a *nonparametric* model usually has a large number of parameters that do not in themselves have any direct physical interpretation.

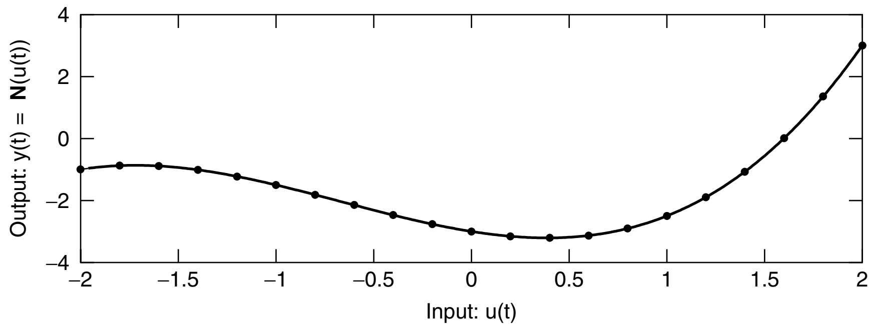

Figure 1.2 A memoryless nonlinear system. A parametric model of this system is $y(t) = -3 - u(t) + u^2(t) - 0.5u^3(t)$. A nonparametric model of the same system could include a list of some of the domain and range values, say those indicated by the dots. The entire curve is also a nonparametric model of the system. While the parametric model is more compact, the nonparametric model is more flexible.

1.2.2 Static and Dynamic Systems

In a static, or *memoryless*, system, the current value of the output depends only on the current value of the input. For example, a full-wave rectifier is a static system since its output, $y(t) = |u(t)|$, depends only on the instantaneous value of its input, $u(t)$.

On the other hand, in a *dynamic* system, the output depends on some or all of the input history. For example, the output at time t of the delay operator,

$$y(t) = u(t - \tau)$$

depends only on the value of the input at the previous time, $t - \tau$.

In contrast, the output of the peak-hold operation

$$y(t) = \max_{\tau \leq t}(u(\tau))$$

retains the largest value of the past input and consequently depends on the entire history of the input.

Dynamic systems can be further classified according to whether they respond to the past or future values of the input, or both. The delay and peak-hold operators are both examples of *causal* systems, systems whose outputs depend on previous, but not future, values of their inputs. Systems whose outputs depend only on future values of their inputs are said to be *anti-causal* or *anticipative*. If the output depends on both the past and future inputs, the system said to be *noncausal* or *mixed causal anti-causal*.

Although physical systems are causal, there are a number of situations where noncausal system descriptions are needed. For example, behavioral systems may display a predictive ability if the input signal is deterministic or a preview is available. For example, the dynamics of a tracking experiment may show a noncausal component if the subject is permitted to see future values of the input as well as its current value.

Sometimes, feedback can produce behavior that appears to be noncausal. Consider the system in Figure 1.3. Suppose that the experimenter can measure the signals labeled $u(t)$ and $y(t)$, but not $w_1(t)$ and $w_2(t)$. Let both $\mathbf{N_1}$ and $\mathbf{N_2}$ be causal systems that include delays. The effect of $w_1(t)$ will be measured first in the "input," $u(t)$, and then later in the

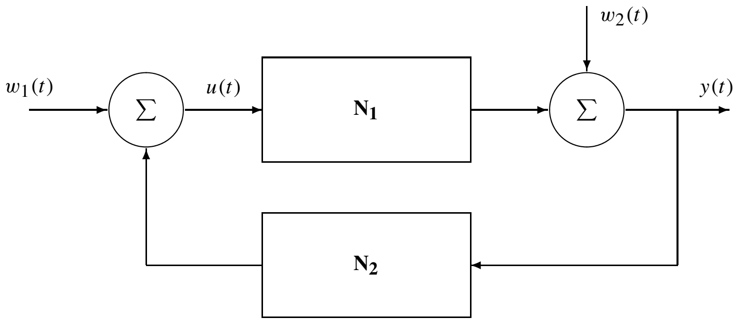

Figure 1.3 A feedback loop with two inputs. Depending on the relative power of the inputs $w_1(t)$ and $w_2(t)$, the system $\mathbf{N_1}$, or rather the relationship between $u(t)$ and $y(t)$, may appear to be either causal, anti-causal, or noncausal.

