

Addendum to: Summary Information for Reasoning About Hierarchical Plans

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Abstract.

Hierarchically structured agent plans are important for efficient planning and acting, and they also serve (among other things) to produce “richer” classical plans, composed not just of a sequence of primitive actions, but also “abstract” ones representing the supplied hierarchies. A crucial step for this and other approaches is deriving precondition and effect “summaries” from a given plan hierarchy. This paper provides mechanisms to do this for more pragmatic and conventional hierarchies than in the past. To this end, we formally define the notion of a precondition and an effect for a hierarchical plan; we present data structures and algorithms for automatically deriving this information; and we analyse the properties of the presented algorithms. We conclude the paper by detailing how our algorithms may be used together with a classical planner in order to obtain abstract plans.

INTRODUCTION

This paper provides effective techniques for automatically extracting abstract actions and plans from a supplied hierarchical agent plan-library. Hierarchically structured agent plans are appealing for efficient acting and planning as they embody an expert’s domain control knowledge, which can greatly speed up the reasoning process and cater for non-functional requirements. Two popular approaches based on such representations are Hierarchical Task Network (HTN) [8, 11] planning and Belief-Desire-Intention (BDI) [19] agent-oriented programming. While HTN planners “look ahead” over a supplied collection of hierarchical plans to determine whether a task has a viable decomposition, BDI agents interleave this process with acting in the real world, thereby trading off solution quality for faster responsiveness to environmental changes. Despite these differences, HTN and BDI systems are closely related in both syntax and semantics, making it possible to translate between the two representations [20, 21].

While HTN planning and BDI execution are concerned with decomposing hierarchical structures (offline and online, respectively), one may perform other kinds of reasoning with them that do not necessarily require or amount to decompositions. For example, [6] and [22, 23] perform reasoning to coordinate the execution of abstract steps, so as to preempt potential negative interactions or exploit positive ones.

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In [7], the authors propose a novel application of such hierarchies to produce “richer” classical plans composed not just of sequences of primitive actions, but also of “abstract” steps. Such abstract plans are particularly appealing in the context of BDI and HTN systems because they respect and re-use the domain control knowledge inherent in such systems, and they provide flexibility and robustness: if a refinement of one abstract step happens to fail, another option may be tried to achieve the step.

A pre-requisite for these kinds of reasoning is the availability of meaningful preconditions and effects for abstract steps (i.e., compound tasks in HTN systems or event-goals in BDI languages). Generally, this information is not supplied explicitly, but embedded within the decompositions of an abstract step. This paper provides techniques for extracting this information automatically. The algorithms we develop are built upon those of [6] and [22, 23], which calculate offline the precondition and effect “summaries” of HTN-like hierarchical structures that define the agents in a multi-agent system, and use these summaries at runtime to coordinate the agents or their plans. The most important difference between these existing techniques and ours is that the former are framed in a propositional language, whereas ours allow for first-order variables. This is fundamental when it comes to practical applicability, as any realistic BDI agent program will make use of variables. A nuance worth mentioning between our work and that of Clement et al. is that the preconditions we synthesise are standard classical precondition formulas (with disjunction), whereas their preconditions are (essentially) two sets of literals: the ones that must hold at the start of *any* successful execution of the entity, and the ones that must hold at the start of *at least one* such execution. Yao et al. [26] extend the above two strands of work to allow for concurrent steps within an agent’s plan, though still not first-order variables.

Perhaps the only work that computes summaries (“external conditions”) of hierarchies specifying first-order variables is [24]. The authors automatically extract a weaker form of summary information (what we call “mentioned” literals) to inform the task selection strategy of the UMCP HTN planner: tasks that can possibly establish or threaten the applicability of other tasks are explored first. They show that even weak summary information can significantly reduce backtracking and increase planning speed. However, the authors only provide insights into their algorithms for computing summaries.

We note that we are only concerned here with how to extract abstract actions (with corresponding preconditions and effects), and eventually abstract plans, from a hierarchical

know-how structure. Consequently, unlike existing useful and interesting work [12, 9, 2, 1], our approach does not directly involve *guiding a planner* toward finding a suitable primitive plan. We also do not aim to build new “macro” actions from sample primitive solution plans, as done in [3], for example.

Thus, the contributions of this paper are as follows. First, we develop formal definitions for the notions of a precondition and an effect of an (abstract) event-goal. Second, we develop algorithms and data structures for deriving precondition and effect summaries from an event-goal’s hierarchy. Unlike past work, we use a typical BDI agent programming language framework; in doing so, we allow for variables in agent programs—an important requirement in practical systems. Our chosen BDI agent programming language cleanly incorporates the syntax and semantics of HTN planning as a built-in feature, making our work immediately accessible to both communities. Finally, we show how derived event-goal summaries may be used together with a classical planner in order to obtain abstract plans (which can later be further refined, if desired, to meet certain properties [7]).

THE HIERARCHICAL FRAMEWORK

Our definitions, algorithms, and results are based on the formal machinery provided by the CANPlan [21] language and operational semantics. While designed to capture the essence of BDI agent-oriented languages, it directly relates to other hierarchical representations of procedural knowledge, such as HTN planning [8, 11], both in syntax and semantics.

A CANPlan BDI agent is created by the specification of a *belief base* \mathcal{B} , i.e., a set of ground atoms, a *plan-library* Π , i.e., a set of plan-rules, and an *action-library* Λ , i.e., a set of action-rules. A plan-rule is of the form $e(\mathbf{v}):\psi \leftarrow P$, where $e(\mathbf{v})$ is an *event-goal*, \mathbf{v} is a vector of distinct variables, ψ is the *context condition*, and P is a *plan-body* or *program*.³ The latter is made up of the following components: primitive actions (*act*) that the agent can execute directly; operations to add (+b) and remove (-b) beliefs; tests for conditions (? ϕ); and event-goal programs (!e), which are simply event-goals combined with the label “!”. These components are composed using the sequencing construct $P_1; P_2$. While the original definition of a plan-rule also included declarative goals and the ability to specify partially ordered programs [25], we leave out these constructs here and focus only on an AgentSpeak-like [18], typical BDI agent programming language.

There are also additional constructs used by CANPlan internally when attaching semantics to constructs. These are the programs *nil*, $P_1 \triangleright P_2$, and $(\{\psi_1 : P_1, \dots, \psi_n : P_n\})$. Intuitively, *nil* is the empty program, which indicates that there is nothing left to execute; program $(\{\psi_1 : P_1, \dots, \psi_n : P_n\})$ represents the plan-rules that are relevant for some event-goal; and program $P_1 \triangleright P_2$ realises failure recovery: program P_1 should be tried first, failing which P_2 should be tried. The complete language of CAN, then, is described by the grammar

$$\begin{aligned} P ::= & \text{ nil } | \text{ act } | ?\phi | +b | -b | !e | P_1; P_2 | P_1 \triangleright P_2 | \\ & (\psi_1 : P_1, \dots, \psi_n : P_n). \end{aligned}$$

The behaviour of a CANPlan agent is defined by a set of derivation rules in the style of Plotkin’s structural single-step

³ In [21] an event-goal is of the form $e(\mathbf{t})$ where \mathbf{t} is a vector of terms. Here, we replace \mathbf{t} with \mathbf{v} and assume WLOG that $\forall t_i \in \mathbf{t}, \psi \supset (t_i = v_i)$, where $v_i \in \mathbf{v}$.

operational semantics [16]. The *transition relation* on a configuration is defined using one or more derivation rules. Derivation rules have an *antecedent* and a *conclusion*: the antecedent can either be empty, or it can have transitions and auxiliary conditions; the conclusion is a single transition. A *transition* $C \xrightarrow{} C'$ within a rule denotes that configuration C yields configuration C' in a single execution step, where a configuration is the tuple $\langle \mathcal{B}, \mathcal{A}, P \rangle$ composed of a belief base \mathcal{B} , a program P , and the sequence \mathcal{A} of actions executed so far. Construct $C \xrightarrow{t} C'$ denotes a transition of type t , where $t \in \{\text{bdi}, \text{plan}\}$; when no label is specified on a transition both types apply. Intuitively, **bdi**-type transitions are used for the standard BDI execution cycle, and **plan**-type transitions for (internal) deliberation steps within a *planning* context. By distinguishing between these two types of transitions, certain rules can be disallowed from being used in a planning context, such as those dealing with BDI-style failure handling.

We shall describe three of the CANPlan derivation rules. The rule below states that a configuration $\langle \mathcal{B}, \mathcal{A}, !e \rangle$ evolves into a configuration $\langle \mathcal{B}, \mathcal{A}, (\{\Delta\}) \rangle$ (with no changes to \mathcal{B} and \mathcal{A}) in one **bdi**- or **plan**-type execution step, with $(\{\Delta\})$ being the set of all relevant plan-rules for e , i.e., the ones whose handling event-goal unifies with e ; *mgu* stands for “most general unifier” [13]. From $(\{\Delta\})$, an applicable plan-rule—one whose context condition holds in \mathcal{B} —is selected by another derivation rule and the associated plan-body scheduled for execution.

$$\frac{\Delta = \{\psi_i \theta : P_i \theta \mid e' : \psi_i \leftarrow P_i \in \Pi \wedge \theta = \text{mgu}(e, e')\}}{\langle \mathcal{B}, \mathcal{A}, !e \rangle \xrightarrow{} \langle \mathcal{B}, \mathcal{A}, (\{\Delta\}) \rangle}$$

The **Plan** construct incorporates HTN planning as a built-in feature of the semantics. The main rule defining the construct states that a configuration $\langle \mathcal{B}, \mathcal{A}, \text{Plan}(P) \rangle$ evolves into a configuration $\langle \mathcal{B}', \mathcal{A}', \text{Plan}(P') \rangle$ in one **bdi**-type execution step if the following two conditions hold: (i) configuration $\langle \mathcal{B}, \mathcal{A}, P \rangle$ yields configuration $\langle \mathcal{B}', \mathcal{A}', P' \rangle$ in one **plan**-type execution step, and (ii) it is possible to reach a *final* configuration $\langle \mathcal{B}'', \mathcal{A}'', \text{nil} \rangle$ from $\langle \mathcal{B}', \mathcal{A}', P' \rangle$ in a finite number of **plan**-type execution steps. Thus, executing the single **bdi**-type step necessitates zero or more internal “look ahead” steps that check for a successful HTN execution of P .

