

IDENTIFICATION OF NONLINEAR PHYSIOLOGICAL SYSTEMS

IEEE Press Series in Biomedical Engineering

The focus of our series is to introduce current and emerging technologies to biomedical and electrical engineering practitioners, researchers, and students. This series seeks to foster interdisciplinary biomedical engineering education to satisfy the needs of the industrial and academic areas. This requires an innovative approach that overcomes the difficulties associated with the traditional textbook and edited collections.

Metin Akay, *Series Editor*
Dartmouth College

Advisory Board

Thomas Budinger
Ingrid Daubechies
Andrew Daubenspeck
Murray Eden
James Greenleaf

Simon Haykin
Murat Kunt
Paul Lauterbur
Larry McIntire
Robert Plonsey

Richard Robb
Richard Satava
Malvin Teich
Herbert Voigt
Lotfi Zadeh

Editorial Board

Eric W. Abel
Dan Adam
Peter Adlassing
Berj Bardakjian
Erol Basar
Katarzyna Blinowska
Bernadette Bouchon-Meunier
Tom Brotherton
Eugene Bruce
Jean-Louis Coatrieux
Sergio Cerutti
Maurice Cohen
John Collier
Steve Cowin
Jerry Daniels
Jaques Duchene
Walter Greenleaf
Daniel Hammer
Dennis Healy

Gabor Herman
Helene Hoffman
Donna Hudson
Yasemin Kahya
Michael Khoo
Yongmin Kim
Andrew Laine
Rosa Lancini
Swamy Laxminarayan
Richard Leahy
Zhi-Pei Liang
Jennifer Linderman
Richard Magin
Jaakko Malmivuo
Jorge Monzon
Michael Neuman
Banu Onaral
Keith Paulsen
Peter Richardson

Kris Ropella
Joseph Rosen
Christian Roux
Janet Rutledge
Wim L. C. Rutten
Alan Sahakian
Paul S. Schenker
G. W. Schmid-Schönbein
Ernest Stokely
Ahmed Tewfik
Nitish Thakor
Michael Unser
Eugene Veklerov
Al Wald
Bruce Wheeler
Mark Wiederhold
William Williams
Andy Yagle
Yuan-Ting Zhang

A list of books in the IEEE Press Series in Biomedical Engineering can be found on page 262.

IDENTIFICATION OF NONLINEAR PHYSIOLOGICAL SYSTEMS

DAVID T. WESTWICK
ROBERT E. KEARNEY



IEEE Engineering in Medicine
and Biology Society, *Sponsor*



IEEE Press Series on Biomedical Engineering
Metin Akay, *Series Editor*



IEEE PRESS



A JOHN WILEY & SONS, INC., PUBLICATION

IEEE Press
445 Hoes Lane
Piscataway, NJ 08854

IEEE Press Editorial Board
Stamatis V. Kartalopoulos, *Editor in Chief*

M. Ajay	R. J. Herrick	M. S. Newman
J. B. Anderson	R. F. Hoyt	M. Padgett
J. Baker	D. Kirk	W. D. Reeve
J. E. Brewer	R. Leonardi	S. Tewksbury
M. E. El-Hawary		G. Zobrist

Kenneth Moore, *Director of IEEE Press*
Catherine Faduska, *Senior Acquisitions Editor*
Christina Kuhnen, *Associate Acquisitions Editor*

IEEE Engineering in Medicine and Biology Society, *Sponsor*
EMB-S Liaison to IEEE Press, Metin Akay

Technical Reviewers

Metin Akay
Robert F. Kirsch
John A. Daubenspeck

Copyright © 2003 by the Institute of Electrical and Electronics Engineers. All rights reserved.

Published simultaneously in Canada.

No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning, or otherwise, except as permitted under Section 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, 978-750-8400, fax 978-750-4470, or on the web at www.copyright.com. Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, (201) 748-6011, fax (201) 748-6008, e-mail: permreq@wiley.com.

