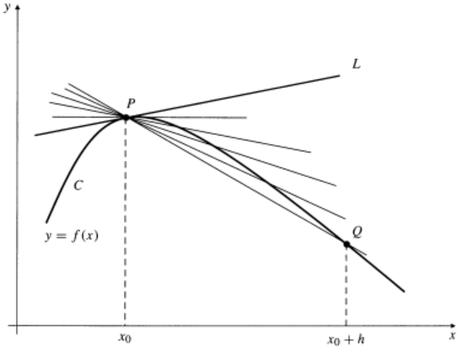
MATH 101 P.S.

October 25, 2021



Suppose that the function f is continuous at $x = x_0$ and that

$$\lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h} = m$$

exists. Then the straight line having slope m and passing through the point $P = (x_0, f(x_0))$ is called the tangent line (or simply the tangent) to the graph of $y = f(x)$ at P. An equation of this tangent is

$$y = m(x - x_0) + y_0$$

Question 8 from section 2.1

In Exercises $1-12$, find an equation of the straight line tangent to the given curve at the point indicated.
$y = \frac{1}{\sqrt{x}}$ at $x = 9$

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$y=\frac{1}{\sqrt{x}}$ at $x=9$
The slope of $y=\frac{1}{\sqrt{x}}$ at $x=9$ is

$$m=\lim_{h\to 0}\frac{1}{h}\left(\frac{1}{\sqrt{9+h}}-\frac{1}{3}\right)$$

$$=$$

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The slope of $y=\frac{1}{\sqrt{x}}$ at $x=9$ is

$$\begin{aligned}
m &= \lim_{h\to 0}\frac{1}{h}\left(\frac{1}{\sqrt{9+h}}-\frac{1}{3}\right)\\
&= \lim_{h\to 0}\frac{3-\sqrt{9+h}}{3h\sqrt{9+h}}\cdot\frac{3+\sqrt{9+h}}{3+\sqrt{9+h}}\\
&= \lim_{h\to 0}\frac{9-9-h}{3h\sqrt{9+h}(3+\sqrt{9+h})}\\
&= -\frac{1}{3(3)(6)}=-\frac{1}{54}
\end{aligned}$$

The tangent line at $\left(9,\frac{1}{3}\right)$ is $y=\frac{1}{3}-\frac{1}{54}(x-9)$, or $y=\frac{1}{2}-\frac{1}{54}x$

Question 10 from section 2.1

In Exercises $1 - 12$, find an equation of the straight line tangent to the given curve at the point indicated. $y = \sqrt{5 - x^2}$ at $x = 1$

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The slope of $y=\sqrt{5-x^2}$ at $x=1$ is

$$\begin{aligned} m &= \lim_{h\to 0} \frac{\sqrt{5-(1+h)^2}-2}{h} \\ &= \end{aligned}$$

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In Exercises $1-12$, find an equation of the straight line tangent to the given curve at the point indicated. $y=\sqrt{5-x^2}$ at $x=1$
The slope of $y=\sqrt{5-x^2}$ at $x=1$ is

$$\begin{aligned} m &= \lim_{h\to 0}\frac{\sqrt{5-(1+h)^2}-2}{h} \\ &= \lim_{h\to 0}\frac{5-(1+h)^2-4}{h\left(\sqrt{5-(1+h)^2}+2\right)} \\ &= \lim_{h\to 0}\frac{-2-h}{\sqrt{5-(1+h)^2+2}} = -\frac{1}{2} \end{aligned}$$

The tangent line at (1,2) is $y=2-\frac{1}{2}(x-1)$, or $y=\frac{5}{2}-\frac{1}{2}x$

Question 16 from section 2.1

Do the graphs of the functions f in Exercises $13 - 17$ have tangent lines at the given points? If yes, what is the tangent line?
$f(x) = \left|x^2 - 1\right|$ at $x = 1$

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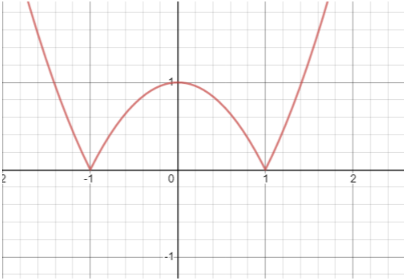

The slope of $f(x) = \left|x^2 - 1\right|$ at $x = 1$ is
$m = \lim_{h\to 0} \frac{\left|(1+h)^2-1\right|-|1-1|}{h} = \lim_{h\to 0} \frac{\left|2h+h^2\right|}{h}$ which does not exist, and is not $-\infty$ or ∞. The graph of f has no tangent at $x = 1$.