Unlike plan-rules, any given action program will have exactly one associated action-rule in the action-library Λ . Like a STRIPS operator, an action-rule $\text{act} : \psi \leftarrow \Phi^+; \Phi^-$ is such that act is a symbol followed by a vector of distinct variables, and all variables free in ψ , Φ^+ (the add list) and Φ^- (the delete list) are also free in act . We additionally expect any action-rule $\text{act} : \psi \leftarrow \Phi^-; \Phi^+$ to be *coherent*: that is, for all ground instances $\text{act}\theta$ of act , if $\psi\theta$ is consistent, then $\Phi^+\theta \cup \{\neg b \mid b \in \Phi^-\theta\}$ is consistent. For example, while the rule R corresponding to an action $\text{move}(X, Y)$ with precondition $\text{at}(X) \wedge \neg \text{at}(Y)$ (or $X \neq Y$) and postcondition $\neg \text{at}(X) \wedge \text{at}(Y)$ is coherent, the same rule with precondition *true* is not, as there will then be a ground instance of R such that its precondition is consistent but its postcondition is not: both its add and delete lists contain the same atom.

ASSUMPTIONS

We shall now introduce some of the definitions used in the rest of the paper and concretise the rest of our assumptions. As usual, we use \mathbf{x} and \mathbf{y} to denote vectors of distinct variables,

and \mathbf{t} to denote a vector of (not necessarily distinct) terms. Moreover, since the language of CANPlan allows variables in programs, we shall frequently make use of the notion of a *substitution* [13], which is a finite set $\{x_1/t_1, \dots, x_n/t_n\}$ of elements where x_1, \dots, x_n are distinct variables and each t_i is a term with $t_i \neq x_i$. We use $E\theta$ to denote the expression obtained from any expression E by simultaneously replacing each occurrence of x_i in E with t_i , for all $i \in \{1, \dots, n\}$.

We assume that the plan-library does not have recursion. Formally, we assume that a *ranking* exists for the plan-library, i.e., that it is always possible to give a child a smaller rank (number) than its parent. We define a ranking as follows.

Definition 1 (RANKING) A *ranking* for a plan-library Π is a function $\mathcal{R}_\Pi : E_\Pi \mapsto \mathbb{N}_0$ from event-goal types mentioned in Π to natural numbers, such that for all event-goals $e_1, e_2 \in E_\Pi$ where e_2 is the same type as some $e_3 \in \text{children}(e_1, \Pi)$, we have that $\mathcal{R}_\Pi(e_1) > \mathcal{R}_\Pi(e_2)$. ■

In addition, we define the following two related notions: first, given an event-goal type e , $\mathcal{R}_\Pi(e)$ denotes the *rank* of e in Π ; and second, given any event-goal $e(\mathbf{t})$ mentioned in Π , we define $\mathcal{R}_\Pi(e(\mathbf{t})) = \mathcal{R}_\Pi(e(\mathbf{x}))$ (where $|\mathbf{x}| = |\mathbf{t}|$), i.e., the rank of an event-goal is equivalent to the rank of its type. In order that these and other definitions also apply to event-goal *programs*, we sometimes blur the distinction between event-goals e and event-goal programs $!e$.

Finally, we assume that context conditions are written with appropriate care. Specifically, if there is no environmental interference, whenever a plan-rule is applicable it should be possible to successfully execute the associated plan-body without any failure and recovery; this disallows rules such as $e : \text{true} \leftarrow ?\text{false}$. Our definition makes use of the notion of a *projection*: given any configuration $\langle \mathcal{B}, \mathcal{A}, P \rangle$, we define the *projection* of the first component of the tuple as $C|_{\mathcal{B}}$, the second as $C|_{\mathcal{A}}$, and the third as $C|_P$.

Definition 2 (COHERENT LIBRARY) A plan-library Π is *coherent* if for all rules $e : \psi \leftarrow P \in \Pi$, ground instances $e\theta$ of e , and belief bases \mathcal{B} , whenever $\mathcal{B} \models \psi\theta\theta'$ (where $\psi\theta\theta'$ is ground) there is a successful HTN execution $C_1 \dots C_n$ of $P\theta\theta'$ (relative to Π) with $C_1|_{\mathcal{B}} = \mathcal{B}$. A *successful HTN execution* of a program P relative to a plan-library is any finite sequence of configurations $C_1 \dots C_n$ such that $C_1|_P = P$, $C_n|_P = \text{nil}$, and for all $0 < i < n$, $C_i \xrightarrow{\text{plan}} C_{i+1}$. ■

Intuitively, the term *HTN execution* simply denotes a BDI execution in which certain BDI-specific derivation rules associated with failure and recovery have not been used.

SUMMARY INFORMATION

We can now start to define what we mean by preconditions and postconditions/effects of event-goals; some of these definitions are also used later in the algorithms. As a first step we define these notions for our most basic programs.

⁴ We define the function $\text{children}(\hat{e}, \Pi) = \{e \mid e' : \psi \leftarrow P \in \Pi, \hat{e} \text{ and } e' \text{ are the same type, } P \text{ mentions } !e\}$, where two event-goals are the *same type* if they have the same predicate symbol and arity. The *type* of an event-goal $e(\mathbf{t})$ is defined as $e(\mathbf{x})$, where $|\mathbf{x}| = |\mathbf{t}|$.

A basic program is either an *atomic program* or a *primitive program*. Formally, a program P is an *atomic program* (or simply *atomic*) if $P = !e \mid \text{act} \mid +b \mid -b \mid ?\phi$, and P is a *primitive program* if P is an atomic program that is not an event-goal program. Then, like the postcondition of a STRIPS action, the *postcondition* of a primitive program is simply the atoms that will be added to and removed from the belief base upon executing the program. Formally, the *postcondition* of a primitive program P relative to an action-library Λ , denoted $\text{post}(P, \Lambda)$, is the set of literals $\text{post}(P, \Lambda) =$

$$\begin{cases} \emptyset & \text{if } P = ?\phi, \\ \{b\} & \text{if } P = +b, \\ \{-b\} & \text{if } P = -b, \\ \Phi^+\theta \cup \{-b \mid b \in \Phi^-\theta\} & \text{if } P = \text{act} \text{ and there exists} \\ & \text{an } \text{act}' : \psi \leftarrow \Phi^+; \Phi^- \in \Lambda \\ & \text{such that } \text{act} = \text{act}'\theta. \end{cases}$$

The postcondition of a test condition is the empty set because executing a test condition does not result in an update to the belief base. The postcondition of an action program is the combination of the add list and delete list of the associated action-rule, after applying the appropriate substitution.

While this notion of a postcondition as applied to a primitive program is necessary for our algorithms later, we do not also need the matching notion of a *precondition* of a primitive program. Such preconditions are already accounted for in context conditions of plan-rules, by virtue of our assumption (Definition 2) that the latter are coherent. What we do require, however, is the notion of a *precondition* as applied to an event-goal. This is defined as any formula such that whenever it holds in some state there is at least one successful HTN execution of the event-goal from that state.

Definition 3 (PRECONDITION) A formula ϕ is said to be a *precondition* of an event-goal $!e$ (relative to a plan- and an action-library) if for all ground instances $!e\theta$ of $!e$ and belief bases \mathcal{B} , whenever $\mathcal{B} \models \phi\theta$, there exists a successful HTN execution $C_1 \dots C_n$ of $!e\theta$, where $C_1|_{\mathcal{B}} = \mathcal{B}$. ■

Unlike the postcondition of a primitive program, the postcondition of an event-goal program—and indeed any arbitrary program P —is non-deterministic: it depends on what plan-rules are chosen to decompose P . There are, nonetheless, certain effects that will be brought about irrespective of such choices. We call these *must literals*: literals that hold at the end of *every* successful HTN execution of P .

Definition 4 (MUST LITERAL) Let P be a program and l a literal where its variables are free in P . Then, l is a *must literal* of P (relative to a plan- and action-library) if for any ground instance $P\theta$ of P and successful HTN execution $C_1 \dots C_n$ of $P\theta$, we have that $C_n|_{\mathcal{B}} \models l\theta$. ■

A desirable consequence of the two definitions above is that any given set of must literals of an event-goal, like the postcondition of an action, is consistent whenever the event-goal's precondition is consistent.

Theorem 1 Let e be an event-goal, ϕ a precondition of e (relative to a plan-library Π and an action-library Λ), and L^{mt} a set of must literals of e (relative to Π and Λ). Then, for all ground instances $e\theta$ of e , if $\phi\theta\theta'$ is consistent for some ground substitution θ' , then so is $L^{mt}\theta$.

PROOF. We prove this by contradiction. First, note that since $L^{mt}\theta$ is a set of ground literals (by Definition 4 and because $e\theta$ is ground), if $L^{mt}\theta$ is consistent, then for all literals $l, l' \in L^{mt}$ it is the case that $l\theta \neq \overline{l}\theta$ (i.e., $l\theta$ is not the complement of $\overline{l}\theta$). Now let us assume that the theorem does not hold. This means that there must exist a ground instance $e\theta$ of e , such that $\phi\theta\theta'$ is consistent for some ground substitution θ' , but $l\theta = \overline{l}\theta$ for some $l, l' \in L^{mt}$.