"output," $y(t)$. However, the effect of the other input, $w_2(t)$, will be noted in $y(t)$ first, followed by $u(t)$. Thus, the delays in the feedback loop create what appears to be noncausal system behavior. Of course the response is not really noncausal, it merely appears so because neither $u(t)$ nor $y(t)$ was directly controlled. Thus, inadequate experimental design can lead to the appearance of noncausal relationships between signals.

In addition, as will be seen below, there are cases where it is advantageous to reverse the roles of the input and output. In the resulting analysis, a noncausal system description must be used to describe the inverse system.

1.2.3 Linear and Nonlinear Systems

Consider a system, **N**, and let $y(t)$ be the response of the system due to the input $u(t)$. Thus,

$$y(t) = \mathbf{N}(u(t))$$

Let c be a constant scalar. Then if the response to the input $c \cdot u(t)$ satisfies

$$\mathbf{N}(c \cdot u(t)) = c \cdot y(t) \tag{1.5}$$

for any constant c, the system is said to obey the principle of *proportionality* or to have the *scaling* property.

Consider two pairs of inputs and their corresponding outputs,

$$y_1(t) = \mathbf{N}(u_1(t))$$
$$y_2(t) = \mathbf{N}(u_2(t))$$

If the response to the input $u_1(t) + u_2(t)$ is given by

$$\mathbf{N}(u_1(t) + u_2(t)) = y_1(t) + y_2(t) \tag{1.6}$$

then the operator **N** is said to obey the *superposition* property. Systems that obey both superposition and scaling are said to be *linear*.

Nonlinear systems do not obey superposition and scaling. In many cases, a system will obey the superposition and scaling properties approximately, provided that the inputs lie within a restricted class. In such cases, the system is said to be operating within its "linear range."

1.2.4 Time-Invariant and Time-Varying Systems

If the relationship between the input and output does not depend on the absolute time, then the system is said to be *time-invariant*. Thus, if $y(t)$ is the response to the input $u(t)$ generated by a time-invariant system, its response due to $u(t-\tau)$, for any real τ, will be $y(t-\tau)$. Thus, a time-invariant system must satisfy

$$\mathbf{N}(u(t)) = y(t) \Rightarrow \mathbf{N}(u(t-\tau)) = y(t-\tau) \qquad \forall \tau \in \mathbb{R} \tag{1.7}$$

Systems for which equation (1.7) does not hold are said to be *time-varying*.

1.2.5 Deterministic and Stochastic Systems

In a deterministic system, the output, $y(t)$, depends only on the input, $u(t)$. In many applications, the output measurement is corrupted by additive noise,

$$z(t) = y(t) + v(t) = \mathbf{N}(u(t)) + v(t) \tag{1.8}$$

where $v(t)$ is independent of the input, $u(t)$. Although the measured output, $z(t)$, has both deterministic and random components, the system (1.8) is still referred to as deterministic, since the "true" output, $y(t)$, is a deterministic function of the input.

Alternatively, the output may depend on an unmeasurable process disturbance, $w(t)$,

$$y(t) = \mathbf{N}(u(t), w(t)) \tag{1.9}$$

where $w(t)$ is a white, Gaussian signal that cannot be measured. In this case, the system is said to be *stochastic*, since there is no "noise free" deterministic output. The process noise term, $w(t)$, can be thought of as an additional input driving the dynamics of the system. Measurement noise, in contrast, only appears additively in the final output. Clearly, it is possible for a system to have both a process disturbance and measurement noise, as illustrated in Figure 1.4, leading to the relation

$$z(t) = y(t) + v(t) = \mathbf{N}(u(t), w(t)) + v(t) \tag{1.10}$$

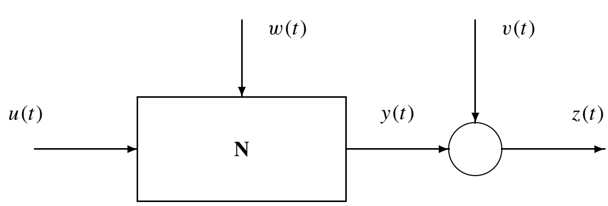

Figure 1.4 Block diagram of a system including a process disturbance, $w(t)$, and measurement noise, $v(t)$.

