Logic, Computability and Incompleteness

Introduction

Course: Logic, Computability and Incompleteness

PHIL10133/PHIL11114

Thurs 14:10 – 16:00

Lecturer: Paul Schweizer

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Room 5.13 DSB

Office Hours: Tuesday 3-4pm

(and by appointment)

Aims and General Description

- Logic and computation are intimately related, and this course will focus on some landmark results of 20th Century metatheory linking the two fields, as well as their Philosophical implications.
- In particular, we'll look at
- Turing machines and their formalization in first-order logic, linking uncomputability and the halting problem to the undecidability of first-order logic.
- We will then study recursive functions and their construction, followed by first-order formalizations of arithmetic, particularly Robinson arithmetic and Peano arithmetic.

Aims and General Description

- We will then turn to the topic of the arithmetization of syntax and the diagonal lemma, before proceeding to prove some of the main limitative results concerning formal systems, in particular
- Gödel's two incompleteness theorems, along with allied results including Tarski's Theorem and Löb's Theorem.
- Will also study some basic metalogical properties of first order systems, including soundness, completeness and compactness
- And we'll look at the classic Löwenheim-Skolem results in model theory.

Syllabus

- Topic 1: Cardinality, Enumerability, Diagonalization
- Topic 2: Turing Machines and Computability
- Topic 3: Recursive Functions
- Topic 4: First-Order Logic Revisited
- Topic 5: Uncomputability and Undecidability
- Topic 6: Completeness, Compactness and Löwenheim-Skolem
- Topic 7: Formal Arithmetic
- Topic 8: Diagonal Lemma, Gödel and Tarski Theorems
- Topic 9: Provability Predicates and Löb's Theorem
- Topic 10: Computational Complexity
- Topic 11: TBA

Text and Reading List

- Our primary text will be Boolos and Jeffrey's *Computability and Logic*. We will use the 'canonical' 3rd edition. Scans of the relevant chapters are in designated files on the Learn site.
  - **Topic 1:** B&J ch 1,2.
  - **Topic 2:** B&J ch 3,6.
  - **Topic 3:** B&J ch 7,8
  - **Topic 4:** B&J ch 9.
  - **Topic 5:** B&J ch 5,10.
  - **Topic 6:** B&J ch 11,12,13.
  - **Topic 7:** B&J ch 14.
  - **Topic 8:** B&J ch 15
  - **Topic 9:** B&J ch 16.
  - **Topic 10:** TBA.

Feedback and Assessment

- Feedback and formative assessment will be provided via two (compulsory) exercise sets to be assigned during the course of the semester.
- Final mark will be based on a standard 2 hour exam held in the Spring diet.