MAT/MATH 101, Final, Group A, 10.01.2022, 17:30 - 19:15

İsim ve Soyisim (Name and Surname) : _____

Öğrenci No(Student ID number): : _____

Bölüm (Department) : _____

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* No calculators, no talking and no questions (Hesap makinası, konuşmak ve soru sormak yasaktır).
* This is a multiple choice exam. In the table below, please fill in the circle corresponding to the correct answer in each question. (Bu çoktan seçmeli bir sınavdır. Verilen tabloda her soru için doğru şıkkı işaretleyiniz.)
* Only the answers on the front page will be considered when calculating exam grade. (Sınav notu hesaplanırken sadece ön sayfadaki cevaplar dikkate alınacaktır.)
* Each question is 10 points (Her soru 10 puandır).

Questions (Group **A**)	Answers
1	(A) (B) (C) (D) (E)
2	(A) (B) (C) (D) (E) (F)
3	(A) (B) (C) (D) (E) (F)
4	(A) (B) (C) (D) (E) (F)
5	(A) (B) (C) (D) (E) (F)
6	(A) (B) (C) (D) (E)
7	(A) (B) (C) (D) (E) (F)
8	(A) (B) (C) (D) (E) (F)
9	(A) (B) (C) (D) (E)
10	(A) (B) (C) (D)
Total	

Questions- Group A

1. What is the coordinates of the point on the line $y = 2x + 1$ which is closest to the point $(2, 1)$?
($y = 2x + 1$ doğrusu üzerinde $(2, 1)$ noktasına en yakın noktanın koordinatları nedir?)

A) $(\frac{1}{5}, \frac{7}{5})$ B) $(\frac{4}{5}, \frac{13}{5})$ C) $(\frac{2}{5}, \frac{9}{5})$ D) $(\frac{3}{5}, \frac{11}{5})$ E) $(\frac{6}{5}, \frac{17}{5})$

The square of the distance between $(2,1)$ and a point $(x,y) = (x, 2x+1)$ which lies on $y = 2x+1$ is given by;

$$f(x) = (x-2)^2 + ((2x+1)-1)^2 = x^2 - 4x + 4 + 4x^2 = 5x^2 - 4x + 4$$

Hence we want to find the minimum value of f on the interval $(-\infty, \infty)$. $f'(x) = 10x - 4$ so $f'(x) = 0$ when $x = \frac{2}{5}$. ($f(x)$ has no singular point.) And $f''(x) = 10$, $f''(\frac{2}{5}) = 10 > 0$ thus f has a local minimum value at $x = \frac{2}{5}$. Since f has only one critical point in $(-\infty, \infty)$, f has, in fact, an absolute minimum value at $x = \frac{2}{5}$.

$y = 2(\frac{2}{5}) + 1 = \frac{9}{5}$ so the point $(\frac{2}{5}, \frac{9}{5})$ is the point on the line $y = 2x+1$ which is closest to $(2,1)$.

2. Determine $f(5)$ if f is a continuous function that satisfies $\int_0^{4x+\sin(\pi x)} f(t)dt = x$ for all x.

(Her x için $\int_0^{4x+\sin(\pi x)} f(t)dt = x$ eşitliğini sağlayan sürekli f fonksiyonu için $f(5)$ değerini bulunuz.)

A) 0 B) $\frac{1}{4}$ C) $\frac{3}{4}$ D) $\frac{27}{16}$ E) $\frac{27}{8}$ F) $\frac{81}{32}$

$4x+\sin(\pi x)$

$$\frac{d}{dx}\left[\int_0 f(t)\,dt\right] = \frac{d}{dx}(x)$$

$$f(4x+\sin(\pi x))(4+\pi\cos(\pi x)) = 1$$

Observe that $4x+\sin(\pi x) = 5$ only when $x = \frac{3}{2}$ since $4x+\sin(\pi x)$ is increasing. When $x = \frac{3}{2}$;

$$f(4\cdot\frac{3}{2} + \sin(\frac{3\pi}{2}))(4 + \pi\cos(\frac{3\pi}{2})) = 1$$

$$f(5) = \frac{1}{4}\cdot 1 = \frac{1}{4}$$

3. Evaluate the integral $\int_{-2}^{-1} \frac{2}{x^2+4x+5}\,dx$.

(Verilen integrali hesaplayınız.)

A) $\pi - \ln(\pi/2)$ B) $\ln(2)+4$ C) $\ln(1/2)+4\pi$ D)2π E) $\pi/2$ F) π

$$\int_{-2}^{-1} \frac{2}{x^2+4x+4+1}\,dx = \int_{-2}^{-1} \frac{2}{(x+2)^2+1}\,dx \qquad x+2=u,\ dx=du$$

$$= \int_{0}^{1} \frac{2}{u^2+1}\,du$$

$$= 2(\arctan 1 - \arctan 0)$$

$$= 2\left(\frac{\pi}{4}-0\right) = \frac{\pi}{2}\ //$$

4. Evaluate $\int_0^1 t^7 e^{-t^4}\,dt$.

($\int_0^1 t^7 e^{-t^4}\,dt$ integralini hesaplayınız.)

