MAT/MATH 102, Final , Group A, 06.06.2022, 17:30 - 19:30

İsim ve Soyisim (Name and Surname) : ______________________

Öğrenci No(Student ID number): : ______________________

Bölüm (Department) : ______________________

* This is a closed book and closed notes exam (Bu sınavda kitap ve not kullanılamaz).
* No calculators, no talking and no questions (Hesap makinası, konuşmak ve soru sormak yasaktır).

*This is a multiple chqice exam. In the table below, please fill in the circle corresponding to the correct answer in each question. (Bu çoktan seçmeli bir sınavdır. Verilen tabloda her soru için doğru şıkkı işaretleyiniz.)

*Only the answers on the front page will be considered when calculating exam grade. (Sınav notu hesaplanırken sadece ön sayfadaki cevaplar dikkate alınacaktır.)

* Each question is 10 points (Her soru 10 puandır).
* Your exam time is 120 minutes. Good luck. (Sınav süreniz 120 dakikadır. Başarılar.)

Questions (Group A)	Answers
1	A B C D E
2	A B C D E F
3	A B C D E
4	A B C D E
5	A B C D E F
6	A B C D E
7	A B C D E F
8	A B C D E F
9	A B C D E F
10	A B C D E
Total	

Questions- Group A

1. Find the integral $\iint_D (x+y)\sin(x-y)dA$ if D is the region bounded by the lines $y=x$, $y=x-8$, $y=-x$, $y=-x+6$.

(D bölgesi $y=x$, $y=x-8$, $y=-x$, $y=-x+6$ doğruları ile sınırlandırılmış bölge olmak üzere $\iint_D (x+y)\sin(x-y)dA$ integralini hesaplayınız.)

A) $9(1+\cos 8)$ B) $18(1+\cos 8)$ C) $9(1-\cos 8)$ D) $18(1-\cos 8)$ E) $3(1+\cos 8)$

$$\left.\begin{aligned} u &= x+y \\ v &= x-y \end{aligned}\right\} \quad D: \quad \begin{aligned} x-y=0 &\Rightarrow v=0 \\ x-y=8 &\Rightarrow v=8 \end{aligned} \qquad \begin{aligned} x+y=0 &\Rightarrow u=0 \\ x+y=6 &\Rightarrow u=6 \end{aligned}$$

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1-1=-2$$

$$\left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \frac{1}{|-2|} = \frac{1}{2}$$

$$I = \iint_D (x+y)\sin(x-y)\,dA = \frac{1}{2}\int_0^8\int_0^6 u\sin v\,du\,dv$$

$$= \frac{1}{2}\int_0^8 \left(\frac{u^2}{2}\sin v\right)\Big|_0^6 dv = \frac{1}{2}\int_0^8 18\sin v\,dv$$

$$= 9(1-\cos 8)$$

2. Find the equation of the tangent plane to the graph of $f(x, y) = \arctan\left(\frac{y}{x}\right)$ at the point $(4\sqrt{3}, 4)$.

(Verilen fonksiyonun grafiğine $(4\sqrt{3}, 4)$ noktasında teğet olan düzlemin denklemini bulunuz.)

A) $\frac{-1}{16}(x - 4\sqrt{3}) + \frac{\sqrt{3}}{15}(y - 4) + (z - \frac{\pi}{3}) = 0$

B) $\frac{\sqrt{3}}{16}(x - 4\sqrt{3}) - \frac{1}{16}(y - 4) - (z - \frac{\pi}{3}) = 0$

C) $\frac{-1}{16}(x - 4\sqrt{3}) + \frac{\sqrt{3}}{16}(y - 4) + (z - \frac{\pi}{6}) = 0$

D) $\frac{\sqrt{3}}{16}(x - 4\sqrt{3}) - \frac{1}{16}(y - 4) + (z - \frac{\pi}{3}) = 0$

E) $\frac{\sqrt{3}}{16}(x - 4\sqrt{3}) - \frac{1}{16}(y - 4) - (z - \frac{\pi}{6}) = 0$

F) $\frac{-1}{16}(x - 4\sqrt{3}) + \frac{\sqrt{3}}{16}(y - 4) - (z - \frac{\pi}{6}) = 0$

$$f_x = \frac{1}{1+\frac{y^2}{x^2}} \cdot \frac{-y}{x^2} = \frac{-y}{x^2+y^2}$$

$$f_y = \frac{1}{1+\frac{y^2}{x^2}} \cdot \frac{1}{x} = \frac{x}{x^2+y^2}$$

$$f_x(4\sqrt{3},4) = \frac{-4}{(4\sqrt{3})^2+4^2} = -\frac{1}{12+4} = -\frac{1}{16}$$

$$f_y(4\sqrt{3},4) = \frac{4\sqrt{3}}{(4\sqrt{3})^2+4^2} = \frac{\sqrt{3}}{16}$$

$$f(4\sqrt{3},4) = \arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

The equation of tangent plane;

$$-\frac{1}{16}(x-4\sqrt{3}) + \frac{\sqrt{3}}{16}(y-4) - \left(z-\frac{\pi}{6}\right) = 0$$

3. For what value(s) of the constant $k \neq 0$ is the function $f(x, y, z) = e^{-(3x+4y)} \sin(kz)$ a harmonic function? Note that a harmonic function f satisfies $f_{xx} + f_{yy} + f_{zz} = 0$.

($f(x, y, z) = e^{-(3x+4y)} \sin(kz)$ fonksiyonu $k \neq 0$ olacak şekilde k nın hangi değer ya da değerleri için harmonik fonksiyondur. Not: $f_{xx} + f_{yy} + f_{zz} = 0$ eşitliğini sağlayan fonksiyona harmonik fonksiyon denir.)

A) $k = \pm 5$ B) $k = \pm 1$ C) $k = 7$ D) $k = 3$ E) $k = \pm 4$

$f_x = -3e^{-(3x+4y)} \sin(kz)$

$f_{xx} = 9e^{-(3x+4y)} \sin(kz)$

$f_y = -4e^{-(3x+4y)} \sin(kz)$

$f_{yy} = 16e^{-(3x+4y)} \sin(kz)$

$f_z = ke^{-(3x+4y)} \cos(kz)$

$f_{zz} = -k^2 e^{-(3x+4y)} \sin(kz)$

$f_{xx} + f_{yy} + f_{zz} = e^{-(3x+4y)} \sin(kz)(25 - k^2) = 0$

$25 - k^2 = 0$

$\boxed{k = \pm 5}$

4. If $x = u^3 - uv$ and $y = 3uv + 2v^2$ are solved for u and v in terms of x and y, evaluate $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial x}$ at the point $u = -1$ and $v = 2$.

(u ve v; x ve y bağlı olmak üzere $x = u^3 - uv$ ve $y = 3uv + 2v^2$; u ve v için çözülebilir olsun. $\frac{\partial u}{\partial x}$ ve $\frac{\partial v}{\partial x}$ ifadelerini $u = -1$ ve $v = 2$ noktasındaki değerini bulunuz.)

