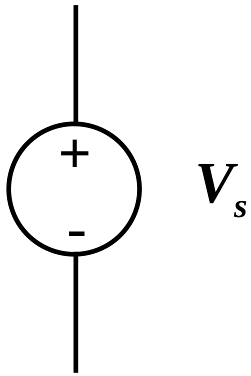
Basic Circuit Elements

"Developing models, which provide an understanding that is imperfect, but adequate, for solving practical problems lies at the heart of engineering."

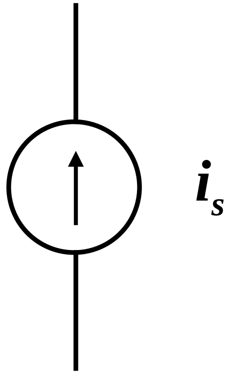
Sources: 'Ideal Sources'

<u>Ideal Voltage Source</u>



Maintains ***constant V*** across terminals regardless of current

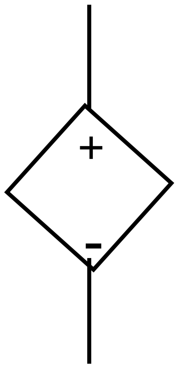
<u>Ideal Current Source</u>



Maintains ***constant I*** through terminals regardless of voltage across them

- These are ***Independent*** sources! Their value does not depend on anything.
- <u>Specifications</u>: **Value** and **Polarity**

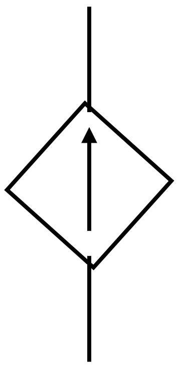
Sources: Dependent Sources



$$V_S = \mu \cdot V_X$$

OR

$$V_S = \rho \cdot i_X$$



$$i_S = \alpha \cdot V_X$$

OR

$$i_S = \beta \cdot i_X$$

Four variations:

μ and β are dimensionless

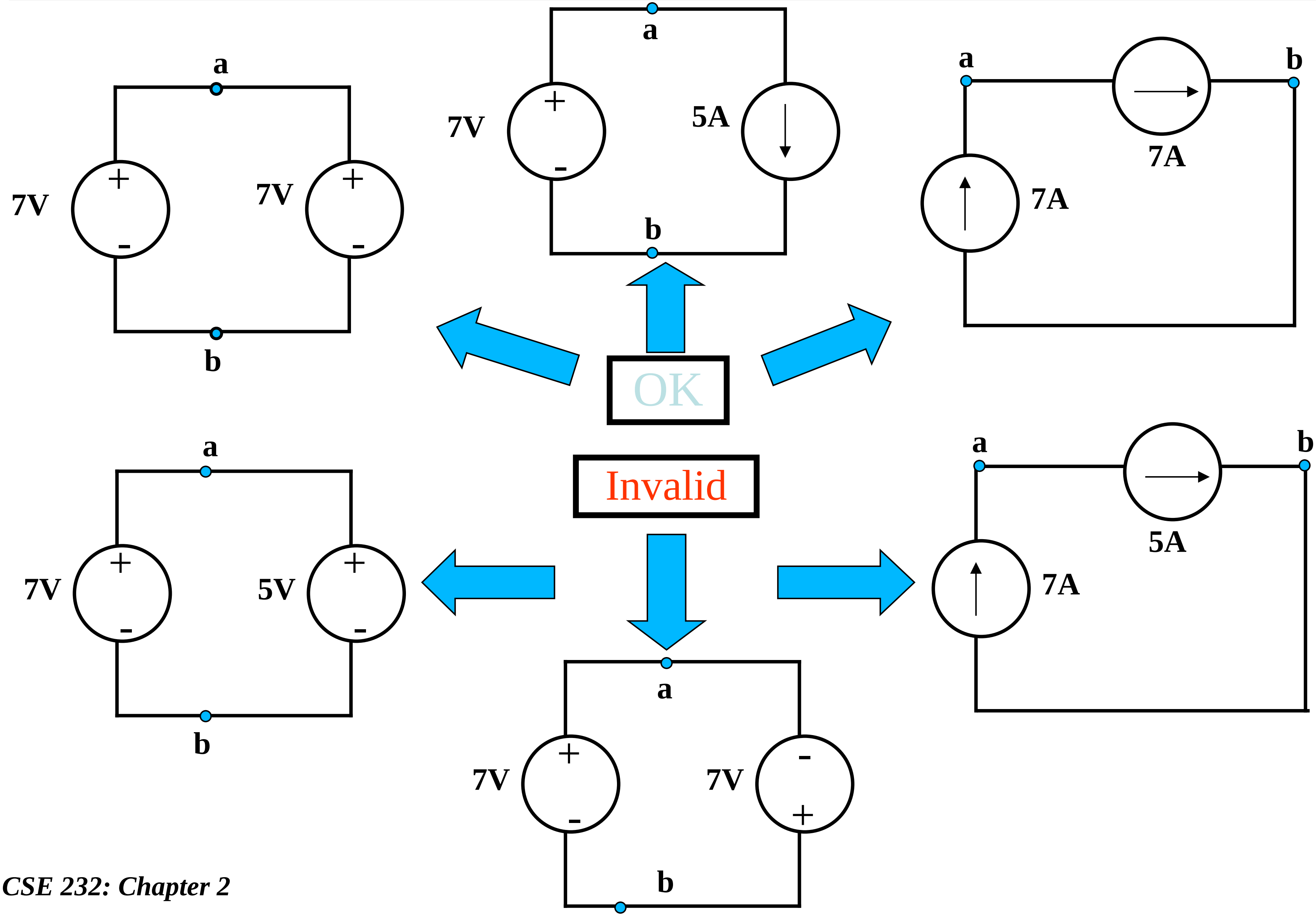
ρ is in (V/A)

α is in (A/V)

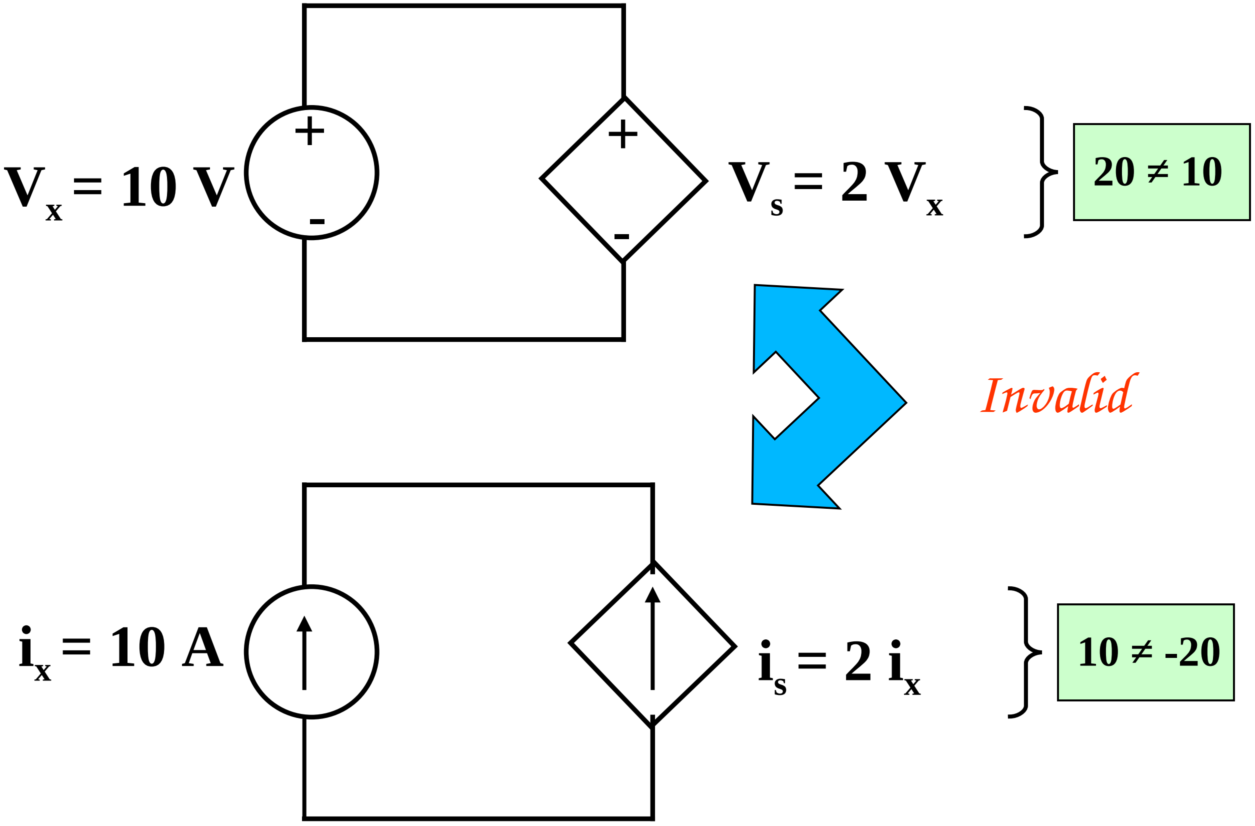
Value depends on ***V*** or ***i*** elsewhere in the circuit

