

Modeling Normative Multi-Agent Systems from a Kelsenian Perspective

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Abstract. Standard Deontic Logic (SDL) has been used as the underlying logic to model and reason over Multi-Agent Systems governed by norms (NorMAS). It is known that SDL is not able to represent contrary-to-duty (CTD) scenarios in a consistent way. That is the case, for example, of the so-called Chisholm paradox, which models a situation in which a conditional obligation that specifies what must be done when a primary obligation is violated holds. In SDL, the set of sentences that represent the Chisholm paradox derives inconsistent sentences. Due to the *autonomy* of the software agents of a NorMAS, norms may be violated and the underlying logic used to model the NorMAS should be able to represent *violation scenarios*. The contribution of this paper is three-fold: (i) we present how Kelsenian thinking, from his jurisprudence in the context of legal ontologies, and Intuitionist Hybrid Logic can be adopted in the modeling of NorMAS, (ii) discuss how this approach overcomes limitations of the SDL and (iii) present a discussion about normative conflict identification according to Hill's functional taxonomy, that generalizes from standard identification by impossibility-of-joint-compliance test.

Keywords: Normative Multi-Agent Systems; Norms; Contrary-to-duties; CTD Paradoxes; Kelsenian Jurisprudence; Intuitionistic Hybrid Logic

1 Introduction

Standard Deontic Logic (SDL) is the traditional logic for reasoning about normative aspects, such as, obligations, permissions and prohibitions [29]. This logic has been widely applied to formalize Normative Multi-Agent Systems (NorMAS) [4], which are Multi-Agent Systems (MAS) that adopt norms as a means to coordinate and restrict the behavior of software agents, aiming at achieving the overall goal of the system. In NorMAS, norms can be viewed as a regulatory mechanism that guides software agents, by establishing the desirable/ideal behavior of them.

It is known that there are scenarios that only can be represented in SDL by sets of sentences that derive inconsistent sentences or derive sentences with a counterintuitive reading [25]. Such scenarios are called contrary-to-duty (CTD)

paradoxes. The CTD paradoxes that are counterintuitive in the common sense reading are not considered so critical as the CTD paradoxes that derive inconsistent sentences [24]. The Chisholm paradox [9] is a scenario that results in an inconsistent theory when formalized in SDL. This paradox models a situation in which: (1) there is a primary obligation stating what ought to be done; (2) there is a compatible-with-duty obligation that says what must be done when the primary obligation is fulfilled; (3) there is a conditional obligation (also called CTD obligation) that determines what must be done if the primary obligation is violated; and (4) there is a factual claim that indicates that the primary obligation was violated. Note that a CTD obligation is an obligation that is only enforced when a violation occurs.

The SDL formalization of the Chisholm paradox is the conjunction of the following sentences:

$$\begin{aligned} O(p) && (1) \\ O(p \Rightarrow q) && (2) \\ \neg p \Rightarrow O(\neg q) && (3) \\ \neg p && (4) \end{aligned}$$

where O is the deontic modality for obligation, p is a propositional variable and so is q .

SDL has the following axioms and inference rules:

- TAUT** All the tautologies of classical propositional logic.
- K** $O(p \Rightarrow q) \Rightarrow (O(p) \Rightarrow O(q))$
- D** $O(p) \Rightarrow \neg O(\neg p)$
- MP** if $\vdash p$ and $\vdash p \Rightarrow q$ then $\vdash q$
- OB-NEC** if $\vdash p$ then $\vdash O(p)$.

As for the proof of inconsistency, from (2) and **K** we get $O(p) \Rightarrow O(q)$, and then from (1) and **MP**, we get $O(q)$, but by **MP** alone we get $O(\neg q)$ from (3) and (4). From these two conclusions, by Propositional Calculus, we get $O(q) \wedge O(\neg q)$, contradicting the SDL principle that obligations cannot conflict [29], formally $\vdash \neg(O(q) \wedge O(\neg q))$, i.e., q cannot be obligatory and forbidden simultaneously.

