RECALL

Theorem: Let V be a vector space with operation $+$ and $\cdot$ and let W be a non-empty subset of V. Then W is a subspace of V if and only if the following conditions hold:

a) If u and v are any vectors in W, then $u+v$ is in W.

b) If c is any real number and u is any vector in W then $c.u$ is in W.

Ex: Let $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \mid z = 0 \right\}$. Is W a subspace of $\mathbb{R}^3$?

Solution: • Observe that W is nonempty, since $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \in W$

a) Let $\begin{bmatrix} x_1 \\ y_1 \\ 0 \end{bmatrix}, \begin{bmatrix} x_2 \\ y_2 \\ 0 \end{bmatrix} \in W$. Then we have

$$\begin{bmatrix} x_1 \\ y_1 \\ 0 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \\ 0 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \\ 0 \end{bmatrix}$$

where $x_1 + x_2 \in \mathbb{R}$

$y_1 + y_2 \in \mathbb{R}$

Lets call $x_1 + x_2 = x_3 \in \mathbb{R}$

$y_1 + y_2 = y_3 \in \mathbb{R}$

$$\Rightarrow \begin{bmatrix} x_3 \\ y_3 \\ 0 \end{bmatrix} \in W.$$

b) Let $\begin{bmatrix} x \\ y \\ 0 \end{bmatrix} \in W$ and c a scaler then

$$c \cdot \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} cx \\ cy \\ 0 \end{bmatrix} \in W \text{ since } \begin{array}{l} c \cdot x \in \mathbb{R} \\ c \cdot y \in \mathbb{R} \end{array}$$

Thus W is a non-empty subset of $\mathbb{R}^3$, closed under addition and scaler multiplication of vectors, so W is a subspace of $\mathbb{R}^3$.

Ex: The set of all vectors of the form $\begin{bmatrix} a \\ b \\ 1 \end{bmatrix}$ is a subspace of $\mathbb{R}^3$ or not?

Solution: Let $W = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \in \mathbb{R}^3 \mid c = 1 \right\}$ be the set of all vectors of the form $\begin{bmatrix} a \\ b \\ 1 \end{bmatrix}$.

Observe that it is not empty since $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \in W$

a) Let $\begin{bmatrix} a_1 \\ b_1 \\ 1 \end{bmatrix}, \begin{bmatrix} a_2 \\ b_2 \\ 1 \end{bmatrix} \in W$. Then

$$\begin{bmatrix} a_1 \\ b_1 \\ 1 \end{bmatrix} + \begin{bmatrix} a_2 \\ b_2 \\ 1 \end{bmatrix} = \begin{bmatrix} a_1 + a_2 \\ b_1 + b_2 \\ 2 \end{bmatrix} \quad \begin{array}{l} a_1 + a_2 \in \mathbb{R} \\ b_1 + b_2 \in \mathbb{R} \end{array}$$

Lets call $a_1 + a_2 = a_3$
$b_1 + b_2 = b_3$

$$= \begin{bmatrix} a_3 \\ b_3 \\ 2 \end{bmatrix} \notin W.$$

Thus W is not closed under $+$
So W is not a subspace of $\mathbb{R}^3$.

RECALL

Definition: The vectors $v_1, \dots, v_k$ in a vector space V are said to be "linearly dependant" if there exist constant $a_1, \dots, a_k$ not all zero, such that

$$\sum_{j=1}^{k} a_j v_j = a_1 v_1 + \dots + a_n v_n = 0$$

Otherwise $v_1, \dots, v_k$ are called "Linearly independant".
That is $v_1, \dots, v_k$ are linearly independant if whenever

$a_1 v_1 + \dots + a_n v_n = 0$

$$a_1 = \dots = a_k = 0$$

Ex: Determine whether $S = \{(1,0), (0,1)\}$ is linearly independant

Solution: Write

$$c_1(1,0) + c_2(0,1) = (0,0)$$

or

$$c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ (if $c_1 = c_2 = 0$ then it is linearly independant)

$$\begin{bmatrix} c_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow c_1 = c_2 = 0.$$

Hence $\{(1,0), (0,1)\}$ is linearly independant.

