MATH 110, Fall 2013
Tutorial #4
October 2, 2013

Today's main problems

Consider the vectors

$$\vec{a} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} -1 \\ -1 \\ 0 \\ -1 \end{bmatrix} \qquad \vec{c} = \begin{bmatrix} 3 \\ 4 \\ 1 \\ 2 \end{bmatrix} \qquad \vec{d} = \begin{bmatrix} 1 \\ 3 \\ 2 \\ -1 \end{bmatrix}$$

1. Is the set $\{\vec{a}, \vec{b}, \vec{c}, \vec{d}\}$ linearly independent?
2. Describe $\text{span}\{\vec{a}, \vec{b}, \vec{c}, \vec{d}\}$ as a point, line, plane, or hyperplane and give its formula in vector form.
3. If $A = [\vec{a}|\vec{b}|\vec{c}|\vec{d}]$ is the matrix whose columns are the vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$, what is the rank of A?
4. You, while stuck in your moon base, discover that you urgently need to transmit the vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ back to earth. However, energy is very limited. It takes 100 units of energy to transmit the coordinates of a single vector. It takes only 1 unit of energy to transmit a linear combination (e.g. $\vec{x} = 7\vec{y} - 2\vec{z} + \vec{w}$). What is the minimum amount of energy required to transmit the vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ back to earth?

Further Questions

Consider the plane $\mathcal{P}$ given by the equation

$$2x - y + z = 0.$$

5. Find vectors $\vec{u}$ and $\vec{v}$ so that $\mathcal{P} = \text{span}\{\vec{u}, \vec{v}\}$.
6. Can you find vectors $\vec{u}, \vec{v}, \vec{w}$ so that $\mathcal{P} = \text{span}\{\vec{u}, \vec{v}, \vec{w}\}$? If you can, find such vectors. Otherwise, explain why it cannot be done.

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Challenge questions

Consider the plane $\mathcal{P}'$ given by the equation

$$2x - y + z = 4.$$

7. Can you find vectors $\vec{u}$ and $\vec{v}$ so that $\mathcal{P}' = \text{span}\{\vec{u}, \vec{v}\}$?

8. Describe span $\mathcal{P}'$.

9. Recall the vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ from before. Again you are stuck on a moon base and need to transmit the vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ to earth, but you've come up with a new encoding scheme. With this new scheme it costs 50 units of energy to transmit the components of a vector. Further, if you transmit the linear combination $\vec{x} = t\vec{y} + r\vec{y} + q\vec{w}$ it takes $|t| + |r| + |q|$ units of energy. What is the least amount of energy you can use to transmit the vectors to earth?

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Tutorial #4. Instructions for TAs

Objectives

Span and linear independence are the first abstract concepts we encounter in this course and as such, we'd like to see how they fit with the concrete algorithms and equations we've been learning. More specifically, the goal is to see how span, linear independence, and reduced row echelon form of a matrix all relate to each other.

Hidden objectives

Span and linear independence are easy once you understand them, but it takes a lot experience working problems to understand the ins and outs.

Suggestions

Start by asking the class what the definition of span is and then what the definition of linear independence is. Putting these definitions on the board should help people out.

Wrapup

Choose a question that most of the class has started but not yet finished, or a question that people particularly struggled with.

Solutions

9. 101.2 units.