A) $\frac{e-2}{4e}$ B) $\frac{e+2}{4}$ C) $\frac{-e-2}{4}$ D) $\frac{e+2}{4e}$ E) $\frac{e-2}{4}$ F) $\frac{-e-2}{4e}$

Let $w = -t^4$ then $dw = -4t^3$. Hence $I = \int_0^1 t^7 e^{-t^4}\,dt$

$$= \frac{1}{4}\int_0^{-1} we^w\,dw \Rightarrow$$

Let $u=w$ and $dv = e^w dw$ By integration by parts;

$du = dw$ and $v = e^w$

$$I = \frac{1}{4}\left(we^w\Big|_0^{-1} - \int_0^{-1} e^w dw\right) = \frac{1}{4}\left(we^w - e^w\right)_0^{-1}$$

$$= \frac{-2e^{-1}+1}{4}$$

$$= \frac{e-2}{4e}$$

I
//

5. Evaluate the integral $\int \frac{12}{x^3 - 2x^2}\, dx$. *(Verilen integrali hesaplayınız.)*

A) $12(3x^2 - 4x)\ln|x^3 - 2x^2| + C$
B) $-3\ln|x| + 2x^{-3} + 3\ln|x-2| + C$
C) $-6\ln|x| + 6\ln|x-2| + C$
D) $-3\ln|x| + 6x^{-1} + 3\ln|x-2| + C$
E) $-\ln|x| - 6x^{-2} + 3\ln|x-2| + C$
F) $-6\ln|x| + 2\ln|x-2| - x^{-1} + C$

$$\frac{12}{x^3-2x^2} = \frac{12}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$$

$$\Rightarrow \quad 12 = A(x^2-2x) + B(x-2) + Cx^2$$

$$A + C = 0$$
$$-2A + B = 0$$
$$-2B = 12 \Rightarrow B = -6,\ A = -3,\ C = 3$$

$$I = \int \left(-\frac{3}{x}\right) dx + \int -\frac{3}{x^2}\, dx + \int \frac{3}{x-2}\, dx = -3\ln|x| + 6x^{-1} + 3\ln|x-2| + C$$

6. Find the area of the plane region bounded by curves $y = x^4$ and $y = \sqrt[4]{x}$.

($y = x^4$ ve $y = \sqrt[4]{x}$ eğrileriyle sınırlanan bölgenin alanını bulunuz.)

A) $\frac{3}{5}$ B) $\frac{4}{5}$ C) $\frac{15}{17}$ D) $\frac{16}{17}$ E) $\frac{4}{3}$

$y = x^4$ and $y = \sqrt[4]{x}$ intersect at $x = 0$ and $x = 1$.

$$A = \int_0^1 \left(\sqrt[4]{x} - x^4\right) dx = \frac{1}{\frac{1}{4}+1} - \frac{1}{4+1} = \frac{3}{5}$$

7. Evaluate $\int_{-\infty}^{\infty} \frac{dx}{25e^x + e^{-x}}$. *(İntegrali hesaplayınız.)*

A) 5π B) $\frac{\pi}{5}$ C) $\frac{\pi}{10}$ D) $\frac{\pi}{2}$ E) $\frac{\pi}{15}$ F) $\frac{\pi}{25}$

$$\int_{-\infty}^{\infty} \frac{dx}{25e^x + \frac{1}{e^x}} = \int_{-\infty}^{\infty} \frac{e^x}{(5e^x)^2 + 1}\, dx = \frac{1}{5}\int_0^{\infty} \frac{du}{u^2+1}$$

$5e^x = u$

$5e^x dx = du$

$$= \frac{1}{5} \lim_{l\to\infty} \int_0^{l} \frac{du}{u^2+1} = \frac{1}{5} \lim_{l\to\infty} \left(\arctan(u)\right)_0^{l}$$

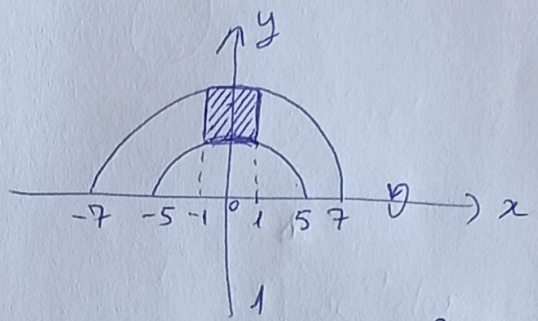
$$= \frac{1}{5} \lim_{l\to\infty} \left(\arctan(l) - \arctan(0)\right)$$

$$= \frac{1}{5}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{10}$$

8. Find the volume of the solid formed by rotating the area between the arcs of the circles $x^2+y^2 = 49$, $x^2 + y^2 = 25$ and above the x-axis in the range of $[-1, 1]$, around the x-axis.

($[-1, 1]$ aralığında $x^2 + y^2 = 49$ ve $x^2 + y^2 = 25$ çemberlerinin x ekseni üstündeki yayları arasında kalan bölgenin x ekseni etrafında döndürülmesi ile oluşan katı cismin hacmini bulunuz.)

A) 5π B) 7π C) 12π D) 22π E) 24π F) 48π



$7 > 5$, $y_1(x) = \sqrt{49 - x^2}$, $y_2(x) = \sqrt{25 - x^2}$

$$V_x = \pi \int_{-1}^{1} (y_1(x))^2 dx - \pi \int_{-1}^{1} (y_2(x))^2 dx$$

$$= \pi \int_{-1}^{1} (49 - 25)\, dx = 2 \cdot \pi \cdot 24 = 48\pi$$

— A —

9. Describe the limit $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{\pi}{n} \sin\frac{k\pi}{n}$ as the area of a certain region in the xy-plane and evaluate the limit.

($\lim_{n\to\infty} \sum_{k=1}^{n} \frac{\pi}{n} \sin\frac{k\pi}{n}$ limitini xy-düzleminde bir bölgenin alanı olarak ifade ediniz ve limiti hesaplayınız.

