Lab 08: Frequency Response: Bandpass and Nulling Filters

ECE 2610: Intro to Signals and Systems

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Introduction

This lab focuses on the design and implementation of FIR filters to isolate or eliminate specific frequency components from a signal. The two main components to achieve this include nulling filters and bandpass filter design. Nulling filters are used to completely suppress selected frequencies from a complex signal by strategically placing zeros in the filter's frequency response. Bandpass filters, on the other hand, are designed to allow a specific frequency band to pass while zeroing out others. The end goal of the lab was to zero out frequencies with a cascaded filter, then reconstruct the approximated signal with convolution by hand to match the filtered output. The students were also tasked with constructing a bandpass filter and testing its different properties to determine the most accurate filter for nulling out certain frequencies.

Background

Useful Formulas and Theory

$$y[n] = x[n] - 2\cos(\hat{\omega})\, x[n-1] + x[n-2]$$

Equation 1: Nulling Filter

Nulling filters are a type of FIR filter designed to eliminate or "null" specific frequency components from a signal. These filters are particularly useful for the removal of unwanted interference or to isolate a desired frequency range. Unlike bandpass filters that allow a range of frequencies to pass, nulling filters create a zero in the frequency response at the target frequency. When a signal contains a specific frequency component that needs to be eliminated, a nulling filter is designed such that it sets that frequency component to zero, while leaving other parts of the signal intact. To eliminate multiple frequency components, cascaded nulling filters are often used.

$$h[n] = \frac{2}{L}\cos(\hat{\omega}n),\ 0 \le n < L$$

Equation 2: Bandpass Filter

A bandpass filter (BPF) is a type of filter that allows frequencies within a certain range to pass through, while nulling frequencies outside that range. In contrast to lowpass or highpass filters, which only allow frequencies below or above a specific cutoff frequency, a bandpass filter targets a specific range of frequencies, defined by a center point, and L which determines the length of the filer or the bandwidth. The larger L is, the narrower the passband, which means the filter becomes more selective in what frequencies pass. Passband is defined as the range of frequencies where the filter allows signals to pass through with minimal altering (the gain is close to 1). A passband filter is designed to allow the desired frequencies to pass with as little distortion as possible. Stopband is the range of frequencies where the filter almost nulls the signal (the gain is close to 0). The filter is designed to reject these frequencies, often to remove noise or interference. This makes bandpass filters ideal for isolating specific frequency components of a signal that lie within the desired range of the filter. This is especially useful in signal processing when there is a need to focus on a particular frequency range, such as in communications, audio processing, and radio transmission.

$$H(e^{j\hat{\omega}}) = \sum_{n=0}^{L-1} h[n]\, e^{-j\hat{\omega}n}$$

Equation 3: Frequency Response

The frequency response of a filter describes how the filter responds to sinusoidal signals at different frequencies. The magnitude of the frequency response tells how much each frequency component is amplified by the filter. The angle of the frequency provides information about the phase shift introduced by the filter at each frequency.

$$G(\widehat{\omega}) = |H\left(e^{j\widehat{w}}\right)|$$

Equation 4: Gain Equation

The gain of a filter at a specific frequency is defined as the magnitude of the frequency response at that frequency. The gain at a given frequency tells how much the filter amplifies a signal component at that frequency.

Procedure

Task 1- Design a nulling filter to cancel out two frequencies in a summing sinusoid.

Task2- Mathematically derive the new output

Task 3- Plot both theoretical and experimental plots over each other

Task 4- Greate a bandpass filter and compute the gain at 3 distinct frequencies

Task 5- Plot the filter at different lengths

Task 6- Design a bandpass filter that will greatly reduce frequency components at $\omega = 0.3\pi$ and $\omega = 0.7\pi$.

