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Кафедра «Системы обработки информации и управления»

Лабораторная работа №4
по дисциплине
«Методы машинного обучения»
на тему

«Реализация алгоритма Policy Iteration»

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Москва — 2023 г.

1. Цель лабораторной работы

ознакомление с базовыми методами обучения с подкреплением.

2. Задание

На основе рассмотренного на лекции примера реализуйте алгоритм Policy Iteration для любой среды обучения с подкреплением (кроме рассмотренной на лекции).

3. Текст программы

```
from matplotlib import pyplot as plt
import numpy as np
from scipy.stats import poisson
import seaborn as sns

"""
Hyper parameters
<int> MAX_NUM: 每个停车场能停的最大数目（20）
<int> lam1_rent: 停车场 1 租车λ值（3）
<int> lam1_return: 停车场 1 还车λ值（3）
<int> lam2_rent: 停车场 2 租车λ值（4）
<int> lam2_return: 停车场 2 还车λ值（2）
<int> MAX_ACTION: 最大移动汽车数目（5）
<int> CAR_COST: 移动车辆的代价（2）
<int> CAR_EARNING: 租车的收入（10）
<float> DISCOUNT: 收益折扣（0.9）
<np.array> actions: 动作集合（-5，-4，…，4，5）
<int> POISSON_UPPER_BOUND: 限制泊松分布产生请求数目的上限
```

```
<dict> poisson_cache: 存储每个（n，λ）对应的泊松概率，key 为 n*(POISSON_UPPER_BOUND-1)+lam

"""

MAX_NUM = 20

lam1_rent = 3

lam1_return = 3

lam2_rent = 4

lam2_return = 2

MAX_ACTION = 5

DISCOUNT = 0.9

CAR_COST = 2

CAR_EARNING = 10

actions = np.arange(-MAX_ACTION, MAX_ACTION + 1)

POISSON_UPPER_BOUND = 11

poisson_cache = dict()
```

```
def poisson_prob(n, lam):

    global poisson_cache

    key = n * (POISSON_UPPER_BOUND - 1) + lam

    if key not in poisson_cache:

        poisson_cache[key] = poisson.pmf(n, lam)

    return poisson_cache[key]
```

```
class dp:

    def __init__(self):

        self.v = np.ones((MAX_NUM + 1, MAX_NUM + 1), float)

        self.actions = np.zeros((MAX_NUM + 1, MAX_NUM + 1), int)
```

```python
        self.gama = DISCOUNT

        self.delta = 0

        self.theta = 0.01

        pass
```

```python
    def state_value(self, state, action, state_value, constant_returned_cars):
        """
```

```python
        :param state: 状态定义为每个地点的车辆数

        :param action: 车辆的移动数量[-5,5]，负：2->1，正：1->2

        :param state_value: 状态价值矩阵

        :param constant_returned_cars: 将换车的数目设定为泊松均值，替换为泊松概率分布

        :return:

        """

        # initial total return

        returns = 0.0
```

```python
        # 移动车辆产生负收益

        returns -= CAR_COST * abs(action)
```

```python
        # 移动后的车辆总数不能超过 20

        NUM_OF_CARS_1 = min(state[0] - action, MAX_NUM)

        NUM_OF_CARS_2 = min(state[1] + action, MAX_NUM)
```

```python
        # 遍历两地全部的可能概率下（截断泊松概率）租车请求数目

        for rent_1 in range(POISSON_UPPER_BOUND):
```

```
            for rent_2 in range(POISSON_UPPER_BOUND):
                # prob 为两地租车请求的联合概率，概率为泊松分布
                prob = poisson_prob(rent_1, lam1_rent) * poisson_prob(rent_2, lam2_rent)
                # 两地原本汽车数量
                num_of_cars_1 = NUM_OF_CARS_1
                num_of_cars_2 = NUM_OF_CARS_2
                # 有效租车数目必须小于等于该地原有的车辆数目
                valid_rent_1 = min(num_of_cars_1, rent_1)
                valid_rent_2 = min(num_of_cars_2, rent_2)
                # 计算回报，更新两地车辆数目变动
                reward = (valid_rent_1 + valid_rent_2) * CAR_EARNING
                num_of_cars_1 -= valid_rent_1
                num_of_cars_2 -= valid_rent_2
                # 如果还车数目为泊松分布的均值
                if constant_returned_cars:
                    # 两地的还车数目均为泊松分布均值
                    returned_cars_1 = lam1_return
                    returned_cars_2 = lam2_return
                    # 还车后总数不能超过车场容量
                    num_of_cars_first_loc = min(num_of_cars_1 + returned_cars_1, MAX_NUM)
                    num_of_cars_second_loc = min(num_of_cars_2 + returned_cars_2, MAX_NUM)
                    # 核心：
                    # 策略评估：V(s) = p(s',r|s,π(s))[r + γV(s')]
                    returns += prob * (reward + DISCOUNT * state_value[num_of_cars_first_loc, num_of_cars_second_loc])
```

```
                # 否则计算所有泊松概率分布下的还车空间
                else:
                    for returned_cars_first_loc in range(POISSON_UPPER_BOUND):
                        for returned_cars_second_loc in
range(POISSON_UPPER_BOUND):
                            prob_return = poisson_prob(
                                returned_cars_first_loc, lam1_return) *
poisson_prob(
                                returned_cars_second_loc, lam2_return)
                            num_of_cars_first_loc_ = min(num_of_cars_1 +
returned_cars_first_loc, MAX_NUM)
                            num_of_cars_second_loc_ = min(num_of_cars_2 +
returned_cars_second_loc, MAX_NUM)
                            # 联合概率为【还车概率】*【租车概率】
                            prob_ = prob_return * prob
                            returns += prob_ * (reward + DISCOUNT *
                                            state_value[num_of_cars_first_loc
_, num_of_cars_second_loc_])
        return returns
```

```
    def policy_iteration(self, constant_returned_cars=True):
        """
```

```
        :param constant_returned_cars:
        :return:
        """
        # 设置迭代参数
        iterations = 0
```

```
        # 准备画布大小，并准备多个子图
        _, axes = plt.subplots(2, 3, figsize=(40, 20))
        # 调整子图的间距，wspace=0.1 为水平间距，hspace=0.2 为垂直间距
        plt.subplots_adjust(wspace=0.1, hspace=0.2)
        # 这里将子图形成一个 1*6 的列表
        axes = axes.flatten()
        while True:
            # 使用 seaborn 的 heatmap 作图
            # flipud 为将矩阵进行垂直角度的上下翻转，第 n 行变为第一行，第一行变为第 n 行，
如此。
            # cmap:matplotlib 的 colormap 名称或颜色对象；
            # 如果没有提供，默认为 cubehelix map (数据集为连续数据集时) 或 RdBu_r (数
据集为离散数据集时)
            fig = sns.heatmap(np.flipud(self.actions), cmap="rainbow",
ax=axes[iterations])
```

```
            # 定义标签与标题
            fig.set_ylabel('# cars at first location', fontsize=30)
            fig.set_yticks(list(reversed(range(MAX_NUM + 1))))
            fig.set_xlabel('# cars at second location', fontsize=30)
            fig.set_title('policy {}'.format(iterations), fontsize=30)
```

```
            # policy evaluation (in-place) 策略评估（in-place）
