MATH 110, Fall 2013
Tutorial #7
October 23, 2013

Today's main problems

1.

$$S_1 = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : x_1 + 3x_2 - x_3 = 0 \right\} \qquad S_2 = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} : x_1 x_2 = 0 \right\}$$

$$S_3 = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : x_1 + 3x_2 - x_3 = 0 \text{ and } -x_1 + x_2 + 2x_3 = 0 \right\}$$

$$S_4 = \text{A line in } \mathbb{R}^7 \text{ through } \vec{0} \text{ with direction } \vec{d}$$

   (a) Is S_1 a subspace of $\mathbb{R}^3$?
   (b) Is S_2 a subspace of $\mathbb{R}^2$?
   (c) Is S_3 a subspace of $\mathbb{R}^3$?
   (d) Is S_4 a subspace of $\mathbb{R}^7$?

2. Consider

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 3 & 0 \\ 3 & -1 & -5 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} 7 \\ -1 \\ 2 \end{bmatrix} \qquad \vec{w} = \begin{bmatrix} 1 & -3 & -3 \end{bmatrix}$$

   (a) Determine whether or not $\vec{b}$ is in the column space of A.
   (b) Determine whether or not $\vec{w}$ is in the row space of A.
   (c) Determine whether or not $\vec{v}$ is in the null space of A.
   (d) Find the dimension of the row space, column space, and null space.

Further Questions

3. Give a basis for the row space, column space, and null space of A where

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -1 \end{bmatrix}$$

4. Let $V = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{v}_5\}$ where

$$\vec{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 5 \\ 2 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} -2 \\ 3 \\ 1 \\ 0 \end{bmatrix} \qquad \vec{v}_3 = \begin{bmatrix} 4 \\ -5 \\ 9 \\ 4 \end{bmatrix} \qquad \vec{v}_4 = \begin{bmatrix} 0 \\ 4 \\ 2 \\ -3 \end{bmatrix} \qquad \vec{v}_5 = \begin{bmatrix} -7 \\ 18 \\ 2 \\ -8 \end{bmatrix}$$

   Find a subset of V that is a basis for span(V) and express the remaining vectors as linear combinations of this basis.

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Challenge questions

5. Show that the intersection of any two subspaces is also a subspace.
6. Completely classify all conditions under which the union of two subspaces a subspace.
7. If X is a subspace, $\text{proj}_X \vec{v}$ is defined to be the vector in X whose distance from $\vec{v}$ is smallest. Show that if $X = \text{span}\{\vec{d}\}$, then $\text{proj}_X \vec{v}$ agrees with $\text{proj}_{\vec{d}} \vec{x}$.
8. If $X = \text{span}\{\vec{u}_1, \vec{u}_2\}$ is a plane, can you come up with an algorithm for computing $\text{proj}_X \vec{v}$? (Note that $\vec{u}_1, \vec{u}_2$ may not be in $\mathbb{R}^3$.)

MATH 110, Fall 2013
Tutorial #7. Instructions for TAs

Objectives

Hidden objectives

Suggestions

They may find set notation confusing, and some may think that $\{\vec{v}\}$ and $\text{span}\{\vec{v}\}$ are two ways of saying the same thing.

I would suggest briefly talking about how to interpret set notation with S_1 as an example. "A plane is a set of infinitely many vectors pointing from the origin to the surface if the plane," etc.

It's a good idea to get the axioms of a subspace (not of an abstract vector space, just a subspace) written on the board at the beginning of class and remind them that if they want to show that something isn't a subspace, they only need to find a single violation of one of the axioms. If something is a subspace, they need to show it works for all vectors, which will involve picking arbitrary vectors that satisfy the conditions specified in the set. This is going to be very, very hard for them.

This tutorial has quite a few problems on it and can be quite time consuming. Remind them that they are not expected to get through both the main problems and the further problems in the tutorial time. (In fact, it wouldn't be unreasonable if it took the full time just for question 1).

Wrapup

Choose a question that most of the class has started but not yet finished, or a question that people particularly struggled with.

Solutions

1.