Transistors and operational amplifiers (op amps) are modeled with this

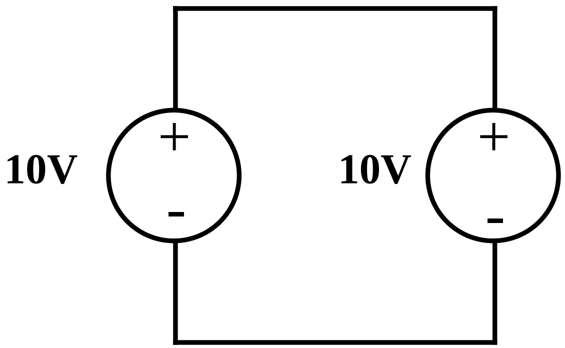
Limited Interconnections – Independent Sources



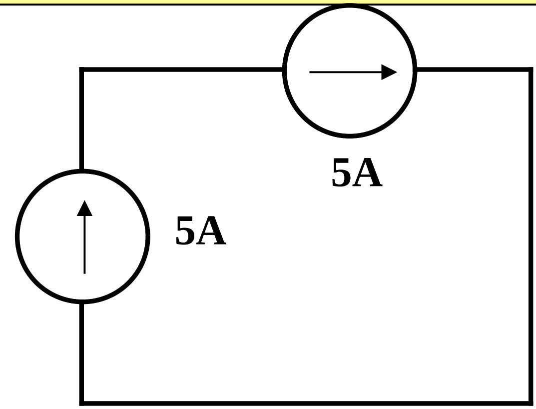
Limited Interconnections – Dependent Sources



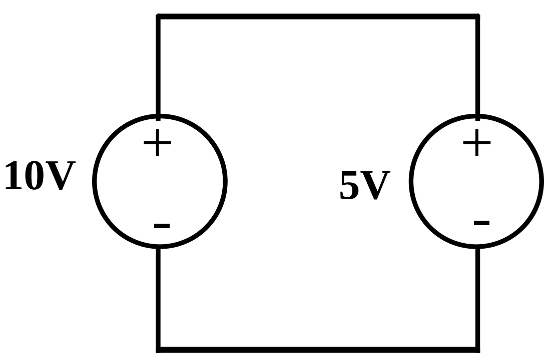
Which are valid connections?



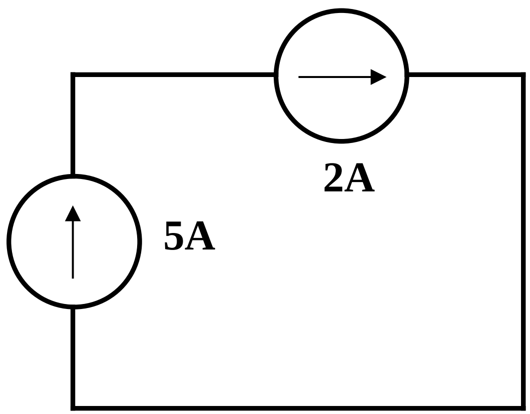
(a)



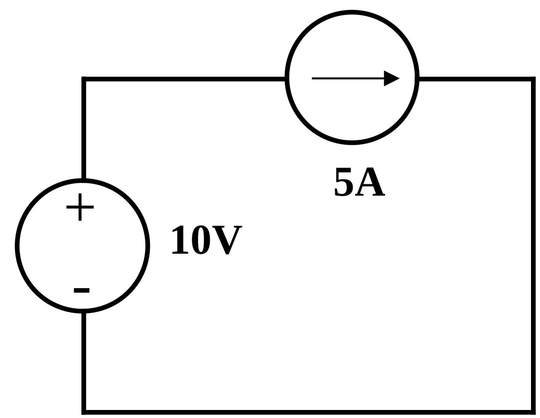
(b)



(c)

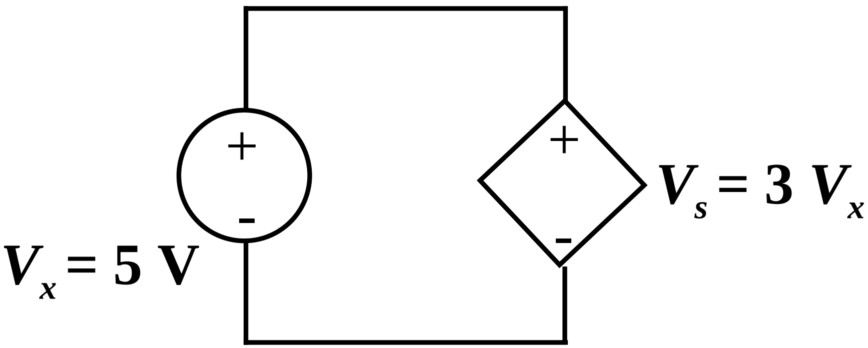


(d)

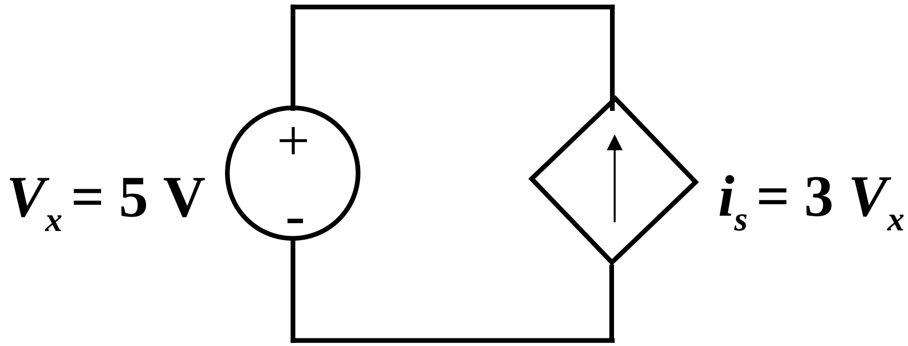


(e)

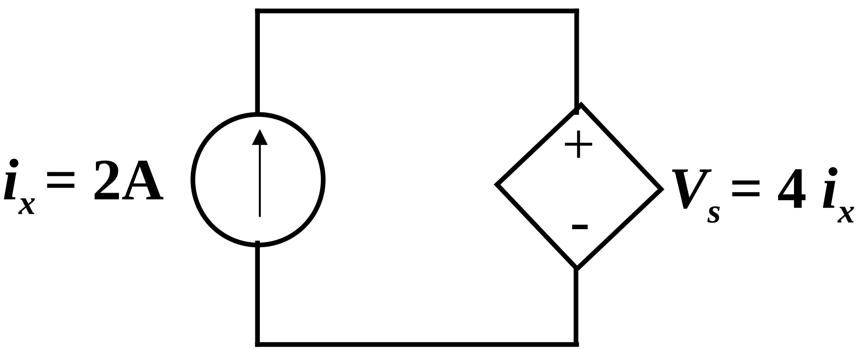
Which are valid connections?



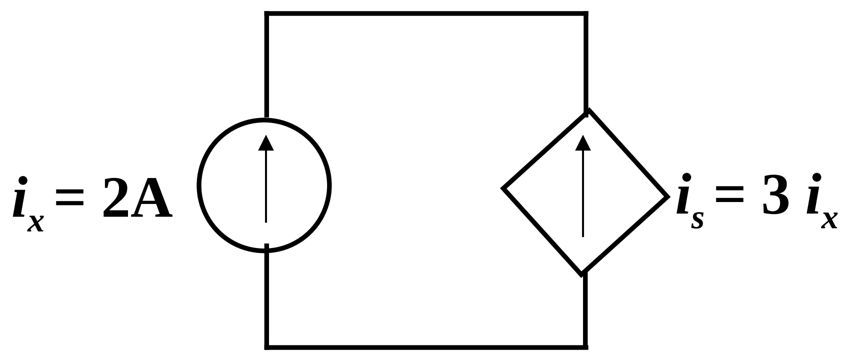
(a)



(b)



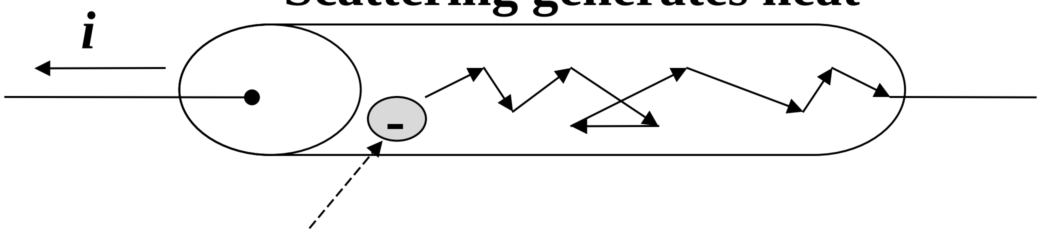
(c)



(d)