            # 未改进前，第一轮 policy 全为 0，即[0，0，0...]
            while True:
                old_value = self.v.copy()
                for i in range(MAX_NUM + 1):
```

```python
                    for j in range(MAX_NUM + 1):
                        # 更新 V（s）
                        new_state_value = self.state_value([i, j], self.actions[i, j], self.v, constant_returned_cars)
                        # in-place 操作
                        self.v[i, j] = new_state_value
                # 比较 V_old(s)、V(s)，收敛后退出循环
                max_value_change = abs(old_value - self.v).max()
                print('max value change {}'.format(max_value_change))
                if max_value_change < 1e-4:
                    break
```

```python
            # policy improvement
            # 在上一部分可以看到，策略 policy 全都是 0，如不进行策略改进，其必然不会收敛到实际最优策略。
            # 所以需要如下策略改进
            policy_stable = True
            # i、j 分别为两地现有车辆总数
            for i in range(MAX_NUM + 1):
                for j in range(MAX_NUM + 1):
                    old_action = self.actions[i, j]
                    action_returns = []
                    # actions 为全部的动作空间，即[-5、-4...4、5]
                    for action in actions:
                        if (0 <= action <= i) or (-j <= action <= 0):
                            action_returns.append(self.state_value([i, j], action, self.v, constant_returned_cars))
                        else:
```

```python
                        action_returns.append(-np.inf)
                # 找出产生最大动作价值的动作
                new_action = actions[np.argmax(action_returns)]
                # 更新策略
                self.actions[i, j] = new_action
                if policy_stable and old_action != new_action:
                    policy_stable = False
        print('policy stable {}'.format(policy_stable))
```

```python
        if policy_stable:
            fig = sns.heatmap(np.flipud(self.v), cmap="rainbow", ax=axes[-1])
            fig.set_ylabel('# cars at first location', fontsize=30)
            fig.set_yticks(list(reversed(range(MAX_NUM + 1))))
            fig.set_xlabel('# cars at second location', fontsize=30)
            fig.set_title('optimal value', fontsize=30)
            break
```

```python
        iterations += 1
    # plt.title('Policy Iteration')
    plt.savefig('./policy_iteration.png')
    plt.show()
    plt.close()
    return
```

```python
def value_iteration(self, constant_returned_cars=True):
    """
```

```
        :param constant_returned_cars:

        :return:

        """

        # 设置迭代参数

        iterations = 0
```

```
        # 准备画布大小，并准备多个子图

        _, axes = plt.subplots(1, 3, figsize=(40, 10))

        # 调整子图的间距，wspace=0.1 为水平间距，hspace=0.2 为垂直间距

        plt.subplots_adjust(wspace=0.1, hspace=0.2)

        # 这里将子图形成一个 1*6 的列表

        axes = axes.flatten()

        while True:

            # 使用 seaborn 的 heatmap 作图

            # flipud 为将矩阵进行垂直角度的上下翻转，第 n 行变为第一行，第一行变为第 n 行，
如此。

            # cmap:matplotlib 的 colormap 名称或颜色对象；

            # 如果没有提供，默认为 cubehelix map (数据集为连续数据集时) 或 RdBu_r (数
据集为离散数据集时)

            fig = sns.heatmap(np.flipud(self.actions), cmap="rainbow",
ax=axes[iterations])
```

```
            # 定义标签与标题

            fig.set_ylabel('# cars at first location', fontsize=30)

            fig.set_yticks(list(reversed(range(MAX_NUM + 1))))

            fig.set_xlabel('# cars at second location', fontsize=30)

            fig.set_title('policy {}'.format(iterations), fontsize=30)
```

```
        # value iteration 价值迭代
        while True:
```

```
            old_value = self.v.copy()
            for i in range(MAX_NUM + 1):
                for j in range(MAX_NUM + 1):
                    action_returns = []
                    # actions 为全部的动作空间，即[-5、-4...4、5]
                    for action in actions:
                        if (0 <= action <= i) or (-j <= action <= 0):
                            action_returns.append(self.state_value([i, j],
action, self.v, constant_returned_cars))
                        else:
                            action_returns.append(-np.inf)
                    # 找出产生最大动作价值的动作
                    max_action = actions[np.argmax(action_returns)]
                    # 更新 V（s）
                    new_state_value = self.state_value([i, j], max_action,
self.v, constant_returned_cars)
                    # in-place 操作
                    self.v[i, j] = new_state_value
            # 比较 V_old(s)、V(s)，收敛后退出循环
            max_value_change = abs(old_value - self.v).max()
            print('max value change {}'.format(max_value_change))
            if max_value_change < 1e-4:
                print(iterations)
                break
```

```
        # policy improvement
        policy_stable = True
        # i、j 分别为两地现有车辆总数
        for i in range(MAX_NUM + 1):
            for j in range(MAX_NUM + 1):
                old_action = self.actions[i, j]
                action_returns = []
                # actions 为全部的动作空间，即[-5、-4...4、5]
                for action in actions:
                    if (0 <= action <= i) or (-j <= action <= 0):
                        action_returns.append(self.state_value([i, j], action,
self.v, constant_returned_cars))
                    else:
                        action_returns.append(-np.inf)
                # 找出产生最大动作价值的动作
                new_action = actions[np.argmax(action_returns)]
                # 更新策略
                self.actions[i, j] = new_action
                if policy_stable and old_action != new_action:
                    policy_stable = False
        print('policy stable {}'.format(policy_stable))
```

```
        if policy_stable:
            fig = sns.heatmap(np.flipud(self.v), cmap="rainbow", ax=axes[-1])
            fig.set_ylabel('# cars at first location', fontsize=30)
            fig.set_yticks(list(reversed(range(MAX_NUM + 1))))
```

```python
                fig.set_xlabel('# cars at second location', fontsize=30)

                fig.set_title('optimal value', fontsize=30)

                break
```

