

MATH 217 Linear Algebra and Differential Equations
MIDTERM EXAM

1. (20 points) Find the eigenvalues and the eigenvectors of the matrix

$$A = \begin{bmatrix} -4 & -2 \\ 3 & 3 \end{bmatrix}.$$

2. (20 points) Find the inverse of the given matrix using its determinant and adjoint.

$$\begin{bmatrix} -1 & 2 & 3 \\ 2 & -1 & -2 \\ -2 & 5 & 4 \end{bmatrix}.$$

3. (10 points) a) Using the Kronecker-Capelli theorem determine whether the following system is consistent:

$$\begin{cases} 2x_1 + x_2 - x_3 = 1 \\ x_1 - 3x_2 + 4x_3 = 2 \\ 11x_1 - 12x_2 + 17x_3 = 3 \end{cases}$$

- (10 points) b) Solve the following system of equations by Cramer's rule:

$$\begin{cases} 5x - y - z = 0 \\ x + 2y + 3z = 14 \\ 4x + 3y + 2z = 16 \end{cases}$$

4. (20 points) Find the null space and, where applicable, the inverse of each of the following linear transformations on \mathbb{R}^2 :

a) $A(x_1, x_2) = A(x_1 + x_2, x_1 + x_2)$

b) $A(x_1, x_2) = A(x_1 + x_2, x_1 - x_2)$

5. (20 points) Test the following equation for exactness and solve it if it is exact:

$$(\cos x - x \sin x + y^2)dx + 2xy dy = 0.$$