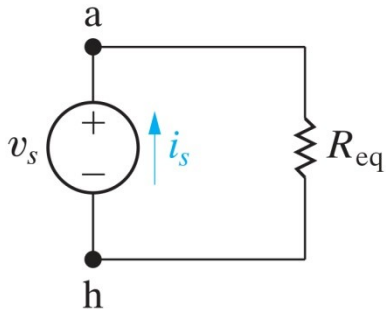
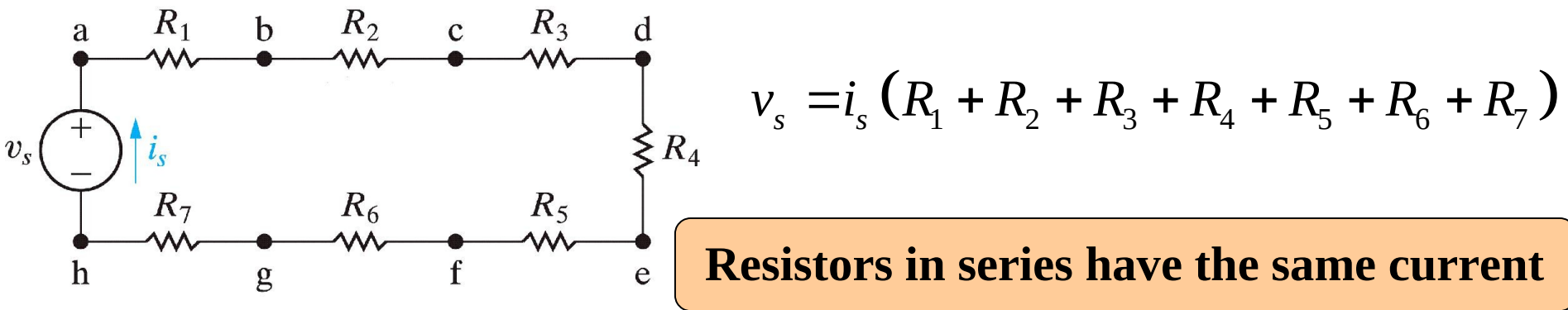
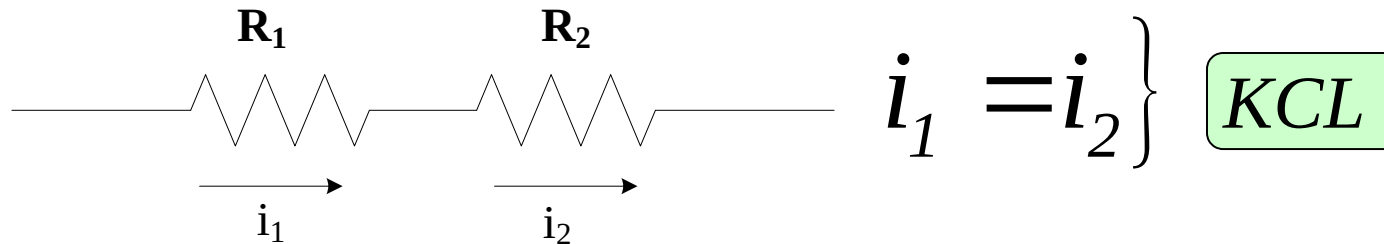


Week2

Resistive Circuits

Series Connection

Two Elements Connected at a Single Node



$$v_s = i_s R_{eq}$$

$$R_{eq} = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7$$

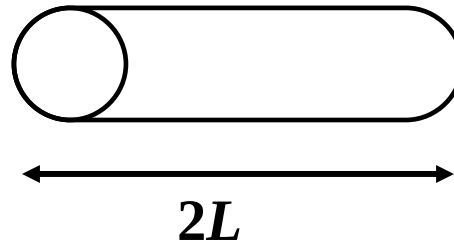
Figure: 03-03
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Resistors in Series

Can be used to simplify circuits



Same as

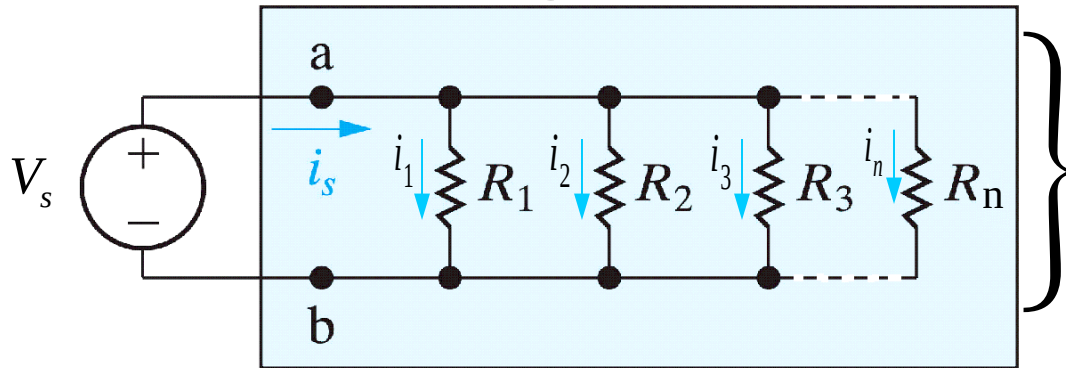


$$R = \rho \cdot \frac{L}{A}$$

Parallel Connections

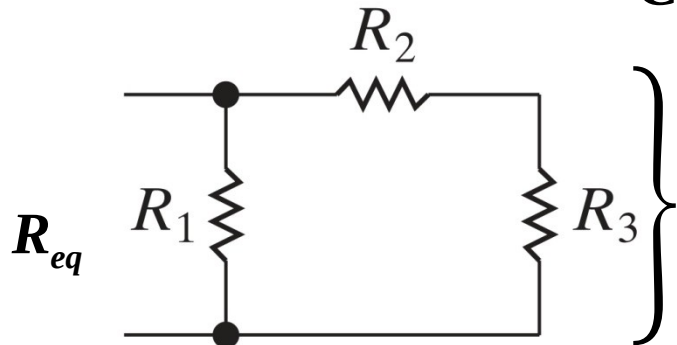
Elements connected at a single node pair

2 nodes in
this circuit



All Resistors
In Parallel

All the resistors have the same voltage, v_s , across them
Currents are different



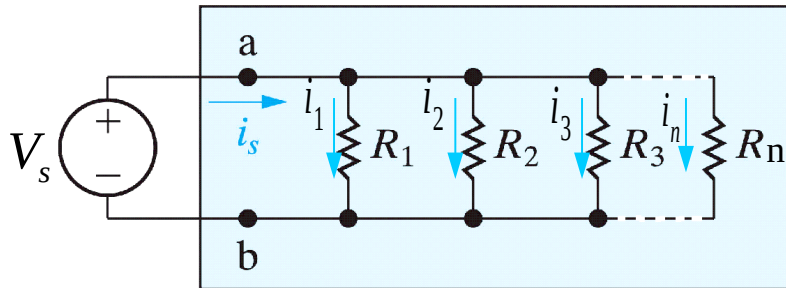
R_1 is not “||” R_3

$$R_1 \parallel (R_2 + R_3) = R_{eq}$$

R_2 and R_3 are in series

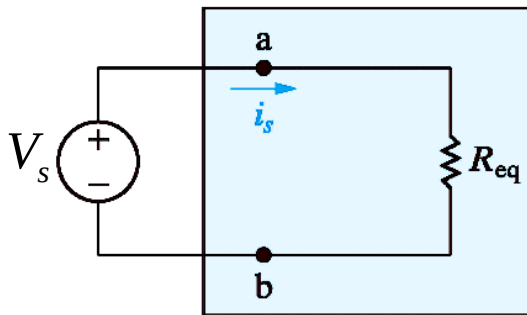
Figure: 03-06
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Equivalent Resistance of Resistors in Parallel



For $n=4$

$$i_s = i_1 + i_2 + i_3 + i_4 \quad \left. \vphantom{i_s} \right\} \text{KCL} \quad (1)$$



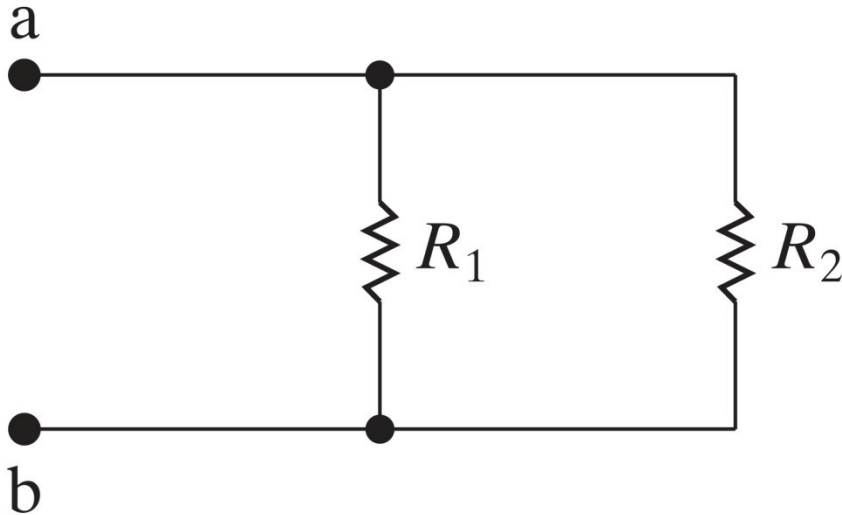
$$V_s = i_1 R_1 = i_2 R_2 = i_3 R_3 = i_4 R_4 \quad (2) \quad \left. \vphantom{V_s} \right\} \text{Ohm's Law}$$

$$i_s = \frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3} + \frac{V_s}{R_4} \quad \left. \vphantom{i_s} \right\} \begin{array}{l} \text{Solve for } i\text{'s in } (2) \\ \text{and then substitute into } (1) \end{array}$$

$$\frac{i_s}{V_s} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{R_{eq}}$$

Solve for i_s/V_s

Special Case: 2 Resistors in Parallel



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$\therefore R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

Figure: 03-08

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$R_1 (\Omega)$	$R_2 (\Omega)$	$R_{eq} (\Omega)$
1000	1000	$500 = 0.5R_1 = 0.5R_2$
1000	500	333.333
10,000	100	99.01 (approx. 100)
10^6	100	99.99 (approx. 100)

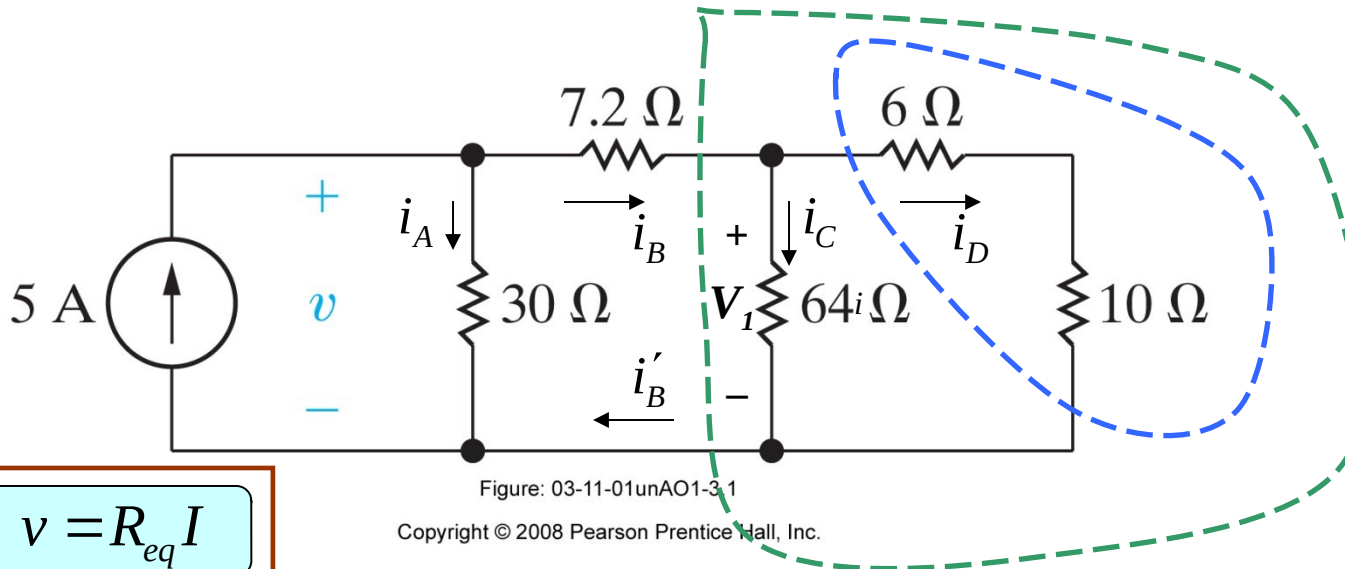
$$R_{eq} < R_1$$

and

$$R_{eq} < R_2$$

} \approx **Smallest**

Drill Exercise: Find Voltage v



Side Note

$$\begin{aligned} i_B &= i_C + i_D \quad \text{KCL} \\ i'_B &= i_C + i_D \quad \text{KCL} \\ i'_B &= i_B \end{aligned}$$