A) region under $y = \cos x$, above $y = 0$, between $x = \frac{\pi}{2}$ and $x = \pi$: area is 2
($y = \cos x$ eğrsinin altında $y = 0$ ın üstünde, $x = \frac{\pi}{2}$ ve $x = \pi$ nin arasında kalan alan 2)

B) region under $y = \cos x$, above $y = 0$, between $x = 0$ and $x = \pi$:area is 1
($y = \cos x$ eğrsinin altında $y = 0$ ın üstünde, $x = 0$ ve $x = \pi$ nin arasında kalan alan 1)

C)region under $y = \cos x$, above $y = 0$, between $x = 0$ and $x = \frac{\pi}{2}$: area is 1
($y = \cos x$ eğrsinin altında, $y = 0$ ın üstünde, $x = 0$ ve $x = \frac{\pi}{2}$ nin arasında kalan alan 1)

D) region under $y = \sin x$, above $y = 0$, between $x = 0$ and $x = \pi$:area is 2
($y = \sin x$ eğrsinin altında $y = 0$ ın üstünde, $x = 0$ ve $x = \pi$ nin arasında kalan alan 2)

E) region under $y = \sin x$, above $y = 0$, between $x = 0$ and $x = \frac{\pi}{2}$: area is 1
($y = \sin x$ eğrsinin altında $y = 0$ ın üstünde, $x = 0$ ve $x = \frac{\pi}{2}$ nin arasında kalan alan 1)

Solution:

$$\lim_{n\to\infty} \sum_{k=1}^{\infty} \frac{\pi}{n} \sin\frac{k\pi}{n} \qquad \begin{matrix} a=0 \\ b=\pi \end{matrix} \qquad \frac{b-a}{n} = \frac{\pi}{n}$$

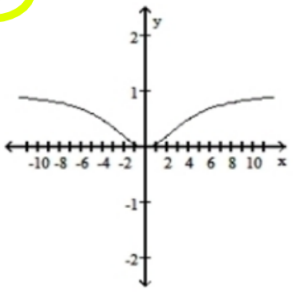
$$f\left(a + \frac{k(b-a)}{n}\right) = f\left(\frac{k\pi}{n}\right) = \sin\frac{k\pi}{n} \qquad f(x) = \sin x$$

$$\int_0^{\pi} \sin x\,dx = -\cos x\Big|_0^{\pi} = -\cos\pi + \cos 0$$
$$= 1+1 = 2$$

-A-

10) $f(x) = \dfrac{x^2}{x^2+16}, \quad -\infty < x < \infty$

B)



f is increasing on $[0, \infty)$ and decreasing on $(-\infty, 0]$. f is concave up on $\left(-\frac{4}{\sqrt{3}}, \frac{4}{\sqrt{3}}\right)$ and concave down on $\left(-\infty, -\frac{4}{\sqrt{3}}\right)$ and $\left(\frac{4}{\sqrt{3}}, \infty\right)$. f has inflection points at $x = -\frac{4}{\sqrt{3}}$ and $x = \frac{4}{\sqrt{3}}$.

(f ; $[0, \infty)$ da artandır ve $(-\infty, 0]$ da azalandır. f ; $\left(-\frac{4}{\sqrt{3}}, \frac{4}{\sqrt{3}}\right)$ da yukarı konkavdır ve $\left(-\infty, -\frac{4}{\sqrt{3}}\right)$ ve $\left(\frac{4}{\sqrt{3}}, \infty\right)$ de aşağı konkavdır. f; $x = -\frac{4}{\sqrt{3}}$ ve $x = \frac{4}{\sqrt{3}}$ de büküm noktalarına sahiptir.)

Solution: $f(x) = \dfrac{x^2}{x^2+16}$

$$f'(x) = \frac{2x(16+x^2) - 2x^3}{(16+x^2)^2} = 0 \rightarrow 32x + 2x^3 - 2x^3 = 0$$

$$32x = 0$$

$$\boxed{x = 0}$$

critical point

		0	
f'	−		+

$[0, \infty)$ increasing

$(-\infty, 0]$ decreasing

$$f''(x) = \frac{512 - 96x^2}{(16+x^2)^3} = 0 \qquad 512 - 96x^2 = 0$$

$$\boxed{x = \mp \frac{4}{\sqrt{3}}}$$

inflection points

		$-4/\sqrt{3}$		$4/\sqrt{3}$	
f''	−		+		−
	concave down		concave up		concave down

MAT/MATH 101, Final, Group B, 10.01.2022, 17:30 - 19:15

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Questions (Group **B**)	Answers
1	(A) B C D E
2	A B (C) D E F
3	A B C (D) E F
4	A B C D E (F)
5	A B C D (E) F
6	(A) B C D E
7	A B C D (E) F
8	A B C (D) E F
9	A (B) C D E
10	A (B) C D
Total	

Questions- Group B

1. What is the coordinates of the point on the line $y = 2x + 1$ which is closest to the point $(3, 2)$?
($y = 2x + 1$ doğrusu üzerinde $(3, 2)$ noktasına en yakın noktanın koordinatları nedir?)

A) $(1, 3)$ B) $(-2, -3)$ C) $(-1, -1)$ D) $(0, 1)$ E) $(2, 5)$

2. Determine $f(5)$ if f is a continuous function that satisfies $\int_0^{4x+\sin(\pi x)} f(t)dt = x^2$ for all x.

(Her x için $\int_0^{4x+\sin(\pi x)} f(t)dt = x^2$ eşitliğini sağlayan sürekli f fonksiyonu için $f(5)$ değerini bulunuz.)