Limit of Liability/Disclaimer of Warranty: While the publisher and author have used their best efforts in preparing this book, they make no representations or warranties with respect to the accuracy or completeness of the contents of this book and specifically disclaim any implied warranties of merchantability or fitness for a particular purpose. No warranty may be created or extended by sales representatives or written sales materials. The advice and strategies contained herein may not be suitable for your situation. You should consult with a professional where appropriate. Neither the publisher nor author shall be liable for any loss of profit or any other commercial damages, including but not limited to special, incidental, consequential, or other damages. For general information on our other products and services please contact our Customer Care Department within the U.S. at 877-762-2974, outside the U.S. at 317-572-3993 or fax 317-572-4002. Wiley also publishes its books in a variety of electronic formats. Some content that appears in print, however, may not be available in electronic format.

Library of Congress Cataloging-in-Publication Data:

Westwick, D. T. (David T.)

Identification of nonlinear physiological systems / D.T. Westwick, R.E. Kearney.

p. cm.—(IEEE Press series on biomedical engineering)

“IEEE Engineering in Medicine and Biology Society, Sponsor.”

Includes bibliographical references and index.

ISBN 0-471-27456-9 (cloth)

1. Physiology—Mathematical models. 2. Nonlinear systems. I. Kearney, Robert E., 1947- II. IEEE Engineering in Medicine and Biology Society. III. Title. IV. IEEE Press series in biomedical engineering.

QP33.6.M36W475 2003

612/.01/5118—dc21

2003043255

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

CONTENTS

Preface	xi
1 Introduction	1
1.1 Signals / 1	
1.1.1 Domain and Range / 2	
1.1.2 Deterministic and Stochastic Signals / 2	
1.1.3 Stationary and Ergodic Signals / 3	
1.2 Systems and Models / 3	
1.2.1 Model Structure and Parameters / 4	
1.2.2 Static and Dynamic Systems / 5	
1.2.3 Linear and Nonlinear Systems / 6	
1.2.4 Time-Invariant and Time-Varying Systems / 7	
1.2.5 Deterministic and Stochastic Systems / 7	
1.3 System Modeling / 8	
1.4 System Identification / 8	
1.4.1 Types of System Identification Problems / 9	
1.4.2 Applications of System Identification / 11	
1.5 How Common are Nonlinear Systems? / 11	
2 Background	13
2.1 Vectors and Matrices / 13	
2.2 Gaussian Random Variables / 14	
2.2.1 Products of Gaussian Variables / 15	
2.3 Correlation Functions / 16	

2.3.1	Autocorrelation Functions / 16
2.3.2	Cross-Correlation Functions / 18
2.3.3	Effects of Noise / 20
2.3.4	Estimates of Correlation Functions / 21
2.3.5	Frequency Domain Expressions / 22
2.3.6	Applications / 23
2.3.7	Higher-Order Correlation Functions / 25
2.4	Mean-Square Parameter Estimation / 25
2.4.1	Linear Least-Squares Regression / 26
2.4.2	Properties of Estimates / 27
2.5	Polynomials / 29
2.5.1	Power Series / 29
2.5.2	Orthogonal Polynomials / 30
2.5.3	Hermite Polynomials / 31
2.5.4	Tchebyshev Polynomials / 32
2.5.5	Multiple-Variable Polynomials / 33
2.6	Notes and References / 35
2.7	Problems / 36
2.8	Computer Exercises / 36

3 Models of Linear Systems

39

3.1	Linear Systems / 39
3.2	Nonparametric Models / 40
3.2.1	Time Domain Models / 41
3.2.2	Frequency Domain Models / 43
3.3	Parametric Models / 46
3.3.1	Parametric Frequency Domain Models / 46
3.3.2	Discrete-Time Parametric Models / 48
3.4	State-Space Models / 52
3.4.1	Example: Human Ankle Compliance—Discrete-Time, State-Space Model / 54
3.5	Notes and References / 54
3.6	Theoretical Problems / 55
3.7	Computer Exercises / 56

4 Models of Nonlinear Systems

57

4.1	The Volterra Series / 57
4.1.1	The Finite Volterra Series / 59
4.1.2	Multiple-Input Systems / 62
4.1.3	Polynomial Representation / 64
4.1.4	Convergence Issues ^(†) / 65