Question 20 from section 2.1

Find all points on the curve $y = x^3 - 3x$ where the tangent line is parallel to the x -axis.

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Question 20 from section 2.1

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The slope of $y = x^3 - 3x$ at $x = a$ is

$$\begin{aligned} m &= \lim_{h \to 0} \frac{1}{h} \left[(a+h)^3 - 3(a+h) - \left(a^3 - 3a \right) \right] \\ &= \lim_{h \to 0} \frac{1}{h} \left[a^3 + 3a^2 h + 3ah^2 + h^3 - 3a - 3h - a^3 + 3a \right] \\ &= \lim_{h \to 0} \left[3a^2 + 3ah + h^2 - 3 \right] = 3a^2 - 3 \end{aligned}$$

At points where the tangent line is parallel to the x -axis, the slope is zero, so such points must satisfy $3a^2 - 3 = 0$. Thus, $a = \pm 1$. Hence, the tangent line is parallel to the x -axis at the points (1,-2) and (-1,2).

Question 24 from section 2.1

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The curves $y = kx^2$ and $y = k(x-2)^2$ intersect at $(1, k)$ The slope of $y = kx^2$ at $x = 1$ is

$$m_1 = \lim_{h \to 0} \frac{k(1+h)^2 - k}{h} = \lim_{h \to 0}(2+h)k = 2k$$

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The slope of $y = k(x-2)^2$ at $x = 1$ is
$m_2 = \lim_{h \to 0} \frac{k(2-(1+h))^2 - k}{h} = \lim_{h \to 0} (-2+h)k = -2k$

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$m_2 = \lim_{h\to 0} \frac{k(2-(1+h))^2-k}{h} = \lim_{h\to 0}(-2+h)k = -2k$ The two curves intersect at right angles if $2k = -1/(-2k)$, that is, if $4k^2 = 1$, which is satisfied if $k = \pm 1/2$

Question 32 from section 2.1

Let $P(x)$ be a polynomial. If a is a real number, then $P(x)$ can be expressed in the form

$$P(x) = a_0 + a_1(x-a) + a_2(x-a)^2 + \cdots + a_n(x-a)^n$$

for some $n \geq 0$. If $\ell(x) = m(x-a) + b$, show that the straight line $y = \ell(x)$ is tangent to the graph of $y = P(x)$ at $x = a$ provided $P(x) - \ell(x) = (x-a)^2 Q(x)$, where $Q(x)$ is a polynomial.

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The slope of $P(x)$ at $x = a$ is

$$m = \lim_{h \to 0} \frac{P(a+h) - P(a)}{h}$$

since $P(a+h) = a_0 + a_1 h + a_2 h^2 + \cdots + a_n h^n$ and $P(a) = a_0$, the slope is

$$\begin{aligned} m &= \lim_{h \to 0} \frac{a_0 + a_1 h + a_2 h^2 + \cdots + a_n h^n - a_0}{h} \\ &= \lim_{h \to 0} a_1 + a_2 h + \cdots + a_n h^{n-1} = a_1 \end{aligned}$$

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Thus the line $y = \ell(x) = m(x-a) + b$ is tangent to $y = P(x)$ at $x = a$ if and only if $m = a_1$ and $b = a_0$ that is, if and only if

$$\begin{aligned} P(x) - \ell(x) &= a_2(x-a)^2 + a_3(x-a)^3 + \cdots + a_n(x-a)^n \\ &= (x-a)^2 \left[a_2 + a_3(x-a) + \cdots + a_n(x-a)^{n-2}\right] \\ &= (x-a)^2 Q(x) \end{aligned}$$

where Q is a polynomial.