Ohm’s Law

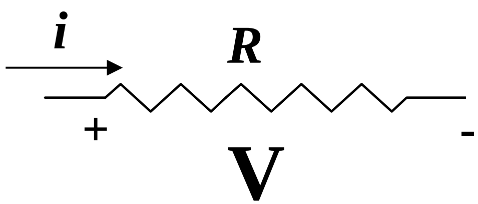
Scattering generates heat



electron

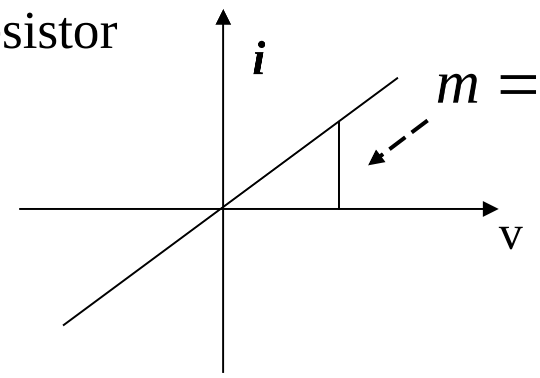
Resistance

Capacity of a material to impede flow of current



Resistor

i-v characteristic of Resistor



$$m = \frac{1}{R}$$

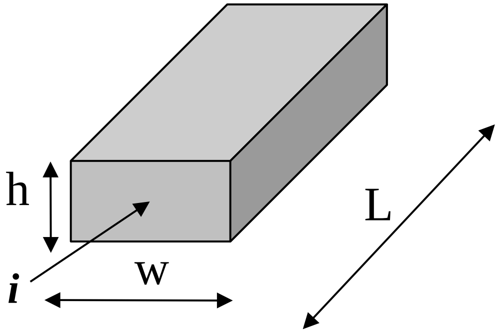
Ohm’s Law

$$i = \left(\frac{1}{R}\right) \cdot V$$

V in **Volts (*V*)**
i in **Amps (*A*)**
R in **Ohms (Ω)**

$$V = i \cdot R$$

Resistance and Conductance



Ideal R is **constant**

R can vary with time (***t***) and temperature (**T**)

Not in this class

$$R = \rho \cdot \frac{L}{A}$$

ρ is Resistivity (Ω -*cm*)

Conductance

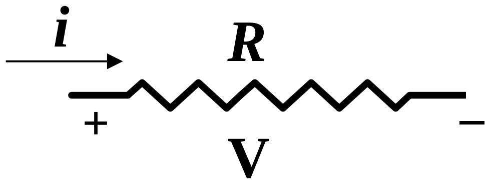
Unit: Siemens (***S***) or mhos(℧)

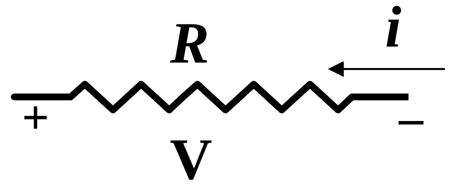
If ***R* = 10Ω** then ***G* = 0.1 ℧**

$$G = \frac{1}{R}$$

Power at Terminals of Resistor

Passive sign convention





Find Power

Ohm's Law

$\boldsymbol{V = i\,R}$

$p = V\,i$

$p = (iR)\,i$

Ohm's Law

$\boldsymbol{V = -i\,R}$

$p = -V\,i$

$p = -(-iR)\,i$

In either case,

$\boldsymbol{p = i^2 R = i^2/G}$

Always positive

Side Note

$p = Vi$

$p = V\,(V/R)$

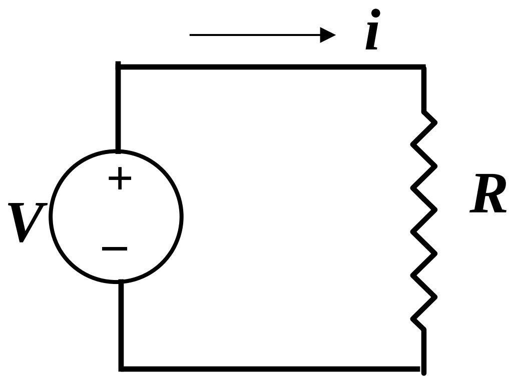
$\boldsymbol{p = V^2/R = V^2 G}$

R always **absorbs** power

Therefore, it is **passive**.

Side Note

Example: Find Power absorbed by the resistor



$V = 100\ (V)$ ➡ $i = V / R = 100 / 10 = 10\ (A)$

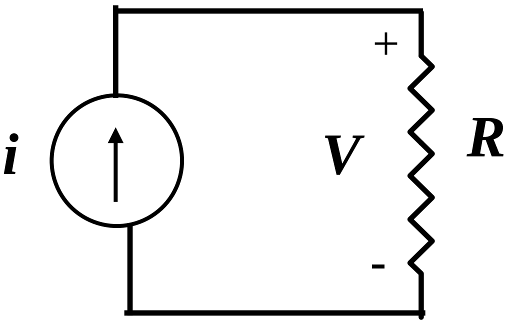
$R = 10(\Omega)$

$p = V\,i \qquad = i^2 R \qquad = V^2 / R$

$= (100)(10) = 10^2(10) = 100^2/10$

$\mathbf{= 1000\ (W)}$

$= \boxed{\mathbf{1\ (kW)}}$



$i = 5(A)$ ➡ $V = i\,R = 5\ (10) = 50\ (V)$

$R = 10(\Omega)$

$p = V\,i \qquad = i^2 R \qquad = V^2 / R$

$= (50)(5) = 5^2(10) = 50^2/10 \quad = \boxed{\mathbf{250\ (W)}}$

Models for Actual Circuit Components

- **Battery:** Voltage Source
- **Lamp:** Resistor
- **Wire:** Resistor or Ignore
- **Conducting Path:** Resistor or Ignore

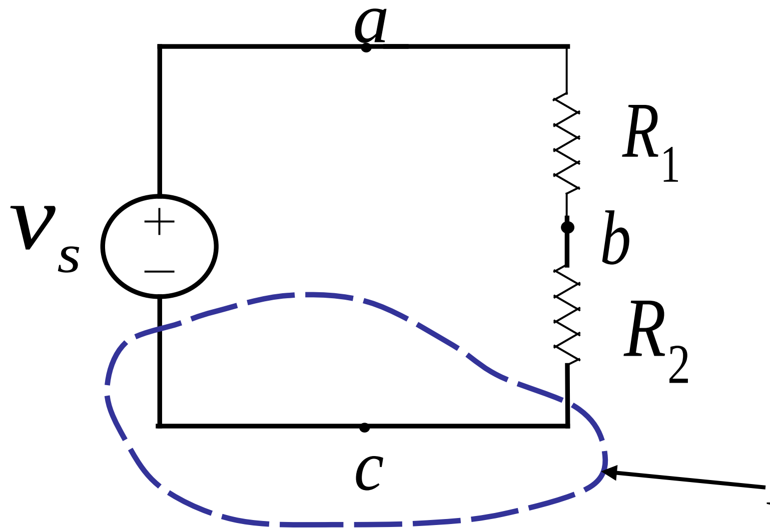
- <u>**Not Necessarily a Unique Answer**</u>

Kirchhoff's Laws

Gustav Robert Kirchhoff

Published in 1845 as a student

<u>Node:</u> Point where 2 or more circuit elements meet

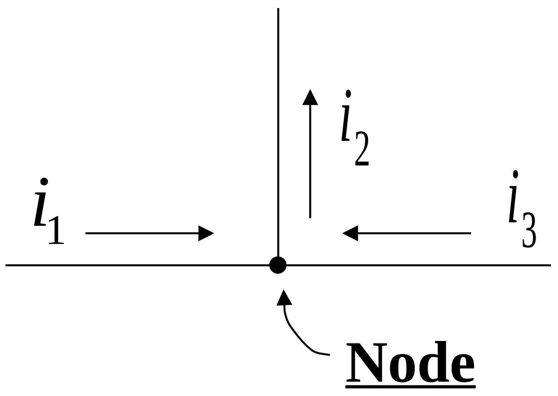


Nodes are a, b, and c

whole section of wire is a node !

Kirchhoff's Current Law: (KCL)

"Algebraic" Relationship is Important



i_1 and i_3 are "entering" the node.
i_2 is "leaving" the node.

Currents leaving a node are "Algebraically" opposite in "sign" to currents entering a node.