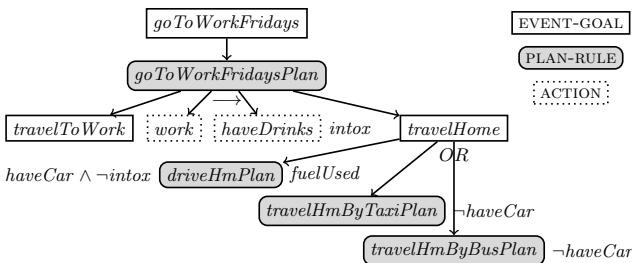
Since $\phi\theta\theta'$ is consistent, there is at least one truth assignment for its (ground) atoms that satisfies $\phi\theta\theta'$. Suppose \mathcal{B} is the set of ground atoms consisting only of those mentioned in $\phi\theta\theta'$ that were assigned the truth value *true*. Observe that $\mathcal{B} \models \phi\theta\theta'$. Then, according to Definition 3 (Precondition), there must exist a successful HTN execution $C_1 | \dots | C_n$ of $e\theta$ such that $C_1|_{\mathcal{B}} = \mathcal{B}$. Moreover, since l, l' are must literals of e and $e\theta$ is ground, by Definition 4, both $l\theta$ and $l'\theta$ are also ground. By the same definition, $C_n|_{\mathcal{B}} \models l\theta$ and $C_n|_{\mathcal{B}} \models l'\theta$. However, according to our assumption, $l\theta = \overline{l}\theta$, and therefore $C_n|_{\mathcal{B}} \models \overline{l}\theta$. This contradicts the fact that $C_n|_{\mathcal{B}} \models l'\theta$, because $C_n|_{\mathcal{B}}$ is ground due to our assumption in Section “ASSUMPTIONS” that Π is coherent. \square

In addition to must literals, there are two related notions. The first, called *may summary conditions* in [6], defines literals that hold at the end of at least one successful HTN execution of the program, and the second, weaker notion defines literals that are simply mentioned in the program or in one of its “descendant” programs; such literals may or may not be brought about when the program executes. It is this second notion, called *mentioned literals*, that we use.

Definition 5 (MENTIONED LITERAL) If P is a program, its *mentioned literals* (relative to a plan-library Π and an action-library Λ), denoted $mnt(P)$, is the set $mnt(P) =$

$$\left\{ \begin{array}{ll} post(P, \Lambda) & \text{if } P = +b \mid -b \mid act \mid ?\phi, \\ mnt(P_1) \cup mnt(P_2) & \text{if } P = P_1; P_2, \\ \{l\theta' \mid e' : \psi \leftarrow P' \in \Pi, & \text{if } P = !e, \\ e = e'\theta, l \in mnt(P'), & \\ \theta' \text{ is any substitution} \} & \end{array} \right. \blacksquare$$

We use this weaker notion because the stronger notion of a may summary condition in [6] is not suitable for our approach, which reasons about plans that will not be interleaved with one another—i.e., plans that will be scheduled as a sequence. For example, consider the figure below, which shows a plan-library for going to work on Fridays, possibly one belonging to a larger library from an agent-based simulation. The expressions to the left and right sides of actions/plan-rules are their preconditions and postconditions, respectively.



Observe that *fuelUsed* is actually never asserted in the context of the hierarchy shown, because literal $\neg intorx$ (“not intoxicated”) in the context condition of *driveHmPlan* is contradicted by literal *intox*. However, the algorithms in [6] will still classify *fuelUsed* as a may summary condition of plan *goToWorkFridaysPlan*, because some other plan may have a step asserting $\neg intorx$ —perhaps a step that involves staying overnight in a hotel nearby—that can be ordered to occur between *haveDrinks* and *travelHome*.

Since we cannot rely on such steps, we settle for a weaker notion—mentioned literals—than the corresponding definition of a may summary condition. By our definition there can be literals that are mentioned in some plan-body but in fact can never be asserted, because of interactions that preclude the particular plan-body which asserts that literal from being applied. We avoid the approach of disallowing interactions like the one shown above in order to use the stronger notion of a may summary condition because such interactions are natural in BDI and HTN domains: event-goals such as *travelHome* are, intuitively, meant to be self-contained “modules” that can be “plugged” into any relevant part of a hierarchical structure in order to derive all or just some of their capabilities.

Finally, we conclude this section by combining the above definitions of must and mentioned literals to form the definition of the *summary information* of a program.

Definition 6 (SUMMARY INFORMATION) If P is a program, its *summary information* (relative to a plan-library and an action-library) is a tuple $\langle P, \phi, L^{mt}, L^{mn} \rangle$, where ϕ is a precondition of P if P is an event-goal program, and $\phi = \epsilon$ otherwise; L^{mt} is a set of must literals of P ; and L^{mn} is a set of mentioned literals of P . \blacksquare

EXTRACTING SUMMARY INFORMATION

With the formal definitions now in place, in this section we provide algorithms to extract summary information for event-goals in a plan-library. Moreover, we illustrate the algorithms with an example, and analyse their properties.

Basically, we extract summary information from a given plan-library and action-library by propagating up the summary information of lower-level programs, starting from the leaf-level ones in the plan-library, until we eventually obtain the summary information of all the top-level event-goals.

To be able to identify must literals, we need to be able to determine whether a given literal is definitely undone, or *must undone*, and possibly undone, or *may undone* in a program. Informally, a literal l is must undone in a sequence P of atomic programs if the literal’s negation is a must literal of some atomic program in P . Formally, then, given a program P and the set Δ of summary information of all atomic programs in P , a literal l is *must undone* in P relative to Δ , denoted $Must\text{-}Undone}(l, P, \Delta)$, if there exists an atomic program P' in P and a literal $l' \in L^{mt}$, with $\langle P', \phi, L^{mt}, L^{mn} \rangle \in \Delta$, such that $l = \overline{l}'$, that is, l is the complement of l' .

Similarly, we can informally say that a literal l is may undone in a program P if there is a literal l' that is a mentioned (or must) literal of some atomic program in P such that l' may become the negation of l after variable substitutions. Formally, given a program P and the set Δ of summary infor-

Algorithm 1 Summ(Π, Λ)

Require: Plan-library Π and action-library Λ .
Ensure: Set of summary info. of event-goal types in Π .

- 1: $\Delta \leftarrow \{\langle P, \epsilon, \text{post}(P, \Lambda), \text{post}(P, \Lambda) \rangle \mid P \text{ is a primitive program mentioned in } \Pi\}$
- 2: $E \leftarrow \{e(\mathbf{x}) \mid e \text{ is an event-goal mentioned in } \Pi\}$
- 3: **for** $i \leftarrow \min(R)$ **to** $\max(R)$ **where**
 $R = \{\mathcal{R}_\Pi(e) \mid e \in E\}$ **do** // Recall $\mathcal{R}_\Pi(e)$ is the rank of e
for each $e \in E$ such that $\mathcal{R}_\Pi(e) = i$ **do**
- 5: $\Delta \leftarrow \Delta \cup \{\text{SummPlan}(P, \Pi, \Lambda, \Delta) \mid e' : \psi \leftarrow P \in \Pi, e' = e\theta\}$
- 6: $\Delta \leftarrow \Delta \cup \{\text{SummEvent}(e, \Pi, \Delta)\}$
- 7: **return** $\Delta \setminus \{u \mid u \in \Delta, u \text{ is not the summary information of an event-goal}\}$

Algorithm 2 SummPlan($P, \Pi, \Lambda, \Delta_{in}$)

Require: Plan-body P ; plan-library Π ; action-library Λ ; and the set Δ_{in} of summary information of primitive programs and event-goal types mentioned in P .

Ensure: The summary information of P .

- 1: $\Delta \leftarrow \Delta_{in} \cup \{\langle !e(\mathbf{x}), \phi, L^{mt}, L^{mn} \rangle \theta \mid !e(\mathbf{t}) \text{ occurs in } P, \langle e(\mathbf{x}), \phi, L^{mt}, L^{mn} \rangle \in \Delta_{in}, e(\mathbf{t}) = e(\mathbf{x})\theta \}$
// We assume variables in L^{mn} are appropriately renamed
- 2: Let $P = P_1; P_2; \dots; P_n$ where each P_i is atomic
- 3: $L_P^{mt} \leftarrow \{l \mid l \in L^{mt}, \langle P_i, \phi, L^{mt}, L^{mn} \rangle \in \Delta, i \in \{1, \dots, n\}, \neg \text{May-Undone}(l, P_{i+1}; \dots; P_n, \Delta)\}$
- 4: $L_P^{mn} \leftarrow \{l \mid l \in L^{mt} \cup L^{mn}, \langle P_i, \phi, L^{mt}, L^{mn} \rangle \in \Delta, i \in \{1, \dots, n\}, \neg \text{Must-Undone}(l, P_{i+1}; \dots; P_n, \Delta)\}$
- 5: **return** $\langle P, \epsilon, L_P^{mt}, L_P^{mn} \rangle$

Algorithm 3 SummEvent($e(\mathbf{x}), \Pi, \Delta$)

Require: Event-goal type $e(\mathbf{x})$; plan-library Π ; and the set Δ of summary information of plan-bodies of plan-rules $e' : \psi \leftarrow P \in \Pi$ such that $e' = e(\mathbf{x})\theta$.

Ensure: The summary information of $e(\mathbf{x})$.

