

Interactive Restless Multi-armed Bandit Game and Swarm Intelligence Effect

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Abstract We obtain the conditions for the emergence of the swarm intelligence effect in an interactive game of restless multi-armed bandit (rMAB). A player competes with multiple agents. Each bandit has a payoff that changes with a probability p_c per round. The agents and player choose one of three options: (1) Exploit (a good bandit), (2) Innovate (asocial learning for a good bandit among n_I randomly chosen bandits), and (3) Observe (social learning for a good bandit). Each agent has two parameters (c, p_{obs}) to specify the decision: (i) c , the threshold value for Exploit, and (ii) p_{obs} , the probability for Observe in learning. The parameters (c, p_{obs}) are uniformly distributed. We determine the optimal strategies for the player using complete knowledge about the rMAB. We show whether or not social or asocial learning is more optimal in the (p_c, n_I) space and define the swarm intelligence effect. We conduct a laboratory experiment (67 subjects) and observe the swarm intelligence effect only if (p_c, n_I) are chosen so that social learning is far more optimal than asocial learning.

Keywords Multi-armed bandit, Swarm intelligence, Interactive game, Experiment, Optimal strategy

§1 Introduction

The trade-off between the exploitation of good choices and the exploration of unknown but potentially more profitable choices is a well-known problem^{6, 10, 5)}. A multi-armed bandit (MAB) provides the most typical environment for studying this trade-off. It is defined by sequential decision making among multiple choices that are associated with a payoff. The MAB problem involves the maximization of the total reward for a given period or budget. In a variety of circumstances, exact or approximated optimal strategies have been proposed^{2, 7, 11, 1, 13)}.

Recently, the MAB has also provided a good environment for the trade-off between social and asocial learning¹⁰⁾. Here, social learning is learning through observation or interaction with other individuals, and asocial learning is individual learning^{8, 10, 6, 12)}. The advantage of social learning is its cost compared with asocial learning. The disadvantage is its error-prone nature, as the information obtained by social learning might be outdated or inappropriate. In order to clarify the optimal strategy in the environment with the two trade-offs, Rendell et al. held a computer tournament using a restless multi-armed bandit (rMAB)¹⁰⁾. Here, restless means that the payoff of each bandit changes over time. There are 100 bandits in an rMAB, and each bandit has a distinct payoff independently drawn from an exponential distribution. The probability that a payoff changes per round is p_c . An agent has three options for each round: Innovate, Observe, and Exploit. Innovate and Observe correspond to asocial and social learning, respectively. For Innovate, an agent obtains the payoff information of one randomly chosen bandit. For Observe, an agent obtains the payoff information of n_O randomly chosen bandits that were exploited by the agents during the previous round. Compared to the information obtained by Innovate, that obtained by Observe is older by one round. For Exploit, an agent chooses a bandit that he has already explored by Innovate or Observe and obtains a payoff. In an rMAB environment, it is extremely difficult for agents to optimize their choices^{9, 10)}. The outcome of the tournament was that the winning strategies relied heavily on social learning. This contradicted previous studies in which the optimal strategy is a mixed one that relies on some combination of social and asocial learning. In the tournament, the cost for Observe was not very

low, as approximately 50% of the choices of Observe returns information that the agents already knew. The results of the tournament imply the inadvertent filtering of information when an agent chooses Observe, as the agents choose the best bandit during Exploit.

In this paper, we discuss whether social or asocial learning is optimal in an rMAB, where a player competes with many agents. We answer to the question why social learning is so adaptive in Rendell's tournament. We suppose that the cost of Innovate becomes higher than that of Observe in the tournament. In order to reduce the cost of Innovate, we control the exploration range n_I for Innovate, and agents obtain the best information about the bandits among n_I randomly chosen bandits. An rMAB is characterized by two parameters, p_c and n_I . We compare the average payoffs of the optimal strategies when only Innovate, only Observe, and both are available for learning using the complete knowledge of an rMAB and the information of the bandits exploited by agents. We determine the region in which each type of learning is optimal in the (n_I, p_c) plane and show that Observe is more adaptive than Innovate for $n_I = 1$. We define the swarm intelligence effect as the increase in the average payoff compared with the payoffs of the optimal strategies where only asocial learning is available. We have conducted a laboratory experiment where 67 human subjects competed with multiple agents in an rMAB. If the parameters are chosen in the region where social learning is far more optimal than asocial learning, we observe the swarm intelligence effect.