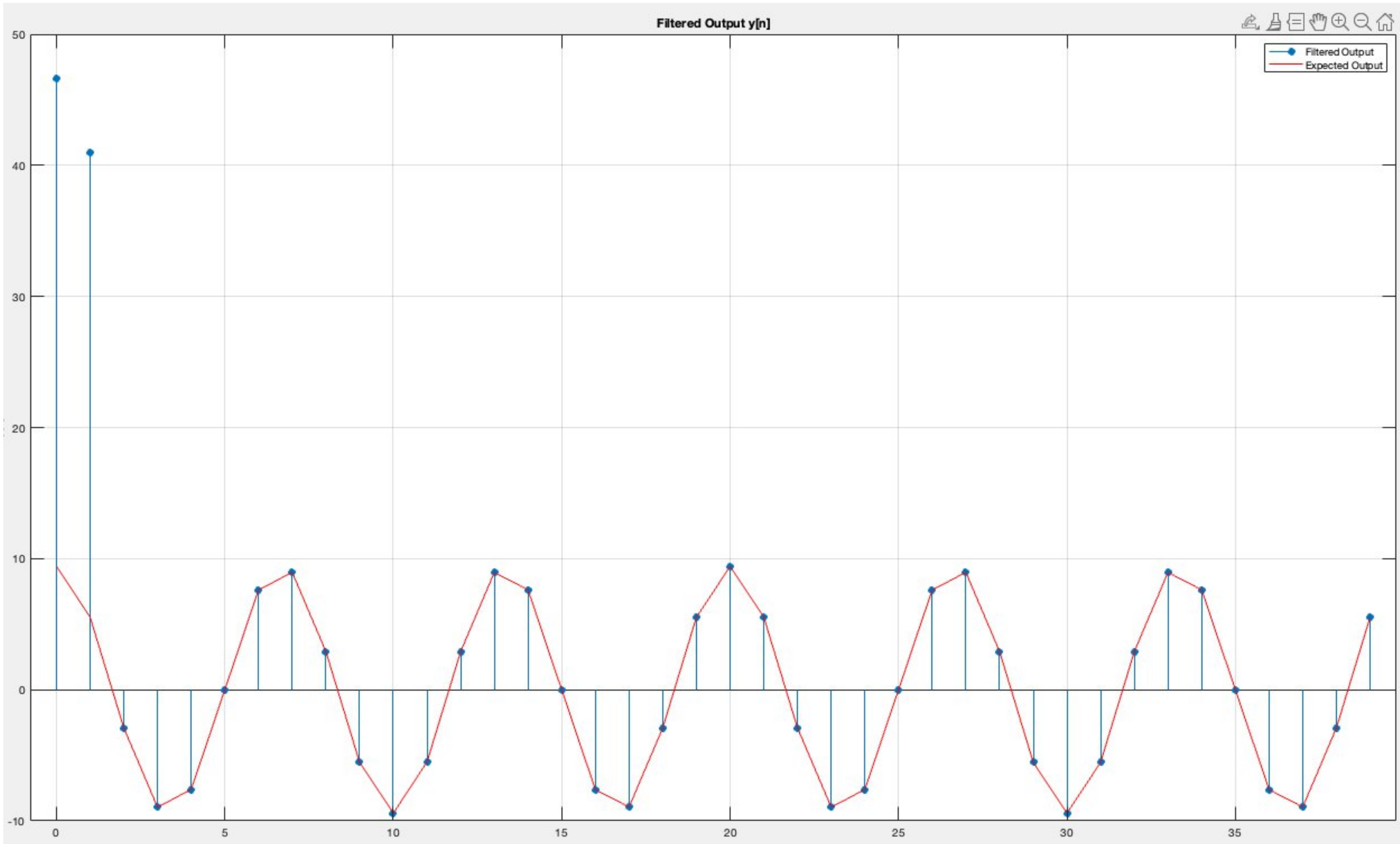
Task 7- Use the filter to null frequencies in the original sum of sinusoids

Task 8- Make a plot of the frequency response for the filter

Results

Filter coefficients- [1,-2cos(.44pi),1], [1,-2cos(.7pi),1]