- 1: $\phi \leftarrow \text{false}, L^{mt} \leftarrow \emptyset, L^{mn} \leftarrow \emptyset$, and $S \leftarrow \emptyset$
// L^{mt}, L^{mn} are sets of literals and S is a set of sets of literals
- 2: **for** each $e(\mathbf{y}) : \psi \leftarrow P \in \Pi$ such that $e(\mathbf{x}) = e(\mathbf{y})\theta$ **do**
- 3: $\phi \leftarrow \phi \vee \psi\theta$
// Relevant variables in ψ and L_P^{mt}, L_P^{mn} below are renamed
- 4: $S \leftarrow S \cup \{L_P^{mt}\theta\}$, where $\langle P, \epsilon, L_P^{mt}, L_P^{mn} \rangle \in \Delta$
- 5: $L^{mn} \leftarrow L^{mn} \cup L_P^{mn}\theta$
- 6: **if** $S \neq \emptyset$ **then** // Obtain the must literals of $e(\mathbf{x})$
- 7: $L^{mt} \leftarrow \bigcap S$
- 8: $L^{mt} \leftarrow \{l \mid l \in L^{mt}, \text{variables occurring in } l \text{ also occur in } e(\mathbf{x})\}$
- 9: **return** $\langle e(\mathbf{x}), \phi, L^{mt}, L^{mn} \rangle$

mation of all atomic programs in P , a literal l is *may undone* in P relative to Δ , denoted $\text{May-Undone}(l, P, \Delta)$, if there exists an atomic program P' in P , a substitution θ , and a literal $l' \in L^{mn}$ ⁵, with $\langle P', \phi, L^{mt}, L^{mn} \rangle \in \Delta$, such that $l\theta = l'\bar{l}'\theta$.

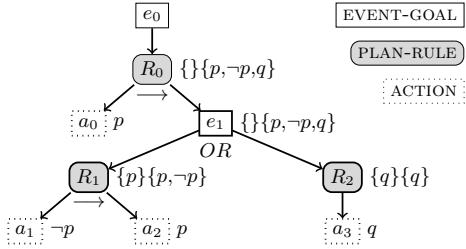
Algorithm 1. This is the top-level algorithm for computing the summary information Δ of event-goal types occurring in the plan-library. The algorithm works bottom up, by summarising first the leaf-level entities of the plan-library—the primitive programs (line 1)—and then repetitively summarising plan-bodies (Algorithm 2) and event-goals (Algorithm 3) in increasing order of their levels of abstraction (lines 3–6).

Algorithm 2. This algorithm summarises the given plan-body P by referring to the set Δ_{in} containing the summary information tuples of programs in P . First, the algorithm obtains the summary information of each event-goal program in the plan-body from the summary information of the corre-

⁵ variables occurring in l' are renamed to those not occurring in l

sponding event-goal types in Δ_{in} (line 1). This involves substituting variables occurring in relevant summary information tuples in Δ with the corresponding terms occurring in the event-goal program being considered. Second, the algorithm computes the set of must literals (L_P^{mt}) and the set of mentioned literals (L_P^{mn}) of the given plan-body P , by determining, from the must and mentioned literals of atomic programs in P , which literals will definitely hold and which ones will only possibly hold on successful executions of P (lines 3 and 4). More precisely, a must literal l of an atomic program P_i in $P = P_1; \dots; P_n$ is classified as a must literal of P only if l is not may undone in $P_{i+1}; \dots; P_n$ (line 3). Otherwise, l is classified as only a mentioned literal of P , provided l is not also must undone in $P_{i+1}; \dots; P_n$ (line 4). The reason we do not summarise literals that are must undone is to avoid missing must literals in cases where they are possibly undone but then later (definitely) reintroduced, as we illustrate below.

Suppose, on the contrary, that the algorithm *does* summarise mentioned literals that are must undone. Then, given the plan-library below, the algorithm would (hypothetically) compute the summary information denoted by the two sets attached to each node, the one on the left being its set of must literals and the one on the right its set of mentioned literals.



Observe that literal p asserted by a_0 is not recognised as a must literal of R_0 simply because it is may undone by mentioned literal $\neg p$ of e_1 (asserted by a_1), despite the fact that action a_2 of R_1 also subsequently adds p . On the other hand, our algorithm does recognise p as a must literal of R_0 by not including $\neg p$ in the set of mentioned literals of R_1 (line 4).

Algorithm 3. This algorithm summarises the given event-goal type $e(\mathbf{x})$ by referring to the set Δ containing the summary information tuples associated with the plan-bodies of plan-rules handling $e(\mathbf{x})$. In lines 2 and 3, the algorithm takes the precondition of the event-goal as the disjunction of the context conditions of all associated plan-rules.⁶ Then, the algorithm obtains the must and mentioned literals of the event-goal by respectively taking the intersection of the must literals of associated plan-rules (lines 4 and 7), and the union of the mentioned literals of associated plan-rules (line 5). Applying substitution θ in line 4 helps recognise must literals of $e(\mathbf{x})$, by ensuring that variables occurring in the summary information of its associated plan-bodies have consistent names with respect to $e(\mathbf{x})$.

An illustrative example

We shall illustrate the three algorithms with the example of a simple agent (like the ones in [5]) exploring the surface of

⁶ We do not need to “propagate up” context conditions as we do with plan-bodies’ summary information because higher-level context conditions account for lower-level ones due to Definition 2.

Mars. A part of the agent's domain is depicted as a hierarchy in Figure 1. The hierarchy's top-level event-goal is to explore a given soil location Y from current location X . This is achieved by plan-rule R_0 , which involves navigating to the location and then doing a soil experiment. Navigation is achieved by rules R_1 and R_2 , which involve moving to the location, possibly after calibrating some of the rover's instruments. Doing a soil experiment involves the two sequential event-goals of getting soil results for Y and transmitting them to the lander. Specifically, the former is refined into actions such as determining moisture content and average soil particle size, and transmitting results involves either establishing a connection with the lander, sending it the results, and then terminating the connection, or if the lander is not within range, navigating to it and uploading the soil results. The table in Figure 1 shows the summary information computed by our algorithms for elements in the figure's hierarchy. Below, we describe some of the more interesting values in the table.

Plan-body P_7 . Must literals $\neg at(Y)$ and $at(L)$ of P_7 are derived from those of $nav(X, Y)$, after renaming variables X and Y to respectively Y and L in line 1 of Algorithm 2.

Plan-body P_4 . While $hSS(Y)$ is a must literal of P_4 's primitive action $pickSoil(Y)$, the literal is must undone by P_4 's last primitive action $dropSoil(Y)$. Thus, $hSS(Y)$ is not a must (nor mentioned) literal of P_4 . On the other hand, literal $\neg hSS(Y)$ is indeed a must literal of P_4 , along with literals $hMC(Y)$ and $hPS(Y)$, both of which are derived from the summary information of event-goal $analyseSoil(Y)$.

Plan-body P_0 . While $\neg at(X)$ is a must literal of event-goal $nav(X, Y)$, it is only a mentioned literal of P_0 because it is may undone in event-goal $doSoilExp(Y)$; specifically, its mentioned literal $at(L)$ is such that $\neg at(X)\theta = \neg at(L)\theta$ for $\theta = \{X/L\}$. Similarly, must literal $at(Y)$ of $nav(X, Y)$ is also may undone in $doSoilExp(Y)$.

Event-goal $transmitRes(Y)$. Since literal $rT(Y)$ is a must literal of both of the event-goal's associated plan-bodies P_6 and P_7 , and Y also occurs in the event-goal, the literal is classified as a must literal of the event-goal. Recall that this means that for any ground instance $transmitRes(Y)\theta$ of the event-goal, literal $rT(Y)\theta$ holds at the end of any successful HTN execution of $transmitRes(Y)\theta$.

Soundness and Completeness

We shall now analyse the properties of the algorithms presented. We show that they are sound, and we then discuss completeness. First, it is not difficult to see that the presented algorithms terminate, and that they run in polynomial time.

Theorem 2 *Algorithm 1 always terminates, and runs in polynomial time on the number of symbols occurring in $\Pi \cup \Lambda$.*

PROOF. Let n_e be the total number of event-goal types occurring in Π and $n = 1$. Since a ranking function does exist for Π , we can rank it as follows. For each event-goal type e occurring in Π that does not also occur in a plan-body mentioned in Π , we first set n to $n + n_e$, and then recursively assign the rank n to e and $n - 1$ to its children event-goal types that are either not already ranked or have a higher rank.

Since the remaining algorithms are not recursive, the only non-trivial part is the algorithm for computing a unification

in $May\text{-}Undone}(l, P, \Delta)$ (where l is a literal, P is a program and Δ is a set of summary information). This was shown to be linear on the number of symbols occurring in the two literals to be unified [14]. \square

This result is important when the plan-library changes over time, e.g. because the agent learns from past experience, and summary information needs to be recomputed frequently, or when it needs to be computed right at the start of HTN planning, as done in [24].

The next result states that whenever Algorithm 1 (**Summ**) classifies a literal as a must literal of an event-goal, this is guaranteed to be the case, and that the algorithm correctly computes its precondition and mentioned literals. More specifically, any computed tuple, which includes one event-goal type e , formula ϕ and must literals L^{mt} , respects Definitions 3 and 4. Moreover, there is exactly one tuple associated with e .

Theorem 3 *Let Π be a plan-library, Λ be an action-library, e be an event-goal type mentioned in Π , and let $\Delta_{out} = \text{Summ}(\Pi, \Lambda)$. There exists one tuple $\langle e, \phi, L^{mt}, L^{mn} \rangle \in \Delta_{out}$, the tuple is the summary information of e , and $L^{mn} \subseteq mnt(e)$ (recall $mnt(e)$ denotes the mentioned literals of e).*

PROOF. We prove this by induction on e 's rank in Π . First, from our ranking function we obtain a new one \mathcal{R}_Π by making event-goal ranks “contiguous” and start from 0.

