Inductors and Capacitors

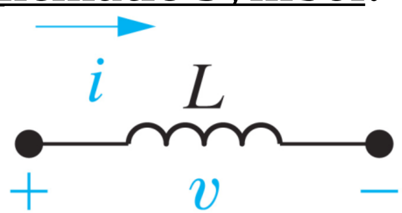
Inductors

- Based on **Magnetic Field** Phenomena
- Stores energy *
  Passive Element

- *Moving Charge ≡ Current
- If ***i*** varies with time, the magnetic field varies with time

For circuit analysis, we need the **Current – Voltage Relationship**

Schematic Symbol: Coil



$$\boxed{v = L\frac{di}{dt}}$$

L **≡ Inductance**
Units ≡ Henrys (*H*)

Compare to Ohm's Law

$$v = Ri$$

Notes on Inductors

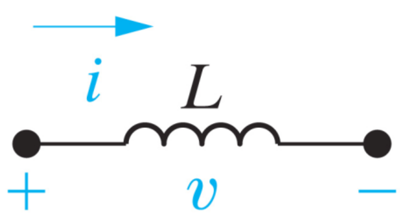
v is proportional to time rate-of-change of ***i***

1. For Constant (DC) Current, $v = 0$
$\therefore$ **Inductor is a "Short" for DC**

$$v = L\frac{di}{dt}$$

1. Current Can't Change Instantly;
Would require ∞ Voltage

$$v = L\frac{di}{dt}$$



$$L = \frac{\mu N^2 A}{S}$$

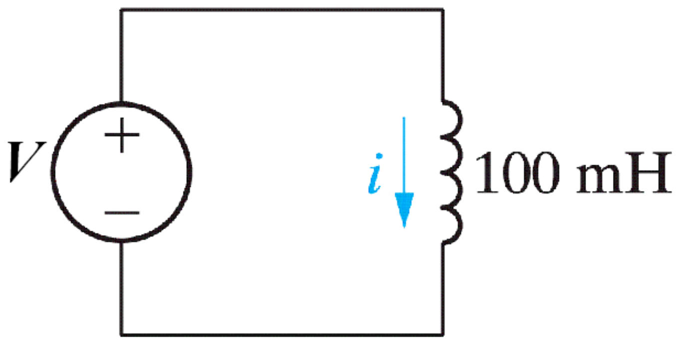
μ **= Permeability**

N **= Number of Turns**

A **= Cross-Sectional Area**

S **= Axial Length**

Current in an Inductor

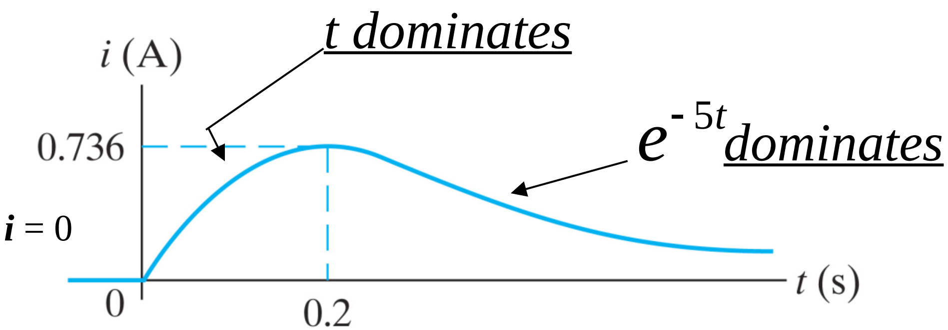


$$V = e^{-5t}(1-5t) \quad t \geq 0$$ **Given**

$$i = 10te^{-5t} \quad t \geq 0$$ **Given**

Must Confirm to $v = L\dfrac{di}{dt}$

Sketch *i*(t)



Sketch *v*(t)

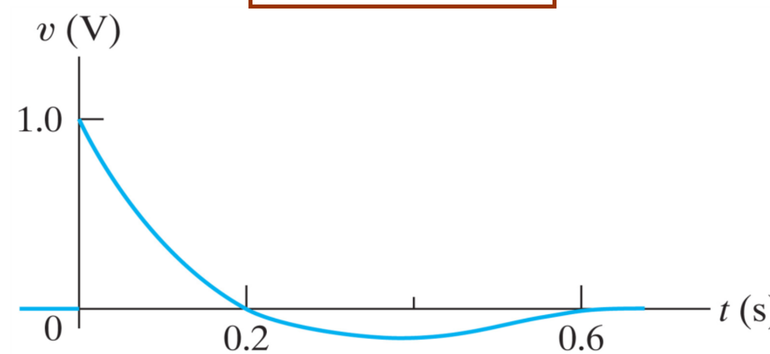


Figure: 06-04Ex6.1

Initially, ***t*** dominates, ***i*** increases.

Eventually, 1/e^{5t} dominates, ***i*** decreases

Current in an Inductor (Contd.)

Find Maximum Current in an Inductor

$$\frac{di}{dt} = 10\left[e^{-5t} + t\left(-5e^{-5t}\right)\right] = 0$$

Take derivative and set to zero

$$= 10e^{-5t}\left[1 - 5t\right] = 0$$

Simplify

$$\boxed{e^{-5t}\left[1 - 5t\right] = 0}$$

$$\therefore \frac{di}{dt} = 0 \quad \text{when} \quad \boxed{t = \frac{1}{5} = 0.2\ (s)}$$

$$i(t = 0.2) = 0.736\ (A)$$

Maximum value

Current in an Inductor (Contd.)

Derive $v(t)$ from $i(t)$

$i(t) = 0$ for $t < 0$ $\quad i(t) = 10te^{-5t}$ for $t \geq 0$

i(t) is Continuous

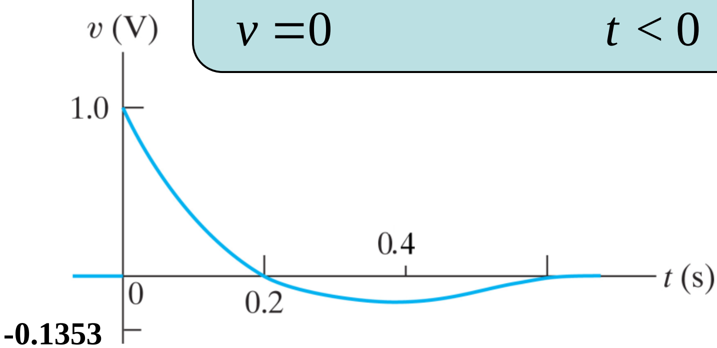
$L = 100mH$

$$\left\{ v = L\frac{di}{dt} = 0.1\left[10e^{-5t}(1-5t)\right] \right\}$$

Take derivative of *i(t)*

$$v = e^{-5t}(1-5t) \qquad t \geq 0$$
$$v = 0 \qquad t < 0$$

Voltage across inductor



$$\left\{ \begin{array}{lll} v(0) = e^{0}(1-0) = 1(V) & & \\ v = 0 & \text{when} & t = 0.2\text{s} \\ v = 0 & \text{when} & t \to \infty \end{array} \right\}$$

v(t) is Discontinuous

<u>Find Minimum</u>

$$\frac{\partial v}{\partial t} = -5e^{-5t}(1-5t) + e^{-5t}(-5)$$

Take derivative and set to zero

$$= 5e^{-5t}(5t-2) = 0$$

$$t = 0.4\text{s} \qquad v_{\min}(0.4) = -0.1353(V)$$

Analytic Expression for Current in an Inductor

$$\boxed{v = L\frac{di}{dt}} \Bigg\} \quad \textbf{Integrate equation}$$

$$Ldi = vdt$$

***L* is Constant**

$$L\int di = \int v d\tau + C \longleftarrow \text{Integration Constant}$$

$$i(t) = \frac{1}{L}\int v d\tau + C' \qquad \left[C' = \frac{C}{L}\right]$$

Lets say τ ranges from $t_0 \to t$

$$\therefore \quad i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + C'$$

$$@ \quad t = t_0 \quad \text{(initial point in time)}, \quad i(t) \equiv i(t_0), \quad \Rightarrow \quad \int_{t_0}^{t_0} v(\tau)d\tau = 0$$