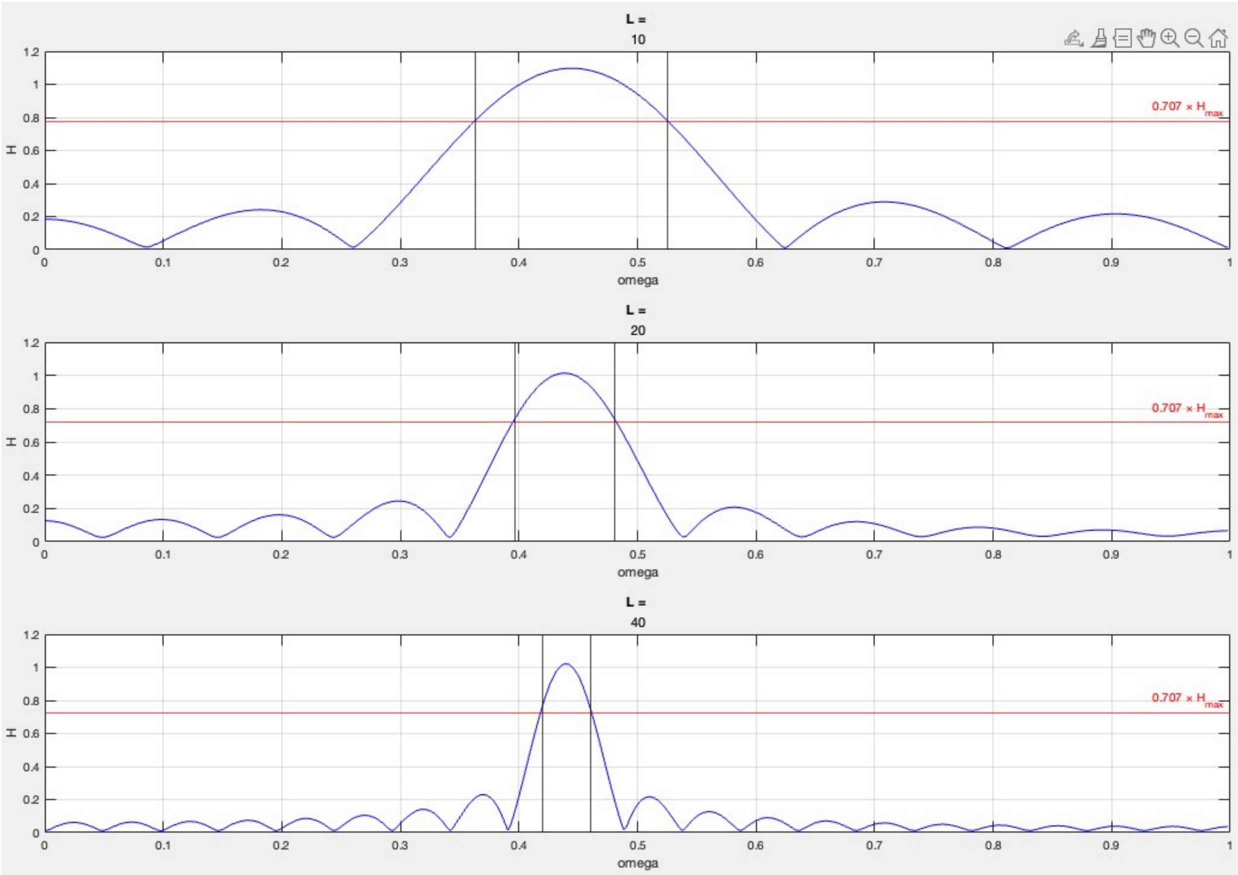
Filtered vs. expected output after nulling filter



Gain at 3 frequencies centered at .44pi

Frequency (rad)	Expected Gain	Measured Gain
0.3pi	Close to 0	0.2744
0.44pi	1	1.0946
0.7pi	Close to 0	0.284