Question 18 from section 2.2

In Exercises $11 - 24$, (a) calculate the derivative of the given function directly from the definition of derivative, and (b) express the result of (a) using differentials. $f(x) = \frac{3}{4}\sqrt{2-x}$

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$$f'(x)=\lim_{h\to 0}\frac{\frac{3}{4}\sqrt{2-(x+h)}-\frac{3}{4}\sqrt{2-x}}{h}$$

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$$f'(x)=\lim_{h\to 0}\frac{\frac{3}{4}\sqrt{2-(x+h)}-\frac{3}{4}\sqrt{2-x}}{h}$$

$$=\lim_{h\to 0}\frac{3}{4}\left[\frac{2-x-h-2+x}{h(\sqrt{2-(x+h)}+\sqrt{2-x})}\right]$$

$$=-\frac{3}{8\sqrt{2-x}}$$

$$df(x)=-\frac{3}{8\sqrt{2-x}}dx$$

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$$y'(x) = \lim_{h\to 0} \frac{\frac{1}{\sqrt{1+x+h}} - \frac{1}{\sqrt{1+x}}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{1+x} - \sqrt{1+x+h}}{h\sqrt{1+x+h}\sqrt{1+x}}$$

$$= \lim_{h\to 0} \frac{1+x-1-x-h}{h\sqrt{1+x+h}\sqrt{1+x}(\sqrt{1+x}+\sqrt{1+x+h})}$$

$$= \lim_{h\to 0} -\frac{1}{\sqrt{1+x+h}\sqrt{1+x}(\sqrt{1+x}+\sqrt{1+x+h})}$$

$$= -\frac{1}{2(1+x)^{3/2}}$$

$$dy = -\frac{1}{2(1+x)^{3/2}}dx$$

Question 27 from section 2.2

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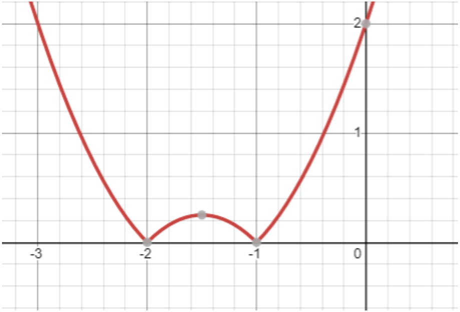

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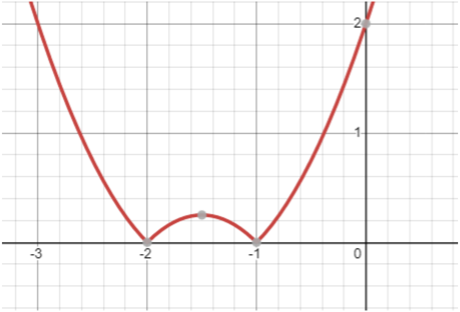

$h(x) = \left|x^2 + 3x + 2\right|$ fails to be differentiable where $x^2 + 3x + 2 = 0$, that is, at $x = -2$ and $x = -1$.

Question 32 from section 2.2

Using the definition of derivative, find equations for the tangent lines to the curves in Exercises $30 - 33$ at the points indicated. $y = \frac{t}{t^2-2}$ at the point where $t = -2$

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The slope of $y = \frac{t}{t^2-2}$ at $t = -2$ and $y = -1$ is

$$\left.\frac{dy}{dt}\right|_{t=-2} = \lim_{h\to 0} \frac{1}{h}\left[\frac{-2+h}{(-2+h)^2-2} - (-1)\right]$$
$$= \lim_{h\to 0} \frac{-2+h+\left[(-2+h)^2-2\right]}{h\left[(-2+h)^2-2\right]} = -\frac{3}{2}$$

Thus, the tangent line has the equation $y = -1 - \frac{3}{2}(t+2)$, that is, $y = -\frac{3}{2}t - 4$

Question 45 from section 2.2

Find an equation of the straight line normal to the curve $y = 1/x$ at the point where $x = a$.

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Slope of $y = \frac{1}{x}$ at $x = a$ is $-\left.\frac{1}{x^2}\right|_{x=a} = \frac{-1}{a^2}$ Normal has slope a^2, and equation $y - \frac{1}{a} = a^2(x - a)$ or $y = a^2x - a^3 + \frac{1}{a}$

Question 47 from section 2.2

There are two distinct straight lines that pass through the point (1,-3) and are tangent to the curve $y = x^2$. Find their equations. Hint: Draw a sketch. The points of tangency are not given; let them be denoted $\left(a, a^2\right)$

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Let the point of tangency be $\left(a, a^2\right)$.