$$\therefore C' = i(t_0)$$

$$\therefore \boxed{i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)}$$

Notes on Inductor Equations

- $$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)\,d\tau + i(t_0)$$

  *** $i(t_0)$ is initial current**

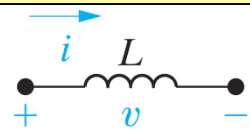
  *** Often $t_0 \equiv 0$ ⇨ $i(0)$**

  *** Often $i(0)$ or $i(t_0) = 0$**
  No initial current

- $$v = L\frac{di}{dt}$$ *** Current Can't Change Instantly**

  *** Current Can be Stored**

Power and Energy



$$p = Vi$$

$$V = L\frac{di}{dt}$$

$$i = \frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0)$$

$$\boxed{p = Vi = Li\frac{di}{dt}}$$

Power

$$w = \int p\,dt$$

Energy definition

$$w = \int Li\frac{di}{dt}dt = \int Li\,di$$

Substitute Power Expression

$$w = L\left[\frac{1}{2}i^2\right]$$

Integrate

$$\therefore \boxed{w = \frac{1}{2}Li^2}$$

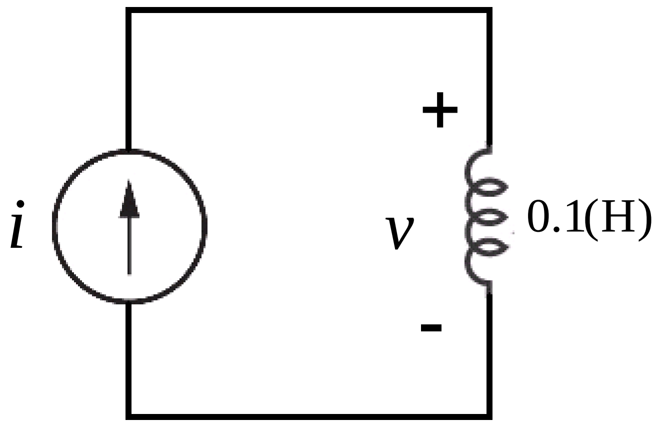
Energy in Inductor

Derivative of w w.r.t. time

Power

$$\frac{dw}{dt} = \frac{1}{2}L(2i)\frac{di}{dt} = Li\frac{di}{dt} = p$$

Power and Work Graphs



$$i = 0, \qquad t < 0$$
$$i = 10t\exp(-5t) \qquad t \geq 0$$

$$\left\{ \begin{array}{l} v = L\dfrac{di}{dt} \\ p = vi \\ w = \dfrac{1}{2}Li^2 \end{array} \right\}$$

Curves come from these equations.

Notes on Curves

① **Peak in** $i(t)$ **corresponds to** $v(t) = 0$ **, since** $i_{\max}$ **occurs when** $di/dt = 0$

② $v(t) \equiv$ **Negative when** $di/dt \equiv$ Slope $\equiv$ Negative

③ **Power = Voltage × Current (Competing Processes)**

$\{ \quad \downarrow \qquad\qquad \uparrow \quad \}$

Power and Work Graphs (Contd.)

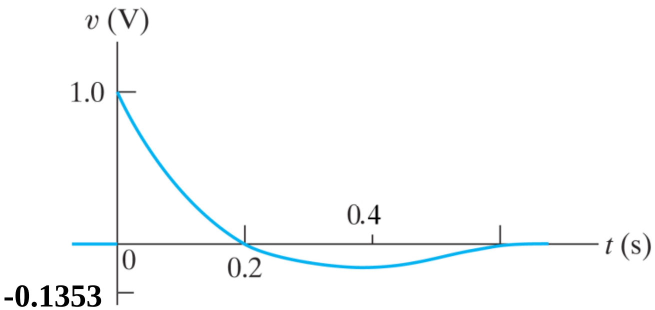
$p = 0$ **when** $v = 0$

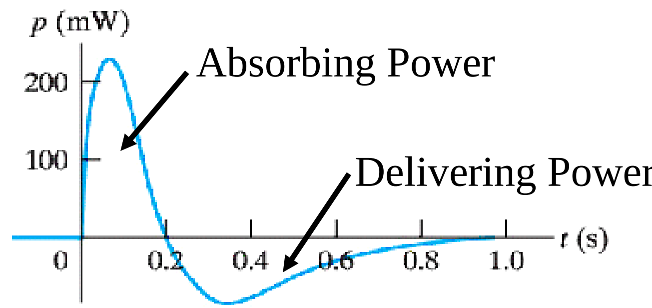
$p < 0$ **(negative) when** $v < 0$

Power peaks when decrease in v overwhelms increase in i

④ Power and Energy Comparison

Time	**Power**	**Energy**
$t = 0$	$p = 0$	$w = 0$
$0 \le t \le 0.2$	$p > 0$ **"Absorbed"**	**Increasing; "Stored"**
$0.2 \le t \le \infty$	$p < 0$ **"Delivered"**	**Decreasing; "Extracted"**





$p = vi$

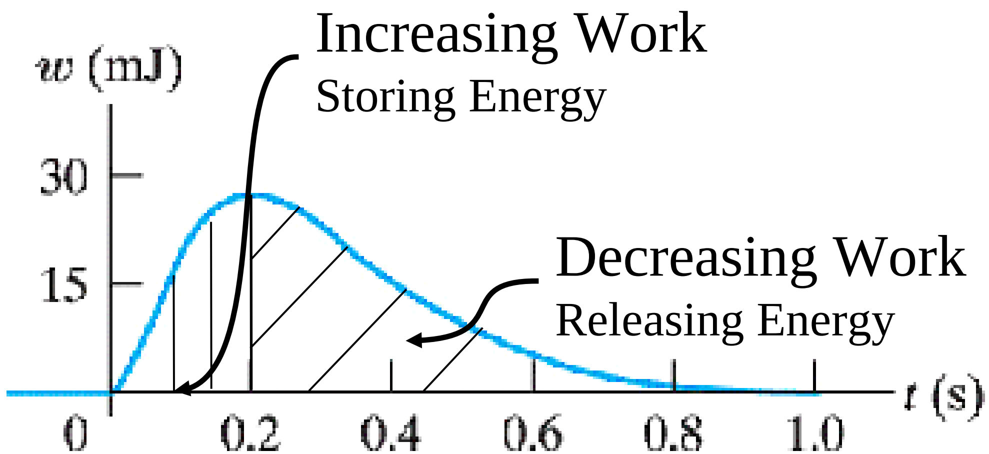
$i(t) \ge 0$ for $t > 0$

when $v > 0$ absorbing power

when $v < 0$ delivering power

Power and Work Graphs (Contd.)

$$w = \int p\,dt$$



Integrate Power from the previous slide

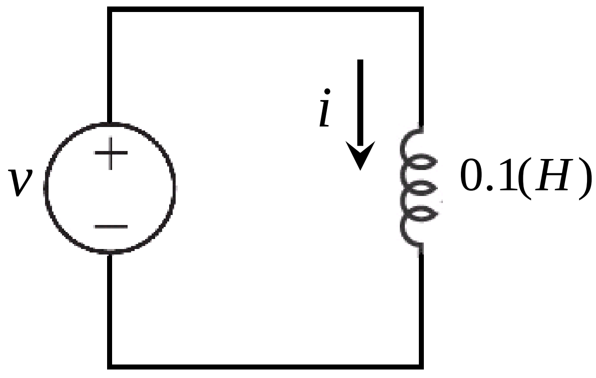
5 $$w = \frac{1}{2} L i^2$$

***w* is Maximum when *i* is Maximum**

Example: Graph the Power and Work

$$v = 0 \quad \text{for } t < 0$$

$$v = 20t\,e^{-10t} \quad \text{for } t \geq 0$$



$$\left\{ \begin{aligned} & i = \frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0) \\ & p = vi \qquad\qquad i(t_0) = 0 \\ & w = \frac{1}{2}Li^2 \end{aligned} \right\} \text{Given}$$

$$i = \frac{1}{0.1}\int_0^t 20\tau e^{-10\tau}\,d\tau + 0$$

$$\left\{ \begin{array}{c} \textbf{Integrate by parts} \\ \textbf{or} \\ \textbf{Use integral tables} \end{array} \right\}$$

$$\boxed{i = 2(1 - 10te^{-10t} - e^{-10t}) \qquad t \geq 0} \Bigg\} \text{ Analytical expression for } i(t)$$

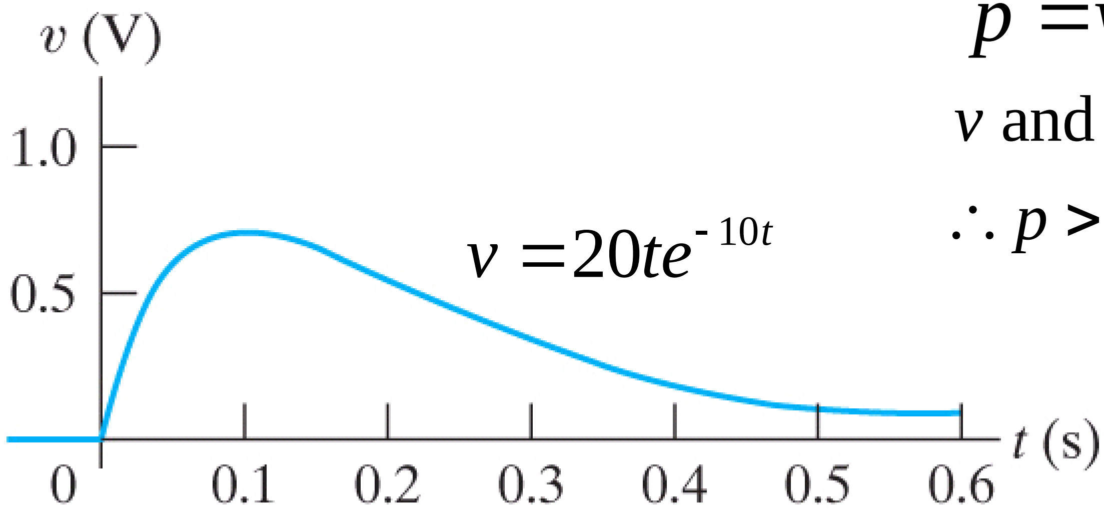
$$i(t = 0) = 0 \qquad\qquad i(t \to \infty) \to 2(A)$$