A) $\frac{\partial u}{\partial x} = 5, \quad \frac{\partial v}{\partial x} = 6$

B) $\frac{\partial u}{\partial x} = 3, \quad \frac{\partial v}{\partial x} = -5$

C) $\frac{\partial u}{\partial x} = 2, \quad \frac{\partial v}{\partial x} = -7$

D) $\frac{\partial u}{\partial x} = -5, \quad \frac{\partial v}{\partial x} = 6$

E) $\frac{\partial u}{\partial x} = -8, \quad \frac{\partial v}{\partial x} = 1$

$u = u(x,y)$
$v = v(x,y)$

$x = u^3 - uv$

$y = 3uv + 2v^2$

for $u=-1$ and $v=2$;

$$1 = 3u^2\frac{\partial u}{\partial x} - v.\frac{\partial u}{\partial x} - u.\frac{\partial v}{\partial x}$$

$$0 = 3v\frac{\partial u}{\partial x} + 3u\frac{\partial v}{\partial x} + 4v\frac{\partial v}{\partial x}$$

$$1 = 3\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial x}$$

$$0 = 6\frac{\partial u}{\partial x} - 3\frac{\partial v}{\partial x} + 8\frac{\partial v}{\partial x} = 6\frac{\partial u}{\partial x} + 5\frac{\partial v}{\partial x} \Rightarrow \frac{\partial u}{\partial x} = -\frac{5}{6}\frac{\partial v}{\partial x}$$

Thus; $$1 = -\frac{5}{6}\frac{\partial v}{\partial x} + \frac{\partial v}{\partial x} = \frac{1}{6}\frac{\partial v}{\partial x} \Rightarrow \frac{\partial v}{\partial x} = 6$$

$$\frac{\partial u}{\partial x} = -5$$

5. Find $\nabla f(a,b)$ for the differentiable function $f(x,y)$ given the directional derivatives

$$D_{\mathbf{u}}f(a,b) = 3\sqrt{2} \quad \text{and} \quad D_{\mathbf{v}}f(a,b) = 5$$

where $\mathbf{u} = \frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j})$ and $\mathbf{v} = \frac{1}{5}(3\mathbf{i}-4\mathbf{j})$.

($f(x,y)$ diferansiyellenebilir fonksiyonu için $\mathbf{u} = \frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j})$ ve $\mathbf{v} = \frac{1}{5}(3\mathbf{i}-4\mathbf{j})$ olmak üzere

$$D_{\mathbf{u}}f(a,b) = 3\sqrt{2} \quad \text{ve} \quad D_{\mathbf{v}}f(a,b) = 5$$

yönlü türev değerleri verildiğine göre $\nabla f(a,b)$ vektörünü bulunuz.)

A) $\frac{2}{7}\mathbf{i} - \frac{16}{7}\mathbf{j}$ B) $\frac{36}{7}\mathbf{i} - \frac{8}{7}\mathbf{j}$ C) $\frac{23}{7}\mathbf{i} - \frac{9}{7}\mathbf{j}$ D) $\frac{10}{7}\mathbf{i} - \frac{10}{7}\mathbf{j}$ E) $7\mathbf{i} - \mathbf{j}$ F) $\mathbf{i} - 7\mathbf{j}$

$\vec{u}$ and $\vec{v}$ are unit vectors, then

$$D_{\vec{u}} f(a,b) = \nabla f(a,b) \cdot \vec{u} = \frac{1}{\sqrt{2}}\left(f_1(a,b) + f_2(a,b)\right) = 3\sqrt{2}$$

$$D_{\vec{v}} f(a,b) = \nabla f(a,b) \cdot \vec{v} = \frac{1}{5}\left(3f_1(a,b) - 4f_2(a,b)\right) = 5$$

$$4 \,/\, f_1(a,b) + f_2(a,b) = 6$$

$$3f_1(a,b) - 4f_2(a,b) = 25$$

$$7f_1(a,b) = 49$$

$$f_1(a,b) = 7$$

$$f_2(a,b) = -1$$

$$\nabla f(a,b) = 7\vec{i} - \vec{j}$$

6. Find the local extremum value of the function $f(x,y) = xy - x^2 - y^2 - 3x - 3y + 1$.

($f(x,y) = xy - x^2 - y^2 - 3x - 3y + 1$ fonksiyonunun yerel (lokal) ekstrem değerini bulunuz.)

A) 20 local minimum (yerel minimum)
B) 20 local maximum (yerel maksimum)
C) 10 local minimum (yerel minimum)
D) 10 local maximum (yerel maksimum)
E) 30 local maximum (yerel maksimum)

$f_1(x,y) = y - 2x - 3 = 0 \Rightarrow y = 2x+3$

$f_2(x,y) = x - 2y - 3 = 0 \Rightarrow x - 2(2x+3) - 3 = 0$

$x - 4x - 6 - 3 = 0$

$-3x = 9$

$x = -3$

$y = 2(-3)+3 = -3$

The only critical point is $(-3,-3)$

$A = f_{11}(-3,-3) = -2$

$B = f_{12}(-3,-3) = 1$

$C = f_{22}(-3,-3) = -2$

$B^2 - AC = 1 - 4 = -3 < 0$ and $A < 0$

thus $f(-3,-3) = 9 - 9 - 9 + 9 + 9 + 1$

$f(-3,-3) = 10$ is the local maximum value

7. Find the maximum value of $f(x,y,z) = x - 2y + 5z$ on the sphere $x^2 + y^2 + z^2 = 30$.
Hint: Use the method of Lagrange Multipliers.

($f(x,y,z) = x - 2y + 5z$ fonksiyonunun $x^2 + y^2 + z^2 = 30$ küresi uzerindeki maximum değerini bulunuz. Yol gösterme: Lagrange Çarpanları Yöntemi'ni kullanınız.)

A) 30 B) -30 C) 25 D) 29 E) 22 F) -25

$f(x,y,z) = x - 2y + 5z$ $\quad g(x,y,z) = x^2 + y^2 + z^2 - 30$

$\nabla f = \lambda \nabla g$

$\vec{i} - 2\vec{j} + 5\vec{k} = \lambda(2x\vec{i} + 2y\vec{j} + 2z\vec{k})$

$1 = 2\lambda x$, $\quad -2 = 2\lambda y$, $\quad 5 = 2\lambda z$ $\quad x = \frac{1}{2\lambda}$, $\quad y = -\frac{1}{\lambda}$, $\quad z = \frac{5}{2\lambda}$

substitute into $g(x)$, we have

$\left(\frac{1}{2\lambda}\right)^2 + \left(-\frac{1}{\lambda}\right)^2 + \left(\frac{5}{2\lambda}\right)^2 = 30 \Rightarrow \frac{30}{4\lambda^2} = 30 \Rightarrow \lambda_1 = -1/2$

$\lambda_2 = 1/2$

for $\lambda_1 = -\frac{1}{2}$, $P_1(-1, 2, -5) \Rightarrow f(P_1) = -30$, for $\lambda_2 = 1/2$, $P_2(1,-2,5) \Rightarrow f(P_2) = 30$

8. Calculate the double integral

$$\int_0^2 \int_0^y y^2 e^{xy}\, dx dy.$$

(Yukarıda verilen iki katlı integrali çözünüz.)