§2 Restless multi-armed bandit interactive game

An interactive rMAB game is a game in which a player competes with 120 agents using an rMAB. The player aims to maximize the total payoff over 103 rounds and obtain a high ranking among all entrants. Below, we term the population of all agents and a player as all entrants. The rMAB has $N = 100$ bandits, and we label them as $n \in \{1, 2, \dots, N = 100\}$. Bandit n has a distinct payoff $s(n)$, and we term the $(n, s(n))$ pair as bandit information. $s(n)$ is an integer drawn at random from an exponential distribution ($\lambda = 1$; values were squared and rounded to give integers mostly falling in the range of 0–10¹⁰). We denote the probability function for $s(n)$ as $\Pr(s(n) = s) = P(s)$ (left figure in Figure 1). We write the expected value of $s(n)$ as $E(S(n))$, and it is approximately 1.68. The payoff of each bandit changes independently between

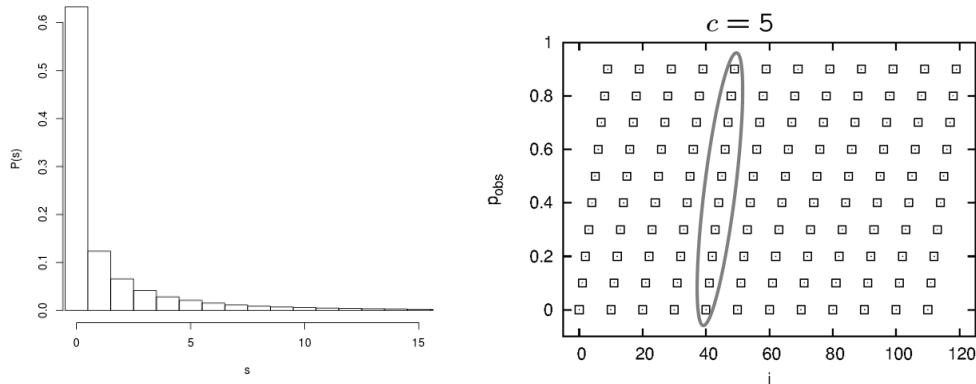


Fig. 1 Left: Plot of $P(s)$. The expected value of s is $E(s) \simeq 1.68$. Right: Parameter assignment for agent $i \in \{1, 2, \dots, 120\}$. $p_{obs}(i) = 0.1 \times (i\%10) \in \{0.0, 0.1, \dots, 0.9\}$. $c(i) = i/10 + 1 \in \{1, 2, \dots, 12\}$.

rounds with a probability p_c , with new payoff drawn at random from the same distribution.

Every entrant has his own repertoire and can store at most three pieces of bandit information. The bandit information has a time stamp when the entrant obtains it. The time stamp is updated when the entrant obtains new bandit information about the bandit. When an entrant obtains more than three pieces of bandit information, the one with the oldest time stamp is erased from the repertoire.

There are three possible moves for the entrants: Innovate, Observe, and Exploit. Innovate and Observe are learning processes to obtain bandit information. Exploit is the exploitation process that obtains some payoff.

- Innovate is individual learning, and an entrant obtains bandit information. n_I bandits are chosen at random among $N = 10^2$ bandits, and the bandit information with the maximum payoff is provided to the entrant. If there are several bandits with the same maximum payoff, one of them is chosen at random.
- Observe is social learning, and an entrant obtains the bandit information exploited by an agent during the previous round. If there are many agents who exploited a bandit, an agent is randomly chosen among them, and its bandit information is provided to the entrant. If there are no such

agents, no bandit information is provided. The information obtained by Observe is one round older than that obtained by Innovate.

- Exploit is the exploitation of a bandit. An entrant chooses a bandit from his repertoire and exploits the bandit. Even if the bandit information is $(n, s(n))$, as the information changes with a probability p_c per round, he does not necessarily receive the payoff $s(n)$.

The repertoire is updated after a move. For Innovate, the bandit information with the maximum payoff s_I among n_I randomly chosen bandits is provided to the entrant. We denote the distribution function of s_I as $P_I(s) = \Pr(s_I = s)$. Intuitively, s_I is chosen in the region of upper probability $1/n_I$ of $P(s)$. We denote the expectation value of s_I as $E(s_I)$. If $n_I > 1$, $E(s_I) > E(s)$ holds. For example, $E(s_I) \simeq 9.63$ for $n_I = 10$. By controlling n_I , we can change the cost of Innovate.

2.1 Agent strategy

We explain the strategy of the agents. The most important factor in the performance of the strategies in Rendell's tournament was the proportion of Observe in learning¹⁰⁾. The high performance of Observe originated from the inadvertent filtering of bandit information, as the agents exploited the best bandit in their repertoires. If the agents choose at random, Observe does not provide good bandit information. We take these facts into account and introduce a simple strategy for the agents with two parameters c and p_{obs} .

- c : every agent has a threshold value c . If there is no bandit in one's repertoire whose payoff is greater than c , the agent will learn by Innovate or Observe.
- p_{obs} : an agent chooses Observe with a probability p_{obs} when he learns.

We label 120 agents as $i \in \{1, 2, \dots, I = 120\}$. Agent i has the parameters $(c(i), p_{obs}(i))$. $c(i)$ is given as the quotient $i/10$ plus one. $p_{obs}(i)$ is the remainder of $i \% 10$ multiplied by 0.1. The assignment of (c, p_{obs}) to agent i is represented in the right figure in Figure 1.