¿ **Conventions** ¿

(a) $i_2 - i_1 - i_3 = 0$ — *i* leaving considered positive

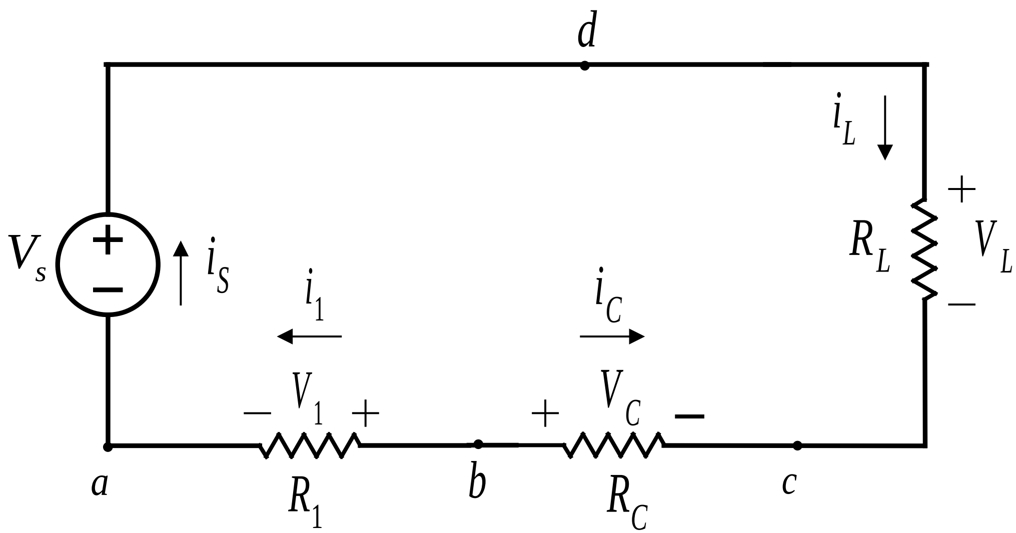
(b) $i_1 + i_3 - i_2 = 0$ — *i* entering considered positive

Note a "considered" positive current could be negative

(c) $i_2 = i_1 + i_3$ — (c) is equivalent to (a) and (b)

Kirchhoff's Voltage Law: (KVL)

The algebraic sum of all the voltages around any closed path in a circuit equals zero.

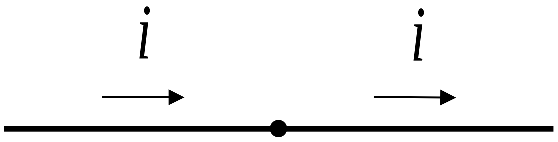


$$\underbrace{-V_S + V_L - V_C + V_1 = 0}_{\text{KVL}}$$

KVL

Voltages and Currents have been defined in the circuit

Sign Convention



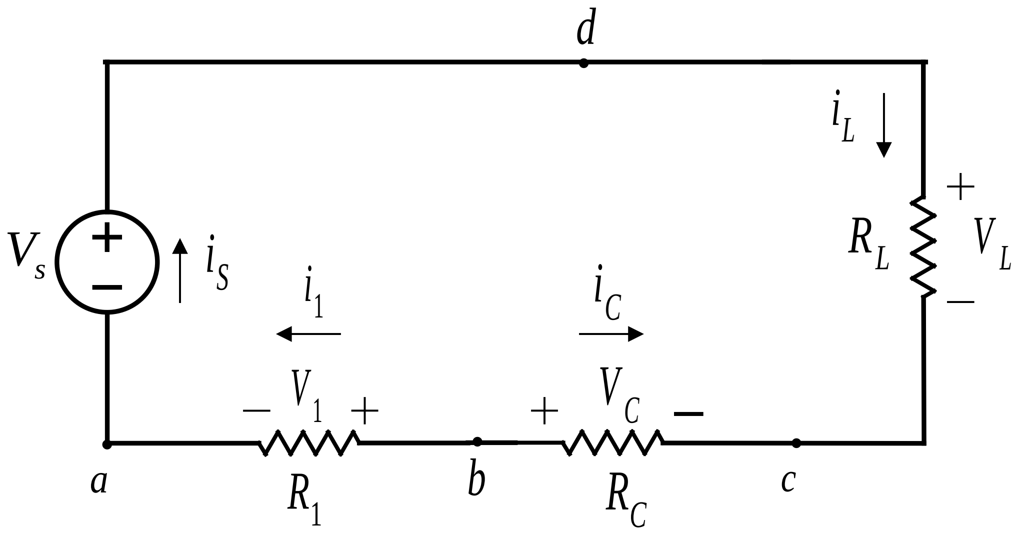
Is i entering or leaving the node ?

$$i - i = 0$$
$$\Rightarrow i = i$$

KCL Stated Another Way

$$\sum i\text{'s entering node} = \sum i\text{'s leaving node}$$

Flashlight Circuit: Find the currents in the circuit



Voltages and Currents have been defined in the circuit

Ohm's Law

$$V_1 = i_1 R_1 \quad (1)$$

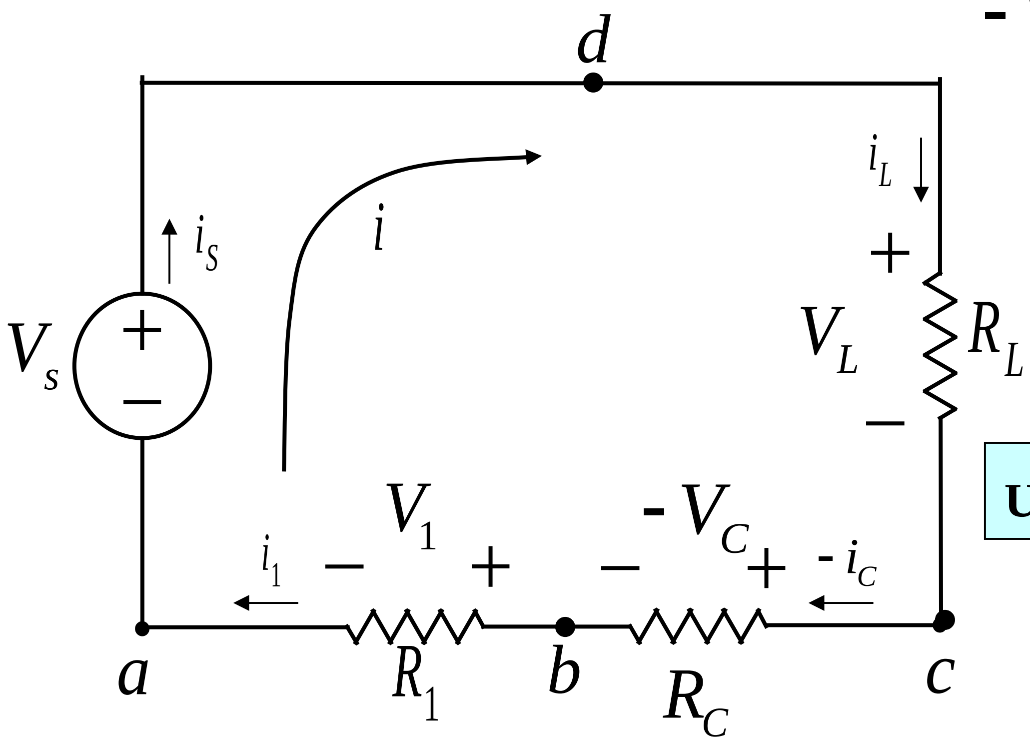
$$V_C = i_C R_C \quad (2)$$

$$V_L = i_L R_L \quad (3)$$

(contd.)

Flashlight Circuit (Contd.)

Circuit can be redrawn as:



Start from node @

$$-V_S + V_L - V_C + V_1 = 0 \quad \text{KVL}$$

(1) $$V_S = V_L + V_1 - V_C$$

<u>Currents are all the same; so define:</u>

(2) $$i \equiv i_S = i_1 = i_L = -i_C$$

Using Ohm's Law and (2), (1) becomes

$$V_S = iR_L + \overbrace{iR_C}^{-V_C} + iR_1$$

(3) $$V_S = i(R_L + R_C + R_1) \quad \text{After Simplifying}$$

(Contd.)

Flashlight Circuit (Contd.)

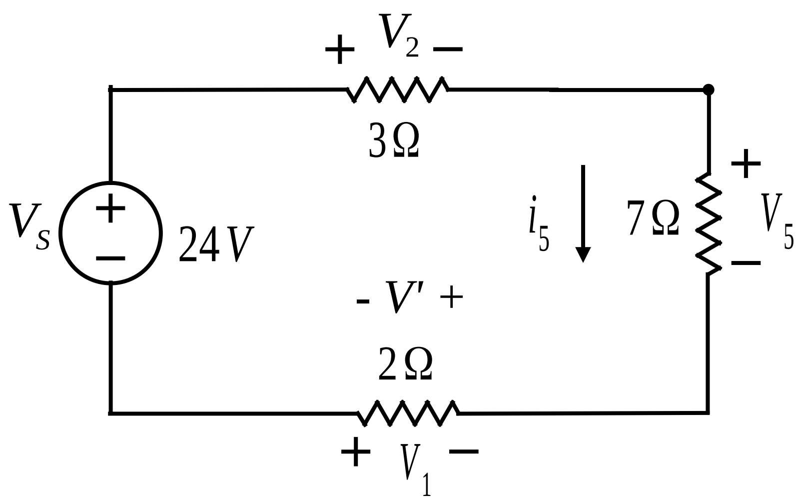
$$i = \frac{V_S}{R_L + R_C + R_1}$$ Solve 3 for *i*

Using Ohm's Law and calculated *i*, we obtain V_1, V_C, and V_L

$$V_1 = iR_1 \qquad -V_C = iR_C \qquad V_L = iR_L$$

$$i = i_S = i_1 = i_L = -i_C$$

Example: Find Current in the Circuit



$V_1, V_2, V_5,$ **and** i_5

have been defined in the circuit

a) Find i_5 (current is the same in all elements.)