Bandpass Filter at L = 10,20,40

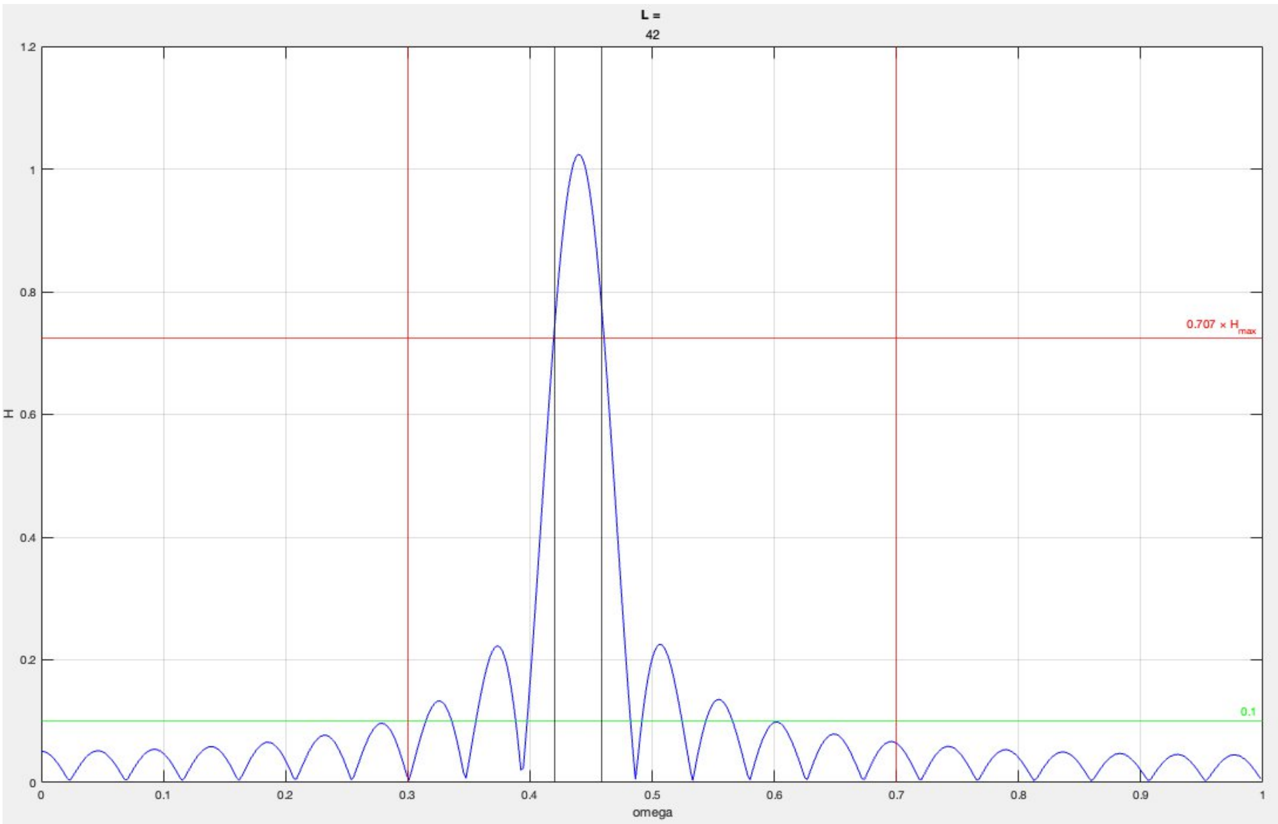


Inverse relationship between filter length and passband width