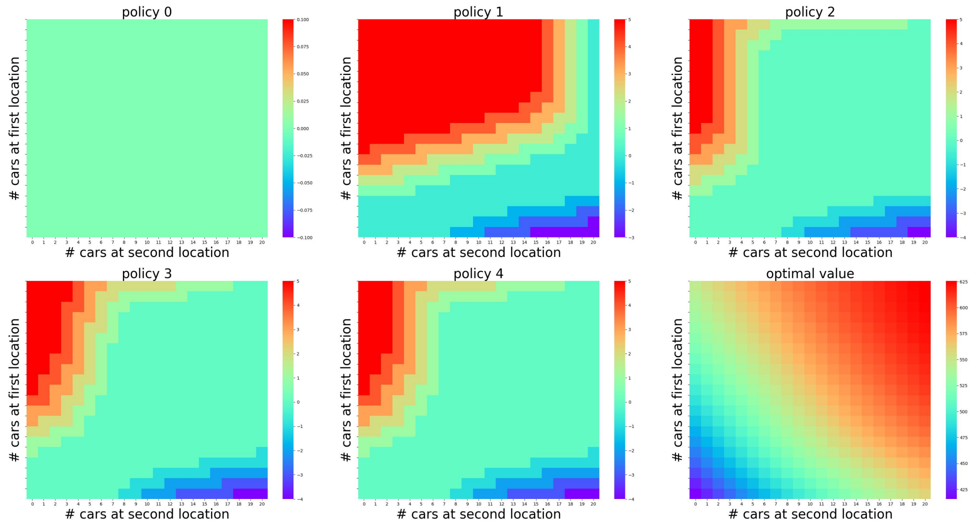
```python
            iterations += 1
```