1.3 SYSTEM MODELING

In many cases, a mathematical model of a system can be constructed from "first principles." Consider, for example, the problem of modeling a spring. As a first approximation, it might be assumed to obey Hooke's law and have no mass so that it could be described by

$$y = -ku \tag{1.11}$$

where the output, y, is the force produced, the input, u, is the displacement, and k is the spring constant. If the spring constant were known, then equation (1.11) would constitute a mathematical model of the system. If the spring constant, k, was unknown, it could be estimated experimentally. Whether or not the assumptions hold, equation (1.11) is a model of the system (but not necessarily a good model). If it yields satisfactory predictions of the system's behavior, then, and only then, can it be considered to be a good model. If it does not predict well, then the model must be refined, perhaps by considering the mass of the spring and using Newton's second law to give

$$y(t) = -ku(t) + m\frac{d^2u(t)}{dt^2} \tag{1.12}$$

Other possibilities abound; the spring might be damped, behave nonlinearly, or have significant friction. The art of system modeling lies in determining which terms are likely to be significant, and in limiting the model to relevant terms only. Thus, even in this simple case, constructing a mathematical model based on "first principles" can become unwieldy. For complex systems, the approach can become totally unmanageable unless there is a good understanding of which effects should and should not be incorporated into the model.

1.4 SYSTEM IDENTIFICATION

The system identification approach to constructing a mathematical model of the system is much different. It assumes a general form, or structure, for the mathematical model and then determines the parameters from experimental data. Often, a variety of model structures are evaluated, and the most successful ones are retained. For example, consider the spring system described in the previous section. If it were assumed to be linear, then a linear differential equation model, such as

$$\begin{aligned}\frac{d^n y(t)}{dt^n} &+ a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}} + \cdots + a_1\frac{dy(t)}{dt} + a_0 y(t)\\ &= b_m\frac{d^m u(t)}{dt^m} + b_{m-1}\frac{d^{m-1}u(t)}{dt^{m-1}} + \cdots + b_1\frac{du(t)}{dt} + b_0 u(t)\end{aligned} \tag{1.13}$$

could be postulated. It would then be necessary to perform an experiment, record $u(t)$ and $y(t)$, compute their derivatives, and determine the coefficients $a_0 \ldots a_{n-1}$ and $b_0 \ldots b_m$. Under ideal conditions, many of the coefficients would be near zero and could be removed from the model. Thus, if the system could be described as a massless linear spring, then equation (1.13) would reduce to equation (1.11) once all extraneous terms were removed.

The scheme outlined in the previous paragraph is impractical for a number of reasons. Most importantly, numerical differentiation amplifies high-frequency noise. Thus, the numerically computed derivatives of the input and output, particularly the high-order derivatives, will be dominated by high-frequency noise that will distort the parameter estimates. Thus, a more practical approach to estimating the system dynamics from input–output measurements is required.

First, note that a system need not be represented as a differential equation. There are many possible parametric and nonparametric representations or model structures for both linear and nonlinear systems. Parameters for many of these model structures can be estimated reliably from measured data. In general, the model structure will be represented by an operator, **M**, having some general mathematical form capable of representing a wide variety of systems. The model itself will depend on a list of parameters, the vector $\boldsymbol{\theta}$. From this viewpoint, the system output may be written as

$$y(t, \boldsymbol{\theta}) = \mathbf{M}(\boldsymbol{\theta}, u(t)) \tag{1.14}$$

where it is assumed that the model structure, **M**, and parameter vector, $\boldsymbol{\theta}$, *exactly* represent the physical system. Thus, the physical system, **N**, can be replaced with an exact model, $\mathbf{M}(\boldsymbol{\theta})$.

The objective of system identification is to find a suitable model structure, **M**, and corresponding parameter vector, $\boldsymbol{\theta}$, given measurements of the input and output. Then, the identified model will have a parameter vector, $\hat{\boldsymbol{\theta}}$, and generate

$$\hat{y}(t) = \mathbf{M}(\hat{\boldsymbol{\theta}}, u(t)) \tag{1.15}$$

where $\hat{y}(t)$ is an estimate of the system output, $y(t)$. Similarly, $\mathbf{M}(\hat{\boldsymbol{\theta}}, u(t))$ represents the model structure chosen together with a vector of estimated parameters. The system identification problem is then to choose the model structure, **M**, and find the corresponding parameter vector, $\hat{\boldsymbol{\theta}}$, that produces the model output, given by equation (1.15), that best predicts the measured system output.