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Let the point of tangency be $\left(a, a^2\right)$. Slope of tangent is $\frac{d}{dx}x^2\Big|_{x=a} = 2a$

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Let the point of tangency be $\left(a, a^2\right)$. Slope of tangent is $\frac{d}{dx}x^2\Big|_{x=a} = 2a$
This is the slope from $\left(a, a^2\right)$ to $(1, -3)$,

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This is the slope from $\left(a, a^2\right)$ to $(1, -3)$, so
$\frac{a^2+3}{a-1} = 2a$, and $\quad a^2+3 = 2a^2-2a \quad a^2-2a-3 = 0 \quad a = 3$ or -1

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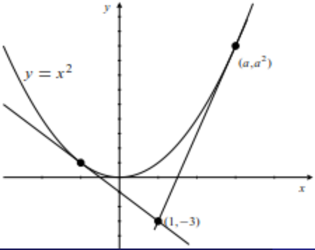
This is the slope from $\left(a, a^2\right)$ to $(1, -3)$, so
$\frac{a^2+3}{a-1} = 2a$, and $\qquad a^2+3 = 2a^2-2a \qquad a^2-2a-3 = 0 \qquad a = 3$ or -1
The two tangent lines are (for $a = 3$): $y - 9 = 6(x - 3)$ or $6x - 9$
(for $a = -1$) : $y - 1 = -2(x + 1)$ or $y = -2x - 1$



Question 48 from section 2.2

Find equations of two straight lines that have slope -2 and are tangent to the graph of $y = 1/x$

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Find equations of two straight lines that have slope -2 and are tangent to the graph of $y = 1/x$
The slope of $y = \frac{1}{x}$ at $x = a$ is

$$\left.\frac{dy}{dx}\right|_{x=a} = -\frac{1}{a^2}$$

If the slope is -2, then $-\frac{1}{a^2} = -2$, or $a = \pm\frac{1}{\sqrt{2}}$ Therefore, the equations of the two straight lines are $y = \sqrt{2} - 2\left(x - \frac{1}{\sqrt{2}}\right)$ and $y = -\sqrt{2} - 2\left(x + \frac{1}{\sqrt{2}}\right)$ or $y = -2x \pm 2\sqrt{2}$

Question 8 from section 2.3

Find the derivative of $y = 3\sqrt[3]{t^2} - \frac{2}{\sqrt{t^3}}$

Question 8 from section 2.3

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$$y = 3\sqrt[3]{t^2} - \frac{2}{\sqrt{t^3}} = 3t^{2/3} - 2t^{-3/2}$$

$$\frac{dy}{dt} = 2t^{-1/3} + 3t^{-5/2}$$

Question 11 from section 2.3

Find the derivative of $y = \sqrt{x}\left(5 - x - \frac{x^2}{3}\right)$

Question 11 from section 2.3

Find the derivative of $y = \sqrt{x}\left(5 - x - \frac{x^2}{3}\right)$

$y = \sqrt{x}\left(5 - x - \frac{x^2}{3}\right) = 5\sqrt{x} - x^{3/2} - \frac{1}{3}x^{5/2}$ $y' = \frac{5}{2\sqrt{x}} - \frac{3}{2}\sqrt{x} - \frac{5}{6}x^{3/2}$

Question 18 from section 2.3

Find the derivative of $g(u) = \frac{u\sqrt{u}-3}{u^2}$

Question 18 from section 2.3

Find the derivative of $g(u) = \frac{u\sqrt{u}-3}{u^2}$
$g(u) = \frac{u\sqrt{u}-3}{u^2} = u^{-1/2} - 3u^{-2}$ $g'(u) = -\frac{1}{2}u^{-3/2} + 6u^{-3} = \frac{12-u\sqrt{u}}{2u^3}$

