CSE 211: Discrete Mathematics **(Due: 30/10/22)**

Homework #1

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to bkarakas2018@gtu.edu.tr
- Use LaTeX. You can work on the tex file shared with you in the assignment document.
- Submit both the tex and pdf files into Homework1. Name of the files should be "*SurnameName_Id.tex*" and "*SurnameName_Id.pdf*".

Problem 1: Sets (3+3+3+3+3=15 points)

Which of the following sets are equal? Show your work step by step.

(a) {t : t is a root of x^2 − 6x + 8 = 0}

(b) {y : y is a real number in the closed interval [2, 3]}

(c) {4, 2, 5, 4}

(d) {4, 5, 7, 2} - {5, 7}

(e) {q: q is either the number of sides of a rectangle or the number of digits in any integer between 11 and 99}

(Solution)

5 sets are going to be named as A,B,C,D and E respectively for the sake of clarity.

- (a) Let us find roots of the equation to find elements of the set. Roots of the equation y=$x^2 - 6x + 8$ are 4 and 2. Therefore, A = {4,2}
- (b) B = {2, ..., 3} - $\mathbb{C}$
- (c) C = {4,2,5,4}
- (d) D = {4,2} 5 and 7 are not included.
- (e) E = {4,2}

Set A, set D and set E are considered as equal set of elements since they contain the same elements. Therefore;

$$A = D = E$$

Problem 2: Cardinality of Sets (2+2+2+2=8 points)

What is the cardinality of each of these sets? Explain your answers.

(a) $\{\emptyset\}$

(b) $\{\emptyset, \{\emptyset\}\}$

(c) $\{\emptyset, \{\emptyset, \{\emptyset\}\}\}$

(d) $\{\emptyset, \{\emptyset, \{\emptyset, \{\emptyset\}\}\}\}$

(Solution)

4 sets are going to be named as A,B,C,D respectively for the sake of clarity. Cardinality of a set is the number of objects in the set. This object can be a set, an individual element, set of subsets and even an empty set.

- (a) The only element in set A is an empty set, therefore $|A| = 1$
- (b) There are two elements in set B, first one is an empty set, second one is a set holding an empty set, therefore $|B| = 2$
- (c) There are two elements in set C, first one is an empty set, second one is a set containing two elements, therefore $|C| = 2$
- (d) There are two elements in set D, first one is an empty set, second one is a set containing two elements, therefore $|D| = 2$

Problem 3: Cartesian Product of Sets (15 points)

Explain why (A $\times$ B) $\times$ (C $\times$ D) and A $\times$ (B $\times$ C) $\times$ D are not the same.

(Solution)

Let us assume that;

- A = {a: a $\in$ A}
- B = {b: b $\in$ B}
- C = {c: c $\in$ C}
- D = {d: d $\in$ D}

There are set A, set B, set C and set D, which all are non-empty. Therefore,

- A x B = {(a,b): a $\in$ A, b $\in$ B}
- C x D = {(c,d): c $\in$ C, d $\in$ D}

Therefore,

- (A x B) x (C x D) = {(a,b), (c,d): (a,b) $\in$ (A x B), (c,d) $\in$ (C x D)}

Additionally,

- B x C = {(b,c): b $\in$ B, c $\in$ C}
- A x (B x C) = {(a, (b,c)): a $\in$ A, (b,c) $\in$ (B x C)}

Therefore,

- A x (B x C) x D = $\{(a,(b,c),d)\colon a \in A, (b,c) \in (B \times C), d \in D\}$

As a result,

- $((a,b),(c,d)) \neq (a,(b,c),d)$
- (A x B) x (C x D) $\neq$ A x (B x C) x D

Problem 4: Cartesian Product of Sets in Algorithms (25 points)

Let A, B and C be sets which have different cardinalities. Let (p, q, r) be each triple of A $\times$ B $\times$ C where p $\in$ A, q $\in$ B and r $\in$ C. Design an algorithm which finds all the triples that are satisfying the criteria: p $\leq$ q and q $\geq$ r. Write the pseudo code of the algorithm in your solution.

For example: Let the set A, B and C be as A = { 3, 5, 7 }, B = { 3, 6 } and C = { 4, 6, 9 }. Then the output should be : { (3, 6, 4), (3, 6, 6), (5, 6, 4), (5, 6, 6) }.

(Note: Assume that you have sets of A, B, C as an input argument.)

(Solution)

Algorithm 1: Pseudo Code of Your Algorithm

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Input: The sets of A, B, C
for keep going through the elements of set A until all elements in set A has been gone through once do
    for going through the elements of set B until all elements in set B has been gone through once do
        for going through the elements of set C until all elements in set C has been gone through once do
            if element of set A is less than or equal to element of set B and element of set B is greater
              than or equal to element of set C is true then
                print particular element in set A, set B and set C respectively and put parentheses and
                  commas as indicated in the expected output.
            else
                keep going through the sets' elements
            end
        end
    end
end
```

Problem 5: Functions (16 points)

If f and f ∘ g are one-to-one, does it follow that g is one-to-one? Justify your answer.

(Solution)

Let us assume $g(a) = g(b)$. Unless a is equivalent to b, g cannot be a one-to-one function. Therefore, we will show a is equivalent to b or not.

- Let us take the function f of each side of the previous equation:

$$f(g(a)) = f(g(b))$$

- Using the definition of composition:

$$\mathrm{f} \circ \mathrm{g(a)} = \mathrm{f} \circ \mathrm{g(b)}$$

- Since it is already said that f ∘ g is one-to-one, a is equivalent to b. By the definition of one-to-one function, we have proved that g is a one-to-one function.

Problem 6: Functions (7+7+7=21 points)

Determine whether the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is onto if

(a) $f(m, n) = 2m - n$

(b) $f(m, n) = m^2 - n^2$

(c) $f(m, n) =\mid m \mid - \mid n \mid$

(Solution)

- (a) Given any integer n, we have f(0,n) = -n, therefore the function is onto.

- (b) Given any integer n, we have f(0,n) = $-n^2$. $n^2 \geq 0$ and $-n^2 \leq 0$. As seen, the range does not contain any positive integers, therefore the function is not onto.

- (c) Given any integer m, we have f(m,0) = $|m|$. $|m| \geq 0$. As seen, the range does not contain any negative integers. Therefore, the function is not onto.

Problem 7: Functions (Bonus 20 points)

Suppose that f is a function from A to B, where A and B are finite sets with $\mid A \mid = \mid B \mid$. Show that f is one-to-one if and only if it is onto.

(Solution)

- Since f is one-to-one, every element in A is mapped to a distinct element in B, which means if f(x) = f(y), x=y for x,y $\in$ A.
- We will use method of contradiction to prove it. Let us say f is not onto, which means there is at least one element in set B with no preimage in set A. Therefore, $|B|$ will be at least one greater than $|A|$, which contradicts $|A| = |B|$. Therefore, f is onto.