The impossibility of representing CTD scenarios is a relevant limitation of SDL since in many applications norms can be violated. This kind of reasoning is needed in NorMAS because software agents are endowed with autonomy and sometimes the actual behavior of an agent deviates from the ideal behavior (according to the norms), i.e., an agent may choose not to comply with the system's norms in order to achieve its own goals [4,6]. Then, it is natural to define CTD obligations in MAS telling what should be done when a norm is violated. Applications that deal with fault-tolerance, for instance, also need to

represent CTD scenarios, i.e., once an obligation is violated, a corrective action must be performed [8].

Another challenge, together with the modeling of CTD scenarios, that must be taken into account in NorMAS, is that the system must be able to deal with conflicts among norms. Two norms are in conflict, for instance, when one obliges an agent to perform an action that is being prohibited by other norm simultaneously. There is a famous example in the literature illustrating this situation, where a soldier is obliged to kill but he has a (moral) obligation to not to kill. When there is a normative conflict, the agent cannot choose to comply with both norms and whatever the agent does or refrain from doing will lead to a norm violation [28]. In MAS, considering that an agent can play different roles, it is common that conflicts of this nature arise between norms associated with the different roles [10]. (Several techniques to deal with normative conflicts are nicely summarized in [27].)

In this paper we propose a novel approach to formalize NorMAS that overcomes limitations of SDL. We represent norms from a *Kelsenian perspective* with Intuitionistic Hybrid Logic (IHL) [3] as underlying logic. The Kelsenian perspective does not assign truth-values to norms. Rather, they are understood as *worlds* (in a Kripke structure) where properties of a MAS hold or not. In Hybrid Logic terminology, norms are understood as nominals. The second contributions of this paper is a discussion on how NorMAS with a Kelsenian interpretation of norms, or Kelsenian NormAS for short, deal with normative conflicts while considering Hill's taxonomy [19].

The remainder of this paper is organized as follows. In Section 2 we present some approaches that also do not consider SDL as a suitable way to represent normative systems. Section 3 presents the concept of norm according to the Kelsenian Jurisprudence and shows how Intuitionistic Hybrid Logic (IHL) complies with Kelsenian requirements. In Section 4, we present an example of CTD scenario in a NorMAS and discuss how it is modeled in our approach. Section 5 describes how our approach represents normative conflicts taking into account the Hill's taxonomy for normative conflicts. Section 6 concludes this paper and presents directions for future work.

2 Related Work

The approach described in [7] describes a deontic action logic with stratified norms for designing and reasoning about fault-tolerant systems. When a violation occurs in these systems, a non-desirable system state is reached and CTD norms are applied to recover the system. In order to be able to represent CTD norms, the logic described considers that there are different levels of norms in the specification, and violations of norms are tolerated only at some levels.

In [13], the third author and others propose Intuitionistic Description Logic (iALC), for reasoning about laws. iALC is a notation variant of IHL, designed to cope with the ontological requirements in [12]. In particular, iALC handles Commitment III, discussed in Section 3, very nicely as iALC concepts represent different normative systems (understood as a collection of norms) quite naturally. However, as opposed to iALC, IHL has support for (action-based) temporal reasoning. A characteristic that we believe makes it more suitable to model and reason about NormAS than iALC.

There is a line of thinking that agrees with the idea a norm should not have a truth-value, which we adopt in this paper. In [22,16,5], for instance, the authors advocate that SDL is not a suitable logic for modeling normative systems. In addition to the impossibility to represent CTD in a consistent way they agree that it makes no sense to assign norms truth values because norms are *non-descriptive*, i.e., obligations and prohibitions (imperative norms) demand and allow behavior, respectively. They can be applied or not, can be followed or not and can be evaluated based on other norms (when a norm is judged based on a moral code, for instance). Some approaches in the literature have proposed to *reconstruct* SDL as a logic of normative propositions (e.g. [23,1]) or as a logic of imperatives (e.g. [14,15]) to cope with this problem, as pointed out in [5]. We agree with their point of view that norms should not have truth value. However, we do not follow the proposal of reconstructing SDL. We chose a different approach, based on *Intuitionistic Hybrid Logics* (IHL), where norms are nominals, as discussed in Section 3.

