GTU, Fall 2020, MATH 101

A quadratic equation is an equation of degree two.

A quadratic equation is an equation of degree two.

For example, $x^2 + 3x - 4 = 0$ is a quadratic equation in one variable, namely x.

Preliminaries, Graphs of Quadratic Equations

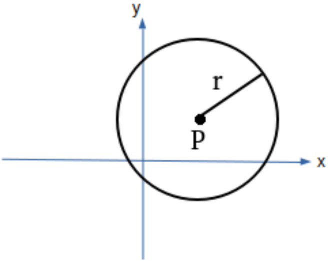
A quadratic equation is an equation of degree two.

For example, $x^2 + 3x - 4 = 0$ is a quadratic equation in one variable, namely x.

But now we will consider quadratic equations in two variables such as $y = x^2 - 3x$, $x^2 + 2x + y^2 = 5$, ...

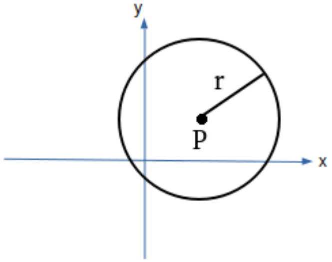
$*$ A circle is the set of all points whose distance to a given point $P(a, b)$ is the same.

$*$ A circle is the set of all points whose distance to a given point $P(a, b)$ is the same.



$*$ $P(a, b)$ is the center,
and r is the radius of the circle.

* A circle is the set of all points whose distance to a given point $P(a, b)$ is the same.



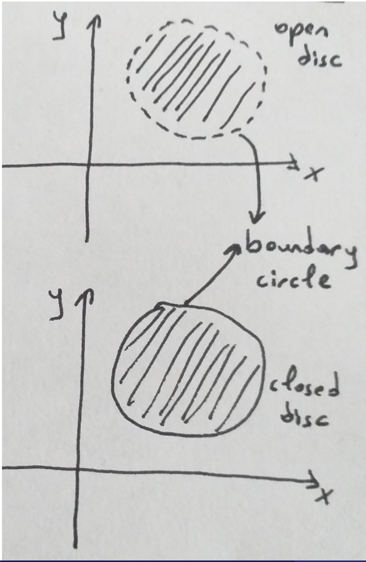
* $P(a, b)$ is the center,
and r is the radius of the circle.
* Let's
find the equation of the circle.
Recall that the distance between a point (x, y) and $P(a, b)$ is

$$r = \sqrt{(x-a)^2 + (y-b)^2}$$

So the circle is represented by

$$(x-a)^2 + (y-b)^2 = r^2$$

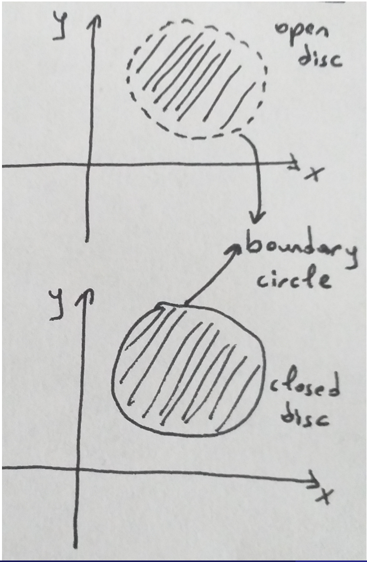
* Exercise: Let C be the circle with equation $x^2 + y^2 + 2x - 4y - 10 = 0$. Find the center and the radius of C. Sketch C.



$*$ If we only take

the *interior* of a circle then we obtain an *open disc*, so it is given by the inequality

$$(x-a)^2+(y-b)^2<r^2$$



∗ If we only take the *interior* of a circle then we obtain an *open disc*, so it is given by the inequality

$$(x-a)^2+(y-b)^2<r^2$$

∗ The inequality

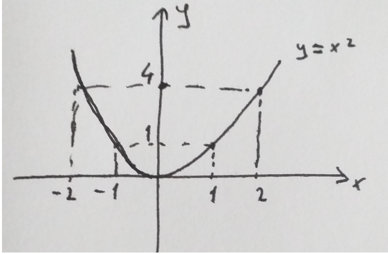
$$(x-a)^2+(y-b)^2\leq r^2$$

represents a closed disc. The circle $(x-a)^2+(y-b)^2=r^2$ is the *boundary* of the disc.

