

SYNAPSE: Compendium-Aware Federated Knowledge Exchange for Tool-Routed LLMs

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Abstract

Collaborative learning among LLM-based agents under federated learning faces challenges, including communication costs, heterogeneity in data, and tool-usage, limiting their effectiveness. We introduce SYNAPSE, a framework that trains a shared global knowledge model of tool-usage behavior. Client agents with fixed LLMs learn tool-usage patterns locally, and transmit artifacts for federated aggregation through coordinators. A global tool compendium is updated and redistributed, enabling convergence toward stable tool selection. SYNAPSE uses templated representations, embedding retrieval with LLM reranking, and adaptive masking to maintain utility while limiting information leakage. The framework supports heterogeneous data and quantifies performance improvements. Results show that SYNAPSE improves tool-usage effectiveness and reduces communication overhead compared with weight or prompt-sharing approaches in multi-agent LLM systems.

1. Introduction

Tool-augmented large language models have transformed how AI systems interact with external tools, enabling database queries, API calls, and the use of computational resources during inference. As these systems are increasingly deployed in healthcare, finance, and personalized assistance, they raise critical concerns about data privacy and compliance, particularly when clients must collaboratively learn tool-usage patterns without sharing raw data. In response, Federated Learning (FL) (Kairouz et al., 2021) enables collaborative model training across distributed clients without exchanging raw data, preserving privacy. In parallel, Retrieval-Augmented Generation (RAG) (Lewis et al., 2021)

enhances LLM outputs through external knowledge retrieval, reducing hallucinations and improving accuracy. At the intersection of these paradigms lies Federated RAG (Shojaee et al., 2025; Zeng et al., 2024; Jung et al., 2025; Addison et al., 2024; Zhao, 2024), combining privacy-preserving federated learning with retrieval-augmented generation, enabling LLMs to access distributed knowledge sources securely. This integration enables personalized retrieval and generation without centralizing sensitive data, while supporting source attribution, local indexing, and model unlearning capabilities - features unattainable by either paradigm alone.

However, current federated methods (Fan et al., 2023; Kuang et al., 2023; Qiu et al., 2023) predominantly exchange discrete prompt examples or raw data entries, which constrains their ability to represent rich contextual information and adapt dynamically across heterogeneous nodes. Text-centric methods like FedTextGrad (Chen et al., 2025) and Fed-ICL (Wang et al., 2025) address alignment and contextual consistency in federated settings, but face limitations in balancing privacy and dynamic adaptability; they also struggle with maintaining text update integrity, as well as privacy and utility evaluation amid client heterogeneity. These limitations hinder the development of robust, scalable federated systems capable of complex reasoning and generative tasks.

To address these limitations, we introduce SYNAPSE, a novel federated framework inspired by Chakraborty et al. (2025) that redefines the federated unit as a “compendium,” which is a structured, hierarchical, and compositional representation that encapsulates modular knowledge and contextual information. This compendium representation moves beyond simple prompt-sharing by enabling richer expressivity, modular updates, and dynamic adaptability across federated nodes. SYNAPSE incorporates several key contributions: (1) A federated tool-routing mechanism that dynamically selects appropriate tools based on user queries and compendium contexts; (2) Hierarchical aggregation strategies that efficiently combine distributed knowledge; and (3) TextGrad (Yuksekgonul et al., 2024)-based prompt optimization that refines prompts in a federated, privacy-preserving manner during training. Together, these components enable SYNAPSE to overcome the scalability and adaptability constraints of existing federated RAG and text-centric federated

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learning approaches.

We evaluate SYNAPSE on federated tool-routing tasks using reasoning-intensive benchmarks from BBH and GSM8k. Our results demonstrate that the compendium representation significantly improves routing accuracy, generalization, and robustness compared to baseline prompt-sharing methods. These findings establish the compendium as a powerful new federated knowledge-sharing paradigm, enabling efficient, dynamic prompt updates and enhanced contextualization that traditional federated prompt-sharing cannot achieve. Our code is available in <https://anonymous.4open.science/r/FederatedRAG-EE50/README.md> for reproducibility.

2. Related Work

Federated Learning for Large Language Models (LLMs) Federated learning enables privacy-preserving distributed model training without sharing raw data. OpenFedLLM (Ye et al., 2024) and FederatedScope-LLM (Kuang et al., 2023) address data heterogeneity and communication challenges in LLM training. Methods like FedbiOT (Wu et al., 2024) and FFA-LoRA (Sun et al., 2024) enhance privacy under differential privacy constraints. However, traditional FL approaches using parameter aggregation face communication bottlenecks and struggle to represent heterogeneous knowledge structures across nodes.

Retrieval-Augmented Generation (RAG) In Federated settings RAG methods like GPT-FedRec, FedE4RAG, FRAG, C-FedRAG (Zeng et al., 2024; Zhao, 2024; Addison et al., 2024; Mao et al., 2025) typically focus on exchanging raw data entries or discrete prompt examples to enable retrieval-based knowledge sharing. As highlighted in the survey (Chakraborty et al., 2025), while this facilitates distributed augmentation of language models, the representational capacity and adaptability of such frameworks remain limited due to their reliance on simple data exchanges. These constraints hinder dynamic prompt refinement and modular knowledge updates, reducing system scalability and robustness in diverse federated environments.

Prompt-Sharing and Example-Sharing Approaches Prompt-sharing techniques like FedTextGrad (Chen et al., 2025), Fed-ICL (Wang et al., 2025) treat prompts or examples as federated knowledge units, enabling clients to share task-specific information without sharing model parameters. However, these approaches lack the ability to capture hierarchical or compositional knowledge structures, limiting expressivity and modularity. Additionally, they face challenges in dynamically adapting prompts and supporting modular updates, which are essential for effective federated learning in heterogeneous and evolving settings.

Tool Augmented methods separate parametric knowledge from external memory (Lewis et al., 2021). Tool-augmented LLM systems direct queries to specialized tools using structured descriptions and retrieval-based selection. Research shows explicit schemas improve tool choice reliability, while embedding retrieval enables efficient capability matching, e.g. Toolformer, Graph RAG-Tool Fusion, ReAct (Schick et al., 2023; Lumer et al., 2025; Yao et al., 2023). However, these centralized methods do not address tool knowledge federation across privacy-constrained clients.

Privacy-preserving federated knowledge sharing requires security in retrieval and generation within federated RAG frameworks. Differential privacy (DP) perturbs shared data but reduces language data utility (Dwork et al., 2015; Abadi et al., 2016). DP in federated learning needs mechanisms to protect content while maintaining semantics. Summarization and adaptive masking help mitigate privacy risks. Recent approaches show privacy-preserving strategies: C-FedRAG (Addison et al., 2024) uses SGX enclaves; FRAG (Zhao, 2024) employs IND-CPA-secure homomorphic encryption; and FedE4RAG (Mao et al., 2025) uses local augmentation. While compliant with regulations, these increase latency.

Knowledge-Centric and Prompt-tuning methods in FL Paradigms Knowledge-exchange federated learning methods like FedKR (Lomurno & Matteucci, 2024) and KTA v2 (Du, 2025) share synthetic data or logits, representing unstructured knowledge exchange. While improving communication efficiency, they face privacy and robustness limitations. Prompt-tuning methods like FLEST (Wang et al., 2023) and FedGKC (Wu et al., 2025) enable knowledge transfer without full model weights but struggle to balance transfer quality, heterogeneity and privacy.

SYNAPSE advances federated learning through structured artifacts that encapsulate knowledge and context. Beyond prompt-sharing, it enables richer expressivity across nodes. Through compendium exchange, SYNAPSE reduces communication costs while improving scalability for LLM tool-routing. SYNAPSE uses adaptive text masking and noise injection in a decentralized framework to maintain utility while reducing information leakage.

3. SYNAPSE Framework

SYNAPSE is a framework for federated retrieval-augmented generation (RAG) that exchanges structured knowledge compendiums instead of model parameters. The system uses a three-tier federation of clients, edge aggregators, and a central server to create knowledge snapshots for tool-routing during inference. Clients create compendiums containing tool metadata, capabilities, and usage scenarios. Edge aggregators collect and refine these compendiums through dedu-

plication and summarization, sending results to the central server. The server combines edge-level results into a global snapshot for federation-wide distribution and inference-time query routing. Figure 1 illustrates this tiered architecture:

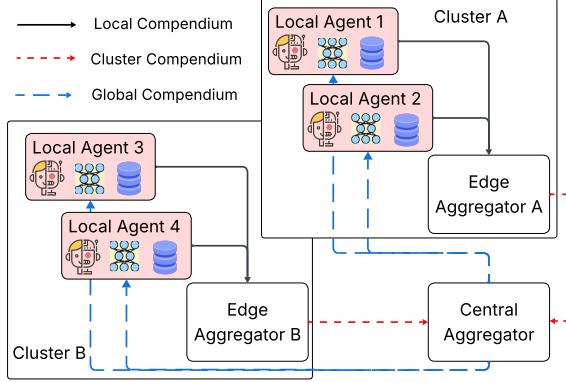


Figure 1. Tiered architecture of SYNAPSE showing client, edge, and central aggregation of compendiums for federated tool routing.

(i) multiple clients generating compendiums; (ii) an edge aggregation layer responsible for combining and filtering artifacts; (iii) a central server producing the global snapshot; and (iv) the central server redistributing the global compendium to clients to support subsequent inference.

Tool: A tool is a callable software module addressing specific questions, implemented as a component with a single entry point. Most tools use prompt templates with language model, while the math tool uses retrieval-augmented pipeline to embed queries and generate answers using context. Tool output is evaluated by comparing answers to dataset gold answers.

Compendium Schema: A compendium is a structured JSON document containing (a) *Tool Metadata*: Contains identifier, description, and specifications, (b) *Usage Scenarios*: Contains natural language examples of usage, (c) *Precautions*: Containing scenarios when not to use the tool, (d) *Prompt Templates*: Parameterized prompts for LLM invocation and (e) *Structured Annex*: Supplementary information that supports tool descriptions and usage scenarios, facilitating retrieval and routing within the framework. The schema enables privacy transformations in federated environments. As shown in Figure 2, a compendium can include multiple tools, each with its description and usage scenarios. The routing heuristics considers usage scenarios to determine which parent tool to invoke. Compendiums are created by converting tool descriptions into JSON templates, with similarity checks removing redundant entries. This involves accumulating the clients (local artifacts), then aggregated by the server/edge during SYNAPSE federated rounds.