2.2 Game environment

A player participated in a game and competed with N agents. However, the game did not advance on a real-time basis. Agents had already participated in the game for 1000 rounds. When a player participated in the game, 103 sequential rounds were randomly chosen from the 1000 rounds, and he competed

Now 4/100 rounds, your ranking is 33/121

The score is 19 pts

Please select from blow.

Innovate

No.10,Point.8

No.33,Point.11

No.41,Point.8

Observe

logout

Fig. 2 The interactive rMAB game online interface. A human player is presented with the present round $t/100$, his ranking among 121 entrants (one player and 120 agents), and his repertoire. He must choose one among Innovate, Exploit a bandit, and Observe. In his repertoire, only $(n, s(n))$ is shown. The bandit information from left to right indicates the newest to oldest information, respectively.

with agents for 103 rounds. We denote the round by $t \in \{-2, -1, 0, 1, 2, \dots, T = 100\}$. The scores of the player and agents were set to zero. The agents had already stored at most three pieces of bandit information in their repertoires. The player had three rounds to learn the rMAB. He could choose Innovate or Observe for three rounds and stored at most three pieces of bandit information in his repertoire. After three rounds, the rMAB game started. As the agents had already finished the game, they could not observe the information of the player. On the other hand, the player could observe the information of the agents.

The game environment was constructed as a website. The information of the agents for 1000 rounds was stored in a database of the website. The player used a tablet (7 inch) and participated in the game through a web browser. The player had to learn for three rounds and stored at most three pieces of bandit information in his repertoire. Afterwards, the game started. Figure 2 shows the interface of the rMAB game. For the present round t , the ranking and score are shown on the screen. The player had to choose an action among Innovate, Exploit, and Observe. For Exploit, the player had to choose which bandit he would exploit in his repertoire. Then, the payoff and new ranking were shown on the screen, and the game proceeded to the next round.

For the parameters (n_I, p_c) of the rMAB, we adopted the next four combinations. We call the combinations A, B, C, and D.

- A: $(n_I, p_c) = (1, 0.1)$. p_c is small and the change in the payoff of a bandit is slow. As $n_I = 1$, $E(s_I) = E(s)$, and it is difficult to find a bandit with high payoff with Innovate.
- B: $(n_I, p_c) = (10, 0.1)$. p_c is small, as in A. As $n_I = 10$, $E(s_I) \simeq 9.63$ is large, and good bandit information can be obtained with Innovate.
- C: $(n_I, p_c) = (1, 0.2)$. p_c is large, and the bandit information changes frequently. As $n_I = 1$, it is difficult to obtain good bandit information with Innovate.
- D: $(n_I, p_c) = (10, 0.2)$. p_c is large, as in C. As $n_I = 10$, good bandit information can be obtained with Innovate.

2.3 Experimental procedure

The experiment reported here were conducted at the Information Science room at Kitasato University. The subjects included students from the university, mainly from the School of Science. The number of subjects S was 67. Each subject participated in the game at most four times.

The subjects entered a room and sat down on a chair. After listening to a brief explanation about the experiment and reward, they signed a consent document for participation in the experiment. Afterwards, they logged into the experiment website using the IDs written on the consent document. The game environment was chosen among the four cases A, B, C, and D, and they started their games. After $100 + 3$ rounds, the game ended. The subjects logged into the website again to participate in a new game. Within the allotted time of approximately 40 min, most subjects participated in the game at least three times. Subjects were paid upon being released from the experiment.

There were slight differences in the experimental setup and rewards among the subjects. For the first 21 subjects (July 2014), there was no participation fee. The reward was completely determined by the number of times that they entered the Top 20 among the $120 + 1$ entrants in each game. Their rewards were a prepaid card of 300 yen (approximately \$2.50) for each placement within the Top 20. The subject could choose the game environment at the start of the game. They could choose each environment at most once, and the average number of subjects in each environment is approximately 19. They did not know the parameters of each environment. For the last 46 subjects (December

2014) there was a 1050 yen (approximately \$9) participation fee in addition to the performance-related reward. The reason for the change in the reward is to recruit more subjects. They were asked to play the game at least three times during the allotted time. The game environment was randomly chosen by the experimental program. The average number of subjects in each environment is approximately 37. A total of 67 subjects participated in the experiment, and we gathered data from approximately 56 subjects for each game environment.

§3 Optimal strategy and swarm intelligence effect

We estimate the expected payoff of the optimal strategies for the player in the rMAB game. Here, optimal means to maximize the expected total payoff in a total of $100 + 3$ rounds. For the first three rounds ($t \in \{-2, -1, 0\}$), the player could choose Innovate or Observe. After that, he could choose all three options. The optimal choice for round t is defined as the choice that maximizes the expected payoff obtained during the remaining $T - t$ rounds.

We assume that the player has the complete knowledge about the rMAB game. More concretely, he knows p_c , $E(s)$, and $E(s_I)$ about the rMAB. Furthermore, he knows the bandit information exploited during the previous round. We denote the average value of the payoff of the exploited bandit at round $t - 1$ as $\bar{O}(t)$. If the player chooses Observe for round t , the expected value of the payoff of the obtained bandit information is $\bar{O}(t)$. $\bar{O}(t)$ depends on the agents' choices in the background. It is usually the most difficult quantity to estimate for the player in the game, as it depends on the strategies of the agents. With this information, we estimated the expected value of the payoff per round for the remaining rounds for each choice.