$$v = R_{eq} I$$

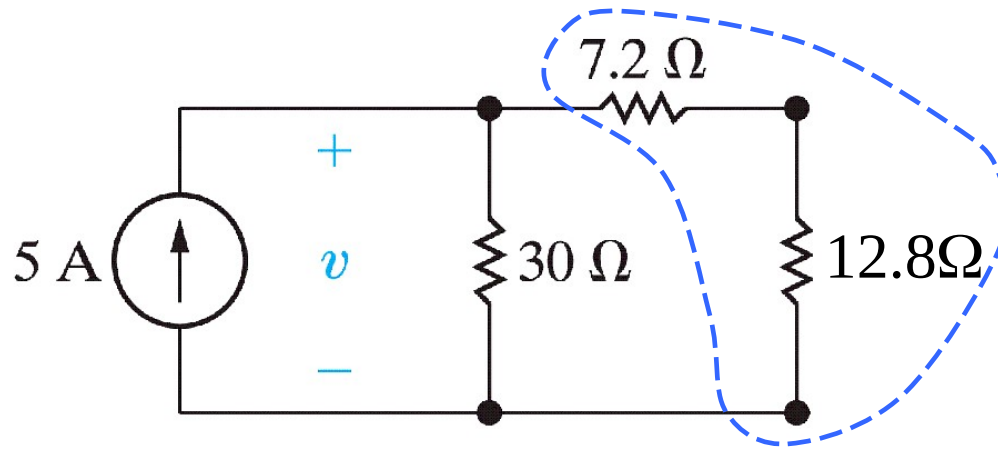
Note: $I = 5(A)$

Find R_{eq}

$$10\Omega + 6\Omega = 16(\Omega) \quad \text{Add series resistors}$$

$$16\Omega \parallel 64\Omega = \frac{16 \times 64}{16 + 64} = 12.8(\Omega) \quad \text{Simplify Parallel Resistors}$$

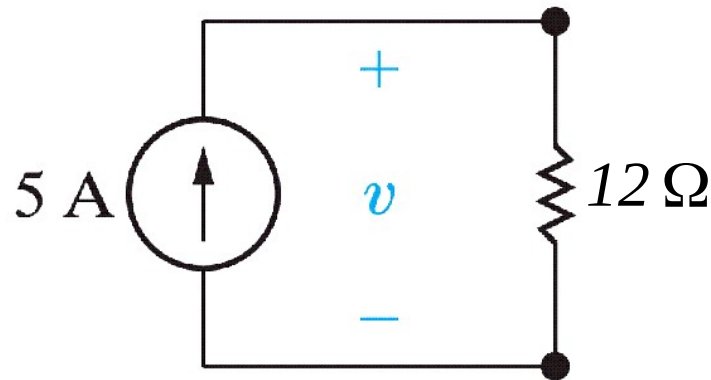
Drill Exercise (Contd.)



$$12.8\Omega + 7.2\Omega = 20(\Omega) \left\{ \begin{array}{l} \text{Add series resistors} \end{array} \right.$$

$$20 \parallel 30 = \frac{20 \times 30}{20 + 30} = 12(\Omega) \left\{ \begin{array}{l} \text{Simplify} \\ \text{Parallel} \\ \text{Resistors} \end{array} \right.$$

Drill Exercise (Contd.)



$$v = IR \longrightarrow a) \quad v = 5A(12\Omega) = 60(V)$$

$$p = Iv \longrightarrow b) \quad p_{del} = (5A)(60V) = 300(W) \quad \left. \vphantom{p = Iv} \right\} \quad p > 0$$

Same Problem: What is the Power delivered to the 10Ω Resistor?

$v = 60(V)$ } Found in the previous slide

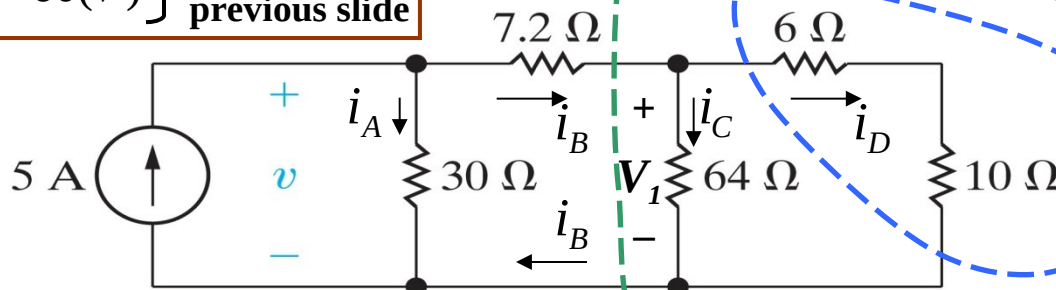


Figure: 03-11-01unAO1-3.1

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$$1 \quad 5(A) = i_A + i_B \quad \text{KCL}$$

$$2 \quad i_A = \frac{v}{30\Omega} = \frac{60}{30} = 2(A) \quad \text{Ohm's Law}$$

$$i_B = 5(A) - i_A = 5 - 2 = 3(A) \quad \text{Substitute 2 into 1}$$

$$v = i_B(7.2\Omega) + V_1 \quad \text{Use KVL to find } V_1$$

$$V_1 = v - i_B(7.2\Omega) = 60 - 3(7.2) = 38.4(V) \quad \text{Solve for } V_1$$

$$i_D = V_1 / (10\Omega + 6\Omega) = 38.4 / 16 = 2.4(A) \quad \text{Use Ohm's Law to find } i_C$$

$$p_{10\Omega} = i_D^2 \cdot 10 = (2.4)^2 10$$

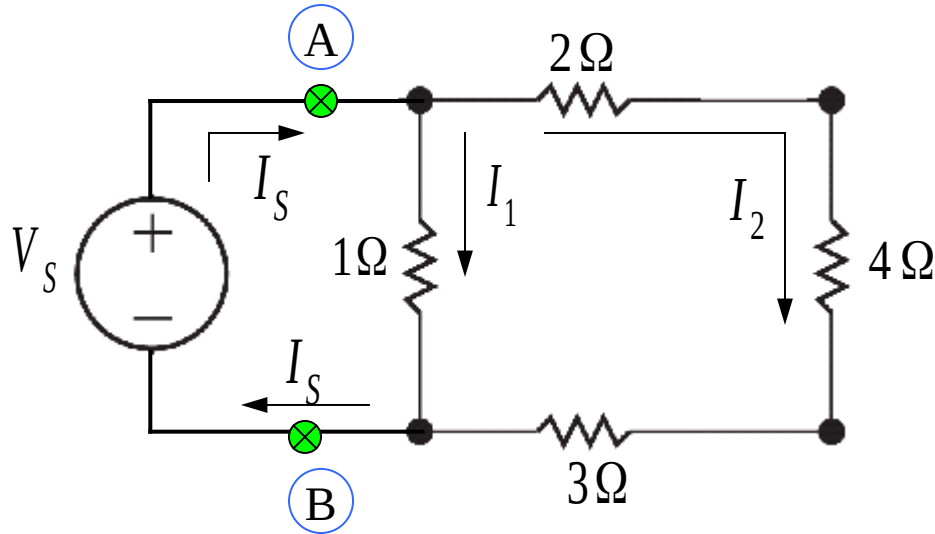
$$p_{10\Omega} = 57.6(W) \quad \text{Find Power}$$

In General

When solving problems at this point,
Ask how I can use:

- Ohms Law
- KVL
- KCL
- R_{eq}
- Knowledge of Current and Voltage
 - Current same in series
 - Voltage same in parallel

Equivalent Resistance Examples



What is R_{eq} at (A) and (B)

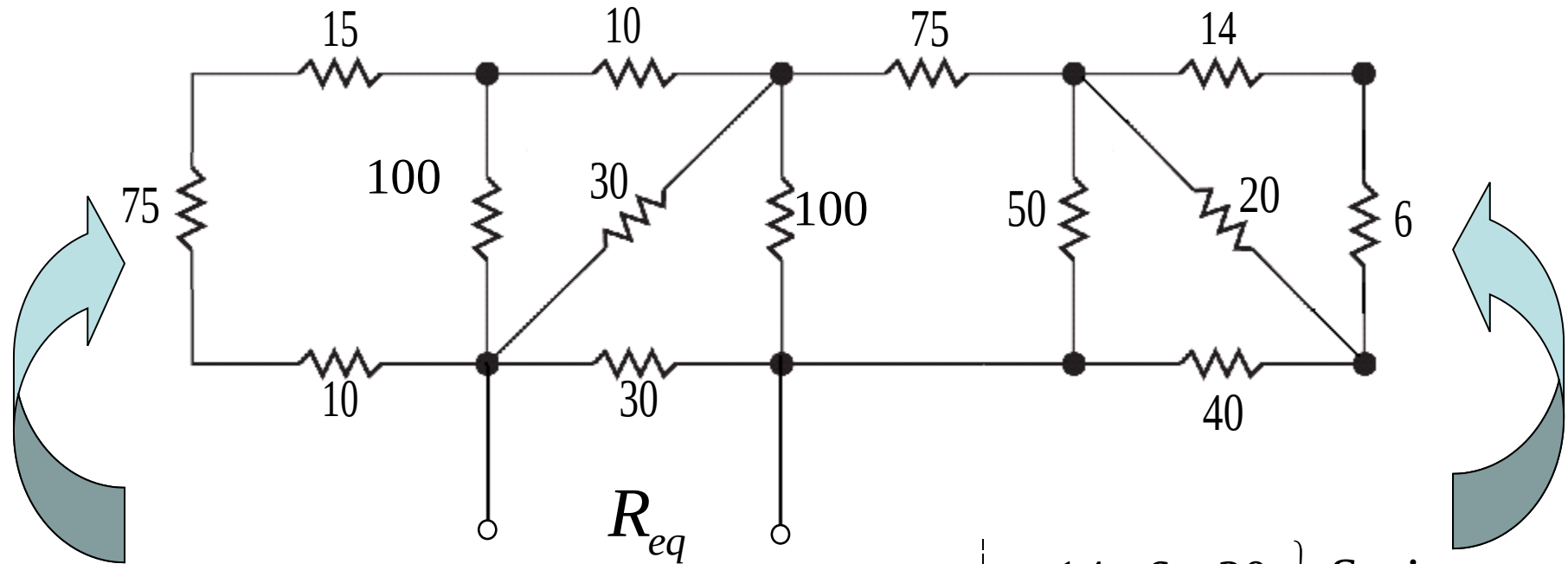
$$R_{eq} = 1 \parallel (2 + 3 + 4) = \frac{1 \cdot (2 + 3 + 4)}{1 + (2 + 3 + 4)} = \frac{9}{10} (\Omega)$$

