MATH 110, Fall 2013
Tutorial #8
October 30, 2013

Today's main problems

1. The transformation $\mathcal{S} : \mathbb{R}^2 \to \mathbb{R}^2$ is given by $\mathcal{S}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ y \end{bmatrix}$ is linear.

   (a) Find the standard matrix representation for the transformation $\mathcal{S}$.

   (b) Explain why this linear transformation is called a *horizontal scaling*. Find the factor of this scaling. Is this transformation stretching or contracting vectors in $\mathbb{R}^2$?

   (c) Describe in words what $\mathcal{S}^{-1}$ does to vectors in $\mathbb{R}^2$.

2. Give an example of a *vertical scaling* that contracts vectors in $\mathbb{R}^3$ by a factor of 3.

3. Let T denote the triangle with vertices give by the vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \qquad \vec{v}_3 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

   (a) Draw the triangle T.

   (b) Draw the result of $\mathcal{S}(T)$.

   (c) $\mathcal{C}$ contracts vertically by a factor of 3. Draw $\mathcal{C}(T)$.

   (d) Draw $\mathcal{SC}(T)$. How does this compare to $\mathcal{CS}(T)$? Using this fact, what can you say about the matrix representations of $\mathcal{S}$ and $\mathcal{C}$?

Further Questions

4. $\mathcal{R} : \mathbb{R}^2 \to \mathbb{R}^2$ rotates vectors counterclockwise by $90°$.

   (a) Compute the matrix representation of $\mathcal{R}$ and $\mathcal{R}^{-1}$.

   (b) Compute the matrix representation of the transformation $\mathcal{W}$ that rotates vectors in $\mathbb{R}^2$ clockwise by $90°$.

   (c) How do $\mathcal{R}^{-1}$ and $\mathcal{W}$ relate?

5. Consider the matrix P that projects vectors in $\mathbb{R}^2$ onto the line $y = 2x$.

   (a) Draw the image of P applied to the triangle T from problem 3.

   (b) If you write down the matrix for P, would it be invertible? Why or why not?

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Challenge questions

$\mathcal{R}_t$ rotates vectors in $\mathbb{R}^2$ counterclockwise by t radians.

6. Write the matrix representation, R, of $\mathcal{R}_t$.
7. Write the matrix representation of $\mathcal{R}_t^{-1}$. (It's easier to think about this one instead of row reducing).
8. How does $\mathcal{R}_t^{-1}$ compare to R^T?.
9. Suppose V is an $n \times n$ matrix whose columns are orthogonal and are all unit vectors. Explain how V^T relates to V^{-1}. Can you relate this to the rotation matrices you've been playing around with?

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Tutorial #8. Instructions for TAs

Objectives

Hidden objectives

Suggestions

When they start scaling the triangle in number 3, make sure they scale from the origin not just draw wider and shorter triangles (like they will be tempted to).

Wrapup

Choose a question that most of the class has started but not yet finished, or a question that people particularly struggled with.

Solutions

1.