MATH 110, Fall 2013
Tutorial #10
November 20, 2013

Today's main problems

$\mathcal{V} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ where

$$\vec{v}_1 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} -4 \\ 0 \\ 3 \end{bmatrix} \qquad \vec{v}_3 = \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}$$

1. (a) Show that $\mathcal{V}$ is an orthogonal basis. Is it an orthonormal basis?
   (b) Create an orthonormal basis $\mathcal{B} = \{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$ by "fixing" the vectors in $\mathcal{V}$ so they are orthonormal.
   (c) Write the vector $\vec{w} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ as a linear combination of vectors in the $\mathcal{B}$ basis.

2. Consider the three planes $\mathcal{P}_1 = \text{span}\{\vec{v}_1, \vec{v}_2\}$, $\mathcal{P}_2 = \text{span}\{\vec{v}_2, \vec{v}_3\}$, $\mathcal{P}_3 = \text{span}\{\vec{v}_1, \vec{v}_3\}$.
   (a) Find the normal vectors of the planes $\mathcal{P}_1, \mathcal{P}_2$, and $\mathcal{P}_3$.
   (b) Find the projection of $\vec{w} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ onto $\mathcal{P}_1, \mathcal{P}_2$, and $\mathcal{P}_3$.
   (c) Write $\vec{w} = \vec{a} + \vec{b}$ where $\vec{a} \in \mathcal{P}_1$ and $\vec{b}$ is perpendicular to $\mathcal{P}_1$.

Further Questions

3. Let $B = [\vec{b}_1|\vec{b}_2|\vec{b}_3]$ and $V = [\vec{v}_1|\vec{v}_2|\vec{v}_3]$ where $\vec{b}_i$ and $\vec{v}_i$ are from problem 1.
   (a) Compute B^{-1} (Hint, this is easy. No computation required!)
   (b) Compute V^{-1} (This requires a little thinking, but not much computation).

4. V is a subspace of $\mathbb{R}^9$ and P is a matrix that projects vectors in $\mathbb{R}^9$ onto V. Further, $\text{rank}(P) = 3$.
   (a) How many vectors are in a basis for V?
   (b) Let $\vec{v} \in \mathbb{R}^9$ and $\vec{w} = \vec{v} - P\vec{v}$. What is the angle between $\vec{w}$ and any vector in V?
   (c) What is $\text{rank}(I - P)$?

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Challenge questions

Recall that the trace of a matrix is the sum of the diagonal entries and that trace(A) =trace(B) if A and B are similar.

5. Let P be a projection matrix. Show that trace(P) is always an integer (Hint, think about what a projection matrix must be similar to).

6. Show that in fact trace(P) is the dimension of the subspace that P projects onto.

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Tutorial #10. Instructions for TAs

Objectives

Hidden objectives

Suggestions

Wrapup

Choose a question that most of the class has started but not yet finished, or a question that people particularly struggled with.

Solutions

1.