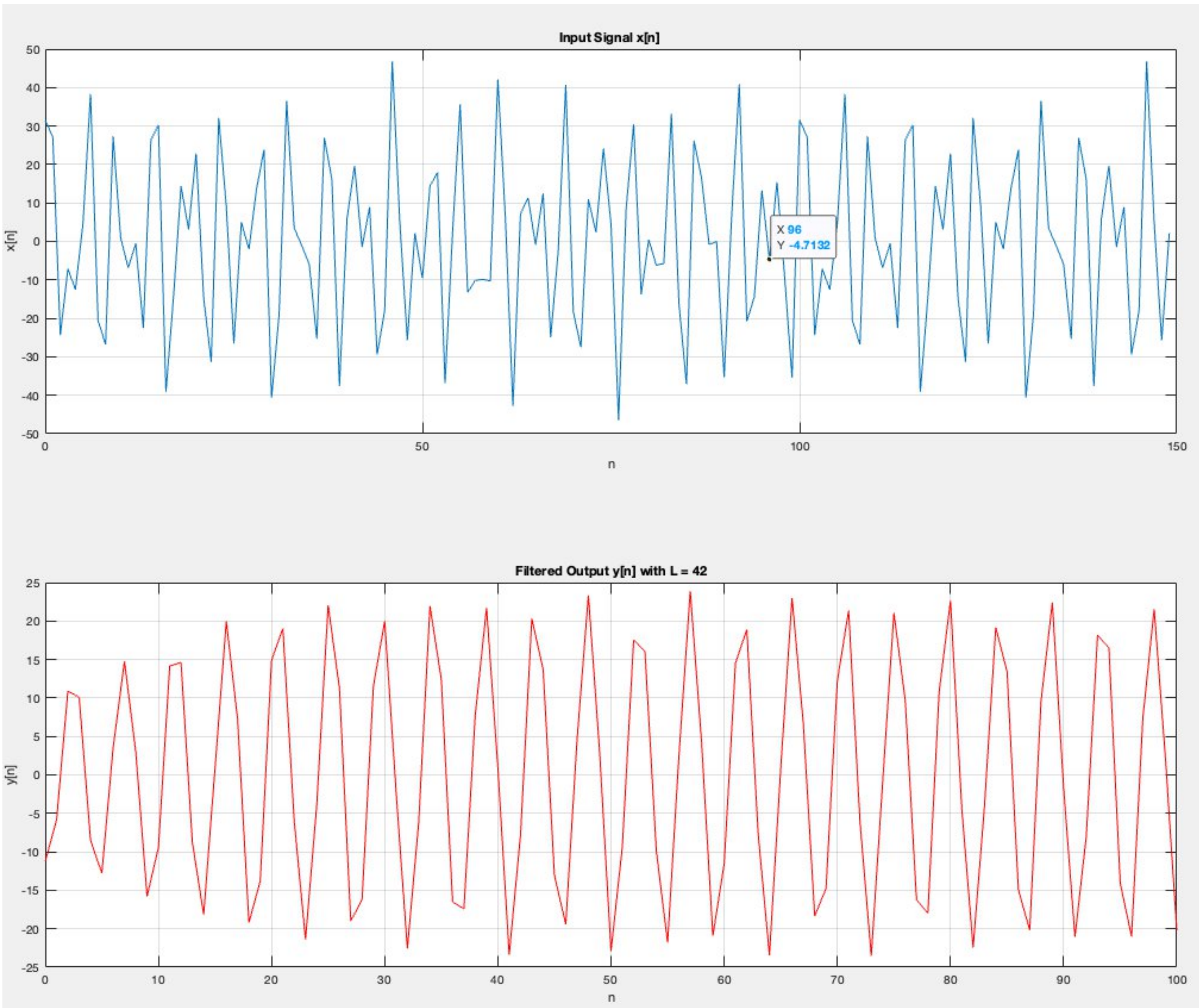
Filter size (L)	Passband width
10	0.1621
20	0.0840
40	0.0410

