1. $y' - \frac{2y}{x} = 2x\sqrt{y}$

$\frac{y'}{\sqrt{y}} - \frac{2\sqrt{y}}{x} = 2x$, $\sqrt{y} = z$, $\frac{y'}{\sqrt{y}} = 2z'$

$2z' - \frac{2z}{x} = 2x$, $\mu(x) = e^{-\int \frac{1}{x} dx} = \frac{1}{x}$

$z' - \frac{z}{x} = x$

$\left(z \cdot \frac{1}{x}\right)' = x \cdot \frac{1}{x}$

$\frac{z}{x} = x + C \Rightarrow z = x^2 + Cx$

$y = z^2 = x^2(x+C)^2$

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2. $2(\cos^2 y \cdot \cos 2y - x)dy - \sin 2y\, dx = 0$

$N = 2\cos^2 y \cos 2y - 2x \quad , \quad M = -\sin 2y$

$\frac{\partial N}{\partial x} = -2 \quad , \quad \frac{\partial M}{\partial y} = -2\cos 2y$

$$-\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = \frac{-2 + 2\cos 2y}{-\sin 2y} = 2\,\frac{1 - \cos 2y}{\sin 2y} =$$

$$= 2\,\frac{2\sin^2 y}{2\sin y \cos y} = 2\tan y$$

$$\mu(y) = e^{\int 2\tan y\, dy} = e^{-2\int \frac{d\cos y}{\cos y}} = e^{-2(\ln \cos y)} = \cos^{-2} y$$

$$2\left(\cos 2y - \frac{x}{\cos^2 y}\right)dy - \frac{\sin 2y}{\cos^2 y}\, dx = 0$$

$$2\left(\cos 2y - \frac{x}{\cos^2 y}\right)dy - 2\,\frac{\sin y}{\cos y}\, dx = 0$$

$$\frac{\partial v}{\partial y} = 2\cos 2y - \frac{2x}{\cos^2 y} \Rightarrow v = \sin 2y - \frac{2x\sin y}{\cos y} + c(x)$$

$$\frac{\partial v}{\partial x} = -\frac{2\sin y}{\cos y} + c'(x) = -\frac{2\sin y}{\cos y} \Rightarrow c(x) = const$$

$$v = \sin 2y - 2x\,\frac{\sin y}{\cos y} + c = 0$$

3.

$$xy'' - y' = x^3$$

$$xy'' - y' = 0 \quad , \quad y' = v$$

$$xv' - v = 0 \Rightarrow \frac{dv}{dx} = \frac{v}{x} \Rightarrow \frac{dv}{v} = \frac{dx}{x}$$

$$\Rightarrow v = x c_1 \Rightarrow y' = c_1 x \Rightarrow y = \frac{c_1}{2} x^2 + c_2$$

$\{1, x^2\}$ - fundamental system

$$y_g = c_1(x) y_1 + c_2(x) y_2 = c_1(x) + c_2(x) x^2$$

$$\begin{cases} 1 \frac{dc_1}{dx} + x^2 \frac{dc_2}{dx} = 0 \\ 0 \cdot \frac{dc_1}{dx} + 2x \frac{dc_2}{dx} = x^2 \end{cases} \Rightarrow \frac{dc_2}{dx} = \frac{x}{2} \Rightarrow c_2(x) = \frac{x^2}{4} + \gamma_2$$

$$\frac{dc_1}{dx} = -\frac{x^3}{2} \Rightarrow c_1(x) = -\frac{x^4}{8} + \gamma_1$$

$$y_g = \gamma_1 + \gamma_2 x^2 + \frac{x^4}{8}$$

4. $y'' + y = 4\sin x$

$y'' + y = 0 \quad y = kx \Rightarrow k^2 = -1 \quad k = \pm i$

$y_1 = \cos x \; , \; y_2 = \sin x$

$S_1 = \{\cos x, \sin x\} \Rightarrow S_2 = \{x\sin x, x\cos x\}$

$y_p = x(A\cos x + B\sin x)$

$y_p'' = 2(-A\cos x + B\sin x) - x(A\cos x + B\sin x)$

$y_p'' + y_p = 2(-A\cos x + B\sin x) = 4\sin x$

$-2A = 4 \; , \; B = 0 \Rightarrow A = -2 \; , \; B = 0$

$y_p = -2x\cos x$

$y_g = C_1\cos x + C_2\sin x - 2x\cos x$

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5.

$$x^2y'' + 3xy' + y = 0$$

$$x = e^t \quad , \quad t = \ln x$$

$$x\frac{dy}{dx} = x\frac{dy}{dt}\cdot\frac{dt}{dx} = x\frac{dy}{dt}\cdot\frac{1}{x} = \frac{dy}{dt}$$

$$x^2\frac{d^2y}{dx^2} = x^2\frac{d}{dx}\left(\frac{dy}{dx}\right) = x^2\frac{d}{dx}\left(\frac{dy}{dt}\cdot\frac{1}{x}\right) =$$

$$= x^2\left(\frac{d^2y}{dt^2}\cdot\frac{1}{x^2} - \frac{dy}{dt}\cdot\frac{1}{x^2}\right) = \frac{d^2y}{dt^2} - \frac{dy}{dt}$$

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} + 3\frac{dy}{dt} + y = 0$$

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 0$$

$$y(t) = e^{kt}$$

$$k^2 + 2k + 1 = 0$$

$$(k+1)^2 = 0$$

$$y(t) = (c_1 + c_2 t)e^{-t}$$

$$y(x) = (c_1 + c_2 \ln x)x^{-1}$$