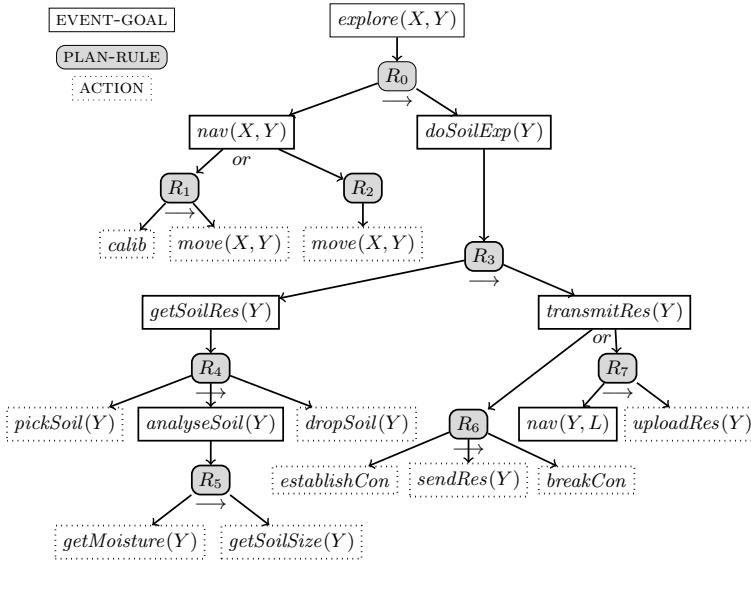
[**Base Case**] Let e be an event of rank 0 in Π , that is, $\mathcal{R}_\Pi(e) = 0$. Observe from the definition of a ranking for a plan-library (Definition 1) that if $\mathcal{R}_\Pi(e) = 0$, then $children(e, \Pi) = \emptyset$. This entails that for all plan-rules $e' : \psi \leftarrow P \in \Pi$ such that $e = e'\theta$, no event-goals are mentioned in P . There are two cases to consider.

Case 1.1. In the special case where no such plan-rule exists, then the call to procedure **SummEvent**(e, Π, Δ) in line 6 of procedure **Summ**(Π, Λ) returns tuple $\langle e, \text{false}, \emptyset, \emptyset \rangle$, which is indeed the summary information of e .

Case 1.2. Now consider the case where there are one or more plan-rules in Π such that for all $e' : \psi \leftarrow P \in \Pi$ it is the case that $e = e'\theta$ but no event-goals are mentioned in P . Let P_{all} denote the (non-empty) set of all such plan-bodies. Then, we know from Lemma 6 that, due to line 1 in the algorithm, there is exactly one tuple $\langle P', \epsilon, L_{P'}^{mt}, L_{P'}^{mn} \rangle \in \Delta$ for each primitive program P' mentioned in each plan-body $P \in P_{all}$, such that the tuple is the summary information of P' .

Next, observe that before reaching line 6 of procedure **SummEvent**(Π, Λ), procedure **SummPlan**(P, Π, Λ, Δ) is called in line 5 for each plan-body $P \in P_{all}$. Then, from Lemma 7, we know that, on the completion of line 5, there is exactly one tuple $\langle P, \epsilon, L_P^{mt}, L_P^{mn} \rangle \in \Delta$ for each plan-body $P \in P_{all}$ such that the tuple is the summary information of P . Finally, from Lemmas 8 and 9, we can conclude that on the completion of line 6 of the algorithm (i.e., after calling procedure **SummEvent**(e, Π, Δ)), there is exactly one tuple $\langle e, \phi_e, L_e^{mt}, L_e^{mn} \rangle \in \Delta$ such that the tuple is the summary information of e . Therefore, the theorem holds for the base case.

[**Induction Hypothesis**] Assume that the theorem holds if $\mathcal{R}_\Pi(e) \leq k$, for some $k \in \mathbb{N}_0$.



Program	Must Literals	Mentioned Literals
<i>calib</i>	<i>cal</i>	-
<i>move(X, Y)</i>	$\neg at(X), at(Y)$	-
<i>pickSoil(Y)</i>	<i>hSS(Y)</i>	-
<i>dropSoil(Y)</i>	$\neg hSS(Y)$	-
<i>getMoisture(Y)</i>	<i>hMC(Y)</i>	-
<i>getSoilSize(Y)</i>	<i>hPS(Y)</i>	-
<i>establishCon</i>	<i>cE</i>	-
<i>sendRes(Y)</i>	<i>rT(Y)</i>	-
<i>breakCon</i>	$\neg cE$	-
<i>uploadRes(Y)</i>	<i>rT(Y)</i>	-
<i>P₁</i>	$\neg at(X), at(Y), cal$	-
<i>P₂</i>	$\neg at(X), at(Y)$	-
<i>P₅</i>	<i>hMC(Y), hPS(Y)</i>	-
<i>P₄</i>	<i>hMC(Y), hPS(Y),</i> $\neg hSS(Y)$	-
<i>P₆</i>	<i>rT(Y), \neg cE</i>	-
<i>P₇</i>	$\neg at(Y), at(L),$ <i>rT(Y)</i>	<i>cal</i>
<i>P₃</i>	<i>rT(Y), hMC(Y),</i> <i>hPS(Y), \neg hSS(Y)</i>	$\neg cE, \neg at(Y), at(L),$ <i>cal</i>
<i>P₀</i>	<i>rT(Y), hMC(Y),</i> <i>hPS(Y), \neg hSS(Y)</i>	$\neg cE, at(Y), \neg at(Y),$ <i>at(L), cal, \neg at(X)</i>
<i>nav(X, Y)</i>	$\neg at(X), at(Y)$	<i>cal</i>
<i>analyseSoil(Y)</i>	Same as <i>P₅</i>	-
<i>getSoilRes(Y)</i>	Same as <i>P₄</i>	-
<i>transmitRes(Y)</i>	<i>rT(Y)</i>	$\neg cE, \neg at(Y), at(L),$ <i>cal</i>
<i>doSoilExp(Y)</i>	Same as <i>P₃</i>	Same as <i>P₃</i>
<i>explore(X, Y)</i>	Same as <i>P₀</i>	Same as <i>P₀</i>

Figure 1: Must and mentioned literals (right) of atomic programs and plan-bodies in the hierarchy (left). The rightmost column only shows mentioned literals that are not also must literals. Abbreviations in the table are as follows: *cal* = calibrated, *hSS* = haveSoilSample, *hMC* = haveMoistureContent, *hPS* = haveParticleSize, *cE* = connectionEstablished, *rT* = resultsTransmitted, and variable *L* = Lander. Rule *R₇*'s context condition binds *L* to the lander's location. Each plan-body *P_i* corresponds to rule *R_i* in the hierarchy.

[Inductive Step] Suppose $\mathcal{R}_\Pi(e) = k + 1$. Let $P_{all} = \{P \mid e' : \psi \leftarrow P \in \Pi, e = e\theta\}$. There are two cases to consider.

Case 2.1. First, there is no plan-body $P \in P_{all}$ such that there is an event-goal mentioned in P (i.e., all plan-bodies in P_{all} mention only primitive programs). Thus, $children(e, \Pi) = \emptyset$. If $P_{all} \neq \emptyset$, then the proof for this case is the same as *Case 1.2* in the Base Case above.

Case 2.2. On the other hand, if $P_{all} = \emptyset$, then the proof for this case is the same as *Case 1.1* in the Base Case above.

Case 2.3. The third case is that $P_{all} \neq \emptyset$ and there exists a plan-body $P \in P_{all}$ such that an event-goal is mentioned in P . Then, let E_{all} denote the (non-empty) set of event-goal types of all event-goals mentioned in all plan-bodies $P \in P_{all}$. From Definition 1 (Ranking), for all event-goals $e' \in E_{all}$, $\mathcal{R}_\Pi(e') < \mathcal{R}_\Pi(e) \leq k$. Then, by the induction hypothesis, for each $e' \in E_{all}$, there is exactly one tuple $\langle e', \phi_{e'}, L_{e'}^{mt}, L_{e'}^{mn} \rangle \in \Delta_{out}$ such that the tuple is the summary information of e' . It is not difficult to see from procedure **Summ** that all such tuples exist in Δ_{out} because the value returned by procedure **SummEvent**(e', Π, Δ) is added to set Δ in line 6, for each event-goal $e' \in E_{all}$. After this, since e has a higher rank than all $e' \in E_{all}$, procedure **Summ**(Π, Λ) will repeat its outer loop at least one more time (note that there may be other event-goals mentioned in Π with the same rank as e or higher). The procedure will then call **SummEvent**(e, \dots), by which point procedure **SummPlan**(P, \dots) will have already been called for each plan-body $P \in P_{all}$, and in turn, each such call will only have occurred after procedure **SummEvent**(e', \dots) is called for each event-goal $e' \in E_{all}$.

Then, by Lemmas 6 and 7 and the induction hy-

pothesis, it follows that on the completion of the call to **SummPlan**(P, \dots) in line 5 for each $P \in P_{all}$, there is exactly one tuple $\langle P, \epsilon, L_P^{mt}, L_P^{mn} \rangle \in \Delta$ such that the tuple is the summary information of P . Finally, from Lemmas 8 and 9, we can conclude that after calling procedure **SummEvent**(e, Π, Δ) in line 6, there is exactly one tuple $\langle e, \phi_e, L_e^{mt}, L_e^{mn} \rangle \in \Delta$ such that the tuple is the summary information of e . Therefore, the theorem holds. \square

Next, we discuss completeness. The theorem below states that any precondition computed by Algorithm 3 is complete: i.e., given any state from where there is a successful HTN execution of an event-goal, the precondition extracted for the event-goal will hold in that state. This theorem only concerns Algorithm 3 because we can compute preconditions of event-goals without needing to compute preconditions of plans.