* A parabola is represented by an equation $y = ax^2 + bx + c$ or $x = ay^2 + by + c$ where $a \neq 0$.

Preliminaries, Graphs of Quadratic Equations - Parabolas

* A parabola is represented by an equation $y = ax^2 + bx + c$ or $x = ay^2 + by + c$ where $a \neq 0$.
* The simplest example is $y = x^2$ whose graph is



* We can obtain the graph of any parabola by using only $y = x^2$ (!).

Preliminaries, Graphs of Quadratic Equations - Shifting, Scaling and Reflecting

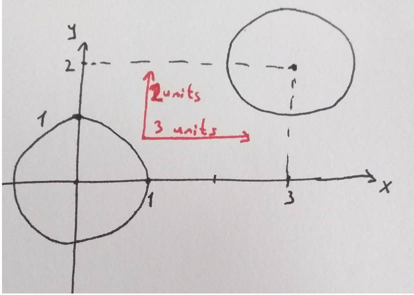
- The graphs of some equations can be obtained form the graphs of similar functions.

Preliminaries, Graphs of Quadratic Equations - Shifting, Scaling and Reflecting

* The graphs of some equations can be obtained form the graphs of similar functions.
* For example, consider the circles $x^2 + y^2 = 1$ and $(x-3)^2 + (y-2)^2 = 1$. Both have radius 1 but different centers.

Preliminaries, Graphs of Quadratic Equations - Shifting, Scaling and Reflecting

* The graphs of some equations can be obtained form the graphs of similar functions.
* For example, consider the circles $x^2 + y^2 = 1$ and $(x-3)^2 + (y-2)^2 = 1$. Both have radius 1 but different centers.
* We replace x by $x-3$ and y by $y-2$ in $x^2 + y^2 = 1$. So the circle $x^2 + y^2 = 1$ is *shifted* right by 3 units and up by 2 units.



Preliminaries, Graphs of Quadratic Equations - Shifting, Scaling and Reflecting

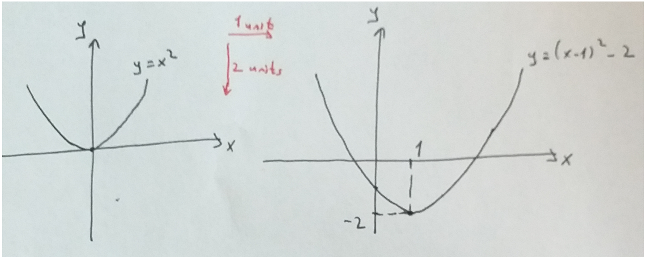
* This also applies to parabolas or to any *graph*.

Preliminaries, Graphs of Quadratic Equations - Shifting, Scaling and Reflecting

* This also applies to parabolas or to any *graph*.
* Example: Consider the parabola $y = x^2 - 2x - 1$. We can write it as $y + 2 = (x - 1)^2$.

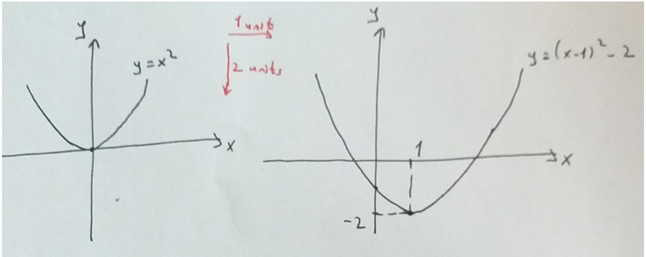
Preliminaries, Graphs of Quadratic Equations - Shifting, Scaling and Reflecting

* This also applies to parabolas or to any *graph*.
* Example: Consider the parabola $y = x^2 - 2x - 1$. We can write it as $y + 2 = (x - 1)^2$.
* So we shift $y = x^2$ to right by 1 unit and down by 2 units (why?)



Preliminaries, Graphs of Quadratic Equations - Shifting, Scaling and Reflecting

* This also applies to parabolas or to any *graph*.
* Example: Consider the parabola $y = x^2 - 2x - 1$. We can write it as $y + 2 = (x-1)^2$.
* So we shift $y = x^2$ to right by 1 unit and down by 2 units (why?)