Question 23 from section 2.3

Find the derivative of $s = \frac{1+\sqrt{t}}{1-\sqrt{t}}$

Question 23 from section 2.3

Find the derivative of $s = \frac{1+\sqrt{t}}{1-\sqrt{t}}$

$$\frac{ds}{dt} = \frac{(1-\sqrt{t})\frac{1}{2\sqrt{t}} - (1+\sqrt{t})\left(-\frac{1}{2\sqrt{t}}\right)}{(1-\sqrt{t})^2}$$

$$= \frac{1}{\sqrt{t(1-\sqrt{t})^2}}$$

Question 32 from section 2.3

Find the derivative of $f(x) = \frac{(\sqrt{x}-1)(2-x)\left(1-x^2\right)}{\sqrt{x}(3+2x)}$

Question 32 from section 2.3

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$$f(x) = \frac{(\sqrt{x}-1)(2-x)\left(1-x^2\right)}{\sqrt{x}(3+2x)}$$

$$= \left(1 - \frac{1}{\sqrt{x}}\right) \cdot \frac{2-x-2x^2+x^3}{3+2x}$$

$$f'(x) = \left(\frac{1}{2}x^{-3/2}\right)\frac{2-x-2x^2+x^3}{3+2x} + \left(1 - \frac{1}{\sqrt{x}}\right)$$

$$\times \frac{(3+2x)\left(-1-4x+3x^2\right) - \left(2-x-2x^2+x^3\right)(2)}{(3+2x)^2}$$

$$= \frac{(2-x)\left(1-x^2\right)}{2x^{3/2}(3+2x)}$$

$$+ \left(1 - \frac{1}{\sqrt{x}}\right)\frac{4x^3+5x^2-12x-7}{(3+2x)^2}$$

Question 40 from section 2.3

Find $\frac{d}{dt}((1+t)(1+2t)(1+3t)(1+4t))\Big|_{t=0}$

Question 40 from section 2.3

Find $\frac{d}{dt}((1+t)(1+2t)(1+3t)(1+4t))\Big|_{t=0}$

$\frac{d}{dt}[(1+t)(1+2t)(1+3t)(1+4t)]\Big|_{t=0}$
$= (1)(1+2t)(1+3t)(1+4t) + (1+t)(2)(1+3t)(1+4t) +$
$(1+t)(1+2t)(3)(1+4t) + (1+t)(1+2t)(1+3t)(4)|_{t=0}$
$= 1+2+3+4 = 10$

Question 44 from section 2.3

Find the equations of all horizontal lines that are tangent to the curve $y = x^2\left(4 - x^2\right)$

Question 44 from section 2.3

Find the equations of all horizontal lines that are tangent to the curve
$y = x^2\left(4 - x^2\right)$
If $y = x^2\left(4 - x^2\right)$, then
$y' = 2x\left(4 - x^2\right) + x^2(-2x) = 8x - 4x^3 = 4x\left(2 - x^2\right)$ The slope of a horizontal line must be zero, so $4x\left(2 - x^2\right) = 0$, which implies that $x = 0$ or $x = \pm\sqrt{2}$ At $x = 0, y = 0$ and at $x = \pm\sqrt{2}, y = 4$ Hence, there are two horizontal lines that are tangent to the curve. Their equations are $y = 0$ and $y = 4$

Question 46 from section 2.3

Find the coordinates of points on the curve $y = \frac{x+1}{x+2}$ where the tangent line is parallel to the line $y = 4x$.

Question 46 from section 2.3

Find the coordinates of points on the curve $y = \frac{x+1}{x+2}$ where the tangent line is parallel to the line $y = 4x$.
If $y = \frac{x+1}{x+2}$, then

$$y' = \frac{(x+2)(1) - (x+1)(1)}{(x+2)^2} = \frac{1}{(x+2)^2}$$

In order to be parallel to $y = 4x$, the tangent line must have slope equal to 4, i.e.,

$$\frac{1}{(x+2)^2} = 4, \quad \text{or } (x+2)^2 = \frac{1}{4}$$

Hence $x + 2 = \pm\frac{1}{2}$, and $x = -\frac{3}{2}$ or $-\frac{5}{2}$. At $x = -\frac{3}{2}$ $y = -1$, and at $x = -\frac{5}{2}, y = 3$ Hence, the tangent is parallel to $y = 4x$ at the points $\left(-\frac{3}{2}, -1\right)$ and $\left(-\frac{5}{2}, 3\right)$