A) 0 B) $\frac{1}{4}$ C) $\frac{3}{4}$ D) $\frac{27}{16}$ E) $\frac{27}{8}$ F) $\frac{81}{32}$

3. Evaluate the integral $\int_{-3}^{-2} \frac{3}{x^2+6x+10}\,dx$.

(Verilen integrali hesaplayınız.)

A) $\ln(3)+9$ B) 3π C) $\ln(3/4)+9\pi$ D)$3\pi/4$ E) $3\pi/2-\ln(3\pi/4)$ F) $3\pi/2$

4. Evaluate $\int_0^1 t^9 e^{2t^5}\,dt$.

($\int_0^1 t^9 e^{2t^5}\,dt$ integralini hesaplayınız.)

A) $\frac{e^2+1}{5}$ B) $\frac{-e^2-1}{20}$ C) $\frac{-e^2-1}{5}$ D)$\frac{e^2-1}{5}$ E) $\frac{e^2-1}{20}$ F) $\frac{e^2+1}{20}$

5. Evaluate the integral $\int \frac{4}{x^3 - x^2}\, dx$. *(Verilen integrali hesaplayınız).*

A) $-\ln|x| - 4x^{-2} + 4\ln|x-1| + C$
B) $-4\ln|x| + \frac{4}{3}x^{-3} + 4\ln|x-1| + C$
C) $-4\ln|x| + 4\ln|x-1| + C$
D) $4(3x^2 - 2x)\ln|x^3 - x^2| + C$
E) $-4\ln|x| + 4x^{-1} + 4\ln|x-1| + C$
F) $-4\ln|x| + \ln|x-1| - x^{-1} + C$

6. Find the area of the plane region bounded by curves $y = x^6$ and $y = \sqrt[6]{x}$.

($y = x^6$ ve $y = \sqrt[6]{x}$ eğrileriyle sınırlanan bölgenin alanını bulunuz.)

A) $\frac{5}{7}$ B) $\frac{6}{7}$ C) $\frac{35}{37}$ D) $\frac{36}{37}$ E) $\frac{6}{5}$

7. Evaluate $\int_{-\infty}^{\infty} \frac{dx}{16e^x + e^{-x}}$. *(İntegrali hesaplayınız.)*

A) 4π B) $\frac{\pi}{2}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{6}$ E) $\frac{\pi}{8}$ F) $\frac{\pi}{16}$

8. Find the volume of the solid formed by rotating the area between the arcs of the circles $x^2+y^2 = 36$, $x^2 + y^2 = 25$ and above the x-axis in the range of $[-1, 1]$, around the x-axis.

($[-1, 1]$ aralığında $x^2 + y^2 = 36$ ve $x^2 + y^2 = 25$ çemberlerinin x ekseni üstündeki yayları arasında kalan bölgenin x ekseni etrafında döndürülmesi ile oluşan katı cismin hacmini bulunuz.)

A) 5π B) 6π C) 11π D) 22π E) 24π F) 48π

B

9. Describe the limit $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{2}{n}\left(1+\frac{2k\pi}{n}\right)$ as the area of a certain region in the xy-plane and evaluate the limit.

($\lim_{n\to\infty} \sum_{k=1}^{n} \frac{2}{n}\left(1+\frac{2k\pi}{n}\right)$ limitini xy-düzleminde bir bölgenin alanı olarak ifade ediniz ve limiti hesaplayınız.)

A) $1+\pi$ (the area of the trapezoidal region under $y=1+2\pi x$, above $y=0$ from $x=0$ to $x=1$)
($x=0$ ve $x=1$; $y=0$ ın üstünde $y=1+2\pi x$ nin altında; yamuk bölgenin alanı: $1+\pi$)

B) $2+2\pi$ (the area of the trapezoidal region under $y=1+\pi x$, above $y=0$ from $x=0$ to $x=2$)
($x=0$ ve $x=2$; $y=0$ ın üstünde $y=1+1\pi x$ nin altında; yamuk bölgenin alanı: $2+2\pi$)

C) $2+\frac{\pi}{2}$ (the area of the trapezoidal region under $y=2+\pi x$, above $y=0$ from $x=0$ to $x=1$)
($x=0$ ve $x=1$; $y=0$ ın üstünde $y=2+\pi x$ nin altında; yamuk bölgenin alanı: $2+\frac{\pi}{2}$)

D) $2+4\pi$ (the area of the trapezoidal region under $y=1+2\pi x$, above $y=0$ from $x=0$ to $x=2$)
($x=0$ ve $x=2$; $y=0$ ın üstünde $y=1+2\pi x$ nin altında; yamuk bölgenin alanı: $2+4\pi$)

E) $4+2\pi$ (the area of the trapezoidal region under $y=2+\pi x$, above $y=0$ from $x=0$ to $x=2$)
($x=0$ ve $x=2$; $y=0$ ın üstünde $y=2+\pi x$ nin altında; yamuk bölgenin alanı: $4+2\pi$)

