

Mat-Math 101 Review for Final

1. (Integration)

a)

$$\int_0^{\pi/2} \sin(2x) \sqrt{\sin^2 x + 1} dx$$

b)

$$\int_1^e \sqrt{x} \ln x dx$$

c)

$$\int \frac{x}{x^2 + 3x + 2} dx$$

d)

$$\int_1^e \frac{1}{x\sqrt{\ln x}} dx$$

e)

$$\int_0^{\infty} x e^{-x} dx$$

2. (Comparison) Determine if the following integral is convergent or not.

$$\int_1^{\infty} \frac{x-1}{\sqrt{x^5+2}} dx$$

3. (Definition of integral) Evaluate the limit $\lim_{n \rightarrow \infty} \frac{2}{n} \sum_{k=1}^n \frac{1}{1 + (2k/n)^2}$ by expressing it as a suitable definite integral.

4 (Fund. Thm. of Calculus) Find $F'(x)$ if $F(x) = \int_x^{e^x+1} (\ln(t-1))^{2018} dt$

5 (Area - Volume) Let R be the region bounded $y = \sqrt{1-x^2}$, the line $x+y=1$ in the first quadrant.

a) Find the area of R (Hint: Use an inverse trigonometric substitution).

- b) Find the volume of the solid obtained by rotating R about the x -axis.
- c) Find the volume of the solid obtained by rotating R about the y -axis.
- 6 (Min. - Max. Problem)** Find all points (x, y) satisfying $y^2 - x^2 = 1$ and $0 \leq x \leq 3$ that are farthest away from the point $(2, 0)$.