3 Kelsenian Normative Multi-Agent Systems

The Theory of Law, also called Jurisprudence, is interested in to determine the meaning of the concept of *law*. Basically there are two views concerning this concept. The first view states that a law is a *fact* that can be perceived by our senses. It is an object of natural sciences, such as, physics and chemistry. The second view states that a law is a norm, i.e., it is a rule that establishes what ought to be done. The Pure Theory of Law adopts the view that the law is a norm [20].

Kelsenian Jurisprudence [21], in a nutshell, advocates that “the law”, in legal terms, or the norms in a NormAS, is a set of individual regulatory statements, each of them created to enforce a positively desired behavior in the system. In [12], the authors propose the following requirements (or ontological commitments) from an analysis of Kelsenian Jurisprudence. Table 1 interprets the three ontological commitments in [12] in the context of NormAS. Henceforth, we will refer to the contents of Table 1 as *Kelsenian regulation*.³

³ In this paper, we discuss Commitments (I) and (II) only, without losing consistency of presentation. Commitment (III) requires a broaden presentation on the interconnections of normative systems.

- (I) Individuals are norms;
- (II) There is a transitive and reflexive relationship between individuals and norms that reflects a precedence relationship between norms;
- (III) There are normative connections between individual norms in different normative systems or between different agent organizations in the same normative system.

Table 1. Kelsenian regulation

Ontological commitment (I) is fulfilled by the choice of Hybrid Logic [26,2] (an extension to Modal Logics that adds a new sort of propositional symbols which are true at *exactly* one possible world), and the representation of norms as nominals. In Hybrid Logics, there is a new kind of operator, called satisfaction operator $@_a$. It allows for the declaration of satisfaction statements $a : \varphi$ (sometimes written $@_a\varphi$), denoting that the formula φ is true at the point to which the nominal a refers to. More formally, the nominal a represents a *possible world* in the Kripke semantics of IHL.

Before justifying the need for intuitionism, let us first recall the models of Classical Hybrid Logic. They are Kripke structures

$$(W, R, V)$$

where W is a non-empty set of worlds, R is a world accessibility relation $R \subseteq W \times W$ and $V : W \times AP \rightarrow \{0, 1\}$, where AP is the set of atomic propositions, together with a function g , called an assignment, that, to each nominal, assigns an element of W . An assignment g' is an *a-variant* of g if g' agrees with g on all nominals save possibly a . The relation $M, g, w \models \varphi$ is defined by structural induction in the language of the Classical Hybrid Logic, where M is a model, g is an assignment, w is an element of W , and φ is a formula. The satisfaction relation for Classical Hybrid Logics is as follows:

$$\begin{aligned} M, g, w \models p &\Leftrightarrow V(w, p) = 1, \\ M, g, w \models a &\Leftrightarrow w = g(a), \\ M, g, w \models \varphi \wedge \psi &\Leftrightarrow M, g, w \models \varphi \text{ and } M, g, w \models \psi, \\ M, g, w \models \varphi \Rightarrow \psi &\Leftrightarrow M, g, w \models \varphi \text{ implies } M, g, w \models \psi, \\ M, g, w \models \neg\varphi &\Leftrightarrow M, g, w \not\models \varphi, \\ M, g, w \models \Box\varphi &\Leftrightarrow \text{for any element } v \text{ of } W \text{ such that } wRv, \text{ it is} \\ &\quad \text{the case that } M, g, v \models \varphi, \\ M, g, w \models a : \varphi &\Leftrightarrow M, g, g(a) \models \varphi. \end{aligned}$$

From the perspective of Kelsenian regulation, Classical Hybrid Logic is not enough because commitment (II) requires a pre-order among norms. Ordering is a natural way to prevent norm conflicts [27] as it imposes a precedence between norms. Classical Hybrid Logic, however, does not offer it, as opposed to *Intuitionistic Hybrid Logic*.