Theorem 4 Let Π be a plan-library, Λ an action-library, e an event-goal type mentioned in Π , and let $\langle e, \phi, L^{mt}, L^{mn} \rangle \in \text{Summ}(\Pi, \Lambda)$. For all ground instances $!e\theta$ of $!e$ and belief bases \mathcal{B} such that there exists a successful HTN execution $C_1 \dots C_n$ of $!e\theta$ with $C_1|_{\mathcal{B}} = \mathcal{B}$, it is the case that $\mathcal{B} \models \phi\theta$.

PROOF. If $!e\theta$ has a successful HTN execution, then there is also a plan-rule $e' : \psi \leftarrow P \in \Pi$ associated with e such that $\mathcal{B} \models \psi'\theta$ holds, where ψ' is an appropriate renaming of variables in ψ . Since ψ' is a disjunct of ϕ (line 3 of Algorithm 3), it follows that $\mathcal{B} \models \phi\theta$. (See also proof of Lemma 9.) \square

There are, however, situations where the algorithms do not detect all *must literals* of an event-goal. The underlying reason

for this is that we do not reason about (FOL) precondition formulas; specifically, we do not check entailment, because this is semi-decidable in general [10]. In what follows, we use examples to characterise the four cases in which the algorithms are unable to recognise must literals, and show how some of the cases can be averted.

The first case was depicted in our example about going to work on Fridays: by Definition 4, literal $\neg\text{haveCar}$ is a must literal of $\text{goToWorkFridaysPlan}$, but Algorithm 2 classifies it as only a mentioned literal, as it cannot infer that the context condition of rule driveHmPlan is contradicted by literal intox , and therefore that driveHmPlan can never be applied.

The second case is where a literal is a must literal simply because it is entailed by a context condition. For example, take an event-goal $\text{mov}(P, T, L)$ that is associated with one plan-rule, whose context condition checks whether package P is in truck T , i.e., $\text{in}(P, T)$, and whose plan-body moves the truck to location L . Observe that $\text{in}(P, T)$ is a must literal of $\text{mov}(P, T, L)$ by definition, but since $\text{in}(P, T)$ does not occur in the plan-body, Algorithm 2 does not consider the literal. We do not expect this to be an issue in practice, however, because such literals are accounted for by the event-goal’s (extracted) precondition.

The third case is where must literals are “hidden” due to the particular variable/constant symbols chosen by the domain writer when encoding literals. For example, given the following two plan-rules for an event-goal that sends an email from F to T , literal $\text{sent}(T)$ is only a mentioned literal of $\text{sendMail}(F, T)$ according to Algorithm 3 (line 7 in particular), but a must literal of it by definition:

$$\begin{aligned} \text{sendMail}(F, T) : (F \neq T) &\leftarrow +\text{addedSignature} ; +\text{sent}(T), \\ \text{sendMail}(F, T) : (F = T) &\leftarrow +\text{sent}(F). \end{aligned}$$

Nonetheless, by changing $+sent(F)$ to $+sent(T)$, which then mentions the same variable symbol as the first plan-body, $\text{sent}(T)$ is identified by the algorithm as a must literal of $\text{sendMail}(F, T)$. In general, such “hidden” must literals can be disclosed by choosing terms with appropriate care.

Finally, while Algorithm 2 “conservatively” classifies any must literal that is may undone as a may literal, it could still be a must literal by definition. For example, given an event-goal $\text{move}(X, Y)$, suppose that the following plan-rule is the only one relevant for the event-goal:

$$\text{move}(X, Y) : \text{at}(X) \wedge \neg\text{at}(Y) \leftarrow -\text{at}(X) ; +\text{at}(Y).$$

Then, by Definition 4, both $\neg\text{at}(X)$ and $\text{at}(Y)$ are must literals of the event-goal, but only $\text{at}(Y)$ is its must literal according to Algorithm 2, because it cannot infer that the context condition entails $X \neq Y$.⁷ While the algorithm does fail to detect some must literals in such domains, this can sometimes be averted by encoding the domain differently. For example, the above rule can be encoded as an action-rule instead, in which case Algorithm 1 (in line 1) will classify $\neg\text{at}(X)$ (and $\text{at}(Y)$) as a must literal of $\text{move}(X, Y)$, under the assumption that action-rules are coherent.

⁷ Note that if the context condition is just $\text{at}(X)$, then, by definition, $\text{at}(Y)$ would indeed be the only must literal of the event-goal, because it would then be possible for X and Y to have the same value, and for $\text{at}(Y)$ to “undo” $\neg\text{at}(X)$.

AN APPLICATION TO PLANNING

One application of the algorithms presented is to create abstract planning operators that may be used together with primitive operators and a classical planner in order to obtain abstract (or “hybrid”) plans. While [7] focuses on algorithms for extracting an “ideal” abstract plan from an abstract plan that is supplied, here we give the details regarding how a first abstract plan may be obtained.

To get abstract operators Λ^a from a plan-library Π and an action-library Λ , we take the set $\Delta = \text{Summ}(\Pi, \Lambda)$ and create an (abstract) operator for every summary information tuple $\langle e, \phi, L^{mt}, L^{mn} \rangle \in \Delta$. To this end, we take the operator’s name as e , appended with its arity and combined with any additional variables occurring in ϕ ; the operator’s precondition as ϕ ; and its postcondition as the set of must literals L^{mt} .

Since mentioned literals of event-goals are not included in their associated abstract operators, it is crucial that we ascertain whether these literals will cause unavoidable conflicts in an abstract plan found. For example, consider the classical planning problem with initial state p and goal state r , and the abstract plan $e_1 \cdot e_2$ consisting of two event-goals (or abstract operators). Suppose e_1 and e_2 have the following plan-rules:

$$e_1 : \text{true} \leftarrow +p; +q \quad e_1 : \text{true} \leftarrow -p; +q \quad e_2 : p \wedge q \leftarrow +r$$

Notice that the postconditions (must literals) of abstract operators e_1 and e_2 are respectively q and r , and that $e_1 \cdot e_2$ is a classical planning solution for the given planning problem. However, when this plan is executed, if e_1 is decomposed using its second plan-rule, this will cause (mentioned literal) $\neg p$ to be brought about, thereby invalidating the context condition of e_2 (which requires p).

To check for such cases, we present the following simple polynomial-time algorithm. Suppose that $P = P_1; \dots; P_n$ is the program corresponding to a classical planning solution $P'_1 \cdot \dots \cdot P'_n$ for some planning problem, where each (ground) P_i is either an action or event-goal. Then, we say that P is *correct* relative to Δ if for any (ground) literal l occurring in the precondition of any P_i , the following condition holds: if l is not must undone and it is may undone (relative to Δ) in the preceding subplan $P_1; \dots; P_{i-1}$ by some mentioned literal l' of a step P_k in the subplan,⁸ then literal l , or its complement, is also must undone (relative to Δ) in the steps $P_{k+1}; \dots; P_{i-1}$. Otherwise, P is said to be *potentially incorrect*. Interestingly, the situation where l is must undone in $P_1; \dots; P_{i-1}$ is not unacceptable because it cannot invalidate the (possibly disjunctive) precondition of P_i , given that $P_1; \dots; P_n$ corresponds to a solution for some classical planning problem. The following theorem states that, as expected, a correct program P will always have at least one successful HTN decomposition.

Theorem 5 *Let Π be a plan-library, Λ an action-library, $\Delta = \text{Summ}(\Pi, \Lambda)$, and P the program corresponding to a solution for the classical planning problem $\langle \mathcal{B}, \mathcal{B}_g, \Lambda \cup \Lambda^a \rangle$, where \mathcal{B} and \mathcal{B}_g are belief bases representing respectively initial and goal states. Then, if P is correct (relative to Δ), there*

⁸ We rely here on a slightly extended version of the definition of may undone from before, to have the exact step (P_k) and literal (l') responsible for the “undoing”. Moreover, observe that literals l and l' are obtained by applying the same substitution that the planner applied to obtain P'_i and P'_k , respectively.

is a successful HTN execution $C_1 \cdot \dots \cdot C_n$ of P such that $C_1|_{\mathcal{B}} = \mathcal{B}$, $C_1|_P = P$ and $C_n|_{\mathcal{B}} \models \mathcal{B}_g$.

PROOF. If there is no such successful execution, since $P = P_1; \dots; P_n$ is a classical planning solution, there must be a mentioned literal of some P_i that intuitively “conflicts” with a literal occurring in the precondition of some P_j , with $j > i$. The classical planner will not have taken such conflicts into account, but according to the definition of what it means for P to be correct, such a mentioned literal cannot exist. \square

If we find that P is potentially incorrect, we then determine whether it is *definitely incorrect*, i.e., whether there are conflicts that are unavoidable. To this end, we look for a successful HTN decomposition of P , failing which the plan is discarded and the process repeated with a new abstract plan.

DISCUSSION & FUTURE WORK

We have presented definitions and sound algorithms for summarising plan hierarchies which, unlike past work, are defined in a typical and well understood BDI agent-oriented programming language. By virtue of its syntax and semantics being inherently tied to HTN planning, our work straightforwardly applies to HTN planners such as SHOP [15]. Our approach is closely related to [6], the main differences being that we support variables in agent programs, and we reason about non-concurrent plans. While these do make a part of our approach incomplete, we have shown how this can sometimes be averted by writing domains with appropriate care. Crucially, we have handled variables “natively”, without grounding them on a finite set of constants. We concluded with one application of our algorithms, showing how they can be used together with a classical planner in order to obtain abstract plans.