Example $V_s = I_s R_{AB} \equiv I_s R_{eq}$

Note: Depending on where you take R_{eq} ,

You will have to do a different calculation

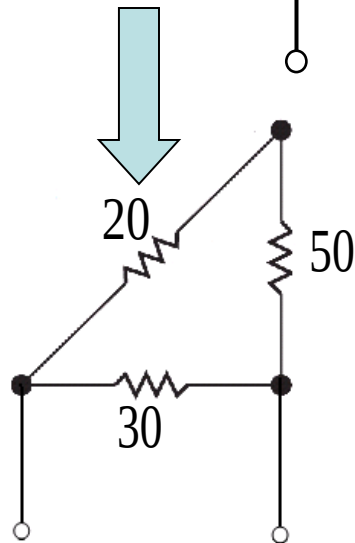
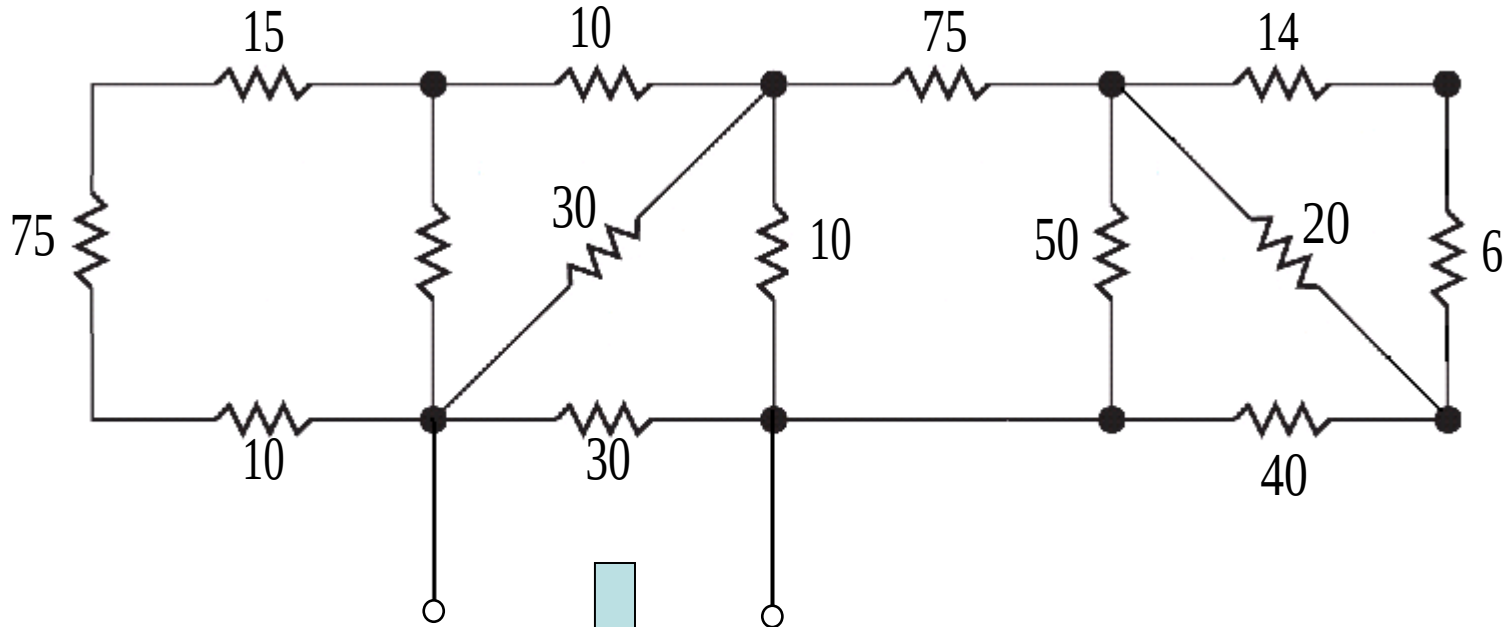
Example: Find R_{eq}



Series $\left\{ \begin{array}{l} 75 + 15 + 10 = 100 \end{array} \right.$
Parallel $\left\{ \begin{array}{l} 100 \parallel 100 = 50 \end{array} \right.$
Series $\left\{ \begin{array}{l} 50 + 10 = 60 \end{array} \right.$
Parallel $\left\{ \begin{array}{l} 60 \parallel 30 = \frac{60(30)}{60+30} = \frac{1800}{90} = 20 \end{array} \right.$

$14 + 6 = 20$ } *Series*
 $20 \parallel 20 = 10$ } *Parallel*
 $10 + 40 = 50$ } *Series*
 $50 \parallel 50 = 25$ } *Parallel*
 $25 + 75 = 100$ } *Series*
 $100 \parallel 100 = 50$ } *Parallel*

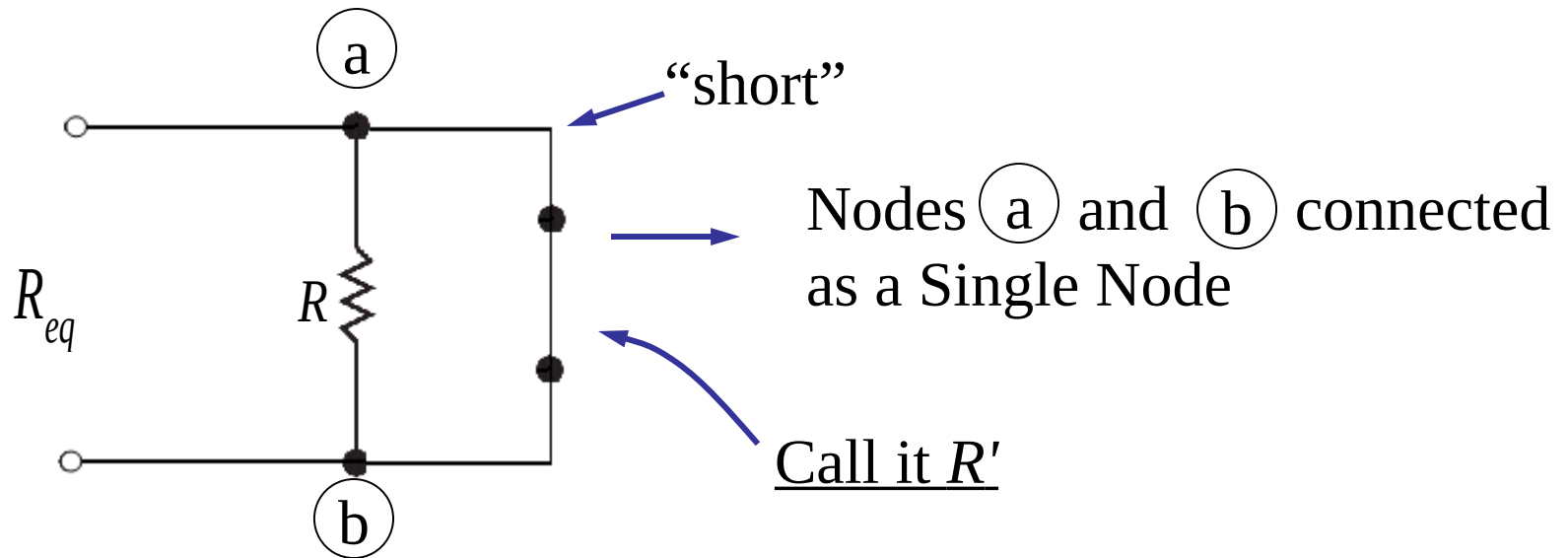
Example: Find R_{eq}



$$R_{eq} = (20 + 50) \parallel 30 = \frac{70(30)}{70 + 30} = \frac{2100}{100}$$

$$R_{eq} = 21(\Omega)$$

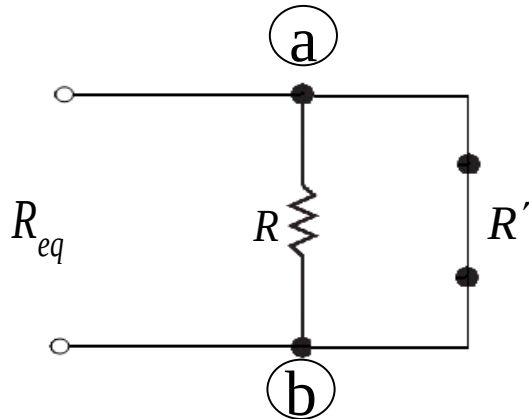
Notes on short circuits



- All current flows through “wire”
- R is “Shorted Out”

- Short wall socket
- Short power supply

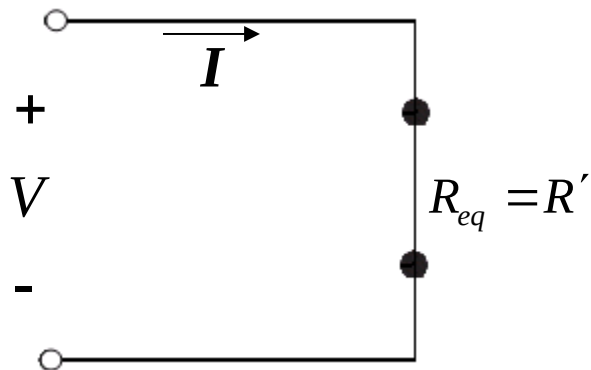
Mathematical Viewpoint



Assume the wire is a perfect conductor

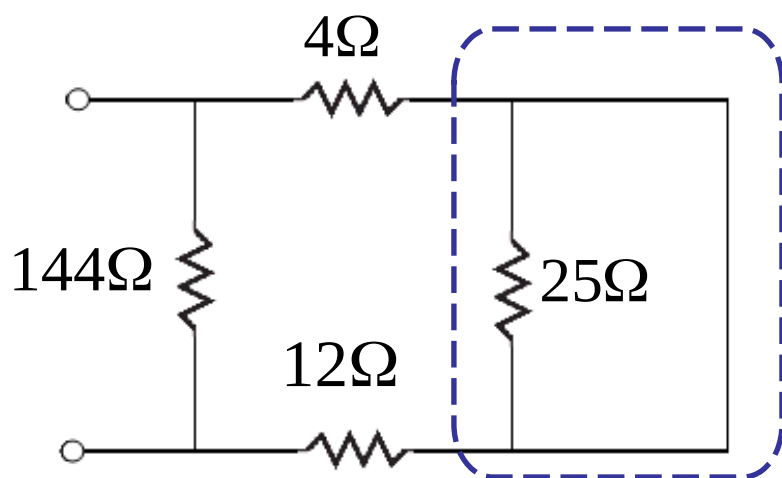
$$R_{eq} = R \parallel R' = \frac{R \cdot R'}{R + R'} \Rightarrow$$

$$R_{eq} = \frac{R(0)}{R + 0} = 0$$

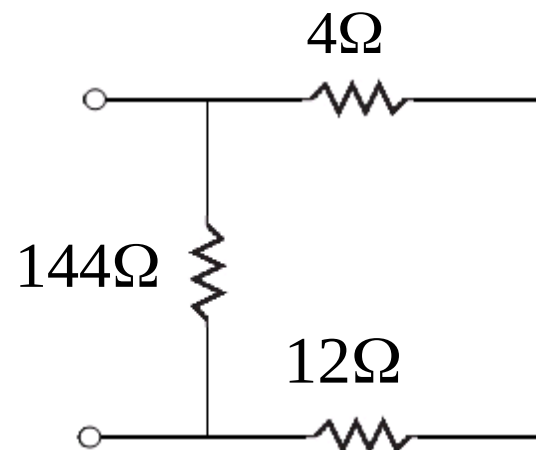


$$I = \frac{V}{R'} \quad \text{as } R' \rightarrow 0 \quad \text{and } I \rightarrow \infty$$

Example: Find R_{eq}



!

