

MAT/MATH 101, Vize (Midterm), Group A, 04.12.2021, 10:00 - 11:45

İsim ve Soyisim (Name and Surname) : _____

Öğrenci No(Student ID number): _____

Bölüm (Department) : _____

* This is a closed book and closed notes exam (Bu sınavda kitap ve not kullanılamaz).

* No calculators, no talking and no questions (Hesap makinası, konuşmak ve soru sormak yasaktır).

* This is a multiple choice exam. In the table below, please fill in the circle corresponding to the correct answer in each question. (Bu çoktan seçmeli bir sınavdır. Aşağıdaki tabloda her soru için doğru şıkkı işaretleyiniz.)

*Only the answers on the front page will be considered when calculating exam grade. (Sınav notu hesaplanırken sadece ön sayfadaki cevaplar dikkate alınacaktır.)

Questions (Group A)	Answers
1	(A) <input checked="" type="radio"/> (B) <input type="radio"/> (C) <input type="radio"/> (D) <input type="radio"/> (E) <input type="radio"/>
2	(A) <input type="radio"/> (B) <input type="radio"/> (C) <input type="radio"/> (D) <input checked="" type="radio"/> (E) <input type="radio"/>
3	(A) <input checked="" type="radio"/> (B) <input type="radio"/> (C) <input type="radio"/> (D) <input type="radio"/> (E) <input type="radio"/>
4	(A) <input type="radio"/> (B) <input type="radio"/> (C) <input checked="" type="radio"/> (D) <input type="radio"/> (E) <input type="radio"/> (F) <input type="radio"/>
5	(A) <input type="radio"/> (B) <input type="radio"/> (C) <input type="radio"/> (D) <input type="radio"/> (E) <input checked="" type="radio"/>
6	(A) <input checked="" type="radio"/> (B) <input type="radio"/> (C) <input type="radio"/> (D) <input type="radio"/> (E) <input type="radio"/>
7	(A) <input type="radio"/> (B) <input type="radio"/> (C) <input type="radio"/> (D) <input checked="" type="radio"/> (E) <input type="radio"/>
8	(A) <input type="radio"/> (B) <input type="radio"/> (C) <input checked="" type="radio"/> (D) <input type="radio"/> (E) <input type="radio"/> (F) <input type="radio"/>
9	(A) <input type="radio"/> (B) <input type="radio"/> (C) <input checked="" type="radio"/> (D) <input type="radio"/> (E) <input type="radio"/>
10	(A) <input checked="" type="radio"/> (B) <input type="radio"/> (C) <input type="radio"/> (D) <input type="radio"/> (E) <input type="radio"/>
Total	

Questions- Group A

1. Find numbers a and b so that $f(x)$ is continuous at every point.

$$f(x) = \begin{cases} 21, & x < -5 \\ ax + b, & -5 \leq x \leq 2 \\ -14, & x > 2 \end{cases}$$

($f(x)$ fonksiyonu her noktada sürekli olacak şekilde a ve b değerlerini bulunuz.)

- A) $a = -5$
 B) $a = -5$
 C) $a = 21$
 D) $a = -5$
 E) $a = 21$
 b = -24 b = -4 b = -14 b = -14 b = -5

$$\lim_{x \rightarrow -5^-} f(x) = 21 = -5a + b$$

$$\lim_{x \rightarrow 2^+} f(x) = -14 = 2a + b$$

$$a = -5$$

$$b = -4$$

2. For what values of the numbers a and b is the curve $f(x) = e^{ax^2+bx}$ tangent to the line $y = e$ at the point $(x, y) = (1, e)$.

(a ve b nin hangi değerleri için $y = e$ doğrusu $(x, y) = (1, e)$ noktasında $f(x) = e^{ax^2+bx}$ eğrisine teğettir.)

- A) $a = -5$
 B) $a = -4$
 C) $a = -3$
 D) $a = -2$
 E) $a = -1$
 b = 10 b = 8 b = 6 b = 4 b = 2

$$f'(x) = e^{ax^2+bx} \cdot (2ax + b), \quad (2ax + b) = \text{slope} = 0 \quad (x=1)$$

$$2a + b = 0$$

$$f(1) = e \Rightarrow e^{a+b} = e \Rightarrow a+b = 1$$

$$\Rightarrow a = -1, b = 2$$

3. Let $F(x) = f(3 + 2\sqrt{x})$. If $f'(7) = -2$, find $F'(4)$.

($F(x) = f(3 + 2\sqrt{x})$ olsun. $f'(7) = -2$ ise $F'(4)$ değerini bulunuz.)

A) -1

B) $-\frac{1}{2}$

C) 0

D) $\frac{1}{2}$

E) 1

$$F'(x) = f'(3 + 2\sqrt{x}), 2 \cdot \frac{1}{2} x^{-\frac{1}{2}}$$

$$x=4$$

$$F'(4) = f'(7), 4^{-\frac{1}{2}} = -2 \cdot \frac{1}{2} = -1$$

4. Let $x^2y^5 + 2x = 7y + 1$. Find $y'|_{(1,1)}$.

($x^2y^5 + 2x = 7y + 1$ ise, $y'|_{(1,1)}$ değerini bulunuz.)

