1. Bryan: row reduction changes the eigenvalues. You should use the characteristic polynomial like Rachel.

Rachel: you're good!

2.
$$\begin{bmatrix} 2 & 1 & 2 \\ 0 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}, \text{ and so } \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} \text{ is an eigenvector with eigen-}$$

value 1, but using the row reduced form $\begin{bmatrix} 2 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ -1 \end{bmatrix},$

so $\begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$ is not an eigenvector with eigenvalue 1 for the row reduced matrix. \therefore using the row reduced form doesn't work.

3. Row reduction changes the column space of a matrix. Since an eigenvector needs to satisfy $A\vec{x} = \lambda \vec{x}$, changing the column space will change $A\vec{x}$.