We assume that there are M pieces of bandit information in the player's repertoire at round t . We denote them as $(n_m, s_m, t_m), m \in \{1, \dots, M\}$. Here, t_m is the round during which the player obtained the information. When, the player obtains information from Innovate or obtains updated information from Exploit at t' , $t_m = t'$. If the player obtains information from Observe at t' , $t_m = t' - 1$, as Observe returns the bandit information from the previous round, $t' - 1$.

We denote the expected value of the payoff per round for exploiting

bandit n_m from t to T as $E_m(t)$. This quantity is estimated as

$$\begin{aligned} E_m(t) &= \mathbb{E}(s) + \frac{1}{T-t+1} \sum_{t'=t}^T (s_m - \mathbb{E}(s))(1-p_c)^{t'-t_m} \\ &= \mathbb{E}(s) + \frac{(1-(1-p_c)^{T-t+1})(s_m - \mathbb{E}(s))(1-p_c)^{t-t_m}}{p_c(T-t+1)} \end{aligned} \quad (1)$$

where $(1-p_c)^{t'-t_m}$ is the probability that the bandit information does not change from t_m until t' . During this period, the payoff is s_m . If the bandit information changes until t' , the probability for it is $1-(1-p_c)^{t'-t_m}$, and the expected payoff of the bandit is given by $\mathbb{E}(s)$. By summing these values and dividing by the number of rounds $T-t+1$, we obtain the above expression.

We denote the expected payoff per round for Innovate as $I(t)$. For Innovate, a player does not receive any payoff. He only obtains bandit information, and the expected value of the payoff of the obtained bandit information is $\mathbb{E}(s_I)$. We estimate the expected value of the payoff by Innovate by assuming that the player continues to exploit the new bandit with the payoff $\mathbb{E}(s_I)$ from round $t+1$ to T as

$$I(t) = \frac{T-t}{T-t+1} \mathbb{E}(s) + \frac{(1-(1-p_c)^{T-t})(\mathbb{E}(s_I) - \mathbb{E}(s))(1-p_c)}{p_c(T-t+1)}. \quad (2)$$

As the player loses one round because of Innovate, the prefactor in front of $\mathbb{E}(s)$ and the power of $(1-p_c)$ are reduced to $(T-t)/(T-t+1)$ and $(T-t)$ as compared with those in eq.(1). If $n_I = 1$, $\mathbb{E}(s_I) = \mathbb{E}(s)$, the second term vanishes, and Innovate is almost worthless. For cases in which all of the payoffs of the bandit information in one's repertoire are zero or less than $\mathbb{E}(s)$, it might be optimal to choose Innovate. Otherwise, instead of losing one round and obtaining bandit information with a payoff $\mathbb{E}(s)$, it is optimal to choose Exploit with the maximum expected payoff. If p_c is large, even if all the payoffs in one's repertoire is zero, $(1-p_c)^{t-t_m}$ can be negligibly small, and it is optimal to choose Exploit. When $n_I > 1$ and p_c are not very large, Innovate might be optimal.

Likewise, we estimate the expected payoff per round for Observe, which we denote as $O(t)$. For Observe, a player obtains bandit information with a payoff $\bar{O}(t)$. The age of the information is one round older than the information obtained by Innovate. We change $\mathbb{E}(s_I)$ to $\bar{O}(t)$ in eq.(2). Accounting for the age of the new bandit information, we estimate $O(t)$ as

$$O(t) = \frac{T-t}{T-t+1} E(s) + \frac{(1 - (1-p_c)^{T-t})(\bar{O}(t) - E(s))(1-p_c)^2}{p_c(T-t+1)}. \quad (3)$$

Comparing $I(t)$ and $O(t)$, which is more optimal depends on p_c and $E(s_I) - \bar{O}(t)$. If p_c is small and $1 - p_c \simeq 1$, the magnitude of the relationship between $E(s_I)$ and $\bar{O}(t)$ determines which is more optimal.

The optimal strategy is to choose the action with maximum expected payoff during every round $t \in \{-2, -1, \dots, T = 100\}$. For example at $t = T$, the last round of the game, as $I(T) = O(T) = 0$ holds, it is optimal to choose Exploit for bandit m with the maximum $E_m(T)$. In the first three rounds where the player can choose only Innovate or Observe, if both p_c and n_I are small, $E(s_I) < \bar{O}(t)$ usually holds. Observe is more optimal than Innovate in this case. The situation is the same in later rounds, and the optimal strategy is a combination of Exploit and Observe. Conversely, if both p_c and n_I are large, even if $\bar{O}(t) \simeq E(s_I)$, $(1 - p_c) < 1$ and $I(t) > O(t)$ hold.

We estimate the expected payoff per round for several “optimal” strategies with a restriction on the choice of learning. We consider three strategies, and an Exploit-only strategy as a control strategy.

