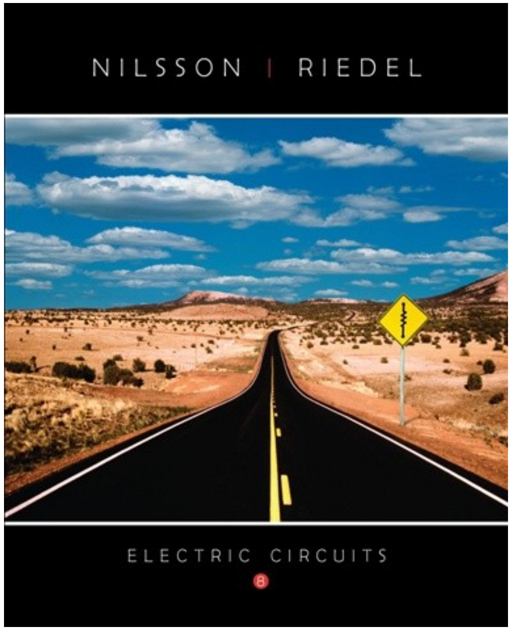
Week 3

Analysis Techniques

Textbook (for Circuits only)



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Electric Circuits

Pearson Prentice Hall

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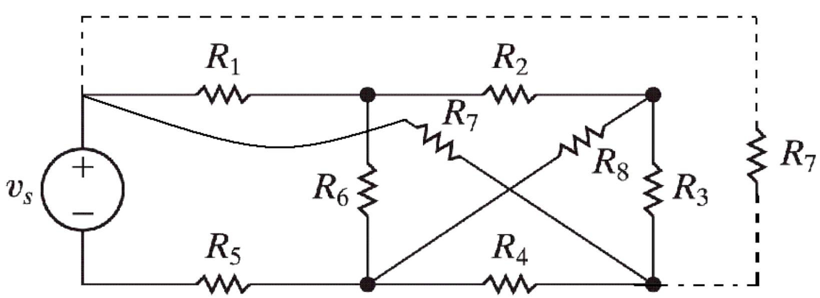
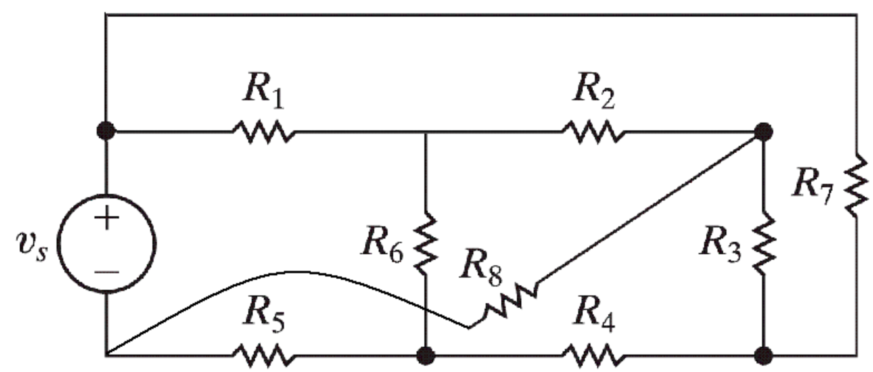
The Heart of the Course

Nodal and Mesh analysis techniques are used to solve circuits

<u>Planar Circuits:</u> Can be drawn on a plane with no crossing branches





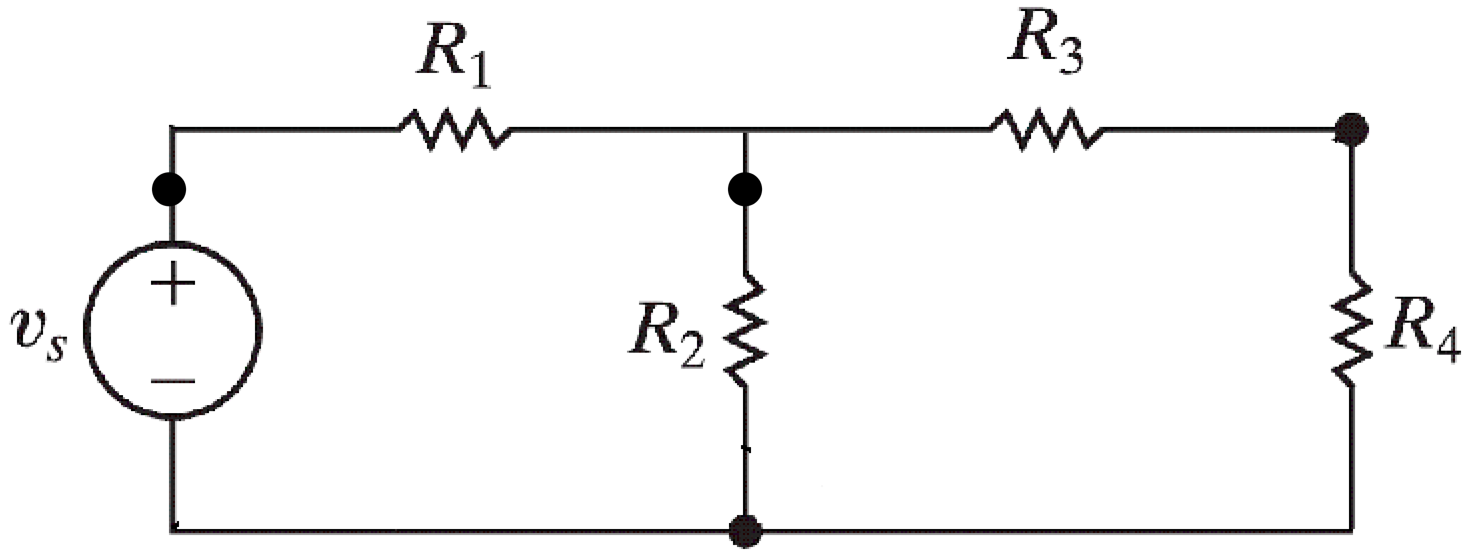
<u>Planar</u>
Nodal, Mesh

<u>Non Planar</u>
Nodal Only

Most Important Terms

Node	Point where 2 or more circuit elements join.
Loop	Closed Path. Start node = end node.
Mesh	Loop not enclosing another loop.
Path	Trace of adjoining elements, with no element included more than once.
Branch	Path that connects 2 nodes.
Essential Node	Node where 3 or more elements join. (n_e)
Essential Branch	Branch connecting 2 essential nodes. (b_e)

Simple Illustration



How many Nodes? **4**

How many Branches? **5**

What are the Essential Nodes and Branches?

2 **1**

Simultaneous Equations

- **b_e = # of essential branches in the circuit**
- **b_e = # of independent unknown currents**

- **n_e = # of essential nodes in the circuit**

- **We can derive (n_e-1) independent equations with KCL applied to (n_e-1) nodes**

- **We can derive the other $b_e - (n_e-1)$ independent equations with KVL applied to $b_e - (n_e-1)$ loops or meshes**

Example

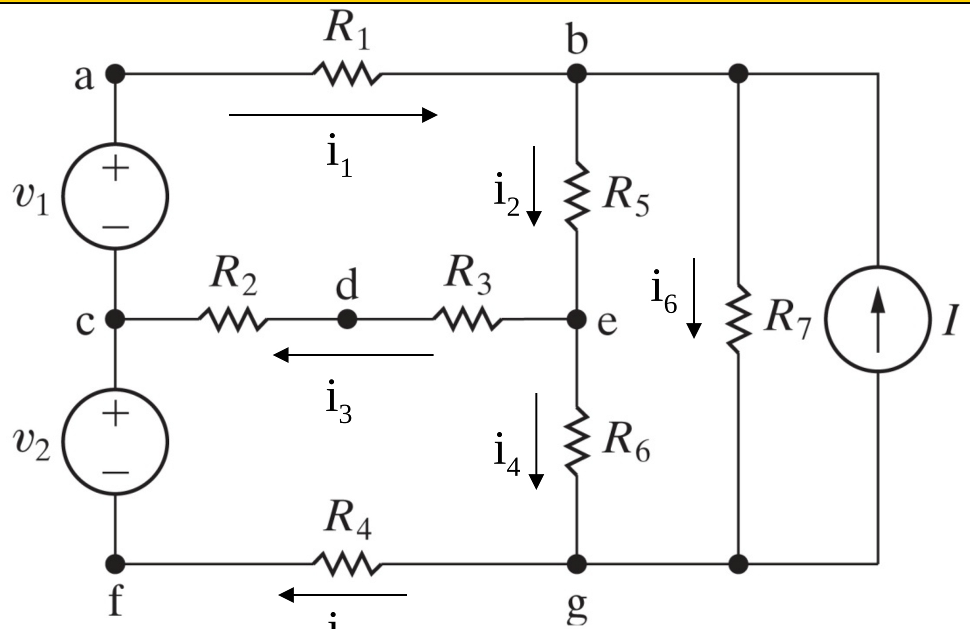


Figure: 04-03Ex4.1

Essential Nodes

b c e g $\Bigg\}$ $n_e = 4$

3 or more elements connected

Current is divided at these nodes

Essential Branches **(Total of 6)**

(c-a-b), (b-g), (c-d-e), (c-f-g), (b-e), (e-g) $\Bigg\}$ $b_e = 6$

Connects 2 essential nodes with none in between

There are six independent current variables $\Bigg\}$ $b_e = 6$

Example (Contd.)

- KCL ⇨ $(n_e - 1) = 4 - 1$ = <u>3 Independent Equations</u>
- KVL ⇨ the remaining: $b_e - (n_e - 1) = 6 - (4 - 1)$

  = <u>3 Independent Equations</u>
- Need b_e = 6 independent equations. Six independent current variables

$$\sum i_{in} = \sum i_{out} \quad \text{KCL}$$

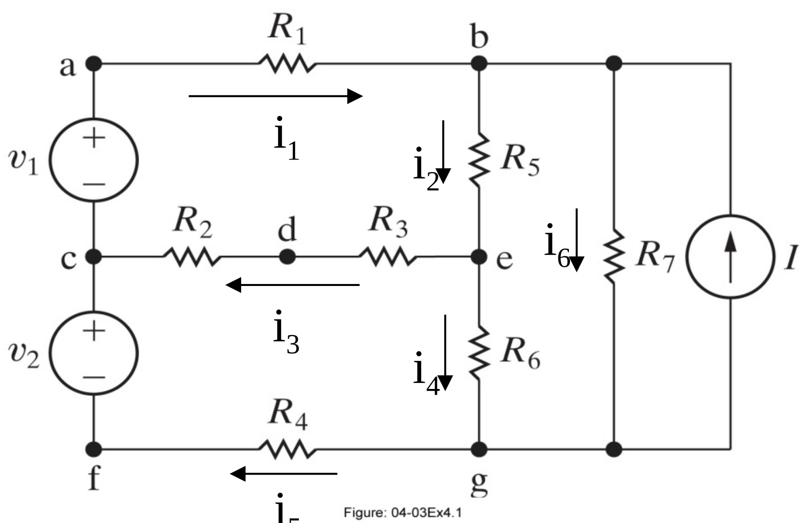


Figure: 04-03Ex4.1

KCL

Node	In		Out	Eq.
b	$i_1 + I$	=	$i_2 + i_6$	1
c	$i_3 + i_5$	=	i_1	2
e	i_2	=	$i_3 + i_4$	3

Example (Contd.)

KVL: 4 meshes are in the circuit, but we will use only 3 meshes

(a – b – e – d – c – a) $V_1 = i_1R_1 + i_2R_5 + i_3 (R_2 + R_3)$ 4

(c – d – e – g – f – c) $V_2 = -i_3 (R_2 + R_3) + i_4R_6 + i_5R_4$ 5

(b – e – g – b) $i_2R_5 + i_4R_6 – i_6R_7 = 0$ 6

- Don't use (b – g – b) across the I-source because we don't know the voltage drop

- <u>We now have 6 equations to solve for 6 unknowns</u>

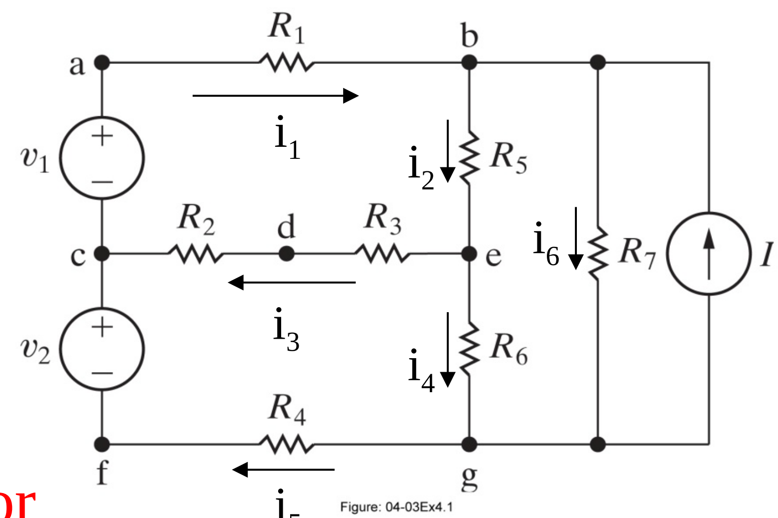


Figure: 04-03Ex4.1

Introduce New Techniques

If you set the problem up right, you don't need as many equations

Node Voltages Requires only $n_e - 1$ equations

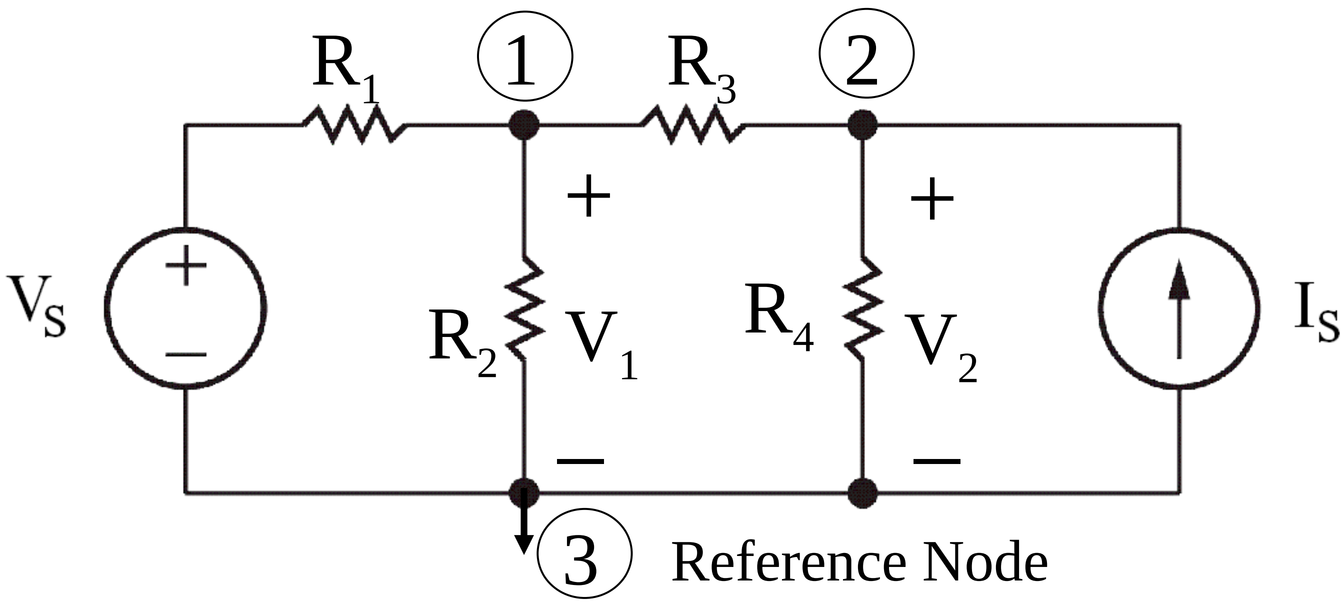
OR

Mesh Currents Requires only $b_e - (n_e - 1)$ equations

The Node-Voltage Method

Construct (n_e – 1) Node Voltage Equations

n_e: Number of Essential nodes



3 Essential nodes, $n_e = 3$

$(n_e - 1) = 3 - 1 = 2$ } **Number of Equations**

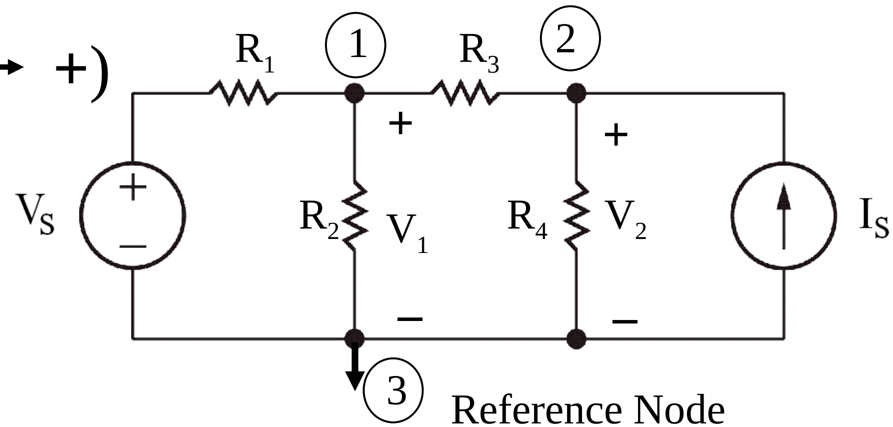
Choose reference node. (Usually node with most branches)