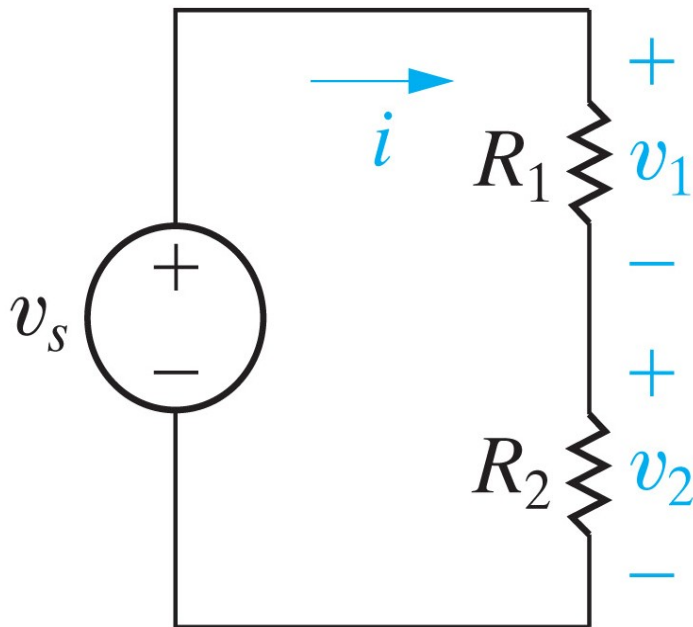


$$R_{eq} = (4 + 12) \parallel 144$$

$$R_{eq} = 14.4(\Omega)$$

Voltage Division

Voltage is “proportionally” distributed among resistors in a circuit.



$$v_s = i(R_1 + R_2) \quad \left. \vphantom{v_s = i(R_1 + R_2)} \right\} \text{Ohm's Law}$$

$$\textcircled{1} \quad i = v_s / (R_1 + R_2) \quad \left. \vphantom{i = v_s / (R_1 + R_2)} \right\} \text{Ohm's Law}$$

$$v_1 = iR_1 = v_s \frac{R_1}{R_1 + R_2}$$

$$v_2 = iR_2 = v_s \frac{R_2}{R_1 + R_2}$$

Use Ohm's
Law
and

$\textcircled{1}$

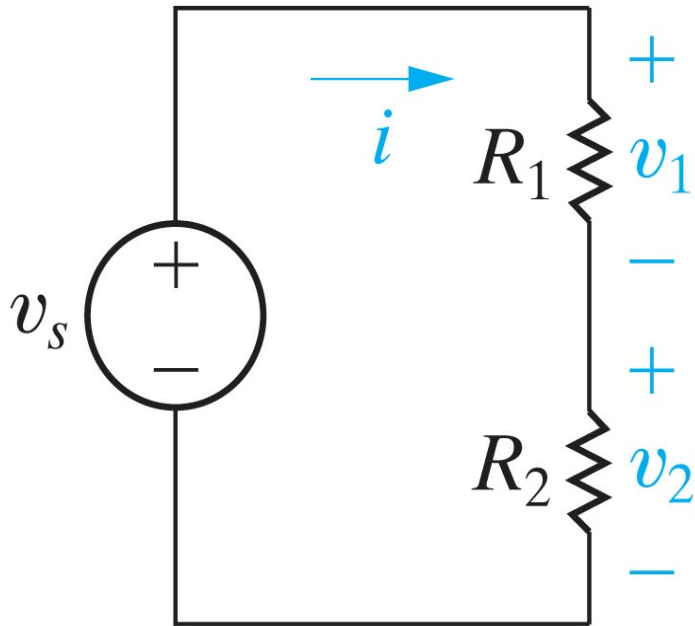
Bigger R 's have larger voltages



Voltage Division



Voltage Division (Contd.)

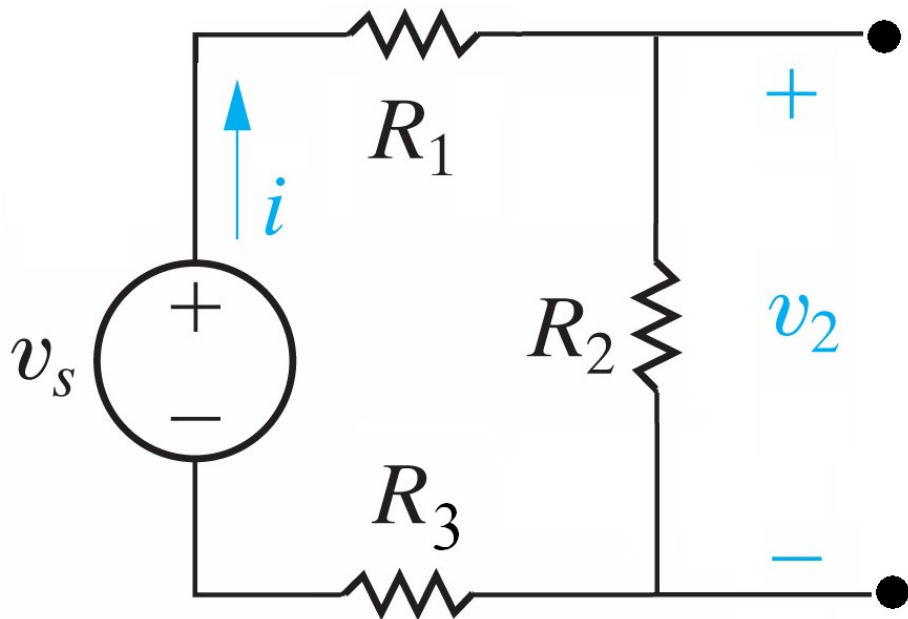


v_1 is a fraction $\left(\frac{R_1}{R_1 + R_2} \equiv \frac{R_1}{R_{Total}} \right)$ of v_s

Special Case

$$R_1 = R_2; v_1 = v_2 = \frac{1}{2} v_s$$

Example: Use Voltage Division to Find v_2



$$v_s = 10V \quad \text{Given}$$

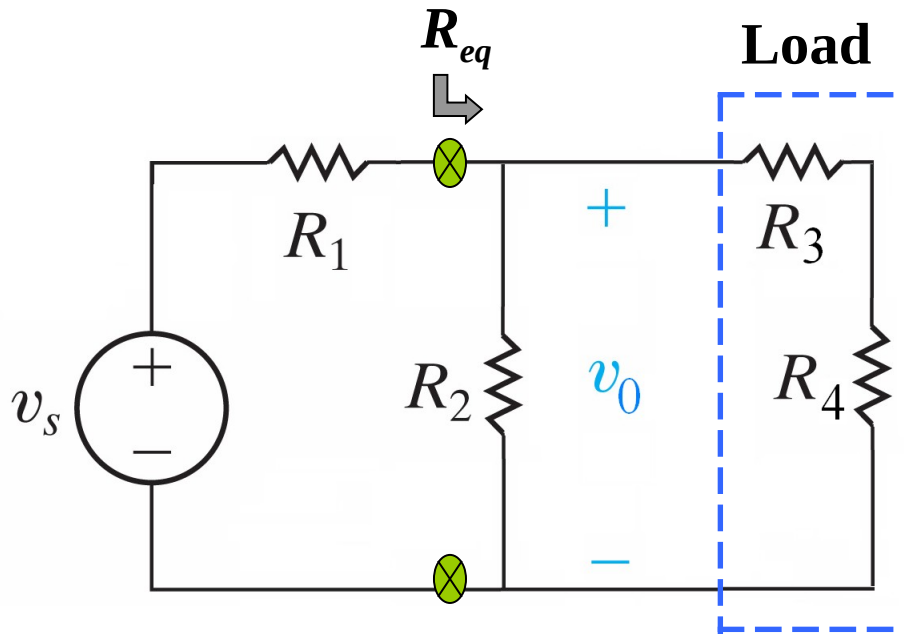
$$R_1 = 3\Omega; R_2 = 4\Omega; R_3 = 3\Omega$$

$$v_2 = v_s \left(\frac{R_2}{R_1 + R_2 + R_3} \right) \quad \left\{ \begin{array}{l} \text{Voltage} \\ \text{across } R_2 \end{array} \right\}$$
$$= 10 \left(\frac{4}{3 + 4 + 3} \right) = 10 \left(\frac{4}{10} \right) = 4(V)$$

$$v_1 = v_s \left(\frac{R_1}{R_1 + R_2 + R_3} \right) = 10 \left(\frac{3}{10} \right) = 3(V) \quad \left\{ \begin{array}{l} \text{Voltage across } R_1 \end{array} \right\}$$

$$v_3 = v_s \left(\frac{R_3}{R_1 + R_2 + R_3} \right) = 10 \left(\frac{3}{10} \right) = 3(V) \quad \left\{ \begin{array}{l} \text{Voltage across } R_3 \end{array} \right\}$$

Example: Find v_o



$$\left. \begin{array}{l} v_s = 100V \\ R_1 = 5K\Omega; R_2 = 50K\Omega \\ R_3 = 10K\Omega; R_4 = 15K\Omega \end{array} \right\} \text{Given}$$

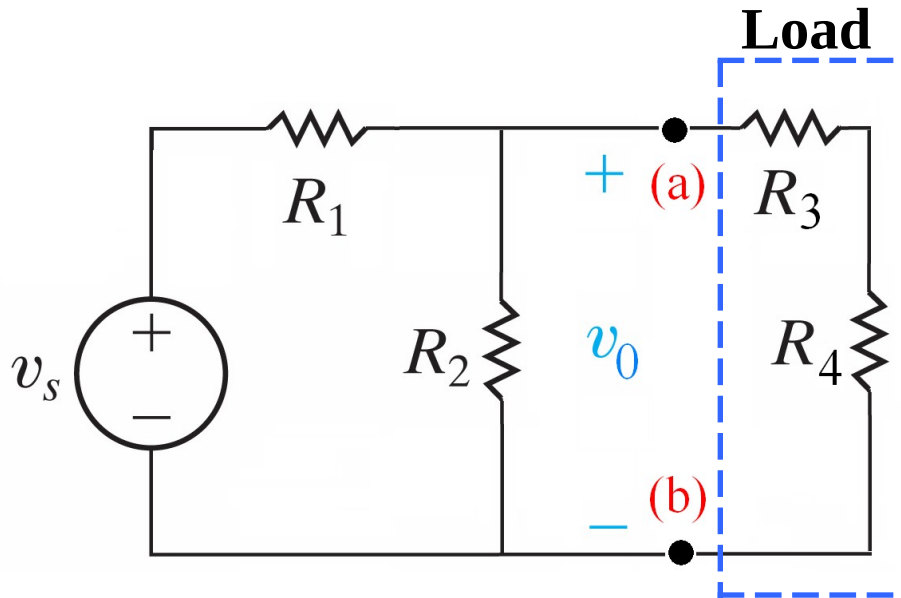
$$v_o = v_s \frac{R_{eq}}{R_1 + R_{eq}} \left. \right\} \text{Voltage Division}$$

Find R_{eq}

$$\begin{aligned} R_{eq} &= R_2 \parallel (R_3 + R_4) = 50K \parallel (10K + 15K) \\ &= \frac{50K(25K)}{50K + 25K} = 16.67(K\Omega) \end{aligned}$$

$$\begin{aligned} v_o &= 100[16.67 / (5 + 16.67)] \\ v_o &= 100(0.769) = 76.9(V) \end{aligned}$$

A Side Note



Load: Anything that draws power from a circuit.