A) $\frac{e^4}{2}$ B) $\frac{e^4-1}{2}$ C) $\frac{e^4}{2}-1$ D) $\frac{e^4-3}{2}$ E) $\frac{e^4}{2}-3$ F) $\frac{e^4-5}{2}$

$$\int_0^2\int_0^y y^2 e^{xy}\,dx\,dy = \int_0^2 y^2 dy \left(\frac{e^{xy}}{y}\right)_{x=0}^{x=y}$$

$$= \int_0^2 y^2\left(\frac{e^{y^2}}{y} - \frac{1}{y}\right) dy$$

$y^2 = u$

$2y\,dy = du$

$$= \int_0^2 y\left(e^{y^2}-1\right) dy = \frac{1}{2}\int_0^4 \left(e^u - 1\right) du$$

$$= \frac{1}{2}\left(e^u - u\right)_0^4 = \frac{1}{2}\left[\left(e^4-4\right)-1\right]$$

$$= \frac{1}{2}\left(e^4-5\right)$$

9.

$$R = \{\,(x,y):\ 1 \le x^2+y^2 \le 4,\ y \ge 0,\ x \ge \sqrt{3}y\,\}$$

Find the area of the region R given above. (Yukarıda verilen R bölgesininin alanını bulunuz.)

A) $\pi/6$ B) $\pi/4$ C) $\pi/2$ D) π E) $6\pi/5$ F) $5\pi/4$

$$\text{Area of } R = \int_0^{\pi/6}\int_1^2 r\,dr\,d\theta$$

$$= \int_0^{\pi/6} \left(\frac{r^2}{2}\right)_1^2 d\theta$$

$$= \frac{\pi}{6}\left(2-\frac{1}{2}\right) = \frac{3}{2}\frac{\pi}{6} = \frac{\pi}{4}$$

$1 \le x^2+y^2 \le 4$ repitsents the region bold. by the circles $r=1$ and $r=2$ so

$1 \le r \le 2$

$y \ge 0$, $x \ge \sqrt{3}y \Rightarrow 0 < \theta \le \frac{\pi}{6}$

10. Find the volume of the solid that is bounded above by the hemisphere $z = \sqrt{25 - x^2 - y^2}$, below by the xy-plane ($z = 0$), and laterally by the cylinder $x^2 + y^2 = 9$.
Hint: Use triple integration in cylindrical coordinates.

($z = \sqrt{25 - x^2 - y^2}$ yarı küresi ile üstten, xy düzlemi ($z = 0$) ile alttan ve yanal olarak $x^2 + y^2 = 9$ silindiri ile sınırlanan katı cismin hacmini bulunuz.
Yol gösterme: Silindirik koordinatlarda üç katlı integral kullanınız.)

A) $\frac{61}{3}\pi$ B) $\frac{183}{3}\pi$ C) $\frac{122}{3}\pi$ D) $\frac{196}{3}\pi$ E) $\frac{98}{3}\pi$

$x^2+y^2+z^2=25 \Rightarrow z^2=25-r^2 \Rightarrow z=\sqrt{25-r^2}$

$x^2+y^2=9 \Rightarrow r^2=9 \Rightarrow 0 \le r \le 3$

$0 \le \theta \le 2\pi$

$$V=\int_0^{2\pi}\int_0^{3}\int_0^{\sqrt{25-r^2}} r\,dz\,dr\,d\theta = \frac{122}{3}\pi$$

MAT/MATH 102, Final , Group B, 06.06.2022, 17:30 - 19:30

İsim ve Soyisim (Name and Surname) : ____________________

Öğrenci No(Student ID number): : ____________________

Bölüm (Department) : ____________________

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* No calculators, no talking and no questions (Hesap makinası, konuşmak ve soru sormak yasaktır).
*This is a multiple choice exam. In the table below, please fill in the circle corresponding to the correct answer in each question. (Bu çoktan seçmeli bir sınavdır. Verilen tabloda her soru için doğru şıkkı işaretleyiniz.)
*Only the answers on the front page will be considered when calculating exam grade. (Sınav notu hesaplanırken sadece ön sayfadaki cevaplar dikkate alınacaktır.)
* Each question is 10 points (Her soru 10 puandır).
* Your exam time is 120 minutes. Good luck. (Sınav süreniz 120 dakikadır. Başarılar.)

Questions (Group **B**)	Answers
1	A B C D E
2	A B C D E F
3	A B C D E
4	A B C D E
5	A B C D E F
6	A B C D E
7	A B C D E F
8	A B C D E F
9	A B C D E F
10	A B C D E
Total	

Questions- Group B

1. Find the integral $\iint\limits_D (x+y)\sin(x-y)dA$ if D is the region bounded by the lines $y=x$, $y=x-10$, $y=-x$, $y=-x+9$.

(D bölgesi $y=x$, $y=x-10$, $y=-x$, $y=-x+9$ doğruları ile sınırlandırılmış bölge olmak üzere $\iint\limits_D (x+y)\sin(x-y)dA$ integralini hesaplayınız.)

A) $\frac{81}{8}(1+\cos 10)$ B) $\frac{81}{4}(1+\cos 10)$ C) $\frac{81}{2}(1+\cos 10)$ D) $\frac{81}{2}(1-\cos 10)$ E) $\frac{81}{4}(1-\cos 10)$

2. Find the equation of the tangent plane to the graph of $f(x,y) = \arctan\left(\frac{y}{x}\right)$ at the point $(3\sqrt{3}, 3)$.

(Verilen fonksiyonun grafiğine $(3\sqrt{3}, 3)$ noktasında teğet olan düzlemin denklemini bulunuz.)

A) $\frac{-1}{12}(x - 3\sqrt{3}) + \frac{\sqrt{3}}{12}(y-3) - (z - \frac{\pi}{6}) = 0$

B) $\frac{\sqrt{3}}{12}(x - 3\sqrt{3}) + \frac{-1}{12}(y-3) - (z - \frac{\pi}{6}) = 0$

C) $\frac{-1}{12}(x - 3\sqrt{3}) + \frac{\sqrt{3}}{12}(y-3) + (z - \frac{\pi}{3}) = 0$

D) $\frac{\sqrt{3}}{12}(x - 3\sqrt{3}) - \frac{1}{12}(y-3) - (z - \frac{\pi}{3}) = 0$

E) $\frac{-1}{12}(x - 3\sqrt{3}) + \frac{\sqrt{3}}{12}(y-3) + (z - \frac{\pi}{6}) = 0$

F) $\frac{\sqrt{3}}{12}(x - 3\sqrt{3}) - \frac{1}{12}(y-3) + (z - \frac{\pi}{3}) = 0$

3. For which value(s) of the constant $k \neq 0$ is the function $f(x,y,z) = e^{(4x+3y)}\sin(kz)$ a harmonic function? Note that a harmonic function f satisfies $f_{xx} + f_{yy} + f_{zz} = 0$.