Smallest L value to reduce .3 and .7 frequency = 42

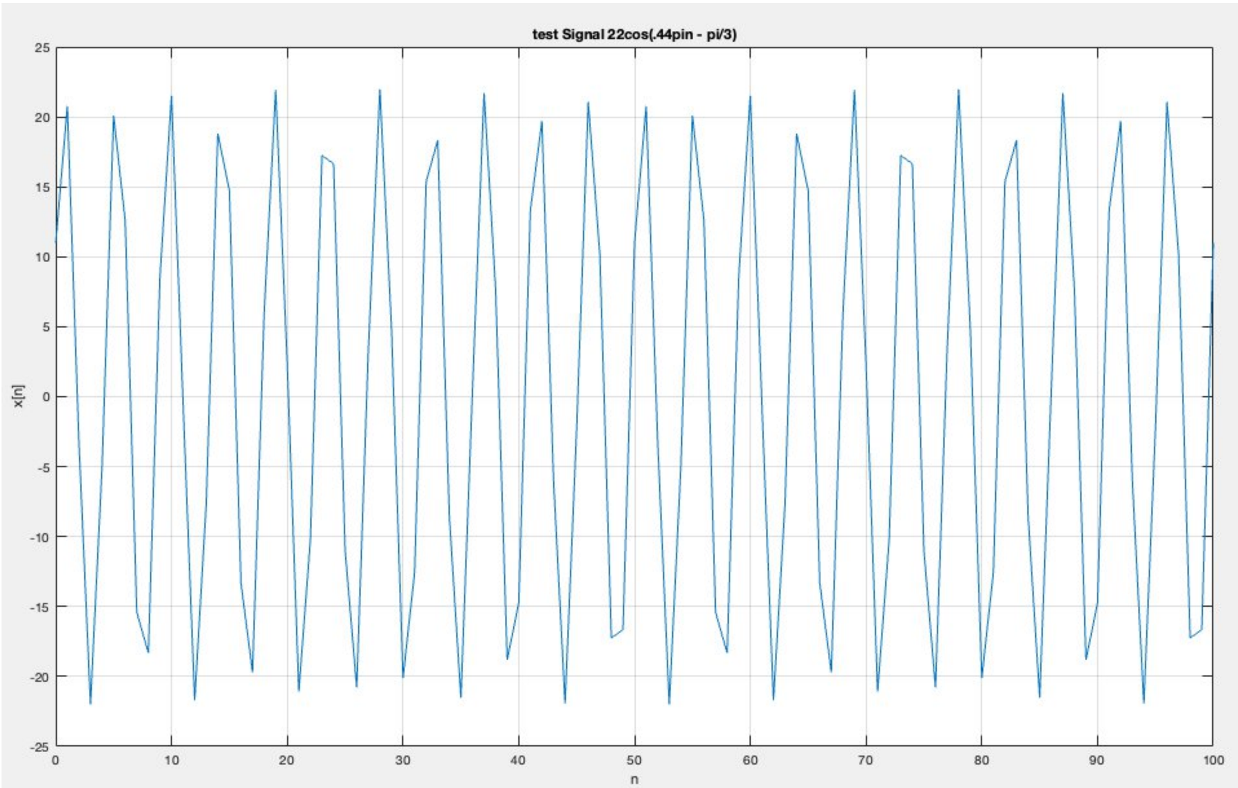
Magnitude graph of bandpass filter at L = 42



Applying filter to x[n] cleans up sinusoid, but leaves some discrepancies



What the signal should look like with 0.3 and 0.7 frequencies zeroed



Discussion and Analysis

The plot from cascading two FIR nulling filters confirmed the expectation that frequency components at 0.44pi and 0.7pi could be effectively zeroed out. By convolving the individual filters to create a single combined nulling filter, a clear suppression of these two components in the output signal was shown. The filtered output signal matched the calculated expectation of the sinusoid. Distortion occurred in the first few samples of the filtered sinusoid. This transient behavior lasted approximately five samples, consistent with the length of the combined FIR filter. Although the whole length of the output was not present in the graph, it is probable that distortion also was present in the last few n values of the graph. This would be due to the natural overhang that occurs during convolution. Comparative plots between the filtered signal and the calculated waveform demonstrated a precise match, confirming that the nulling filters were effective by way of convolution. The convolution not only caused an issue with the start of the sinusoid, it also magnified the amplitude in both the theoretical and filtered plot of the output. This is likely due to not a perfect nulling taking effect by the filter, as there is always some data lost during convolution. Even though precise amplitude nulling did not take place, all phase shift was effectively canceled out. This is shown in the graph above at n = 5 starting the cos wave at 0. The remaining term of the input had no phase shift, while the other two did. Although this filter did not perfectly cancel out terms in the input, it cleaned up the sinusoid into a steady wave and managed to cancel all phase that was previously present in the input.