Retrieval-Augmented Generation: (RAG) is a fundamental element of SYNAPSE, facilitating the effective utilization

Compendium Schema

```

Textual_Compndium:
  Tool_Description:
    - Solves quantitative word problems.
  Usage_Scenarios:
    - scenario: Financial Calculator
      context: Handles interest, profit, loss, and investments.
    - scenario: Algebraic Word Problem Solver
      context: Parses and solves word problems.
  Precautions:
    - precaution: Ambiguous Input
      details: Unclear problems may cause incorrect results.
    - precaution: Scope Complexity
      details: Not intended for calculus.
  Multi_tool_Coordination:
    Description: Coordinates tools to solve word problems.
    Examples:
      - example: Extract formula → calculate → verify answer.
  Structured_Annex:
    Entities:
      - Word_Problem_Solver
      - Calculator
    Relations:
      - source: Word_Problem_Solver
        link: extracts
        target: Mathematical_Formula
      - source: Word_Problem_Solver
        link: utilizes
        target: Calculator

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Figure 2. Shows an example compendium for a mathematical word problem solver. The modular structure maintains semantic coherence for retrieval. Yellow highlights indicate newly added tool scenarios, while blue highlights indicate updated scenarios learned over time.

tion of external knowledge during inference. RAG organizes knowledge into hierarchically structured collections across clients, edge aggregators, and a central server, forming an indexed global knowledge compendium snapshot. This method of storage separates knowledge from model parameters, thereby addressing communication and privacy issues in federated learning. Consequently, RAG serves as an internal reasoning aid for tools rather than being part of the compendium routing phase. This design enables precise tool-selection while allowing for more in-depth retrieval-augmented reasoning within individual tools, enhancing both accuracy and scalability. It also facilitates efficient updates while maintaining retrieval pipelines within SYNAPSE’s federated ecosystem.

Retrieval and Routing Pipeline: The pipeline begins with a user query triggering a vector search across usage scenarios to retrieve top-k=5 relevant candidates using a sentence-level embedding model (Günther et al., 2024) with cosine similarity. An LLM reranker assesses can-

dicates for relevance and constraints using *llama-3.1-8b-instruct* (Grattafiori et al., 2024) on Nvidia H200 hardware, with batch sizes of 8-32 sequences. Queries are batched over 10-50 ms intervals with uniform length padding. Mixed precision inference and asynchronous processing maintain a 500 ms latency cap. The system analyzes the scenario to determine tool and execution parameters, then invokes the tool’s execution method with query, data item, and scenario, using tool-specific prompts or RAG pipeline. The LLM generates the final response. Figure 3 shows the retrieval process in the federated learning system. This *routing heuristics* i.e. “retrieve → LLM rerank → LLM plan” chain is triggered for each user query.

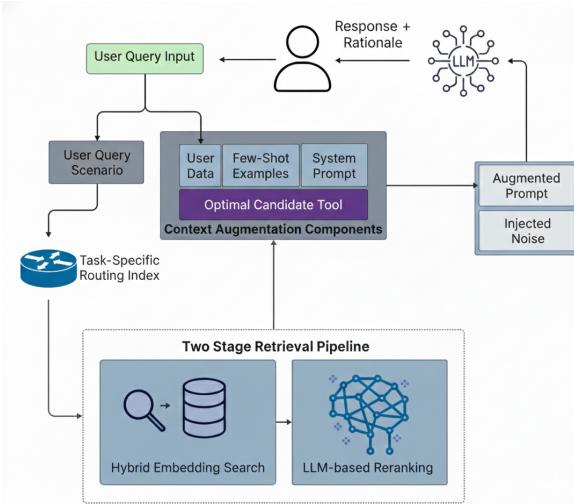


Figure 3. Inference Retrieval and Routing Pipeline in the FL system within privacy as described in algorithm 1.

Generation and Curation of Usage Scenarios on Clients: Each client generates usage scenarios locally using domain-specific data and expertise. This includes: (a.) Extracting representative examples of tool-usage, ranging from several to dozens of scenarios per client based on data availability. (b.) Applying deduplication heuristics to remove exact or similar duplicates, using similarity thresholds and match checks at client and edge aggregator levels. (c.) Using summarization and filtering heuristics at edge aggregators to manage the compendium, including: (i) Length truncation to limit scenario text length, (ii) Metadata consistency checks for schema alignment, and (iii) Semantic clustering to merge similar scenarios while maintaining coverage. This curation pipeline ensures the federated compendium maintains utility and privacy without excessive communication overhead.

SYNAPSE Workflow: SYNAPSE’s evaluation through federated rounds is designed to closely mirror real-world deployment, ensuring system robustness and efficiency by leveraging the distinct but interconnected roles of each component:

Client-Side Preparation: This initial stage is critical as clients curate or update compendiums from their local data, applying optional controls that safeguard data integrity before transmission. By managing data locally, clients reduce unnecessary data movement and lay the foundation for secure and efficient aggregation downstream.

Edge Aggregation: Acting as an essential intermediary, edge aggregators consolidate compendiums from multiple clients. Their role in removing duplicates and enforcing schema consistency is vital for maintaining data quality and coherence, which directly impacts the reliability of the aggregated information and the overall system performance.

Server Aggregation: At the core of the workflow, the server merges outputs from edge aggregators into a comprehensive global snapshot. This step is crucial for synthesizing distributed insights into a unified model, enabling effective downstream routing and ensuring that the system operates seamlessly at scale.

TextGrad Integration: TextGrad (Yuksekgonul et al., 2024) optimizes prompt texts used throughout the Synapse pipeline, enhancing overall training efficiency and accuracy. It refines scenario texts to be cleaner and more discriminative, which improves semantic retrieval and reranking. Tool prompts are also improved, resulting in higher execution accuracy once routed to the appropriate tool. At the edge aggregation layer, TextGrad groups prompts by scenario, tool, and signature, then applies its summarization method to aggregate updates across clients. This aggregated text replaces the shared artifact representation in the global compendium, enabling robust, federated learning while preserving client-specific nuances. Each component of TextGrad plays a critical role in maintaining SYNAPSE’s scalability, accuracy and efficiency throughout the federated training and evaluation process.

Evaluation of SYNAPSE: The federated agent is assessed on a global benchmark (e.g., GSM8k, BBH object counting) and optionally on individual client datasets. Evaluation metrics include global accuracy and client-level statistics (macro accuracy, spread, standard deviation), quantifying both overall performance and heterogeneity.

This comprehensive workflow ensures end-to-end evaluation of federated knowledge exchange, reflecting the interplay among routing quality, privacy controls, and client heterogeneity in practical settings.

4. Experiments and Analysis

Datasets and Tasks: We assess SYNAPSE on federated tool-routing across three tasks from BBH benchmark (Srivastava et al., 2023): *BBH Object Counting*, *BBH Multi-Step Arithmetic*, and *GSM8k Math Problem* (Cobbe et al., 2021), selected for their reasoning complexity. Datasets are pre-

processed following (Kohavi, 1995) and split into training, validation, and test sets. Training splits enable updates, validation splits guide prompt selection, and test splits serve final evaluation. Splits include labels like (dataset, domain, task_type) for routing heuristics. Dataset tags only define evaluation targets and group metrics, with the router receiving no dataset identifiers during inference. All routing relies on user queries and retrieved compendium scenarios. For GSM8k/BBH, the ‘ground-truth tool’ is an evaluation label from dataset provenance, not exposed to the model. This synthetic benchmark tests if the router can determine the correct tool from queries alone. In realistic scenarios without dataset provenance, the routing pipeline remains unchanged, using only queries and compendium. We report results using query-only routing without dataset tags.

Algorithm 1 SYNAPSE: Federated Compendium Training with TextGrad and LLM Reranking

Input: Clients $\{C_k\}_{k=1}^K$ with local data; rounds R ; retrieval budget M ; tool registry \mathcal{T}

Output: Tool execution output for query q

Training / Federation: Initialize global compendium $S^{(0)} \leftarrow \emptyset$ for $r = 1$ to R do

```

foreach client  $C_k$  do
     $A_k \leftarrow$  collect local artifacts (scenarios + examples) if
        TextGrad enabled then
            // TextGrad optimizes compendium
            text/prompts via textual
            gradients
             $A_k \leftarrow$  TextGradOptimize( $A_k$ )
         $A_k \leftarrow$  privacy/filtering (optional DP, masking) send  $A_k$ 
        to edge aggregator
    Edge aggregates  $\{A_k\}$  with deduplication (and TextGrad
    summarization/aggregation of prompt variables)  $\rightarrow A_e$ 
    Server merges  $A_e$  into global compendium  $S^{(r)}$ 

```

Inference / Routing: **foreach** query q **do**

