IDENTIFICATION OF NONLINEAR PHYSIOLOGICAL SYSTEMS

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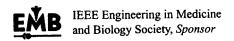
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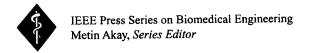
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IDENTIFICATION OF NONLINEAR PHYSIOLOGICAL SYSTEMS

DAVID T. WESTWICK ROBERT E. KEARNEY









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PREFACE

Since it first appeared in 1978, Advanced Methods in Physiological Modeling: The White Noise Approach by P. Z. Marmarelis and M. Z. Marmarelis has been the standard reference for the field of nonlinear system identification, especially as applied in biomedical engineering and physiology. Despite being long out of print, Marmarelis and Marmarelis is still, in many cases, the primary reference. Over the years, dramatic advances have been made in the field, many of which became practical only with the advent of widespread computing power. Many of these newer developments have been described in the three volumes of the series Advanced Methods in Physiological Modeling, edited by V. Z. Marmarelis. While these volumes have been an invaluable resource to many researchers, helping them to stay abreast of recent developments, they are all collections of research articles. As a resource for someone starting out in the field, they are somewhat lacking. It is difficult for a newcomer to the field to see the relationships between myriad contributions. Choosing which approach is best for a given application can be an arduous task, at best.

This textbook developed out of a review article (Westwick and Kearney, 1998) on the same subject. The goal of the review article was to bring the various analyses that have been developed by several groups of researchers into a common notation and framework, and thus to elucidate the relationships between them. The aim of this book was to go one step farther and to provide this common framework along with the background necessary to bring the next generation of systems physiologists into the fold.

In this book, we have attempted to provide the student with an overview of many of the techniques currently in use, and some of the earlier methods as well. Everything is presented in a common notation and from a consistent theoretical framework. We hope that the relationships between the methods and their relative strengths and weaknesses will become apparent to the reader. The reader should be well-equipped to make an informed decision as to which techniques to try, when faced with an identification or modeling problem.

xii PREFACE

We have assumed that readers of this book have a background in linear signals and systems equivalent to that given by a junior year signals and systems course. Background material beyond that level is summarized, with references given to more detailed, pedagogical treatments.

Each chapter has several theoretical problems, which can be solved with pencil and paper. In addition, most of the chapters conclude with some computer exercises. These are intended to give the reader practical experience with the tools described in the text. These computer exercises make use of MATLAB®* and the nonlinear system identification (NLID) toolbox (Kearney and Westwick, 2003). More information regarding the NLID toolbox can be found at www.bmed.mcgill.ca. In addition to implementing all of the system identification tools as MATLAB m-files, the toolbox also contains the data and model structures used to generate the examples that run throughout the text.

Although our primary goal is to educate informed users of these techniques, we have included several theoretical sections dealing with issues such as the generality of some model structures, convergence of series-based models, and so on. These sections are marked with a dagger, †, and they can be skipped by readers interested primarily in practical application of these methods, with little loss in continuity.

The dedication in Marmarelis and Marmarelis reads "To an ambitious breed: Systems Physiologists." We feel that the sentiment reflected in those words is as true today as it was a quarter century ago. The computers are (much) faster, and they will undoubtedly be faster still in a few years. As a result, the problems that we routinely deal with today would have been inconceivable when M & M was first published. However, with increased computational abilities come more challenging problems. No doubt, this trend will continue. We hope that it is an interesting ride.

DAVID T. WESTWICK ROBERT E. KEARNEY

Calgary, Alberta, Canada Montreal, Quebec, Canada May, 2003

^{*}MATLAB is a registered trademark of the MathWorks, Inc.