Solution:

$$\lim_{n\to\infty} \sum_{k=1}^{\infty} \frac{2}{n}\left(1+\frac{2k\pi}{n}\right) \qquad \begin{matrix} a=0 \\ b=2 \end{matrix} \qquad \frac{b-a}{n}=\frac{2}{n}$$

$$f\left(a+\frac{k(b-a)}{n}\right)=f\left(\frac{2k}{n}\right)=\underbrace{1+\frac{2k}{n}\cdot\pi}$$

$$\boxed{f(x)=1+\pi x}$$

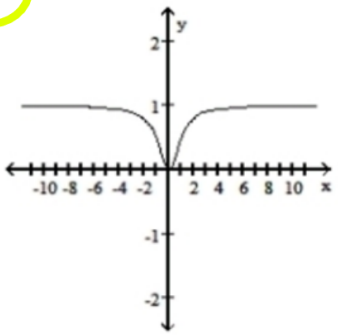
$$\int_0^2 (1+\pi x)dx=\left(x+\pi\frac{x^2}{2}\right)\Big|_0^2$$

$$\Rightarrow 2+2\pi$$

– B –

10. $f(x) = \dfrac{x^2}{x^2+1}$, $-\infty < x < \infty$

B)



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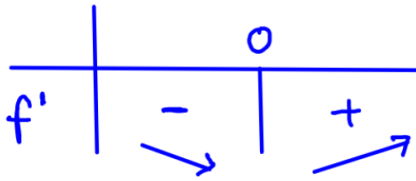
Solution: $f(x) = \dfrac{x^2}{1+x^2}$

$$f'(x) = \frac{2x(1+x^2) - 2x^3}{(1+x^2)^2} = 0 \qquad 2x = 0$$

$\boxed{x = 0}$ critical point

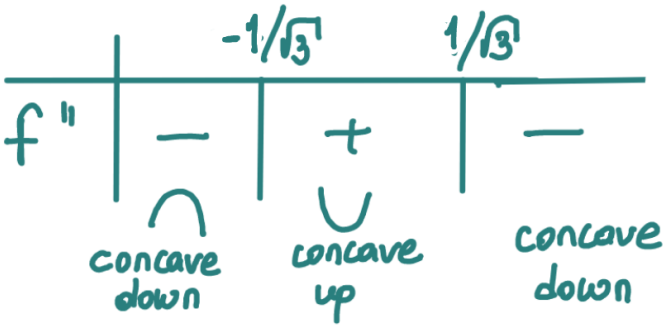


$[0, \infty)$ increasing

$(-\infty, 0]$ decreasing

$$f''(x) = \frac{2-6x^2}{(1+x^2)^3} = 0 \qquad 2-6x^2 = 0 \rightarrow$$

$\boxed{x = \mp \frac{1}{\sqrt{3}}}$ inflection points



MAT/MATH 101, Final, Group C, 10.01.2022, 17:30 - 19:15

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*Only the answers on the front page will be considered when calculating exam grade. (Sınav notu hesaplanırken sadece ön sayfadaki cevaplar dikkate alınacaktır.)
* Each question is 10 points (Her soru 10 puandır).

Questions (Group C)	Answers
1	(A) (B) (C) (D) (E)
2	(A) (B) (C) (D) (E) (F)
3	(A) (B) (C) (D) (E) (F)
4	(A) (B) (C) (D) (E) (F)
5	(A) (B) (C) (D) (E) (F)
6	(A) (B) (C) (D) (E)
7	(A) (B) (C) (D) (E) (F)
8	(A) (B) (C) (D) (E) (F)
9	(A) (B) (C) (D) (E)
10	(A) (B) (C) (D)
Total	

Questions- Group C

1. What is the coordinates of the point on the line $y = 2x + 1$ which is closest to the point $(4, 3)$?

($y = 2x + 1$ doğrusu üzerinde $(4, 3)$ noktasına en yakın noktanın koordinatları nedir?)

A) $(\frac{7}{5}, \frac{19}{5})$ B) $(\frac{2}{5}, \frac{9}{5})$ C) $(\frac{4}{5}, \frac{13}{5})$ D) $(\frac{3}{5}, \frac{11}{5})$ E) $(\frac{8}{5}, \frac{21}{5})$

2. Determine $f(5)$ if f is a continuous function that satisfies $\int_0^{4x+\sin(\pi x)} f(t)dt = x^3$ for all x.

(Her x için $\int_0^{4x+\sin(\pi x)} f(t)dt = x^3$ eşitliğini sağlayan sürekli f fonksiyonu için $f(5)$ değerini bulunuz.)

A) 0 B) $\frac{1}{4}$ C) $\frac{3}{4}$ D) $\frac{27}{16}$ E) $\frac{27}{8}$ F) $\frac{81}{32}$

3. Evaluate the integral $\int_{-4}^{-3} \frac{4}{x^2+8x+17}\,dx$. *(Verilen integrali hesaplayınız.)*

A) 16π
B) $\ln(4)+16$
C) π
D) 4π
E) $2\pi - \ln(\pi)$
F) 2π

4. Evaluate $\int_0^1 t^7 e^{-2t^4}\,dt$.

($\int_0^1 t^7 e^{-2t^4}\,dt$ integralini hesaplayınız.)

A) $\frac{-e^2-3}{4e^2}$ B) $\frac{-e^2-3}{16}$ C) $\frac{e^2-3}{16}$ D) $\frac{e^2-3}{16e^2}$ E) $\frac{-e^2-3}{16e^2}$ F) $\frac{e^2-3}{4e^2}$

5. Evaluate the integral $\int \frac{8}{x^3 - 2x^2}\, dx$. *(Verilen integrali hesaplayınız).*

A) $-4\ln|x| + 2\ln|x-2| - x^{-1} + C$
B) $-2\ln|x| + \frac{4}{3}x^{-3} + 2\ln|x-2| + C$
C) $-4\ln|x| + 4\ln|x-k| + C$
D) $8(3x^2 - 4x)\ln|x^3 - 2x^2| + C$
E) $-\ln|x| - 4x^{-2} + 2\ln|x-2| + C$
F) $-2\ln|x| + 4x^{-1} + 2\ln|x-2| + C$

6. Find the area of the plane region bounded by curves $y = x^8$ and $y = \sqrt[8]{x}$.

($y = x^8$ ve $y = \sqrt[8]{x}$ eğrileriyle sınırlanan bölgenin alanını bulunuz.)