A) -1

B) 1

C) 2

D) 3

E) 0

F) -3

$$2xy^5 + x^2 5y^4 y' + 2 = 7y'$$

$$2 + 5y' + 2 = 7y'$$

$$4 = 2y'$$

$$y' = 2$$

5. Evaluate the derivative of $y = (\cos x)^{x^2}$ at $x = \pi/4$. ($y = (\cos x)^{x^2}$ fonksiyonunun $x = \pi/4$ noktasındaki türevini hesaplayınız.)

A) $\left(\left(\frac{\pi}{4}\right) \ln\left(\frac{1}{2}\right) + \left(\frac{\pi}{4}\right)^2\right) \left(\frac{1}{\sqrt{2}}\right)^{\left(\frac{\pi}{4}\right)}$

B) $\left(\left(\frac{\pi}{2}\right) \ln\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{\pi}{4}\right)^2\right) \left(\frac{1}{\sqrt{2}}\right)^{\left(\frac{\pi}{4}\right)^2}$

C) $\left(\left(\frac{\pi}{2}\right) \ln\left(\frac{1}{2}\right) - \left(\frac{\pi}{4}\right)^2\right) \left(\frac{1}{2}\right)^{\left(\frac{\pi}{4}\right)^2}$

D) $\left(\left(\frac{\pi}{2}\right) \ln\left(\frac{1}{2}\right) + \left(\frac{\pi}{4}\right)^2\right) \left(\frac{1}{2}\right)^{\left(\frac{\pi}{4}\right)^2}$

E) $\left(\left(\frac{\pi}{2}\right) \ln\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{\pi}{4}\right)^2\right) \left(\frac{1}{\sqrt{2}}\right)^{\left(\frac{\pi}{4}\right)}$

F) $\left(\left(\frac{\pi}{2}\right) \ln\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{\pi}{4}\right)^2\right) \left(\frac{1}{\sqrt{2}}\right)^{\left(\frac{\pi}{4}\right)^2}$

$$\ln y = x^2 \ln \cos x$$

$$\frac{y'}{y} = 2x \ln \cos x + x^2 \frac{1}{\cos x} (-\sin x)$$

$$x = \frac{\pi}{4} \Rightarrow$$

$$y' = \left(\frac{\pi}{2} \ln\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{\pi}{4}\right)^2 \right) \left(\frac{1}{\sqrt{2}}\right)^{\left(\frac{\pi}{4}\right)^2}$$

6. Find $\frac{d}{dx}(2x \arcsin x + \sqrt{1-x^2})$.

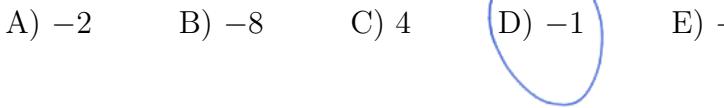
$(\frac{d}{dx}(2x \arcsin x + \sqrt{1-x^2})$ türevini hesaplayınız.)

- A) $2 \arcsin x + \frac{x}{\sqrt{1-x^2}}$ B) $2 \arctan x + \frac{x}{\sqrt{1-x^2}}$ C) $4 \arcsin x + \frac{x}{\sqrt{1-x^2}}$ D) $2 \arcsin x$ E) $2 \arctan x$

$$2 \sin^{-1} x + 2x \cdot \frac{1}{\sqrt{1-x^2}} + \frac{1}{2} (1-x^2)^{-\frac{1}{2}} (-2x)$$

7. The area of a disk is decreasing at a rate $4\sqrt{\pi} \text{ cm}^2/\text{min}$. How fast is the radius of the disk changing when the area is 4 cm^2 ? (Bir dairenin alanı $4\sqrt{\pi} \text{ cm}^2/\text{min}$ hızla azalmaktadır. Alan 4 cm^2 iken, dairenin yarıçapı ne hızla değişir?)

- A) -2 B) -8 C) 4 D) -1 E) -4



$$A = \pi r^2$$

$$A=4 \Rightarrow r = \frac{2}{\sqrt{\pi}}$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$-4\sqrt{\pi} = 2\pi \frac{2}{\sqrt{\pi}} \frac{dr}{dt}$$