Graph of Voltage and Current

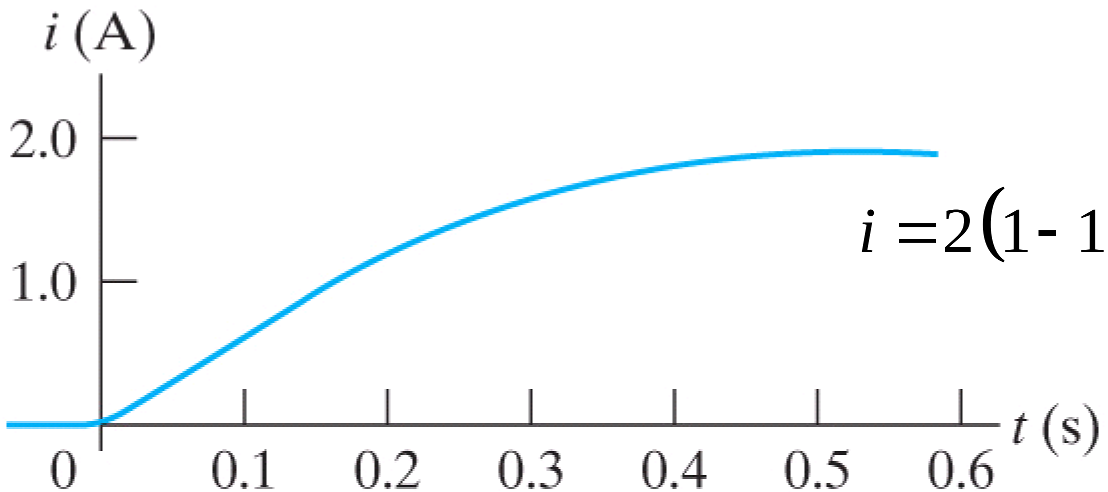
$$p = vi$$

v and i are always positive

$\therefore p > 0 \Rightarrow$ Power is absorbed

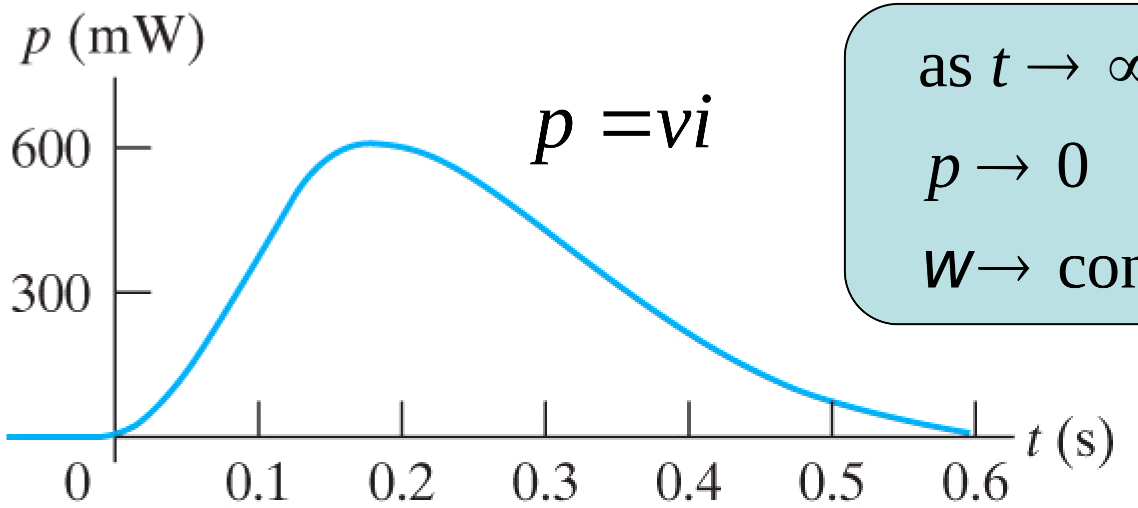


$$v = 20te^{-10t}$$



$$i = 2\left(1 - 10te^{-10t} - e^{-10t}\right)$$

Graph of Power and Work

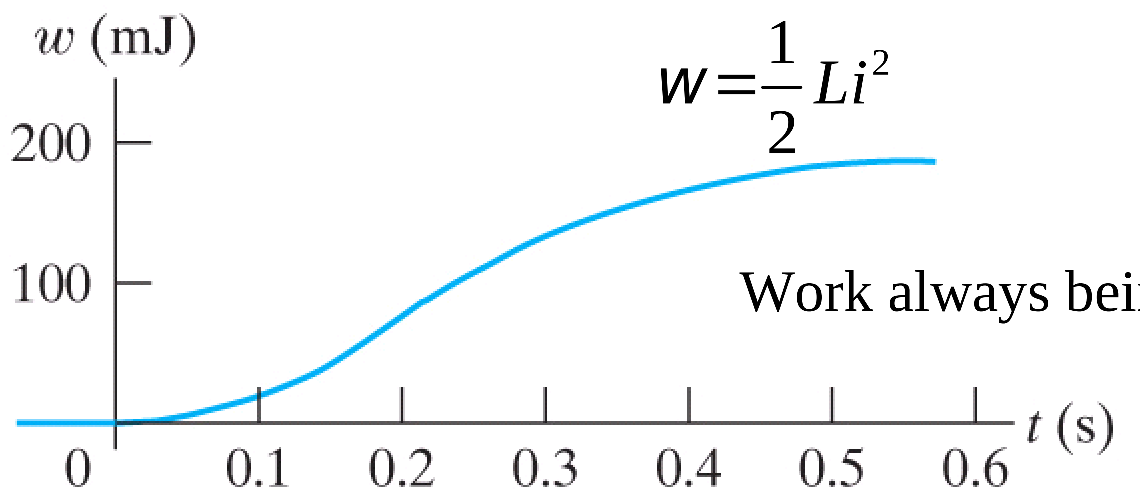


$$p = vi$$

as $t \to \infty$

$p \to 0$

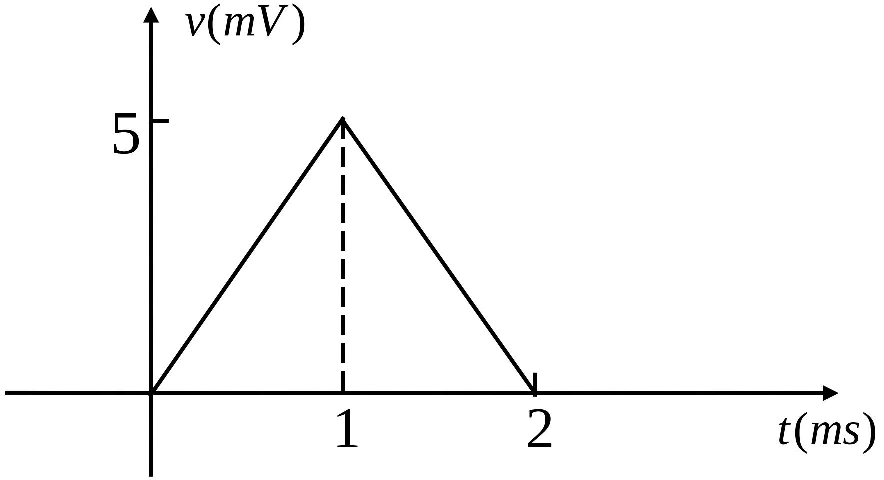
$w \to$ constant



$$w = \frac{1}{2} L i^2$$

Work always being stored

Math Review



How do you write an expression for *v(t)*?

$v(t) = 0 \qquad t < 0$

$v(t) = 0 \qquad t > 2(ms)$

Obvious

- $\left(0 \le t \le 1(ms)\right)$ $\quad y = mx + b$

slope y-intercept

$$m = \frac{\text{Rise}}{\text{Run}} = \frac{5\text{mV}}{1\text{mV}} = 5$$

$$b = 0$$

$$v(t) = 5t \qquad \left(0 \le t \le 1(ms)\right)$$

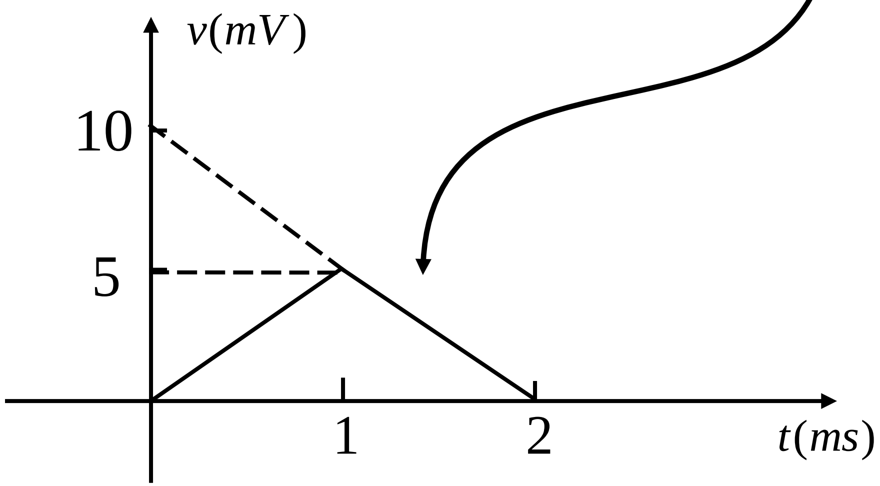
Math Review (Contd.)