KVL $V_S = V_2 + V_5 + V'$ — $V_1 = -V'$ ⟸ forget "V_1" for now

$V_S = 3i_5 + 7i_5 + 2i_5$ — $V_2 = 3i_5,\; V_5 = 7i_5,\; V' = 2i_5$

$V_S = i_5(3+7+2) = 12i_5$ — **Simplify**

$i_5 = \dfrac{24}{12} = 2\,(A)$ — **Solve for i_5**

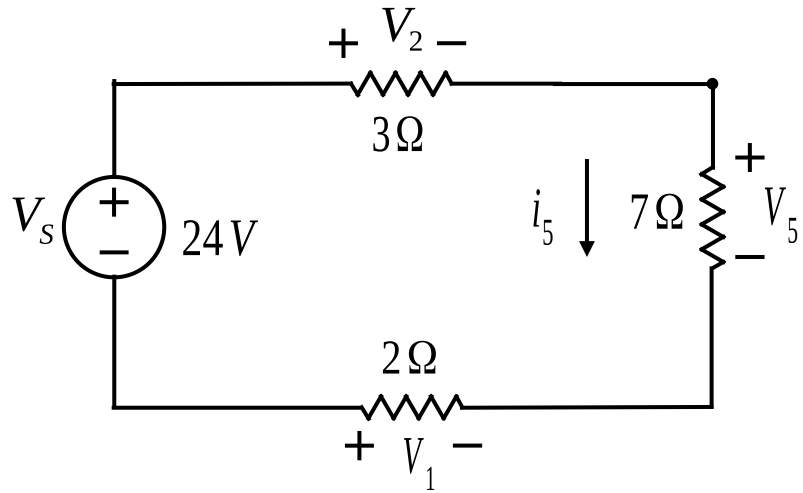
Example: Find Voltages and Power of the Supply (Contd.)

b) $V_1 = -2i_5 = -2(2)$

$V_1 = -4\,(V)$

c) $V_2 = 3i_5 = 3(2)$

$V_2 = 6\,(V)$



d) $V_5 = 7i_5 = 7(2)$

$V_5 = 14\,(V)$

Note: $V_S = V_2 + V_5 - V_1$

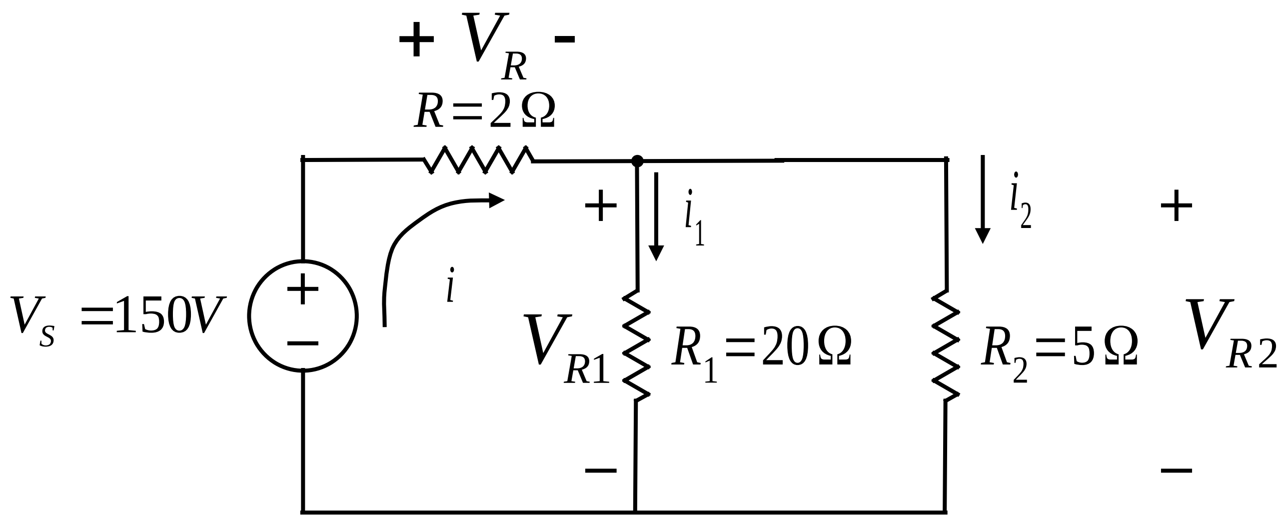
$= 6 + 14 - (-4) = 24(V)$

Sign Convention

e) $p_{24V} = \boxed{-V_S i_5} = -24(2) = -48(W)$

Thus power is extracted from the supply

Example: Find Current *i*



1 $V_S = Ri + R_1 i_1$ — **KVL and Ohm's Law**

2 $R_1 i_1 = R_2 i_2$ — **KVL and Ohm's Law**

3 $i = i_1 + i_2$ — **KCL**

Substitute 3 → 1 ⇨ $V_S = R(i_1 + i_2) + R_1 i_1$ 4

$V_S = (R + R_1) i_1 + R i_2$ 4 — **Collect terms**

Example (Contd.)

Solve 2 for $\boldsymbol{i_2}$ then substitute into 4

$$R_1 i_1 = R_2 i_2 \quad \Rightarrow i_2 = \frac{R_1}{R_2} i_1 \qquad 2$$

$$V_S = (R + R_1)i_1 + R\left(\frac{R_1}{R_2}\right)i_1 = (R + R_1 + \frac{R_1}{R_2} \cdot R)i_1 \qquad 4$$

$$i_1 = \frac{V_S}{R + R_1 + \frac{R \cdot R_1}{R_2}} = \frac{150}{2 + 20 + \frac{2 \cdot (20)}{5}} = \frac{150}{22 + 8} = \frac{150}{30}$$

Solve 4 for $\boldsymbol{i_1}$

$$i_1 = 5(A) \qquad i_2 = \frac{R_1}{R_2} i_1 = \frac{20}{5} \cdot 5 \quad i_2 = 20(A)$$

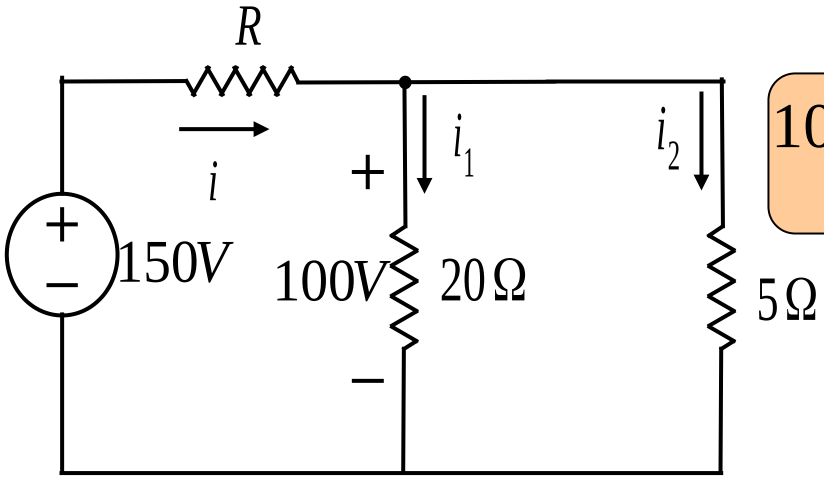
Use 2 for $\boldsymbol{i_2}$

$$3 \longrightarrow i = i_1 + i_2 = 5 + 20 \Rightarrow \boxed{i = 25(A)}$$

Use 3 for $\boldsymbol{i}$

$$\boxed{V_{R1} = V_{R2} = i_1 R_1 = i_2 R_2 = 100(V)} \qquad \boxed{V_R = R \cdot i = 2(25) = 50(V)}$$

Example: Find Resistance *R*



100V dropped across the 20Ω and 5Ω Resistors

Step 1

Ohm's Law

$$i_1 = \frac{100V}{20\Omega} = 5(A)$$

$$i_2 = \frac{100V}{5\Omega} = 20(A)$$

Step 2

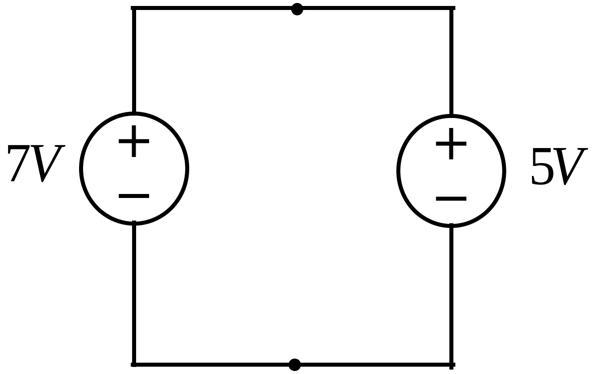
KCL $\{\, i = i_1 + i_2 = 5(A) + 20(A) = 25(A)$

KVL $\{\, 150\text{V} = iR + 100(V)$ $\}$ **Solve for *R***

$$R = \frac{150(V) - 100(V)}{i} = \frac{50(V)}{25(A)} \Rightarrow \boxed{R = 2(\Omega)}$$

Example: Voltage Source

What would happen experimentally if we hooked two voltage sources in parallel with different voltages?