Ex: Is $S=\{(1,0),(0,1),(1,-1)\}$ linearly independant?

Solution: Consider

$$c_1(1,0)+c_2(0,1)+c_3(1,-1)=(0,0)$$ (if $c_1=c_2=c_3=0$ then it is linearly independant)

$$(c_1,0)+(0,c_2)+(c_3,-c_3)=(0,0)$$

$$(c_1+c_3,\ c_2-c_3)=(0,0)$$

$$\begin{cases} c_1+c_3=0 \\ c_2-c_3=0 \end{cases}$$

$$\begin{cases} c_1=-c_3 \\ c_2=c_3 \end{cases}$$

This means $c_1=k$, $c_2=-k$, $c_3=-k$

This system has solution $c_1=k, c_2=-k, c_3=-k$ if $k\neq 0$ then we have a non-trivial solution.

For example $c_1=2$, $c_2=-2$, $c_3=-2$:

$$2(1,0)-2(0,1)-2(1,-1)=(2-2,-2+2)=(0,0)$$

Hence $S=\{(1,0),(0,1),(1,-1)\}$ is linearly dependant, not independant!

Ex: Write $(7,-2,2) \in \mathbb{R}^3$ as a linear combination of $(1,-1,0)$, $(0,1,1)$ and $(2,0,1)$

Solution: We want to find c_1, c_2, c_3 so that

$$(7,-2,2) = c_1(1,-1,0) + c_2(0,1,1) + c_3(2,0,1)$$

$$(7,-2,2) = (c_1,-c_1,0) + (0,c_2,c_2) + (2c_3,0,c_3)$$

$$(7,-2,2) = (c_1+2c_3, -c_1+c_2, c_2+c_3)$$

$$\Rightarrow \begin{cases} c_1 + 2c_3 = 7 \\ -c_1 + c_2 = -2 \\ c_2 + c_3 = 2 \end{cases} \Rightarrow \begin{array}{l} c_1 = +1 \\ c_2 = -1 \\ c_3 = 3 // \end{array}$$

Thus $(7,-2,2) = 1(1,-1,0) - 1(0,1,1) + 3(2,0,1)$

$$= (1,-1,0) + (0,-1,-1) + (6,0,3) //$$

Ex: Show the $\text{span}\{(1,0,1),(-1,2,3),(0,1,-1)\}$ is all of $\mathbb{R}^3$.

Solution: We must show that any vector $(a,b,c) \in \mathbb{R}^3$ can be written as a linear combination of the three given vectors.

$$c_1(1,0,1) + c_2(-1,2,3) + c_3(0,1,-1) = (a,b,c)$$

$$(c_1,0,c_1) + (-c_2,2c_2,3c_2) + (0,c_3,-c_3) = (a,b,c)$$

$$\begin{cases} a = c_1 - c_2 \\ b = 2c_2 + c_3 \\ c = c_1 + 3c_2 - c_3 \end{cases}$$

$$c_1 = \frac{5a+b+c}{6} \quad // \qquad c_2 = \frac{b+c-a}{6} \quad // \qquad c_3 = \frac{a+2b-c}{3} \quad //$$

Hence $(a,b,c) \in \text{span}\{(1,0,1),(-1,2,3),(0,1,-1)\}$ and the span is all of $\mathbb{R}^3$.

RECALL

Definition: The vectors $v_1, \dots, v_k$ in a vector space V are said to form a basis for V if

a) $v_1, v_2, \dots, v_k$ are linearly independent

b) $v_1, v_2, \dots, v_k$ span V

Ex: Show that the set $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^3$.