In the second part of the lab, the design of a bandpass filter centered at 0.44pi demonstrated how filter length influences bandwidth and selectivity of passing frequencies. Using equation 2, a 10-point bandpass filter was created to pass a frequency component at 0.44pi. Frequency response analysis revealed that the filter achieved a peak gain at the center, 0.44pi, of a value very close to 1, while the other values were significantly smaller. Gain, determined from equation 4, is the magnitude of the frequency response. The filter's relatively short length resulted in a wide passband, allowing partial passage of adjacent frequencies. When the filter length was increased to 20 and 40, the passband width narrowed inversely, enhancing the filter's ability to isolate the target, centered frequency. This inverse relationship between passband width and filter length was confirmed by demonstrating different values of L of the filter.

A longer filter was designed with the goal of setting the 0.3pi and 0.7pi components to 0.1 or less. The frequency response showed that increasing L sufficiently minimized the gain at unwanted frequencies. Filtering the original sum-of-sinusoids signal with this narrow filter effectively reduced the unwanted components leaving mostly the desired 0.44pi sinusoid. This part of the graph in terms of the frequency response would be located in the passband section or above 0.707 while the other two frequencies were in the stopband section on or below 0.1. The output closely matched the 0.44pi frequency term in the original input.

The frequency response plot of this filter provided direct insight into the relative amplitudes of the sinusoidal components in the output. Mathematically, the output is the product of the input and the frequency response magnitude. For example, a gain near 1 at 0.44pi maintained the amplitude of that component as any number times 1 is itself, while near-zero gains at 0.3pi and 0.7 pi reduced those amplitude below 2.2 and 0.5. Since the filter did not completely zero out the gains in the stopband section of the graph, a perfect output was not achieved. Adjusting the length of the filter, however, can minimize the stopband so, a near perfect output can be reached. Overall, this exercise demonstrated the practical implementation of a bandpass FIR filter. It also showed its imperfections, and how its frequency response can be used to determine signal output behavior.

Summary and Conclusion

This lab focused on the design and application of FIR filters for frequency-selective filtering. First, it demonstrated how FIR nulling filters can eliminate specific frequency components and how cascading two filters allows for multiple frequency nullification. The practical use of convolution to combine filters connects directly to linear time-invariant systems and their impulse responses.

Secondly, the lab explored how the length of a bandpass FIR filter affects its frequency response- specifically its bandwidth. A longer filter led to a narrower passband and more effective nulling of out-of-band components, showing an inverse relationship between filter length and bandwidth. Although a perfect output was never achieved, altering the filter led to a more precise resemblance of the expected output. This ties into the trade-offs between resolution and accuracy in output and complexity in FIR design.