The Node-Voltage Method

Node Voltage: Voltage Rise (– ⟶ +)

{V_1: between 1 & 3 }

{V_2: between 2 & 3 }

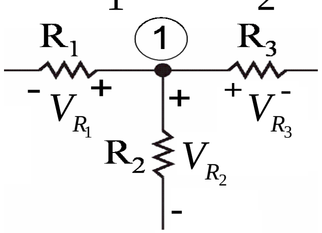


Node Equation for 1

Step 1 Sum the currents Leaving Non-Reference Node = 0 } **KCL**

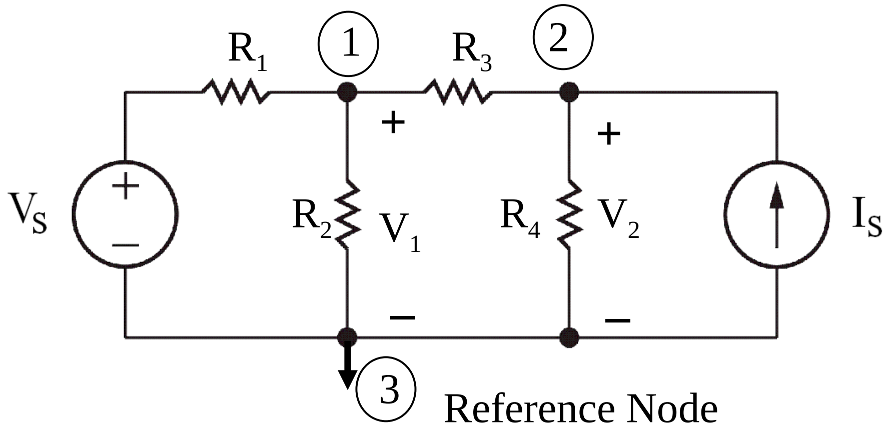
Step 2 Write Currents in Terms of Node Voltages

Step 3 $$\frac{V_{R_1}}{R_1}+\frac{V_{R_2}}{R_2}+\frac{V_{R_3}}{R_3}=0$$ } **KCL**



$$\left.\begin{aligned} V_{R_1} &= V_1 - V_S \\ V_{R_2} &= V_1 \\ V_{R_3} &= V_1 - V_2 \end{aligned}\right\}$$ ***See Circuit***

The Node-Voltage Method



Current Leaving Node 2

$$\frac{V_2 - V_1}{R_3} + \frac{V_2}{R_4} - I_S = 0$$

Node Equation for 2

Current leaving node 2 → opposite polarity across R_3 to that of node 1

Current source produces a current Flowing into 2 ; ⇨ "–" sign

2 Equations and 2 Unknowns. Solve for V_1 and V_2

All other quantities can be computed from V_1 and V_2

Example With Numbers

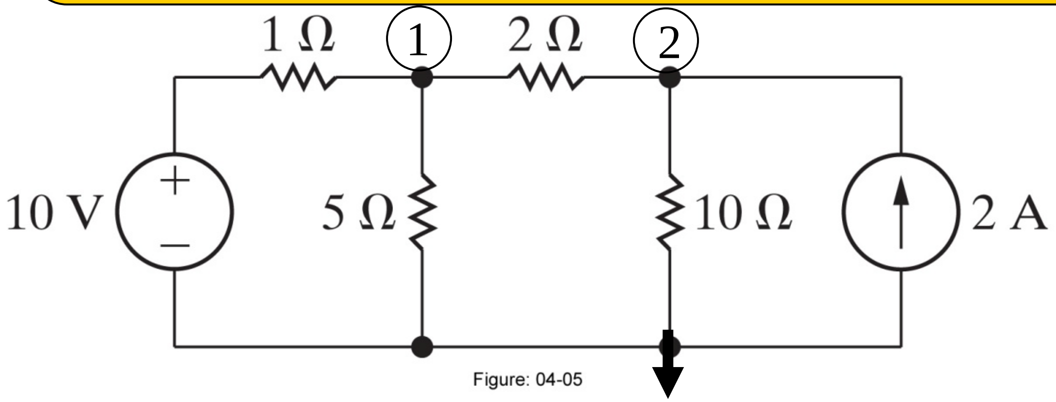


Figure: 04-05

Find essential nodes and solve for node voltages

$$\frac{V_1 - 10}{1} + \frac{V_1}{5} + \frac{V_1 - V_2}{2} = 0 \quad (1)$$

$$\frac{V_2 - V_1}{2} + \frac{V_2}{10} - 2 = 0 \quad (2)$$

Multiply Both Equations by 10

$$10V_1 - 100 + 2V_1 + 5V_1 - 5V_2 = 0$$

$$17V_1 - 5V_2 = 100 \quad (1)$$

$$5V_2 - 5V_1 + V_2 - 20 = 0$$

$$-5V_1 + 6V_2 = 20 \quad (2)$$

Solve the 2 Equations Simultaneously

Example (Contd.)

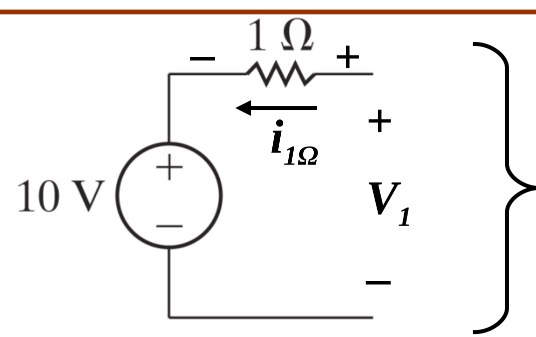
Rearrange

2 $\Rightarrow$ $V_2 = \dfrac{20 + 5V_1}{6}$ $\Rightarrow$ Substitute into 1 $\Rightarrow$ $17V_1 - \dfrac{100 + 25V_1}{6} = 100$

Solve (1) for V_1, then (2) for V_2

$$\boxed{V_1 = 9.09(V) \qquad V_2 = 10.91(V)}$$

Now that we know V_1 and V_2, finding other quantities is straightforward



From the previous slide

Example: $10 = -i_{1\Omega}1 + V_1 \quad \therefore\ i_{1\Omega} = \dfrac{-10 + 9.09}{1} = -0.91(A)$

Example (Contd.)

Previous example is much easier to solve using the Nodal Method

Solve 2 Equations

Compare

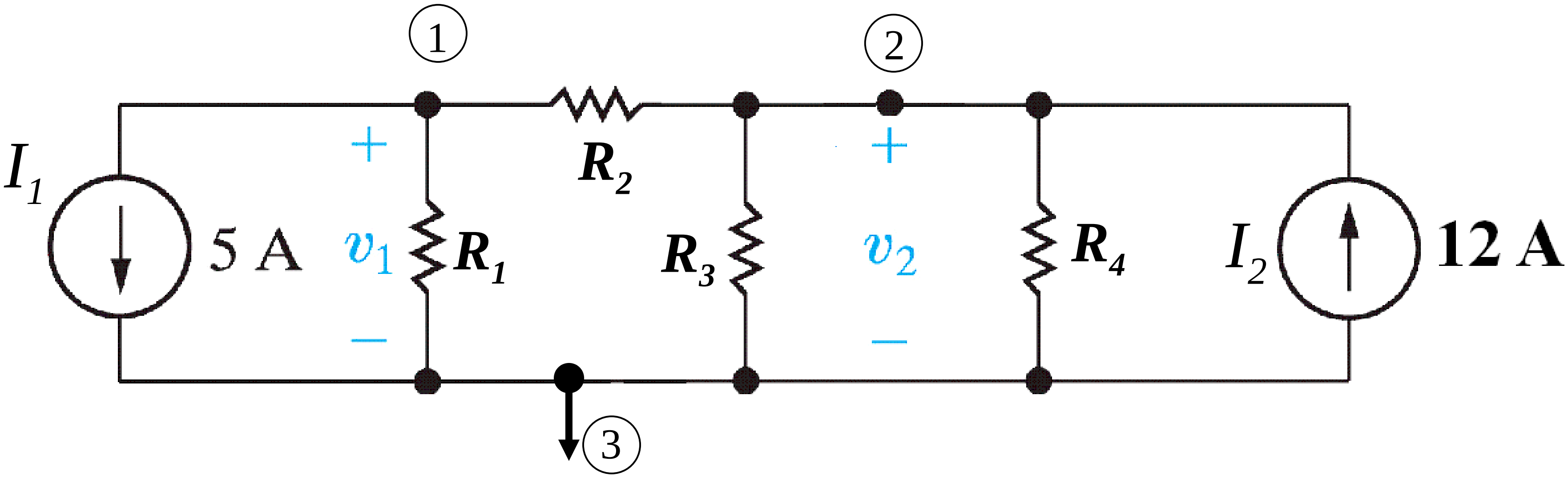
$(n_e - 1) = (3 - 1) = 2$ KCL Equations

$b_e - (n_e - 1) = 4 - 2 = 2$ KVL Equations

Solve 4 Equations for 4 Unknowns

Standard Approach

Drill Exercise: Find the Node Voltages



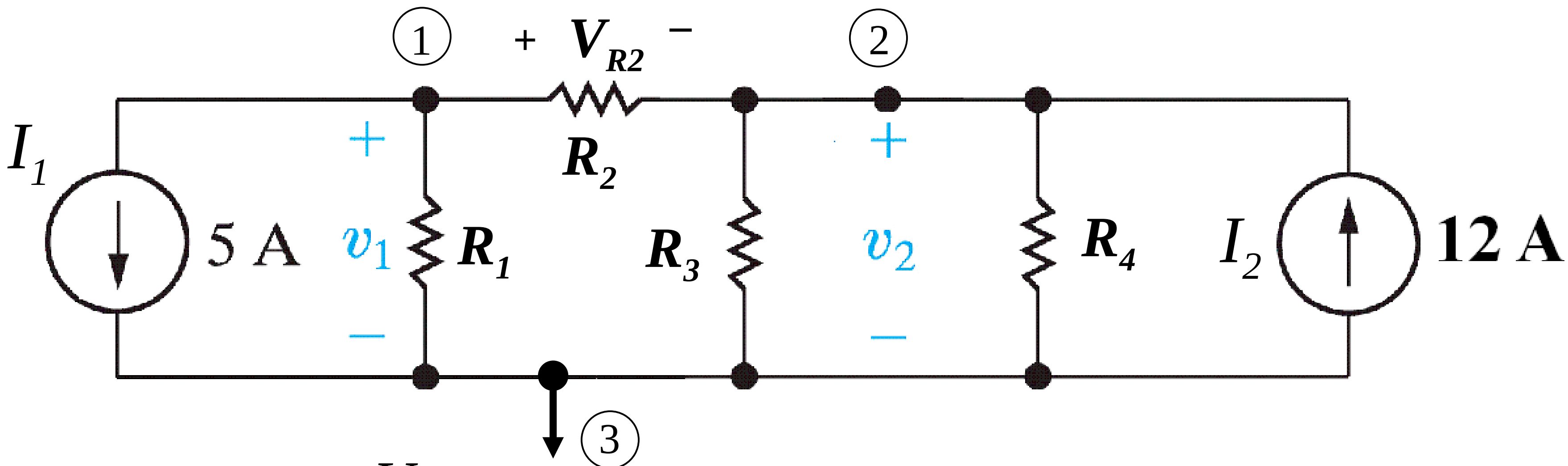
$R_1 = 16\Omega \quad R_2 = 2\Omega \quad R_3 = 20\Omega \quad R_4 = 80\Omega$

$n_e = 3$ Essential Nodes

$(n_e - 1)$ Node Voltage Equations = 2 Node Voltages

Node Voltages V_1 and V_2 Reference Node = 3

Example: Find the node equation for V_1 (Contd.)



1 $$5 + \frac{V_1}{R_1} + \frac{V_{R_2}}{R_2} = 0$$

Currents Out of Node 1

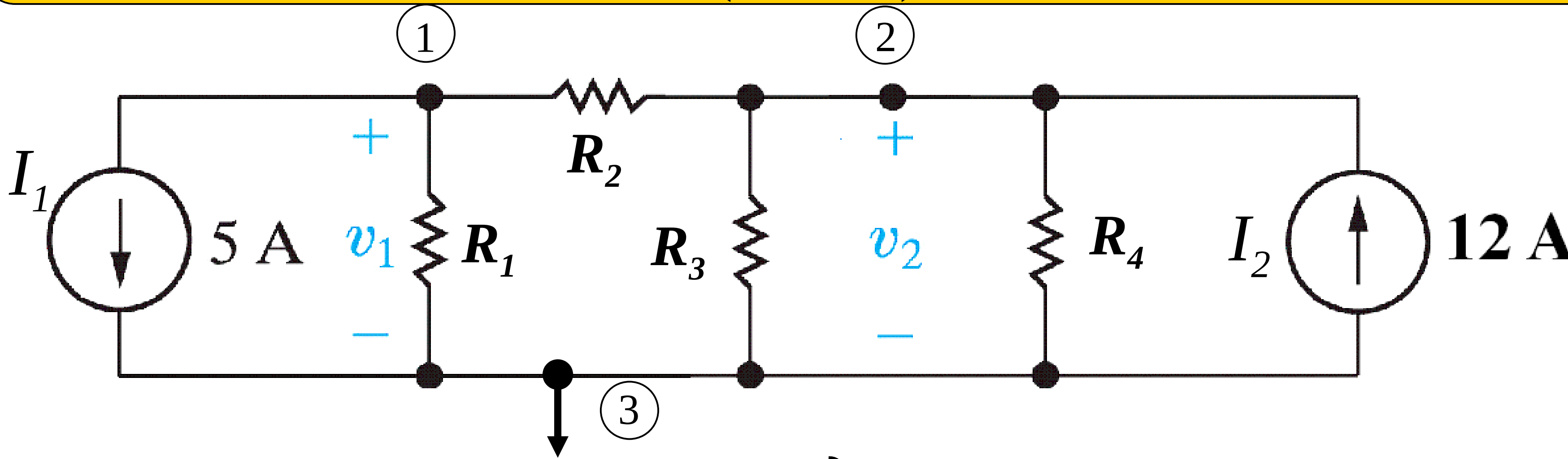
$$5 + \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} = 0$$

$$V_{R_2} = V_1 - V_2$$

1 $$\boxed{5 + \frac{V_1}{16} + \frac{V_1 - V_2}{2} = 0}$$

Node Voltage Equation

Example: Find the node equation for V_2 (Contd.)