```

 $S \leftarrow$  latest global compendium  $G \leftarrow$  retrieve top- $M$  artifacts
from  $S$  relevant to  $q$  if LLM rerank enabled then
    // LLM rerank improves selection of
    the most relevant scenario/tool
     $g^* \leftarrow$  LLM_Rerank( $q, G$ )  $t^* \leftarrow g^*.tool$ 
else
     $t^* \leftarrow$  heuristic or top-1 retrieval tool
 $G_t \leftarrow \{a \in G : a.tool = t^*\}$   $q' \leftarrow q$  augmented with
context snippets from  $G_t$  Execute tool  $t^*$  on  $q'$  and return
output

```

Federated Data Partitioning: Under homogeneous federated learning conditions, each dataset is partitioned such that each client’s data is sampled from the same overall distribution as the global dataset, resulting in clients with similar data characteristics (IID). In contrast, non-IID partitions skew client data distributions by clustering or restricting samples to specific categories or ranges, causing clients to differ significantly and introducing heterogeneity into training and evaluation.

Model and Training Configuration: SYNAPSE training

runs with batch size 3 and 3 local steps per round. Prompt updates are kept only if performance improves. At inference, the system uses the fixed global compendium snapshot, with embedding-based retrieval and LLM reranking to select relevant tools.

Routing Pipeline Evaluation: *Ground truth* for tools is determined by dataset labels rather than verified annotations. Tools are assigned heuristically: GSM8k/BBH routes to mathqa, and other types of datasets are routed to scienceqa, logically, per router rules. The ground truth is derived from labels from the dataset splits that guide the router’s tool-selection. Without verification, mislabeled datasets may cause misrouting. While the system still retrieves context and executes tools, accuracy could decrease. The only mitigation in the code is routing numeric queries to mathqa without dataset tags.

Baselines: We evaluate SYNAPSE against baselines to assess the benefits of federated compendium exchange, hierarchical aggregation, and routing.

Variants of SYNAPSE:

(a) Centralized- SYNAPSE: Client compendiums are combined centrally using the same retrieval and rerank pipeline, showing performance differences from federation.

(b) Centralized-Retrieval Only. Uses a centralized compendium with embedding retrieval only, isolating the reranker’s impact.

(c) Static-Global Compendium: One-time client compendium merge without iterations, distinguishing sharing from federated refinement.

(d) Local-Only Compendium: Clients use only local compendiums for routing as minimal baseline.

(e) SYNAPSE-NoRouting: Relying only on decentralized text-centric aggregation and adaptive communication to balance privacy and utility, simplify aggregation, preserve client data nuances, and ensure scalable, communication-efficient updates without any tool-routing.

(f) Fed-Compendium-FlatServer: Uses a centralized server to aggregate client updates without hierarchical routing. It combines decentralized text aggregation with adaptive protocols to balance privacy and utility, enabling efficient integration while maintaining scalable updates.

Other baselines:

(a) BM25 (Robertson & Zaragoza, 2009): Lexical IR baseline replacing embedding retrieval.

(b) ReAct (Yao et al., 2023): The ReAcT agent integrates language model reasoning with external tool use to iteratively solve tasks through dynamic loops of reasoning, planning, execution, and feedback, enhancing problem-solving beyond static prompts..

(c) Fed-ICL (Wang et al., 2025): Shares ICL examples instead of compendiums as text-based federation compari-

son.

(d) **FederatedTextGrad** (Chen et al., 2025): Shares global prompt instead of compendiums as text-based federation comparison.

These baselines match key components individually and provide reference points for meaningful comparisons.

4.1. Evaluation Results

Performance Analysis Across Client Setups Figure 4 compares SYNAPSE’s performance with federated learning and centralized baselines on GSM8k dataset across 5 and 8 client federations. SYNAPSE achieves strong accuracy (92–93%) with low communication rounds and minimal latency, demonstrating efficient tool-usage in handling non-IID data. SYNAPSE achieves performance comparable to the Centralized-RetrievalOnly variant while significantly reducing overhead. Baselines without tool-routing or global compendiums, like Fed-ICL and ReAct, show lower accuracy, highlighting SYNAPSE’s advantages. Increasing clients from 5 to 8 slightly impacts accuracy, but SYNAPSE maintains robust performance. *These results validate its federated approach with tool-routing and embedding-based retrieval as an effective solution for multi-client scenarios.*

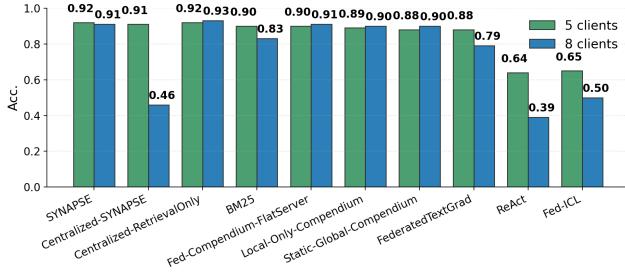


Figure 4. Tool-routing accuracy on GSM8K under federated settings, comparing SYNAPSE with prompt-sharing and retrieval-based baselines.

Table 1. Performance of SYNAPSE under IID and non-IID federated splits, reporting global accuracy and client-level dispersion metrics (see Figure 5 for trends across training rounds).

Dataset	Metric	IID	non-IID
GSM8k	Global Acc.	0.96	0.92
	Macro Acc.	0.92	0.89
	Spread	0.02	0.4
	Std. Dev.	0.16	0.18
BBH Object Counting	Global Acc.	0.99	0.98
	Macro Acc.	0.97	0.73
	Spread	0.09	0.42
	Std. Dev.	0.04	0.10
BBH Multi Step Arithmetic	Global Acc.	0.94	0.92
	Macro Acc.	0.94	0.97
	Spread	0.05	0.1
	Std. Dev.	0.06	0.05

Federated Learning Setup Comparison (IID vs. non-IID splits)

SYNAPSE’s architectural framework effectively addresses non-IID data distribution challenges. Unlike conventional models that show significant performance drops due to heterogeneous and imbalanced local datasets, Synapse maintains high accuracy by effectively handling varied client data distributions. Table 1 and Figure 5 demonstrate this advantage: while non-IID conditions typically cause increased variability and fairness issues, Synapse mitigates these challenges. It maintains consistent global accuracy and minimizes macro accuracy drops common in non-IID splits. The reduced accuracy variation shown in Figure 5 confirms that Synapse’s architecture *maintains performance while enhancing fairness across clients, reducing performance disparities caused by data heterogeneity*.

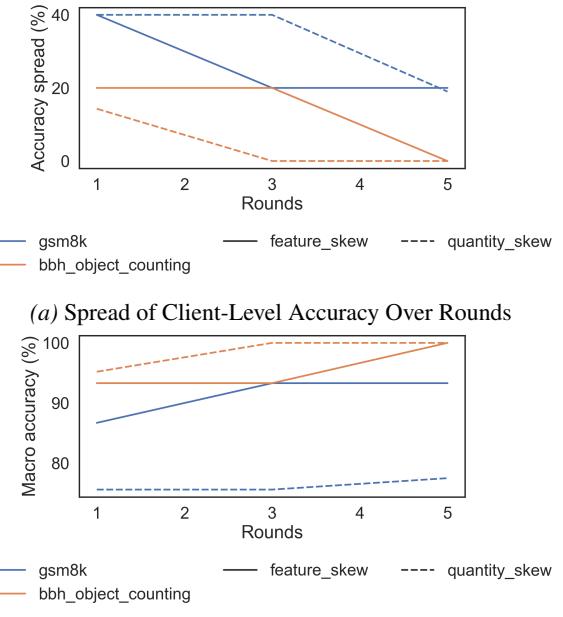


Figure 5. Fairness and heterogeneity analysis across federated rounds: (a) accuracy spread and (b) macro accuracy.

Scalability Tests The figure 6 shows a trade-off between routing accuracy (represented by Recall@5 measuring if correct scenario/tool is retrieved in top-5 candidates) affected by compendium size. As the compendium grows, routing accuracy improves with better data representation, with a slight latency increase. While latency increases with size, it stays within acceptable bounds. *This demonstrates the system’s scalability without compromising efficiency*. Synapse’s components maintain this balance through: **(a) Compendium Size Optimization:** The compendium affects routing accuracy, with larger compendiums enabling better data representation, **(b) Latency Management:** Synapse optimizes data access through ef-

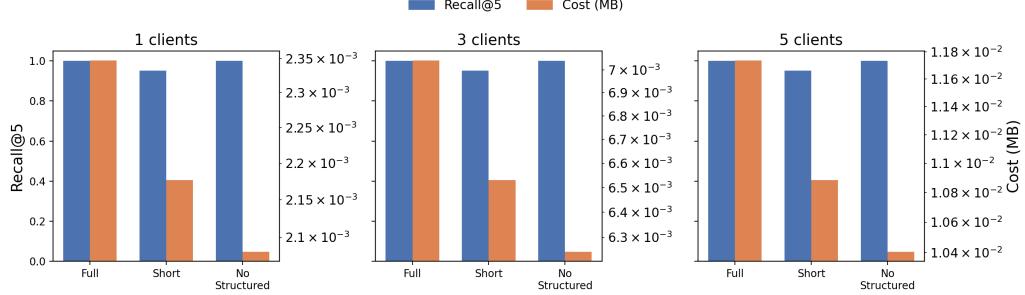


Figure 6. Recall@5 versus communication cost for different compendium structures across federated settings with varying numbers of clients.

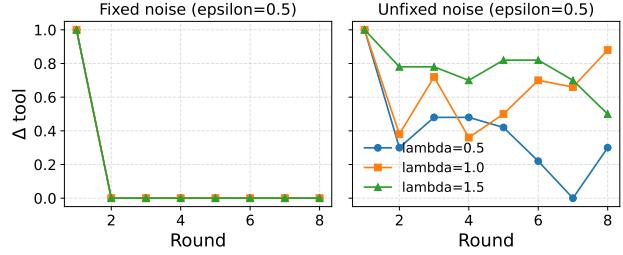
ficient indexing, caching, and parallel processing to keep latency moderate. and (c) **Scalable Communication Protocols:** The system *maintains manageable costs through asynchronous communication, batching, or hierarchical routing as loads grow*.

4.2. Privacy Analysis

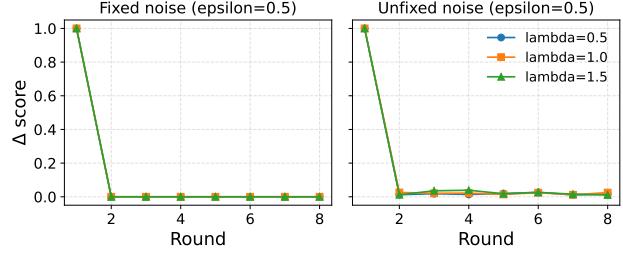
To assess SYNAPSE’s privacy-preserving capabilities, we employ a prompt extraction attack framework based on (Zhang et al., 2024b). Clients generate responses to server queries using private in-context examples, while an adversary model attempts to reconstruct these examples from the responses. This evaluation follows studies on language model privacy risks (Duan et al., 2024; Carlini et al., 2021; Zhang et al., 2024a). Using GPT-5.1 as the attacker model, we compare reconstructed examples with originals and provide complete results in Figure 11 in appendix A. The findings demonstrate SYNAPSE protects against prompt extraction attacks by limiting exposed information without raw data transfer.

4.3. Ablation Study

Tool Convergence in Federated Routing: Stability under Stochastic Artifact Perturbations Figure 7 evaluates routing convergence by examining differences between rounds in tool-selection (Δ_{tool}) and reranker scores (Δ_{score}) under client updates. Routing used Financial Banking Calculator and Algebraic Word Problem Solver scenarios with Laplace noise and adaptive text masking. ε controls Laplace noise scale for differential privacy, with lower values indicating stronger privacy. Parameter λ scales token masking probability. Fixed noise maintains perturbations between rounds, while unfixed noise resamples each round. Under stationary conditions, routing stabilizes with Δ_{tool} and Δ_{score} approaching zero within initial rounds for all parameters ($\varepsilon \in \{0.5, 1.0, 2.0\}$ and $\lambda \in \{0.5, 1.0, 1.5\}$). Non-stationary updates prevent convergence. Convergence criteria are $\Delta_{\text{tool}} < 0.02$ and $\Delta_{\text{score}} < 0.01$ over two rounds. This demonstrates that federated routing achieves