Appendix- Code

```
%% 3.1
% Frequencies to null
omega1 = 0.44 * pi;
omega2 = 0.70 * pi;

% FIR filter coefficients for nulling
b1 = [1, -2*cos(omega1), 1];
b2 = [1, -2*cos(omega2), 1];

b_total = conv(b1, b2);

n = 0:149;

% Input
xn = 5 * cos(0.3 * pi * n) + 22 * cos(0.44 * pi * n - pi/3) + 22 * cos(0.7 * pi * n - pi/4);

% Output
yn = conv(xn, b_total, 'same');

figure;
stem(0:39, yn(1:40), 'filled');
xlabel('n'); ylabel('y[n]');
title('Filtered Output y[n]');
grid on;
hold on;

y_expected = 9.4135 * cos(0.3 * pi * n);


plot(0:39, y_expected(1:40), 'r');
legend('Filtered Output', 'Expected Output');
```

```
%% 3.2
L = 10;
omega_c = 0.44 * pi;
n = 0:L-1;
h = (2/L) * cos(omega_c * n);

[H, w] = freqz(h, 1, 512);

omega_test = [0.3*pi, 0.44*pi, 0.7*pi];
gain = zeros(size(omega_test));

for k = 1:length(omega_test)
    % Find the index the frequencies are at
    index = round(omega_test(k) / pi * length(w));
    gain(k) = abs(H(index));
end

% Display results
fprintf('Gain at 0.3π: %.4f\n', gain(1));
fprintf('Gain at 0.44π: %.4f\n', gain(2));
fprintf('Gain at 0.7π: %.4f\n', gain(3));
```

```
%% b.
L_values = [10, 20, 40];

% Create a figure for plotting
figure;
```

```
for i = 1:length(L_values)
    L = L_values(i);
    n = 0:L-1;
    h = (2/L) * cos(omega_c * n);

    % Convert to frequency response
    [H, w_resp] = freqz(h, 1, 512);
    magH = abs(H);
    Hmax = max(magH);

    % Find passband
    passband_indices = find(magH >= 0.707 * Hmax);
    passband_w = w_resp(passband_indices);

    % Plot frequency response
    subplot(3,1,i);
    plot(w_resp/pi, magH, 'b');
    hold on;
    yline(0.707 * Hmax, 'r', '0.707 × H_{max}');
    xline(min(passband_w)/pi, 'k');
    xline(max(passband_w)/pi, 'k');
    title('L = ', num2str(L));
    xlabel('omega');
    ylabel('H');
    grid on;
end
```

```
%% d.
L = 42;
n = 0:L-1;
h = (2/L) * cos(omega_c * n);

% Convert to frequency response
[H, w_resp] = freqz(h, 1, 512);
magH = abs(H);
Hmax = max(magH);

% Find passband
passband_indices = find(magH >= 0.707 * Hmax);
passband_w = w_resp(passband_indices);

% Plot frequency response
figure;
plot(w_resp/pi, magH, 'b');
hold on;
yline(0.707 * Hmax, 'r', '0.707 × H_{max}');
yline(0.1,'g', '0.1');
xline(min(passband_w)/pi, 'k');
xline(max(passband_w)/pi, 'k');
xline(.7, 'r');
xline(.3, 'r');
title('L = ', num2str(L));
xlabel('omega');
ylabel('H');
grid on
```

```
%% e.
% Filter the signal
yn = conv(xn, h, 'same');
n_x = 0:149;
```

```
% Plot input and filtered output
figure;
subplot(2,1,1);
plot(n_x, xn);
title('Input Signal x[n] ');
xlabel('n'); ylabel('x[n]');
grid on;

subplot(2,1,2);
plot(n_x, yn, 'r');
title(['Filtered Output y[n] with L = ', num2str(L)]);
xlabel('n'); ylabel('y[n]');
xlim([0 100]);
grid on;

n_xtest = 0:100;
xtest = 22 * cos(0.44 * pi * n_xtest - pi/3);
% Plot expected sinusoid
figure;
plot(n_xtest, xtest);
title('test Signal 22cos(.44pin - pi/3)');
xlabel('n'); ylabel('x[n]');
xlim([0 100]);
grid on;
%% f
[H, w] = freqz(h, 1, 0:.01:pi);

% Plot magnitude of freq response
figure;
plot(w/pi, abs(H));
xline(.3, 'k');
xline(.7, 'k');
xlabel('\omega / ');
ylabel('H');
title('Magnitude Response');
grid on;
```

Appendix- Hand Calculations

LAB 8

a. $b_1 = [1, -2\cos(.44\pi), 1]$

$b_2 = [1, -2\cos(.7\pi), 1]$

d. don't count first 5, y_n = 0,0, 7.616, 8.9533, 2.909, -5.5334, -9.4141, -5.334

Amplitude no Phase - all nulled out

different bc nature of convolution leaves disruption at the beginning and end. First five - length of cascaded filter.

3.2

b. passband w = max - min / π = @L=10 $\frac{0.1621}{2} = 0.081$ @L=20

@L=40 $\frac{.081}{2} = 0.0405$