Formulas of Intuitionistic Hybrid Logic are the same as those of Classical Hybrid Logic. However, connectives \vee and \Diamond are *primitive* as they are not intuitionistically definable in terms of the other connectives, contrary to the classical case.

A model for Intuitionistic Hybrid Logic is a tuple

$$(W, \leq, \{D_w\}_{w \in W}, \{\sim_w\}_{w \in W}, \{R_w\}_{w \in W}, \{V_w\}_{w \in W})$$

where W is a non-empty set partially ordered by \leq , for each w , D_w is a non-empty set such that $w \leq v$ implies $D_w \subseteq D_v$, for each w , \sim_w is an equivalence relation on D_w such that $w \leq v$ implies $\sim_w \subseteq \sim_v$, for each w , R_w is a binary relation on D_w such that $w \leq v$ implies $R_w \subseteq R_v$, and for each w , V_w is a function that to each ordinary propositional symbol p assigns a subset of D_w such that $w \leq v$ implies $V_w(p) \subseteq V_v(p)$.

W represents the set of states of knowledge and for each state $w \in W$, D_w denotes the set of possible worlds that are known in such a state. $V_w(p)$ denotes the set of worlds in which the proposition p is known to be true.

Given a model

$$M = (W, \leq, \{D_w\}_{w \in W}, \{\sim_w\}_{w \in W}, \{R_w\}_{w \in W}, \{V_w\}_{w \in W})$$

and an element $w \in W$, a w -assignment is a function g that to each nominal assigns an element of D_w . Note that if g is a w -assignment and $w \leq v$, then g is also a v -assignment (this is used in the clauses below for implication and the \Box operator). The relation $M, g, w, d \models \varphi$ is defined by induction, where w is an element of W , g is a w -assignment, d is an element of D_w , and φ is a formula.

$$\begin{aligned} M, g, w, d \models p &\Leftrightarrow d \in V_w(p), \\ M, g, w, d \models a &\Leftrightarrow d \sim_w g(a), \\ M, g, w, d \models \varphi \wedge \psi &\Leftrightarrow M, g, w, d \models \varphi \text{ and } M, g, w, d \models \psi, \\ M, g, w, d \models \varphi \vee \psi &\Leftrightarrow M, g, w, d \models \varphi \text{ or } M, g, w, d \models \psi, \\ M, g, w, d \models \varphi \Rightarrow \psi &\Leftrightarrow \text{for all } v \geq w, M, g, v, d \models \varphi \text{ implies} \\ &\quad M, g, v, d \models \psi, \\ M, g, w, d \models \perp &\Leftrightarrow \text{falsum} \\ M, g, w, d \models \Box \varphi &\Leftrightarrow \text{for any element } v \geq w, \text{ for all } e \in D_v, \\ &\quad dR_v e \text{ implies } M, g, v, e \models \varphi, \\ M, g, w, d \models \Diamond \varphi &\Leftrightarrow \text{for some } e \in D_w, dR_w e \text{ and } M, g, w, e \models \varphi, \\ M, g, w, d \models a : \varphi &\Leftrightarrow M, g, w, g(a) \models \varphi. \end{aligned}$$

We now define Kelsenian NorMAS in terms of an IHL model. (See Def. 1.) Following Kelsen, for each norm we have a world $w \in W$ in the IHL model. In other words, the pair (W, \leq) captures the normative part of the NorMAS. The MAS description is represented by the tuple $(D, R^{Act, A})$ of the IHL model, where D denotes the set of the states of MAS and $R^{Act, A}$ its (action-labeled) transition relation, for actions in Act , defined as the disjoint union of the relations for each agent in A . (In this paper we consider relation \sim , in the IHL model, to be empty,

without loss of generality, that is, there is no equivalence among states of the MAS.)