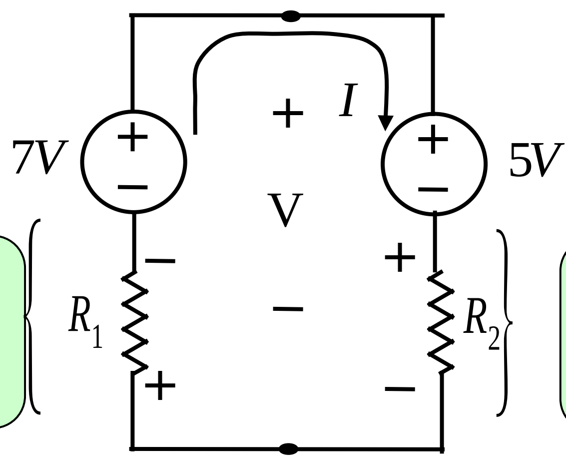


For ideal sources, this is simply not possible

Model Practical Sources

KVL $\left\{ \; V = 7 - IR_1 = 5 + IR_2 \right.$ (1)

Solve for *I* $\left\{ \begin{array}{l} 7 - 5 = IR_1 + IR_2 \\ 2 = I(R_1 + R_2) \end{array} \right.$



Source Resistance

Source Resistance

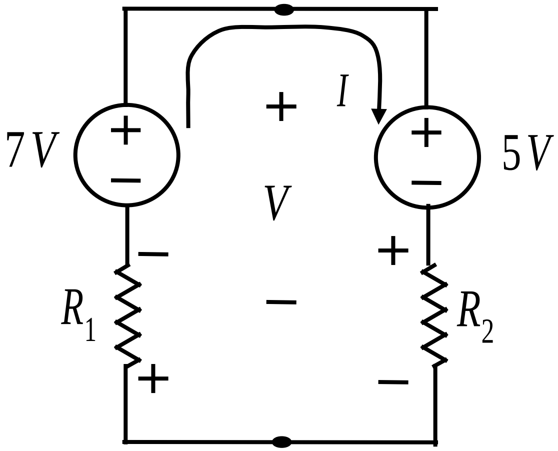
$$\therefore \quad I = \frac{2}{R_1 + R_2} \qquad (2)$$

Example (Contd.)

Substitute (2) into (1)

$$\left\{\begin{aligned} V &= 7 - IR_1 = 7 - \frac{2R_1}{R_1+R_2} \\ V &= 5 + IR_2 = 5 + \frac{2R_2}{R_1+R_2} \end{aligned}\right\}$$

Voltage is distributed according to KVL



if $R_1 = R_2$ $\qquad I = \frac{2}{2R} = \frac{1}{R}$ (3)

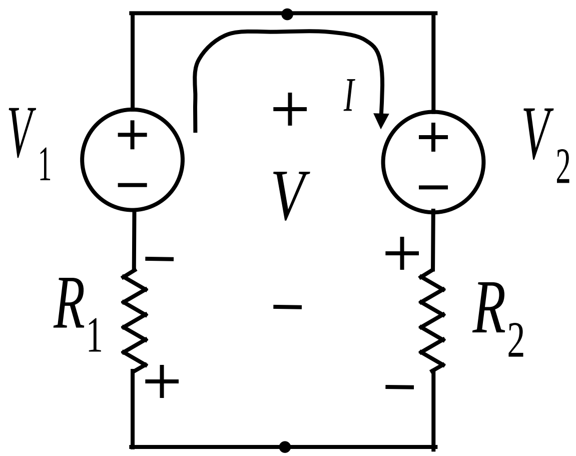
Substitute (3) into (1)

$$\left\{\begin{aligned} V &= 7 - \left(\frac{1}{R}\right) R = 6 \\ V &= 5 + \left(\frac{1}{R}\right) R = 6 \end{aligned}\right.$$

$$V = 6(V)$$

(mid-point)

Redo Example with No Numerical Values



1 2

$$V \equiv V_1 - IR_1 = V_2 + IR_2$$ KVL

$$V_1 - V_2 = I\left(R_1 + R_2\right)$$ Subtract 2 from 1

$$I = \frac{V_1 - V_2}{R_1 + R_2}$$ 3 Solve for *I*

Substitute 3 into 1 and 2

$$V = V_1 - \left(V_1 - V_2\right)\frac{R_1}{R_1 + R_2}$$

$$V = V_2 + \left(V_1 - V_2\right)\frac{R_2}{R_1 + R_2}$$

If $R_1 = R_2 = R$

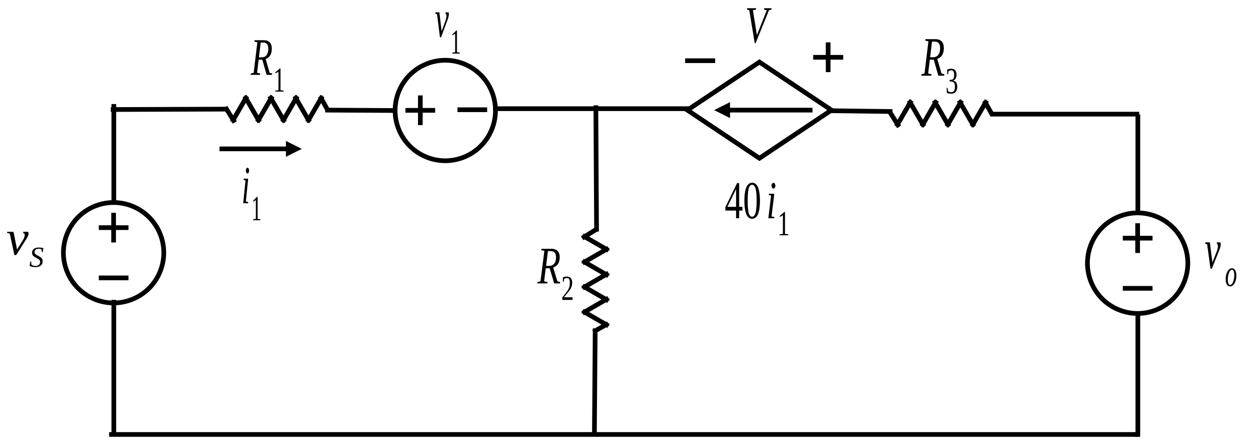
$$V = V_1 - \frac{V_1 - V_2}{2}$$

or

$$V = V_2 + \frac{V_1 - V_2}{2}$$

Average of 2 voltage sources

Example: Circuit with Dependent Sources



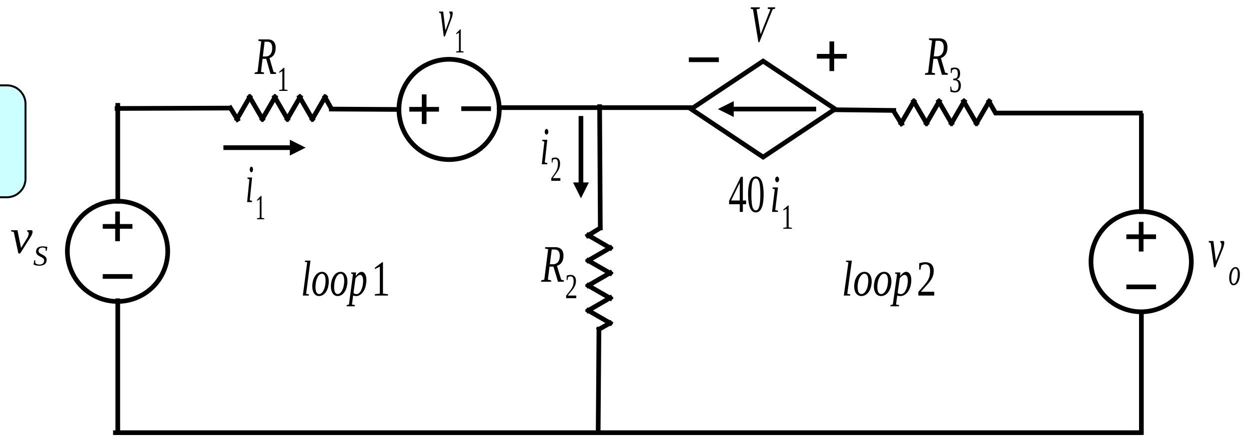
Find i_1 and V

$$v_s = 3(V),\ v_1 = 0.5(V),\ v_o = 10(V),$$
$$R_1 = 29.5(k\Omega),\quad R_2 = 500(\Omega),$$
$$R_3 = 2.4(k\Omega)$$

Given in the Problem

Example: Find i_1 (Contd.)