Often, instead of having the system output, $y(t)$, only a noise corrupted measurement will be available. Usually, this measurement noise is assumed to be additive, random, and statistically independent of the system's inputs and outputs. The goal, then, is to find the model, $\mathbf{M}(\hat{\boldsymbol{\theta}}, u(t))$, whose output, $\hat{y}(t, \hat{\boldsymbol{\theta}})$, "best approximates" the measured output, $z(t)$. The relationship between the system, model, and the various signals, is depicted in Figure 1.5.

1.4.1 Types of System Identification Problems

Figure 1.6 gives a more complete view of a typical system identification experiment. First, the desired test input, labeled $\mu(t)$, is applied to an actuator. In some applications, such as the study of biomechanics, the actuator dynamics may restrict the types of test input which can be applied. In addition, the actuator may be influenced by the noise term, $n_1(t)$. Thus, instead of using the desired input, $\mu(t)$, in the identification, it is desirable to measure the actuator output, $u(t)$, and use it as the input instead. Note, however, that measurements of the input, $\hat{u}(t)$, may contain noise, $n_2(t)$.

Many system identification methods are based, either directly or indirectly, on solving an ordinary least-squares problem. Such formulations are well suited to dealing with

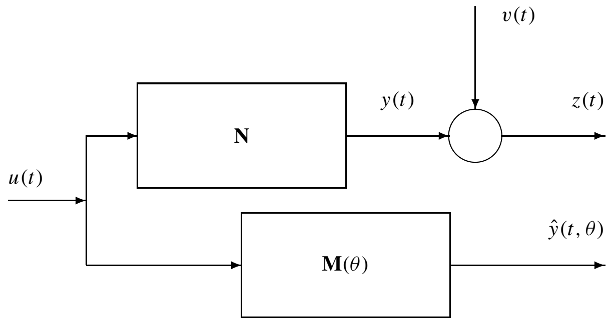

Figure 1.5 The deterministic system identification problem in the presence of measurement noise.

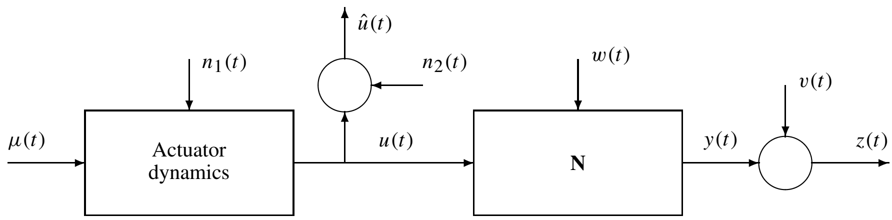

Figure 1.6 A more realistic view of the system being identified, including the actuator, which transforms the ideal input, μ, into the applied input, $u(t)$, which may contain the effects of the process noise term, $n_1(t)$. Furthermore, the measured input, $\hat{u}(t)$, may contain noise, $n_2(t)$. As before, the plant may be affected by process noise, $w(t)$, and the output may contain additive noise, $v(t)$.

noise in the output signals. However, to deal with noise at the input, it is necessary to adopt a "total least-squares" or "errors in the variables" framework, both of which are much more computationally demanding. To avoid this added complexity, identification experiments are usually designed to minimize the noise in the input measurements. In some cases, it may be necessary to adopt a noncausal system description so that the measurement with the least noise may be treated as the input. Throughout this book it will be assumed that $n_2(t)$ is negligible, unless otherwise specified.

The system may also include an unmeasurable process noise input, $w(t)$, and the measured output may also contain additive noise, $v(t)$. Given this framework, there are three broad categories of system identification problem:

- *Deterministic System Identification Problem.* Find the relationship between $u(t)$ and $y(t)$, assuming that the process noise, $w(t)$, is zero. The measured output, $z(t)$, may contain additive noise, $v(t)$. The identification of deterministic systems is generally pursued with the objective of gaining insight into the system function and is the problem of primary interest in this text.
- *Stochastic System Identification Problem.* Find the relationship between $w(t)$ and $y(t)$, given only the system output, $z(t)$, and assumptions regarding the statistics of $w(t)$. Usually, the exogenous input, $u(t)$, is assumed to be zero or constant. This formulation is used where the inputs are not available to the experimenter, or

where it is not evident which signals are inputs and which are outputs. The myriad approaches to this problem have been reviewed by Brillinger (1975) and Caines (1988).

- *Complete System Identification Problem.* Given both the input and the output, estimate both the stochastic and deterministic components of the model. This problem formulation is used when accurate output predictions are required, for example in model-based control systems (Ljung, 1999; Söderström and Stoica, 1989).

