CSE 211: Discrete Mathematics **(Due: 13/10/22)**

Problem Session #1

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Problem 1: Inverse Image of a Set (0 points)

Let f be the function from $\mathbb{R}$ to $\mathbb{R}$ defined by f(x) = x^2. Find

(a) f^{-1} ({ x | 0 < x < 1 })

(b)f^{-1} ({ x | x > 4 })

Problem 2: Injective and Surjective Functions (0 points)

For each of the following functions, give the following information: What is its codomain? What is its image? Is the function onto? Is the function one-to-one?

(a) f : $\mathbb{Z} \to \mathbb{Z}$ such that f(x) = $2 \lfloor \frac{x}{2} \rfloor$

(b) g : $\mathbb{N} \to \mathbb{N}$ such that g(x) = $\frac{x(x+1)}{2}$

(c) f : $\mathbb{R} \to \mathbb{R}$ such that h(ai+b) = a where i = $\sqrt{-1}$

Problem 3: Floor and Ceiling Functions (0 points)

Find these values.

(a) $\lceil \frac{3}{4} \rceil$

(b) $\lfloor \frac{7}{8} \rfloor$

(c) $\lceil \frac{-3}{4} \rceil$

(d) $\lfloor \frac{-7}{8} \rfloor$

(e) $\lceil 3 \rceil$

(f) $\lfloor -1 \rfloor$

(g) $\lfloor \frac{1}{2} \lceil \frac{3}{2} \rceil \rfloor$

(h) $\lfloor \frac{1}{2} \lfloor \frac{5}{2} \rfloor \rfloor$

Problem 4: Inverse of Functions (0 points)

Let S = {1, 2, 3, 4, 5} and let f , g, h : S → S be the functions defined by

- f = {(1, 2), (2, 1), (3, 4), (4, 5), (5, 3)}
- g = {(1, 3), (2, 5), (3, 1), (4, 2), (5, 4)}
- h = {(1, 2), (2, 2), (3, 4), (4, 3), (5, 1)}

Explain why f and g have inverses but h does not. Find f^{-1} and g^{-1}.

Problem 5: Subsets of Functions (0 points)

Let f be a function from A to B. Let S and T be subsets of A. Show that:

(a) f(S) ∪ f(T) = f(S ∪ T)

(b) f(S ∩ T) ⊆ f(S) ∩ f(T)

Problem 6: Subsets of Functions (0 points)

Let f be a function from A to B. Let S and T be subsets of B. Show that:

(a) f^{-1}(S) ∪ f^{-1}(T) =f^{-1}(S ∪ T)

(b)f^{-1}(S ∩ T) = f^{-1}(S) ∩ f^{-1}(T)