Example: $(R_3 + R_4)$ is the “Load”

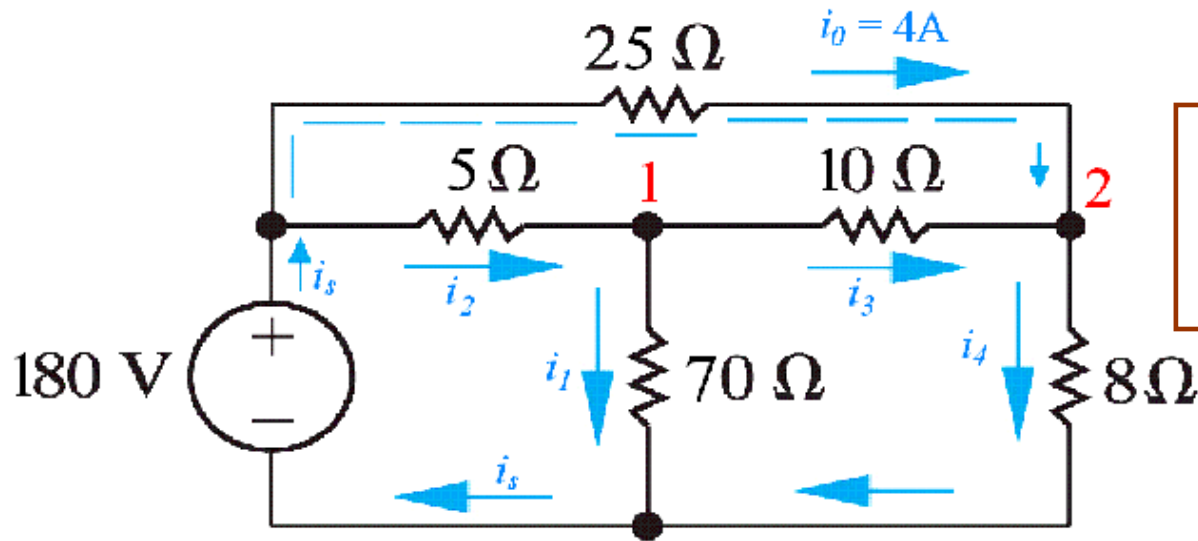
No-Load means that we disconnect the circuit at (a) and (b)

$$v_{0 \text{ (no load)}} = v_s \frac{R_2}{R_1 + R_2} = 100 \left(\frac{50K}{5K + 50K} \right) = 90.91(V)$$

**Disconnect
the load**

“No-Load” value of v_0 is without $(R_3 + R_4)$

Example: Find i_4 ; Given $i_0 = 4(A)$



- Current directions can be arbitrary
- However, these are logical

Solution: Two inner loops have too many unknowns – Try outer loop

$$180 = 25i_0 + 8i_4 \quad \left. \vphantom{180 = 25i_0 + 8i_4} \right\} \text{KVL}$$

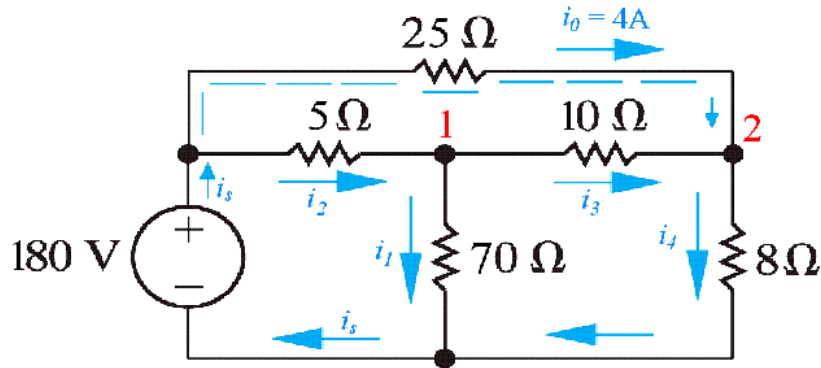
$$i_4 = \frac{180 - 25i_0}{8} = \frac{180 - 25(4)}{8} \quad \left. \vphantom{i_4 = \frac{180 - 25i_0}{8}} \right\} \text{Solve for } i_4$$

$$i_4 = 10 \text{ (A)}$$

(Contd..)

Same Example: Find i_3 and i_1

$i_4 = 10(\text{A})$ } From the previous slide



KCL @ node (2)

$$i_0 + i_3 = i_4$$

KCL

$$i_3 = i_4 - i_0 = 10 - 4$$

Solve for i_3

$$i_3 = 6 (\text{A})$$

KVL for “Bottom-Right” loop has only 1 unknown: **Clockwise from node (1)**

$$10i_3 + 8i_4 - 70i_1 = 0$$

KVL

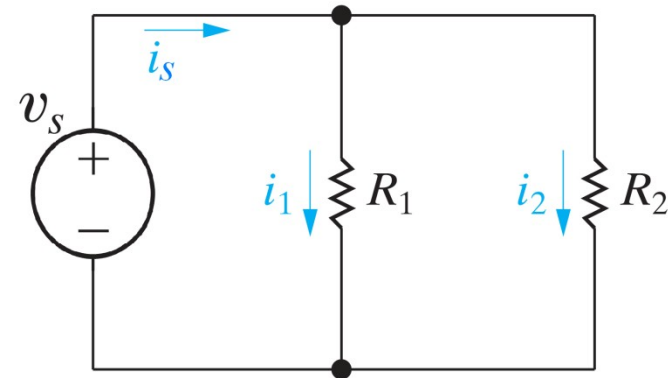
$$i_1 = \frac{10i_3 + 8i_4}{70} = \frac{10(6) + 8(10)}{70}$$

Solve for i_1

$$i_1 = 2 (\text{A})$$

Current Division

Current is “proportionally” distributed through resistors in a circuit



Use ① and ②
to eliminate v_s

$$i_1 = \frac{v_s}{R_1} = i_s \frac{R_2}{R_1 + R_2}$$

$$i_2 = \frac{v_s}{R_2} = i_s \frac{R_1}{R_1 + R_2}$$

★ **Current Division** ★

Ohm's Law:

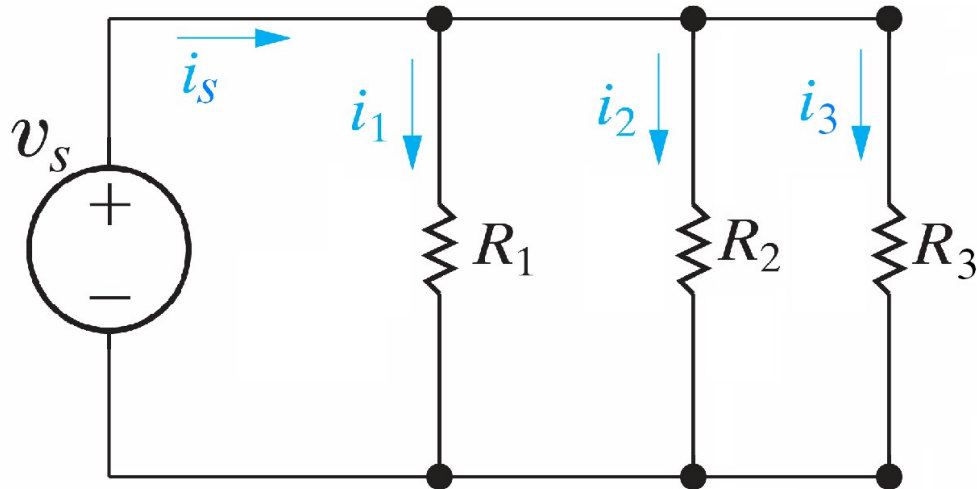
$$① \quad v_s = i_1 R_1 = i_2 R_2 = i_s \overbrace{(R_1 || R_2)}^{R_{eq}}$$

$$② \quad v_s = i_s \overbrace{\frac{R_1 R_2}{R_1 + R_2}}^{R_{eq}}$$

Book uses $\uparrow i_s$
Works either way

- “Fraction” is **OPPOSITE** to that of Voltage Division.
- Current “seeks” the path of least Resistance; i.e., more current flows through the smaller R .
- If $R_1 = R_2$; then, $i_1 = i_2 = 0.5 i_s$
- As R_1 becomes $> R_2$; more current flows through R_2
- i_1 is a fraction, $\left(\frac{R_2}{R_1 + R_2} \equiv \frac{R_2}{R_{Total}} \right)$, of i_s .

Example



Find i_2

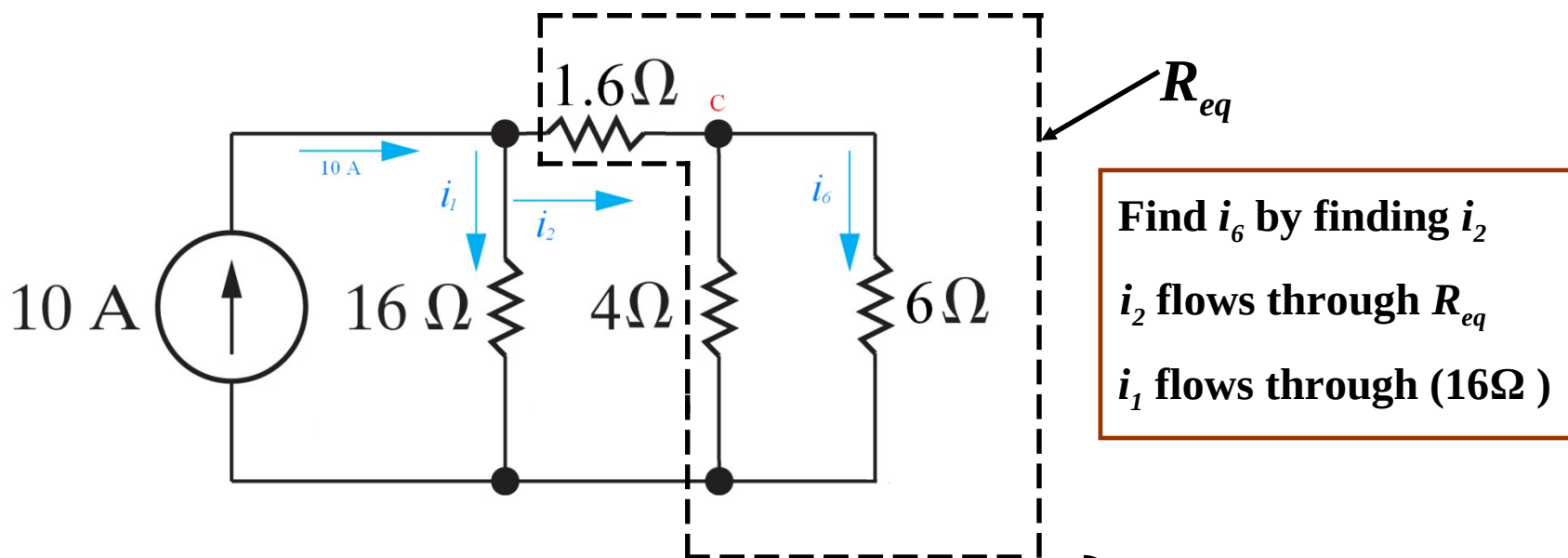
Using Current Division:

$$i_2 = i_s \cdot \left(\frac{R_1 \parallel R_3}{R_1 \parallel R_3 + R_2} \right)$$

⇒ i_2 flows through R_2

⇒ $i' = (i_1 + i_3)$ flows through $(R_1 \parallel R_3)$

Example: Find i_6 using current division



Find i_6 by finding i_2
 i_2 flows through R_{eq}
 i_1 flows through (16 Ω)

$$i_2 = 10 \left(\frac{16\Omega}{16\Omega + R_{eq}} \right)$$

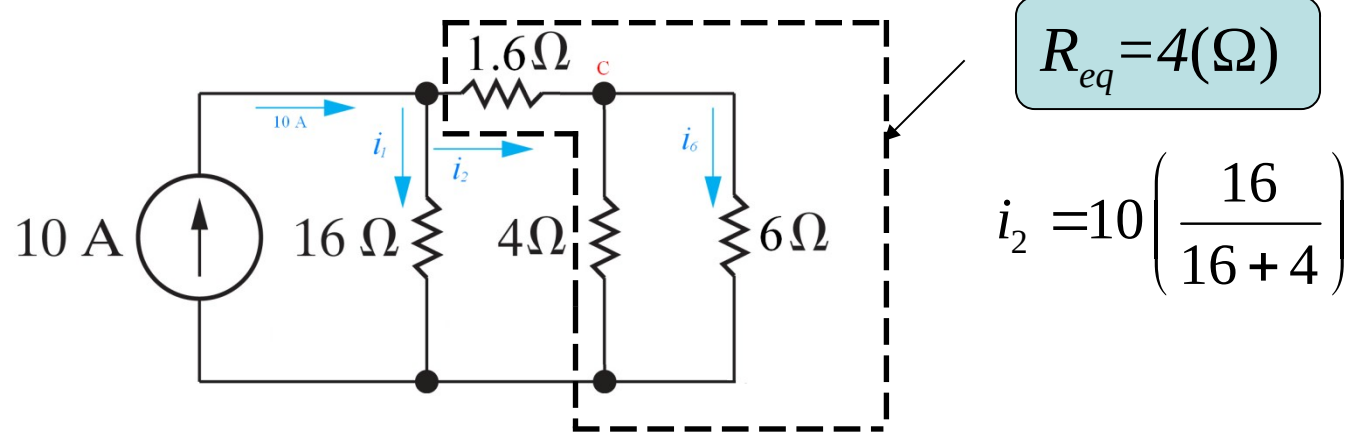
Find i_2 with
current
division

$$R_{eq} = (6 \parallel 4) + 1.6 = \frac{24}{10} + 1.6 = 2.4 + 1.6$$

$$R_{eq} = 4 (\Omega)$$

(Contd..)

Example: Find i_6 using current division (Contd.)