($f(x,y,z) = e^{(4x+3y)}\sin(kz)$ fonksiyonu $k \neq 0$ olacak şekilde k nın hangi değer yada değerleri için harmonik fonksiyondur. Not: $f_{xx} + f_{yy} + f_{zz} = 0$ eşitliğini sağlayan fonksiyona harmonik fonksiyon denir.)

A) $k = 3$ B) $k = \pm 5$ C) $k = \pm 4$ D) $k = 7$ E) $k = \pm 1$

4. If $x = u^3 - uv$ and $y = 3uv + 2v^2$ are solved for u and v in terms of x and y, evaluate $\dfrac{\partial u}{\partial y}$ and $\dfrac{\partial v}{\partial y}$ at the point $u = -1$ and $v = 2$.

(u ve v; x ve y bağlı olmak üzere $x = u^3 - uv$ ve $y = 3uv + 2v^2$; u ve v için çözülebilir olsun. $\dfrac{\partial u}{\partial y}$ ve $\dfrac{\partial v}{\partial y}$ ifadelerini $u = -1$ ve $v = 2$ noktasındaki değerini bulunuz.)

A) $\dfrac{\partial u}{\partial y} = 3, \quad \dfrac{\partial v}{\partial y} = 1$

B) $\dfrac{\partial u}{\partial y} = 3, \quad \dfrac{\partial v}{\partial y} = -7$

C) $\dfrac{\partial u}{\partial y} = 2, \quad \dfrac{\partial v}{\partial y} = -2$

D) $\dfrac{\partial u}{\partial y} = 0, \quad \dfrac{\partial v}{\partial y} = -1$

E) $\dfrac{\partial u}{\partial y} = 1, \quad \dfrac{\partial v}{\partial y} = -1$

5. Find $\nabla f(a,b)$ for the differentiable function $f(x,y)$ given the directional derivatives

$$D_{\mathbf{u}}f(a,b) = 2\sqrt{2} \quad \text{and} \quad D_{\mathbf{v}}f(a,b) = 4$$

where $\mathbf{u} = \frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j})$ and $\mathbf{v} = \frac{1}{5}(3\mathbf{i} - 4\mathbf{j})$.

($f(x,y)$ diferansiyellenebilir fonksiyonu için $\mathbf{u} = \frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j})$ ve $\mathbf{v} = \frac{1}{5}(3\mathbf{i} - 4\mathbf{j})$ olmak üzere

$$D_{\mathbf{u}}f(a,b) = 2\sqrt{2} \quad \text{and} \quad D_{\mathbf{v}}f(a,b) = 4$$

yönlü türev değerleri verildiğine göre $\nabla f(a,b)$ vektörünü bulunuz.)

A) $7\mathbf{i} - \mathbf{j}$ B) $\frac{36}{7}\mathbf{i} - \frac{8}{7}\mathbf{j}$ C) $\frac{23}{7}\mathbf{i} - \frac{9}{7}\mathbf{j}$ D) $\frac{10}{7}\mathbf{i} - \frac{10}{7}\mathbf{j}$ E) $\frac{2}{7}\mathbf{i} - \frac{16}{7}\mathbf{j}$ F) $\mathbf{i} - 7\mathbf{j}$

6. Find the local extremum value of the function $f(x, y) = xy - x^2 - y^2 - 4x - 4y + 4$.

($f(x, y) = xy - x^2 - y^2 - 4x - 4y + 4$ fonksiyonunun yerel (lokal) ekstrem değerini bulunuz.)

A) 20 local minimum (yerel minimum)
B) 10 local maximum (yerel maksimum)
C) 10 local minimum (yerel minimum)
D) 20 local maximum (yerel maksimum)
E) 30 local maximum (yerel maksimum)

7. Find the minimum value of $f(x, y, z) = x - 2y + 5z$ on the sphere $x^2 + y^2 + z^2 = 30$.
Hint: Use the method of Lagrange Multipliers.

($f(x, y, z) = x - 2y + 5z$ fonksiyonunun $x^2 + y^2 + z^2 = 30$ küresi üzerindeki minimum değerini bulunuz.
Yol gösterme: Lagrange Çarpanları Yöntemi'ni kullanınız)

A) -30 B) 30 C) 25 D) 29 E) 22 F) -25

8. Calculate the double integral

$$\int_0^1 \int_0^y y^2 e^{xy}\, dx dy.$$

(Yukarıda verilen iki katlı integrali çözünüz.)

A) $\frac{e}{2} - 3$ B) $\frac{e-5}{2}$ C) $\frac{e}{2} - 2$ D) $\frac{e-3}{2}$ E) $\frac{e}{2}$ F) $\frac{e}{2} - 1$

9.

$$R = \{\, (x, y) :\ 1 \le x^2 + y^2 \le 4,\ x \ge 0,\ x \le \sqrt{3}y \,\}$$

Find the area of the region R given above. (Yukarıda verilen R bölgesininin alanını bulunuz.)

A) $\pi/6$ B) $\pi/4$ C) $\pi/2$ D) π E) $6\pi/5$ F) $5\pi/4$

10. Find the volume of the solid that is bounded above by the hemisphere $z = \sqrt{25 - x^2 - y^2}$, below by the xy$-$plane ($z = 0$), and laterally by the cylinder $x^2 + y^2 = 16$.
Hint: Use triple integration in cylindrical coordinates.

($z = \sqrt{25 - x^2 - y^2}$ yarı küresi ile üstten, xy düzlemi ($z = 0$) ile alttan ve yanal olarak $x^2+y^2 = 16$ silindiri ile sınırlanan katı cismin hacmini bulnuz.
Yol gösterme: Silindirik koordinatlarda üç katlı integral kullanınız.)

A) $\frac{98}{3}\pi$ B) $\frac{122}{3}\pi$ C) $\frac{49}{3}\pi$ D) $\frac{196}{3}\pi$ E) $\frac{61}{3}\pi$

MAT/MATH 102, Final , Group C, 06.06.2022, 17:30 - 19:30

İsim ve Soyisim (Name and Surname) : ______________________________

Öğrenci No(Student ID number): : ______________________________

Bölüm (Department) : ______________________________

* This is a closed book and closed notes exam (Bu sınavda kitap ve not kullanılamaz).
* No calculators, no talking and no questions (Hesap makinası, konuşmak ve soru sormak yasaktır).
*This is a multiple choice exam. In the table below, please fill in the circle corresponding to the correct answer in each question. (Bu çoktan seçmeli bir sınavdır. Verilen tabloda her soru için doğru şıkkı işaretleyiniz.)
*Only the answers on the front page will be considered when calculating exam grade. (Sınav notu hesaplanırken sadece ön sayfadaki cevaplar dikkate alınacaktır.)