Solution: We will show that 1) Linearly independent
2) span

1) Write

$$c_1 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(if $c_1 = c_2 = c_3 = 0$ then it is linearly independent)

$$\begin{bmatrix} 2c_1 \\ c_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 2c_2 \\ c_2 \\ c_2 \end{bmatrix} + \begin{bmatrix} 2c_3 \\ 2c_3 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2c_1 + 2c_2 + 2c_3 \\ c_1 + c_2 + 2c_3 \\ c_2 + c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} 2c_1 + 2c_2 + 2c_3 = 0 \\ c_1 + c_2 + 2c_3 = 0 \\ c_2 + c_3 = 0 \end{cases} \Rightarrow \begin{matrix} c_1 + c_2 + c_3 = 0 \\ c_1 + c_2 + 2c_3 = 0 \end{matrix} \Rightarrow c_3 = 0 \quad c_2 + c_3 = 0 \Rightarrow c_2 = 0$$

$$\Rightarrow c_1 = 0$$

Hence $c_1 = c_2 = c_3 = 0$. Therefore these vectors are linearly independent.

2) Take $(a,b,c) \in \mathbb{R}^3$

We try to write (a,b,c) as a lineer combination of three vectors:

$$c_1 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\begin{bmatrix} 2c_1 \\ c_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 2c_2 \\ c_2 \\ c_2 \end{bmatrix} + \begin{bmatrix} 2c_3 \\ 2c_3 \\ c_3 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\begin{bmatrix} 2c_1 + 2c_2 + 2c_3 \\ c_1 + c_2 + 2c_3 \\ c_2 + c_3 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\begin{cases} 2c_1 + 2c_2 + 2c_3 = a \\ c_1 + c_2 + 2c_3 = b \\ c_2 + c_3 = c \end{cases}$$

$$c_1 + c_2 + c_3 = \frac{a}{2}$$

$$c_2 + c_3 = c$$

$$c_1 = \frac{a}{2} - c = \frac{a-2c}{2}$$

$$\begin{cases} c_2 + c_3 = c \\ c_1 + c_2 + 2c_3 = b \Rightarrow \dfrac{a-2c}{2} + c_2 + 2c_3 = b \end{cases}$$

$$c_2 = \frac{2c - 2b + a}{2} \qquad c_3 = \frac{2b - a}{2}$$

Hence we can write (a,b,c) as a lineer combination of three vectors with coefficient c_1, c_2, c_3.

Hence $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^3$.

Ex: Which of the following sets of vectors are basis for $\mathbb{R}^3$?

a) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$

Solution: Recall that since $\dim(\mathbb{R}^3) = 3$, all basis of $\mathbb{R}^3$ has exactly three linearly independent vectors.

So No! Here we have two vectors, we need three linearly independant vectors.

b) $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$

Solution: No! Here we have four vectors and $\dim(\mathbb{R}^3) = 3$ so at least one of them is a linear combination of the others.

c) $\left\{ \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

Solution: There are 3 vectors, now we need to verify if they are

1) Linearly independant
2) span

1) Write

$$c_1 \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(if $c_1 = c_2 = c_3 = 0$ then it is linearly independant)

$$\begin{bmatrix} 3c_1 - c_2 \\ 2c_1 + 2c_2 + c_3 \\ 2c_1 + c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} 3c_1 - c_2 = 0 \\ 2c_1 + 2c_2 + c_3 = 0 \\ 2c_1 + c_2 = 0 \end{cases}$$

$3c_1 = c_2 \Rightarrow c_1 = \frac{c_2}{3}$

$c_2 + c_3 = 0 \Rightarrow c_2 = -c_3$

$\frac{2c_2}{3} + 2c_2 - c_2 = 0$

$\Rightarrow c_2 = 0$

$c_3 = 0$

$c_1 = 0$

So $c_1 = c_2 = c_3 = 0$. Hence these vectors are linearly independent.

2) Take $(a, b, c) \in \mathbb{R}^3$

We try to write (a, b, c) as a linear combination of three vectors

$$c_1 \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\begin{bmatrix} 3c_1 - c_2 \\ 2c_1 + 2c_2 + c_3 \\ 2c_1 + c_2 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

$$\begin{cases} 3c_1 - c_2 = a \\ 2c_1 + 2c_2 + c_3 = b \\ 2c_1 + c_2 = c \end{cases} \Big\rangle \quad c + c_2 + c_3 = b$$