 $i_2 = 8 \text{ (A)}$
 → Flows through 1.6Ω resistor and into node C

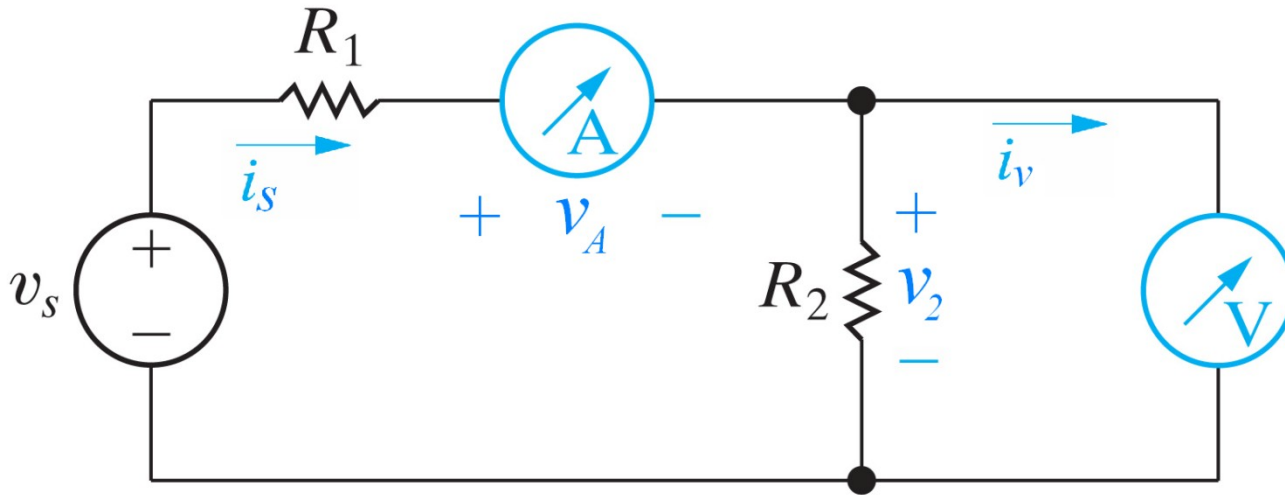
i_2 Flows into node C

$$i_6 = i_2 \left(\frac{4}{4 + 6} \right) = 8 \left(\frac{4}{10} \right)$$

$$i_6 = 3.2 \text{ (A)}$$

Use Current Division
to find i_6

METERS To measure Current and Voltage



**Meters Affect
the Circuit**

(A) in series

Want ≈ 0 Internal Resistance; otherwise, i_s is reduced.

➡ Loads the circuit.

(V) in parallel

Want $\approx \infty$ Internal Resistance; otherwise, v_2 is reduced.

➡ Loads the circuit.

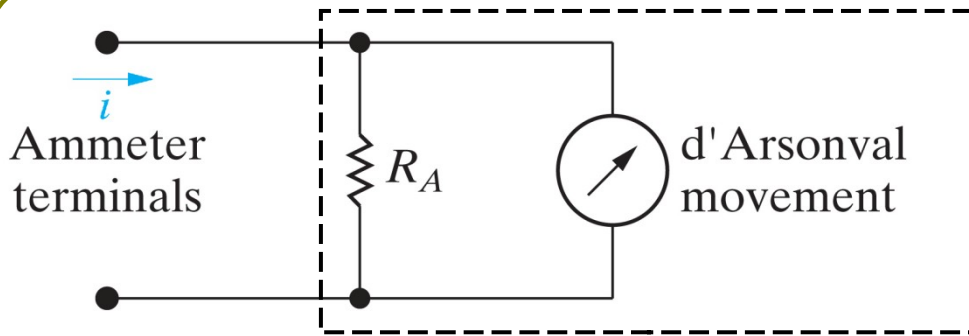
d'Arsonval Meter

Movable coil in the field of a Permanent Magnet
 i in coil \Rightarrow Torque which moves a dial

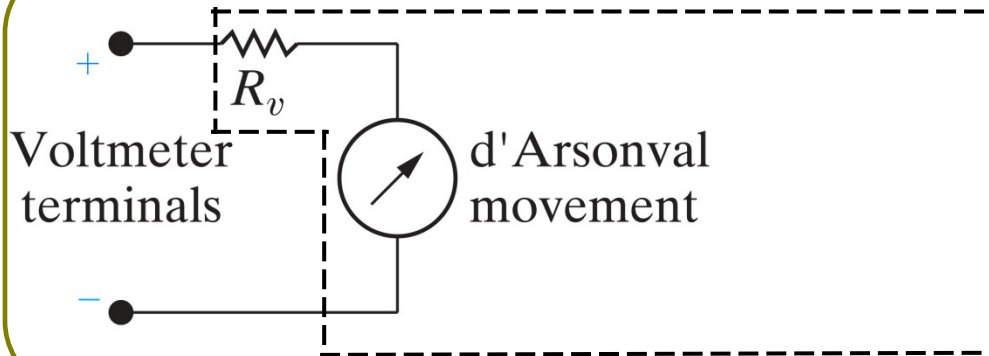
Rating of 50 mV and 1 mA.

1 mA in coil \Rightarrow 50 mV across coil \Rightarrow Full deflection

Limits of d'Arsonval meters can measure



R_A Limits Current Through Movement.
<Current Divider>



R_v Limits Voltage Across Movement.
<Voltage Divider>

Resistor Determines Full-Scale Reading

Example: Find the correct value for R_A

Movement Specification

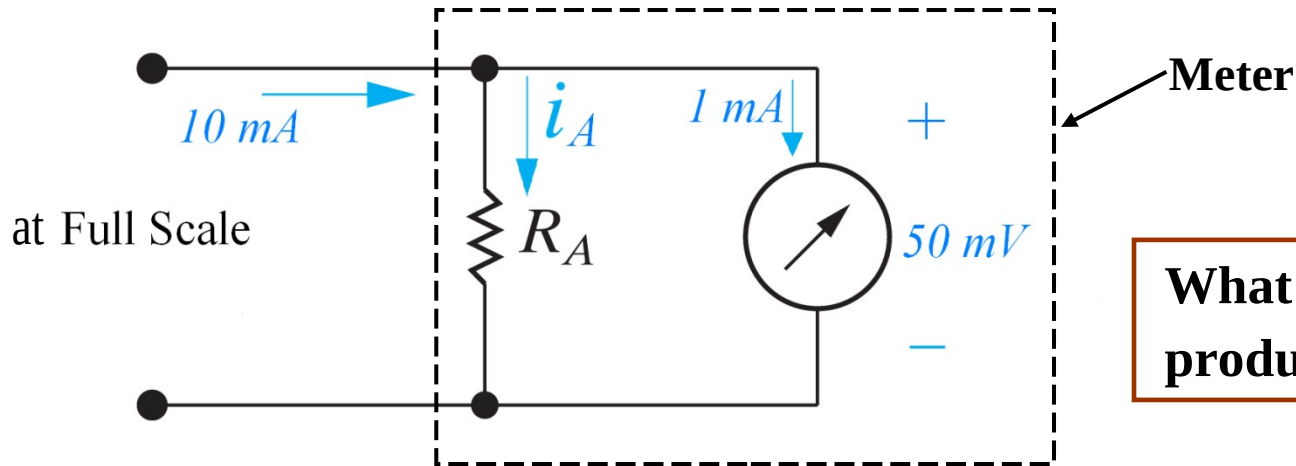
Ammeter Specification

a) Rating

50 (mV) and 1 (mA).

Full-Scale Reading: 10 (mA).

Can measure up to 10 mA



What value of R_A will produce this result?

$$10 \text{ (mA)} = i_A + 1 \text{ (mA)}$$

$$i_A = 10 \text{ (mA)} - 1 \text{ (mA)} = 9 \text{ (mA)}$$

KCL @ Full Scale

$$V_{R_A} = 50 \text{ (mV)} = i_A R_A$$

Ohm's Law

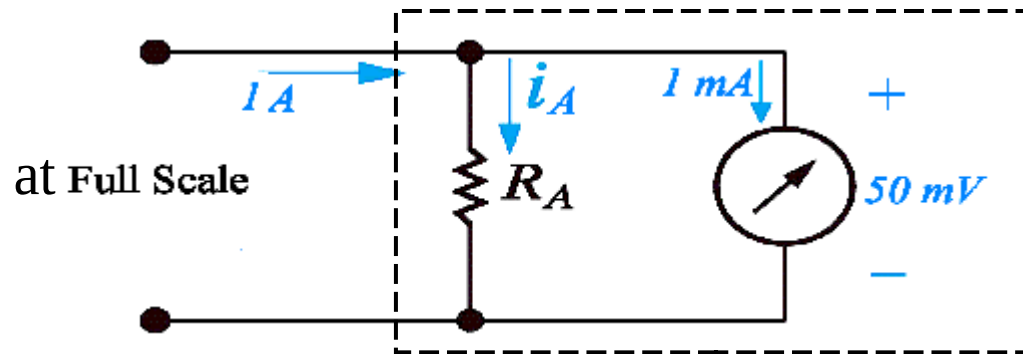
$$R_A = 50 \text{ (mV)} / 9 \text{ (mA)}$$

\Rightarrow

$$R_A = 5.556 \text{ } (\Omega)$$

(Contd..)

Example: Find the correct value for R_A



b) Full-Scale Reading 1A for Ammeter

$$i_A = 1(A) - 1(mA) = 1 - 0.001 = 0.999(A) \quad \text{KCL}$$

$$R_A = V_{R_A} / i_A = 50 (mV) / 999 (mA) \quad \text{Ohm's Law}$$

$$R_A = 50.05 (m\Omega)$$

(Contd..)

Example: Find the Equivalent Resistance of the meter

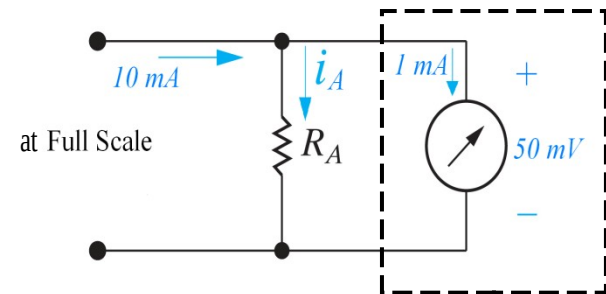
c) R_m is Equivalent Resistance at Ammeter Terminals <10 (mA) Meter>

Effective resistance of movement $\equiv R_{mov} = 50 \text{ (mV)} / 1 \text{ (mA)} = 50 \text{ (}\Omega\text{)}$

$$R_m = R_A \parallel R_{mov} = \frac{R_A R_{mov}}{R_A + R_{mov}} = \frac{5.556(50)}{5.556 + 50} \Rightarrow$$

$$\begin{aligned} R_A &= 5.56 \text{ (}\Omega\text{)} \\ R_{mov} &= 50 \text{ (}\Omega\text{)} \end{aligned}$$

$$R_m = 5 \text{ (}\Omega\text{)}$$



d) 1A Meter

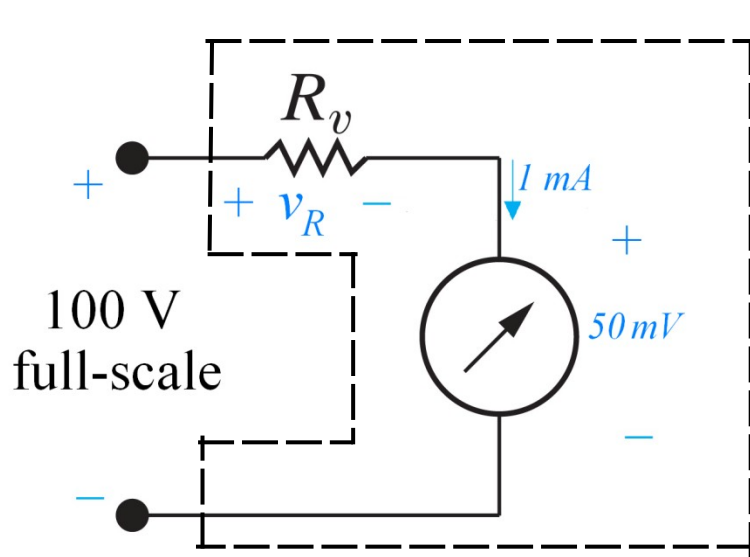
$$R_A = 50.05 \text{ (m}\Omega\text{)}$$

$$R_{mov} = 50 \text{ (}\Omega\text{)}$$

$$R_m = \frac{50.05 \times 10^{-3} (50)}{50.05 \times 10^{-3} + 50}$$

$$R_m = 0.05 \text{ (}\Omega\text{)}$$

Voltmeter Example: d'Arsonval Movement: 50 (mV) @ 1(mA)
Find R_v for Full-scale Reading of 100(V)