1.4.2 Applications of System Identification

There are two general areas of application for models produced by system identification that will be referred to as "control" and "analysis."

In "control" applications, the identified model will be used to design a controller, or perhaps be embedded in a control system. Here, the chief requirements are that the model be compact and easy to manipulate, so that it produces output predictions with little computational overhead. Many control applications use the model "online" to predict future outputs from the histories of both the input and output. Such predictions commonly extend only one time-step into the future. At each time-step the model uses the previous output measurement to correct its estimate of the model's trajectory. Such *one-step-ahead* predictions are often all that is required of a control model. As a result, low-order, linear parametric models of the complete system (i.e., both stochastic and deterministic parts) are often adequate. Since the model's output trajectory is corrected at each sample, the model need not be very accurate. The effects of missing dynamics, or weak nonlinearities, can usually be removed by modeling them as process noise. Similarly, more severe nonlinearities can handled using an adaptive, time-varying linear model. Here, the measured output is used to correct the model by varying its parameters on-line, to track gradual changes in the linearized model.

In "analysis" applications the model is usually employed for *off-line* simulation of the system to gain insight into its functioning. For these applications, the model must be simulated as a *free run*—that is, without access to past output measurements. With no access to prediction errors, and hence no means to reconstruct process noise, the model must be entirely deterministic. Moreover, without the recursive corrections used in on-line models, an off-line model must be substantially more accurate than the on-line models typically used in control applications. Thus, in these applications it is critical for the nonlinearities to be described exactly. Moreover, since simulations are done off-line, there is less need to minimize the mathematical/arithmetic complexity of the model and consequently large, nonlinear models may be employed.

1.5 HOW COMMON ARE NONLINEAR SYSTEMS?

Many physiological systems are highly nonlinear. Consider, for example, a single joint and its associated musculature. First, the neurons that transmit signals to and from the muscles fire with an "all or nothing" response. The geometry of the tendon insertions is such that lever arms change with joint angle. The muscle fibers themselves have nonlinear force–length and force–velocity properties as well as being only able exert force in one direction. Nevertheless, this complex system is often represented using a simple linear model.

In many biomedical engineering applications, the objective of an identification experiment is to gain insight into the functioning of the system. Here, the nonlinearities may play a crucial role in the internal functioning of the system. While it may be possible to linearize the system about one or more operating points, linearization will discard important information about the nonlinearities. Thus, while a controller may perform adequately using a linearized model, the model would provide little insight into the functional organization of the system. Thus, in biomedical applications, it is both common and important to identify nonlinear systems explicitly.

For these reasons, nonlinear system analysis techniques have been applied to a wide variety of biomedical systems. Some of these applications include:

- *Sensory Systems.* These include primitive sensory organs such as the cockroach tactile spine (French and Korenberg, 1989, 1991; French and Marmarelis, 1995; French and Patrick, 1994; French et al., 2001), as well as more evolved sensors such as the auditory system (Carney and Friedman, 1996; Eggermont, 1993; Shi and Hecox, 1991) and the retina (Citron et al., 1988; Juusola et al., 1995; Korenberg and Paarmann, 1989; Naka et al., 1988; Sakuranaga et al., 1985a).
- *Reflex Loops.* Nonlinear system identification techniques have been used to study reflex loops in the control of limb (Kearney and Hunter, 1988; Kearney et al., 1997; Westwick and Kearney, 2001; Zhang and Rymer, 1997) and eye position (the vestibulo-ocular reflex) (Galiana et al., 1995, 2001).
- *Organ Systems.* Similarly, nonlinear feedback loops have been investigated in models of heart rate variability (Chon et al., 1996) and in renal auto-regulation (Chon et al., 1998; Marmarelis et al., 1993, 1999).
- *Tissue Mechanics.* Biological tissues themselves can exhibit nonlinearities. Strips of lung tissue (Maksym et al., 1998; Yuan et al., 1999) and the whole lung (Maksym and Bates, 1997; Zhang et al., 1999) have been shown to include nonlinearities. Skeletal muscle (Crago, 1992; Hunt et al., 1998; Munih et al., 2000) also has strongly nonlinear behavior.

Given the prevalence of nonlinearities in physiological systems, along with the requirement in many biomedical engineering applications to deal explicitly with those nonlinearities, the need for nonlinear system identification methods is clear. Sample applications included in Chapters 6–8 will present results from some of the studies cited above.