CSE211 - PS

10.11.2022

1. Talk about the question i.e. Look at the differences of the followings

$A = \{q : \text{either } q=2 \text{ or } q=4\}$

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A includes either 2 or 4.

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2. Simplify the followings

(a) $\neg(\neg P \wedge \neg Q)$

(b) $(P \wedge Q) \vee (P \wedge \neg Q)$

(c) $\neg(P \wedge \neg Q) \vee (\neg P \wedge Q)$

(d) $(P \wedge R) \vee [\neg R \wedge (P \vee Q)]$

3. $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$ show that

4. $(A \cup B) \setminus B \subseteq A$ show that

*5. Determine $(\neg q \wedge (p \to q)) \to \neg p$ is a tautology.

6. How to represent using logical operators

$A \subseteq B$ and $A \not\subseteq B$

RULES OF INFERENCE

$$\begin{array}{l} p \\ p \to q \\ \hline \therefore q \end{array}$$
therefore

$$\begin{array}{l} \neg q \\ p \to q \\ \hline \end{array} \qquad \begin{array}{l} \neg q \\ \neg q \to \neg p \\ \hline \therefore \neg p \end{array}$$

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

$$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}$$

$$\begin{array}{l} p \wedge q \\ \hline \therefore p \end{array} \qquad \begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array}$$

7. "It is not sunny this afternoon, and it is colder than yesterday", "We will go swimming only if it is sunny", "If we do not go swimming, then we will take a canoe trip", "If we take a canoe trip, then we will be home by sunset."

(¬p, q, p, p, ¬r, s, s)

premises t

$\neg p \wedge q$

$r \to p$

$\neg r \to s$

$\neg p \wedge q$

$\neg p$

$r \to p$

$\neg r$

$\neg r \to s$

s

$s \to t$

t

the premises lead to the conclusion.

"we will be home by sunset.

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8. Check this proof

let a and b are two equal positive integers.

$$a=b \Rightarrow a^2=ab$$
$$\Rightarrow a^2-b^2=ab-b^2$$
$$\Rightarrow (a-b)(a+b)=(a-b)b$$
$$\Rightarrow a+b=b$$
$$\Rightarrow 2b=b$$
$$\Rightarrow 2=1$$

9. Prove that if m and n are integers and mn is even, then m is even or n is even.

$m,n \in \mathbb{Z}$, $mn=2k \quad \because \exists k \in \mathbb{Z}$

Assume the proposition is not true. That means,

$$\neg(\exists t,s \in \mathbb{Z}, (m=2t \vee n=2s)) \equiv \forall t,s \in \mathbb{Z} \ \neg(m=2t \vee n=2s)$$
$$\equiv \forall t,s \in \mathbb{Z} \ (m \neq 2t \wedge n \neq 2s)$$
$$\equiv \exists t,s' \in \mathbb{Z}, \ m=2t'+1 \wedge n=2s'+1$$

$\exists t,s \in \mathbb{Z}$

$$m=2t+1 \wedge n=2s+1 \Leftrightarrow m \cdot n=(2t+1)(2s+1)$$
$$=4ts+2t+2s+1$$
$$=2(2ts+t+s)+1$$
$$=2r+1 \quad \exists r=2ts+t+s \in \mathbb{Z}$$
$$\Leftrightarrow mn \text{ is odd}$$

this is a contradiction ✓

10. Show that if $\underbrace{n \text{ is an integer}}_{p}$ and $\underbrace{n^3+5 \text{ is odd}}_{q}$, then $\underbrace{n \text{ is even}}_{r}$ using

(a) a proof by contraposition $(\neg r \rightarrow \neg(p \wedge q) \equiv \neg r \rightarrow (\neg p \vee \neg q)) \wedge \neg r$

(b) a proof by contradiction $p \wedge q \wedge \neg r \rightarrow C$

11. Prove that there are no solutions in integers x and y to the equation $2x^2+5y^2=14$. (Uniqueness)