2 $$\frac{V_2 - V_1}{\underbrace{R_2}_{}} + \frac{V_2}{R_3} + \frac{V_2}{R_4} - 12 = 0$$ **Currents Out of Node 2**

Opposite to
Node 1 equation

Node 2 Connected to
4 Elements

2 $$\boxed{\frac{V_2 - V_1}{2} + \frac{V_2}{20} + \frac{V_2}{80} - 12 = 0}$$ **Node Voltage Equation**

Example (Contd.)

Solve 2 equations for 2 unknowns

After multiplying 1 by 16

$$V_1 + 8V_1 - 8V_2 = -80 \quad (1)$$

$$\boxed{9V_1 - 8V_2 = -80} \quad (1)$$

After multiplying 2 by 80

$$40V_2 - 40V_1 + 4V_2 + V_2 = -960 \quad (2)$$

$$\boxed{-40V_1 + 45V_2 = 960} \quad (2)$$

Solution

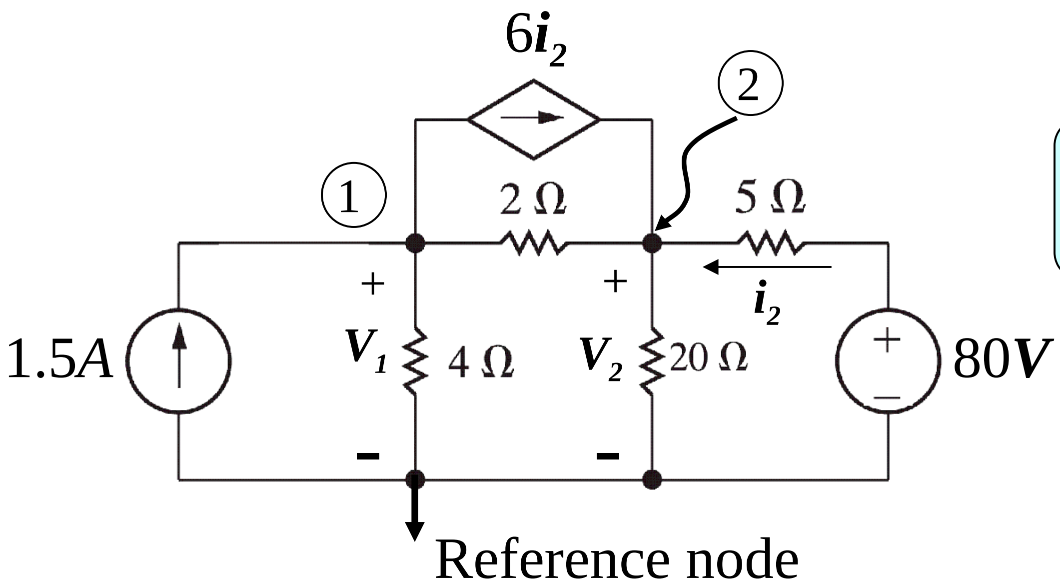
$$\left.\begin{aligned} V_1 &= 48\,(V) \\ V_2 &= 64\,(V) \end{aligned}\right\}$$

Solve two equations for two unknowns

Dependent Sources

1. **How Many Node Voltage Equations?**
2. **Choose "Best" Reference Node.**
3. **Write Independent Node Voltage Equations.**
4. **Find another equation for the dependent source**

Example with a Dependent Source



Need additional equation because of the dependent current source

KVL

$$i_2 = \frac{80 - V_2}{5} \quad (3)$$

(1) $$-1.5 + \frac{V_1}{4} + \frac{V_1 - V_2}{2} + 6i_2 = 0$$

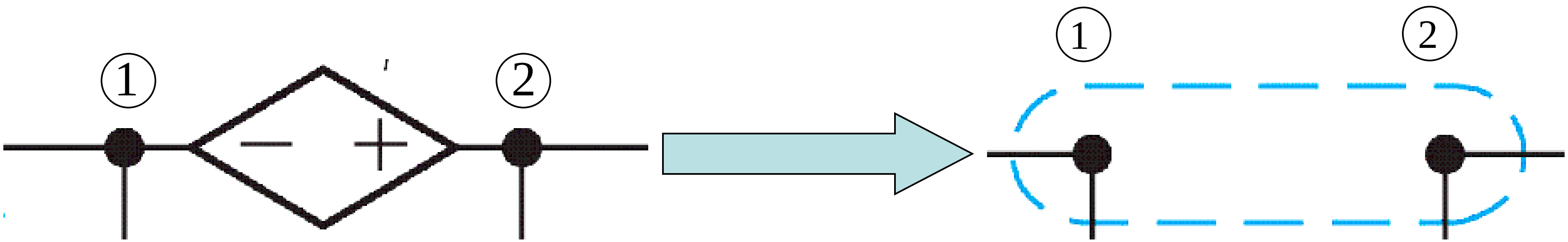
(2) $$\frac{V_2 - V_1}{2} + \frac{V_2}{20} - 6i_2 + \frac{V_2 - 80}{5} = 0$$

Solve 3 equations for 3 unknowns

Results $V_1 = 10(V)$ $V_2 = 60(V)$ $i_2 = 4(A)$

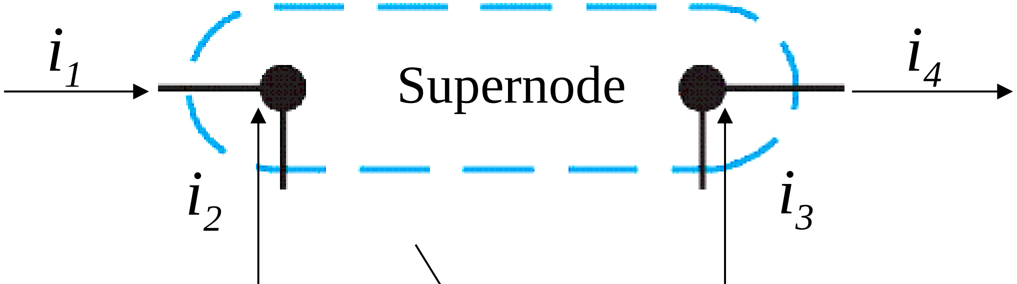
Using the Supernode Concept to deal with a Dependent Voltage Source

Combine 2 Nodes into 1 "Supernode" when a Voltage Source is between them



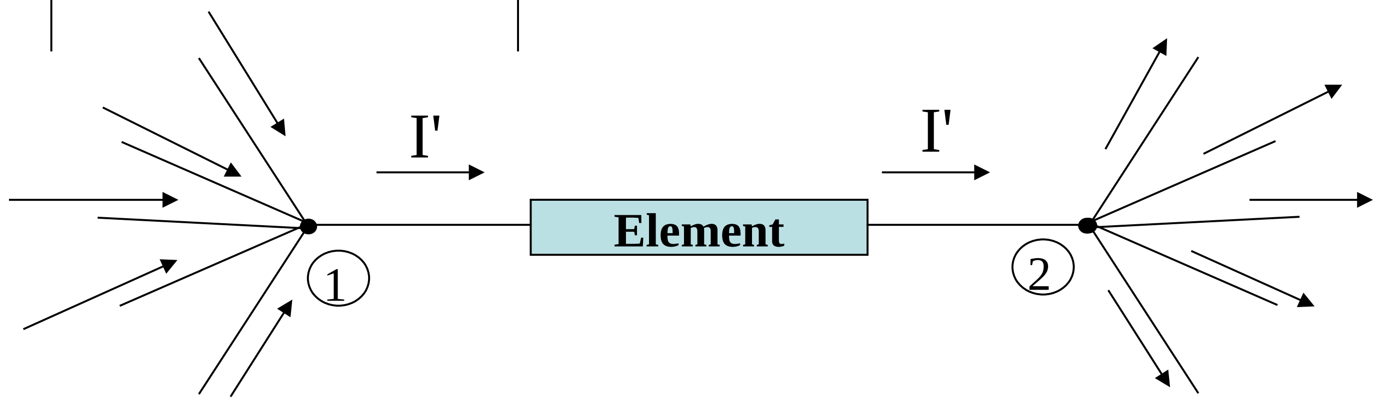
Single Node

KCL Applies to the Supernode



$$i_1 + i_2 + i_3 = i_4$$

Why is this true?



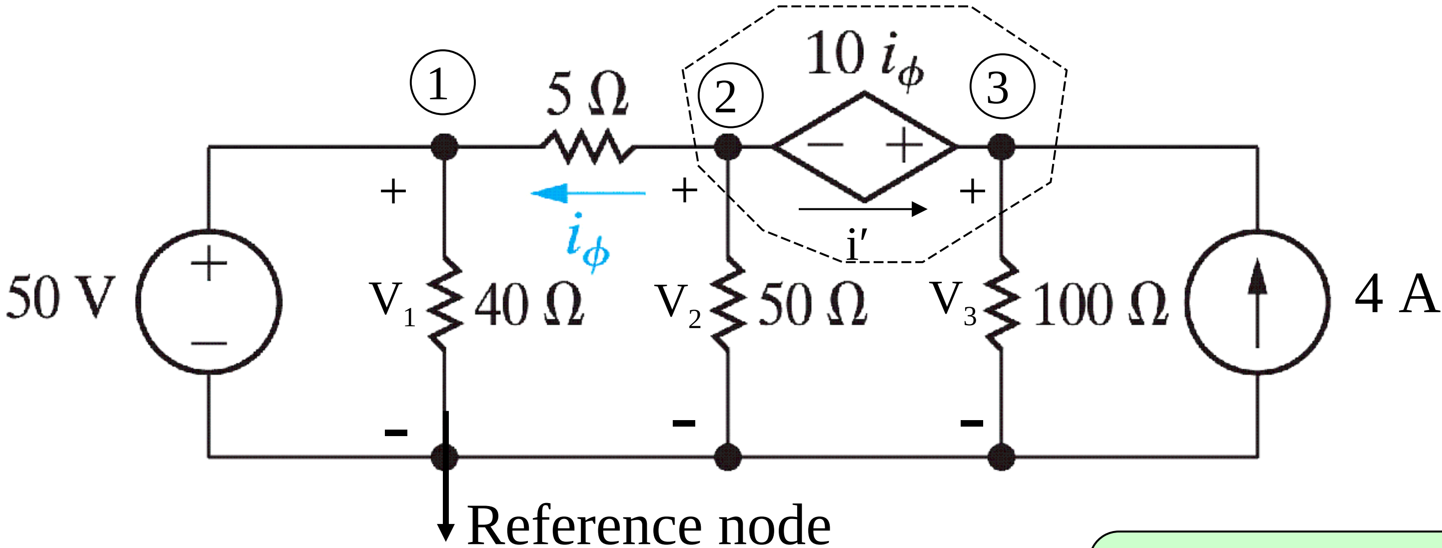
$\sum$ of these currents = I'

$\sum$ of these currents = I'

$$\sum \text{entering } ① = \sum \text{leaving } ②$$

Can combine nodes ① and ② into <u>one</u> supernode, which is treated like a node

Why use the Supernode concept?



Without using the Supernode → 4 Essential Nodes
3 Node Voltage equations

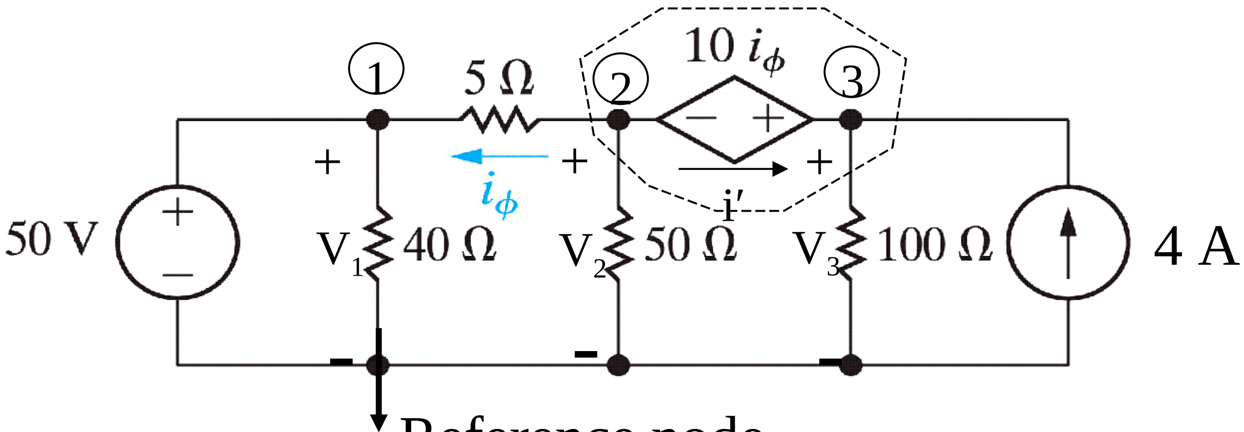
Can't find the current *i'* in terms of the node voltages

Node Equation 2

$$\frac{V_2 - V_2}{5} + \frac{V_2}{50} + (?) = 0$$

Don't know what to put here

Why use the Supernode concept? (Contd.)



Introduce Extra Variable and find extra equation

∴ Introduce <u>new variable</u> ***i′***

(2) $\frac{V_2 - V_1}{5} + \frac{V_2}{50} + i' = 0$ $\Rightarrow$

(3) $-i' + \frac{V_3}{100} - 4 = 0$

(3) $\boxed{i' = \frac{V_3}{100} - 4}$

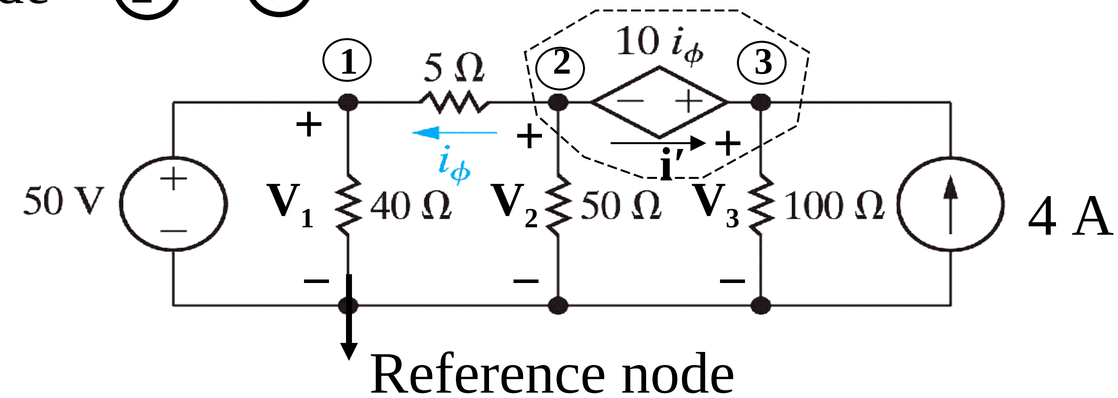
After substituting (3) into (2), becomes

$$\boxed{\frac{V_2 - V_1}{5} - \frac{V_1}{5} + \frac{V_2}{50} + \frac{V_3}{100} - 4 = 0}$$

Can simplify this process with Supernode

Why use the Supernode concept? (Contd.)