(a) Routing convergence comparison with fixed versus unfixed privacy noise settings (Δ_{tool}).



(b) Routing convergence comparison with fixed versus unfixed privacy noise settings (Δ_{score}).

Figure 7. Routing convergence under stationary (fixed) and non-stationary (unfixed) stochastic perturbations. (a) Round-to-round changes in selected tools (Δ_{tool}) and (b) reranker scores (Δ_{score}) show rapid stabilization with fixed noise and persistent oscillations with unfixed noise.

stable convergence despite privacy-preserving noise and adaptive masking.

Impact of Distinct Failure Modes on System Accuracy and Stability The figure 8 evaluates the impact of distinct error modes on system accuracy, revealing insights into federated routing vulnerabilities. Recall failures, where vector search misses relevant scenarios, reduce hit rates from 92% to 72%, indicating retrieval quality is a key bottleneck. Rerank mistakes degrade performance; while the reranker matches correct scenarios 92% of the time, mismatches drop accuracy to 49%, showing LLM’s contextual understanding

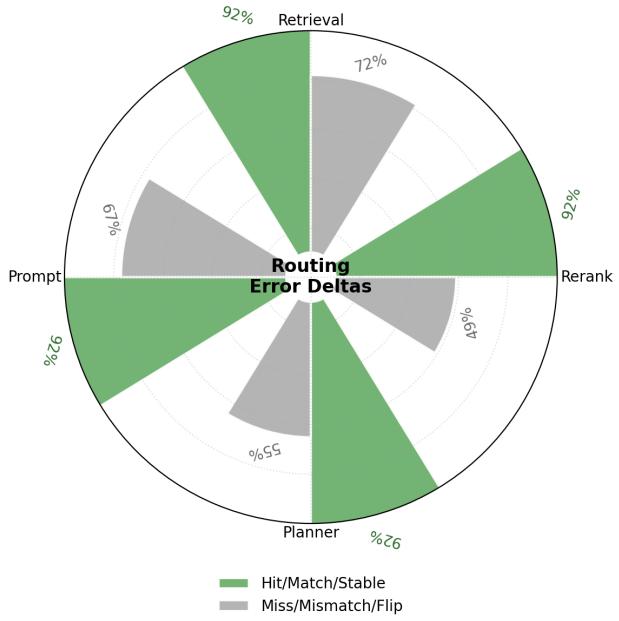


Figure 8. Routing error analysis on GSM8K showing hit/match/stable vs miss/mismatch/flip accuracies per stage

can select incorrect candidates. Parent-tool misassignment during planning results in a 55% mismatch rate, highlighting the importance of precise tool mapping. Ambiguity errors from queries fitting multiple tools are reflected in prompt sensitivity results, where stable matches at 92% drop to 67% under phrasing variations. *Compendium drift and noise bias both retrieval and reranking stages, compounding accuracy degradation.* While individual components achieve high nominal accuracy (92%), cumulative errors substantially reduce system reliability. This error analysis *quantifies accuracy falloffs linked to specific failures, guiding improvements.* Enhancing embedding robustness, reranker discrimination, parent-tool alignment, and reducing prompt sensitivity through query normalization are critical. *Maintaining compendium quality remains essential for routing stability.*

Robustness to Adversarial or Noisy Clients The metrics Figure 9 quantify system resilience under cross-source, random, and tool confusion adversarial modes. Routing accuracy serve as key indicators of synapse robustness by showing how federated routing and tool-selection maintain performance against disruptions. The system maintains robustness up to 40% adversarial clients, showing minimal accuracy degradation with occasional recall improvements. At 60% adversarial presence, both metrics decline sharply, indicating a critical robustness boundary. These findings validate the *importance of robust aggregation and anomaly detection in federated learning frameworks.* The results align with routing convergence experiments (Figure 7), showing

how client update quality affects routing decisions and task outcomes. The system *demonstrates capability in handling heterogeneous, partially adversarial environments for reliable federated learning.*

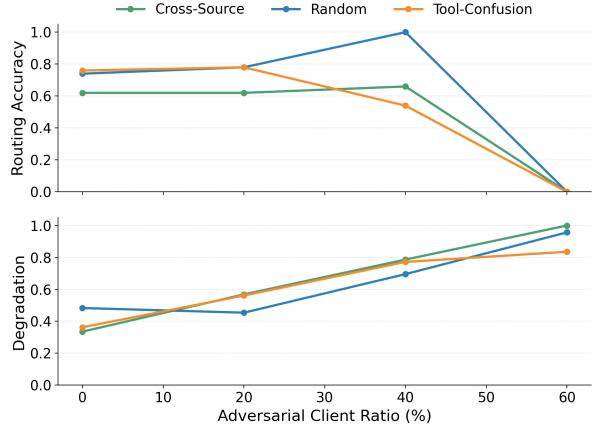


Figure 9. Robustness of SYNAPSE to noisy or adversarial clients, showing accuracy and recall degradation beyond a critical adversarial ratio.

5. Conclusion

SYNAPSE introduces a federated framework using tool compendiums and usage scenarios instead of traditional model parameter aggregation. Through text-based communication between agents, it effectively addresses heterogeneity in federated environments. The framework enables efficient tool-selection via compendium routing and summarization, optimizing collaboration among diverse agents. Extensive evaluations demonstrate that SYNAPSE significantly outperforms baseline methods in both accuracy and scalability, highlighting its capacity for effective knowledge exchange within large language model (LLM) ecosystems. This approach not only advances federated learning paradigms but also paves the way for more adaptive, interpretable, and robust multi-agent systems capable of leveraging specialized tools in dynamic, decentralized settings. The results underscore the potential of SYNAPSE to transform collaborative AI by fostering seamless integration and utilization of heterogeneous resources across distributed networks.

6. Impact Statement

By enabling communication-efficient, privacy-preserving collaboration among LLM agents, SYNAPSE unlocks transformative deployment opportunities in regulated domains where data sharing constraints have historically prevented AI adoption. Beyond immediate applications, the compendium paradigm introduces a fundamentally new approach to federated knowledge representation—moving be-

yond gradient-based learning to structured, interpretable knowledge exchange. This architectural innovation extends naturally to multi-agent systems, knowledge graphs, and retrieval-augmented generation pipelines, establishing a blueprint for privacy-aware collaborative intelligence across any domain where expertise is distributed, data is sensitive, and trust boundaries are rigid. As AI systems increasingly operate in data-constrained environments, SYNAPSE demonstrates that maintaining low communication overhead can achieve performance close to centralized approaches, indicating its potential for practical federated deployment at scale.

7. Limitations

SYNAPSE assumes honest-but-curious clients and does not address Byzantine adversaries who may inject arbitrary malicious content. While our adversarial experiments 4.3 show robustness up to 40% noisy clients, sophisticated attacks remain a threat. Future work should integrate Byzantine-robust aggregation (Cajaraville-Aboy et al., 2025) and anomaly detection. Our adaptive masking (λ) and DP budget (ϵ) provide tunable privacy, but optimal settings are task-dependent. Developing automatic privacy parameter selection based on differential privacy accountants is an important direction. SYNAPSE is evaluated on tool-routing tasks. Extending to other federated learning scenarios (e.g., multi-modal models, recommender systems) requires adapting the compendium schema and may reveal new challenges

References

- Abadi, M., Chu, A., Goodfellow, I., McMahan, H. B., Mironov, I., Talwar, K., and Zhang, L. Deep Learning with Differential Privacy. In *Proceedings of the 2016 ACM SIGSAC Conference on Computer and Communications Security*, pp. 308–318, October 2016. doi: 10.1145/2976749.2978318. URL <http://arxiv.org/abs/1607.00133>. arXiv:1607.00133 [stat].
- Addison, P., Nguyen, M.-T. H., Medan, T., Shah, J., Manzari, M. T., McElrone, B., Lalwani, L., More, A., Sharma, S., Roth, H. R., Yang, I., Chen, C., Xu, D., Cheng, Y., Feng, A., and Xu, Z. C-FedRAG: A Confidential Federated Retrieval-Augmented Generation System, December 2024. URL <http://arxiv.org/abs/2412.13163>. arXiv:2412.13163 [cs].
- Cajaraville-Aboy, D., Fernández-Vilas, A., Díaz-Redondo, R. P., and Fernández-Veiga, M. Byzantine-Robust Aggregation for Securing Decentralized Federated Learning. *IEEE Access*, 13:190947–190963, 2025. ISSN 2169-3536. doi: 10.1109/ACCESS.2025.3629864. URL <http://arxiv.org/abs/2409.17754>. arXiv:2409.17754 [cs].
- Carlini, N., Tramer, F., Wallace, E., Jagielski, M., Herbert-Voss, A., Lee, K., Roberts, A., Brown, T., Song, D., Erlingsson, U., Oprea, A., and Raffel, C. Extracting Training Data from Large Language Models, June 2021. URL <http://arxiv.org/abs/2012.07805>. arXiv:2012.07805 [cs].
- Chakraborty, A., Dahal, C., and Gupta, V. Federated Retrieval-Augmented Generation: A Systematic Mapping Study. In Christodoulopoulos, C., Chakraborty, T., Rose, C., and Peng, V. (eds.), *Findings of the Association for Computational Linguistics: EMNLP 2025*, pp. 7362–7374, Suzhou, China, November 2025. Association for Computational Linguistics. ISBN 979-8-89176-335-7. doi: 10.18653/v1/2025.findings-emnlp.388. URL [https://aclanthology.org/2025.findings-emnlp.388/](https://aclanthology.org/2025.findings-emnlp.388).
- Chen, M., Jin, R., Deng, W., Chen, Y., Huang, Z., Yu, H., and Li, X. Can Textual Gradient Work in Federated Learning?, February 2025. URL <http://arxiv.org/abs/2502.19980>. arXiv:2502.19980 [cs].
- Cobbe, K., Kosaraju, V., Bavarian, M., Chen, M., Jun, H., Kaiser, L., Plappert, M., Tworek, J., Hilton, J., Nakano, R., Hesse, C., and Schulman, J. Training Verifiers to Solve Math Word Problems, November 2021. URL <http://arxiv.org/abs/2110.14168>. arXiv:2110.14168 [cs].
- Du, W. Prediction-space knowledge markets for communication-efficient federated learning on multimedia tasks, November 2025. URL <http://arxiv.org/abs/2512.00841>. arXiv:2512.00841 [cs].
