

**Part 1** *Your answer to this part should be entered/uploaded directly to Crowdmark.*

Rewrite the following informal statements precisely, using linear algebra vocabulary.

- The nullspace is the vectors that disappear.
- Rotating an object doesn't stretch it in any direction.

**(PAR) Part 2** *Prepare your response to this part on the PAR Worksheet. You will bring your writeup of this problem to class and will upload both your draft and a final version to Crowdmark after completing the PAR process.*

Bryan and Rachel are studying for their MAT223 final exam. They are trying to find the eigenvalues of the matrix  $A = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ .

Bryan says the eigenvalues are 2, 1, and  $-1$ , because when you row-reduce,

$$\begin{bmatrix} 2 & 1 & 2 \\ 0 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 2 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix} \xrightarrow{R_3 - 2R_2} \begin{bmatrix} 2 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix},$$

you get an upper-triangular matrix, and so the eigenvalues are the numbers on the diagonal.

Rachel says the eigenvalues are 2 and 1 because when you compute the characteristic polynomial of  $A$ , you get

$$\text{char}(A) = \det \left( \begin{bmatrix} 2 - \lambda & 1 & 2 \\ 0 & 2 - \lambda & -1 \\ 0 & 1 & -\lambda \end{bmatrix} \right) = (2 - \lambda)(1 - \lambda)^2.$$

Since the only roots of  $\text{char}(A)$  are 2 and 1, those are the eigenvalues.

Explain to Bryan and Rachel who's right. Make sure to point out any correct, incorrect, or partially correct statements they made. Your response should be addressed to Bryan and Rachel, and should help them understand the material for their test (that is, don't just say where they went wrong or what they did right, explain *why*).

**Part 3** *Your answer to this part should be entered/uploaded directly to Crowdmark.*

At the start of the term, you responded to the following prompt:

*What type of thinking do you expect to do in MAT223? How is this type of thinking similar or different to the types of thinking you might do in classes for other subjects? Please give some examples.*

Now the course is almost over, review what you wrote at the start of the term (Writing Assignment 1, Part 3). How have your opinions changed or stayed the same since the start of the term? Use at least two specific examples in your response.