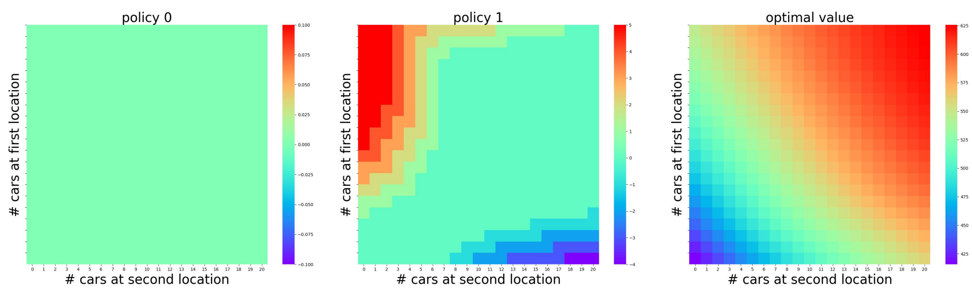
```python
        # plt.title('Value Iteration')

        plt.savefig('./value_iteration.png')

        plt.show()

        plt.close()

        return
```

```python
if __name__ == '__main__':

    model = dp()

    model.policy_iteration(constant_returned_cars=True)
```

```python
    model1 = dp()

    model1.value_iteration(constant_returned_cars=True)

    pass
```

4. Результат


policy 0
policy 1
policy 2
policy 3
policy 4
optimal value
cars at first location
cars at second location


policy 0
policy 1
optimal value
cars at first location
cars at second location