Questions (Group **C**)	Answers
1	(A) (B) (C) (D) (E)
2	(A) (B) (C) (D) (E) (F)
3	(A) (B) (C) (D) (E) (F)
4	(A) (B) (C) (D) (E)
5	(A) (B) (C) (D) (E) (F)
6	(A) (B) (C) (D) (E)
7	(A) (B) (C) (D) (E) (F)
8	(A) (B) (C) (D) (E) (F)
9	(A) (B) (C) (D) (E) (F)
10	(A) (B) (C) (D) (E)
Total	

Questions- Group C

1. D is bounded by $y=x$, $y=x-9$, $y=-x$, $y=-x+9$ and find the integral $\iint\limits_D (x+y)\sin(x-y)dA$.

(D bölgesi $y=x$, $y=x-9$, $y=-x$, $y=-x+9$ doğruları ile sınırlandırılmış bölge olmak üzere $\iint\limits_D (x+y)\sin(x-y)dA$ integralini hesaplayınız.)

A) $\frac{81}{4}(1+\cos 9)$ B) $\frac{81}{2}(1-\cos 9)$ C) $\frac{81}{2}(1+\cos 9)$ D) $\frac{81}{4}(1-\cos 9)$ E) $\frac{81}{8}(1+\cos 9)$

$u=x+y$, $v=x-y$

D: $x-y=0 \Rightarrow v=0$ $x+y=0 \Rightarrow u=0$

$x-y=9 \Rightarrow v=9$ $x+y=9 \Rightarrow u=9$

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -2 \qquad \left|\frac{\partial(u,v)}{\partial(x,y)}\right| = \frac{1}{|-2|} = 1/2$$

$$I = \iint (x+y)\sin(x-y)\,dx\,dy = \frac{1}{2}\int_{v=0}^{9}\int_{u=0}^{9} u.\sin v\,du\,dv$$

$$\frac{1}{2}\int_{v=0}^{9} \sin v.\ \frac{u^2}{2}\Big|_0^9 dv = \frac{81}{4}\int_0^9 \sin v\,dv$$

$$= \frac{81}{4} - \cos v\Big|_0^9 =$$

$$= \frac{81}{4}(-\cos 9 + \cos 0)$$

$$= \frac{81}{4}(1-\cos 9)$$

2. Find the equation of the tangent plane to the graph of $f(x,y) = \arctan\left(\frac{y}{x}\right)$ at the point $(2\sqrt{3}, 2)$.

(Verilen fonksiyonun grafiğine $(2\sqrt{3}, 2)$ noktasında teğet olan düzlemin denklemini bulunuz.)

A) $\frac{-1}{8}(x - 2\sqrt{3}) + \frac{\sqrt{3}}{8}(y-2) + (z - \frac{\pi}{3}) = 0$

B) $\frac{\sqrt{3}}{8}(x - 2\sqrt{3}) - \frac{1}{8}(y-2) - (z - \frac{\pi}{3}) = 0$

C) $\frac{-1}{8}(x - 2\sqrt{3}) + \frac{\sqrt{3}}{8}(y-2) + (z - \frac{\pi}{6}) = 0$

D) $\frac{\sqrt{3}}{8}(x - 2\sqrt{3}) - \frac{1}{8}(y-2) + (z - \frac{\pi}{3}) = 0$

E) $\frac{-1}{8}(x - 2\sqrt{3}) + \frac{\sqrt{3}}{8}(y-2) - (z - \frac{\pi}{6}) = 0$

F) $\frac{\sqrt{3}}{8}(x - 2\sqrt{3}) - \frac{1}{8}(y-2) - (z - \frac{\pi}{6}) = 0$

Setting $u = y/x$

$$f_x = \frac{1}{1+u^2} \cdot \frac{du}{dx} = \frac{1}{1+y^2/x^2} \cdot \frac{-y}{x^2} = \frac{-y}{x^2+y^2}$$

$$f_y = \frac{1}{1+u^2} \cdot \frac{du}{dy} = \frac{1}{1+y^2/x^2} \cdot \frac{1}{x} = \frac{x}{x^2+y^2}$$

$$f_x(2\sqrt{3}, 2) = \frac{-2}{16} = -1/8 \qquad f_y(2\sqrt{3}, 2) = \frac{2\sqrt{3}}{16} = \frac{\sqrt{3}}{8}$$

$$f(2\sqrt{3}, 2) = \arctan\left(\frac{2}{2\sqrt{3}}\right) = \arctan\left(\frac{1}{\sqrt{3}}\right) = \pi/6$$

Tangent plane

$$-\frac{1}{8}(x - 2\sqrt{3}) + \frac{\sqrt{3}}{8}(y-2) - \left(z - \frac{\pi}{6}\right) = 0$$

3. For what value(s) of the constant k is the function $f(x,y,z) = e^{-(6x+8y)}\sin(kz)$ a harmonic function? Note that a harmonic function f satisfies $f_{xx} + f_{yy} + f_{zz} = 0$

($f(x,y,z) = e^{-(6x+8y)}\sin(kz)$ fonksiyonu k nın hangi değer yada değerleri için harmonik fonksiyondur. Not: f harmonik fonksiyonu $f_{xx} + f_{yy} + f_{zz} = 0$ eşitliğini sağlar.)

A) $k = \pm 5$ B) $k = \pm 1$ C) $k = \pm 10$ D) $k = 9$ E) $k = \pm 4$

Solution:

$f_x = -6e^{-(6x+8y)}\sin kz$

$f_{xx} = 36e^{-(6x+8y)}\sin kz$

$f_y = -8e^{-(6x+8y)}\sin kz$

$f_{yy} = 64e^{-(6x+8y)}\sin kz$

$f_z = k.e^{-(6x+8y)}\cos kz$

$f_{zz} = -k^2.e^{-(6x+8y)}\sin kz$

$f_{xx} + f_{yy} + f_{zz} = e^{-(6x+8y)}\sin kz\,(36+64-k^2) = 0$

$100 - k^2 = 0$

$k = \mp 10$

4. If $u = x^2 + y^2$ and $v = x^2 - 2xy^2$ are solved for x and y in terms of u and v, evaluate $\dfrac{\partial x}{\partial u}$ and $\dfrac{\partial y}{\partial u}$ at the point $x = 1$ and $y = 2$.