Two Loops



Given

$v_s = 3(V)$

$v_1 = 0.5(V)$

$v_0 = 10(V)$

$R_1 = 29.5(k\Omega)$

$R_2 = 500(\Omega)$

$R_3 = 2.4(k\Omega)$

Loop 1

$$v_s = i_1 R_1 + v_1 + i_2 R_2 \quad (1)$$

$$i_2 = i_1 + 40 i_1 = 41 i_1 \quad (2)$$

KVL

KCL

$$v_s = i_1 (R_1 + 41 R_2) + v_1$$

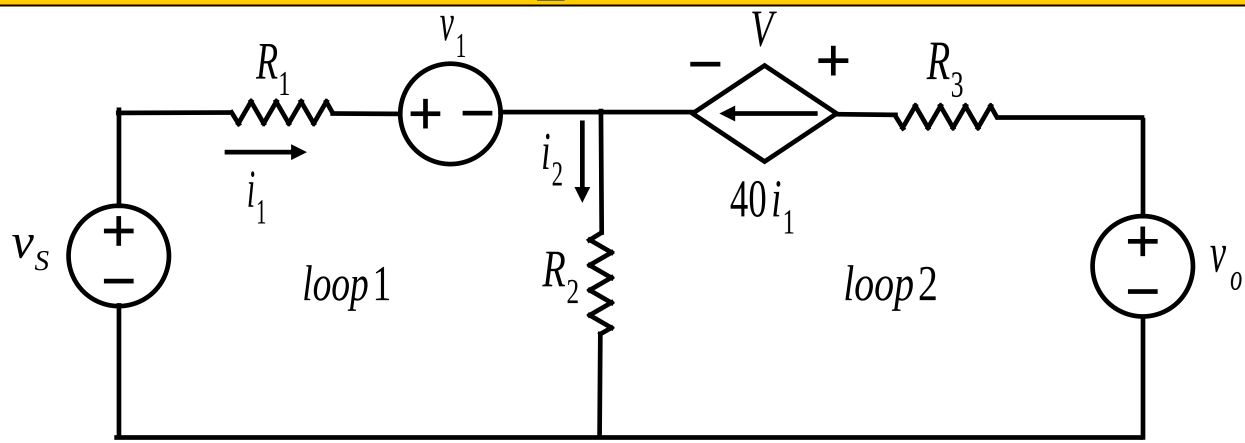
Substitute (2) into (1)

$$i_1 = \frac{v_s - v_1}{R_1 + 41 R_2} = \frac{3 - 0.5}{29.5K + 41(500)} = \frac{2.5}{50K}$$

Solve for i_1

$$i_1 = 50(\mu A) \quad \text{and} \quad i_2 = 41 i_1 = 2.05(mA)$$

Example: Find V (Contd.)



Given

$v_s = 3(V)$

$v_1 = 0.5(V)$

$v_0 = 10(V)$

$R_1 = 29.5(k\Omega)$

$R_2 = 500(\Omega)$

$R_3 = 2.4(k\Omega)$

Loop 2

$$v_o = 40 i_1 R_3 + V + i_2 R_2 \qquad (3)$$

KCL (2)

$$v_0 = i_1(40R_3 + 41R_2) + V$$

Substitute $i_2 = 41i_1$ into (3)

$$V = v_o - i_1\left(40 R_3 + 41 R_2\right)$$

Solve for V

$$= 10 - 50 \times 10^{-6}(40[2.4K] + 41[500])$$

$$= 10 - 5.825$$

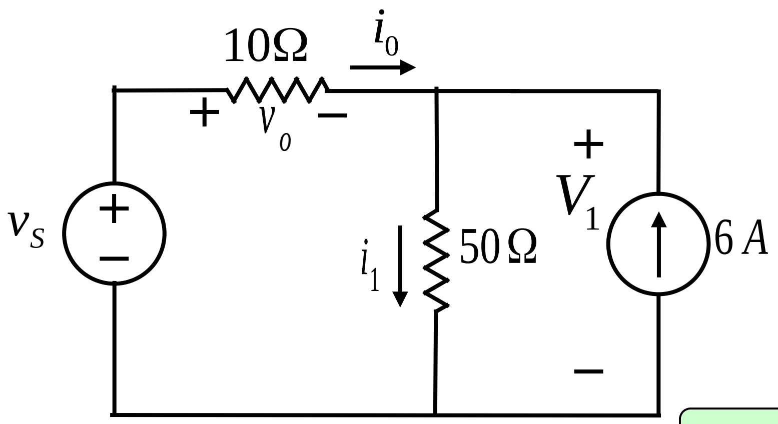
$$V = 4.175(V)$$

Sign Convention

$\boldsymbol{p} > 0$

$$p_{\Diamond} = +V(40 i_1) = 4.175(40)(50 \times 10^{-6}) = +8.35\left(mW\right) \quad \{absorbed\}$$

Example: Find Power of v_s



$v_S = 120(V)$ } Given

KCL $\{ \; i_1 = i_0 + 6$

KVL $\{ \; v_s = 10 i_0 + 50 i_1$

Solve for i_1 and i_0

Result $\quad i_1 = 3(A)$ $\quad i_0 = -3(A)$

$$p_{120} = -120 i_0 = -120(-3(A))$$

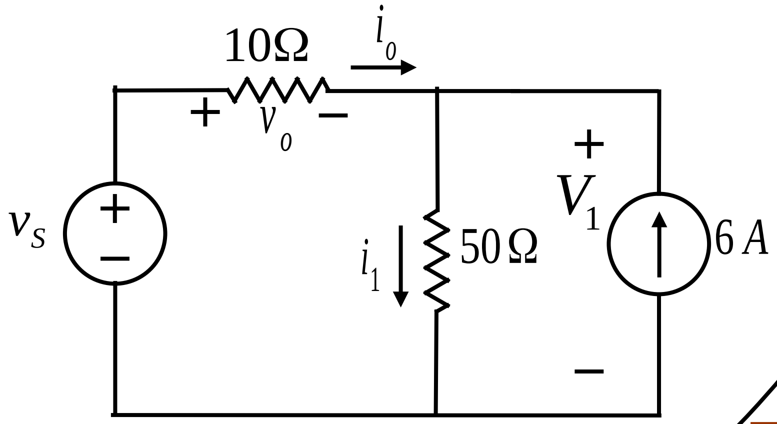
Sign Convention

$$p_{120} = +360(W)$$

$p > 0$

Power is <u>Absorbed</u> by the 120(V) source.

Same Example: Find Power of 6A source



Sign Convention

$$p_6 = -V_1 \cdot 6$$
$$V_1 = +i_1 \cdot 50$$
$$V_1 = +150$$
$$p_6 = -900(W)$$

Ohm's Law

From the previous slide

$$i_1 = 3(A)$$
$$i_0 = -3(A)$$
$$p_{120} = +360(W)$$

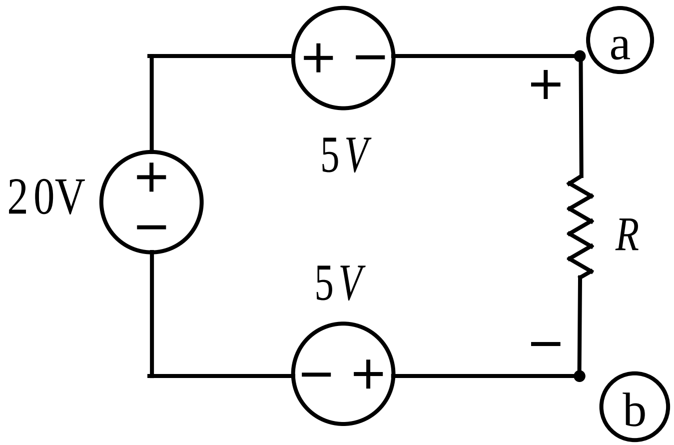
$p < 0$

Power is delivered by 6A source

Conclusion:

In original circuit, the 6A source "overwhelms" the 120V source, and actually supplies power to it!