$\sum$ i Leaving Supernode 2—3



2—3 $$\frac{V_2 - V_1}{5} + \frac{V_2}{50} + \frac{V_3}{100} - 4 = 0$$

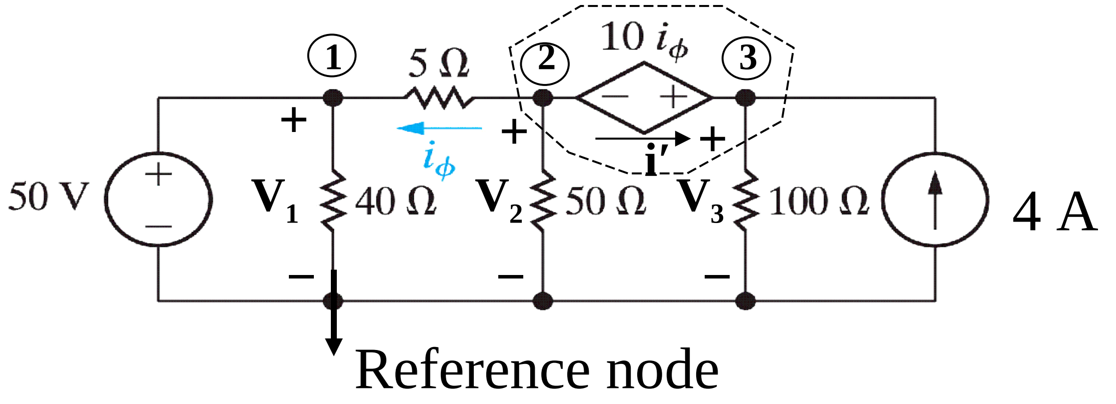
Went from 4 steps to just 1 step

1 Equation and 2 Unknowns

Note $V_1 = 50$

Need to find one more equation

Why use the Supernode concept? (Contd.)



$$V_3 = 10 i_\phi + V_2$$

Use KVL to find another equation

$$i_\phi = \frac{V_2 - V_1}{5}$$

Use Ohm's Law to find expression for i_ϕ

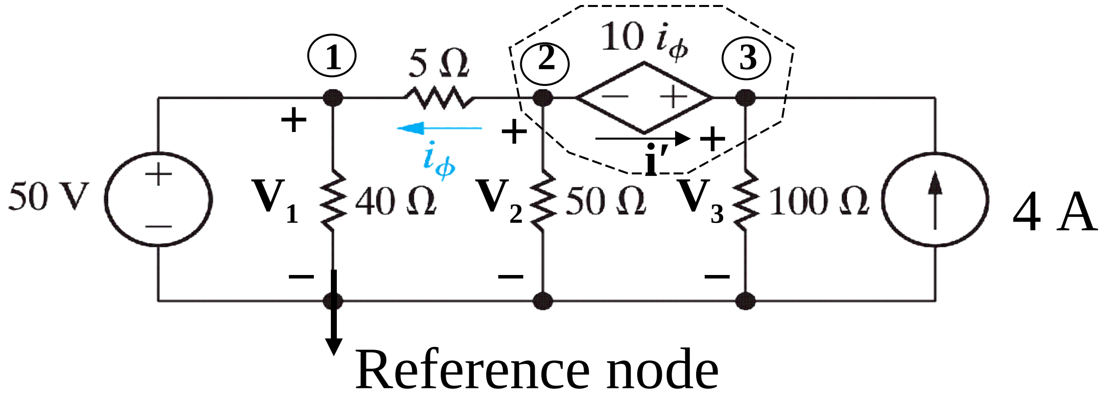
$$V_3 = 2V_2 - 2V_1 + V_2$$

After using expression for i_ϕ

$$V_3 = 3V_2 - 2V_1 = 3V_2 - 2(50)$$

Substitute V_1 = 50

Why use the Supernode concept? (Contd.)



$$V_3 = 3V_2 - 100$$ **New equation**

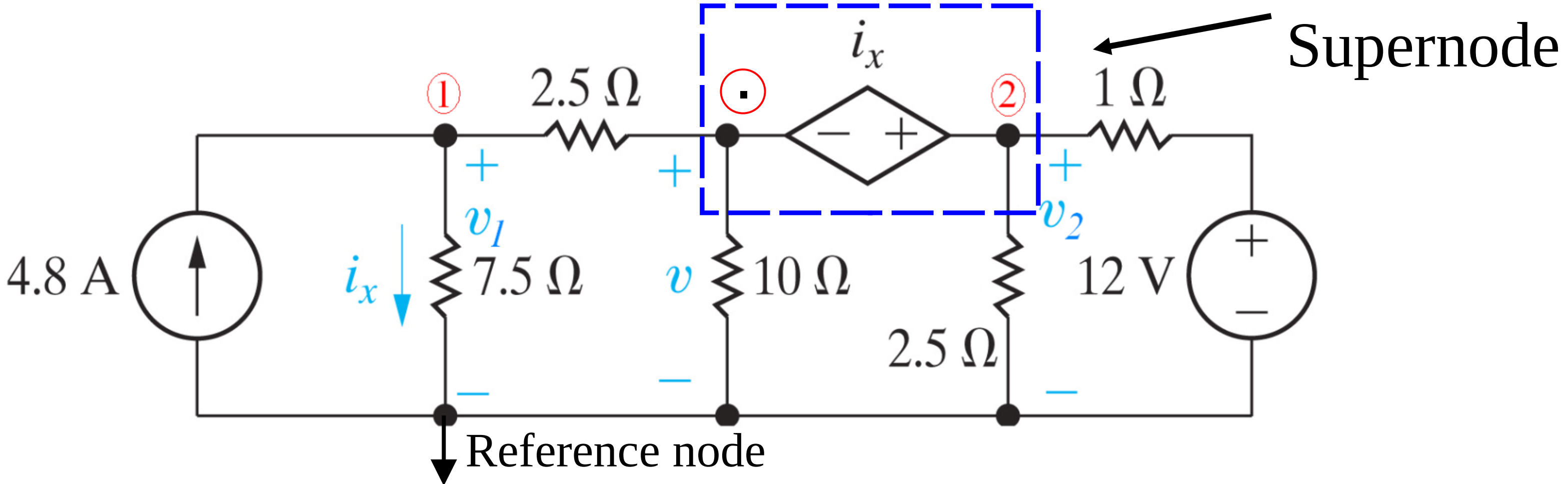
2 — 3 Substitute $V_1 = 50$

$$\frac{V_2 - 50}{5} + \frac{V_2}{50} + \frac{3V_2 - 100}{100} - 4 = 0$$

Solve
2 equations
for
2 unknowns

<u>Results:</u> $V_2 = 60(V)$ $i_\phi = 2(A)$ $V_3 = 80(V)$

Drill Exercise: Find v



$n_e = 4$ } Find 3 equations

$$① \quad -4.8 + \frac{v_1}{7.5} + \frac{v_1 - v}{2.5} = 0$$

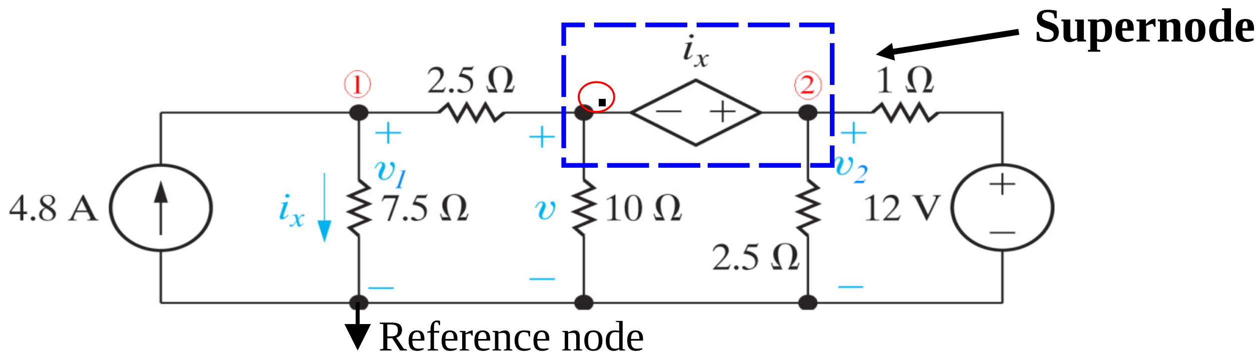
Supernode: 4 Currents Exiting The Supernode

$$\cdot — ② \quad \frac{v - v_1}{2.5} + \frac{v}{10} + \frac{v_2}{2.5} + \frac{v_2 - 12}{1} = 0$$

2 Equations, 3 Unknowns: v, v_1, v_2

(Contd…)

Drill Exercise (Contd.)



Relate i_x to Node Voltages

$$i_x = \frac{v_1}{7.5}$$

Use Ohm's Law

$$v_2 = \underbrace{i_x}_{\text{voltage}} + v$$

Use KVL to find new equation

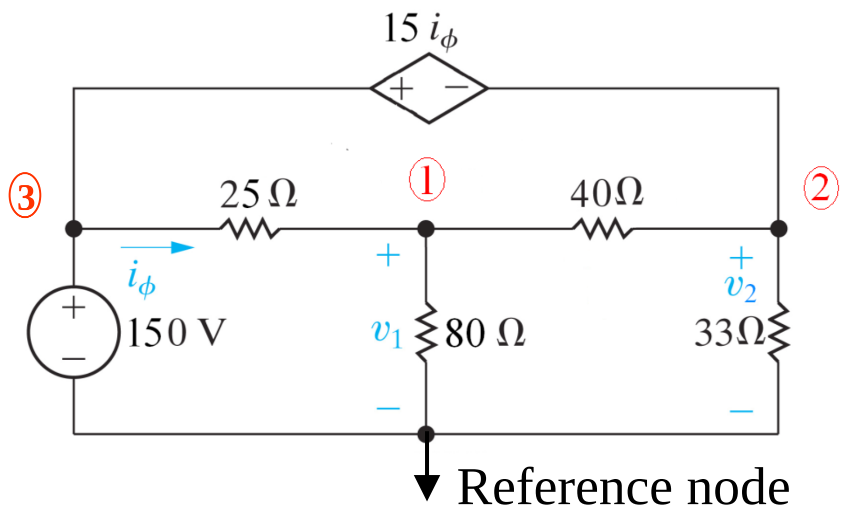
③ $$\boxed{v_2 = \frac{v_1}{7.5} + v}$$

Eliminate i_x to write new equation in terms of node voltages

Answer ⇒ $\boxed{v = 8(V)}$

Solve 3 equations for 3 unknowns
① , ⊙—② and ③

Drill Exercise: Find v_1



$n_e = 4$ } 3 node equations

{ Dealing with a dependent source without using a supernode }

Notice: $v_3 = 150\text{V} \Rightarrow$ 2 Unknowns

$\{ v_2 = -15 i_\varphi + 150 \}$ **KVL around outer loop**

$$i_\varphi = \frac{150 - v_1}{25}$$

Ohm's Law at 25Ω resistor
i_ϕ in terms of v_1

Solved 1 equation for 1 unknown

Node equation

1 $$\frac{v_1 - 150}{25} + \frac{v_1}{80} + \frac{v_1 - v_2}{40} = 0 \Rightarrow \frac{v_1 - 150}{25} + \frac{v_1}{80} + \frac{v_1 - (150 - 15 i_\varphi)}{40} = 0 \Rightarrow \boxed{v_1 = 120(V)}$$

Drill Exercise: Find v_o

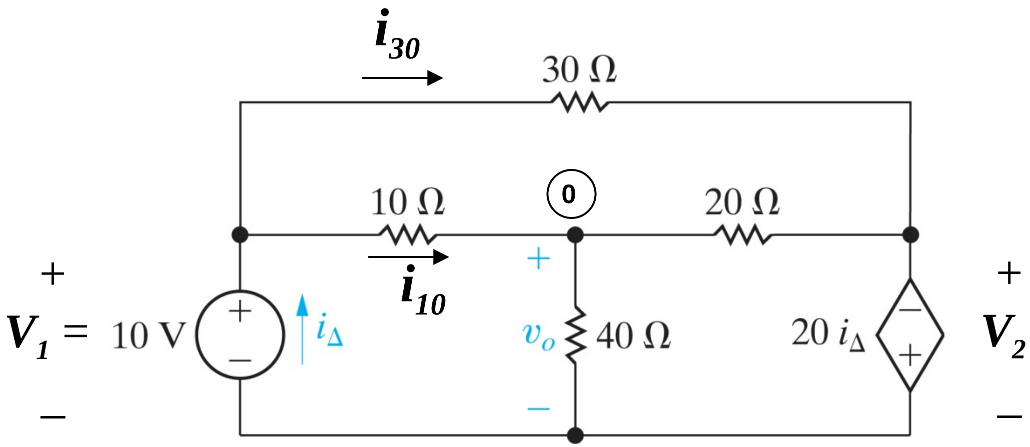


Figure: 04-17-03

Need to account for the dependent source

Need equation for i_Δ

$n_e = 4$; 3 node voltages: v_1, v_2, v_0 $v_1 = 10$ V

$v_2 = -20 i_\Delta$ (dependent source)

$$i_\Delta = i_{10} + i_{30}$$ **KCL**

$$i_\Delta = \frac{10 - v_0}{10} + \frac{10 - v_2}{30}$$

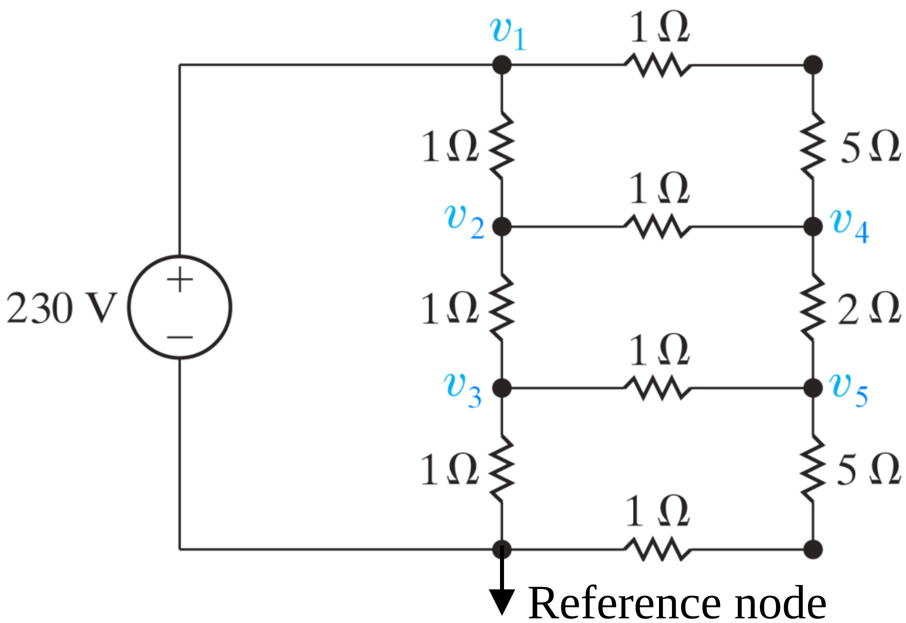
Write currents in terms of Node Voltages

Solve 3 equations for 3 unknowns

$$\frac{v_0 - 10}{10} + \frac{v_0}{40} + \frac{v_0 - v_2}{20} = 0$$

Node Equation at 0

Node Analysis Example



6 essential nodes; therefore, we must find 5 node voltage equations

Reference node at bottom is one of the node voltages

1) $v_1 = 230(V)$ **Due to voltage source**

(2) $$\frac{v_2 - 230}{1} + \frac{v_2 - v_4}{1} + \frac{v_2 - v_3}{1} = 0$$

(3) $$\frac{v_3 - v_2}{1} + \frac{v_3 - v_5}{1} + \frac{v_3}{1} = 0$$

(4) $$\frac{v_4 - 230}{(5+1)} + \frac{v_4 - v_2}{1} + \frac{v_4 - v_5}{2} = 0$$

(5) $$\frac{v_5 - v_4}{2} + \frac{v_5 - v_3}{1} + \frac{v_5}{(5+1)} = 0$$

Simplifying and Putting Equations in Matrix Form

4 equations   4 unknowns

$$3v_2 - v_3 - v_4 + 0v_5 = 230$$
$$-v_2 + 3v_3 + 0v_4 - v_5 = 0$$
$$-6v_2 + 0v_3 + 10v_4 - 3v_5 = 230$$
$$0v_2 - 6v_3 - 3v_4 + 10v_5 = 0$$

$$A = \begin{vmatrix} 3 & -1 & -1 & 0 \\ -1 & 3 & 0 & -1 \\ -6 & 0 & 10 & -3 \\ 0 & -6 & -3 & 10 \end{vmatrix}$$

Put in Matrix Form

$$AX = Y$$

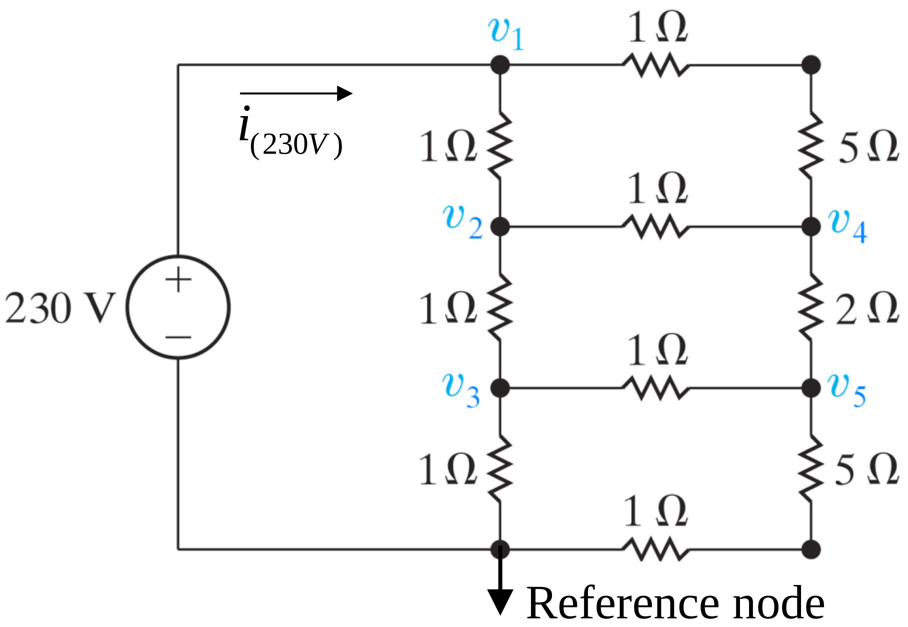
$$X = \begin{bmatrix} v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} \qquad Y = \begin{bmatrix} 230 \\ 0 \\ 230 \\ 0 \end{bmatrix}$$

Solution $X = A^{-1}Y$ **Calculator**

$$X = \begin{bmatrix} 150 \\ 80 \\ 140 \\ 90 \end{bmatrix} \Rightarrow \left.\begin{matrix} v_2 = 150 \\ v_3 = 80 \\ v_4 = 140 \\ v_5 = 90 \end{matrix}\right\}$$

Node Voltages

Node Analysis Example (Contd.)