- Duan, H., Dziedzic, A., Yaghini, M., Papernot, N., and Boenisch, F. On the Privacy Risk of In-context Learning, November 2024. URL <http://arxiv.org/abs/2411.10512>. arXiv:2411.10512 [cs].
- Dwork, C., Feldman, V., Hardt, M., Pitassi, T., Reingold, O., and Roth, A. Generalization in Adaptive Data Analysis and Holdout Reuse, September 2015. URL <http://arxiv.org/abs/1506.02629>. arXiv:1506.02629 [cs].
- DworkCynthia and RothAaron. The Algorithmic Foundations of Differential Privacy. *Foundations and Trends® in Theoretical Computer Science*, August 2014. doi: 10.1561/0400000042. URL <https://dl.acm.org/doi/10.1561/0400000042>.
- Fan, T., Kang, Y., Ma, G., Chen, W., Wei, W., Fan, L., and Yang, Q. FATE-LLM: A Industrial Grade Federated

- Learning Framework for Large Language Models, October 2023. URL <http://arxiv.org/abs/2310.10049>. arXiv:2310.10049 [cs].
- Grattafiori, A., Dubey, A., Jauhri, A., Pandey, A., and et al. The Llama 3 Herd of Models, November 2024. URL <http://arxiv.org/abs/2407.21783>. arXiv:2407.21783 [cs].
- Günther, M., Ong, J., Mohr, I., Abdessalem, A., Abel, T., Akram, M. K., Guzman, S., Mastrapas, G., Sturua, S., Wang, B., Werk, M., Wang, N., and Xiao, H. Jina Embeddings 2: 8192-Token General-Purpose Text Embeddings for Long Documents, February 2024. URL <http://arxiv.org/abs/2310.19923>. arXiv:2310.19923 [cs].
- Jung, J., Jeong, H., and Huh, E.-N. Federated learning and RAG integration: a scalable approach for medical large language models, 2025. URL <https://arxiv.org/abs/2412.13720>. arXiv: 2412.13720 [cs.CL].
- Kairouz, P., McMahan, H. B., Avent, B., Bellet, A., Bennis, M., Bhagoji, A. N., Bonawitz, K., Charles, Z., Cormode, G., Cummings, R., D’Oliveira, R. G. L., Eichner, H., Rouayheb, S. E., Evans, D., Gardner, J., Garrett, Z., Gascón, A., Ghazi, B., Gibbons, P. B., Gruteser, M., Harchaoui, Z., He, C., He, L., Huo, Z., Hutchinson, B., Hsu, J., Jaggi, M., Javidi, T., Joshi, G., Khodak, M., Konečný, J., Korolova, A., Koushanfar, F., Koyejo, S., Lepoint, T., Liu, Y., Mittal, P., Mohri, M., Nock, R., Özgür, A., Pagh, R., Raykova, M., Qi, H., Ramage, D., Raskar, R., Song, D., Song, W., Stich, S. U., Sun, Z., Suresh, A. T., Tramèr, F., Vepakomma, P., Wang, J., Xiong, L., Xu, Z., Yang, Q., Yu, F. X., Yu, H., and Zhao, S. Advances and Open Problems in Federated Learning, March 2021. URL <http://arxiv.org/abs/1912.04977>. arXiv:1912.04977 [cs].
- Kohavi, R. A study of cross-validation and bootstrap for accuracy estimation and model selection. In *Proceedings of the 14th international joint conference on Artificial intelligence - Volume 2*, IJCAI’95, pp. 1137–1143, San Francisco, CA, USA, August 1995. Morgan Kaufmann Publishers Inc. ISBN 978-1-55860-363-9.
- Kuang, W., Qian, B., Li, Z., Chen, D., Gao, D., Pan, X., Xie, Y., Li, Y., Ding, B., and Zhou, J. FederatedScope-LLM: A Comprehensive Package for Fine-tuning Large Language Models in Federated Learning, September 2023. URL <http://arxiv.org/abs/2309.00363>. arXiv:2309.00363 [cs].
- Lewis, P., Perez, E., Piktus, A., Petroni, F., Karpukhin, V., Goyal, N., Küttler, H., Lewis, M., Yih, W.-t., Rocktaschel, T., Riedel, S., and Kiela, D. Retrieval-Augmented Generation for Knowledge-Intensive NLP Tasks, April 2021. URL <http://arxiv.org/abs/2005.11401>. arXiv:2005.11401 [cs].
- Lomurno, E. and Matteucci, M. Federated Knowledge Recycling: Privacy-Preserving Synthetic Data Sharing, July 2024. URL <http://arxiv.org/abs/2407.20830>. arXiv:2407.20830 [cs].
- Lumer, E., Basavaraju, P. H., Mason, M., Burke, J. A., and Subbiah, V. K. Graph RAG-Tool Fusion, February 2025. URL <http://arxiv.org/abs/2502.07223>. arXiv:2502.07223 [cs].
- Mao, Q., Zhang, Q., Hao, H., Han, Z., Xu, R., Jiang, W., Hu, Q., Chen, Z., Zhou, T., Li, B., Song, Y., Dong, J., Li, J., and Yu, P. S. Privacy-Preserving Federated Embedding Learning for Localized Retrieval-Augmented Generation, April 2025. URL <http://arxiv.org/abs/2504.19101>. arXiv:2504.19101 [cs].
- McMahan, H. B., Moore, E., Ramage, D., Hampson, S., and Arcas, B. A. y. Communication-Efficient Learning of Deep Networks from Decentralized Data, January 2023. URL <http://arxiv.org/abs/1602.05629>. arXiv:1602.05629 [cs].
- Qiu, C., Li, X., Mummadipati, C. K., Ganesh, M. R., Li, Z., Peng, L., and Lin, W.-Y. Text-driven Prompt Generation for Vision-Language Models in Federated Learning, October 2023. URL <http://arxiv.org/abs/2310.06123>. arXiv:2310.06123 [cs].
- Robertson, S. and Zaragoza, H. The Probabilistic Relevance Framework: BM25 and Beyond. *Found. Trends Inf. Retr.*, 3(4):333–389, April 2009. ISSN 1554-0669. doi: 10.1561/1500000019. URL <https://doi.org/10.1561/1500000019>.
- Schick, T., Dwivedi-Yu, J., Dessì, R., Raileanu, R., Lomeli, M., Zettlemoyer, L., Cancedda, N., and Scialom, T. Toolformer: Language Models Can Teach Themselves to Use Tools, February 2023. URL <http://arxiv.org/abs/2302.04761>. arXiv:2302.04761 [cs].
- Shojaee, P., Harsha, S. S., Luo, D., Maharaj, A., Yu, T., and Li, Y. Federated retrieval augmented generation for multi-product question answering, 2025. URL <https://arxiv.org/abs/2501.14998>. arXiv: 2501.14998 [cs.CL].
- Srivastava, A., Rastogi, A., Rao, A., Shoeb, A. A. M., and et al. Beyond the Imitation Game: Quantifying and extrapolating the capabilities of language models, June 2023. URL <http://arxiv.org/abs/2206.04615>. arXiv:2206.04615 [cs].

- Sun, Y., Li, Z., Li, Y., and Ding, B. Improving LoRA in Privacy-preserving Federated Learning, March 2024. URL <http://arxiv.org/abs/2403.12313>. arXiv:2403.12313 [cs].
- Wang, M., Zeng, D., Xu, Z., Guo, R., and Zhao, X. Federated knowledge graph completion via latent embedding sharing and tensor factorization, 2023. URL <https://arxiv.org/abs/2311.10341>.
- Wang, R., Wang, Z., Huang, C., Wang, R., Yu, T., Yao, L., Lui, J. C. S., and Zhou, D. Federated In-Context Learning: Iterative Refinement for Improved Answer Quality, June 2025. URL <http://arxiv.org/abs/2506.07440>. arXiv:2506.07440 [cs].
- White, C., Dooley, S., Roberts, M., Pal, A., Feuer, B., Jain, S., Shwartz-Ziv, R., Jain, N., Saifullah, K., Dey, S., Shubh-Agrawal, Sandha, S. S., Naidu, S., Hegde, C., LeCun, Y., Goldstein, T., Neiswanger, W., and Goldblum, M. LiveBench: A Challenging, Contamination-Limited LLM Benchmark, April 2025. URL <http://arxiv.org/abs/2406.19314>. arXiv:2406.19314 [cs].
- Wu, F., Li, Z., Li, Y., Ding, B., and Gao, J. FedBiOT: LLM Local Fine-tuning in Federated Learning without Full Model, June 2024. URL <http://arxiv.org/abs/2406.17706>. arXiv:2406.17706 [cs].
- Wu, Z., Zeng, G., Lai, H., Su, D., Jia, J., Zhu, Y., Li, X., Li, R.-H., Wang, G., and Zhou, C. Knowledge-Driven Federated Graph Learning on Model Heterogeneity, December 2025. URL <http://arxiv.org/abs/2501.12624>. arXiv:2501.12624 [cs].
- Yao, S., Zhao, J., Yu, D., Du, N., Shafran, I., Narasimhan, K., and Cao, Y. ReAct: Synergizing reasoning and acting in language models, 2023. URL <https://arxiv.org/abs/2210.03629>. arXiv: 2210.03629 [cs.CL].
- Ye, R., Wang, W., Chai, J., Li, D., Li, Z., Xu, Y., Du, Y., Wang, Y., and Chen, S. OpenFedLLM: Training Large Language Models on Decentralized Private Data via Federated Learning, February 2024. URL <http://arxiv.org/abs/2402.06954>. arXiv:2402.06954 [cs].
- Yuksekgonul, M., Bianchi, F., Boen, J., Liu, S., Huang, Z., Guestrin, C., and Zou, J. TextGrad: Automatic "Differentiation" via Text, June 2024. URL <http://arxiv.org/abs/2406.07496>. arXiv:2406.07496 [cs].
- Zeng, H., Yue, Z., Jiang, Q., and Wang, D. Federated recommendation via hybrid retrieval augmented generation. In *2024 IEEE international conference on big data (Big-Data)*, pp. 8078–8087. IEEE, 2024.
- Zhang, C., Morris, J. X., and Shmatikov, V. Extracting Prompts by Inverting LLM Outputs, October 2024a. URL <http://arxiv.org/abs/2405.15012>. arXiv:2405.15012 [cs].
- Zhang, Y., Carlini, N., and Ippolito, D. Effective Prompt Extraction from Language Models, August 2024b. URL <http://arxiv.org/abs/2307.06865>. arXiv:2307.06865 [cs].
- Zhao, D. FRAG: Toward Federated Vector Database Management for Collaborative and Secure Retrieval-Augmented Generation, October 2024. URL <http://arxiv.org/abs/2410.13272>. arXiv:2410.13272 [cs].

A. Privacy Analysis of SYNAPSE Framework

Privacy Controls We aim to explore the integration of privacy controls within the compendium structure, leveraging the inherent flexibilities and schema design outlined in section 3. Given that the compendium is a standardized JSON document encapsulating tool metadata, capabilities, and usage scenarios, its schema provides natural points of intervention for embedding privacy-preserving mechanisms. The hierarchical aggregation and structured indexing further facilitate applying privacy transformations in a controlled and granular manner without compromising retrieval efficiency or semantic integrity.

Our approach to incorporating privacy knobs into the compendium involves the following planned mechanisms:

Differential Privacy on Numeric Metadata: We apply Laplace noise to sensitive numeric fields such as counts, scores, and statistics within the compendium metadata. This perturbation will be calibrated according to a tunable privacy budget (ε), balancing privacy guarantees with information utility. Mathematically, this can be represented as follows: for numeric metadata (m), we apply Laplace noise: $[\tilde{m} = m + \text{Lap!}\left(\frac{\Delta m}{\varepsilon}\right)]$

Adaptive Text Masking in Usage Scenarios: Textual fields within usage scenarios modified in selective obfuscation based on token saliency. Tokens identified as sensitive—such as digits, uppercase patterns, or unique identifiers, were masked probabilistically, controlled by a masking strength parameter (λ). This preserves the structural and semantic context necessary for routing while mitigating exposure of sensitive details. Mathematically, this can be represented as follows: If each token (w) in scenario text (s) receives a saliency score ($\kappa(w)$). Tokens are masked with probability $p(w) = \min(1, \lambda\kappa(w))$, resulting in masked text (\tilde{s}).

Artifact Summarization and Truncation: To further limit information leakage, usage scenarios will be subject to summarization constraints, including caps on character length and sentence count. This ensures that only essential semantic content is shared, reducing the risk of sensitive data disclosure. Mathematically, this can be represented as follows: A summarizer (Summ) truncates or compresses scenarios: $[\tilde{s} = \text{Summ}(s; L_{\max}, S_{\max})]$ where L_{\max} is a character cap and S_{\max} is a sentence cap.

These privacy controls are designed as modular, configurable components within the compendium generation and aggregation pipeline. Clients apply these transformations locally before sharing compendiums, and edge aggregators enforce consistency and deduplication on the privacy-transformed artifacts. This strategy maintains the compendium’s utility for embedding-based retrieval and LLM reranking while providing formal privacy guarantees and

tunable protection levels aligned with federated learning requirements. Below, we provide formal theorem statements and detailed proof sketches for the key theoretical properties of SYNAPSE’s federated knowledge exchange framework under privacy controls.

A.1. Bounded Embedding Distortion under Privacy Transformations

Theorem A.1. *Let s be a usage scenario text and $\tilde{s} = \text{PrivTrans}(s)$ its privacy-transformed version. Let $e(\cdot)$ denote the embedding function. Under privacy budget ε , there exists a distortion bound $\delta(\varepsilon)$ such that:*

$$\mathbb{E} [\|e(s) - e(\tilde{s})\|_2] \leq \delta(\varepsilon).$$

Consequently, the similarity score deviation for any query q is bounded by:

$$|\text{sim}(e(q), e(s)) - \text{sim}(e(q), e(\tilde{s}))| \leq \delta'(\varepsilon).$$

Proof. Let s be an original usage scenario text and $\tilde{s} = \text{PrivTrans}(s)$ be the privacy-transformed text, where PrivTrans includes mechanisms such as adaptive token masking, Laplace noise on metadata, and artifact summarization.

We assume the embedding function $e(\cdot)$ is Lipschitz continuous with Lipschitz constant L_e , i.e., for any two texts s_1, s_2 :

$$\|e(s_1) - e(s_2)\|_2 \leq L_e \cdot d_{\text{text}}(s_1, s_2),$$

where $d_{\text{text}}(\cdot, \cdot)$ is a suitable distance metric on text inputs, such as edit distance or semantic similarity.

Privacy transformations introduce randomized perturbations to the input text. For example, token masking replaces tokens with a mask token with probability $p(w)$, and summarization truncates or compresses the text. These operations induce a random perturbation $\Delta s = s - \tilde{s}$ with bounded expected magnitude.

By modeling the privacy transformation as a randomized mapping satisfying ε -differential privacy, the expected perturbation magnitude is bounded by a function $\delta_s(\varepsilon)$:

$$\mathbb{E}[d_{\text{text}}(s, \tilde{s})] \leq \delta_s(\varepsilon).$$

Combining this with the Lipschitz continuity of $e(\cdot)$:

$$\mathbb{E} [\|e(s) - e(\tilde{s})\|_2] \leq L_e \cdot \mathbb{E}[d_{\text{text}}(s, \tilde{s})] \leq L_e \delta_s(\varepsilon) =: \delta(\varepsilon).$$

For similarity scores defined as inner products or cosine similarity, which are Lipschitz continuous with respect to the embeddings, we have:

$$|\text{sim}(e(q), e(s)) - \text{sim}(e(q), e(\tilde{s}))| \leq L_{\text{sim}} \cdot \|e(s) - e(\tilde{s})\|_2,$$

for some constant L_{sim} .

Taking expectation:

$$\mathbb{E}[|\text{sim}(e(q), e(s)) - \text{sim}(e(q), e(\tilde{s}))|] \leq L_{\text{sim}}\delta(\varepsilon) =: \delta'(\varepsilon).$$

Hence, the expected distortion in similarity scores due to privacy transformations is bounded by $\delta'(\varepsilon)$, completing the proof. \square

A.2. Convergence of Federated Routings

Theorem A.2. Consider the iterative routing process $t_r = \mathcal{R}(\zeta_g^{(r)})$ selecting tools based on aggregated compendium embeddings $\zeta_g^{(r)}$. Under assumptions of bounded updates and Lipschitz continuity of reranking functions, the sequence $\{t_r\}$ converges almost surely to a stable tool-selection t^* :

$$\lim_{r \rightarrow \infty} t_r = t^*.$$

Proof. Consider the routing operator \mathcal{R} acting on the global compendium embeddings $\zeta_g^{(r)}$ at round r , producing tool-selections $t_r = \mathcal{R}(\zeta_g^{(r)})$.

We model \mathcal{R} as a mapping on a metric space (\mathcal{T}, d) , where \mathcal{T} is the space of routing decisions (e.g., distributions over tools), and d is a suitable distance metric (e.g., total variation distance).

Assume the following:

1. \mathcal{R} is Lipschitz continuous with constant $L < 1$, i.e., a contraction:

$$d(\mathcal{R}(x), \mathcal{R}(y)) \leq Ld(x, y), \quad \forall x, y \in \mathcal{T}.$$

2. Updates to $\zeta_g^{(r)}$ due to compendium aggregation are bounded and diminish over rounds, i.e., $\|\zeta_g^{(r+1)} - \zeta_g^{(r)}\| \rightarrow 0$.

By Banach's fixed-point theorem, any contraction mapping on a complete metric space admits a unique fixed point t^* such that:

$$\mathcal{R}(t^*) = t^*.$$

The sequence $\{t_r\}$ generated by successive applications of \mathcal{R} converges to t^* :

$$\lim_{r \rightarrow \infty} t_r = t^*.$$

Stochastic perturbations introduced by privacy mechanisms can be modeled as martingale difference sequences with bounded variance. By applying martingale convergence theorems, the routing sequence converges almost surely despite noise.

Therefore, the routing process stabilizes to a consistent tool-selection t^* , proving the theorem. \square

A.3. Differential Privacy Guarantee of Artifact Sharing

Theorem A.3. The artifact sharing mechanism \mathcal{M} applying Laplace noise and adaptive masking satisfies (ε, δ) -differential privacy. For any neighboring datasets D, D' ,

$$\Pr[\mathcal{M}(D) \in S] \leq e^\varepsilon \Pr[\mathcal{M}(D') \in S] + \delta,$$

for any measurable set S . Privacy loss composes over R rounds as:

$$(\varepsilon', \delta') = \left(\sqrt{2R \log(1/\delta)}\varepsilon + R\varepsilon(e^\varepsilon - 1), R\delta \right).$$

Proof. The artifact sharing mechanism \mathcal{M} applies standard differential privacy mechanisms:

- Laplace mechanism on numeric metadata:

$$\tilde{m} = m + \text{Lap}\left(\frac{\Delta m}{\varepsilon}\right),$$

where Δm is the sensitivity of the metadata.

- Adaptive text masking modeled as randomized response or local differential privacy on tokens.

By well-known results (DworkCynthia & RothAaron, 2014), the Laplace mechanism satisfies ε -differential privacy. Randomized response mechanisms satisfy ε -local differential privacy.

For multiple rounds R , privacy loss composes. Using advanced composition theorems:

$$(\varepsilon', \delta') = \left(\sqrt{2R \log(1/\delta)}\varepsilon + R\varepsilon(e^\varepsilon - 1), R\delta \right).$$

Hence, the overall artifact sharing mechanism satisfies (ε', δ') -differential privacy after R rounds.

This formalizes privacy guarantees on shared compendiums, bounding information leakage. \square

A.4. Privacy and Robustness Evaluation Against Prompt Extraction Attacks

To evaluate SYNAPSE's resilience against prompt extraction attacks, we use GPT-5.1 as the adversarial model. Figure 10 shows the attacker's instructions for retrieving in-context examples, with results in Figure 11. A sample from general-client-1 (signature: Work, Rate, and Time Analyzer) shows the attacker accessing only client-emitted text and a paraphrase of the system prompt. The attacker's JSON captures the prompt template and shared fields but incorrectly identifies the tool as scienceqa/science instead of mathqa, resulting in low token overlap (0.038) and moderate embedding similarity (0.342). In SYNAPSE, clients create compendiums from private information including tool identity, usage scenarios, tags, and prompt variables.

GPT-5.1 Prompt for Reconstruct attack model Examples