We expect that the summaries we compute will be useful in other applications that rely on similar information, such as co-ordinating the plans of single [22, 23] and multiple [6] agents, and particularly in improving HTN planning efficiency [24]. There is also potential for using such information as guidance when creating agent plans manually [26].

Interestingly, the application we presented mitigates our restriction that plan-libraries cannot be recursive, as the classical planner can, if necessary, repeat an event-goal in an abstract plan. Nonetheless, allowing recursive plan-libraries is still an interesting avenue for future work. Another useful improvement would be to allow partially ordered steps in plan-bodies (i.e., the construct $P \parallel P'$). Given a plan-library Π , one potential approach to that end is to obtain the plan-library Π' consisting of all linear extensions of plan-rules in Π , and then use Π' as the input into Algorithm 1 (**Summ**). We could use existing, fast algorithms to generate linear extensions [17], or consider simpler plan-rules corresponding to restricted classes of partially ordered sets [4]. Finally, it would be interesting to formally characterise the restricted class of domains in which the presented algorithms are complete.

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Appendix

The lemmas that follow rely on the following definition of what it means for a set of literals to *capture* a program. Intuitively, a set of literals captures a program if any literal resulting from any successful execution of the program is in the set.

Definition 7 (CAPTURING A PROGRAM) Let P be a program and L be a set of literals. Set L *captures* P if and only if for any ground instance P^g of P , successful HTN execution $C_1 \cdot \dots \cdot C_n$ of P^g , and ground literal l such that $C_1|_{\mathcal{B}} \not\models l$ and $C_n|_{\mathcal{B}} \models l$, it is the case that there is a literal $l' \in L$ such that $l = l'\theta$, for some substitution θ . ■

Observe, then, that the (full) set of mentioned literals of a program captures the program.

Lemma 6 Let P be a primitive program (i.e., $P = ?\phi \mid +b \mid -b \mid act$) mentioned in a plan-library Π , and let Λ be an action-library. Given Π and Λ as input for Algorithm 1, at the end of line 1 of the algorithm, there exists exactly one tuple $\langle P, \epsilon, L^{mt}, L^{mn} \rangle \in \Delta$ such that the tuple is the summary information of P and L^{mn} captures P .

PROOF. We consider all possible cases for P .

- Case $P = ?\phi$. Then, $post(P) = \emptyset$, and there is exactly one tuple $\langle P, \epsilon, \emptyset, \emptyset \rangle \in \Delta$. Since no literal is added to the belief base upon the execution of P , and \emptyset is a valid set of must literals (Definition 4), the theorem holds.
- Case $P = +b$. Then, observe that there is exactly one tuple $\langle P, \epsilon, \{b\}, \{b\} \rangle \in \Delta$ on the completion of line 1. Next, let $b\theta$ be any ground instance of b . Then, for all belief bases \mathcal{B} and action sequences \mathcal{A} , the following two conditions hold: (i) $\langle \mathcal{B}, \mathcal{A}, +b\theta \rangle \xrightarrow{\text{plan}} \langle \mathcal{B}' = \mathcal{B} \cup \{b\theta\}, \mathcal{A}, \text{nil} \rangle$, i.e., there is a successful HTN execution of $+b\theta$, and (ii) $\mathcal{B}' \models b\theta$. Therefore, b is a must literal of P .
- Case $P = -b$. This case is proved analogously to the one above.

- Case $P = act$. Notice that according to the definition of an action’s postcondition, act will be unified with the head of some action-rule in Λ (such a unification will always be possible due to our assumption in Section “ASSUMPTIONS” that plan-libraries are written with appropriate care). Then, let $act' : \psi \leftarrow \Phi^+ ; \Phi^- \in \Lambda$, with $act = act'\theta$, be that action-rule. Moreover, let $\hat{\Phi}^+ = \Phi^+\theta$ and $\hat{\Phi}^- = \Phi^-\theta$. Finally, let the set of literals $L_{act} = \hat{\Phi}^+ \cup \{\neg b \mid b \in \hat{\Phi}^-\}$. Observe, then, that there exists exactly one tuple $\langle P, \epsilon, L_{act}, L_{act} \rangle \in \Delta$ on the completion of line 1. Let l be any literal in L_{act} . We will now prove that l is a must literal of act .

First, from the definition of an action-rule, free variables in l will also be free in $P = act$ (a prerequisite in Definition 4). Now, let $act\theta'$ be any ground instance of act , \mathcal{B} any belief base, and \mathcal{A} any action sequence. Then, notice that if $\langle \mathcal{B}, \mathcal{A}, act\theta' \rangle \xrightarrow{\text{plan}} \langle \mathcal{B}' = (\mathcal{B} \setminus \hat{\Phi}^-\theta') \cup \hat{\Phi}^+\theta', \mathcal{A} \cdot act\theta', \text{nil} \rangle$ holds—i.e., there is a successful HTN execution of $act\theta'$ —then $\mathcal{B}' \models l\theta'$ also holds. Thus l is a must literal of P . □

Lemma 7 Let P be a plan-body mentioned in a plan-library Π , and let Λ be an action-library. Let Δ_P be a set of tuples such that:

1. for each primitive program P' mentioned in P , there is exactly one tuple $\langle P', \epsilon, L^{mt}, L^{mn} \rangle \in \Delta_P$ such that the tuple is the summary information of P' and L^{mn} captures P' ; and
2. for the event-goal type e' of each event-goal mentioned in P , there is exactly one tuple $\langle e', \phi, L^{mt}, L^{mn} \rangle \in \Delta_P$ such that the tuple is the summary information of e' , event $e = e'\theta$, and L^{mn} captures e' .

Finally, let $\langle P', \epsilon, L^{mt}, L^{mn} \rangle = \text{SummPlan}(P, \Pi, \Lambda, \Delta_P)$. Then, the tuple is the summary information of P and L^{mn} captures P .

PROOF. Consider line 1 of procedure **SummPlan**. Observe that, together with the second condition in the assumption of the theorem, on the completion of this line, for each event-goal program $!e$ mentioned in P , there is exactly one tuple $\langle !e, \phi_e, L_e^{mt}, L_e^{mn} \rangle \in \Delta$ such that the tuple is the summary information of $!e$. Then, together with the first condition in the assumption of the theorem, we can conclude that on the completion of line 1, for each atomic program P^a mentioned in P , there is exactly one tuple $\langle P^a, \phi_{P^a}, L_{P^a}^{mt}, L_{P^a}^{mn} \rangle \in \Delta$ such that the tuple is the summary information of P^a , and $L_{P^a}^{mn}$ captures P^a . To prove that $\langle P, \epsilon, L^{mt}, L^{mn} \rangle$ is the summary information of P , we will first prove that each literal in L^{mt} is a must literal of P .

Let $P = P_1; P_2; \dots; P_n$, where each P_i is an atomic program. Observe from line 3 of procedure **SummPlan** that the only literals included in the set L_P^{mt} are the literals that are must literals l of atomic programs P_i mentioned in P , where l is not may undone in $P_{i+1}; \dots; P_n$, that is $\neg \text{May-Undone}(l, P_{i+1}; \dots; P_n, \Delta)$. Let l be such a literal and P_i such an atomic program. Next, we prove that l is a must literal of P .

Let us assume the contrary, i.e., that l is not a must literal of P . Then, informally, it must be the case that the complement of l is true at the end of a successful HTN execution

of program $P_{i+1}; \dots; P_n$ —in particular, the complement of l must be true at the end of at least one successful HTN execution of some atomic program mentioned in $P_{i+1}; \dots; P_n$. More precisely, according to Definition 4 (Must Literal), this means that there is an atomic program P_j mentioned in $P_{i+1}; \dots; P_n$, a ground instance P_j^g of P_j , and a successful HTN execution $\langle \mathcal{B}_1, \mathcal{A}_1, P_j^g \rangle \dots \langle \mathcal{B}_m, \mathcal{A}_m, \text{nil} \rangle$ of P_j^g such that $\mathcal{B}_m \models \bar{l}\theta$ for some ground substitution θ , and $\mathcal{B}_1 \not\models \bar{l}\theta$. Then, by Definition 7 (Capturing a Program), and using the fact that the set $L_{P_j}^{mn}$ of tuple $\langle P_j, \phi_j, L_{P_j}^{mt}, L_{P_j}^{mn} \rangle \in \Delta$ captures P_j (by the assumption of the theorem), it must also be the case that there is a literal $l' \in L_{P_j}^{mn}$ such that $l'\theta' = \bar{l}\theta$ for some θ' . Next, we show that this cannot be the case.

Observe that $\neg \text{May-Undone}(l, P_{i+1}; \dots; P_n, \Delta)$ holds according to line 3 in procedure **SummPlan**. Then, from the definition of **May-Undone**, there is no substitution θ'' such that $l\theta'' = \bar{l}\theta''$ holds, or equivalently, such that $\bar{l}\theta'' = l''\theta''$ holds, where l'' is obtained from l' by renaming its variables to those that do not occur in l . Observe that if there is no such θ'' , then there will also not exist two substitutions θ_1 and θ_2 such that $\bar{l}\theta_1 = l''\theta_2$ holds (because otherwise we can always combine them to form $\theta'' = \theta_1 \cup \theta_2$). However, recall from before that $\bar{l}\theta = l'\theta'$ also holds. This means that—since $l\theta$ is ground—we can always rename variables in the pair l', θ' to obtain the pair l'', θ'' with $\bar{l}\theta = l''\theta''$, which means that there are two substitutions θ_1, θ_2 such that $\bar{l}\theta_1 = l''\theta_2$. This contradicts our assumption; therefore, literal l is indeed a must literal of P .