Find power supplied by the source

$i_{(230V)}$ = Current flowing into node (1)

$$i_{(230V)} = \frac{v_1 - v_2}{1} + \frac{v_1 - v_4}{5+1}$$ **KCL**

$$= \frac{230 - 150}{1} + \frac{230 - 140}{6}$$ $v_2 = 150$, $v_4 = 140$

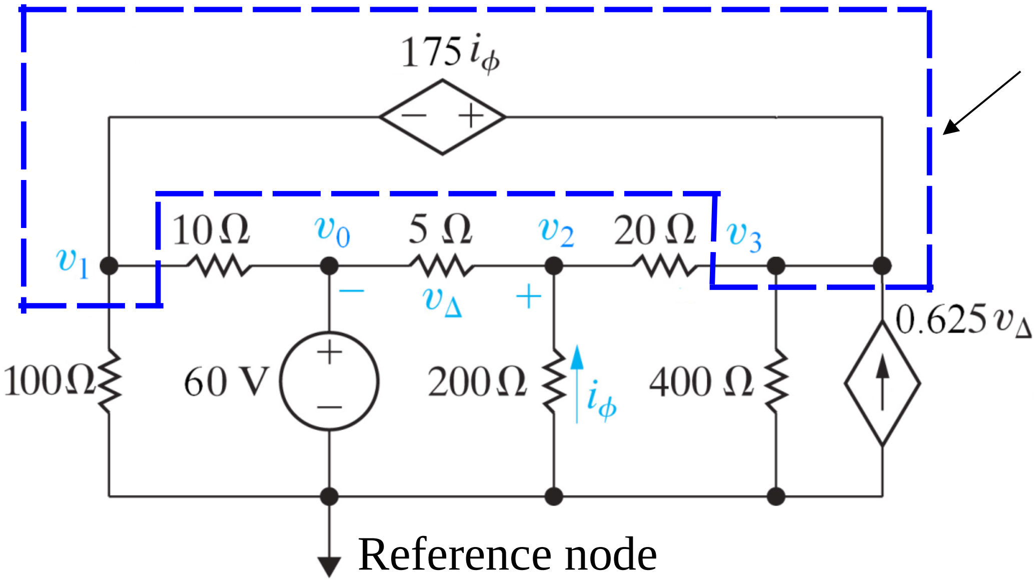
$$= 80 + 15$$

$$\boxed{i_{(230V)} = 95\,(A)}$$

$$p = Vi$$

$\boldsymbol{p}_{(230V)}$ = (230V)(95A) = 21,850 (W) **Supplied**

Example: Find the Node Voltages



Supernode

5 Essential Nodes ➡ 4 Node Voltages

v_0 = 60 V ; due to our choice of Reference Node.

Dependent voltage source between (1) and (3) creates a problem: make this a **Supernode**

Think of supernode as a "Blob"

Σ *i* out of the "blob" = 0

Supernode

1 — 3 Node Equation

$$\frac{v_1}{100}+\frac{v_1-60}{10}+\frac{v_3-v_2}{20}+\frac{v_3}{400}-0.625v_\Delta=0$$

Multiply by 400

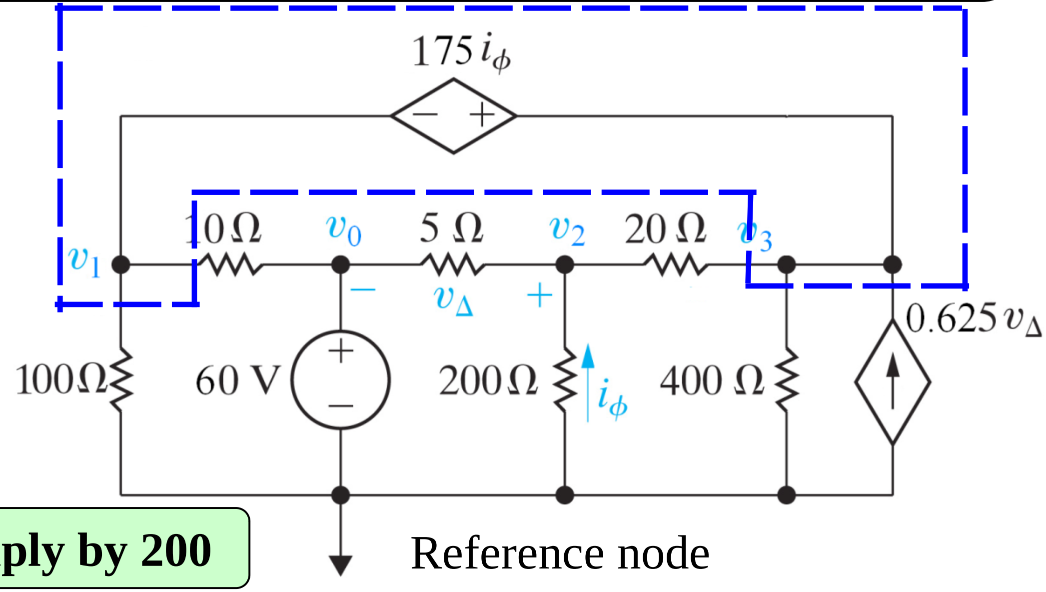
$$4v_1+40v_1-2400+20v_3-20v_2+v_3-250v_\Delta=0$$

Simplify

$\boxed{1}$

$$44v_1-20v_2+21v_3-250v_\Delta=2400$$

Node Analysis Example (Contd.)



Node equation at 2

$$\frac{v_2 - 60}{5} + \frac{v_2}{200} + \frac{v_2 - v_3}{20} = 0$$

Multiply by 200

$$40v_2 - 2400 + v_2 + 10v_2 - 10v_3 = 0$$

Simplify

$$\boxed{51v_2 - 10v_3 = 2400} \quad \boxed{2}$$

$$\boxed{44v_1 - 20v_2 + 21v_3 - 250v_\Delta = 2400} \quad \boxed{1}$$

2 equations

4 unknowns

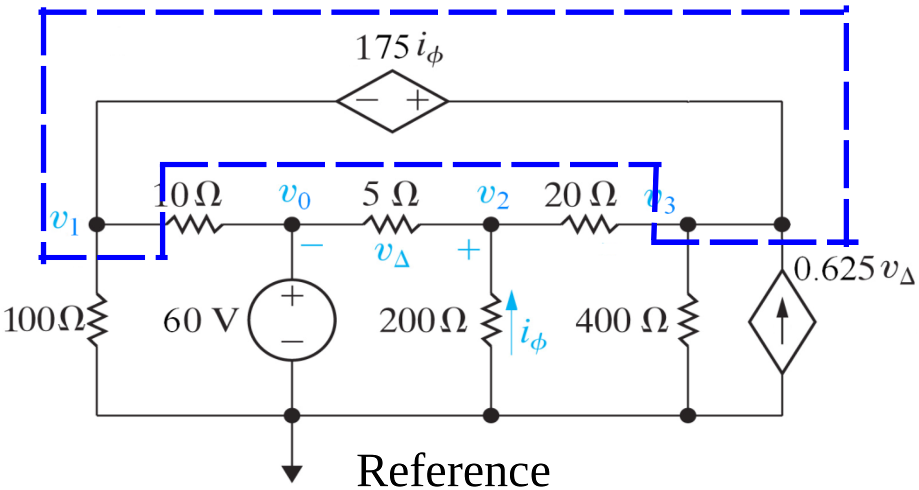
Need two more equations

$$v_\Delta = v_2 - 60 \quad \boxed{3}$$

KVL

Node Analysis Example (Contd.)

Since there is a dependent voltage source between 1 and 3, we can relate v_1 to v_3



a $v_2 = -200 i_\phi$ — **Ohm's Law**

b $v_1 = -175 i_\phi + v_3$ — **KVL**

$$\therefore v_1 = \frac{7}{8} v_2 + v_3 \quad \boxed{4}$$

Eliminate i_ϕ from b

Plugging $\boxed{3}$ and $\boxed{4}$ into $\boxed{1}$, we obtain

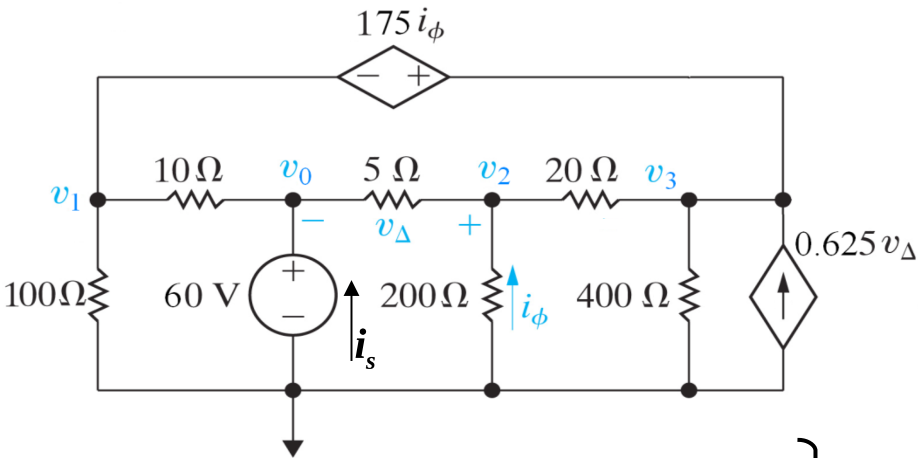
$$-231.5 v_2 + 65 v_3 = -12{,}600 \quad \boxed{5}$$

Solving $\boxed{2}$ and $\boxed{5}$ simultaneously:

v_2 = 30V

v_3 = -87V $\boxed{4} \Rightarrow$ **v_1 = -60.75(V)**

Node Analysis Example (Contd.)



Find Power of the 60V source

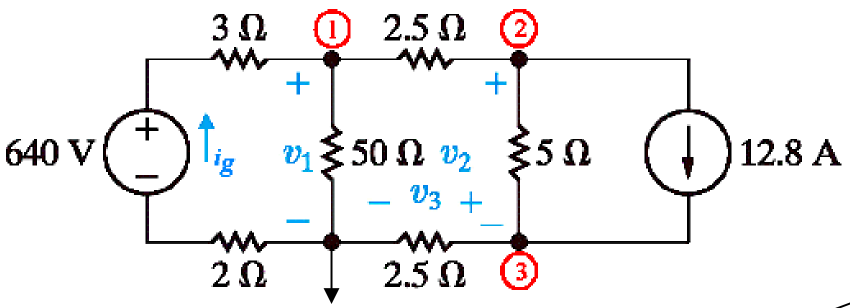
$$p_{60V} = 60V \cdot (i_s)$$ **Power Equation**

$$i_s = \frac{v_0 - v_1}{10} + \frac{v_0 - v_2}{5}$$ **KCL $\Sigma i_{in} = \Sigma i_{out}$ and Ohm's Law**

$$i_s = \frac{60 - (-60.75)}{10} + \frac{60 - 30}{5} = 18.075(A)$$ **Plug in numbers**

$$p_{60V} = 60V \cdot (18.075) = 1084.5(W)$$ **Power Delivered**

Drill Exercise: Find Node Voltages



$$
\begin{cases}
① \quad \dfrac{v_1 - 640}{3+2} + \dfrac{v_1}{50} + \dfrac{v_1 - v_2}{2.5} = 0 \\
② \quad \dfrac{v_2 - v_1}{2.5} + \dfrac{v_2 - v_3}{5} + 12.8 = 0 \\
③ \quad \dfrac{v_3}{2.5} + \dfrac{v_3 - v_2}{5} - 12.8 = 0
\end{cases}
$$

$n_e = 4$ } **3 equations**

Simplify

① $31v_1 - 20v_2 + 0v_3 = 6400$ (multiply by 50)

② $-2v_1 + 3v_2 - v_3 = -64$ (multiply by 5)

③ $0v_1 - v_2 + 3v_3 = 64$ (multiply by 5)

Solution

$v_1 = 380(V); \quad v_2 = 269(V); \quad v_3 = 111(V)$

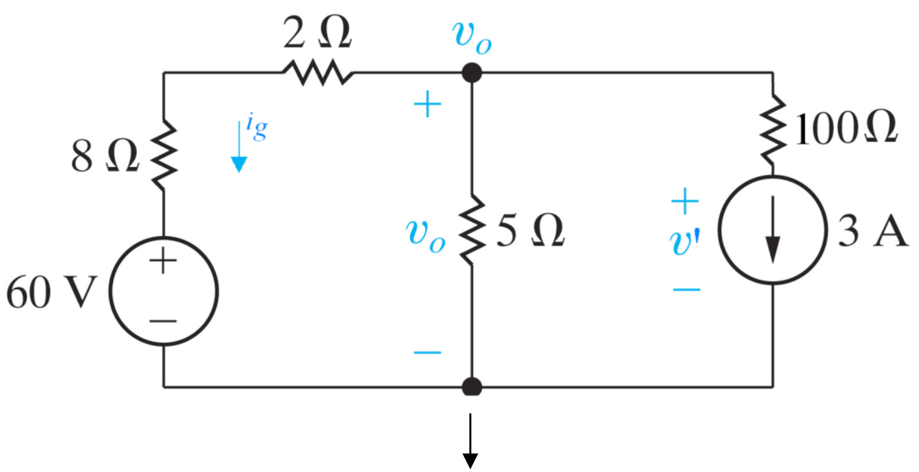
Find power of *640V* source

$$i_g = \frac{640 - v_1}{3+2} = \frac{640 - 380}{5} = 52(A)$$

$$p = vi_g = 640(52) = 32,280(W)$$

Power Delivered

Drill Exercise: Find Power of the Sources



a) 1 node equation

$$\frac{v_0 - 60}{8+2} + \frac{v_0}{5} + 3 = 0$$

$$v_0 - 60 + 2v_0 + 30 = 0$$

$$\boxed{v_0 = 10(V)}$$

b) Find p_{3A}(delivered) **Find v'**

$$v_0 = v_{100\Omega} + v' = 300 + v'$$ Substitute $v_{100\Omega} = 300(V)$

$$v' = v_0 - 300 = 10 - 300$$ **Simplify**

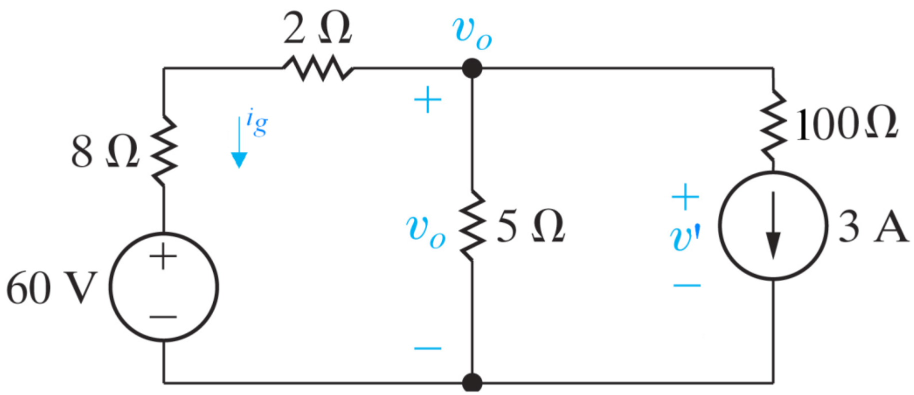
$$\boxed{v' = -290(V)}$$

$$p_{3A} = v'I$$ **Passive sign convention**

$$p_{3A} = -290(3) = -870(W)$$ $p < 0$ Power extracted from

$$p_{3A} = +870(W)$$ *Delivered*

Drill Exercise (Contd.)