A) $\frac{63}{65}$ B) $\frac{8}{9}$ C) $\frac{64}{65}$ D) $\frac{7}{9}$ E) $\frac{64}{63}$

7. Evaluate $\int_{-\infty}^{\infty} \frac{dx}{9e^x + e^{-x}}$. *(İntegrali hesaplayınız.)*

A) 3π B) $\frac{\pi}{2}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{6}$ E) $\frac{\pi}{9}$ F) $\frac{\pi}{18}$

8. Find the volume of the solid formed by rotating the area between the arcs of the circles $x^2+y^2 = 16$, $x^2 + y^2 = 9$ above x-axis in the range of $[-1, 1]$, around the x-axis.

($[-1, 1]$ aralığında $x^2 + y^2 = 16$ ve $x^2 + y^2 = 9$ çemberlerinin x ekseni üstündeki yayları arasında kalan bölgenin x ekseni etrafında döndürülmesi ile oluşan katı cismin hacmini bulunuz.)

A) 14π B) 4π C) 7π D)3π E) 12π F) 25π

-C-

9. Describe the limit $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{2}{n}\sqrt{4-\left(\frac{2k}{n}\right)^2}$ as the area of a certain region in the xy-plane and evaluate the limit.

($\lim_{n\to\infty} \sum_{k=1}^{n} \frac{2}{n}\sqrt{4-\left(\frac{2k}{n}\right)^2}$ limitini xy-düzleminde bir bölgenin alanı olarak ifade ediniz ve limiti hesaplayınız.)

A) π (the area of a quarter of a circular disk of radius 2)
(Yarıçapı 2 olan dairesel diskin dörtte birinin alanı : π)

B) 2π (the area of half of a circular disk of radius 2)
(Yarıçapı 2 olan dairesel diskin yarısını alanı : 2π)

C) 4π (the area of a circular disk of radius 2)
(Yarıçapı 2 olan dairesel diskin alanı : 4π)

D) 8π (the area of half of a circular disk of radius 4)
(Yarıçapı 4 olan dairesel diskin yarısını alanı : 8π)

E) 16π (the area of a circular disk of radius 4)
(Yarıçapı 4 olan dairesel diskin alanı : 16π)

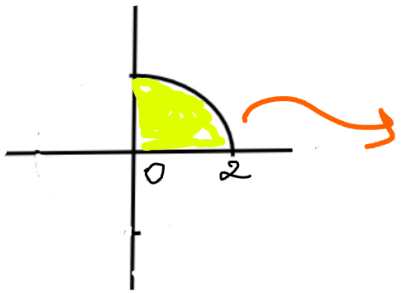
Solution:

$$\lim_{n\to\infty} \sum_{k=1}^{\infty} \frac{2}{n}\sqrt{4-\left(\frac{2k}{n}\right)^2} \qquad \begin{matrix} a=0 \\ b=2 \end{matrix} \qquad \frac{b-a}{n} = \frac{2}{n}$$

$$f\left(a+\frac{k(b-a)}{n}\right) = f\left(\frac{2k}{n}\right) = \sqrt{4-\left(\frac{2k}{n}\right)^2}$$

$$f(x) = \sqrt{4-x^2}$$

$$y = \sqrt{4-x^2} \rightarrow x^2+y^2 = 4$$

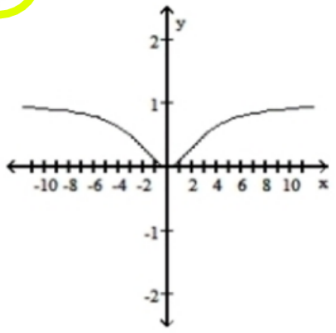


area $\pi r^2 = 4\pi/4 = \pi$

-C-

$$f(x) = \frac{x^2}{x^2+9}, \quad -\infty < x < \infty$$

C)



f is increasing on $[0, \infty)$ and decreasing on $(-\infty, 0]$. f is concave up on $\left(-\frac{3}{\sqrt{3}}, \frac{3}{\sqrt{3}}\right)$ and concave down on $\left(-\infty, -\frac{3}{\sqrt{3}}\right)$ and $\left(\frac{3}{\sqrt{3}}, \infty\right)$. f has inflection points at $x = -\frac{3}{\sqrt{3}}$ and $x = \frac{3}{\sqrt{3}}$.

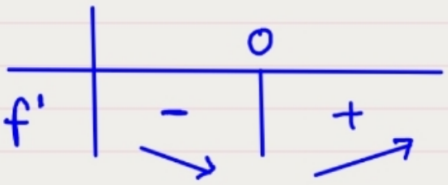
(f; $[0, \infty)$ de artandır ve $(-\infty, 0]$ da azalandır. f; $\left(-\frac{3}{\sqrt{3}}, \frac{3}{\sqrt{3}}\right)$ de yukarı konkavdır ve $\left(-\infty, -\frac{3}{\sqrt{3}}\right)$ ve $\left(\frac{3}{\sqrt{3}}, \infty\right)$ de aşağı konkavdır. f; $x = -\frac{3}{\sqrt{3}}$ ve $x = \frac{3}{\sqrt{3}}$ de büküm noktalarına sahiptir.)