Next, we prove that the set of mentioned literals L^{mn} of program P captures P . First, observe from line 4 of procedure **SummPlan** that all mentioned literals in the summary information of all atomic programs of P are added to the set of mentioned literals of P , unless the literal is must undone. Therefore, all we need to show is that any must or mentioned literal of an atomic program occurring in P that is not included in L_P^{mn} (line 4) is not needed for L_P^{mn} to capture P . Then, let P_i be an atomic program mentioned in P , with $\langle P_i, \phi_i, L_{P_i}^{mt}, L_{P_i}^{mn} \rangle \in \Delta$, such that a must or mentioned literal l of P_i is not added to the set created in line 4 of the procedure, that is, **Must-Undone**($l, P_{i+1}; \dots; P_n, \Delta$) holds. According to the definition of **Must-Undone**, this means that $\bar{l} = l'$ holds, where $l' \in L_{P'}^{mt}$ is a must literal of some atomic program P' mentioned in $P_{i+1}; \dots; P_n$, with $\langle P', \phi_{P'}, L_{P'}^{mt}, L_{P'}^{mn} \rangle \in \Delta$. Next, let $P\theta$ be any ground instance of P . Suppose that a successful HTN execution $\langle \mathcal{B}, \mathcal{A}, P_i\theta \rangle \dots \langle \mathcal{B}_j, \mathcal{A}_j, \text{nil} \rangle$ of $P_i\theta$ exists, such that $\mathcal{B}_j \models l\theta$ holds. Then, since l' is a must literal of P' , it is the case that $\mathcal{B}'_k \models l'\theta$ also holds for any successful HTN execution $\langle \mathcal{B}', \mathcal{A}', P'\theta \rangle \dots \langle \mathcal{B}'_k, \mathcal{A}'_k, \text{nil} \rangle$ of $P'\theta$. However, since $\bar{l}\theta = l'\theta$, literal $l\theta$ is guaranteed to be removed from the belief base by $P'\theta$ during any successful HTN execution of $P\theta$. Therefore, a set of literals that captures P does not need to include mentioned literal l of P_i . (Note, however, that it is still possible that the same literal l'' from the set of must or mentioned literals of some other atomic program P'' occurring after P' in P will be present in the set of literals that captures P , provided l'' is not must undone.) \square

Lemma 8 Let e be the event type of some event-goal mentioned in a plan-library Π . Let Δ be any set such that for

each plan-rule $e' : \psi \leftarrow P \in \Pi$ with $e = e'\theta$, there is exactly one tuple $\langle P, \epsilon, L^{mt}, L^{mn} \rangle \in \Delta$ such that the tuple is the summary information of P and L^{mn} captures P . Finally, let tuple $\langle e', \phi, L^{mt}, L^{mn} \rangle = \text{SummEvent}(e, \Pi, \Delta)$. Then, L^{mt} is a set of must literals of e and L^{mn} captures e .

PROOF. Let literal $l \in L^{mt}$ and let $!e\theta$ be any ground instance of $!e$. If there is no successful HTN execution of $!e\theta$ (relative to Π) then the theorem holds. Otherwise, a successful HTN execution of $!e\theta$ does exist. Then, by the antecedent of the *Sel* transition rule, let $e' : \psi' \leftarrow P' \in \Pi$ be any plan-rule such that $e = e'\theta_r$, where θ_r is a variable renaming substitution for the plan-rule. Moreover, let us take the renamed plan-rule $e : \psi \leftarrow P = e'\theta_r : \psi'\theta_r \leftarrow P'\theta_r$. Then, by the *Event*, *Sel* and other transition rules, the following sequence of transitions must exist for some \mathcal{B}, \mathcal{A} , and set D :

$$\begin{array}{ll} (1) \langle \mathcal{B}, \mathcal{A}, !e\theta \rangle & \xrightarrow{\text{plan}} \langle \mathcal{B}, \mathcal{A}, (\{\psi\theta : P\theta, \dots\}) \rangle, \\ (2) \langle \mathcal{B}, \mathcal{A}, (\{\psi\theta : P\theta, \dots\}) \rangle & \xrightarrow{\text{plan}} \langle \mathcal{B}, \mathcal{A}, P\theta\theta' \triangleright (D) \rangle, \text{ and} \\ (3) \langle \mathcal{B}, \mathcal{A}, P\theta\theta' \triangleright (D) \rangle & \xrightarrow{\text{plan}*} \langle \mathcal{B}', \mathcal{A}', \text{nil} \rangle. \end{array}$$

Therefore, all we need to show is that $\mathcal{B}' \models l\theta$. To this end, since we know that $l \in L^{mt}$, it is not difficult to see from lines 8, 7 and 4 of procedure **SummEvent** that l is a must literal of P (up to the renaming of variables that do not occur in e). By the definition of a must literal (Definition 4), the antecedent of the *Sel* transition rule, and by virtue of the fact that (3) holds above, it follows that $\mathcal{B}' \models l\theta\theta'$ holds; therefore, $\mathcal{B}' \models l\theta$ also holds by the definition of the composition of substitutions and by the fact that $l\theta$ is ground (since $e\theta$ is ground). \square

Lemma 9 Let e be the event type of some event-goal mentioned in a plan-library Π , and let $\langle e', \phi, L^{mt}, L^{mn} \rangle = \text{SummEvent}(e, \Pi, \Delta)$, for some Δ . Then, ϕ is the precondition of e .

PROOF. Suppose we create a set of pairs SP as follows. First, we set $SP = \emptyset$. Next, for each plan-rule $e' : \psi \leftarrow P \in \Pi$ such that $e = e'\theta$, we set SP to $SP \cup \{(\psi\theta, P\theta)\}$, where θ is a renaming substitution that renames all variables occurring in the plan-rule (except those occurring in e' , which may or may not be renamed) to variables not occurring anywhere else. Then, observe that either $\phi = \bigvee_{(\psi, P) \in SP} \psi$, or $\phi = \text{false}$ if $SP = \emptyset$. We shall now show that ϕ is the precondition of e . Let $!e\theta$ be any ground instance of $!e$, \mathcal{B} any belief base, and \mathcal{A} any action sequence. If $SP = \emptyset$, then $\phi = \text{false}$ and $\mathcal{B} \models \phi\theta$ does not hold and the theorem holds. Observe from Definition 3 (Precondition) that there are two cases to consider.

[Case \Rightarrow] Suppose $SP = \emptyset$. Then $\phi = \text{false}$; therefore, $\mathcal{B} \not\models \phi\theta$, and the theorem holds trivially.

Suppose that $\mathcal{B} \models \phi\theta$ holds. Then, $\mathcal{B} \models \psi\theta\theta'$ must also hold for some disjunct ψ of ϕ . Let P be a plan-body such that $(\psi, P) \in SP$. Then, since $\mathcal{B} \models \psi\theta\theta'$, we know from rules *Event* and *Sel* that the following transitions are possible (up to variable renaming) for some set D :

$$\begin{array}{ll} \langle \mathcal{B}, \mathcal{A}, !e\theta \rangle & \xrightarrow{\text{plan}} \langle \mathcal{B}, \mathcal{A}, (\{\psi\theta : P\theta, \dots\}) \rangle, \text{ and} \\ \langle \mathcal{B}, \mathcal{A}, (\{\psi\theta : P\theta, \dots\}) \rangle & \xrightarrow{\text{plan}} \langle \mathcal{B}, \mathcal{A}, P\theta\theta' \triangleright (D) \rangle. \end{array}$$

Finally, by our assumption in Definition 2 we know that there is a successful HTN execution $C_1 \cdot \dots \cdot C_n$ of $P\theta\theta'$ such that $C_1|_{\mathcal{B}} = \mathcal{B}$. Therefore, it follows that there is also a successful HTN execution $C'_1 \cdot \dots \cdot C'_m$ of $\mathbf{!}e\theta$ such that $C'_1|_{\mathcal{B}} = \mathcal{B}$.

[Case \Leftarrow] Suppose $SP = \emptyset$. Then, observe that $\langle \mathcal{B}, \mathcal{A}, \mathbf{!}e\theta \rangle \xrightarrow{\text{plan}} \langle \mathcal{B}, \mathcal{A}, \mathbf{!}e\theta \rangle$ (for any \mathcal{A}). Therefore, there is no successful HTN execution $C_1 \cdot \dots \cdot C_m$ of $\mathbf{!}e\theta$ such that $C_1|_{\mathcal{B}} = \mathcal{B}$, and the theorem holds trivially.

Suppose that there does exist such a successful execution. Then, by the *Event*, *Sel* and other rules, there must exist a pair $(\psi, P) \in SP$ such that $\mathcal{B} \models \psi\theta\theta'$ holds and the following transitions are possible (up to variable renaming) for some D :

$$\begin{array}{ll} \langle \mathcal{B}, \mathcal{A}, \mathbf{!}e\theta \rangle & \xrightarrow{\text{plan}} \langle \mathcal{B}, \mathcal{A}, (\{\psi\theta : P\theta, \dots\}) \rangle, \\ \langle \mathcal{B}, \mathcal{A}, (\{\psi\theta : P\theta, \dots\}) \rangle & \xrightarrow{\text{plan}} \langle \mathcal{B}, \mathcal{A}, P\theta\theta' \triangleright (D) \rangle, \text{ and} \\ \langle \mathcal{B}, \mathcal{A}, P\theta\theta' \triangleright (D) \rangle & \xrightarrow{\text{plan}*} \langle \mathcal{B}', \mathcal{A}', \mathbf{nil} \rangle. \end{array}$$

Since ψ is a disjunct of ϕ , it follows that $\mathcal{B} \models \phi\theta\theta'$ also holds. Therefore, $\mathcal{B} \models \phi\theta$ holds (by the definition of the composition of substitutions). \square