- 4.2 The Wiener Series / 67
 - 4.2.1 Orthogonal Expansion of the Volterra Series / 68
 - 4.2.2 Relation Between the Volterra and Wiener Series / 70
 - 4.2.3 Example: Peripheral Auditory Model—Wiener Kernels / 71
 - 4.2.4 Nonwhite Inputs / 73
- 4.3 Simple Block Structures / 73
 - 4.3.1 The Wiener Model / 73
 - 4.3.2 The Hammerstein Model / 77
 - 4.3.3 Sandwich or Wiener–Hammerstein Models / 79
 - 4.3.4 NLN Cascades / 83
 - 4.3.5 Multiple-Input Multiple-Output Block Structured Models / 87
- 4.4 Parallel Cascades / 87
 - 4.4.1 Approximation Issues([†]) / 89
- 4.5 The Wiener–Bose Model / 91
 - 4.5.1 Similarity Transformations and Uniqueness / 92
 - 4.5.2 Approximation Issues([†]) / 94
 - 4.5.3 Volterra Kernels of the Wiener–Bose Model / 94
 - 4.5.4 Wiener Kernels of the Wiener–Bose Model / 95
 - 4.5.5 Relationship to the Parallel Cascade Model / 97
- 4.6 Notes and References / 100
- 4.7 Theoretical Problems / 100
- 4.8 Computer Exercises / 101

5 Identification of Linear Systems

103

- 5.1 Introduction / 103
 - 5.1.1 Example: Identification of Human Joint Compliance / 103
 - 5.1.2 Model Evaluation / 105
- 5.2 Nonparametric Time Domain Models / 107
 - 5.2.1 Direct Estimation / 107
 - 5.2.2 Least-Squares Regression / 108
 - 5.2.3 Correlation-Based Methods / 109
- 5.3 Frequency Response Estimation / 115
 - 5.3.1 Sinusoidal Frequency Response Testing / 115
 - 5.3.2 Stochastic Frequency Response Testing / 116
 - 5.3.3 Coherence Functions / 117
- 5.4 Parametric Methods / 119
 - 5.4.1 Regression / 119
 - 5.4.2 Instrumental Variables / 120
 - 5.4.3 Nonlinear Optimization / 121
- 5.5 Notes and References / 122
- 5.6 Computer Exercises / 122

6 Correlation-Based Methods 125

- 6.1 Methods for Functional Expansions / 125
 - 6.1.1 Lee–Schetzen Cross-Correlation / 125
 - 6.1.2 Colored Inputs / 140
 - 6.1.3 Frequency Domain Approaches / 144
- 6.2 Block-Structured Models / 149
 - 6.2.1 Wiener Systems / 150
 - 6.2.2 Hammerstein Models / 155
 - 6.2.3 LNL Systems / 162
- 6.3 Problems / 167
- 6.4 Computer Exercises / 167

7 Explicit Least-Squares Methods 169

- 7.1 Introduction / 169
- 7.2 The Orthogonal Algorithms / 169
 - 7.2.1 The Orthogonal Algorithm / 171
 - 7.2.2 The Fast Orthogonal Algorithm / 173
 - 7.2.3 Variance of Kernel Estimates / 180
 - 7.2.4 Example: Fast Orthogonal Algorithm Applied to Simulated Fly Retina Data / 182
 - 7.2.5 Application: Dynamics of the Cockroach Tactile Spine / 186
- 7.3 Expansion Bases / 187
 - 7.3.1 The Basis Expansion Algorithm / 190
 - 7.3.2 The Laguerre Expansion / 191
 - 7.3.3 Limits on α / 192
 - 7.3.4 Choice of α and P / 194
 - 7.3.5 The Laguerre Expansion Technique / 195
 - 7.3.6 Computational Requirements / 195
 - 7.3.7 Variance of Laguerre Kernel Estimates / 195
 - 7.3.8 Example: Laguerre Expansion Kernels of the Fly Retina Model / 196
- 7.4 Principal Dynamic Modes / 198
 - 7.4.1 Example: Principal Dynamic Modes of the Fly Retina Model / 200
 - 7.4.2 Application: Cockroach Tactile Spine / 201
- 7.5 Problems / 205
- 7.6 Computer Exercises / 205