- $\left(1(ms) \le t \le 2(ms)\right)$

$$m = -\frac{5(mV)}{(2-1)(ms)} = -5$$

From the figure

$$b = 10(mV) = 0.01(V)$$



$$\therefore \quad v(t) = -5t + 0.01 \quad (Volts)$$

From the above figure
for $1(ms) \le t \le 2(ms)$

Math Review (Contd.)

Sine Waves

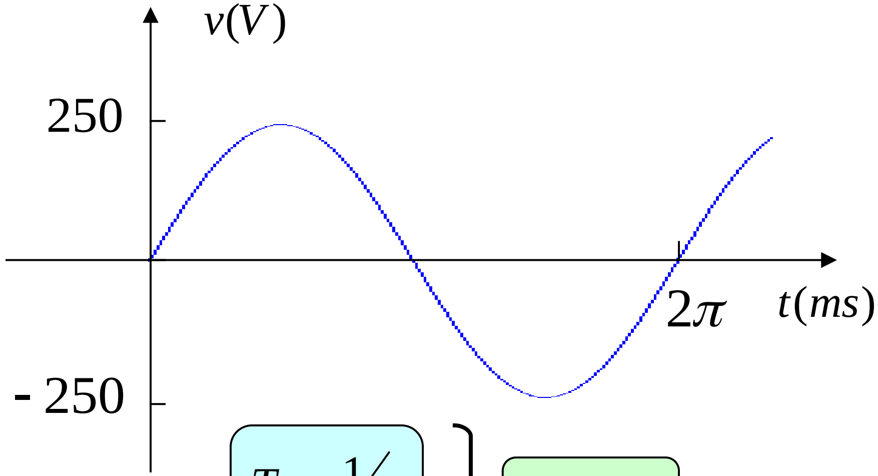
$$v = 250\sin(1000t)$$

$$v = V_m \sin(\omega t)$$

$$V_m = 250$$

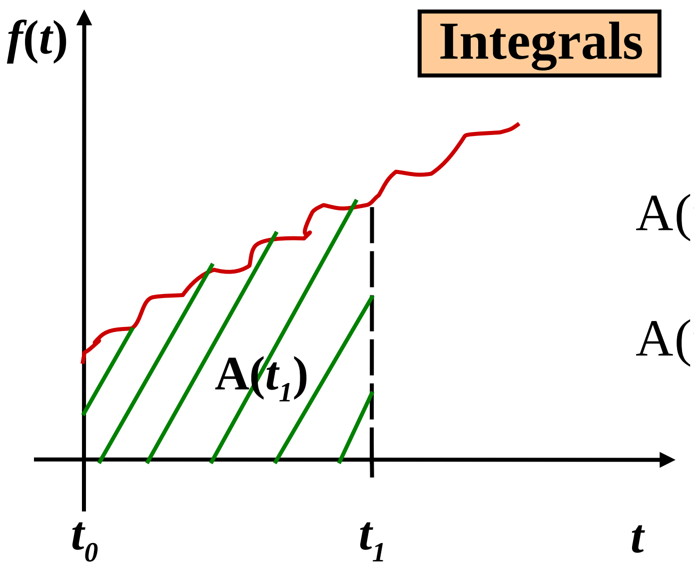
radians $\left\{ \omega = 1000 = 2\pi f \right.$

Hz $\left\{ f = \dfrac{1000}{2\pi} \right.$ $\qquad T = \dfrac{2\pi}{1000} = 2\pi(ms)$



$T = {}^{1}\!/\!{}_{f}$ } **Period**

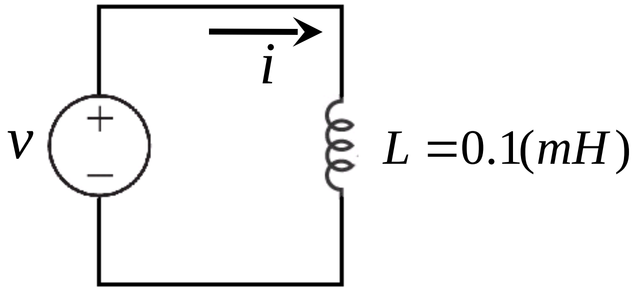
Integrals

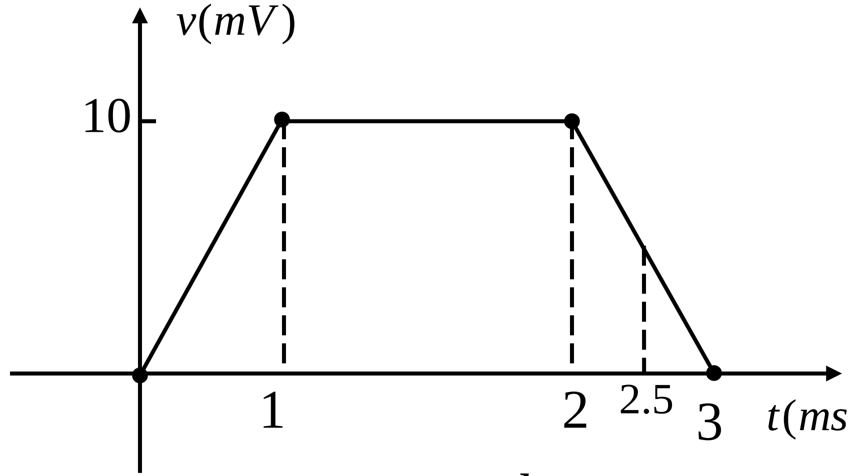


$$A(t_1) = \int_{t_0}^{t_1} f(\tau)d\tau = \text{Specific Area}$$ } **Upper limit is a constant**

$$A(t) = \int_{t_0}^{t} f(\tau)d\tau = A \equiv \text{Function of } t$$ } **Upper limit is a variable**

Inductor Example: Given *v*(*t*), Find *i*(2.5 *ms*)





Given $\quad i(0) = 0 \qquad \dfrac{1}{L} = 10{,}000$

Assume $\boxed{v(t) = 0}$ $\quad t \le 0$

$$y = mx + b$$

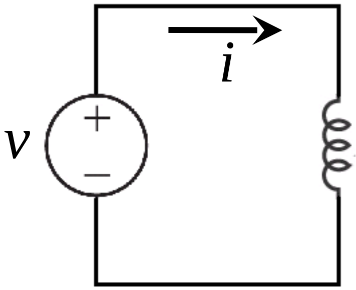
$$\left[\begin{array}{l} m = \dfrac{10(mV)}{1(ms)} = 10 \\ b = 0 \end{array}\right]$$

Mathematical Expression for *v*(*t*)

$$v(t) = \begin{cases} v(t) = 10t(V) \quad \big(0(ms) \le t \le 1(ms)\big) \\ v(t) = 10\text{mV} = 0.01(V) \quad \big(1(ms) \le t \le 2(ms)\big) \\ v(t) = -10t + 0.03(V) \quad \big(2(ms) \le t \le 3(ms)\big) \\ v(t) = 0 \quad \big(t \ge 3(ms)\big) \end{cases}$$

$$\left[\begin{array}{l} m = -\dfrac{10(mV)}{(3-2)(ms)} = -10 \\ b = 10 + 10 + 10 = 30(mV) \end{array}\right]$$

Inductor Example: Find $i(1ms)$ (Contd.)