Full Scale

$$\left. \begin{aligned} V_R &= 100(V) - 50(mV) = 99.95(V) \\ \Rightarrow R_v &= V_R / 1(mA) = 99.95 / 10^{-3} \end{aligned} \right\} \begin{array}{l} \text{KVL} \\ \text{Ohm's Law} \end{array}$$

$$R_v = 99,950 (\Omega)$$

Series

Voltmeter Resistance:

$$\begin{aligned} R_m &= R_v + R_{mov} \\ &= 99,950 + 50(mV) / 1(mA) = 99,950 + 50 \end{aligned}$$

$$R_m = 100 (K\Omega)$$

OR

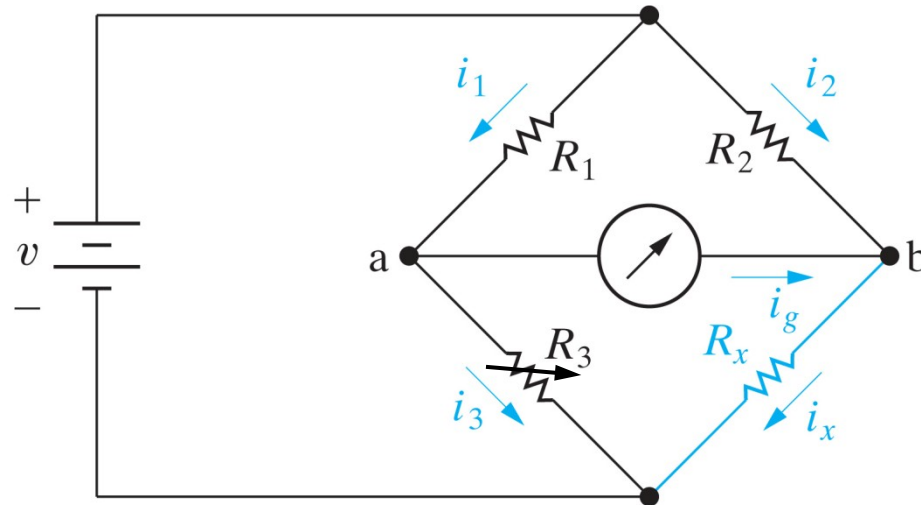
$$R_m = 100(V) / 1(mA) = 100 (K\Omega)$$

Compare to Ammeter:

$$R_m = 5(\Omega), 0.05(\Omega)$$

WHEATSTONE BRIDGE

Used to Measure Resistance



 \equiv detector; d'Arsonval movement

$R_x \equiv$ Unknown Resistance

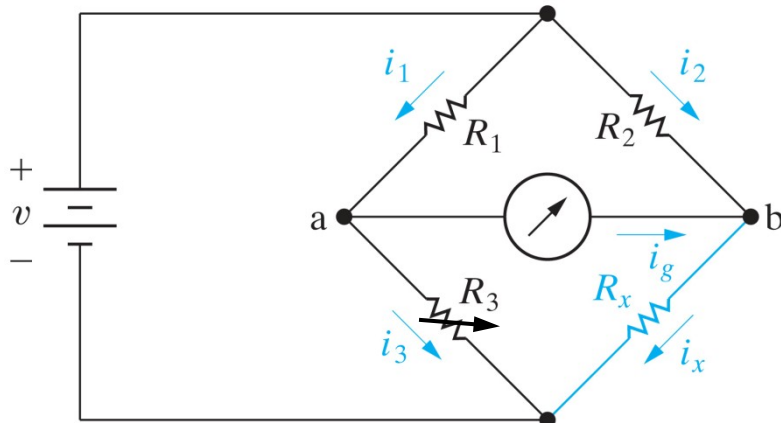
To find R_x :

Adjust R_3 until $i_g = 0$

\longrightarrow Bridge is “Balanced”

When $i_g = 0$, a and b are “effectively” the same node, or $V_{a-b} = 0$

WHEATSTONE BRIDGE: (Contd.) Find R_x when $i_g = 0$



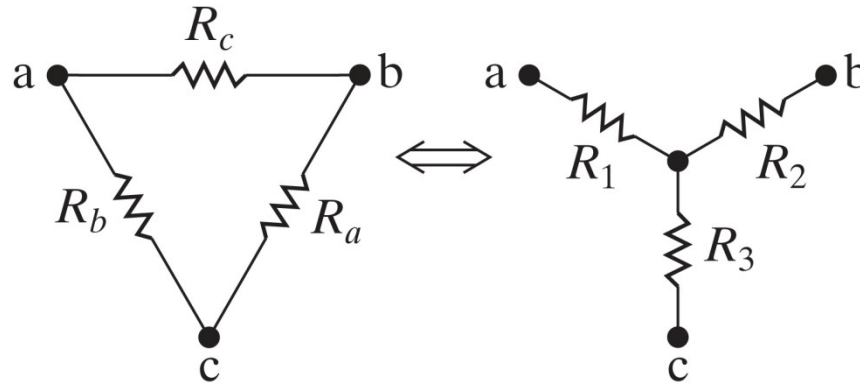
$$(1) \quad i_1 = i_3; \quad i_2 = i_x \quad \left. \vphantom{i_1} \right\} i_g = 0$$

$$(2) \quad i_3 R_3 = i_x R_x; \quad i_1 R_1 = i_2 R_2 \quad \left. \vphantom{i_3} \right\} V_{ab} = 0$$

$$(3) \quad \frac{i_3 R_3}{i_x R_x} = \frac{i_1 R_1}{i_2 R_2} = 1 \quad \left. \vphantom{i_3} \right\} \text{From (2)}$$

$$\text{From (1)} \quad \left\{ \begin{array}{l} i_3 = i_1 \\ i_x = i_2 \end{array} \right\} \Rightarrow \boxed{R_x = \frac{R_2}{R_1} R_3} \quad \left. \vphantom{i_3} \right\} \text{Solve (3) for } R_x$$

DELTA-WYE EQUIVALENT CIRCUITS [Δ - Y]:



In the Δ Circuit	Compared to Y Circuit
[1] $R_{ab} = R_c (R_a + R_b)$	$R_{ab} = R_1 + R_2$ { R_3 is floating }
[2] $R_{bc} = R_a (R_b + R_c)$	$R_{bc} = R_2 + R_3$ { R_1 is floating }
[3] $R_{ca} = R_b (R_c + R_a)$	$R_{ca} = R_1 + R_3$ { R_2 is floating }

Voltages and Currents are the same at each node in both Δ and Y connections

Derivation of Y: resistors in terms of Δ -resistors

1. Solve for R_1

$$R_1 = R_c \parallel (R_a + R_b) - R_2 \quad \left. \vphantom{R_1 = R_c \parallel (R_a + R_b) - R_2} \right\} \text{Using [1]}$$

$$R_1 = R_c \parallel (R_a + R_b) - \left[R_a \parallel (R_b + R_c) - R_3 \right] \quad \left. \vphantom{R_1 = R_c \parallel (R_a + R_b) - [R_a \parallel (R_b + R_c) - R_3]} \right\} \text{Using [2]}$$

$$R_1 = R_c \parallel (R_a + R_b) - R_a \parallel (R_b + R_c) + \left[R_b \parallel (R_c + R_a) - R_1 \right] \quad \left. \vphantom{R_1 = R_c \parallel (R_a + R_b) - R_a \parallel (R_b + R_c) + [R_b \parallel (R_c + R_a) - R_1]} \right\} \text{Using [3]}$$

Solve for
 R_1

$$2R_1 = \frac{R_c(R_a + R_b)}{(R_a + R_b + R_c)} - \frac{R_a(R_b + R_c)}{(R_a + R_b + R_c)} + \frac{R_b(R_c + R_a)}{(R_a + R_b + R_c)}$$

Same
denominators

$$2R_1 = \left[\frac{\cancel{R_a R_c} + R_b R_c - \cancel{R_a R_b} - \cancel{R_a R_c} + R_b R_c + \cancel{R_a R_b}}{R_a + R_b + R_c} \right] \quad \left. \vphantom{2R_1 = \left[\frac{\cancel{R_a R_c} + R_b R_c - \cancel{R_a R_b} - \cancel{R_a R_c} + R_b R_c + \cancel{R_a R_b}}{R_a + R_b + R_c} \right]} \right\}$$

$$2R_1 = \frac{2R_b R_c}{R_a + R_b + R_c}$$

$$\Rightarrow R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

Simplify

Derivation of Y: resistors in terms of Δ -resistors (Contd.)

2. Solve for R_2 : Using [1]

$$R_2 = \frac{R_c(R_a + \cancel{R_b})}{R_a + R_b + R_c} - \frac{\overset{\square \quad \square \quad \square^{R_1} \quad \square \quad \square}{\cancel{R_b} \cancel{R_c}}}{R_a + R_b + R_c} = \boxed{\frac{R_a R_c}{R_a + R_b + R_c}}$$

Using R_1 from previous slide

3. Solve for R_3 : Using [3]

$$R_3 = \frac{R_b(\cancel{R_c} + R_a)}{R_a + R_b + R_c} - \frac{\cancel{R_b} \cancel{R_c}}{\underset{\square \quad \square \quad \square^{R_2} \quad \square \quad \square}{R_a + R_b + R_c}} = \boxed{\frac{R_a R_b}{R_a + R_b + R_c}}$$

Balanced Loads [3- Φ Power Applications]

Loads are balanced when:

$$\Delta : R_a = R_b = R_c \equiv R_\Delta$$

$$Y : R_1 = R_2 = R_3 \equiv R_Y$$

Δ -Y:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{R_\Delta^2}{3R_\Delta} = \frac{R_\Delta}{3}$$

$$\therefore R_1 = R_2 = R_3 = \frac{1}{3} R_\Delta \quad (1)$$

Y- Δ :

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} = \frac{3R_Y^2}{R_Y} = 3R_Y$$

$$\therefore R_a = R_b = R_c = 3R_Y \quad (2)$$

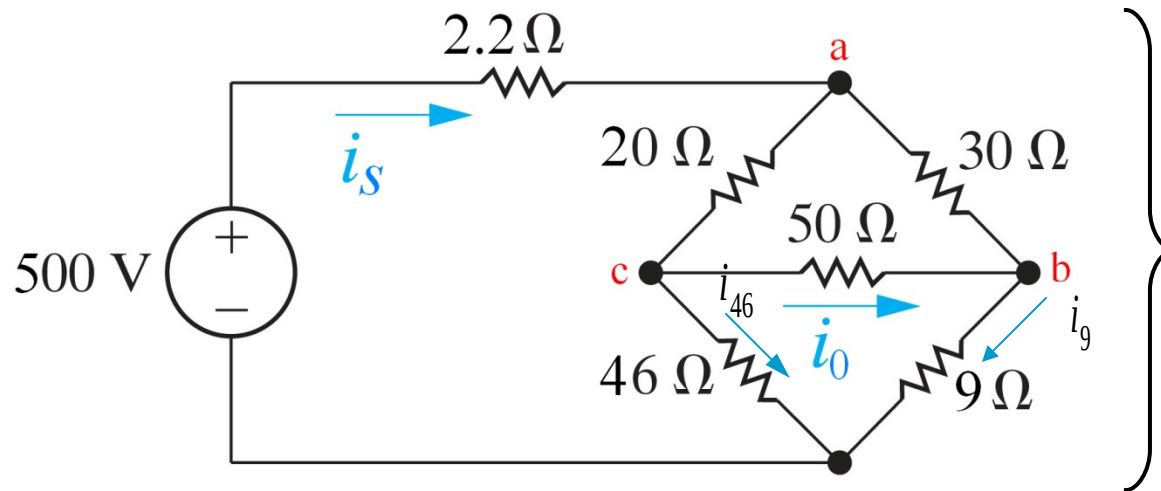
$R_1 = R_Y = \frac{1}{3} R_\Delta$
For “balanced loads”

