MATH 110, Fall 2013
Tutorial #1
September 11, 2013

Today's main problems

1. Knowing the definitions is half the battle in Linear Algebra. Write the definitions of the following (either in words or with a formula).

   - *Vector*:

   - *Dot product*:

   - The *norm* of $\vec{w}$:

   - *Distance between* vectors $\vec{u}$ and $\vec{v}$:

   - $\vec{u}$ and $\vec{v}$ are *orthogonal* when:

   - $\vec{w}$ is a *linear combination* of $\vec{u}$ and $\vec{v}$ when:

   - $\vec{w}$ is a *unit vector* when:

2. For the vectors $\vec{u} = \begin{bmatrix} -2 \\ 1 \\ -2 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$ determine

   (a) the distance between $\vec{u}$ and $\vec{v}$
   (b) a unit vector in the direction $\vec{v}$
   (c) whether $\vec{u}$ and $\vec{v}$ are orthogonal
   (d) the angle between $\vec{u}$ and $\vec{v}$

Further questions

3. Find the projection of $\vec{u}$ onto $\vec{v}$.

4. $L = \{t\vec{v} : t \in \mathbb{R}\}$ is the line in the direction $\vec{v}$ that passes through the origin. Find the intersection of L with the sphere of radius 2 centered at the origin.

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Challenge questions

5. Find a vector that forms a 45° angle with $\vec{v}$ and has length 7.

   Hint: There are many, many such vectors, so you'll have to make some choices.

6. The vector $\vec{w}$ form an angle of 45° with $\vec{v}$ and $\|\mathrm{proj}_{\vec{v}}\vec{w}\| = 10$. The vector $\vec{r}$ satisfies $\|\vec{r}\| = 2\|\vec{w}\|$ and $\|\mathrm{proj}_{\vec{v}}\vec{r}\| = 10$. What are the possible angle(s) between the vectors $\vec{r}$ and $\vec{v}$?

MATH 110, Fall 2013
Tutorial #1. Instructions for TAs

Objectives

Knowing the definitions is half the battle in Linear Algebra. Today we are going to focus on knowing the definitions and how to apply them.

Hidden objectives

Everyone forgets a definition at some point, but the key is to be able to find it when you need it. We'd like students to be resourceful and use each other and their *textbook* to find the definitions they don't know.

Suggestions

Ask students to work on number 1 first and to talk to their neighbors and refer to their textbook and notes if they are unsure. Give them sufficient time come up with definitions before going over them midway through the class. Going over them doesn't need to take much time since they have seen these definitions before, but we want everyone to be on the same page before starting 2.

Further, since this is the first tutorial, make sure to explain that there are many questions and they are not expected to complete them all during tutorial time.

Wrapup

Choose a question that most of the class has started but not yet finished, or a question that people particularly struggled with.

Solutions

1. Knowing the definitions is half the battle in Linear Algebra. Write the definitions of the following (either in words or with a formula).

   - *Vector*: A magnitude and a direction; a list of components.
   - *Dot product*: $\vec{a} \cdot \vec{b} = \|\vec{a}\|\|\vec{b}\| \cos\theta$ where θ is the angle between them; $\vec{a} \cdot \vec{b} = \sum a_i b_i$ where a_i are the components of $\vec{a}$ and b_i are the components of $\vec{b}$.
   - The *norm* of $\vec{w}$: the length of $\vec{w}$; $\|\vec{w}\| = \sqrt{\vec{w} \cdot \vec{w}}$
   - *Distance between* vectors $\vec{u}$ and $\vec{v}$: $\|\vec{u} - \vec{v}\|$
   - $\vec{u}$ and $\vec{v}$ are *orthogonal* when: $\vec{u}$ and $\vec{v}$ are perpendicular; $\vec{u}$ and $\vec{v}$ meet at $90°$; $\vec{u} \cdot \vec{v} = 0$
   - $\vec{w}$ is a *linear combination* of $\vec{u}$ and $\vec{v}$ when: $\vec{w} = a\vec{u} + b\vec{v}$ for some numbers a, b

- $\vec{w}$ is a *unit vector* when: $\|\vec{w}\| = 1$

2. For the vectors $\vec{u} = \begin{bmatrix} -2 \\ 1 \\ -2 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$ determine

   (a) the distance between $\vec{u}$ and $\vec{v}$
   $\sqrt{65} \approx 8.06$

   (b) a unit vector in the direction $\vec{v}$
   $\frac{1}{\sqrt{26}}\vec{v}$

   (c) whether $\vec{u}$ and $\vec{v}$ are orthogonal
   No

   (d) the angle between $\vec{u}$ and $\vec{v}$
   $\cos\theta = \frac{-15}{3\sqrt{26}} \approx -0.98$ so $\theta \approx 169°$

3. Find the projection of $\vec{u}$ onto $\vec{v}$.
   $\frac{1}{26}\begin{bmatrix} -60 \\ 15 \\ -45 \end{bmatrix} \approx \begin{bmatrix} -2.31 \\ .58 \\ -1.73 \end{bmatrix}$

4. $L = \{t\vec{v} : t \in \mathbb{R}\}$ is the line in the direction $\vec{v}$ that passes through the origin. Find the intersection of L with the sphere of radius 2 centered at the origin.

   Two points: $\frac{2}{\sqrt{26}}\begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$ and $\frac{-2}{\sqrt{26}}\begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$