8. Evaluate $\lim_{x \rightarrow \infty} [\cos(\frac{4}{x})]^{x^2}$. (Limiti hesaplayınız.)

- A) e^{-2} B) e^{-4} C) e^{-8} D) e^{-16} E) e^{-18} F) e^{-32}

1^∞

$$y = \cos\left(\frac{4}{x}\right)^{x^2} \quad \ln y = x^2 \ln\left(\cos\left(\frac{4}{x}\right)\right) \rightarrow \infty$$

$$\lim_{x \rightarrow \infty} \ln y = \frac{\ln\left(\cos\left(\frac{4}{x}\right)\right)}{x^2} \quad \frac{0}{0}$$

$$\text{LH} = \lim_{x \rightarrow \infty} \frac{1}{\cos(4/x)} \cdot \frac{-\sin(4/x)(-4)x^{-2}}{-2x^{-3}}$$

$$= \lim_{x \rightarrow \infty} \underbrace{\frac{1}{\cos(4/x)}}_1 \cdot \underbrace{\lim_{x \rightarrow \infty} \frac{\sin(4/x), 8}{-4/x}}_8 = -8$$

9. Find the intervals on which $f(x) = \frac{x^2}{x-2}$ is increasing/decreasing.

($f(x) = \frac{x^2}{x-2}$ fonksiyonunun artan/azalan olduğu aralıkları bulunuz.)

A) increasing on $(0, 2)$ and $(4, \infty)$, decreasing on $(-\infty, 0)$ and $(2, 4)$

((0, 2) ve $(4, \infty)$ aralıklarında artan, $(-\infty, 0)$ ve $(2, 4)$ aralıklarında azalan)

B) increasing on $(0, 2)$ and $(2, 4)$, decreasing on $(-\infty, 0)$ and $(4, \infty)$

((0, 2) ve $(2, 4)$ aralıklarında artan, $(-\infty, 0)$ ve $(4, \infty)$ aralıklarında azalan)

C) increasing on $(-\infty, 0)$ and $(4, \infty)$, decreasing on $(0, 2)$ and $(2, 4)$

(($-\infty, 0$) ve $(4, \infty)$ aralıklarında artan, $(0, 2)$ ve $(2, 4)$ aralıklarında azalan)

D) increasing on $(-\infty, 0)$ and $(2, 4)$, decreasing on $(0, 2)$ and $(4, \infty)$

(($-\infty, 0$) ve $(2, 4)$ aralıklarında artan, $(0, 2)$ ve $(4, \infty)$ aralıklarında azalan)

E) increasing on $(-\infty, 2)$ and $(4, \infty)$, decreasing on $(2, 4)$

(($-\infty, 2$) ve $(4, \infty)$ aralıklarında artan, $(2, 4)$ aralığında azalan)

$$\begin{aligned}f(x) &= x^2 (x-2)^{-1} \\f'(x) &= 2x(x-2)^{-1} + x^2(-1)(x-2)^{-2} \\&= \frac{2x(x-2) - x^2}{(x-2)^2} = \frac{x^2 - 4x}{(x-2)^2} \times (x-4)\end{aligned}$$



10. Find the intervals on which $f(x) = \frac{x^2}{x-2}$ is concave up/down.

($f(x) = \frac{x^2}{x-2}$ fonksiyonunun yukarı/aşağı konkav olduğu aralıkları bulunuz.)

C)

A) concave down on $(-\infty, 0)$ and $(0, 2)$, concave up on $(2, 4)$ and $(4, \infty)$

$((-\infty, 0)$ ve $(0, 2)$ aralıklarında aşağı konkav, $(2, 4)$ ve $(4, \infty)$ aralıklarında yukarı konkav)

B) concave down on $(2, 4)$ and $(4, \infty)$, concave up on $(-\infty, 0)$ and $(0, 2)$

$((2, 4)$ ve $(4, \infty)$ aralıklarında aşağı konkav, $(-\infty, 0)$ and $(0, 2)$ aralıklarında yukarı konkav)

C) concave down on $(-\infty, 0)$ and $(2, 4)$, concave up on $(0, 2)$ and $(4, \infty)$

$((-\infty, 0)$ ve $(2, 4)$ aralıklarında aşağı konkav, $(0, 2)$ ve $(4, \infty)$ aralıklarında yukarı konkav)

D) concave down on $(0, 2)$ and $(4, \infty)$, concave up on $(-\infty, 0)$ and $(2, 4)$

$((0, 2)$ ve $(4, \infty)$ aralıklarında aşağı konkav, $(-\infty, 0)$ ve $(2, 4)$ aralıklarında yukarı konkav)

E) concave down on $(-\infty, 0)$ and $(4, \infty)$, concave up on $(0, 2)$ and $(2, 4)$

$((-\infty, 0)$ ve $(4, \infty)$ aralıklarında aşağı konkav, $(0, 2)$ ve $(2, 4)$ aralıklarında yukarı konkav)

$$f'(x) = (x^2 - 4x)(x-2)^{-2}$$

$$f''(x) = (2x-4)(x-2)^{-2} + (x^2 - 4x)(-2)(x-2)^{-3}$$
$$= (x-2)^{-2} \left((2x-4) - \frac{2(x^2 - 4x)}{x-2} \right)$$

$$= (x-2)^{-2} \left(\frac{2(x^2 - 4x + 4) - 2x^2 + 8x}{x-2} \right)$$
$$= \frac{8}{(x-2)^3} \quad f'' > 0 \text{ if } x > 2$$