(x ve y; u ve v bağlı olmak üzere $u = x^2 + y^2$ ve $v = x^2 - 2xy^2$; u ve v için çözülebilir olsun. $\dfrac{\partial x}{\partial u}$ ve $\dfrac{\partial y}{\partial u}$ ifadelerini $x = 1$ ve $y = 2$ noktasındaki değerini bulunuz.)

A) $\dfrac{\partial x}{\partial u} = -1$ and (ve) $\dfrac{\partial y}{\partial u} = \frac{3}{4}$

B) $\dfrac{\partial x}{\partial u} = 3$ and (ve) $\dfrac{\partial y}{\partial u} = \frac{1}{4}$

C) $\dfrac{\partial x}{\partial u} = 0$ and (ve) $\dfrac{\partial y}{\partial u} = 2$

D) $\dfrac{\partial x}{\partial u} = 2$ and (ve) $\dfrac{\partial y}{\partial u} = 4$

E) $\dfrac{\partial x}{\partial u} = -1$ and (ve) $\dfrac{\partial y}{\partial u} = 0$

Solution:

$u = x^2 + y^2$

$v = x^2 - 2xy^2$

$$1 = 2x\frac{\partial x}{\partial u} + 2y\frac{\partial y}{\partial u} \xrightarrow{x=1,\,y=2} 1 = 2\frac{\partial x}{\partial u} + 4\frac{\partial y}{\partial u}$$

$$0 = 2x\frac{\partial x}{\partial u} - 2y^2\frac{\partial x}{\partial u} - 4xy\frac{\partial y}{\partial u} \xrightarrow{x=1,\,y=2} 0 = -6\frac{\partial x}{\partial u} - 8\frac{\partial y}{\partial u}$$

$$\frac{\partial x}{\partial u} = -1$$

$$\frac{\partial y}{\partial u} = 3/4$$

5. Find $\nabla f(a,b)$ for the differentiable function $f(x,y)$ given the directional derivatives

$$D_{\mathbf{u}} f(a,b) = \sqrt{2} \quad \text{and} \quad D_{\mathbf{v}} f(a,b) = 3$$

where $\mathbf{u} = \frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j})$ and $\mathbf{v} = \frac{1}{5}(3\mathbf{i}-4\mathbf{j})$.

($f(x,y)$ diferansiyellenebilir fonksiyonu için $\mathbf{u} = \frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j})$ ve $\mathbf{v} = \frac{1}{5}(3\mathbf{i}-4\mathbf{j})$ olmak üzere

$$D_{\mathbf{u}} f(a,b) = \sqrt{2} \quad \text{and} \quad D_{\mathbf{v}} f(a,b) = 3$$

yönlü türev değerleri verildiğine göre $\nabla f(a,b)$ vektörünü bulunuz.)

A) $7\mathbf{i}-\mathbf{j}$ B) $\frac{36}{7}\mathbf{i}-\frac{8}{7}\mathbf{j}$ C) $\frac{23}{7}\mathbf{i}-\frac{9}{7}\mathbf{j}$ D) $\frac{10}{7}\mathbf{i}-\frac{10}{7}\mathbf{j}$ E) $\frac{2}{7}\mathbf{i}-\frac{16}{7}\mathbf{j}$ F) $\mathbf{i}-7\mathbf{j}$

Solution: u and v unit vectors

$$D_u f(a,b) = \nabla f(a,b) \cdot u = \frac{1}{\sqrt{2}}\left(f_1(a,b) + f_2(a,b)\right) = \sqrt{2}$$

$$D_v f(a,b) = \nabla f(a,b) \cdot v = \frac{1}{5}\left(3f_1(a,b) - 4f_2(a,b)\right) = 3$$

$$\left.\begin{array}{l} f_1(a,b) + f_2(a,b) = 2 \\ 3f_1(a,b) - 4f_2(a,b) = 15 \end{array}\right\} \quad \begin{array}{l} f_1(a,b) = \frac{23}{7} \\ f_2(a,b) = -9/7 \end{array}$$

$$\boxed{\nabla f(a,b) = \frac{23}{7} i - \frac{9}{7} j}$$

6. Find the local extremum value of the function $f(x, y) = xy - x^2 - y^2 - 5x - 5y + 5$.

($f(x, y) = xy - x^2 - y^2 - 5x - 5y + 5$ fonksiyonunun yerel (lokal) ekstrem değerini bulunuz.)

A) 10 local minimum (yerel minimum)
B) 10 local maximum (yerel maksimum)
C) 20 local maksimum (yerel maximum)
D) 30 local minumum (yerel minimum)
E) 30 local maximum (yerel maksimum)

Solution: $f(x,y) = xy - x^2 - y^2 - 5x - 5y + 5$

$f_x = y - 2x - 5$ $\quad f_y = x - 2y - 5$ critical point $(-5,-5)$

$$\left.\begin{array}{l} A = f_{xx} = -2 \\ B = f_{xy} = 1 \\ C = f_{zz} = -2 \end{array}\right\} B^2 - AC = 1 - 4 = -3 < 0 \text{ and } A < 0$$

local max!

$f(-5,-5) = \cancel{25} - \cancel{25} - \cancel{25} + \cancel{25} + 25 + 5 = 30$

7. Use the method of Lagrange Multipliers to find the maximum value of $f(x, y, z) = x - 2y + 5z$ on the sphere $x^2 + y^2 + z^2 = 120$.

$f(x, y, z) = x - 2y + 5z$ fonksiyonunun $x^2 + y^2 + z^2 = 120$ küresi uzerindeki maximum değerini Lagrange Çarpanları Yöntemi'ni kullanarak bulunuz.

A) 60 B) -30 C) 30 D) -60 E) 52 F) -55

Solution: $f(x,y,z) = x - 2y + 5z$

$g(x,y,z) = x^2 + y^2 + z^2 - 30$

$\nabla f = \lambda \nabla g$

$\vec{i} - 2\vec{j} + 5\vec{k} = \lambda(2x\vec{i} + 2y\vec{j} + 2z\vec{k})$

$$\left.\begin{array}{l} 1 = 2\lambda x \\ -2 = 2\lambda y \\ 5 = 2\lambda z \end{array}\right\} \begin{array}{l} x = \frac{1}{2\lambda} \\ y = -\frac{1}{\lambda} \\ z = \frac{5}{2\lambda} \end{array}$$

substitute into $g(x)$, we have

$\left(\frac{1}{2\lambda}\right)^2 + \left(-\frac{1}{\lambda}\right)^2 + \left(\frac{5}{2\lambda}\right)^2 = 120 \rightarrow \frac{30}{4\lambda^2} = 120 \quad 16\lambda^2 = 1$ $\rightarrow \lambda_1 = -1/4$, $\rightarrow \lambda_2 = 1/4$

(*) For $\lambda_1 = -1/4$ $\quad P_1 = (-2, 4, -10)$ $\qquad$ (*) For $\lambda_2 = 1/4$ $\quad P_2 = (2, -4, 10)$

(*) $f(P_1) = -60$ is the minimum of the function f on the sphere

(*) $f(P_2) = 60$ is the maximum of the function f on the sphere

8. Calculate the iterated integral

$$\int_0^3 dy \int_0^y y^2 e^{xy}\, dx.$$

Yukarıda verilen integralleri çözünüz.