Note on Voltages and Currents

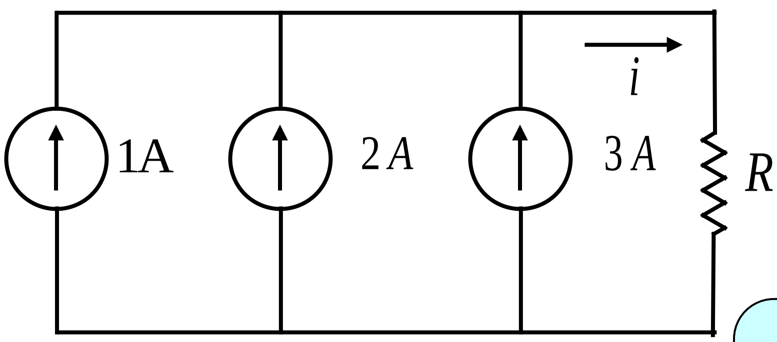


Voltage Across R

$$V_{ab} = -5(V) + 20(V) - 5(V) = 10(V)$$

Voltages in series add

"k" Voltages in parallel <u>must</u> be the same



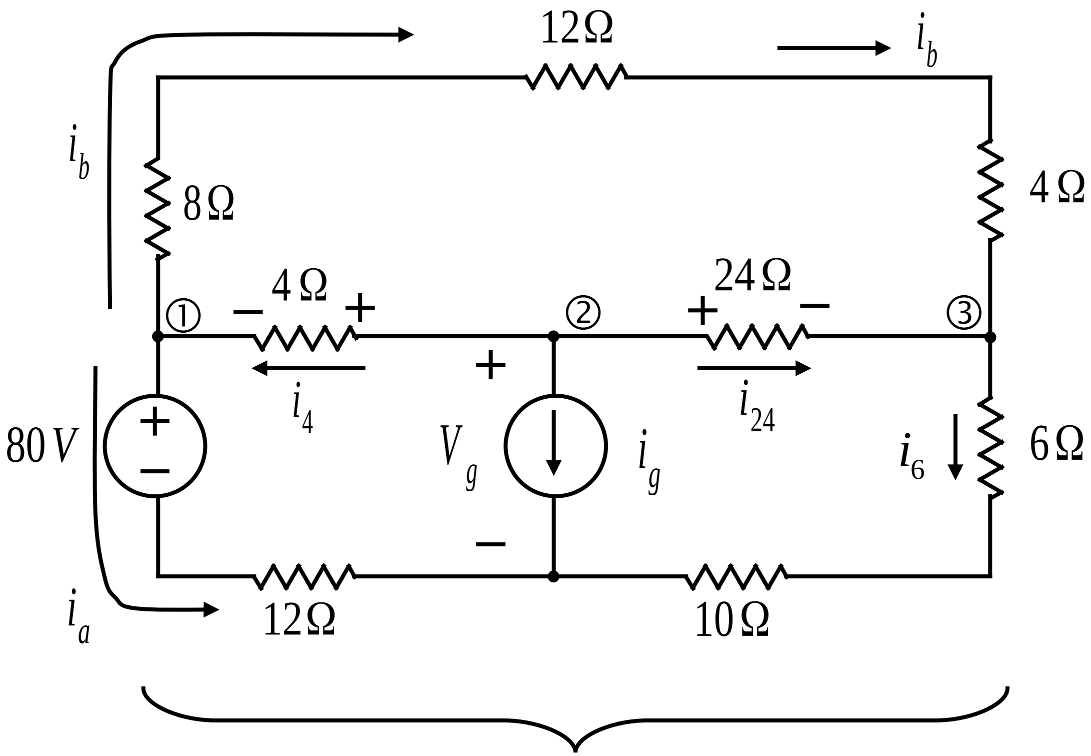
Current Through R

$$i = 1(A) + 2(A) + 3(A) = 6(A)$$

Currents in parallel add

"k" Currents in series <u>must</u> be the same

Example: Find i_g and V_g



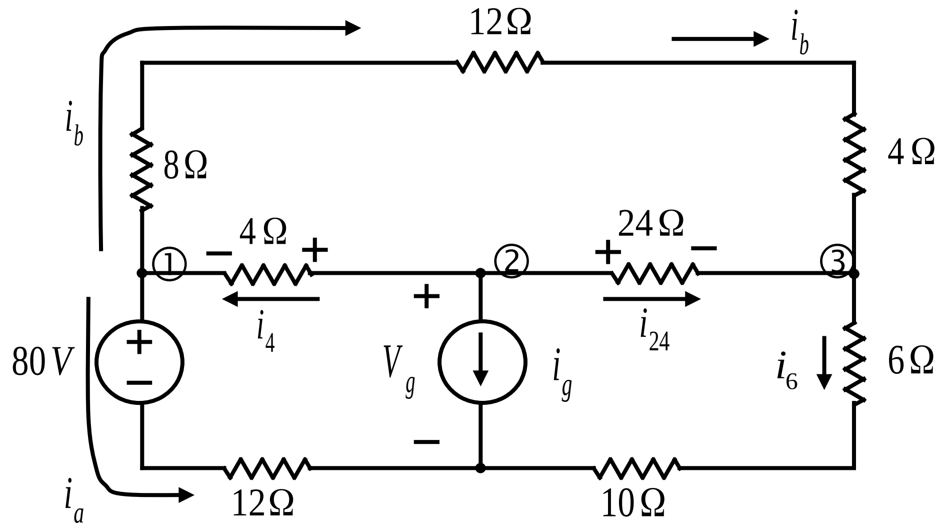
Given

$$i_a = 4(A)$$

$$i_b = 2(A)$$

Currents have been defined in the circuit

Example: Find i_g (Contd.)



Given

$i_a = 4(A)$

$i_b = 2(A)$

a) KCL at node 1

$$i_4 = i_a + i_b = 4 + 2 \Rightarrow \quad i_4 = 6(A)$$

b) KVL around upper loop

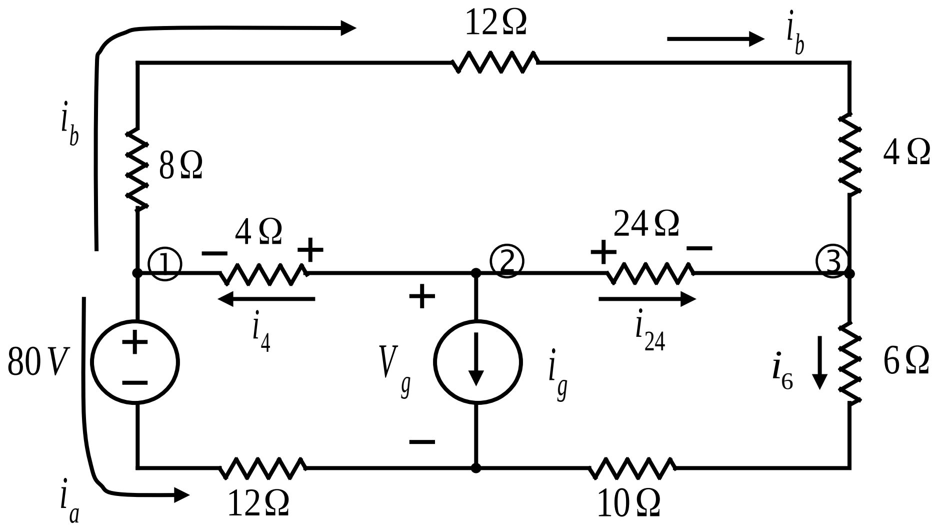
$$i_b(8 + 12 + 4) - 24i_{24} + 4i_4 = 0$$

$$\left.i_{24} = \frac{24i_b + 4i_4}{24} = \underbrace{\frac{24(2) + 4(6)}{24}}_{\text{Substitute for } i_b \text{ and } i_4} = \frac{72}{24}\right\} \text{Solve for } i_{24}$$

$$i_{24} = 3(A)$$

Example: Find i_g (Contd.)

$i_b = 2(A)$
$i_4 = 6(A)$
$i_{24} = 3(A)$



c) KCL at node 2

$$i_4 + i_{24} + i_g = 0$$

$$\left.i_g = -i_4 - i_{24} = -6 - 3 \right\}$$ **Solve for i_g**

$$i_g = -9(A)$$

i_g "really" flows "up"

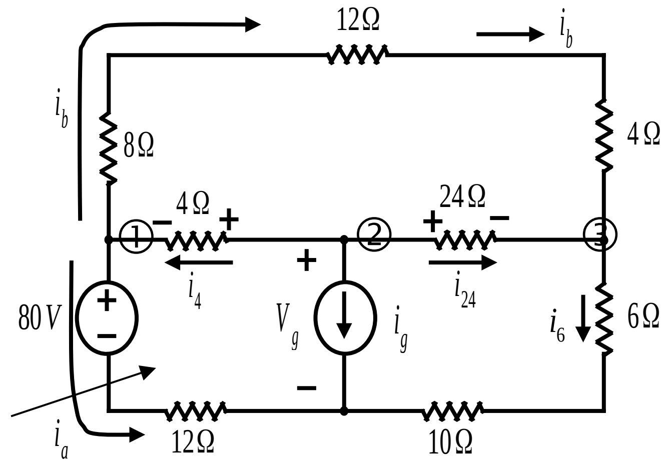
d) KCL at node 3

$$\left.i_b + i_{24} = i_6;\ 2 + 3 = i_6 = 5(A)\right\}$$ **Solve for i_6**

we know all currents now

Example: Find V_g (Contd.)

$$i_a = 4(A)$$
$$i_4 = 6(A)$$



e) KVL

$$V_g = 4i_4 + 80 + 12i_a$$ Bottom LHS

$$= 4(6) + 80 + 12(4)$$ Substitute Currents

$$= 24 + 80 + 48 = 152(V)$$

Sign Convention

f) $p_{80\text{V}} = V \cdot i_a = 80(4) = +320(W)$

$p > 0$

80V source "absorbs" power so do all resistors