c) Find $p_{60V} = (60) i_g$ } **Passive sign convention**

Find i_g $\Rightarrow$ $i_g = \dfrac{v_0 - 60}{8+2} = \dfrac{10-60}{8+2} = -\dfrac{50}{10} = -5(A)$

$p_{60V} = 60(-5) = -300(W)$ } $p < 0$ Power extracted from

$p_{60V} = 300(W)$ } *Delivered*

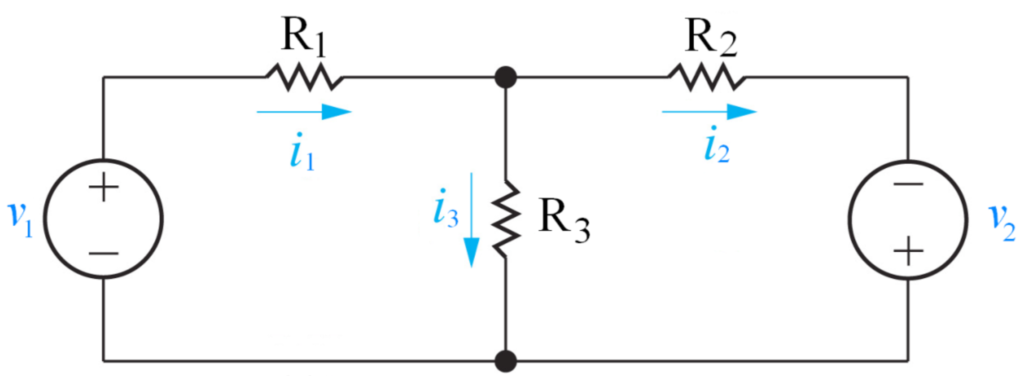
$\boxed{\Sigma p_{dev} = 870 + 300 = 1170(W)}$ } **Total Power Delivered**

Mesh Current Analysis: Planar Circuits

Mesh: Loop with no other loops inside

The circuit can be described by: $b_e - (n_e - 1)$ equations.

Don't worry about this definition.
Need an equation; therefore a mesh current, for each mesh



- i_1, i_2, i_3 **branch currents**
- **3 unknowns**

2 equations 2 unknowns

KVL $v_1 = i_1R_1 + i_3R_3$

KVL $v_2 = i_2R_2 - i_3R_3$

2 equations 3 unknowns

KCL $i_3 = i_1 - i_2$

Additional equation

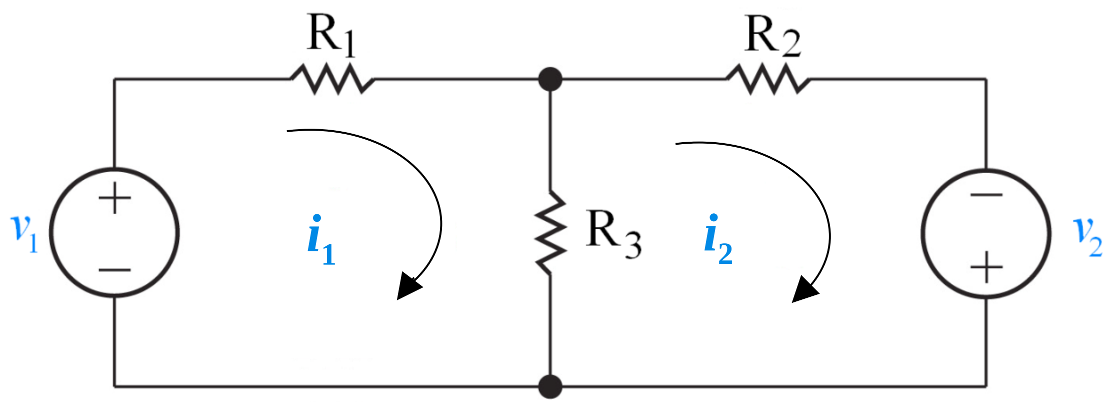
Mesh Equations

$v_1 = i_1(R_1+R_3) - i_2R_3$ (1)

$v_2 = -i_1R_3 + i_2(R_2 + R_3)$ (2)

Mesh Currents

Mesh Current: Current flowing only along the perimeter of a mesh

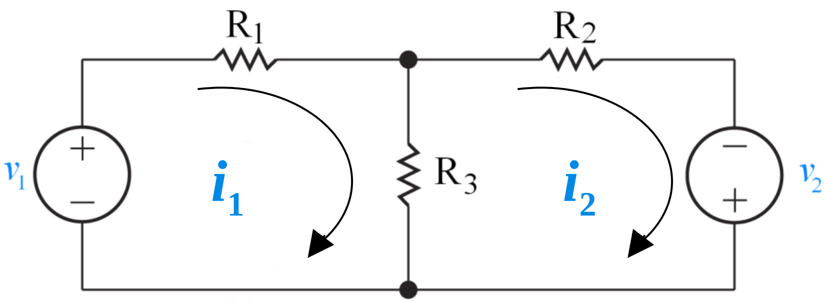


i_1 and i_2 are the mesh currents

m meshes ➡ m currents ➡ m equations!

Mesh Current

Current flowing only along the perimeter of a mesh



i_1, i_2 Mesh Currents

m meshes → **m** currents → **m** equations

Current through any branch is determined by considering the mesh currents flowing in <u>every mesh in which the branch belongs</u>

- $\mathbf{R_1}$ only in Mesh 1 $\qquad$ i_1 identified as branch current
- $\mathbf{R_2}$ only in Mesh 2 $\qquad$ i_2 identified as branch current
- $\mathbf{R_3}$ in both the Meshes $\qquad$ $(i_1 - i_2)$ or $(i_2 - i_1)$ is branch current

Mesh current into a node also flows out of it

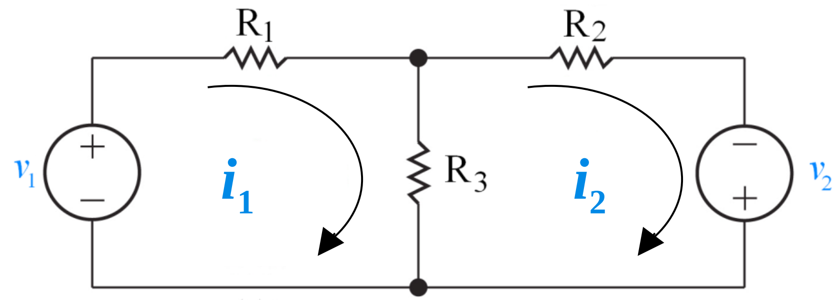
KCL is Automatically Satisfied

Mesh Analysis

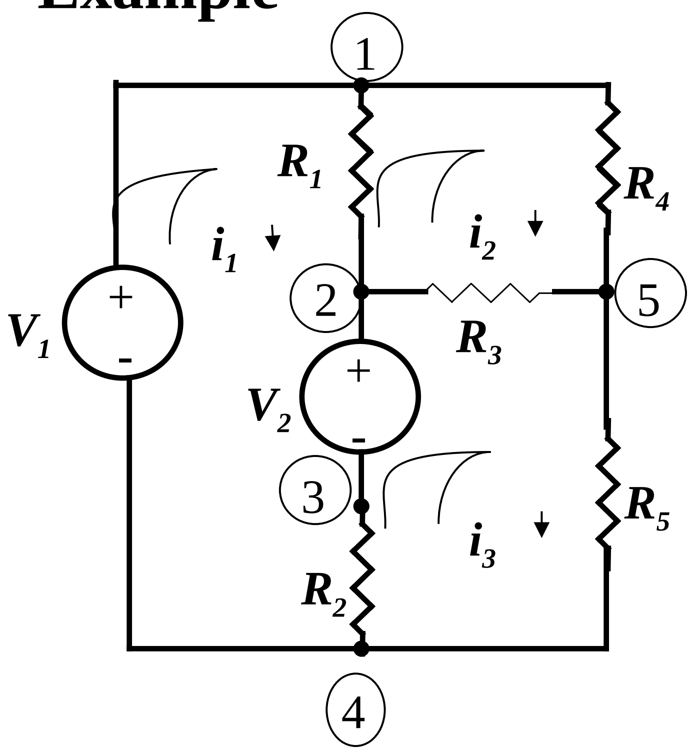
To analyze a circuit, use KVL

$$\left.\begin{array}{ll} ① & V_1 = i_1R_1 + (i_1 - i_2)R_3 \\ ② & V_2 = i_2R_2 + (i_2 - i_1)R_3 \end{array}\right\}$$

Mesh equations found using KVL



Example

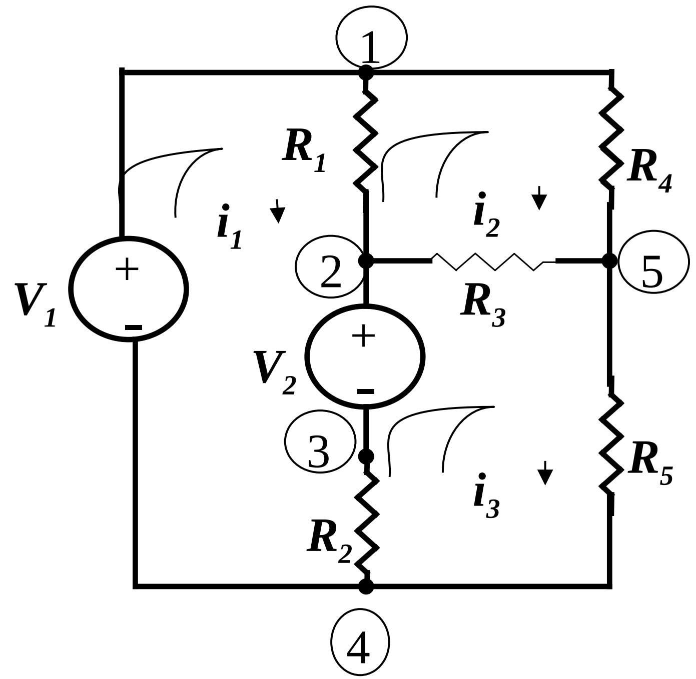


How many meshes and equations in this circuit?

3 meshes

3 equations

Example: Mesh Equations (Contd.)



Mesh 1	$V_1 = (i_1 - i_2)R_1 + V_2 + (i_1 - i_3)R_2$	i_1 is positive in mesh 1
Mesh 2	$i_2R_4 + (i_2 - i_3)R_3 + (i_2 - i_1)R_1 = 0$	i_2 is positive in mesh 2
Mesh 3	$V_2 = (i_3 - i_2)R_3 + i_3R_5 + (i_3 - i_1)R_2$	i_3 is positive in mesh 3

Example: Mesh Equations

$$V_1 - V_2 = (i_1 - i_2)R_1 + (i_1 - i_3)R_2 \qquad \boxed{1}$$

$$0 = (i_2 - i_1)R_1 + (i_2 - i_3)R_3 + i_2 R_4 \qquad \boxed{2}$$

$$V_2 = (i_3 - i_1)R_2 + (i_3 - i_2)R_3 + i_3 R_5 \qquad \boxed{3}$$

Note:

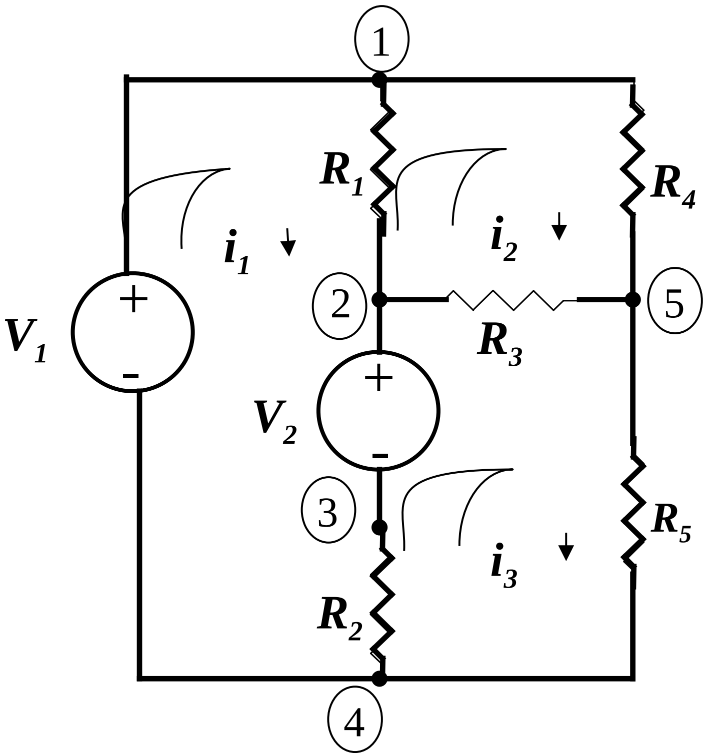
$\mathbf{R_1}$ in meshes $\boxed{1}$ & $\boxed{2}$ $\Rightarrow$ i_1 & i_2 $\mathbf{R_2}$ in meshes $\boxed{1}$ & $\boxed{3}$ $\Rightarrow$ i_1 & i_3

$\mathbf{R_3}$ in meshes $\boxed{2}$ & $\boxed{3}$ $\Rightarrow$ i_2 & i_3 $\mathbf{R_4}$ in mesh $\boxed{2}$ ***only*** $\Rightarrow$ i_2

$\mathbf{R_5}$ in mesh $\boxed{3}$ ***only*** $\Rightarrow$ i_3

Solving Simultaneous Equations

Previous Example with Numbers



$V_1 = 7V$ $\quad$ $V_2 = 6V$

$R_1 = 1\Omega$ $\quad$ $R_2 = 2\Omega$

$R_3 = 3\Omega$ $\quad$ $R_4 = 2\Omega$

$R_5 = 1\Omega$

$$V_1 = (i_1 - i_2)R_1 + V_2 + (i_1 - i_3)R_2 \quad \boxed{1}$$

$$0 = i_2R_4 + (i_2 - i_3)R_3 + (i_2 - i_1)R_1 \quad \boxed{2}$$

$$V_2 = (i_3 - i_2)R_3 + i_3R_5 + (i_3 - i_1)R_2 \quad \boxed{3}$$

Put in matrix form

$$(R_1 + R_2)\mathbf{i_1} - R_1\mathbf{i_2} - R_2\mathbf{i_3} = V_1 - V_2$$

$$(-R_1)\mathbf{i_1} + (R_1 + R_3 + R_4)\mathbf{i_2} - R_3\mathbf{i_3} = 0$$

$$(-R_2)\mathbf{i_1} - R_3\mathbf{i_2} + (R_2 + R_3 + R_5)\mathbf{i_3} = V_2$$

Separate the currents out

Solving Simultaneous Equations

Previous Example with Numbers

Plug Numbers Into the Matrix Form

$$\left.\begin{aligned} 3i_1 - i_2 - 2i_3 &= 1 \\ -i_1 + 6i_2 - 3i_3 &= 0 \\ -2i_1 - 3i_2 + 6i_3 &= 6 \end{aligned}\right\}$$