A) $\frac{e^9-6}{2}$ B) $\frac{e^9-7}{2}$ C) $\frac{e^9-8}{2}$ D) $\frac{e^9}{2}-5$ E) $\frac{e^9-11}{2}$ F) $\frac{e^9-12}{2}$

Solution:

$$\int_0^3 dy \int_0^y \underbrace{y^2 e^{xy} dx} = \int_0^3 y e^{xy} \Big|_{x=0}^{x=y} dx$$

$$\int_0^3 \left[y e^{y^2} - y \right] dy \qquad \begin{array}{l} y^2 = u \\ 2y\,dy = du \end{array} \Rightarrow \frac{1}{2} e^{y^2} - \frac{y^2}{2} \Big|_0^3$$

$$\frac{1}{2} e^9 - \frac{9}{2} - \frac{1}{2} = \frac{1}{2} e^9 - 5 \,//$$

9.

$$R = \{\,(x, y) :\ 1 \le x^2 + y^2 \le 4,\ y \ge 0,\ x \le \sqrt{3}y\,\}$$

Find the area of the region R given above. (Yukarıda verilen R bölgesininin alanını bulunuz.)

A) $\pi/6$ B) $\pi/4$ C) $\pi/2$ D) π E) $6\pi/5$ F) $5\pi/4$

Solution:

$$1 \le x^2 + y^2 \le 4 \Rightarrow \left.\begin{array}{l} r=1 \\ r=2 \end{array}\right\} \text{circles} \quad 1 \le r \le 2$$

$$y \ge 0, \quad x \le \sqrt{3}y \Rightarrow \theta_1 = \pi/6 \quad \theta_2 = \pi$$

$$\int_{\pi/6}^{\pi} \int_1^2 r\, dr\, d\theta = \int_{\pi/6}^{\pi} \frac{1}{2} r^2 \Big|_1^2 d\theta = \frac{3\theta}{2} \Big|_{\pi/6}^{\pi}$$

$$= \frac{3\pi}{2} - \frac{\pi}{4} = \frac{5\pi}{4}$$

10. Find the volume of the solid that is bounded above by the hemisphere $z = \sqrt{25 - x^2 - y^2}$, below by the xy-plane ($z = 0$), and laterally by the cylinder $x^2 + y^2 = 9$.
Hint: Use triple integration in cylindrical coordinates.
($z = \sqrt{25 - x^2 - y^2}$ yarı küresi ile üstten, xy düzlemi ($z = 0$) ile alttan ve yanal olarak $x^2 + y^2 = 9$ silindiri ile sınırlanan katı cismin hacmini bulunuz.
Yol gösterme: Silindirik koordinatlarda üç katlı integral kullanınız.)

A) $\frac{196}{3}\pi$ B) $\frac{122}{3}\pi$ C) $\frac{98}{3}\pi$ D) $\frac{183}{3}\pi$ E) $\frac{61}{3}\pi$

Solution: $x^2+y^2+z^2=25$

$z^2=r^2$

$z^2=25-r^2$

$$\left.\begin{array}{l} z=\sqrt{25-r^2} \\ z=0 \end{array}\right\}$$

$x^2+y^2=9$

$r^2=9 \Rightarrow 0 \le r \le 3$

$0 \le \theta \le 2\pi$

$$\text{Volume} = \iiint_R dv = \int_{\theta=0}^{2\pi} \int_{r=0}^{3} \int_{z=0}^{z=\sqrt{25-r^2}} r\,dz\,dr\,d\theta = \frac{122\pi}{3}$$

MAT/MATH 102, Final , Group D, 06.06.2022, 17:30 - 19:30

İsim ve Soyisim (Name and Surname) : ____________________

Öğrenci No(Student ID number): : ____________________

Bölüm (Department) : ____________________

* This is a closed book and closed notes exam (Bu sınavda kitap ve not kullanılamaz).
* No calculators, no talking and no questions (Hesap makinası, konuşmak ve soru sormak yasaktır).
*This is a multiple choice exam. In the table below, please fill in the circle corresponding to the correct answer in each question. (Bu çoktan seçmeli bir sınavdır. Verilen tabloda her soru için doğru şıkkı işaretleyiniz.)
*Only the answers on the front page will be considered when calculating exam grade. (Sınav notu hesaplanırken sadece ön sayfadaki cevaplar dikkate alınacaktır.)
* Each question is 10 points (Her soru 10 puandır).
* Your exam time is 120 minutes. Good luck. (Sınav süreniz 120 dakikadır. Başarılar.)

Questions (Group **D**)	Answers
1	A B C D E
2	A B C D E F
3	A B C D E
4	A B C D E
5	A B C D E F
6	A B C D E
7	A B C D E F
8	A B C D E F
9	A B C D E F
10	A B C D E
Total	

Questions- Group D

1. Find the integral $\iint\limits_D (x+y)\sin(x-y)dA$ if D is the region bounded by the lines $y=x$, $y=x-2$, $y=-x$, $y=-x+3$.

(D bölgesi $y=x$, $y=x-2$, $y=-x$, $y=-x+3$ doğruları ile sınırlandırılmış bölge olmak üzere $\iint\limits_D (x+y)\sin(x-y)dA$ integralini hesaplayınız.)

A) $\frac{9}{2}(1-\cos 2)$ B) $\frac{9}{2}(1+\cos 2)$ C) $\frac{9}{4}(1-\cos 2)$ D) $\frac{9}{4}(1+\cos 2)$ E) $\frac{9}{8}(1+\cos 2)$

2. Find the equation of the tangent plane to the graph of $f(x,y) = \arctan\left(\frac{y}{x}\right)$ at the point $(\sqrt{3}, 1)$.

(Verilen fonksiyonun grafiğine $(\sqrt{3}, 1)$ noktasında teğet olan düzlemin denklemini bulunuz.)