Definition 1 (Kelsenian NormAS). A Kelsenian Normative Multi-Agent System is an IHL model:

$$\mathcal{K} = (N, \leq, D, \emptyset, \{R_n^{Act, A}\}_{n \in N}, V)$$

where Act is a finite set of actions; and A is a finite set of agents, such that, for all $n \in N$, $R_n^{Act, A} = \bigcup_{i=1}^{|A|} R_i$, with $R_i \subseteq D \times Act \times D$ and $V : D \times AP \rightarrow \{0, 1\}$ with AP the set of atomic propositions.

A state $n_i \in N$ is such that norm n_i is *upheld*. Note that upheld does not mean “the norm holds” as norms have no truth value. It means that R_i complies with n_i , behaving accordingly to n_i .

4 Kelsenian NormAS for a CTD scenario

When we consider Normative MAS (NormAS), scenarios where a norm regulated MAS may misbehave must be accounted for in a proper logical formalization. An example of misbehavior is when negative transitions are specified, assuming a labeled state-transition system formalization of MAS, and a conflict arises between MAS behavior and its regulation. In this section, we present an example that illustrates a situation in which an agent of a NormAS chooses not to comply with a norm and there is a CTD norm saying what should be done if it occurs. We will consider in this example the Contract Net [11] agent interaction protocol, standardized by the Foundation for Intelligent Physical Agents (FIPA).

Informally, the Contract Net protocol specifies that: (i) when an (initiator) agent *realizes* it has a problem to solve, (ii) it may *announce* it to other agents, (iii) which in turn will *bid* for the task of solving (part) of the given problem, (iv) being then notified by the initiator which was the *awarded* agent, that then (v) *expedites* solving the problem. Figure 1 illustrates the protocol.

Let us consider now a subclass of Contract Net implementations regulated by the norms in Table 2.

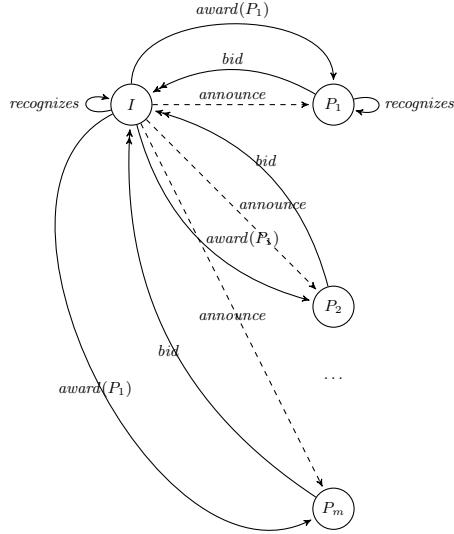


Fig. 1. An instance of the Contract Net protocol

Norm #	Norm description
n_1 .	“Once a problem is announced then all agents must bid.”
n_2 .	“Once a problem is announced, all agents must bid and then an agent must be awarded.”
n_3 .	“If it is not the case that a problem was announced and agents bade for it then there must not be an awarded agent.”

Table 2. A regulated Contract Net

Now, if in an implementation of Contract Net with norms from Table 2, say an electronic commerce system, an agent refuses to perform action *bid*, it would give rise to a misbehaving NormAS, as action *bid*, at the same time, must and does not occur. This scenario is depicted in Figure 2 where P_3 is the misbehaving agent that does not implement action *bid*, with the interrupted or negative transition (loosely dashed, with double tip and a ray symbol denoting interruption) denoting that the agent did not perform action *bid*.