- I+O: The player can choose both Innovate and Observe when learning. In the first three rounds, Innovate is chosen. Then, the action with the highest expected payoff is chosen in the later rounds.
- I: The player can choose Innovate for learning. The other conditions are the same as I+O.
- O: The player can choose Observe for learning. The other conditions are the same as I+O.
- EO: The player can choose Exploit with the maximum expected payoff after the first three rounds.

The expected payoffs per round for these strategies are written as **I+O**, **I**, **O**, and **EO**, respectively. We also denote the expected payoff per round for agent i as **P(i)**. They are estimated by a Monte Carlo simulation. We have performed a simulation of a game in which 120 agents and four players with above strategies participate 10^4 times. As we have explained in the experimental procedure, the agents cannot observe the bandit information exploited by the player. Only player can observe the bandit information of the agents. As there is no interaction between the players, we can estimate the expected payoffs of the four players simultaneously. In the experiment, the player can choose Observe for the first three rounds. With the above strategies, the player can choose Innovate only

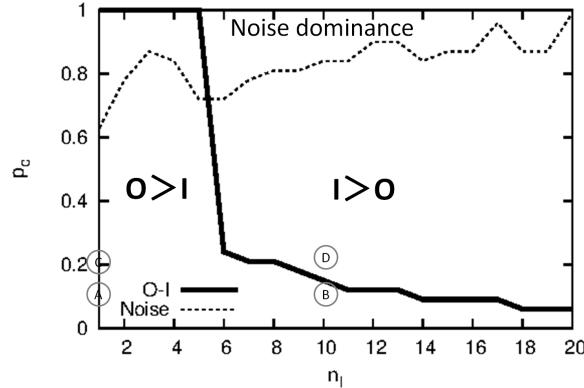


Fig. 3 Optimal learning in (n_I, p_c) . The thick solid line shows the boundary between the region $\mathbf{I} > \mathbf{O}$ and the region $\mathbf{O} > \mathbf{I}$. The dotted line shows the boundary beyond which $\mathbf{EO} \simeq \mathbf{I} + \mathbf{O}$.

for simplicity. The players and agents compete on equal terms.

We summarize the results in Figure 3. In (n_I, p_c) plane, we show which strategy is more optimal, I or O. The thick solid line shows the boundary where $\mathbf{I} = \mathbf{O}$. In the lower-left region $\mathbf{O} > \mathbf{I}$ holds. As n_I and p_c are small, the relationship $\overline{O}(t) > E(s_I)$ holds, and Observe becomes a optimal learning method. In the upper-right region, $\mathbf{I} > \mathbf{O}$ holds. n_I is large, and $E(s_I)$ is greater than or comparable to $\overline{O}(t)$. As p_c is large, the one round delay for exploiting the bandit information obtained by Observe might be crucial. The thin dotted line shows the boundary beyond which $\mathbf{I} + \mathbf{O}$ is comparable with \mathbf{EO} . As p_c is large, the player can obtain comparable payoffs by only exploiting a good bandit in his repertoire. There is neither an exploitation–extrapolation trade-off nor a social–asocial learning trade-off above the dotted line. It is a noise-dominant region.

One can understand why there is no social–asocial learning trade-off in Rendell’s tournament¹⁰⁾. In the tournament, they set $n_I = 1$ and $n_O \geq 1$. Here, n_O is the amount of bandit information obtained by Observe. If $n_I = 1$, as we have explained previously, $E(s_I) = E(s)$ holds. If p_c is small, $\overline{O}(t)$ is usually greater than $E(s)$, as agents exploit the good bandit in their repertoire. Then, an agent can obtain good bandit information by Observe, and Observe becomes an optimal learning method. If p_c is too large, instead of trying to obtain good information with Innovate, it is optimal to wait spontaneous changes in the bandit information in the repertoire. Exploiting a good bandit in one’s

repertoire (EO strategy) is enough, and no other strategy cannot exceed the performance of EO.

In the region where $\mathbf{O} > \mathbf{I}$ and $\mathbf{I} + \mathbf{O} > \mathbf{EO}$, social learning is effective, and a swarm intelligence can emerge. We define the swarm intelligence effect as the increase in the performance compared to \mathbf{I} . In the next section, we estimate the swarm intelligence effect for human subjects. As for the choice of (n_I, p_c) , we have studied four cases A:(1,0.1), B:(10,0.1), C:(1,0.2), and D:(10,0.2). We show the positions for these choices in figure 3. For cases A and C, $\mathbf{O} >> \mathbf{I}$, and one expect to observe the swarm intelligence effect in human subjects. For cases B and D, where $\mathbf{O} \simeq \mathbf{I}$ and $\mathbf{O} < \mathbf{I}$, one does not expect to observe it.