$L = 0.1(mH)$

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$$

Formula for current in an inductor

① $\big(0(ms) \le t \le 1(ms)\big)$: $\quad i(t_0) = 0 \quad t_0 = 0$

$$i(1ms) = \frac{1}{L}\int_0^{1(ms)} v(\tau)d\tau = \frac{1}{L}\text{Area}\Big|_0^{1(ms)} = 10,000\left[\frac{1}{2}(\text{base})(\text{height})\right]$$

Formula for triangle area

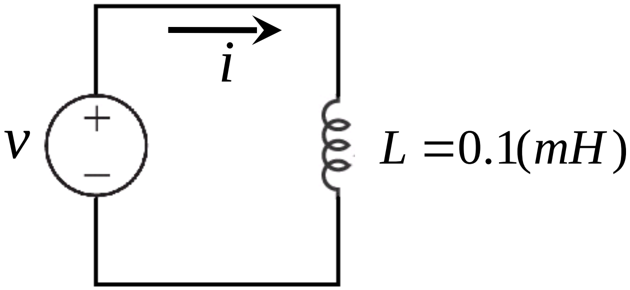
$$i(1ms) = 10,000\left(\frac{1}{2}[0.001][0.01] = 0.05\right)$$

Simplify

$$\therefore\ i(1ms) = 50(mA)$$

We "Charged" *L* for 1(*ms*), and now it has a "stored" current of 50(*mA*)

Inductor Example: Find $i(2ms)$ (Contd.)



$$② \; \left(1(ms) \le t \le 2(ms)\right) \quad t_0 = 1(ms) \quad i(t_0) = i(1ms) = 50(mA)$$ **From the previous slide**

$$i(2ms) = \frac{1}{L}\int_{1(ms)}^{2(ms)} v(\tau)d\tau + i(1)$$ **Use formula again**

$$i(2ms) = \frac{1}{L}\text{Area}\Big|_{1(ms)}^{2(ms)} + 50mA = 10{,}000\left[\text{base} \times \text{height}\right] + 50(mA)$$ **Formula for rectangle area**

$$i(2ms) = 10{,}000[(0.001)(0.01)] + 0.05$$

$$i(2ms) = (0.1 + 0.05)(A)$$ **Plug in the numbers**

$$i(2ms) = 0.15(A) = 150(mA)$$

After 2(ms), we have 150(mA) stored

Inductor Example: Find $i(2.5ms)$ and $i(3ms)$ (Contd.)

③ $\left(2(ms) \le t \le 3(ms)\right)$ $t_0 = 2\left(ms\right)$ $i(t_0) = i(2ms) = 150\left(mA\right)$

$$\left.i(t) = \frac{1}{L}\underbrace{\int_{t_0}^{t} v(\tau)d\tau}_{} + i(t_0)\right\}$$ **General formula**

$$\left.\int_{2\text{ms}}^{t}(-10\tau + 0.03)d\tau = \left[-\frac{10\tau^2}{2} + 0.03\tau\right]\Bigg|_{2\text{ms}}^{t}\right\}$$ **Plug in analytical expression for *v(t)***

$$\left.= \left[(-5t^2 + 0.03t) - (-5[0.002]^2 + 0.03[0.002])\right]\right\}$$ **Plug in limits**

$$\left.= -5t^2 + 0.03t - [4\times 10^{-5}]\right\}$$ **Simplify**

$$\left.i(t) = 10{,}000[-5t^2 + 0.03t - 4\times 10^{-5}] + \overbrace{0.15}^{i(t_0)}\right\}$$ **Add initial conditions**

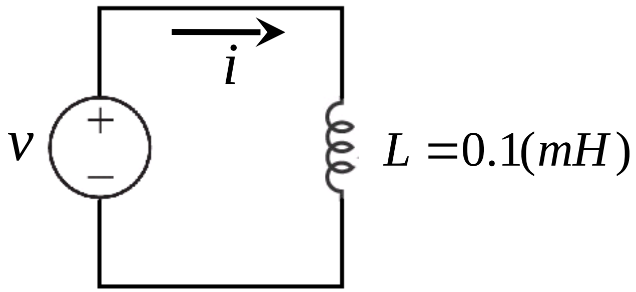
$$\left.i(t) = -50{,}000t^2 + 300t - 0.4 + 0.15\right\}$$ **Simplify**

$$\left.i(t) = -50{,}000t^2 + 300t - 0.25\right\}$$ General expression for $i(t)$ for $2(ms) \le t \le 3(ms)$

$$\left.i(2.5ms) = 0.1875(A) = 187.5(mA)\right\}$$ Let $t = 2.5(ms)$

$$\left.i(3ms) = 0.2(A) = 200(mA)\right\}$$ Let $t = 3(ms)$

Inductor Example (Contd.)



4 What about t > 3(ms) ?

$$\left(3(ms) \le t \le \infty\right) \quad i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$$

General formula

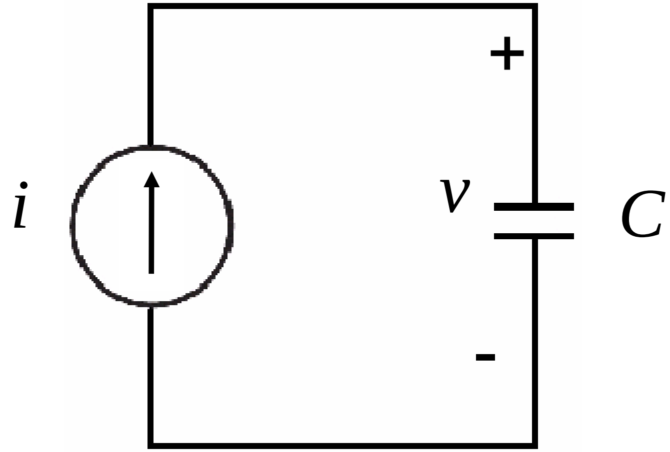
$$v(t) = 0 \qquad \therefore \int v(\tau)d\tau = 0$$

$$t_0 = 3\left(ms\right)$$

$$\therefore \boxed{i(t) = i(3ms) = 200(mA)} \quad \text{for } 3(ms) \le t \le \infty$$

- **Current in the Inductor Remains Constant at 200(mA) from t = 3(ms) $\longrightarrow$ Forever**
- **Stored Current gives Stored Energy**

Dual Problem for Capacitors



$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau + v(t_0)$$

General Equation

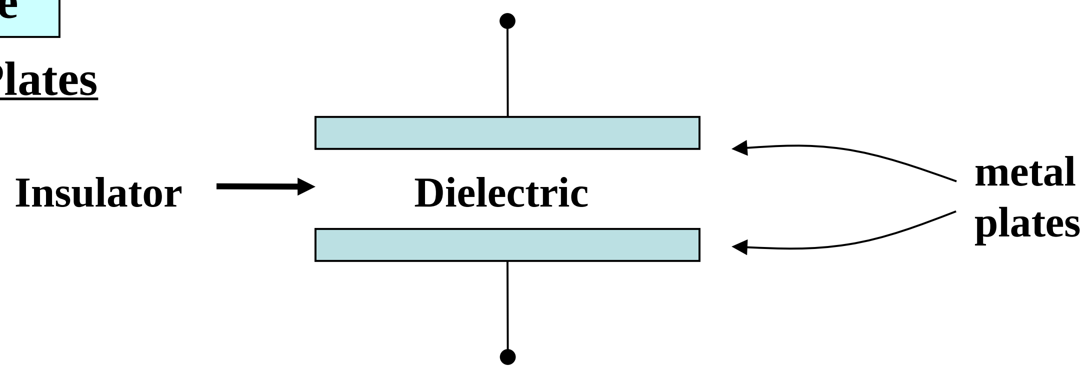
- Use Same Procedure as for the Inductor Example
- Capacitor Charges Up To a Constant Voltage
- Stored Charge ≡ Memory

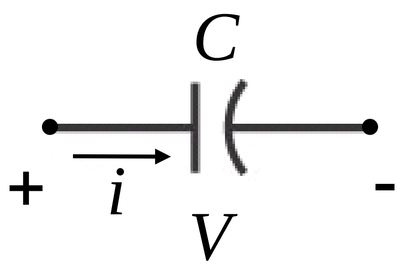
Capacitor

Electric Field Phenomena

Stores Charge

<u>Parallel Plates</u>





C **≡ Capacitance**

Units = Farad (*F*)

Normally $\mu F, pF$

$$\left\{ C = \frac{\varepsilon_0 \varepsilon_r A}{D} \right\}$$

$A = Area$

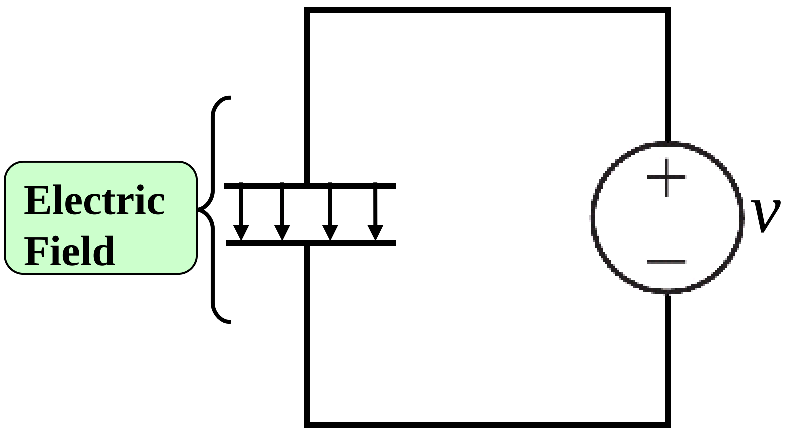
$D = Thickness$

$\varepsilon_0 = Permittivity\ of\ free\ space$

$\varepsilon_r = Relative\ Permittivity$ or $Dielectric\ Constant$

Insulator "Ideally" Prevents Charge from Flowing Through the Device

Where Does Current Come From in a Capacitor?