$$R_\Delta = 3R_Y$$

$$R_a = R_\Delta = 3R_Y$$

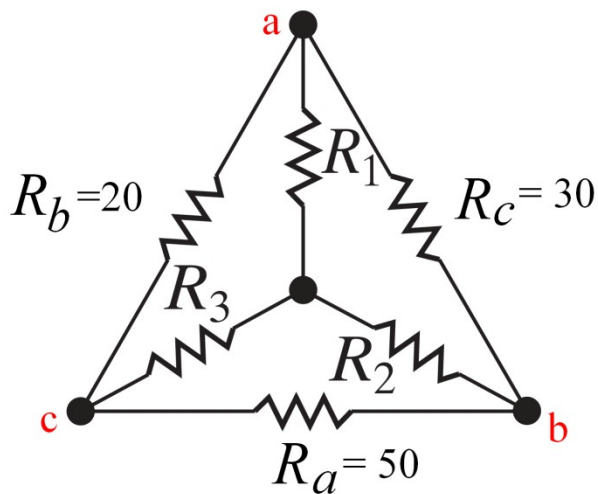
Similarly this can
be derived

Example: Find i_0

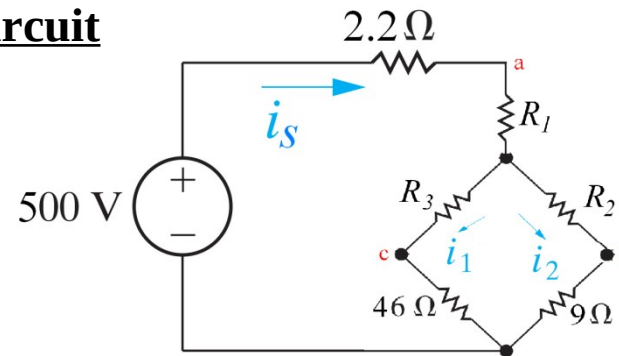


Use Δ -Y transformation to solve the circuit

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{20(30)}{20 + 50 + 30}$$



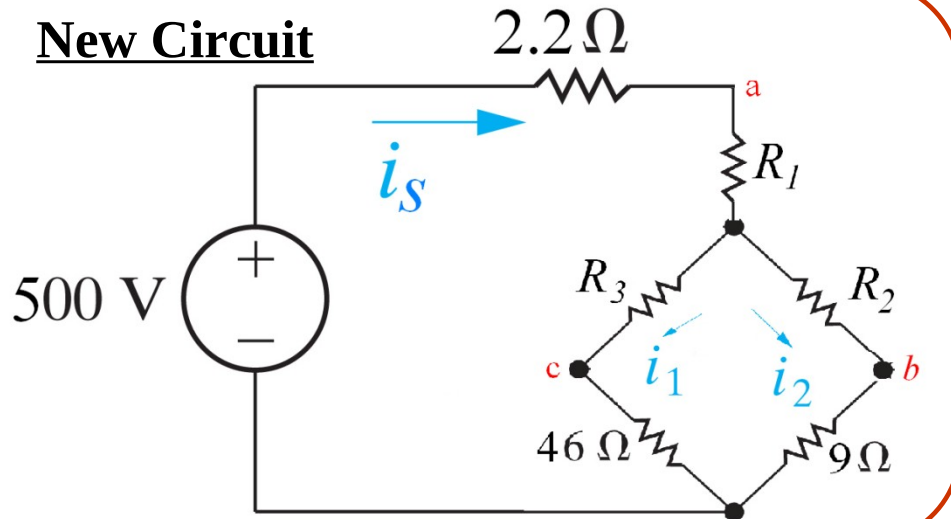
New Circuit



(Contd..)

Example: Find i_o (Contd.)

New Circuit



$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{20(30)}{100} = 6(\Omega)$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{30(50)}{100} = 15(\Omega)$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{50(20)}{100} = 10(\Omega)$$

$$\left. \begin{array}{l} i_1 = i_{46} \\ i_2 = i_9 \end{array} \right\}$$

Must be true for Δ -Y Transformation to be valid

$$R_{eq} = 2.2 + 6 + \frac{(R_3 + 46)(R_2 + 9)}{(R_3 + 46) + (R_2 + 9)} = 25(\Omega)$$

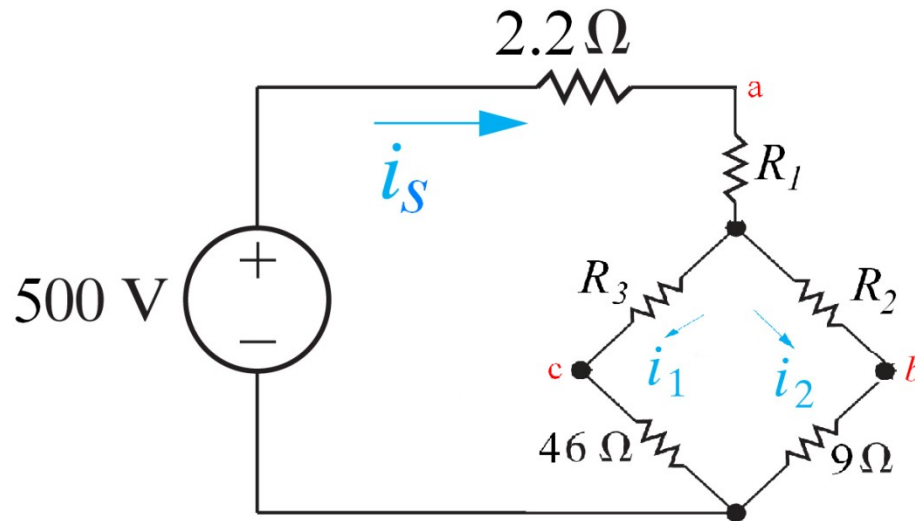
**Simplify
Resistive
Circuit**

$$i_s = \frac{500V}{R_{eq}} = \frac{500}{25} = 20 \text{ (A)}$$

Ohm's Law

(Contd..)

Example: Find i_o (Contd.)



$$\begin{aligned} i_s &= 20(A) \\ R_1 &= 6(\Omega) \\ R_2 &= 15(\Omega) \\ R_3 &= 10(\Omega) \end{aligned}$$

Find all the Currents

$$i_1 = i_s \left[\frac{(R_2 + 9)}{(R_2 + 9) + (46 + R_3)} \right] = 20 \left[\frac{24}{80} \right] = 6 (A)$$

$$i_s = i_1 + i_2 \quad \text{KCL}$$

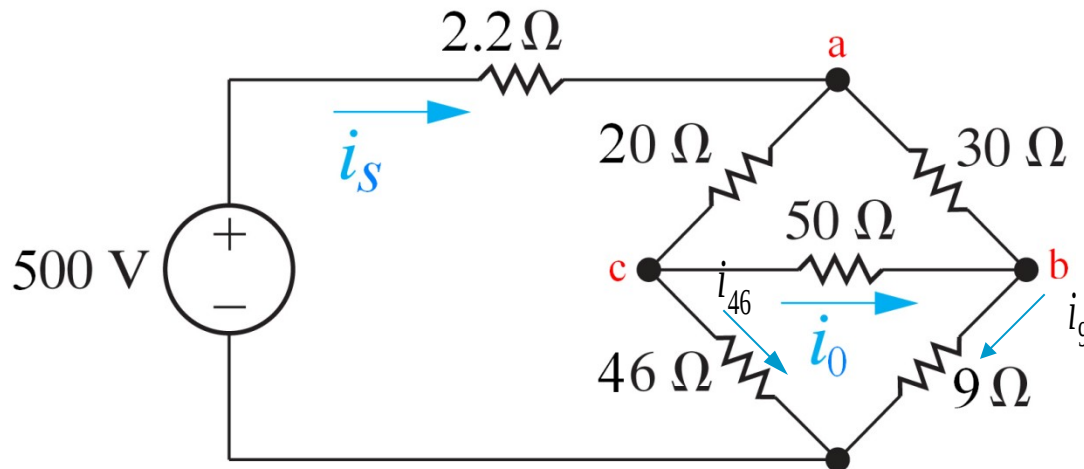
$$i_2 = i_s - i_1 = 20 - 6$$

$$i_2 = 14 (A)$$

Solve for i_2

Find i_1 with
Current Division

Example: Find i_0 (Contd.)



Go back to the original circuit (KVL from c-b through lower delta)

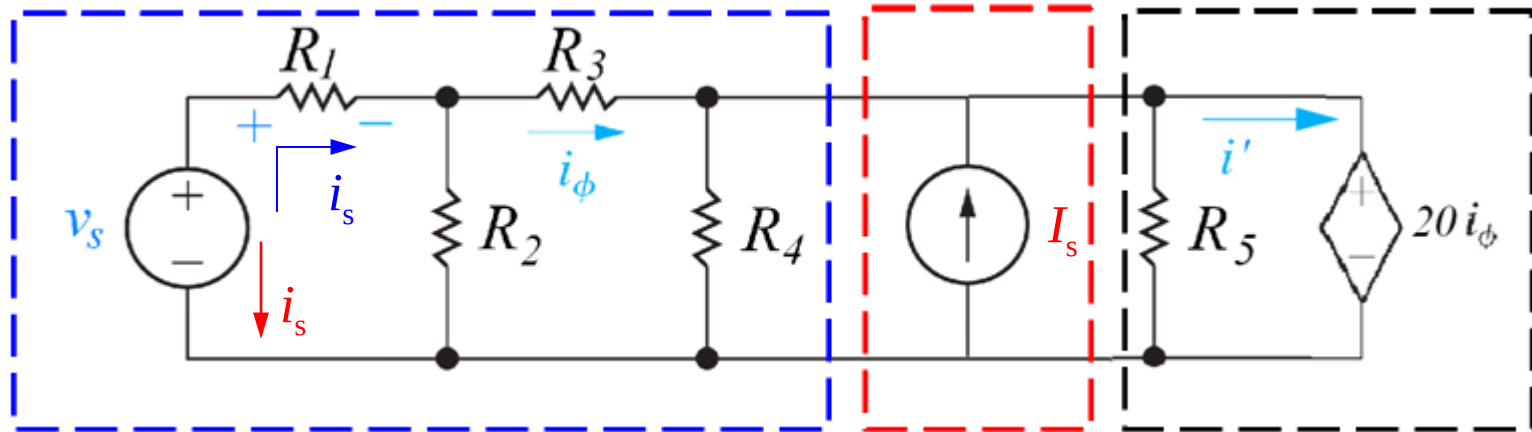
$$(i_{46} \cdot 46) - (i_9 \cdot 9) - i_0 \cdot 50 = 0 \quad \text{KVL}$$

$$\text{Solve for } i_0 \left\{ \begin{array}{l} i_0 = \frac{1}{50} [6(46) - 14(9)] \\ i_0 = 3(A) \end{array} \right. \quad \left. \begin{array}{l} i_{46} = i_1 = 6(A) \\ i_9 = i_2 = 14(A) \end{array} \right.$$

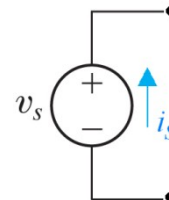
Must
be true
if Δ -Y
transformation
is valid

Special Note on Power

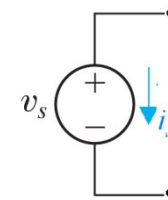
- Power:**
1. Absorbed, Dissipated, Delivered to, extracts
 2. Developed, Produced, Delivered by



- **Resistors:** Always “absorb” power. They can’t “deliver” power.
- **Blue Circuit:** One Energy Source, v_s , (power supply).
 v_s “delivers” power to circuit because i_s goes through a “rise” in voltage.
- **Blue and red Circuit:** If I_s overwhelms v_s , then i_s may reverse.
 v_s “absorbs” power, i_s goes through a “drop” in voltage.
- **Blue, Red, and Black Circuit:** $20i_\phi$ is a source.
it delivers or absorbs depending on polarity of i'



Power
Delivered



Power
Absorbed