3 Equations
3 Unknowns
How do we solve?

Put in Matrix Notation

$$\underbrace{\begin{bmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix}}_{\text{3x3 Square Matrix Of Coefficients}} \underbrace{\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}}_{\text{Column Variables}} = \underbrace{\begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}}_{\text{Column Variables}}$$

Solve The Matrix Equation

$$AX = Y \qquad A = \begin{bmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix} \qquad X = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} \qquad Y = \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$$

$$X = A^{-1}Y$$

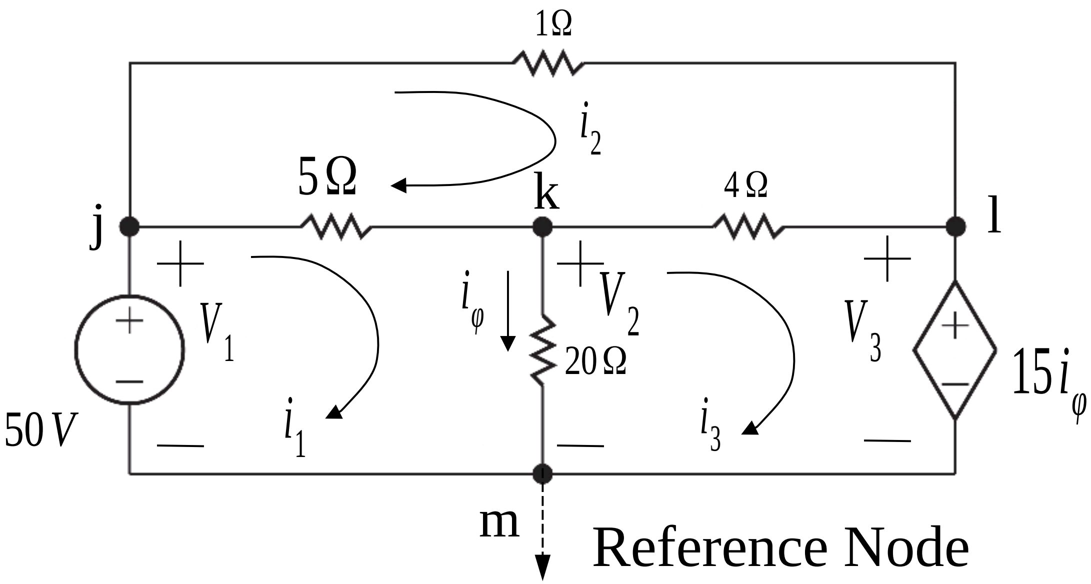
Use Calculator

$$X = \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$$

Answer in vector form

$$i_1 = 3 \ (A) \qquad i_2 = 2 \ (A) \qquad i_3 = 3 \ (A)$$

Dependent Sources AND Choosing Nodal Or Mesh Method: Find $p_{4\Omega}$



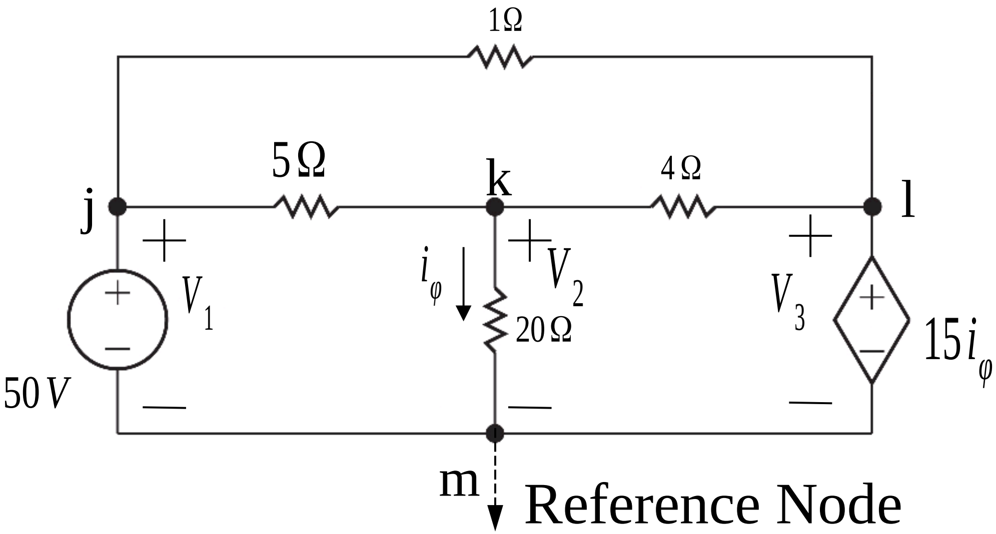
4 essential nodes

3 meshes

Dependent Voltage Source

Book uses Mesh analysis → 3 meshes → 3 equations

Easier to use Node Voltage method



3 node voltages; however,

$V_1 = 50(V)$ Independent Voltage Source

$V_3 = 15 i_\varphi$ Dependent Voltage Source

node (k)

$$\left.\frac{V_2 - 50}{5} + \frac{V_2}{20} + \frac{V_2 - 15 i_\varphi}{4} = 0 \right\}$$ **Multiply by 20** $\Rightarrow$ $4V_2 - 200 + V_2 + 5V_2 - 75 i_\varphi = 0$

$\left.10V_2 - 75 i_\varphi = 200\right\}$ **Simplified Equation**

Ohms Law $\left\{ i_\phi = \frac{V_2}{20} \Rightarrow \right.$ $10V_2 - \frac{75}{20}V_2 = 200$

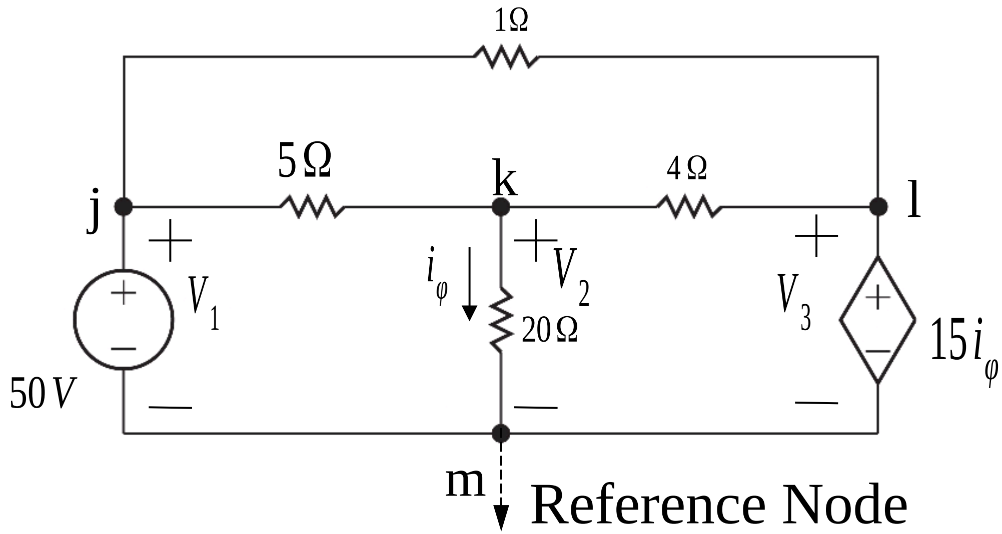
Eliminate i_ϕ

$V_2 = 32(V)$

$i_\varphi = 1.6(A)$

$V_3 = 15 i_\varphi = 24(V)$

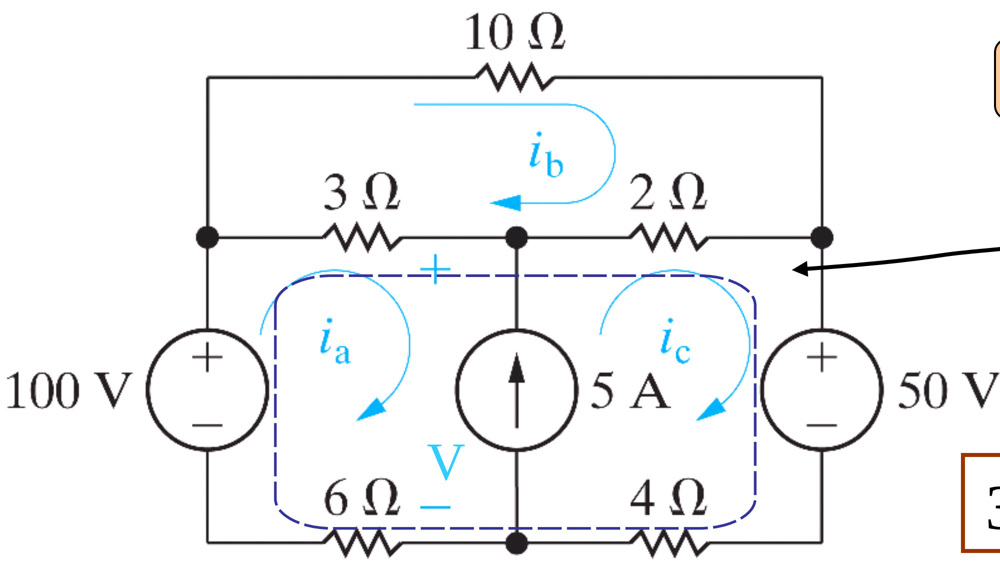
Easier to use Node Voltage method (Contd.)



$$p_{4\Omega} = V_{4\Omega}^2 / 4 \qquad \text{where } V_{4\Omega} = V_2 - V_3 \Big\} \ \boxed{\textbf{KVL}}$$

$$p_{4\Omega} = \frac{(V_2 - V_3)^2}{4} = \frac{(32 - 24)^2}{4} \Rightarrow \boxed{p_{4\Omega} = 16(W)}$$

Mesh analysis with a Current Source



Supermesh

3 "ways" to handle this situation

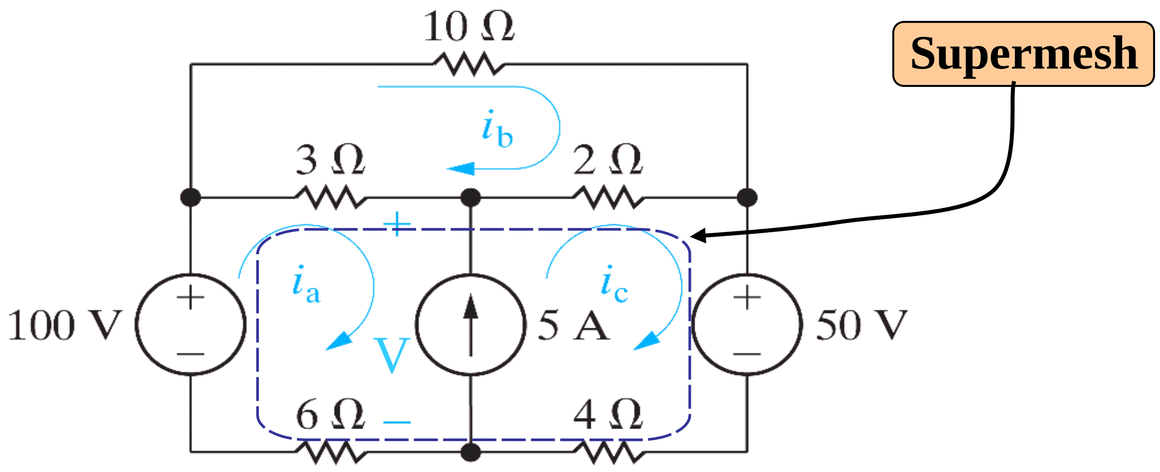
1 Mesh analysis via KVL; however, we must

Introduce unknown voltage, V, across the current source.

KCL

Note that there is a constraint on i_a and i_c $(i_c = 5(A) + i_a)$

Mesh analysis with a Current Source
(Contd.)



MESH

a: $100 = 3(i_a - i_b) + V + 6i_a$

c: $-50 = 2(i_c - i_b) + 4i_c - V$

b: $0 = 10i_b + 2(i_b - i_c) + 3(i_b - i_a)$

$i_c = 5(A) + i_a$ } **Constraint**

Solve 4 Equations for 4 unknowns

Can solve these equations for i_a, i_b, i_c and V

Mesh analysis with a Current Source (Contd.)

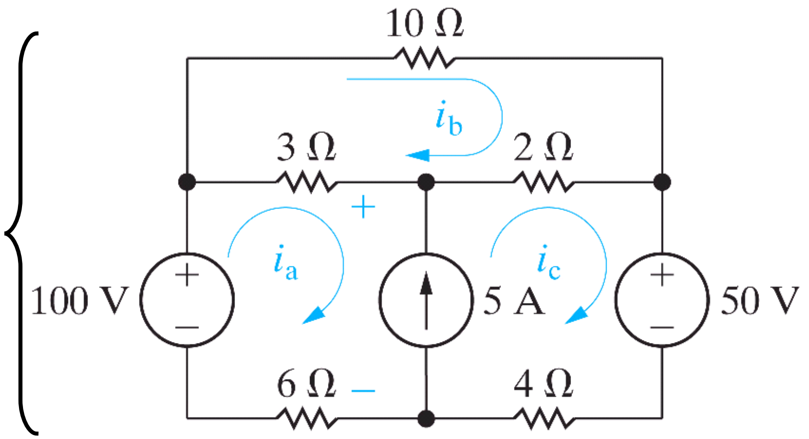
2 **Supermesh:** Use KVL around supermesh, in terms of original mesh currents.

3 **Just Sum Voltages Around the Outer "Loop"**

This is equivalent to a Supermesh

Mesh analysis with a Current Source (Contd.)