- Charge Stored On Each Plate Equal And Opposite
- *V* Changes, *Q* Changes
- ΔQ → Displacement Current

- Displacement Current is Indistinguishable From Conduction Current.
- If Voltage is Constant, Charge is Constant, $i = 0$.

$$i = C\frac{dv}{dt}$$

Compare to Ohm's Law

$$i = \left(\frac{1}{R}\right)v$$

Capacitors Notes: *i* is related to a time rate-of-change in *v*

1 **For Constant (DC) Voltage** ⇨ $i = 0$ $\Big\}$ $\boxed{i = C\dfrac{dv}{dt}}$

< Only Time Varying Voltage → Displacement Current. >

∴ **<u>Capacitor is an "Open" for DC</u>**

2 **<u>Voltage Can't Change Instantly</u>** $\Big\}$ $\boxed{i = C\dfrac{dv}{dt}}$

* **Would Require ∞ Current** *

<u>Voltage Across Capacitor</u>: **< Similar to Procedure for Inductor >**

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau + v(t_0) \Bigg\} \boxed{Integrate\ \ i = C\frac{dv}{dt}}$$

Power and Energy for a Capacitor

Power

$$p = vi = Cv\frac{dv}{dt}$$

$$w = \frac{1}{2}Cv^2$$

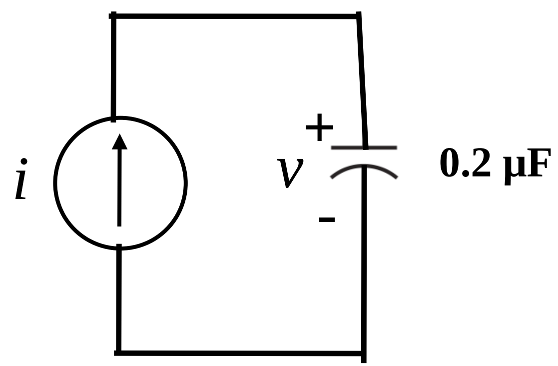
Energy

Duality Between *L* and *C*

Inductor	**Capacitor**
Magnetic field	**Electric field**
$v = L\dfrac{di}{dt}$	$i = C\dfrac{dv}{dt}$
$i = \dfrac{1}{L}\displaystyle\int_{t_0}^{t} v\,d\tau + i(t_0)$	$v = \dfrac{1}{C}\displaystyle\int_{t_0}^{t} i\,d\tau + v(t_0)$
$i_{DC} \Rightarrow v = 0$	$V_{DC} \Rightarrow i = 0$
Short for DC	**Open for DC**
$p = Li\dfrac{di}{dt}$	$p = Cv\dfrac{dv}{dt}$
$w = \dfrac{1}{2}Li^2$	$w = \dfrac{1}{2}Cv^2$

Example

Given *i*(*t*), Find *v*(*t*)



$$i(t) = 0 \qquad t \le 0$$

$$i(t) = 5000t\,(A) \qquad 0 \le t \le 20\mu s$$

$$i(t) = 0.2 - 5000t\,(A) \qquad 20\mu s \le t \le 40\mu s$$

$$i(t) = 0 \qquad t \ge 40\mu s$$

Given

a) $0 \le t \le 20\mu s$

initially uncharged

$$v = \frac{1}{0.2\times10^{-6}}\int_0^t 5000\tau\, d\tau + 0$$

General Equation

$$v(t) = 12.5\times10^{9}t^{2}$$

Analytical Expression

$$v(t = 20\mu s) = 5(V)$$

***t* = 20 (*μs*) in the above equation**

Example

Given $i(t)$, Find $v(t)$ (Contd.)

b) $20\mu s \le t \le 40\mu s$ $\quad t_0 = 20\mu s$

$$v = \frac{1}{0.2\times 10^{-6}} \int_{20\mu s}^{t} (0.2 - 5000\tau)\, d\tau + v(t_0)$$

General Equation

$$v(t_0) = v(20\mu s) = 5\,(V)$$

From part (a)

$$v(t) = -1.25\times 10^{10} t^2 + 10^6 t - 10$$

Analytical Expression

$$\boxed{v(40\mu s) = 10\,(V)}$$

***t* = 40 (*μs*) in the above equation**

Example

Given *i*(*t*), Find *v*(*t*) (Contd.)

c) $t \geq 40\mu s$ $i(t) = 0$

Obtained from part (b)

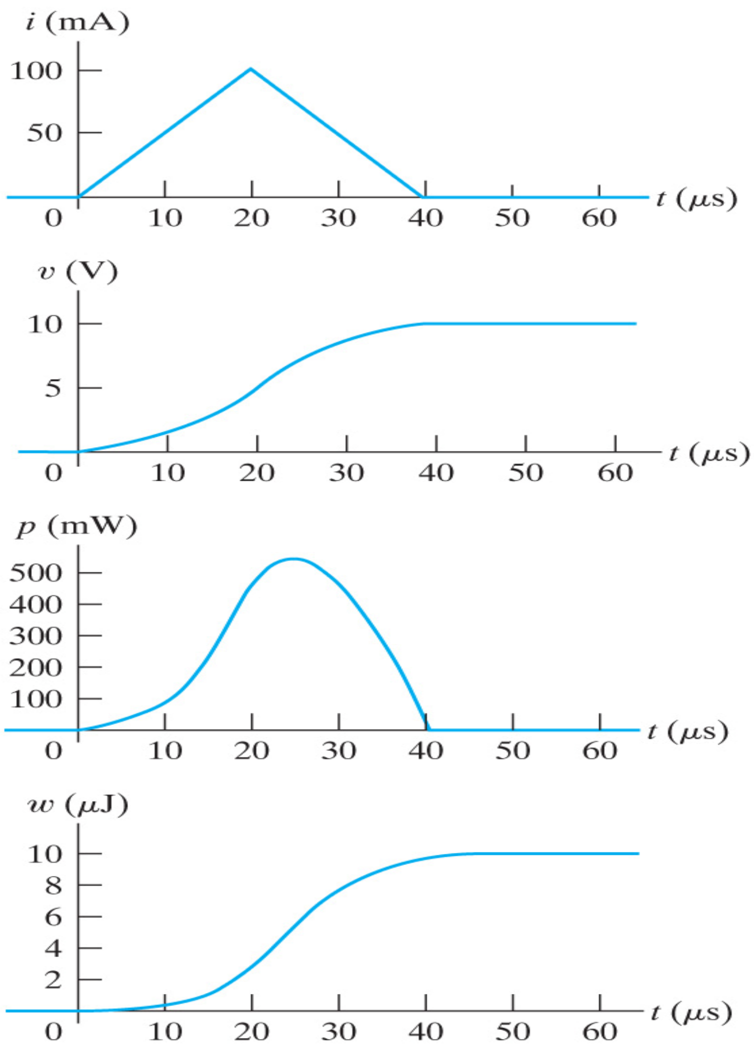
$$v = \frac{1}{0.2 \times 10^{-6}} \int_{40\mu s}^{t} (0)\, d\tau + v(40\mu s)$$

General Equation

$$\boxed{v(t) = v(40\mu s) = 10\,(V)}$$

***v* is constant**

Example Plots



Current through Capacitor

Voltage across Capacitor

Power absorbed by Capacitor

Energy stored in Capacitor

Comments on Example Plots

1. Voltage Constant for $t \geq 40(\mu s)$, $i = 0$

---

2. Since v is Constant for $t \geq 40(\mu s)$

   $w = \frac{1}{2}Cv^2$ is Constant for $t \geq 40(\mu s)$

---

3. $p > 0 \quad 0 < t < 40(\mu s)$

   Power Always Absorbed

Whole industry based on this concept!

Conclusion: We charged up a Capacitor for 40 μs, and the Capacitor remained charged

Example

Given *i*(*t*), Find *v*(*t*) **(Contd.)**

Summary of Results

1 $v(t) = (12.5 \times 10^{9})t^{2}$ $\quad 0 \le t \le 20(\mu s)$

2 $v(t) = (-1.25 \times 10^{10})t^{2} + (10^{6})t - 10$ $\quad 20(\mu s) \le t \le 40(\mu s)$

3 $v(t) = v(40\mu s) = 10(V)$ $\quad t \ge 40(\mu s)$

Example

Given $i(t)$, Find $v(t)$ (Contd.)