Solution:

$$f(x) = \frac{x^2}{x^2+9}$$

$$f'(x) = \frac{2x(x^2+9) - 2x^3}{(9+x^2)^2} = 0 \qquad 18x = 0$$

$$x = 0$$ critical point



$[0, \infty)$ increasing

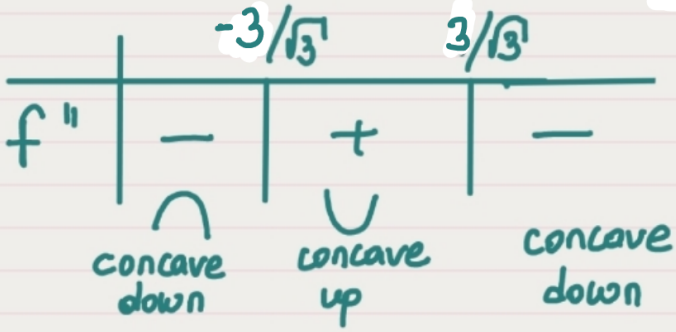
$(-\infty, 0]$ decreasing

$$f''(x) = \frac{-54(x^2-3)}{(9+x^2)^3} = 0$$

$$x^2 = 3$$

$$x = \mp\frac{3}{\sqrt{3}}$$

$x = \mp\frac{3}{\sqrt{3}}$ inflection points



MAT/MATH 101, Final, Group D, 10.01.2022, 17:30 - 19:15

İsim ve Soyisim (Name and Surname) : _____

Öğrenci No(Student ID number): : _____

Bölüm (Department) : _____

* This is a closed book and closed notes exam (Bu sınavda kitap ve not kullanılamaz).
* No calculators, no talking and no questions (Hesap makinası, konuşmak ve soru sormak yasaktır).
* This is a multiple choice exam. In the table below, please fill in the circle corresponding to the correct answer in each question. (Bu çoktan seçmeli bir sınavdır. Verilen tabloda her soru için doğru şıkkı işaretleyiniz.)
* Only the answers on the front page will be considered when calculating exam grade. (Sınav notu hesaplanırken sadece ön sayfadaki cevaplar dikkate alınacaktır.)
* Each question is 10 points (Her soru 10 puandır).

Questions (Group **D**)	Answers
1	(A) (B) (C) (D) (E)
2	(A) (B) (C) (D) (E) (F)
3	(A) (B) (C) (D) (E) (F)
4	(A) (B) (C) (D) (E) (F)
5	(A) (B) (C) (D) (E) (F)
6	(A) (B) (C) (D) (E)
7	(A) (B) (C) (D) (E) (F)
8	(A) (B) (C) (D) (E) (F)
9	(A) (B) (C) (D) (E)
10	(A) (B) (C) (D)
Total	

Questions- Group D

1. What is the coordinates of the point on the line $y = 2x + 1$ which is closest to the point $(5, 4)$?

($y = 2x + 1$ doğrusu üzerinde $(5, 4)$ noktasına en yakın noktanın koordinatları nedir?)

A) $(\frac{3}{5}, \frac{11}{5})$ B) $(\frac{2}{5}, \frac{9}{5})$ C) $(\frac{4}{5}, \frac{13}{5})$ D) $(\frac{11}{5}, \frac{27}{5})$ E) $(\frac{6}{5}, \frac{17}{5})$

2. Determine $f(5)$ if f is a continuous function that satisfies $\int_0^{4x+\sin(\pi x)} f(t)dt = x^4$ for all x.

(Her x için $\int_0^{4x+\sin(\pi x)} f(t)dt = x^4$ eşitliğini sağlayan sürekli f fonksiyonu için $f(5)$ değerini bulunuz.)

A) 0 B) $\frac{1}{4}$ C) $\frac{3}{4}$ D) $\frac{27}{16}$ E) $\frac{27}{8}$ F) $\frac{81}{32}$

3. Evaluate the integral $\int_{-5}^{-4} \frac{5}{x^2+10x+26}\,dx$. *(Verilen integrali hesaplayınız.)*

A) $5\pi/2$
B) $\ln(5)+25$
C) $\ln(5/4)+25\pi$
D) 5π
E) $5\pi/2-\ln(5\pi/4)$
F) $5\pi/4$

4. Evaluate $\int_0^1 t^9 e^{-t^5}\,dt$.

($\int_0^1 t^9 e^{-t^5}\,dt$ integralini hesaplayınız.)

A) $\frac{-e-2}{5}$ B) $\frac{e+2}{5e}$ C) $\frac{e-2}{5}$ D) $\frac{-e-2}{5e}$ E) $\frac{e-2}{5e}$ F) $\frac{e+2}{5}$

5. Evaluate the integral $\int \frac{18}{x^3 - 3x^2}\, dx$. *(Verilen integrali hesaplayınız).*

A) $-6\ln|x| + 6\ln|x-3| + C$
B) $-2\ln|x| + 2x^{-3} + 2\ln|x-3| + C$
C) $-2\ln|x| + 6x^{-1} + 2\ln|x-3| + C$
D) $18(3x^2 - 6x)\ln|x^3 - 3x^2| + C$
E) $-\ln|x| - 6x^{-2} + 2\ln|x-3| + C$
F) $-6\ln|x| + 3\ln|x-3| - x^{-1} + C$

6. Find the area of the plane region bounded by curves $y = x^{10}$ and $y = \sqrt[10]{x}$.

($y = x^{10}$ ve $y = \sqrt[10]{x}$ eğrileriyle sınırlanan bölgenin alanını bulunuz.)