We make a comment about the definition of the swarm intelligence effect. For the estimation of \mathbf{I} , we assume that the player knows p_c , $E(s)$, and $E(s_I)$ and can choose the best option among Exploit and Innovate. The player has to estimate this information from his actions in the real game. If the player cannot choose Observe, his performance cannot exceed \mathbf{I} . The definition of the swarm intelligence effect only provides a lower limit. Toyokawa¹²⁾ defined it as the surplus in performance compared to when the same player can only choose Innovate. Our definition has the advantage that it can be estimated easily without performing an experiment. The same reasoning applies to $\mathbf{I} + \mathbf{O}$. In this case, the player knows everything that is related with his decision making. $\mathbf{I} + \mathbf{O}$ provides an upper limit on the performance of the player in the game.

§4 Experimental results

In this section, we explain the experimental results. We estimate the swarm intelligence effect for human subjects. We perform a regression analysis of the performance of each subjects in each experimental environment.

4.1 Swarm intelligence effect

We calculated the total payoff of each subject for 100+3 rounds in each game environment. We divided the total payoff by 100 and obtained the average payoff per round. For each game environment, we estimated the average value of the average payoffs per round for approximately 56 subjects and denote it as \mathbf{H} . This represents the average performance of human subjects in each case. We compare \mathbf{H} with $\mathbf{I} + \mathbf{O}$, \mathbf{I} , \mathbf{O} , and $\mathbf{P}(i)$ for agent i . Figure 4 show the results for cases A, B, C, and D. We explain the results of each case.

A: Case A is in the region where where $\mathbf{O} > \mathbf{I}$, and Observe is optimal for

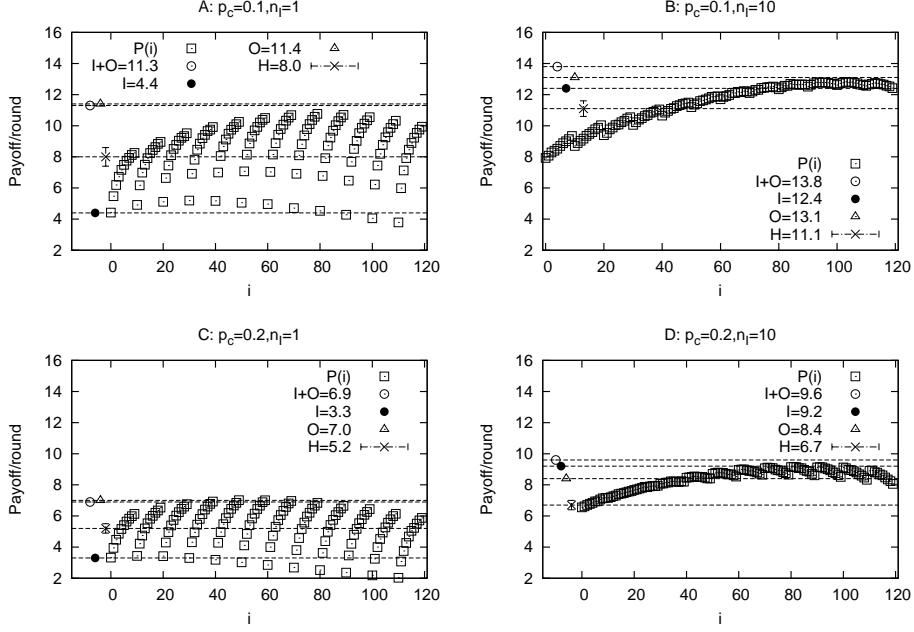


Fig. 4 Plots of $\mathbf{P}(\mathbf{i})$ (\square), $\mathbf{I}+\mathbf{O}$ (\circ), \mathbf{I} (\bullet), \mathbf{O} (\triangle) and \mathbf{H} (\times). A: $(p_c, n_I) = (0.1, 1)$, $\mathbf{H}(54) = 8.0 \pm 0.6$. B: $(p_c, n_I) = (0.1, 10)$, $\mathbf{H}(65) = 11.1 \pm 0.5$. C: $(p_c, n_I) = (0.2, 1)$, $\mathbf{H}(54) = 5.2 \pm 0.3$. D: $(p_c, n_I) = (0.2, 10)$, $\mathbf{H}(52) = 6.7 \pm 0.3$. The number in each parentheses is the number of subjects in each case.

learning. As $\mathbf{I}+\mathbf{O}\simeq\mathbf{O}$ and \mathbf{O} is much greater than \mathbf{I} , one can expect the swarm intelligence effect. In fact, \mathbf{H} , which is plotted with a chain line, is higher than \mathbf{I} . For a fixed value of c , $\mathbf{P}(\mathbf{i})$ increases with p_{obs} . For the dependence of $\mathbf{P}(\mathbf{i})$ on c for a fixed value of p_{obs} , there is a maximum for some c . For $p_{obs} = 0.0$, $\mathbf{P}(\mathbf{i})$ is maximum at $c \sim 4$. For $p_{obs} = 0.9$, $\mathbf{P}(\mathbf{i})$ is maximum at $c \sim 8.5$. The agent can obtain good bandit information by Observe, and they had better to adopt large c .