***Question*: What is the Voltage Across, & the <u>Energy Stored</u> in, the capacitor at $t = 31\ \mu s$?**

Use expression in (2) on the previous slide

$$v(31\mu s) = (-1.25\times 10^{10})(31\times 10^{-6})^2 + (10^6)(31\times 10^{-6}) - 10$$

$$= -12.0125 + 31 - 10$$

$$v(31\mu s) = 8.9875(V)$$

$$w(31\mu s) = \frac{1}{2}Cv^2(31\mu s) = \frac{1}{2}(0.2\times 10^{-6})(8.9875)^2$$

$$w(31\mu s) = 8.078(\mu J)$$

Series and Parallel Combinations

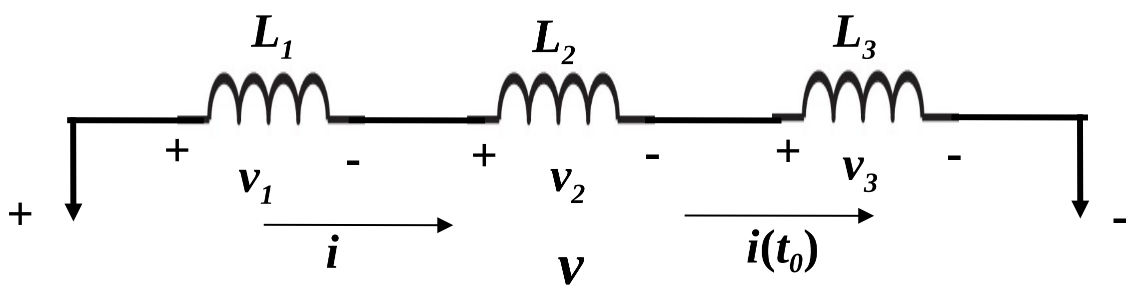
Inductors

Analogous to Resistors.

Longer Inductor → Larger L values

Due to # of Windings

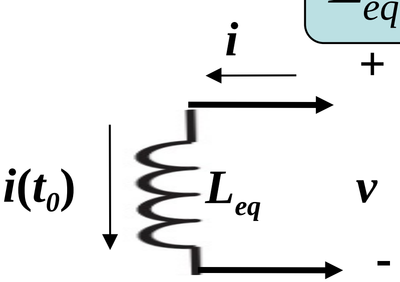
Series



i **same through all inductors**

$$L_{eq} = L_1 + L_2 + L_3$$

Add series inductors



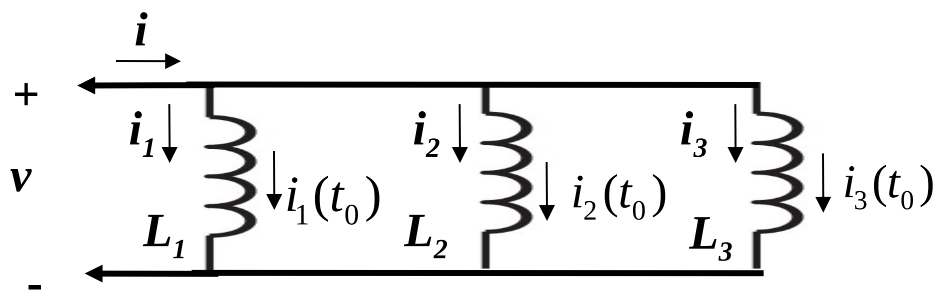
$$v = L_{eq}\frac{di}{dt}$$

$$v = v_1 + v_2 + v_3$$

Series and Parallel Combinations (Contd.)

Inductors

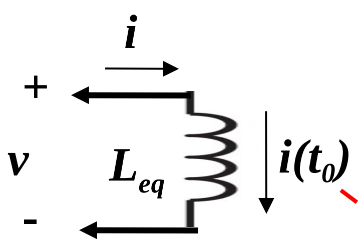
Parallel



v same across all inductors

$$L_{eq} = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \right)^{-1}$$

Add reciprocals and invert



$$i = i_1 + i_2 + i_3 = \frac{1}{L_{eq}} \int_{t_0}^{t} v \, d\tau + i(t_0)$$

$$i(t_0) = i_1(t_0) + i_2(t_0) + i_3(t_0)$$

Inductors Combine like Resistors

Series and Parallel Combinations (Contd.)

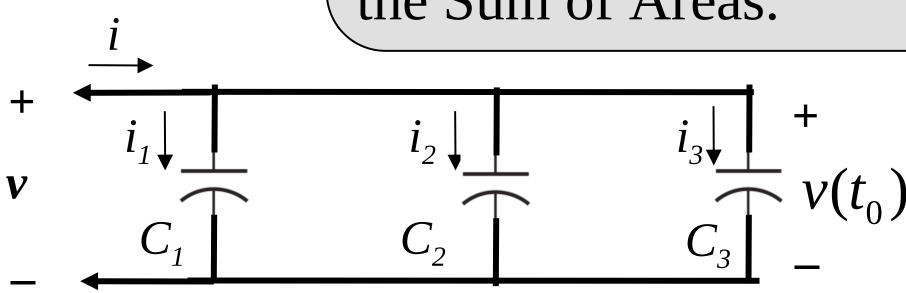
Capacitors → **Dual of Inductor**

Parallel →

Caps Store Charge Between "Plates".

Larger Area → Larger Capacity → Larger C

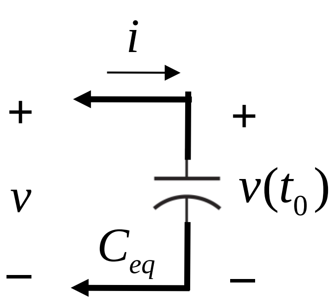
Caps in Parallel Add since Total "Capacity" is Due to the Sum of Areas.



v same across all the Capacitors

$$C_{eq} = C_1 + C_2 + C_3$$

Add in Parallel



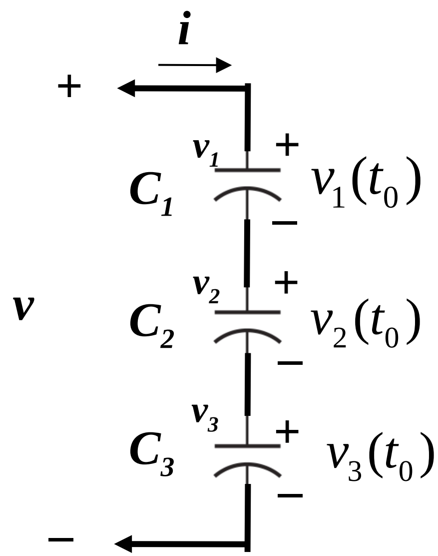
$$i = C_{eq}\frac{dv}{dt}$$

$$i = i_1 + i_2 + i_3$$

Series and Parallel Combinations
(Contd.)

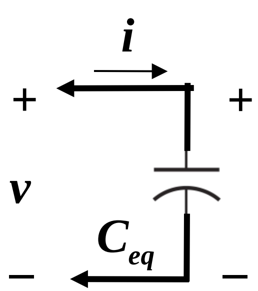
Capacitors

Series



i **same through all the Capacitors**

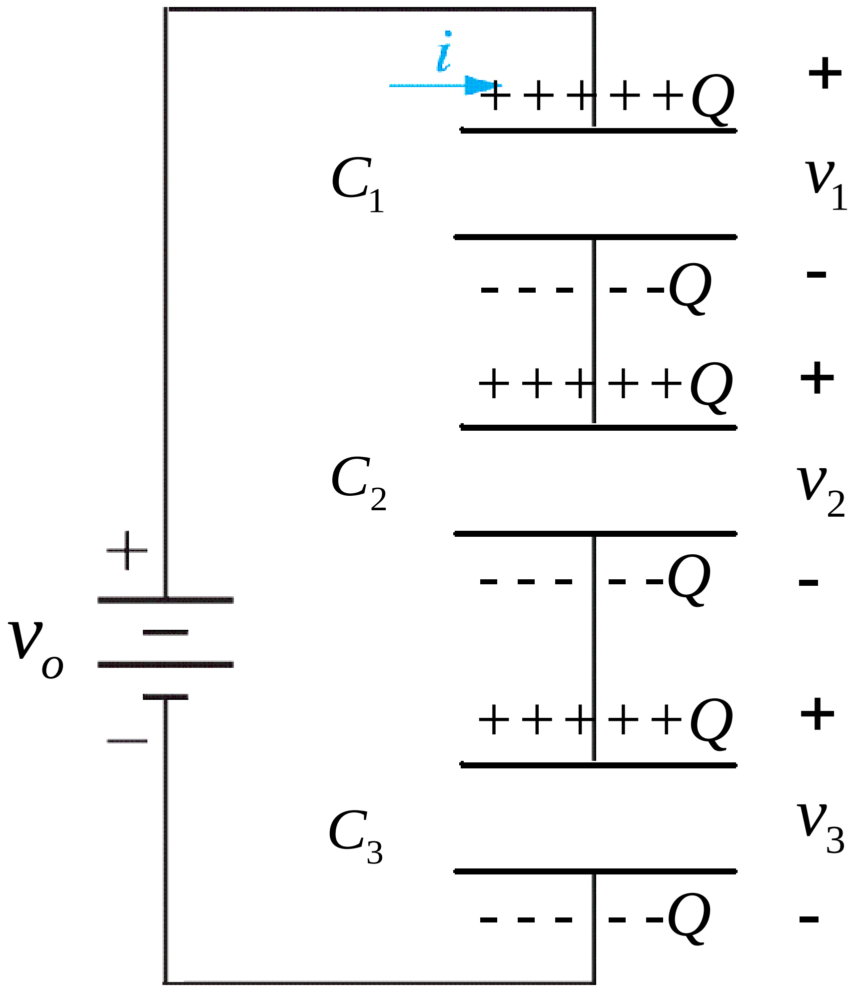
$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right)^{-1}$$