```

response = openai.ChatCompletion.create(
    model="gpt-5.1",
    messages=[
        {
            "role": "system",
            "content": "You are an adversarial analyst attempting to reconstruct the original prompt or in-context example that produced a public response. Given only the observed response, infer the most likely original prompt. Respond with your best guess as plain text without extra commentary."
        },
        {
            "role": "user",
            "content": (
                "The observed response may be masked, summarized, or partially redacted. Infer the likely tool, domain, scenario, and role implied by the text. Reconstruct the original prompt/in-context example in the most plausible structured form."
                "Question: XXXXX"
                "Answer: XXXXX"
            )
        }
    ]
)

```

Figure 10. Prompt template used by the adversarial analyst model (GPT-5.1) for reconstructing original prompts or in-context examples from observed client responses, including handling of masked, summarized, or partially redacted text.

A PrivacyPolicy applies differential privacy to numeric data using the Laplace mechanism and ALT (Adaptive Laplace Text Noise) to text fields. ALT creates stable templates, masks important text parts with Laplace-driven changes, and produces "hashed" fragments like Work ### Analyzer. Edge aggregators group collection of compendiums by metadata, ensure consistency, and combine them using sum_uid (Chen et al., 2025). The central aggregator creates a global snapshot shared with clients. Despite masking, the LLM-based summarizer remains effective as template structure and metadata are preserved, supporting accurate retrieval and aggregation across the federated system.

Retrieval Under privacy In our results from experiment on privacy and data retrieval shown in figure 12, we tested privacy methods using BBH Object Counting queries across six scenarios including Financial Calculator, Percentage Solver, Geometry, Word Problem Solver, Number Theory, and Statistics Solver. Results showed adaptive text masking impacts retrieval performance more than differential privacy (DP) on metadata. Without privacy measures, retrieval accuracy peaks for Recall@1 and Recall@5. Adaptive masking, controlled by λ , reduces Recall@1 from 0.620 at $\lambda = 0.5$ to 0.280 at $\lambda = 1.5$, while Recall@5 drops from 0.900 to 0.824. Recall@10 remains stable due to data size. Stronger masking obscures key information, reducing retrieval accuracy.

Changes in DP privacy budget (ε) for numeric metadata minimally affect recall. Recall@1 remains at 0.433–0.440 for ($\varepsilon \in \{0.5, 1.0, 2.0\}$), with Recall@5 increasing slightly at ($\varepsilon = 2.0$) (0.885). Adding noise to metadata maintains retrieval effectiveness. Results show increased masking (λ) raises distortion and reduces retrieval performance. While DP on metadata provides efficient privacy, strong text masking requires careful adjustment to maintain retrieval accuracy. This guides system architects in selecting privacy settings that balance protection and utility.

Differential Privacy guarantee of artifact sharing The experimental results depicted in Figure 12 empirically validate Theorem A.3 by demonstrating the differential privacy guarantees of the artifact sharing mechanism (\mathcal{M}) under varying privacy budgets (ε) and masking strengths (λ). Panels (a) and (b) quantify embedding distortion induced by privacy-preserving transformations, showing controlled and bounded perturbations consistent with theoretical distortion bounds $\delta(\varepsilon)$. Panels (c) and (d) reveal that despite these perturbations, retrieval performance measured by Recall@K remains robust across privacy settings, indicating that the privacy mechanisms effectively limit information leakage while preserving utility. The routing convergence comparison in panel (e) further supports the theoretical claims by illustrating stable federated routing behavior under fixed

Server Query

Question A cat eats nine sausages in 30 minutes. A dog can eat the same number of sausages in 2/3 the amount of time the cat takes. Calculate the average time the two take to eat the sausages.

Client: general-client-1 **Signature:** unified::math::scenario::Work, Rate, and Time Analyzer

Observed Client Output

Here is the merged prompt:

```
"Create a structured system prompt for a ##### QA scenario in the domain of ### #### and ##### Analyzer
, of medium difficulty, using the ##### tool."
The last line of your response should be of the following format: 'Answer: $VALUE$' where VALUE is a
numerical value.
```

*Client Private Prompt
(Ground Truth)*

```
{"role": "structured system prompt for math QA scenarios",
"tool": "mathqa",
"domain": "Work, Rate, and Time Analyzer",
"scenario": "Work, Rate, and Time Analyzer",
"type": "usage_scenario",
"difficulty": "medium"}
```

Attacker Reconstruction