8 Iterative Least-Squares Methods 207

- 8.1 Optimization Methods / 207
 - 8.1.1 Gradient Descent Methods / 208

8.1.2	Identification of Block-Structured Models /	209
8.1.3	Second-Order Optimization Methods /	212
8.1.4	Jacobians for Other Block Structures /	216
8.1.5	Optimization Methods for Parallel Cascade Models /	219
8.1.6	Example: Using a Separable Volterra Network /	220
8.2	Parallel Cascade Methods /	223
8.2.1	Parameterization Issues /	226
8.2.2	Testing Paths for Significance /	228
8.2.3	Choosing the Linear Elements /	230
8.2.4	Parallel Wiener Cascade Algorithm /	242
8.2.5	Longer Cascades /	242
8.2.6	Example: Parallel Cascade Identification /	243
8.3	Application: Visual Processing in the Light-Adapted Fly Retina /	246
8.4	Problems /	249
8.5	Computer Exercises /	250
References		251
Index		259
IEEE Press Series in Biomedical Engineering		262

PREFACE

Since it first appeared in 1978, *Advanced Methods in Physiological Modeling: The White Noise Approach* by P. Z. Marmarelis and M. Z. Marmarelis has been the standard reference for the field of nonlinear system identification, especially as applied in biomedical engineering and physiology. Despite being long out of print, Marmarelis and Marmarelis is still, in many cases, the primary reference. Over the years, dramatic advances have been made in the field, many of which became practical only with the advent of widespread computing power. Many of these newer developments have been described in the three volumes of the series *Advanced Methods in Physiological Modeling*, edited by V. Z. Marmarelis. While these volumes have been an invaluable resource to many researchers, helping them to stay abreast of recent developments, they are all collections of research articles. As a resource for someone starting out in the field, they are somewhat lacking. It is difficult for a newcomer to the field to see the relationships between myriad contributions. Choosing which approach is best for a given application can be an arduous task, at best.

This textbook developed out of a review article (Westwick and Kearney, 1998) on the same subject. The goal of the review article was to bring the various analyses that have been developed by several groups of researchers into a common notation and framework, and thus to elucidate the relationships between them. The aim of this book was to go one step farther and to provide this common framework along with the background necessary to bring the next generation of systems physiologists into the fold.

In this book, we have attempted to provide the student with an overview of many of the techniques currently in use, and some of the earlier methods as well. Everything is presented in a common notation and from a consistent theoretical framework. We hope that the relationships between the methods and their relative strengths and weaknesses will become apparent to the reader. The reader should be well-equipped to make an informed decision as to which techniques to try, when faced with an identification or modeling problem.

We have assumed that readers of this book have a background in linear signals and systems equivalent to that given by a junior year signals and systems course. Background material beyond that level is summarized, with references given to more detailed, pedagogical treatments.

Each chapter has several theoretical problems, which can be solved with pencil and paper. In addition, most of the chapters conclude with some computer exercises. These are intended to give the reader practical experience with the tools described in the text. These computer exercises make use of MATLAB®* and the nonlinear system identification (NLID) toolbox (Kearney and Westwick, 2003). More information regarding the NLID toolbox can be found at www.bmed.mcgill.ca. In addition to implementing all of the system identification tools as MATLAB m-files, the toolbox also contains the data and model structures used to generate the examples that run throughout the text.

Although our primary goal is to educate informed users of these techniques, we have included several theoretical sections dealing with issues such as the generality of some model structures, convergence of series-based models, and so on. These sections are marked with a dagger, †, and they can be skipped by readers interested primarily in practical application of these methods, with little loss in continuity.

The dedication in Marmarelis and Marmarelis reads “To an ambitious breed: Systems Physiologists.” We feel that the sentiment reflected in those words is as true today as it was a quarter century ago. The computers are (much) faster, and they will undoubtedly be faster still in a few years. As a result, the problems that we routinely deal with today would have been inconceivable when *M & M* was first published. However, with increased computational abilities come more challenging problems. No doubt, this trend will continue. We hope that it is an interesting ride.

DAVID T. WESTWICK
ROBERT E. KEARNEY

Calgary, Alberta, Canada
Montreal, Quebec, Canada
May, 2003

*MATLAB is a registered trademark of the MathWorks, Inc.