What we have just described is an example of the CTD scenarios that become paradoxical when Deontic Logic is chosen as the underlying logic to model and reason on NormAS. More precisely, to see that the misbehaved Contract Net implementation in Figure 2 is an instance of the so-called Chisholm paradox

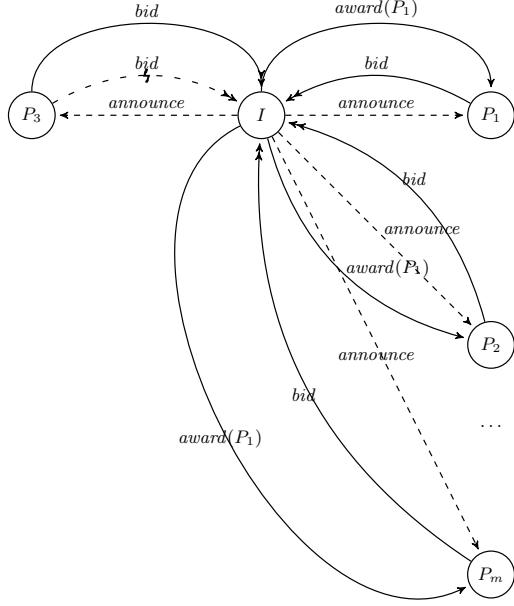


Fig. 2. A misbehaving Contract Net implementation

(see Section 1) we just need to replace the proposition p from the Chisholm paradox for ‘‘Once a problem is announced then all agents bid’’, and proposition q for ‘‘an agent is awarded’’. When action bid is not performed by an agent, thus leading to the negative transition in Figure 2, ‘‘all agents must bid’’ becomes false and so does p .

Example 1 (Kelsenian NormAS for the misbehaving Contract Net). The Kelsenian NormAS for the Contract Net instance in Figure 2 is the tuple

$$\mathcal{K} = (N, \leq, D, \emptyset, \{R_n^{Act, A}\}_{n \in N}, V)$$

where $N = \{n_1, n_2, n_3, n_1 \sqcap n_2, n_1 \sqcap n_2 \sqcap n_3\}$, where operation \sqcap denotes the *meet* operation of IHL’s underlying Heyting algebra [18]. (Kripke models for Intuitionistic Logics are Heyting algebras, and therefore lattices.) A state n_i is such that norm n_i is upheld. The ordering \leq is as pictured in Figure 3, $D = \{\text{Recognized}, \text{Announced}, \text{Bade}, \text{Awarded}\}$, relation R is pictured in Figure 2, $A = \{I, P_1, P_2, \dots, P_n\}$, $Act = \{\text{recognize}, \text{announce}, \text{bid}, \text{award}\}$, and $V = \emptyset$. Let p denote ‘‘Once a problem is announced then all agents bid’’.

Figure 3 pictures an IHL model for a NormAS that is an instance of the Chisholm paradox. In this model each norm is represented by a nominal (world), then $n_1 \models \top$ and $n_2 \models \top$ indicate the existence of norms n_1 and n_2 in the NormAS.

The meet of these two worlds $n_1 \sqcap n_2 \models \top$ is a world in which it is obligatory to comply with n_1 and with n_2 . The nominal n_3 is the world in which $\neg p$ holds (and then proposition p does not hold), that is, it represents a case where the agency does not comply with the first norm of Table 2.⁴

The least element of (N, \leq) is $n_1 \sqcap n_2 \sqcap n_3$, that is, the meet of n_1 , n_2 and n_3 denoting a world which is not a model for “Once a problem is announced then all agents bid” since it is ensured that $\neg p$ holds.