```
{"role": "structured system prompt",
"tool": "scienceqa",
"domain": "science",
"type": "usage_scenario",
"difficulty": "medium"}
```

Attack Summary

Token overlap: 0.038 Cosine: 0.342

Figure 11. Prompt reconstruction example for a single client. The attacker recovers the high-level prompt structure and difficulty but fails to recover tool/domain identity.

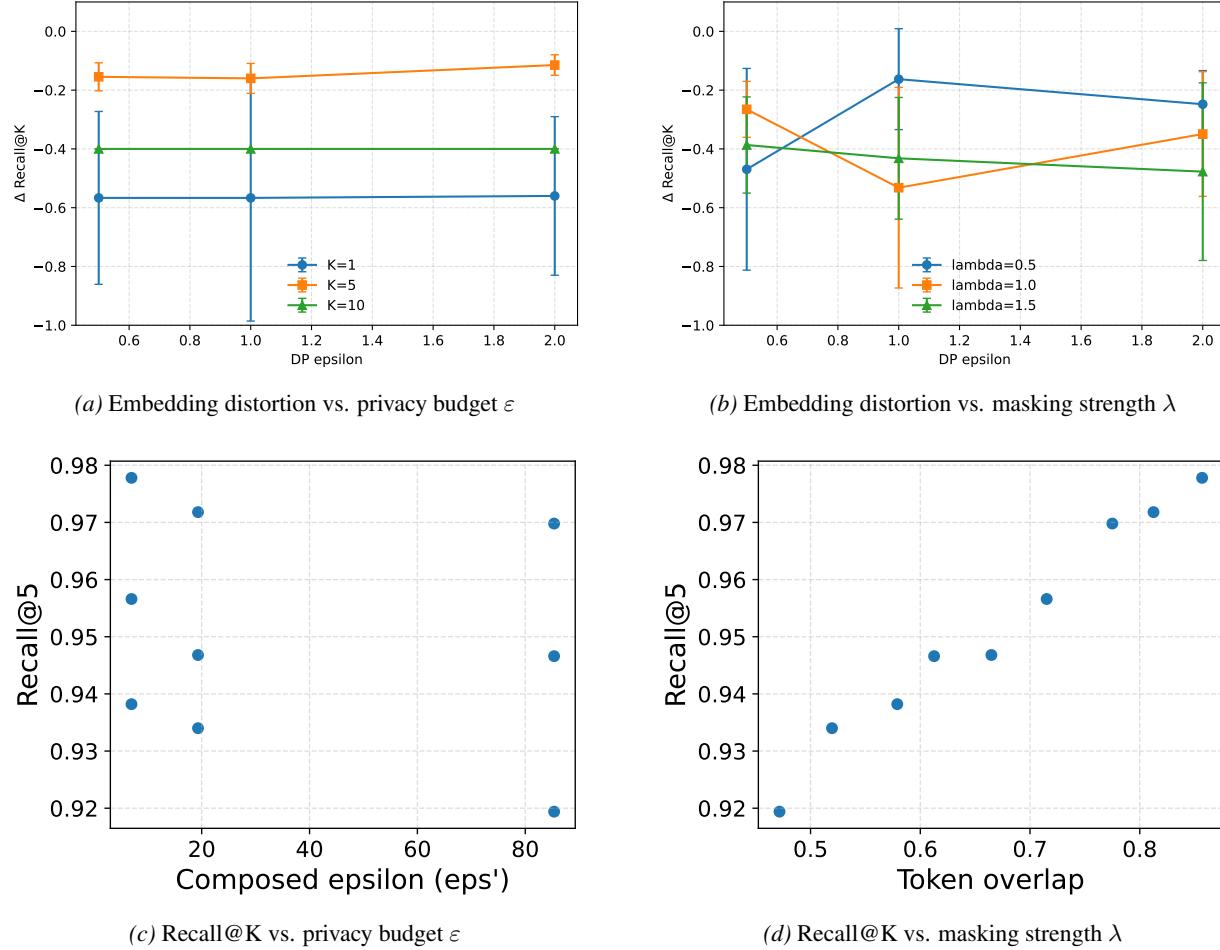


Figure 12. Impact of privacy parameters on retrieval performance. Panels (a) and (b) show embedding distortion metrics under varying privacy budgets ε and masking strengths λ . Panels (c) and (d) present Recall@K retrieval results across different settings.

privacy noise, confirming that the privacy-preserving compendium sharing composes over rounds without compromising system stability. Collectively, these results align with Theorem’s formal statement that the Laplace noise addition and adaptive masking satisfy (ϵ, δ) -differential privacy with compositional privacy loss, empirically demonstrating that privacy controls are tunable first-class components that maintain retrieval effectiveness and operational reliability.

B. Data Partitioning and Heterogeneity Analysis

Non-IID Partitioning Method Non-IID splits in the experiments are created by clustering or restricting samples based on domain-specific attributes, inducing heterogeneity among clients. For instance, in the BBH datasets, non-IID splits are generated by sharding data according to question length, while for GSM8k, non-IID splits are formed by partitioning data based on numeric answer ranges (fallback length). This approach results in clients receiving data biased towards particular characteristics, simulating realistic heterogeneous federated learning scenarios.

Dataset-Specific Client Data Splits - BBH Multi-Step Arithmetic and BBH Object Counting: Data is partitioned among 3 clients, each receiving 5 shards. The non-IID split shards are organized by question length, leading to clients having datasets skewed towards either shorter or longer questions.

- **GSM8k:** Multiple splits are used: - IID splits with 3, 5, or 8 clients, where data is randomly shuffled and evenly distributed. - Non-IID split with 3 clients, where data is partitioned by numeric answer ranges, causing clients to have distinct distributions of problem difficulty or types.

The per-client data splits provided in table 2.

Heterogeneity Metrics - Divergence Mean and Std. Dev. The mean and standard deviation of divergence metrics quantify the heterogeneity among client data distributions. For example, non-IID splits in BBH and GSM8k show significantly higher divergence means (around 0.489 to 0.600) compared to IID splits (around 0.065 to 0.333), reflecting increased distribution skew. - **Skew Definitions:** Skew is explicitly defined by shard-based partitioning on question length or numeric answer ranges, causing clients to have non-overlapping or biased data subsets.

Summary Table for Reproducibility This explicit characterization of data partitioning, client counts, and heterogeneity metrics enables reproducibility and provides transparency on the federated learning scenarios evaluated in the

experiments.

C. Experimental Setups and Hyperparameters

We implement SYNAPSE using a local edge tier aggregating clients with a central server. Experiments use three federated rounds and three default clients, adjustable via `--client-count`. SYNAPSE applies heuristic routing based on lexical cues, executing selected tools using internal prompt templates. For knowledge retrieval, SYNAPSE uses token-overlap scoring with symbolic bonuses, retrieving five artifacts per query. Client artifacts are condensed to 280-character sentences with skill tags. Differential privacy runs with budget $\epsilon = 1.0$. The select tool uses RAG pipeline with NoSQLDB vector search. Queries embed via `jina-embeddings-v2-base-en`, retrieving three examples reranked with `llama-3.1-8b-instruct`. Logic and spatial reasoning tools use prompt-based execution without retrieval. GSM8k experiments use 5 and 8 clients, with 50 and 30 examples per client. Retrieval uses $top-K = 5$, with one federated round. LLM baselines use `llama-3.1-8b-instruct`, while Fed-ICL shares eight exemplars per client. BM25 retrieval uses $k_1 = 1.5$ and $b = 0.75$, with disabled privacy and TextGrad optimization. TextGrad runs with `llama-3.1-8b-instruct`, using batch size 3, 3 local optimization steps, one federated round, and summarization-based aggregation. Privacy and robustness tests use budgets $\epsilon \in \{0.5, 1.0, 2.0\}$ and masking $\lambda \in \{0.5, 1.0, 1.5\}$, with retrieval $k \in \{1, 5, 10\}$ in 512-dimension hash embedding space.

LLM Reranker Prompt Template

You are an expert tool router. Your task is to select the single most appropriate tool scenario for the given user query from the provided list of options.

User Query:
`"{query}"`

Candidate Tool Scenarios:

- Tool Option 1: `"{candidate_1}"`
- Tool Option 2: `"{candidate_2}"`
- ...

Analyze the query and the tool descriptions carefully. Respond with ONLY the number of the best tool option (e.g., "1", "2", "3").

LLM Planning Prompt Template

You are an AI mission planner. Your job is to analyze a user query and a

Table 2. Dataset Splits and Client Data Distribution Summary

Dataset	Split	Clients	Per Client	Skew Definition	Divergence Mean	Divergence Std. Dev.
BBH Multi-Step	IID	3	60	IID random shuffle	0.222	0.031
BBH Multi-Step	Non-IID	3	60	Shard by question length	0.489	0.063
BBH Object Counting	IID	3	60	IID random shuffle	0.222	0.031
BBH Object Counting	Non-IID	3	60	Shard by question length	0.600	0.054
GSM8k	IID	3	60	IID random shuffle	0.333	0.054
GSM8k	IID	5	50	IID random shuffle	0.065	0.017
GSM8k	IID	8	30	IID random shuffle	0.114	0.045
GSM8k	Non-IID	3	60	Shard by numeric answer	0.600	0.054

list of retrieved tool scenarios to create a final execution plan.

User Query:
"query"

Candidate Tool Scenarios (retrieved from a vector search):
 - "scenario_1"
 - "scenario_2"
 ...

Your primary task is to determine which main tool, 'mathqa' or 'scienceqa' or 'mmluqa' or 'truthfulqa', is the correct one to handle this query, based on which tool owns the most relevant scenario from the candidate list. Then, create a JSON object describing your plan.

JSON Response Format:
 {
 "plan_rationale": "A brief explanation of why you chose the parent tool, referencing the most relevant candidate scenario.",
 "primary_tool": {
 "scenario_name": "The full name of the best matching tool scenario from the candidate list.",
 "parent_tool_name": "The final parent tool name. This value MUST be similar to 'mathqa' or 'scienceqa' or 'logicqa' or 'spatialqa'."
 }
 }

Respond with ONLY the valid JSON object.

D. Experiments on more challenging and high-complexity tasks with SYNAPSE

To evaluate the performance of SYNAPSE on tasks with higher complexity and reasoning challenges, we conducted

experiments using GPT-4o on datasets extracted from LiveBench (White et al., 2025). These datasets include tasks that test logical inference, spatial reasoning, and mathematical abstraction, providing a rigorous benchmark for assessing the robustness of our SYNAPSE.

Experimental Setup. We evaluated SYNAPSE on two categories of tasks: reasoning and advanced mathematical problems. For reasoning tasks, we utilized Web of Lies (Version 2), an enhanced dataset that introduces deductive red herrings to challenge logical rigor; Zebra Puzzle, a deductive reasoning task involving multiple constraints across variables like colors and nationalities; and a Spatial Dataset, requiring the model to reason about numerical and positional attributes of solid, regular heptagons. For mathematical tasks, we employed the AMPS Hard Dataset, designed to test advanced symbolic manipulation and mathematical reasoning through challenging, randomized problem distributions.

Results. Table 3 shows centralized configurations outperforming federated approaches on reasoning tasks—Spatial (0.53 vs. 0.40), Web of Lies (0.37 vs. 0.30), and Zebra Puzzle (0.33 vs. 0.27). However, for AMPS Hard mathematical tasks, federated learning surpasses the centralized baseline (0.50 vs. 0.46), indicating better handling of client-specific variations in numerical problems. Table 4 shows prompt

Table 3. Performance of Centralized and Federated Configurations on Reasoning and Mathematical Tasks. Best test accuracies are reported for each method across the datasets.

Category	Dataset	Central Server	Federated
Reasoning	Spatial	0.53	0.40
	Web of Lies	0.37	0.30
	Zebra Puzzle	0.33	0.27
Math	AMPS Hard	0.46	0.50

transfer results from LLaMA 3.2-11B to 3.2-3B. Transferred prompts improve performance for Object Counting (+0.03) and Multi-step Arithmetic (+0.15), but decrease GSM8k performance (-0.08), suggesting task-dependent effectiveness.

These findings reveal trade-offs in federated learning and prompt transfer. While federated methods excel in mathe-

matical tasks, reasoning tasks remain challenging. Prompt transfer can improve efficiency across models but requires task-specific optimization to prevent performance losses.

Table 4. Results of Prompt Transferability from LLaMA 3.2-11B to LLaMA 3.2-3B. The table reports the performance of prompts optimized on the larger model and directly transferred to the smaller model, compared to initial prompts without optimization. Performance metrics are prediction accuracy on the task (higher is better).

Task	Initial Prompt	Transferred Prompt	Performance Change
Object Counting	0.66	0.69	+0.03
Multi-step Arithmetic	0.51	0.66	+0.15
GSM8k	0.80	0.72	-0.08

E. Communication Cost Analysis

Table 5 summarizes communication costs for experiments on GSM8k dataset, comparing SYNAPSE variants with prompt-sharing and weight-sharing baselines. The compendium size per client per round ranges from 1,067 bytes for Static-Global Compendium to 5,334 bytes for federated compendium methods. The weight-sharing baseline using Llama-3.1-8b-instruct model in fp32 precision requires 64 GB per client per round. This baseline, FedAvg (McMahan et al., 2023), aggregates full model weights from clients and serves as a comparative benchmark. SYNAPSE’s compendium-based knowledge exchange demonstrates significant communication efficiency by using compact, structured compendiums instead of large model weights, showing advantages in bandwidth-constrained environments.

Discussion Communication volume analysis shows substantial savings with SYNAPSE. Fed-Compendium-NoRouting requires 26.7 MB (5 clients) to 42.7 MB (8 clients), while weight-sharing baselines need hundreds of gigabytes (320 GB for 5 clients, 512 GB for 8 clients). Prompt-sharing baselines have intermediate costs but remain significantly higher than SYNAPSE compendium exchange. Per-client per-round bandwidth requirements are reduced by over four orders of magnitude compared to weight-sharing, enabling lower network consumption, reduced latency, and decreased costs. *These results show SYNAPSE’s compendium-based exchange achieves superior communication efficiency without accuracy loss, enabling deployment in bandwidth-constrained scenarios.*

Table 5. Communication Costs of SYNAPSE and Baselines on GSM8k

Baseline	Clients	Avg. Bytes / Client / Round	Total Bytes (All Clients)
Static-Global Compendium	5	1,067	5,334
Fed-Compendium-NoRouting	5	5,334	26,670
Fed-Compendium-FlatServer	5	5,334	26,670
Fed-ICL	5	1,104	5,521
Weight-Share (Llama-3.1-8b fp32)	5	64×10^9	320×10^9
Static-Global Compendium	8	667	5,334
Fed-Compendium-NoRouting	8	5,334	42,672
Fed-Compendium-FlatServer	8	5,334	42,672
Fed-ICL	8	1,106	8,848
Weight-Share (Llama-3.1-8b fp32)	8	64×10^9	512×10^9

F. Result Example

F.1. GSM8k Benchmark Evaluation Using llama-3.1-8b-instruct as the Client Model

<p>User Query</p> <p>If Ann is 9 years old and her brother is twice her age, how old will her brother be in 3 years?</p> <p>Retrieved SYNAPE compendium entries</p> <ul style="list-style-type: none"> • Financial and Banking Calculator • Percentage and Proportion Solver • Geometry: Shapes and Measurement • General Logic and Counting • Work, Rate, and Time Analyzer <p>Abridged reasoning</p> <p>Brother is twice Ann's age: $2 \times 9 = 18$. In 3 years: $18 + 3 = 21$.</p> <p>Final Answer</p> <p>21</p> <p>Ground Truth</p> <p>21</p>	<p>User Query</p> <p>Doctor Jones works 9 hours. Rounds take 20 minutes per inpatient (9 inpatients) and he has 10 appointments at 30 minutes each. How many hours are left to update records?</p> <p>Retrieved SYNAPE compendium entries</p> <ul style="list-style-type: none"> • Work, Rate, and Time Analyzer • Financial and Banking Calculator • Percentage and Proportion Solver • Geometry: Shapes and Measurement • General Logic and Counting <p>Abridged reasoning</p> <p>Rounds: $9 \times 20 = 180$ min. Appointments: $10 \times 30 = 300$ min. Total scheduled: 480 min; total available: $9 \times 60 = 540$ min. Remaining: $540 - 480 = 60$ min = 1 hour.</p> <p>Final Answer</p> <p>1 hour</p> <p>Ground Truth</p> <p>1 hour</p>
<p>User Query</p> <p>Jamal's phone holds $6 \times$ Brittany's capacity. Brittany's capacity is $50 \times$ the number of ducks in Jamal's photo. If Jamal holds 1800 photos, how many ducks are in the photo?</p> <p>Retrieved SYNAPE compendium entries</p> <ul style="list-style-type: none"> • Work, Rate, and Time Analyzer • Financial and Banking Calculator • Percentage and Proportion Solver • Geometry: Shapes and Measurement • Algebraic Word Problem Solver <p>Abridged reasoning</p> <p>Let $J=1800$ and $J=6B \Rightarrow B=300$. Also $B=50D \Rightarrow D=300/50=6$.</p> <p>Final Answer</p> <p>6 ducks</p> <p>Ground Truth</p> <p>6 ducks</p>	<p>User Query</p> <p>John attends a cooking class 4 times/week for 2 hours each time for 6 weeks. He learns one recipe per 1.5 hours. How many recipes does he learn?</p> <p>Retrieved SYNAPE compendium entries</p> <ul style="list-style-type: none"> • Work, Rate, and Time Analyzer • Financial and Banking Calculator • Percentage and Proportion Solver • Geometry: Shapes and Measurement • Algebraic Word Problem Solver <p>Abridged reasoning</p> <p>Weekly hours: $4 \times 2 = 8$. Total hours: $8 \times 6 = 48$. Recipes: $48/1.5 = 32$.</p> <p>Final Answer</p> <p>32 recipes</p> <p>Ground Truth</p> <p>32 recipes</p>

Figure 13. Qualitative examples of SYNAPSE solving math queries. For each query, SYNAPSE retrieves relevant compendium artifacts, reranks candidates for relevance, and synthesizes an answer via an LLM generation step.