$$v = v_1 + v_2 + v_3 = \frac{1}{C_{eq}} \int_{t_0}^{t} i \, d\tau + v(t_0)$$

$$v(t_0) = v_1(t_0) + v_2(t_0) + v_3(t_0)$$

“Voltage Division” for Capacitors



Q’s are the same due to charge conservation

$$\boldsymbol{Q = Cv}$$

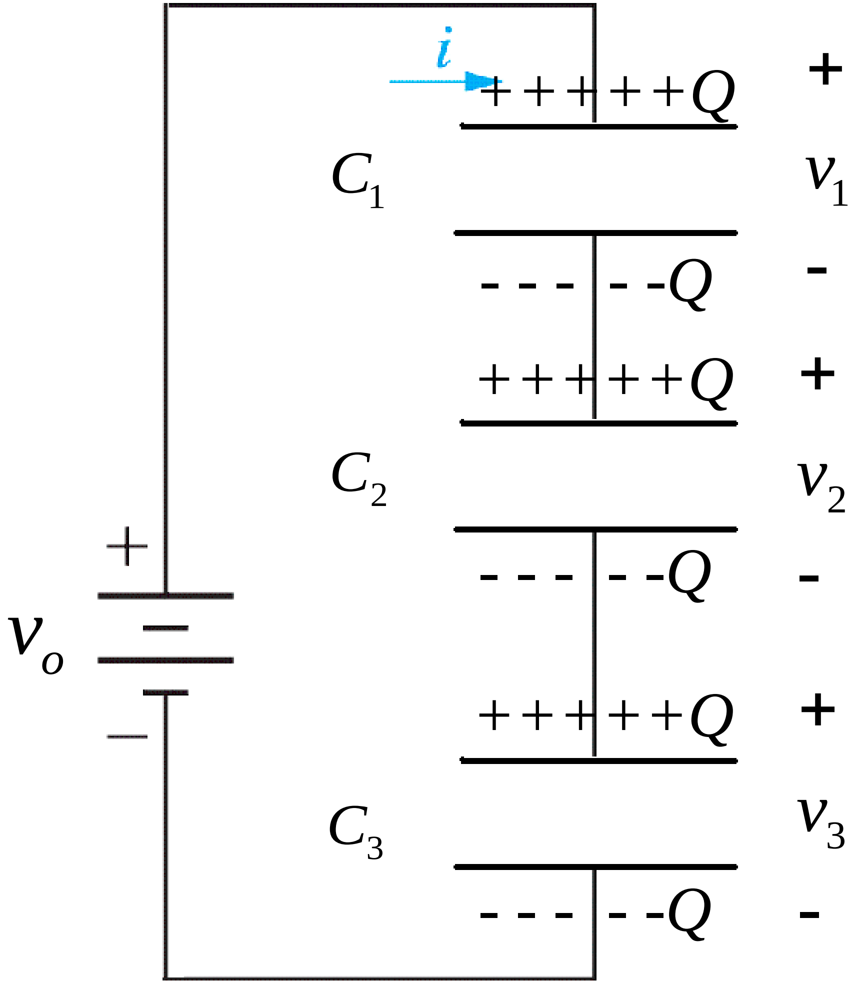
$$\therefore Q = C_1 v_1 = C_2 v_2 = C_3 v_3$$

$$Q = C_{eq} v_o$$

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right)^{-1}$$

$$v_o = v_1 + v_2 + v_3$$

“Voltage Division” for Capacitors
(Contd.)



$$Q = C_{eq} v_0 \quad \}$$ From previous slide

$$v_1 = \frac{Q}{C_1} = \frac{C_{eq}}{C_1} v_o$$

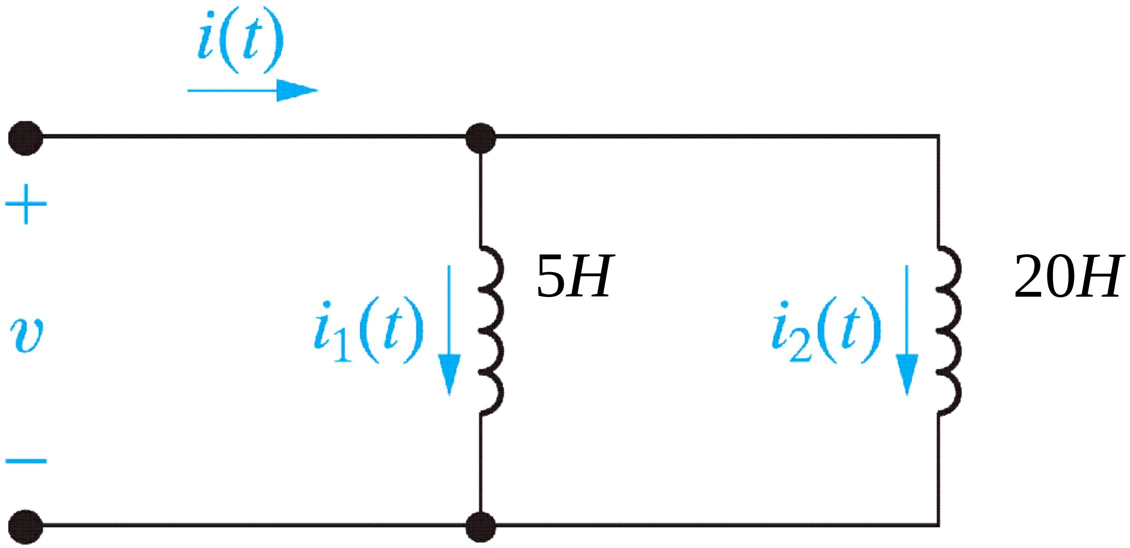
$$v_2 = \frac{Q}{C_2} = \frac{C_{eq}}{C_2} v_o$$

$$v_3 = \frac{Q}{C_3} = \frac{C_{eq}}{C_3} v_o$$

“**Voltage Division**”

Example

Given *v*(*t*), *i*$_1$(0), and *i*$_2$(0); Find *i*(*t*)



$$i_1(0) = -2(A)$$

$$i_2(0) = +4(A)$$

$$v = -40e^{-5t}(V) \qquad t \geq 0$$

a) $$L_{eq} = \frac{L_1 L_2}{L_1 + L_2} = \frac{5(20)}{5+20} = \frac{100}{25}$$

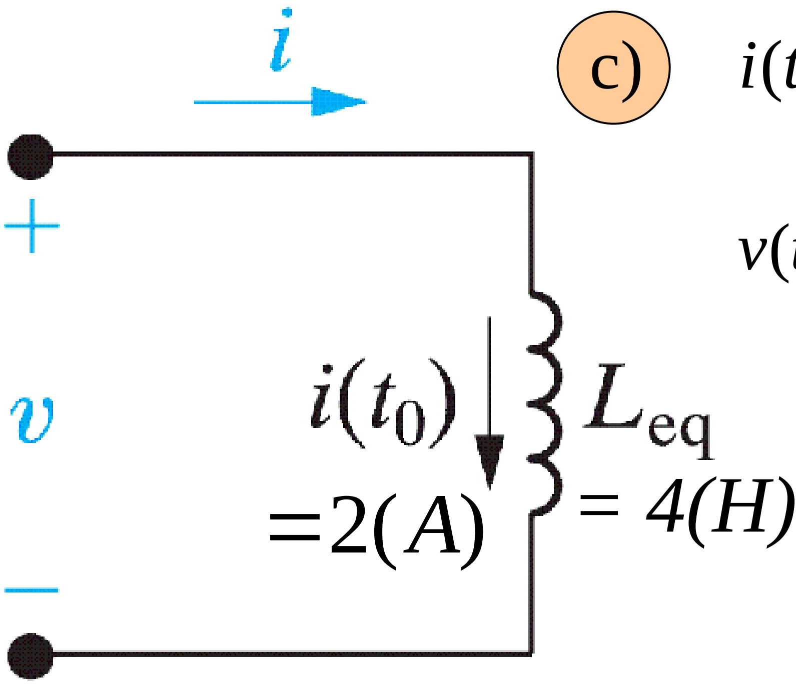
$$L_{eq} = 4(H)$$

b) $$i(t_o) = i_1(t_o) + i_2(t_o) = -2 + 4$$

$$i(t_0) = +2(A)$$

Example

Given $v(t)$, $i_1(0)$, and $i_2(0)$; Find $i(t)$ (Contd.)



c) $$i(t) = \frac{1}{L_{eq}} \int_0^t v(\tau)d\tau + i(t_o)$$ **General Equation**

$$v(t) = -40e^{-5t}(V)$$

Plug in $v(t)$ and $i(t_0)$

$$i(t) = \frac{1}{4}\int_0^t (-40e^{-5\tau})d\tau + 2$$

$$i(t) = -10\left[-\frac{1}{5}e^{-5\tau}\right]_0^t + 2 = 2\left[e^{-5t} - e^0\right] + 2$$ **Integrate and Simplify**

$$i(t) = 2e^{-5t} \qquad t \geq 0$$