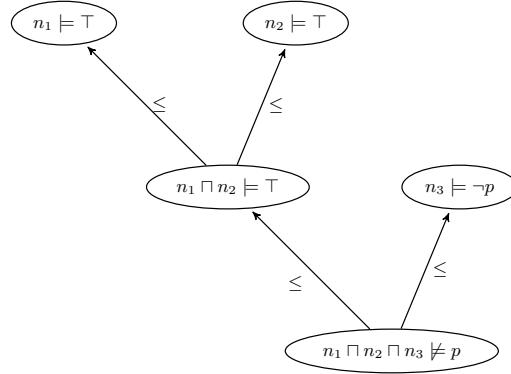


Fig. 3. A Kelsenian NorMAS for a normative contradiction

5 Hill’s taxonomy and Kelsenian NorMAS

In [19] H. Hamner Hill proposes a taxonomy for normative conflict, an important reference in the normative systems literature in the context of norm conflict identification and resolution [27]. Hill argues, developing ideas by Kelsen and others, that normative conflicts identified by impossibility-of-joint-compliance test, when it is impossible for a norm subject to comply with both of a pair of norms, is too restrictive. Let us briefly recall why.

The impossibility-of-joint-compliance test can only be applied to norms that one can construct obedience statements for, that is, a statement that certifies compliance of a norm subject with a given norm. For example, let us consider norm n_1 from Table 2 from Section 4, that says “Once a problem is announced then all agents must bid”. Its obedience statement would be “A problem has been announced and all agents bade”. Therefore, impossibility-of-joint-compliance test

⁴ Note that not being a model for p and $\neg p$ holding in a state are different things. In the former case state $n_1 \sqcap n_2 \sqcap n_3$ is *not* a model for p while in the latter case state n_3 *is* a model for $\neg p$, that is, $\neg p$ is satisfiable.

only applies to deontic imperatives, i.e., it cannot detect conflicts involving permissions.

There are, however, scenarios where: (i) deontic permissions conflict with deontic permissions, (ii) there are regulatory modalities other than the deontic modalities, as in power-conferring norms [17], and (iii) conflicts considering non-deontic norms.

Quoting Hill [19, pg. 238 and 239]:

For Kelsen, a normative conflict is a clash of forces, forces which operate in different directions in a single point. (...) The generic phenomenon of normative conflict occurs when norms interact in ways that the function of one or more of the norms involved is thwarted.

In order to be able to detect conflicts involving permissions, Hill replaced the concept of obedience by the concept of conformity. For each norm that indicates permission, there are two conformity statements, as follows: (i) the first conformity statement indicates that the agent chose to do what is being allowed by the permission; and (ii) the second conformity statement indicates that the agent chose not to do what is being allowed by the permission. Then, considering a scenario in which there is a prohibition stating that a cannot be done and there is a permission stating that a can be done. In this example, there is a conflict between the prohibition and the first conformity statement of the permission.

Hill defines a functional taxonomy of normative conflicts: (i) normative contradiction, (ii) normative collision, and (iii) normative competition. Normative contradictions are “a purely deontic phenomena” where only deontic norms may contradict each other. In normative collisions, deontic imperatives and deontic permissions collide, mixtures of deontic and non-deontic norms collide or only non-deontic norms collide. Normative competition regards norm conflicts from distinct normative systems. In what follows, we discuss about Kelsenian NorMAS for normative contradictions and normative collisions between deontic imperatives and deontic permissions. In this paper we do not consider normative competitions since we are only interested in dealing with conflicts in one NorMAS.

Normative contradiction. Normative contradictions are conflicts that can be detected by the impossibility-of-joint-compliance test. They come from scenarios where the norm subject finds oneself faced with duties one cannot fulfil. Normative contradictions violate the principle of consistency of SDL that says: $O(a) \Rightarrow \neg O(\neg a)$. This kind of conflict usually occurs between norms imposed by different authorities. In SDL, normative contradiction is a conflict in the form:

$$O(a) \wedge O(\neg a)$$

Note that, the so-called contrary-to-duty paradoxes, when SDL is used to formalize them, result in this kind of conflict. One such contrary-to-duty paradox

is Chisholm paradox which is embodied in the “misbehaved” Contract Net implementation in Figure 2. When formalized in Deontic Logic, it gives rise to an inconsistent theory. An example of Kelsenian NorMAS model for a normative contradiction is illustrated in Figure 3.

