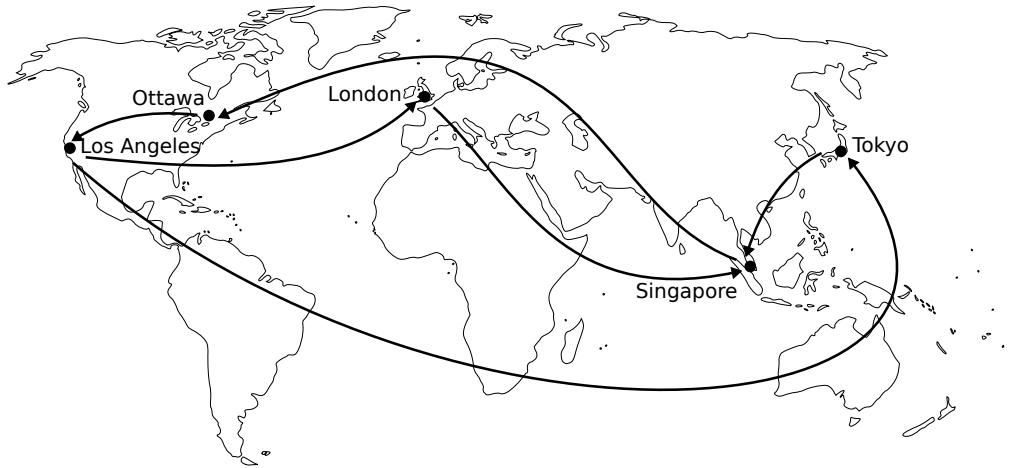# Today's main problems

The map shows the direct, one-way flights offered by the Pacific Rim air shipping company.

1. Write down a *transition matrix* A with entries $A_{i,j}$ which are the number of ways to take exactly one flight from city i to city j.

   (a) What do the diagonal entries tell you about the available flights?

   (b) Should $A_{i,j} = A_{j,i}$? Explain.

2. Write down a transition matrix B with entries $B_{i,j}$ which are the number of ways to take exactly two flights from city i to city j.

   (a) Compute A^2 and compare with B.

   (b) What information does the 1st row of A give you about flights?

   (c) What information does the 2nd column of A give you about flights?

   (d) Based upon your last two answers what does the $1, 2$ entry of A^2 tell you about flights?

3. Compute A^3 by any method.

   (a) Notice that the diagonal entries of A, A^2 and A^3 are all zero. What is the first number n so that the diagonal entries of A^n are non-zero?

   (b) For what numbers n do you expect A^n to have a non-zero diagonal entry?

Further Questions

4. Find a matrix C with entries $C_{i,j}$ which are the number of ways to fly from i to j in at most 3 flights.

   (a) If you can fly at most three times, are all trips between different cities possible?

   (b) Based on the trips of at most three flights which city would make the best hub? (You may want to pick different cities for your outgoing and your incoming hubs.)

Challenge questions

A matrix A is called *primitive* if for large enough powers A^n has all positive entries.

5. Suppose that company A has round-robin delivery service for 6 cities. (That is there are routes from cities $1 \to 2 \to 3 \to 4 \to 5 \to 6 \to 1$.) Let A be the transition matrix for company A. Is A primitive?

6. Suppose there were a looping route from $1 \to 1$ in addition. Would A be primitive?

7. Suppose a company B, with shipping routes $1 \to 2 \to 3 \to 1$, buys company A and so company B may now use its original routes or any of the newly aquired routes (Note, the self-loop is not included). Let B be the transition matrix for company B. Is B primitive?

8. Suppose C is a transition matrix for a shipping company with two round-robin shipping routes that intersect at exactly one city. Give a condition on the round-robin routes to ensure that C is primitive.

Objectives

We've been working hard at learning algorithms and matrix arithmetic. It's time to see matrices appear in an unexpected context and have some fun!

Hidden objectives

We've learned matrix multiplcation from one perspective, but it's time to think deeply about what is really going on combinatorically.

Suggestions

Wrapup

Choose a question that most of the class has started but not yet finished, or a question that people particularly struggled with.

Solutions

1.