$$3c_1 - \cancel{c_2} = a$$
$$2c_1 + \cancel{c_2} = c$$

$$5c_1 = a + c$$
$$c_1 = \frac{a+c}{5} \; //$$

$$2\left(\frac{a+c}{5}\right) + c_2 = c$$
$$c_2 = c - \left(\frac{2a+2c}{5}\right)$$
$$c_2 = \frac{3c - 2a}{5} \; //$$

$$c + c_2 + c_3 = b \Rightarrow$$

$$c + \frac{3c-2a}{5} + c_3 = b \Rightarrow c_3 = b - c - \left(\frac{3c-2a}{5}\right)$$

$$c_3 = \frac{5b - 8c + 2a}{5} \; //$$

Hence we can write (a, b, c) as a linear combination of three vectors with coefficient c_1, c_2, c_3.

Hence $\left\{ \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^3$.

RECALL

Definition: Let V and W be two vector spaces. A function

$$T : V \longrightarrow W$$

is called a "Linear transformation" of V into W, if following two properties are true for all $u, v \in V$ and scalers c.

1) $T(u+v) = T(u) + T(v)$

2) $T(cu) = cT(u)$

Ex: Let $T(v_1, v_2, v_3) = (2v_1 + v_2,\ 2v_2 - 3v_1,\ v_1 - v_3)$

a) Compute $T(-4, 5, 1)$.

Solution: We know that

$T(v_1, v_2, v_3) = (2v_1 + v_2,\ 2v_2 - 3v_1,\ v_1 - v_3)$ So

$T(-4, 5, 1) = (2.(-4) + 5,\ 2.5 - 3.(-4),\ -4 - 1)$

$= (-3, 22, -5)$ //

b) Compute the preimage of $W = (4, 1, -1)$

Solution: Suppose (v_1, v_2, v_3) is in the preimage of $(4, 1, -1)$.

Then

$(2v_1 + v_2,\ 2v_2 - 3v_1,\ v_1 - v_3) = (4, 1, -1)$

$$\begin{cases} 2v_1 + v_2 = 4 \\ 2v_2 - v_1 = 1 \\ v_1 - v_3 = -1 \end{cases} \qquad \begin{aligned} v_1 &= 1 + v_3 \\ \Rightarrow v_1 &= 2v_2 - 1 \\ v_1 &= \frac{4 - v_2}{2} \end{aligned} \qquad \begin{aligned} \Rightarrow 2v_2 - 1 &= \frac{4 - v_2}{2} \\ 4v_2 - 2 &= 4 - v_2 \\ 5v_2 = 6 \quad v_2 &= 6/5 \end{aligned}$$

$v_1 = 2v_2 - 1 = 2 \cdot \frac{6}{5} - 1 = \frac{12}{5} - 1 = \frac{7}{5} \Rightarrow v_1 = \frac{7}{5}$ //

$v_1 = 1 + v_3 \Rightarrow \frac{7}{5} = 1 + v_3 \Rightarrow v_3 = \frac{2}{5}$ //

Hence preimage of $(4,1,-1)$ is $\left(\frac{7}{5}, \frac{6}{5}, \frac{2}{5}\right)$ //

<u>Ex</u>: Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that

$T(1,0,0) = (2,4,-1)$

$T(0,1,0) = (1,3,-2)$

$T(0,0,1) = (0,-2,2)$

Compute $T(-2,4,-1)$.

<u>Solution</u>: We have

$(-2,4,-1) = -2(1,0,0) + 4(0,1,0) - 1(0,0,1)$

Hence $T(-2,4,-1) = -2\underbrace{T(1,0,0)}_{(2,4,-1)} + 4\underbrace{T(0,1,0)}_{(1,3,-2)} - \underbrace{T(0,0,1)}_{(0,-2,2)}$

$= -2(2,4,-1) + 4(1,3,-2) - (0,-2,2)$

$= (-4+4, 0, -8+12-2, 2-8-2)$

$= (0, 2, -8)$ //

RECALL

Definition: Let V, W be two vector spaces and $T: V \to W$ a linear transformation

1) Then the kernel of T, $\ker(T)$, is the set of $v \in V$ such that $T(v) = 0$.