B: Case B is in the region where $\mathbf{O}>\mathbf{I}$. However, it is near the boundary for $\mathbf{I}=\mathbf{O}$, and the difference between \mathbf{O} and \mathbf{I} is small. One cannot expect the swarm intelligence effect. In fact, \mathbf{H} is below \mathbf{I} . As $\mathbf{I}\simeq\mathbf{O}$, subjects could not improve their performance by Observe. One see $\mathbf{I}+\mathbf{O}>\mathbf{O}$, and the difference between $\mathbf{I}+\mathbf{O}$ and \mathbf{O} is small. As $n_I = 10$ is large, Innovate is frequently more optimal than Observe. For example, if an agent finds that the payoff of good bandit information changes to zero in round t

by Exploit, one can suspect that Observe does not provide good bandit information. In particular, if the bandit is good, and the payoff is high, one can assume that many agents also exploited the bandit. Then Observe should provide bandit information with zero payoff in round $t + 1$ with a high probability.

P(i) is an increasing function of p_{obs} when c is small. When c is large, **P(i)** does not depend on c very much. When c is small, agents can easily obtain bandit information whose payoff is greater than c . Then, the agent exploits the not so good bandit. On the other hand, if the agent obtains bandit information by Observe, he can obtain good bandit information, as the bandit's payoff exceeds the other agents' c . Observe is more optimal than Innovate when c is small. However, when c is large, the agent can obtain good bandit information by Innovate, as n_I is large. By Observe, the agent can obtain good bandit information, and there is not a big difference in the performance of **I** and **O**. As a result, **P(i)** does not depend very much on p_{obs} when c is large.

- C: Case C is in the region where $\mathbf{O} > \mathbf{I}$, $\mathbf{I} + \mathbf{O} \simeq \mathbf{O}$, and \mathbf{O} is much greater than \mathbf{I} , as with case A. One can observe the swarm intelligence effect because \mathbf{H} is greater than \mathbf{I} . Because p_c is large, the expected payoffs and average payoff of the subjects are lower than those for case A.
- D: Case D is in the region where $\mathbf{I} > \mathbf{O}$, and Innovate is optimal for learning. As $\mathbf{I} + \mathbf{O} > \mathbf{I}$, Observe is optimal in some cases. When c is large, **P(i)** is a decreasing function of p_{obs} . As both p_c and n_I are large, instead of obtaining good bandit information by Observe, Innovate succeeds in obtaining new and good bandit information. When c is small, as in case B, the agent can obtain better bandit information by Observe than Innovate. One cannot observe the swarm intelligence effect, as in case B.

4.2 Regression analysis of the performance of individual subjects

We perform a statistical analysis of the variation in the payoffs of the subjects in the four cases. We examined the factors that made strategies successful by using a linear multiple regression analysis. In Rendell's tournament, there were five predictors in the best-fit model for the performance of the strategies¹⁰⁾. Among them, we considered three predictors: r_{learn} , the proportion of moves that involved learning of any kind; r_{obs} , the proportion of learning moves

that were Observe; and Δt_{learn} , the average round between learning moves. Other predictors were the variance in the number of rounds to first use of Exploit and a qualitative predictor of whether or not the agent program estimates p_c . For the latter, we suppose that human subjects estimated p_c , or they could notice whether the frequency of the change in bandit information is high or low. For the former predictor, it is impossible to estimate it, as the subjects participated in the game at most once for each case. We do not include these two predictors in the regression model. We denote the average payoff per rounds for subject j as $payoff(j)$. The multiple linear regression model is written as $payoff(j) = a_0 + a_1 \cdot r_{learn}(j) + a_2 \cdot r_{obs}(j) + a_3 \cdot \Delta t_{learn}(j)$. We select the model with maximum \tilde{R}^2 . The results are summarized in Table 1.

Table 1 Parameters of the linear multiple regression model predicting the average payoff per round in each game environment. From the second to fourth columns, the intercepts and regression coefficients for r_{learn} , r_{obs} , and Δt_{learn} are shown. n.s. for $p > 0.05$, * for $p < 0.05$, ** for $p < 10^{-2}$, *** for $p < 10^{-3}$ and **** for $p < 10^{-4}$.

Case	Intercept	r_{learn}	r_{obs}	Δt_{learn}	\tilde{R}^2
A(53)	10.0 (****)	-11.0(**)	3.3 ($p = 0.16$)	n.s.	0.253
B(65)	14.1 (****)	-10.7 (*)	n.s.	n.s.	0.076
C(54)	8.37 (***)	-8.2 (**)	3.0 (*)	-0.49 ($p = 0.06$)	0.186
D(52)	8.7 (****)	-7.2 (**)	1.4 ($p = 0.23$)	n.s.	0.144
ALL(224)	12.0 (****)	-13.8 (****)	1.3 ($p = 0.15$)	n.s.	0.246

r_{learn} had a negative effect on the performance of the subjects, as in Rendell's tournament. This result suggests that it is suboptimal to invest too much time in learning, as one cannot obtain any payoffs for learning. For r_{obs} , the results are not consistent with the results of Rendell's tournament. There, the predictor had a strong positive effect, which reflected the fact that the best strategy was to almost exclusively choose Observe rather than Innovate. In our experiment, the predictor seems to have a positive effect for cases A, C, and D. For cases A and C, it is consistent with the results in the previous section because $\mathbf{O} > \mathbf{I}$, and Observe is more optimal than Innovate. In case D, as \mathbf{H} is much less than both \mathbf{I} and \mathbf{O} , obtaining good bandit information from the agents by Observe might improve the performance.