Normative collision. Normative collisions are conflicts that do not involve logic inconsistency but involve functional incompatibility. In SDL, they are conflicts in the form:

$$O(\neg a) \wedge P(a)$$

or similarly,

$$O(a) \wedge P(\neg a)$$

where P denotes the deontic modality for permission.

Following the same approach for deontic imperatives, in our approach deontic permissions do not have truth value: there are nominal states denoting when the agency avails itself of a permission and otherwise.

We can illustrate this situation by considering Contract Net instance from Figure 1, the norms from Table 2, and a permission *prescribing* that “An agent is allowed not to bid”. Note that this normative system is slightly different from the “misbehaving” Contract Net instance in Figure 2. There, the negative transition is part of the *description* of the system (the behavior of the system) and here the permission not to act is at the *prescription* (what it should do) level. There is clearly a normative collision here between norm n_1 and the given permission, if the agent choose to do what is being permitted (not to bid). Figure 3 pictures the IHL model for the normative collision example where world n_i denotes norm n_i , with $i \in \{1, 2, 3\}$, n_4 denotes the state where the agency avails itself of the given permission and $\overline{n_4}$ otherwise. The resulting Kelsenian NorMAS is a lattice with combinations of n_i , n_4 , and $\overline{n_4}$, as depicted in Figure 4. (All arrows denote relation \leq .)

The infimum of the lattice is $n_1 \sqcap n_2 \sqcap n_3 \sqcap n_4 \sqcap \overline{n_4} \models \perp$ denoting a state of the Kelsenian NorMAS where no property holds since a join between states n_4 and $\overline{n_4}$ results in a state where agents chose not to bid and chose to bid making it impossible to uphold norms n_1 to n_3 . In particular, either norm n_1 or n_2 can not be upheld when agents choose not to bid (n_4) as they must when a problem is announced. (A similar argument is used to justify state $n_3 \sqcap \overline{n_4} \models \perp$.) On the other hand, when either n_1 or n_2 can be upheld in a state when agents may bid and therefore $(n_1 \sqcup n_2) \sqcap \overline{n_4} \models \top$. Norms are upheld in states where single norms alone are upheld (not a join or meet of worlds) or a permission is upheld, such as n_1 or n_4 . The join of all the worlds (not to be confused with their conjunction) represents the supremum of the lattice.



Fig. 4. A Kelsenian NormMAS for a normative collision

6 Conclusion

Regulatory mechanisms are required in order to deal with the autonomy and (possible) heterogeneity of software agents that compose a Multi-Agent System (MAS). Norms can be applied in such systems as way of coordination. They are social constraints that establish explicitly, for instance, which actions a software agent is permitted, prohibited or obliged to perform.

Standard techniques for modeling and reasoning on Normative Multi-Agent Systems (NorMAS) fall prey of the interpretation that norms are formulas. When Deontic Logic is chosen as underlying formalism for NorMAS, CTD scenarios cannot be represented because its SDL formalization is inconsistent. This is a relevant problem in NorMas, where autonomous software agents should be able to violate norms. In order to cope with this problem, the approach presented in this paper adopts Intuitionistic Hybrid Logic as underlying logic, meeting the commitments of Kelsenian Jurisprudence.

An important issue in systems regulated by multiple norms, is the possibility of conflicts among norms. We discuss the possible kinds of conflicts, according

to the Hill’s taxonomy, that may arise in a system governed by multiple norms and show how the approach presented allows us to construct models for conflicts involving deontic imperatives and deontic permissions in an elegant and simple way. In order to exemplify our IHL model, we use a Contract Net scenario that is a FIPA protocol for multi-agent communication.

Future work includes further developing our framework, in particular regarding the normative connections from Commitment III possibly in the directions of iALC where relations between concepts denote classes of norms. Another direction is the automation of our approach to simulate and (bound) model check Kelsenian NormAS where the model is the semantics of a description in an appropriate specification language.

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