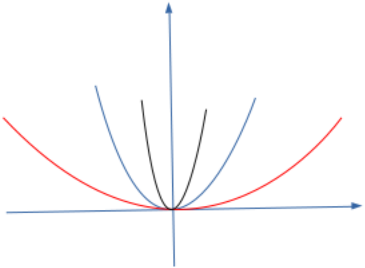


* We used *horizantal* and vertical shifting.

* We may also use scaling.

Preliminaries, Graphs of Quadratic Equations - Shifting, Scaling and Reflecting

* We may also use scaling.
* For example the graphs of $y = 2x^2$, $y = x^2$, and $y = x^2/2$ are very similar to each other, but scaled.



Preliminaries, Graphs of Quadratic Equations - Shifting, Scaling and Reflecting

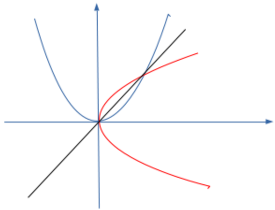
* Now consider the parabola $x = y^2$. It is obtained from $y = x^2$ by interchanging x and y.

Preliminaries, Graphs of Quadratic Equations - Shifting, Scaling and Reflecting

* Now consider the parabola $x = y^2$. It is obtained from $y = x^2$ by interchanging x and y.
* What does this mean geometrically?

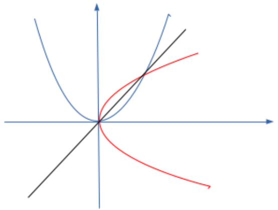
Preliminaries, Graphs of Quadratic Equations - Shifting, Scaling and Reflecting

* Now consider the parabola $x = y^2$. It is obtained from $y = x^2$ by interchanging x and y.
* What does this mean geometrically?
* We need to reflect $y = x^2$ about the line $y = x$ to obtain $x = y^2$ (why?)



Preliminaries, Graphs of Quadratic Equations - Shifting, Scaling and Reflecting

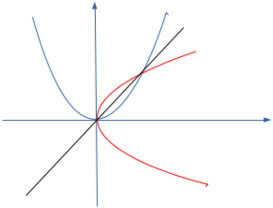
* Now consider the parabola $x = y^2$. It is obtained from $y = x^2$ by interchanging x and y.
* What does this mean geometrically?
* We need to reflect $y = x^2$ about the line $y = x$ to obtain $x = y^2$ (why?)



* This reflection rule holds for any parabola.

Preliminaries, Graphs of Quadratic Equations - Shifting, Scaling and Reflecting

* Now consider the parabola $x = y^2$. It is obtained from $y = x^2$ by interchanging x and y.
* What does this mean geometrically?
* We need to reflect $y = x^2$ about the line $y = x$ to obtain $x = y^2$ (why?)



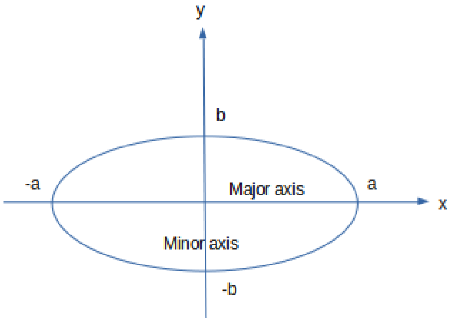
* This reflection rule holds for any parabola.
* Exercise: Sketch the parabolas $y = 2x^2 + 4x - 5$ and $x = y^2 + 4y + 3$.

Preliminaries, Graphs of Quadratic Equations - Ellipse and Hyperbola

$\ast$ Another basic quadratic equation is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

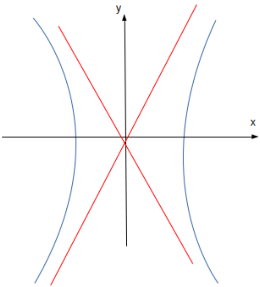
which represents an ellipse.



Preliminaries, Graphs of Quadratic Equations - Ellipse and Hyperbola

* The general equation for a hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



Preliminaries, Graphs of Quadratic Equations - Ellipse and Hyperbola

∗ The general equation for a hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

∗ The lines $\frac{x}{a} \pm \frac{y}{b} = 0$
are called the asymptotes of the hyperbola.