A) $\frac{9}{11}$ B) $\frac{10C}{101}$ C) $\frac{100}{9}$ D) $\frac{10}{11}$ E) $\frac{99}{101}$

7. Evaluate $\int_{-\infty}^{\infty} \frac{dx}{4e^x + e^{-x}}$. *(İntegrali hesaplayınız.)*

A) 2π B) $\frac{\pi}{4}$ C) $\frac{\pi}{2}$ D) $\frac{\pi}{8}$ E) $\frac{\pi}{10}$ F) $\frac{\pi}{12}$

8. Find the volume of the solid formed by rotating the area between the arcs of the circles $x^2+y^2 = 9$, $x^2 + y^2 = 4$ on the x-axis in the range of $[-1, 1]$, around the x-axis.

($[-1, 1]$ aralığında $x^2 + y^2 = 9$ ve $x^2 + y^2 = 4$ çemberlerinin x ekseni üstündeki yayları arasında kalan bölgenin x ekseni etrafında döndürülmesi ile oluşan katı cismin hacmini bulunuz.)

A) 4π B) 9π C) 10π D) 13π E) 20π F) 24π

_D_

9. Describe the limit $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n}\sqrt{1-\left(\frac{k}{n}\right)^2}$ as the area of a certain region in the xy-plane and evaluate the limit.

($\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n}\sqrt{1-\left(\frac{k}{n}\right)^2}$ limitini xy-düzleminde bir bölgenin alanı olarak ifade ediniz ve limiti hesaplayınız.)

A) $\frac{\pi}{4}$ (the area of a quarter of a circular disk of radius 1)
(Yarıçapı 1 olan dairesel diskin dörtte birinin alanı : $\frac{\pi}{4}$)

B) $\frac{\pi}{2}$ (the area of half of a circular disk of radius 1)
(Yarıçapı 1 olan dairesel diskin yarısını alanı : $\frac{\pi}{2}$)

C) π (the area of a circular disk of radius 1)
(Yarıçapı 1 olan dairesel diskin alanı : π)

D) 2π (the area of half of a circular disk of radius 2)
(Yarıçapı 2 olan dairesel diskin yarısını alanı : 2π)

E) 4π (the area of a circular disk of radius 2)
(Yarıçapı 2 olan dairesel diskin alanı : 4π)

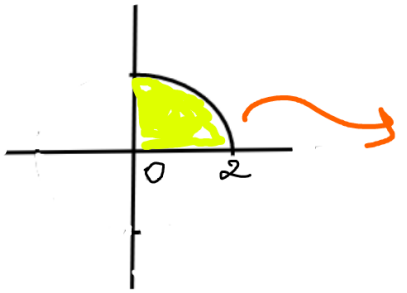
Solution:

$$\lim_{n\to\infty} \sum_{k=1}^{\infty} \frac{1}{n}\sqrt{1-\left(\frac{k}{n}\right)^2} \qquad \begin{matrix} a=0 \\ b=1 \end{matrix} \qquad \frac{b-a}{n} = \frac{1}{n}$$

$$f\left(a+\frac{k(b-a)}{n}\right) = f\left(\frac{k}{n}\right) = \sqrt{1-\left(\frac{k}{n}\right)^2}$$

$$f(x) = \sqrt{1-x^2}$$

$$y = \sqrt{1-x^2} \rightarrow x^2+y^2=1$$

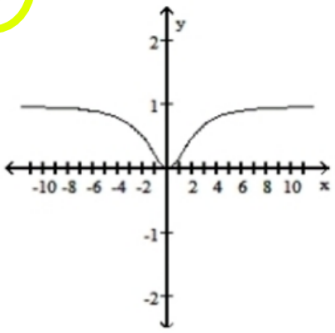


area $\pi r^2 = \pi/4$

-D-

10. $f(x) = \dfrac{x^2}{x^2+4}$, $-\infty < x < \infty$

D)



f is increasing on $[0, \infty)$ and decreasing on $(-\infty, 0]$. f is concave up on $\left(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$ and concave down on $\left(-\infty, -\frac{2}{\sqrt{3}}\right)$ and $\left(\frac{2}{\sqrt{3}}, \infty\right)$. f has inflection points at $x = -\frac{2}{\sqrt{3}}$ and $x = \frac{2}{\sqrt{3}}$.

($f; [0, \infty)$ de artandır ve $(-\infty, 0]$ da azalandır. $f; \left(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$ de yukarı konkavdır ve $\left(-\infty, -\frac{2}{\sqrt{3}}\right)$ ve $\left(\frac{2}{\sqrt{3}}, \infty\right)$ de aşağı konkavdır. $f; x = -\frac{2}{\sqrt{3}}$ ve $x = \frac{2}{\sqrt{3}}$ de büküm noktalarına sahiptir.)

Solution:

$$f(x) = \frac{x^2}{x^2+4}$$

$$f'(x) = \frac{8x}{(4+x^2)^2} = 0 \qquad 8x = 0 \qquad \boxed{x = 0}$$

critical point

		0	
f'	−		+

$[0, \infty)$ increasing

$(-\infty, 0]$ decreasing

$$f''(x) = \frac{-8(3x^2-4)}{(4+x^2)^3} = 0$$

$$3x^2 - 4 = 0$$

$$x^2 = \mp\frac{2}{\sqrt{3}}$$

$$\boxed{x = \mp\frac{2}{\sqrt{3}} \text{ inflection points}}$$

		$-2/\sqrt{3}$		$2/\sqrt{3}$	
f''	−		+		−
	∩ concave down		∪ concave up		concave down