GTU, Fall 2020, MATH 101

Limits at Infinity and Infinite Limits

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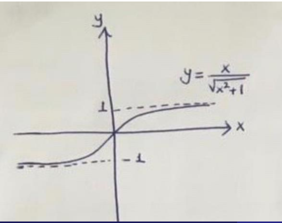

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Similarly we can give the definition of $\lim\limits_{x \to -\infty} f(x) = L$.

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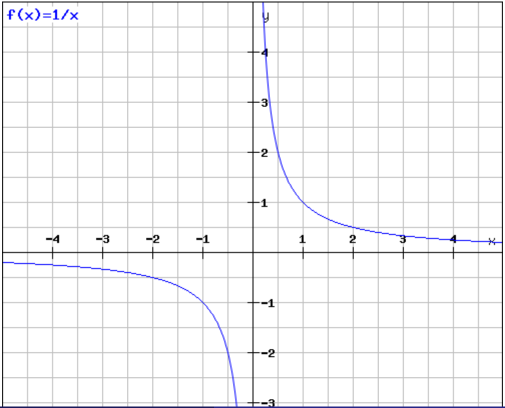

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$\ast$ Note that $\sqrt{x^2} = |x|$.

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- In general arithmetics does hold for:

$$"\infty + \infty" \quad "\infty . \infty" \quad "L . \infty" \quad (L \neq 0)$$

but many situations are undefined:

$$"\infty - \infty" \quad "\frac{\infty}{\infty}" \quad "0 . \infty" \quad "\frac{0}{0}" \quad "1^{\infty}" \quad "\infty^{0}" \quad "0^{\infty}$$

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$*$ Note that $\sqrt{x^2}=x$ because $x>0$ as $x\to\infty$.

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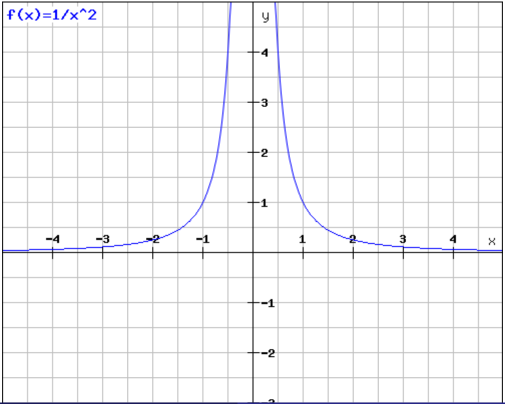

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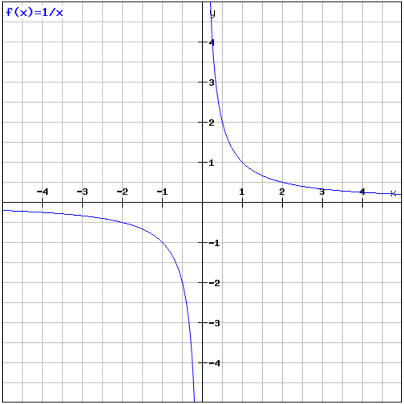

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The factor in the parentheses approaches 2 as x approaches $\pm\infty$,so the behaviour of the polynomial depend on its highest-degree term.

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Solution:

Divide the numerator and denominator by highest power of x appearing in the denominator:

$$\lim_{x \to -\infty} \frac{4x^3+2x^2+5}{x^2+7} = \lim_{x \to -\infty} \frac{4x+2+5/x^2}{1+7/x^2} = \frac{\lim\limits_{x \to -\infty} \left(4x+2+5/x^2\right)}{1} = -\infty$$