A) $\frac{\sqrt{3}}{4}(x - \sqrt{3}) - \frac{1}{4}(y - 1) - (z - \frac{\pi}{6}) = 0$

B) $\frac{-1}{4}(x - \sqrt{3}) + \frac{\sqrt{3}}{4}(y - 1) - (z - \frac{\pi}{6}) = 0$

C) $\frac{-1}{4}(x - \sqrt{3}) + \frac{\sqrt{3}}{4}(y - 1) + (z - \frac{\pi}{6}) = 0$

D) $\frac{\sqrt{3}}{4}(x - \sqrt{3}) - \frac{1}{4}(y - 1) + (z - \frac{\pi}{3}) = 0$

E) $\frac{-1}{4}(x - \sqrt{3}) + \frac{\sqrt{3}}{4}(y - 1) + (z - \frac{\pi}{3}) = 0$

F) $\frac{\sqrt{3}}{4}(x - \sqrt{3}) - \frac{1}{4}(y - 1) - (z - \frac{\pi}{3}) = 0$

3. For what value(s) of the constant $k \neq 0$ is the function $f(x, y, z) = e^{(8x+6y)} \sin(kz)$ a harmonic function? Note that a harmonic function f satisfies $f_{xx} + f_{yy} + f_{zz} = 0$

($f(x, y, z) = e^{(8x+6y)} \sin(kz)$ fonksiyonu $k \neq 0$ olacak şekilde k nın hangi değer yada değerleri için harmonik fonksiyondur. Not: $f_{xx} + f_{yy} + f_{zz} = 0$ eşitliğini sağlayan fonksiyona harmonik fonksiyon denir.)

A)$k = \pm 4$ B) $k = \pm 1$ C) $k = \pm 5$ D) $k = \pm 10$ E) $k = 9$

4. If $u = x^2 + y^2$ and $v = x^2 - 2xy^2$ are solved for x and y in terms of u and v, evaluate $\dfrac{\partial x}{\partial v}$ and $\dfrac{\partial y}{\partial v}$ at the point $x = 1$ and $y = 2$.

(x ve y; u ve v bağlı olmak üzere $u = x^2 + y^2$ ve $v = x^2 - 2xy^2$; u ve v için çözülebilir olsun. $\dfrac{\partial x}{\partial v}$ ve $\dfrac{\partial y}{\partial v}$ ifadelerini $x = 1$ ve $y = 2$ noktasındaki değerini bulunuz.)

A) $\dfrac{\partial x}{\partial v} = -1, \quad \dfrac{\partial y}{\partial v} = \dfrac{3}{4}$

B) $\dfrac{\partial x}{\partial v} = \dfrac{-1}{2}, \quad \dfrac{\partial y}{\partial v} = \dfrac{1}{4}$

C) $\dfrac{\partial x}{\partial v} = \dfrac{3}{2}, \quad \dfrac{\partial y}{\partial v} = \dfrac{1}{2}$

D) $\dfrac{\partial x}{\partial v} = 0, \quad \dfrac{\partial y}{\partial v} = \dfrac{1}{4}$

E) $\dfrac{\partial x}{\partial v} = \dfrac{-1}{2}, \quad \dfrac{\partial y}{\partial v} = 0$

5. Find $\nabla f(a,b)$ for the differentiable function $f(x,y)$ given the directional derivatives

$$D_{\mathbf{u}}f(a,b) = -\sqrt{2} \quad \text{and} \quad D_{\mathbf{v}}f(a,b) = 2$$

where $\mathbf{u} = \frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j})$ and $\mathbf{v} = \frac{1}{5}(3\mathbf{i}-4\mathbf{j})$.

($f(x,y)$ diferansiyellenebilir fonksiyonu için $\mathbf{u} = \frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j})$ ve $\mathbf{v} = \frac{1}{5}(3\mathbf{i}-4\mathbf{j})$ olmak üzere

$$D_{\mathbf{u}}f(a,b) = -\sqrt{2} \quad \text{and} \quad D_{\mathbf{v}}f(a,b) = 2$$

yönlü türev değerleri verildiğine göre $\nabla f(a,b)$ vektörünü bulunuz.)

A) $7\mathbf{i}-\mathbf{j}$ B) $\dfrac{36}{7}\mathbf{i}-\dfrac{8}{7}\mathbf{j}$ C) $\dfrac{23}{7}\mathbf{i}-\dfrac{9}{7}\mathbf{j}$ D) $\dfrac{10}{7}\mathbf{i}-\dfrac{10}{7}\mathbf{j}$ E) $\dfrac{2}{7}\mathbf{i}-\dfrac{16}{7}\mathbf{j}$ F) $\mathbf{i}-7\mathbf{j}$

6. Find the local extremum value of the function $f(x, y) = xy - x^2 - y^2 - 6x - 6y + 4$.

($f(x, y) = xy - x^2 - y^2 - 6x - 6y + 4$ fonksiyonunun yerel (lokal) ekstrem değerini bulunuz.)

A) 30 local minimum (yerel minimum)
B) 30 local maximum (yerel maksimum)
C) 40 local maximum (yerel maksimum)
D) 40 local minimum (yerel minimum)
E) 50 local maximum (yerel maksimum)

7. Find the minimum value of $f(x, y, z) = x - 2y + 5z$ on the sphere $x^2 + y^2 + z^2 = 120$.
Hint: Use the method of Lagrange Multipliers.

($f(x, y, z) = x - 2y + 5z$ fonksiyonunun $x^2 + y^2 + z^2 = 120$ küresi üzerindeki minimum değerini bulunuz.
Yol gösterme: Lagrange Çarpanları Yöntemi'ni kullanınız.)

A) 30 B) -30 C) 60 D) 52 E) -60 F) -55

8. Calculate the double integral

$$\int_0^4 \int_0^y y^2 e^{xy}\, dx dy.$$

(Yukarıda verilen iki katlı integrali çözünüz.)

A) $\dfrac{e^{16}}{2} - 7$ B) $\dfrac{e^{16} - 19}{2}$ C) $\dfrac{e^{16}}{2} - 8$ D) $\dfrac{e^{16}}{2} - 9$ E) $\dfrac{e^{16}}{2} - 10$ F) $\dfrac{e^{16} - 17}{2}$

9.

$$R = \{\, (x, y) : \; 1 \leq x^2 + y^2 \leq 4, \; x \geq 0, \; x \geq \sqrt{3}y \,\}$$

Find the area of the region R given above. (Yukarıda verilen R bölgesininin alanını bulunuz.)

A) $\pi/6$ B) $\pi/4$ C) $\pi/2$ D) π E) $6\pi/5$ F) $5\pi/4$

10. Find the volume of the solid that is bounded above by the hemisphere $z = \sqrt{25 - x^2 - y^2}$, below by the xy−plane ($z = 0$), and laterally by the cylinder $x^2 + y^2 = 16$.
Hint: Use triple integration in cylindrical coordinates.

($z = \sqrt{25 - x^2 - y^2}$ yarı küresi ile üstten, xy düzlemi ($z = 0$) ile alttan ve yanal olarak $x^2 + y^2 = 16$ silindiri ile sınırlanan katı cismin hacmini bulnuz.
Yol gösterme: Silindirik koordinatlarda üç katlı integral kullanınız.)

A) $\frac{196}{3}\pi$ B) $\frac{98}{3}\pi$ C) $\frac{122}{3}\pi$ D) $\frac{61}{3}\pi$ E) $\frac{49}{3}\pi$