§5 Conclusion

In this paper, we attempt to clarify the optimal strategy in a two trade-offs environment. Here, the two trade-offs are the trade-off of exploitation–exploration and that of social–asocial learning. For this purpose, we have developed an interactive rMAB game, where a player competes with multiple agents. The player and agents choose an action from three options: Exploit a bandit, Innovate to obtain new bandit information, and Observe the bandit information exploited by other agents. The rMAB has two parameters, p_c and n_I . p_c is the probability for a change in the environment. n_I is the scope of exploration for asocial learning. The agents have two parameters for their decision making, p_{obs} and c . p_{obs} is the probability for Observe when the agents learn, and c is the threshold value for Exploit.

We have estimated the average payoff of the optimal strategy with some restrictions on learning and complete knowledge about rMAB and the bandit information exploited during the previous round. We consider three types of optimal strategies, **I+O**, **I**, and **O**, where both Innovate and Observe, Innovate, and Observe are available. In the (n_I, p_c) plane, we have derived the strategy that is more optimal, either **O** or **I**. Furthermore, we have defined the swarm intelligence effect as the surplus of the performance of **I**. The estimate of the swarm intelligence effect provides only a lower bound for it; however, the estimation is easy and objective. We also point out that the swarm intelligence effect can be observed in the region of the (n_I, p_c) plane where **O** is more optimal than **I**. We have performed an experiment with 67 subjects and have gathered approximately 56 samples for the four cases of (n_I, p_c) . If (n_I, p_c) are chosen in the region where **O** is far more optimal than **I**, we have observed the swarm intelligence effect. If (n_I, p_c) are chosen near the boundary of the two regions or in the region where **I** is more optimal than **O**, we did not observe the swarm intelligence effect. We have performed a regression analysis of the performance of each subject in each case. Only the proportion of learning is the effective factor in the four cases. In contrast, the proportion of the use of Observe for learning is not significant.

As the agent’s decision making algorithm is too simple, it is difficult to believe that the conditions for the emergence of the swarm intelligence in Figure 3 are general. In addition, the analysis of the human subjects is too superficial, as we only studied the correlation between the performance and some predictive factors. With these points in mind, we make three comments about future problems.

The first one is a more elaborate and autonomous model of the decision making in an rMAB environment. The algorithm needs to estimate p_c , $E(s)$, $E(s_I)$, and $O(t)$ for round t on the basis of the data that the agent has obtained through his choices. Then, the agent can choose the most optimal option during each round and maximize the expected total payoff on the basis of these estimates. This is an adaptive autonomous agent model. With this model, we can understand the decision making of humans in the rMAB game more deeply. It is impossible to understand human decision making completely with experimental data. On the basis of the model, we can detect the deviation in human decision making and propose a decision making model for a human that can be tested in other experiments.

The second one is the collective behavior of the above adaptive autonomous agents or humans. It is necessary to clarify how the conditions for the emergence of the swarm intelligence effect would change. In the case of a population of adaptive autonomous agents, they would estimate the optimal value of p_{obs} for the environment (n_I, p_c) and collectively realize the optimal value. The optimal strategy should be neither **I** nor **O** but a mixed strategy of Innovate and Observe. Then, the condition for the emergence of the swarm intelligence effect is that the performance of **I+O** is equal to that of **I**. If the performance of the former is greater than that of the latter for any (n_I, p_c), the swarm intelligence effect can always emerge, except for the noise-dominant region. After that, we can study the conditions with human subjects experimentally. A human subject participates in the rMAB game as a player, as in this study, or many human players participate in the game to compete with each other. The target is how and when humans collectively solve the rMAB problem.

The third one is the design of an environment in which swarm intelligence works. In this study, we choose the rMAB interactive game and study the conditions for the emergence of the swarm intelligence effect for a player. However, there are many degrees of freedom in the design of the game. For example, when an agent observes, there are many degrees of freedom regarding how bandit information is provided to the agent. In the present game environment, the probability that a bandit exploited in the previous round is chosen is proportional to the number of agents who have exploited it. Instead, we can consider an environment in which the bandit information of the most exploited bandit is provided, the bandit information of the agents who are near the agent is provided, or the player can choose a bandit by showing him the number of agents

who have exploited the bandit. We think these changes should affect the choice and performance of the player. It was shown experimentally that by providing subjective information about a bandit, the performance of the subjects diminished¹²⁾. We think that the interaction between the design of the environment and the decision making, performance, and swarm intelligence effect should be a very important problem in the industrial usage of the swarm intelligence effect.

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