$$\ker(T) = \{ v \in V \mid T(v) = 0 \}$$

2) The range of T, range(T) is given by

$$\text{range}(T) = \{ w \in W \mid w = T(v) \text{ for some } v \in V \}$$

Ex: Find $\ker(T)$ where $T: \mathbb{R}^3 \to \mathbb{R}^2$ is defined by

$T((v_1, v_2, v_3)) = (v_1 + v_2, v_2 - v_3)$

Solution: Since $\ker(T) = \{ v \in \mathbb{R}^3 \mid T(v) = 0 \}$ we must show

$T((v_1, v_2, v_3)) = (0, 0)$

$(v_1 + v_2, v_2 - v_3) = (0, 0)$

$$\begin{cases} v_1 + v_2 = 0 \\ v_2 - v_3 = 0 \end{cases} \qquad \begin{cases} v_1 = -v_2 \\ v_2 = v_3 \end{cases} \qquad \begin{aligned} v_1 &= -k \\ v_2 &= k \\ v_3 &= -k \end{aligned}$$

Hence

$$\ker(T) = \{ v \in \mathbb{R}^3 \mid v = k(-1, 1, 1) \} = \text{span}\{(-1, 1, 1)\}$$

Ex: Define $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ such that

$$T((a,b,c)) = (a-b+c,\ 2a+b-c,\ -a-2b+2c)$$

Determine range (T).

Solution: Let $w = (w_1, w_2, w_3)$ be in range (T).

Thus $w = T((a,b,c))$ for some vector $(a,b,c) \in \mathbb{R}^3$.

Thus

$$T((a,b,c)) = (a-b+c,\ 2a+b-c,\ -a-2b+2c) = (w_1, w_2, w_3)$$

$$a-b+c = w_1$$

$$2a+b-c = w_2$$

$$-a-2b+2c = w_3$$

$$\underbrace{a-b+c}_{= w_1} = w_2 + w_3 \qquad \Rightarrow w_1 = w_2 + w_3$$

So if $w = (w_1, w_2, w_3)$ is to be the range (T), then $w_1 = w_2 + w_3$. That is

$$\text{range}(T) = \{ w = (w_1, w_2, w_3) \in \mathbb{R}^3 \mid w_1 = w_2 + w_3 \}$$

Ex: Let $T: \mathbb{R} \to \mathbb{R}$. Define $T(x) = mx$ where ~~[illegible]~~ m is a fixed real number. Show that T is a linear transformation.

Solution: We must show the following two conditions:

1) Let $x, y \in \mathbb{R}$. Then

$$T(x+y) = m(x+y) = mx + my$$
$$= T(x) + T(y)$$

$$\Rightarrow T(x+y) = T(x) + T(y) //$$

2) $T(rx) = m(rx) = (mr)(x) = (rm)(x) = r(mx)$
$$= rT(x)$$

$$\Rightarrow T(rx) = rT(x) //$$

Hence T is a linear transformation.

Ex: Let $T: \mathbb{R} \to \mathbb{R}$. Define $T(x) = mx + b$ where m and b are real numbers and $b \neq 0$. Show that T is not a linear transformation.

Solution: We must show the following two conditions:

1) Let $x, y \in \mathbb{R}$. Then

$$T(x+y) = m(x+y) + b = mx + my + b$$

$$T(x+y) = mx + my + b$$

Now lets check

$$T(x) + T(y) = mx + b + my + b$$

$$= mx + my + 2b$$

Hence

$$\left.\begin{aligned} T(x+y) &= mx + my + b \\ T(x) + T(y) &= mx + my + 2b \end{aligned}\right\} \neq \text{ since } b \neq 0$$

Hence T is not a Linear transformation

<u>Ex</u>: Is $U=\left\{\begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 \mid x+y=0,\ x-y=1\right\}$ a subspace of $\mathbb{R}^2$?

<u>Solution</u>: No!

Remember that in order for U to be a subspace of $\mathbb{R}^2$,

it must contain the zero vector, which in this case is

$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$. But $\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is not a solution to the system

$$x+y=0$$
$$x-y=1$$

Hence $\begin{bmatrix} 0 \\ 0 \end{bmatrix} \notin U$ implying that U is not a subspace.