REVIEW

(1) $$\lim_{x\to-\infty} \frac{|x^3-1|}{2x^3+x-1}$$

$$\lim_{x\to-\infty} \frac{-x^3+1}{2x^3+x-1}$$

$$\lim_{x\to-\infty} \frac{\cancel{x^3}\left(-1+\frac{1}{x^3}\right)}{\cancel{x^3}\left(2+\frac{1}{x^2}-\frac{1}{x^3}\right)} = -\frac{1}{2}$$

(2) $$\lim_{x\to\frac{\pi}{4}} \frac{\tan x-1}{x-\frac{\pi}{4}} = \lim_{x\to\frac{\pi}{4}} \frac{\tan x-\tan\left(\frac{\pi}{4}\right)}{x-\frac{\pi}{4}}$$

$$= (\tan x)'\Big|_{x=\frac{\pi}{4}} = (\sec^2 x)\Big|_{x=\frac{\pi}{4}} = \left(\frac{1}{\cos x}\right)^2\Big|_{x=\frac{\pi}{4}}$$

$$= \frac{1}{\frac{2}{4}} = \frac{4}{2} = 2$$

Note: $$f'(x) = \lim_{z\to x} \frac{f(z)-f(x)}{z-x}$$

(3) $$f(x) = \begin{cases} \frac{\sin x}{x}, & x<0 \\ |x+a|, & x\geq 0 \end{cases}$$

Find all values of a for which $f(x)$ is continuous.

$$\lim_{x\to0^-} \frac{\sin x}{x} \left(=\frac{0}{0}\right)$$ By L'Hospital's Rule

$$= \lim_{x\to0^-} \frac{\cos x}{1} = 1 \ldots \quad (1)$$

$$\lim_{x\to0^+} |x+a| = ?$$

if $a>0$ then $\lim_{x\to 0^+} x+a = a$ ... 2

if $a<0$ then $\lim_{x\to 0^+} -x-a = -a$

By 1 and 2 and
$f(0) = |a| = \lim_{x\to 0} f(x)$ we have ;

If $a>0$ $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

$$1 = a = a$$

In this case $\boxed{a=1}$.

If $a<0$ $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

$$1 = -a = -a$$

In this cove $\boxed{a=-1}$

When $a=1$ and $a=-1$ $f(x)$ is continuous.

4 Show that the equation $1+\cos x_0 - 2^{\sin x} = 0$ has a solution in the interval $[0, \pi/2]$.

Let $f(x) = 1+\cos x - 2^{\sin x}$

for $x=0 \Rightarrow f(0) = 1+\cos(0) - 2^{\sin(0)} = 1+1-1 = 1 > 0$

for $x=\frac{\pi}{2} \Rightarrow f(\frac{\pi}{2}) = 1+\cos(\frac{\pi}{2}) - 2^{\sin(\frac{\pi}{2})} = 1+0-2^1 = -1 < 0$

$f(x)$ is a continuous function on $[0, \frac{\pi}{2}]$ and 0 is a number between $f(0)=1$ and $f(\frac{\pi}{2})=-1$ there must exists $c \in (0, \frac{\pi}{2})$ such that $f(c)=0$.

(5) Compute the derivatives of $f(x)$ in each part.

(a) $f(x) = \sin(e^{\sqrt{x}})$

$f(x) = (\tan x)^x$, $x \in (0, \frac{\pi}{2})$ (Hint: Use logarithmic diff.)

a) $f'(x) = \cos(e^{\sqrt{x}})(e^{\sqrt{x}})' = \cos(e^{\sqrt{x}})\, e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$ //

b) $y = f(x) = (\tan x)^x$, $x \in (0, \frac{\pi}{2})$

$\ln y = x \ln(\tan x)$ differentiating implicitly;

$\frac{1}{y} \cdot y' = 1 \cdot \ln(\tan x) + x \cdot \frac{1}{\tan x} \cdot \sec^2 x$

$y' = (\tan x)^x \left(\ln(\tan x) + \frac{x \sec^2 x}{\tan x} \right)$

$y' = (\tan x)^x \left(\ln(\tan x) + \frac{x}{\cos x \sin x} \right)$

(6) Evaluate $\lim_{x \to 1^+} x^{1/(1-x)}$. $(= 1^\infty)$ Indeterminate form

$y = x^{\frac{1}{1-x}}$

$\ln y = \frac{1}{1-x} \ln x$

$\lim_{x \to 1^+} \ln y = \lim_{x \to 1^+} \left(\frac{1}{1-x} \right) \ln x \left(= \frac{0}{0} \right)$ By L'Hospital's Rule;

$= \lim_{x \to 1^+} \frac{\frac{1}{x}}{-1} = -1$ //

Thus, $\lim_{x \to 1^+} x^{1/(1-x)} = e^{-1} = \frac{1}{e}$ //

(7) The volume of a right circular cyclinder is $60\ cm^3$ and is increasing at $2\ cm^3/min$ at a time when the radius is 5cm and is increasing at 1 cm/min. How fast is the height of the cyclinder changing at that time?

$$V = \pi r^2 h = 60 \qquad h = \frac{60}{\pi r^2}$$

$$\frac{dV}{dt} = \pi\left(2r\frac{dr}{dt}\cdot h + r^2\cdot\frac{dh}{dt}\right)$$

$$r = 5 \qquad \frac{dV}{dt} = 2 \qquad \frac{dh}{dt} = ?$$

$$\frac{dr}{dt} = 1$$

$$2 = \pi\left(2.5.1.\frac{60}{\pi r^2} + 5^2\cdot\frac{dh}{dt}\right)$$

$$2 = \pi\left(\frac{600}{\pi.25} + 25\frac{dh}{dt}\right)$$

$$2 = 24 + 25\pi\frac{dh}{dt}$$

$$\frac{dh}{dt} = -\frac{22}{25\pi} \ //$$

(8) $f(x) = f(2-3f(4-5x))$ and $f(-1) = 1$ and $f'(-1) = \frac{1}{\sqrt{3}}$.

Find the tangent line of $f(x)$ passing through $x = 1$.

$$f'(x) = f'(2-3f(4-5x)).\ (2-3f(4-5x))'$$

$$f'(x) = f'(2-3f(4-5x))\ (-3f'(4-5x))\ (-5)$$

$$m_T = f'(1) = f'(2-3\underbrace{f(-1)}_{1})\left((-3)(-5)\underbrace{f'(-1)}_{\frac{1}{\sqrt{3}}}\right)$$

$$= f'(-1)\ 15.\frac{1}{\sqrt{3}} = \frac{1}{(\sqrt{3})^2}.15 = 5 \ //$$

when $x=1 \Rightarrow f(1) = f(2-3f(-1)) = f(-1) = 1$

Thus the point is $(1,1)$ and the $m_T = 5$.

Equation of tangent line is;

$$(y-1) = 5(x-1)$$

$$y = 5x - 5 + 1$$

$$\boxed{y = 5x - 4}$$

(9) $y\sin x = x^3 + \cos y$ find $\frac{dy}{dx}$.

$$\frac{d}{dx}(y\sin x) = \frac{d}{dx}(x^3 + \cos y)$$ Differentiating implicitly

$$\frac{dy}{dx}\sin x + y\cos x = 3x^2 + (-\sin y)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{3x^2 - y\cos x}{\sin x + \sin y}$$