MATH 110, Fall 2013
Tutorial #9
November 6, 2013

Today's main problems

$$A = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & -6 \\ 3 & -4 \end{bmatrix} \qquad C = \begin{bmatrix} -3 & 4 \\ -3 & 5 \end{bmatrix}$$

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad \vec{v}_3 = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \qquad \vec{v}_4 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \vec{v}_5 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

1. For each $\vec{v}_i$, identify whether $\vec{v}_i$ is an eigenvector of A, B, or C. If so, find the corresponding eigenvalue.

2. Find all eigenvectors and eigenvalues of C.

3. $F = \begin{bmatrix} 1 & 2 & 1 \\ -1 & -1 & 3 \\ 4 & 0 & k \end{bmatrix}$

   (a) Compute $\det(F)$.

   (b) For what values of k is F invertible?

   (c) For what values of k is zero an eigenvalue of F?

Further Questions

4. $R = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

   (a) Explain what R does geometrically.

   (b) Compute the eigenvalues and eigenvectors of R.

   (c) Explain geometrically why you should or shouldn't expect real eigenvectors or eigenvalues for R. Can you generalize this result?

5. Let $E = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

   (a) Find the eigenvectors and eigenvalues of E.

   (b) List the algebraic and geometric multiplicity of each eigenvalue.

   (c) Can you form a basis for $\mathbb{R}^3$ consisting of eigenvectors of E?

6. For the matrix F, you know $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is an eigenvector with eigenvalue 3 and $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ is an eigenvector with eigenvalue 2.

   (a) Is there a basis for $\mathbb{R}^2$ consisting of eigenvectors of F?

   (b) Write $\vec{v} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$ in the eigenbasis.

   (c) Compute $F\vec{v}$.

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Challenge questions

7. Find the matrix F from the previous problem.

8. For a matrix A, $\vec{u}, \vec{v}$ are eigenvectors with different eigenvalues. Show that $\vec{u}$ and $\vec{v}$ are linearly independent.

9. Prove that if A is a 4×4 matrix with eigenvalues $2, 1, 0, -7$, then there is a basis for $\mathbb{R}^4$ consisting of eigenvectors of A.

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Tutorial #9. Instructions for TAs

Objectives

Hidden objectives

Suggestions

Wrapup

Choose a question that most of the class has started but not yet finished, or a question that people particularly struggled with.

Solutions

1.