New approach to solve the same problem



Write KVL around loops that excludes the current source

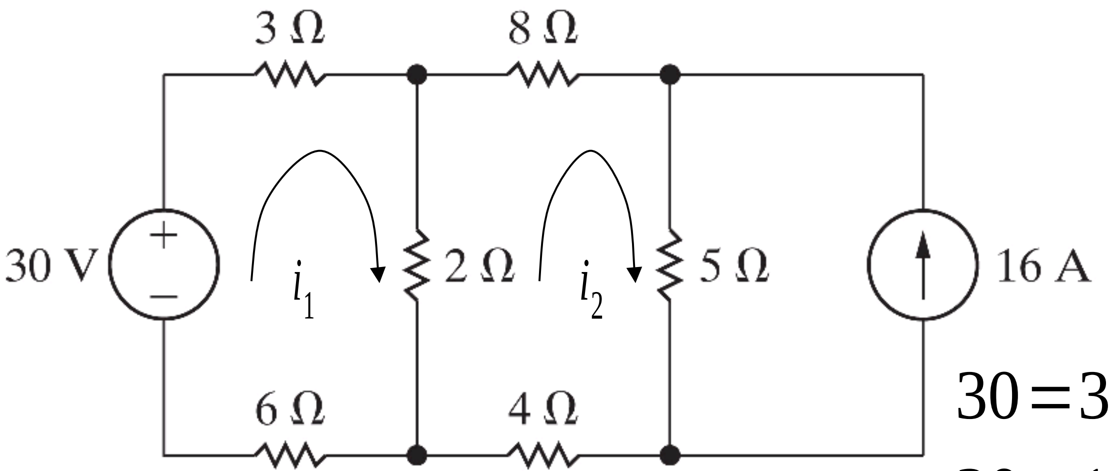
$$100 = 3(i_a - i_b) + 2(i_c - i_b) + 50 + 4i_c + 6i_a$$ **KVL around supermesh**

$$10i_b + 2(i_b - i_c) + 3(i_b - i_a) = 0$$ **KVL around mesh b**

$$i_c = 5 + i_a$$ **Constraint equation**

3 Equations, 3 unknowns, easy to solve

Drill Exercise: Find $p_{2\Omega}$



$$30 = 3i_1 + 2(i_1 - i_2) + 6i_1$$ **Mesh i_1**

$$30 = 11i_1 - 2i_2$$ **Simplified equation**

$$8i_2 + 5(i_2 + 16) + 4i_2 + 2(i_2 - i_1) = 0$$ **Mesh i_2**

$$-80 = -2i_1 + 19i_2$$ **Simplified equation**

Find Power

$$p_{2\Omega} = (i_1 - i_2)^2 2 = (2+4)^2 2 = 72\,(W)$$

$$i_1 = 2\,(A)$$

$$i_2 = -4\,(A)$$

Solve
2 equations
for
2 unknowns

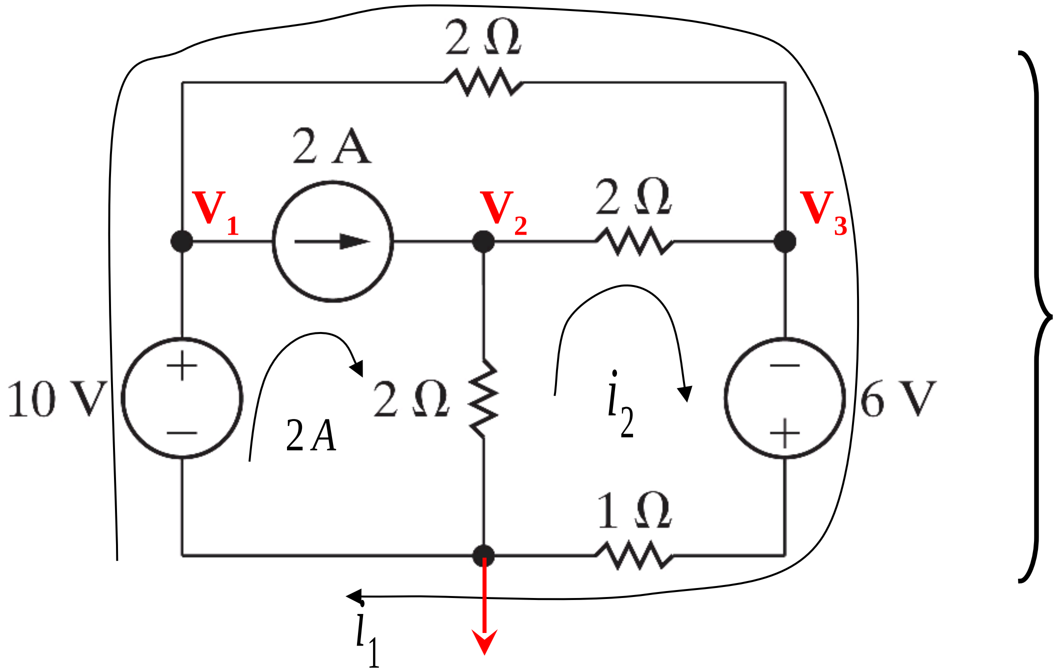
Nodal Versus Mesh Analysis

1) Which method yields the least number of equations?

2) <u>Supernodes:</u> Voltage source between essential nodes

3) <u>Supermesh:</u> Use loop excluding any current source which is part of 2 meshes

Drill Exercise



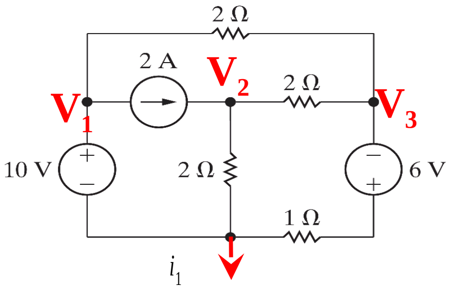
Use Node or Mesh

Only 2 mesh currents unknown. <3 meshes.>

However, 1 node voltage is also known.

How Would You Setup Nodal Equations?

Drill Exercise (Contd.)



a $$-2+\frac{V_2}{2}+\frac{V_2-V_3}{2}=0$$ Node Equation at V_2

a $$-4+V_2+V_2-V_3=0$$ Multiply by 2

a $$2V_2-V_3=4$$ Simplify

b $$\frac{V_3-V_2}{2}+\frac{V_3-10}{2}+\frac{V_3+6}{1}=0$$ Node Equation at V_3

b $$V_3-V_2+V_3-10+2V_3+12=0$$ Multiply by 2

b $$-V_2+4V_3=-2 \Rightarrow V_2=4V_3+2$$ Simplify

Drill Exercise (Contd.)

$$2(4V_3 + 2) - V_3 = 4$$

Substitute b **into** a

$$8V_3 + 4 - V_3 = 4$$

Simplify

$$7V_3 = 4 - 4$$

Simplify

$$\boxed{V_3 = 0}$$

$$V_2 = 4(0) + 2$$

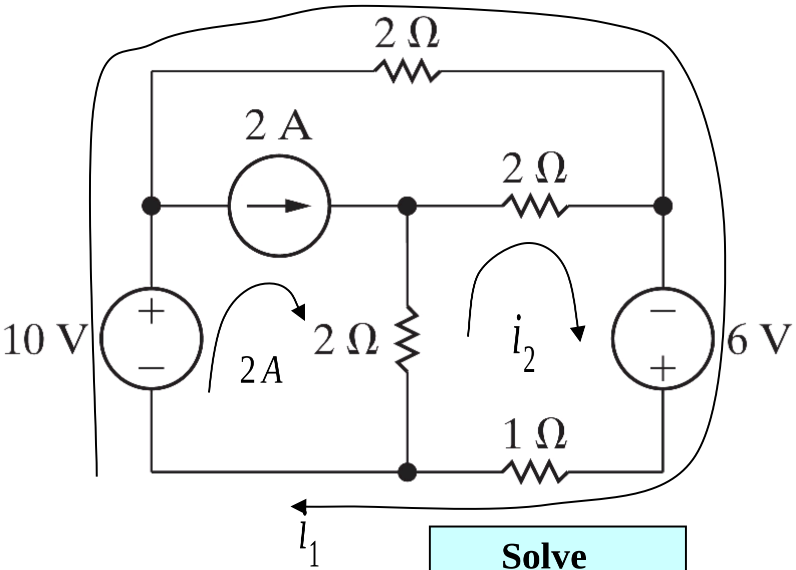
Substitute V_3 = 0 into b

$$\boxed{V_2 = 2(V)}$$

Drill Exercise

Solution Manual Method for the same problem

- Supermesh (outer loop) current ≡ i_1
- Mesh currents ≡ i_2 and 2A source
- No mesh current defined in "Top" loop



Outer loop (Supermesh)

(1) $10 = 2i_1 - 6 + 1(i_1 + i_2)$ } **KVL**

(1) $3i_1 + i_2 = 16$ } **Simplify**

(2) $6 = 1(i_1 + i_2) + 2(i_2 - 2) + 2i_2$ } **KVL for Mesh 2**

(2) $\boxed{i_1 + 5i_2 = 10}$ } **Simplify**

Solve 2 equations for 2 unknowns

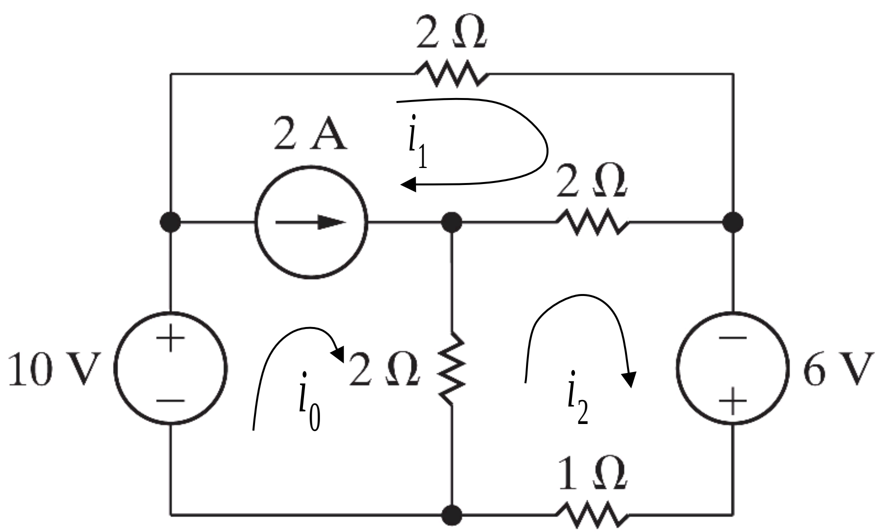
Solution

$i_1 = 5(A)$

$i_2 = 1(A)$

Drill Exercise

Third Method for the same problem



2A current source presents

a difficulty

Use “Supermesh” or “outer” loop

Define 3 mesh currents, as shown above

Write KVL Loop Equation Around “Outer Loop”

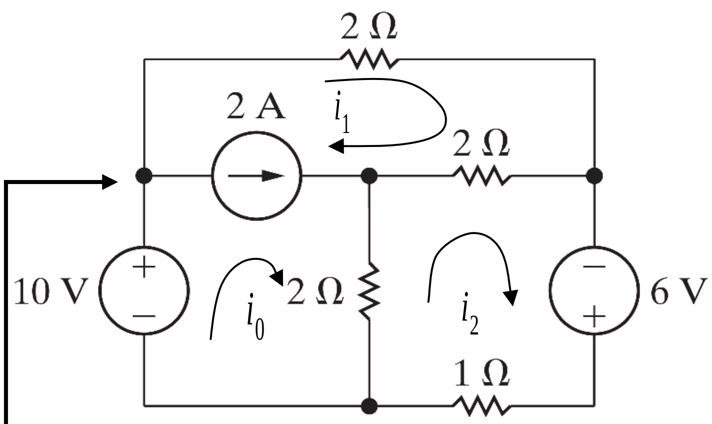
$$(1) \quad 10 = 2i_1 - 6 + 1i_2$$

$$(1) \quad 2i_1 + 1i_2 = 16$$

Simplify

Drill Exercise (Contd.)

Third Method for the same problem



KVL Around Mesh "2"

$$(2)\quad 6 = 1i_2 + 2(i_2 - i_0) + 2(i_2 - i_1) \quad \textbf{KVL}$$

$$(2)\quad 6 = 5i_2 - 2i_0 - 2i_1 \quad \textbf{Simplify}$$

$$(2)\quad 2i_0 + 2i_1 - 5i_2 = -6 \quad \textbf{Simplify}$$

Since we don't know the voltage across the 2A current source, we can not write a KVL for Mesh "0".

2 Equations, 3 Unknowns. Need another equation.
Use **KCL** to relate Mesh Currents.

$$(3)\quad i_0 = 2 + i_1 \quad \textbf{Constraint}$$

Solve
3 equations
for
3 unknowns

Drill Exercise (Contd.)

Third Method for the same problem

So, our equations are

$$2i_1 + i_2 = 16 \quad ①$$

$$2i_0 + 2i_1 - 5i_2 = -6 \quad ②$$

$$i_0 = i_1 + 2 \quad ③$$

③ → ② ⇒ $4i_1 - 5i_2 = -10$ ④

Solving ① and ④ simultaneously

$$i_1 = 5(A)$$

$$i_2 = 6(A)$$

$$i_0 = i_1 + 2 = 5 + 2$$

$$\boxed{i_0 = 7(A)}$$

Drill Exercise Comparison

Third Method Versus Solution Manual Method

$i_1 = 5(A)$ **- Same**

$i_2 = 1(A)$ **- Different**

$\Bigg\}$ **This is OK**

Note: Current Through 10(V) Source is 7(A)

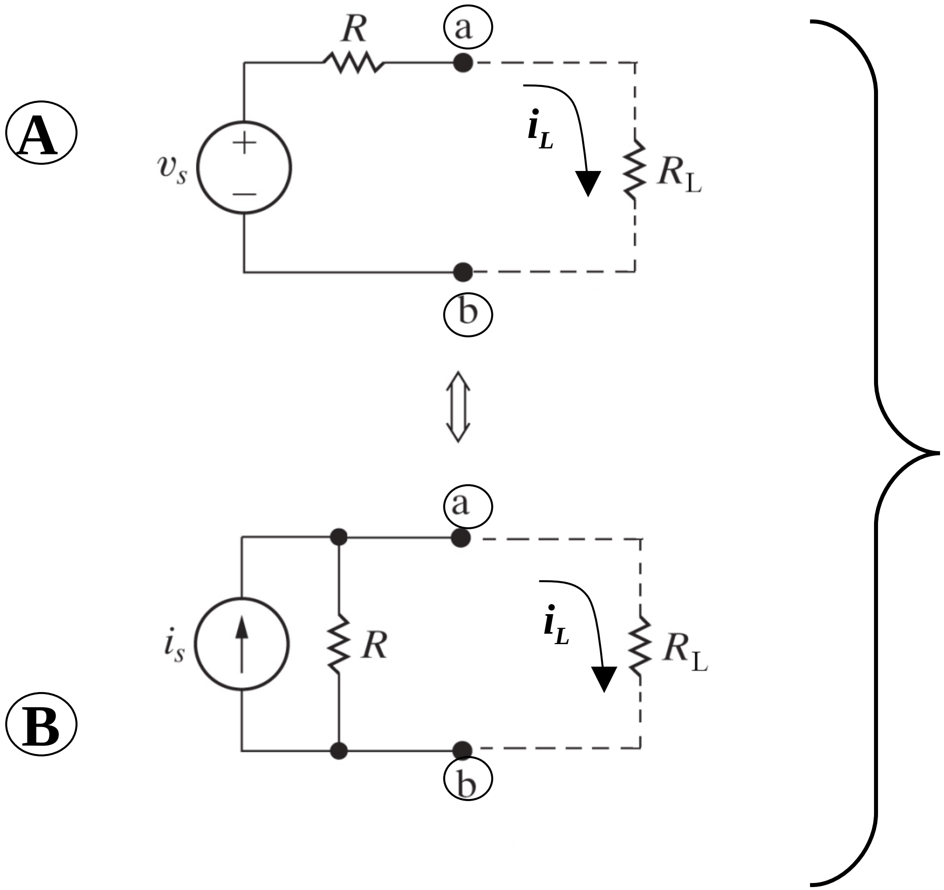
Solution Manual: $\underline{I_{10\text{V}}} = i_1 + 2\text{A} = 5 + 2 = \underline{7(A)}$

Third Method: $\underline{I_{10\text{V}}} = i_0 = \underline{7(A)}$

All Net Currents Are Consistent

Source Transformation

- **Another Simplification Technique**
- **Analogous to Combining Resistors**

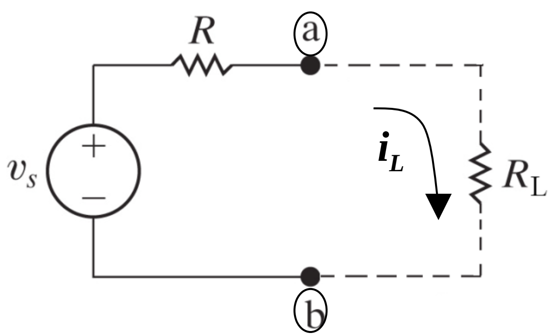


Equivalence:

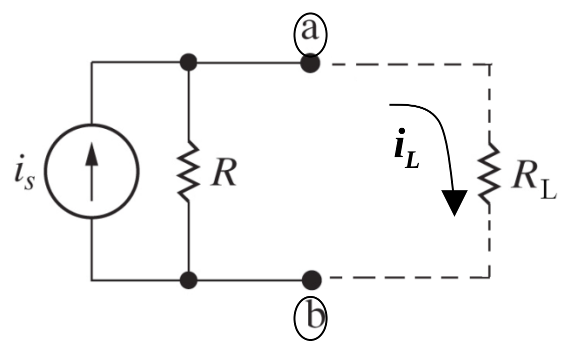
If R_L "see's" the same current flow and voltage drop in either circuit.

What is the Relationship Between V_s and i_s ?

Circuit A



Circuit B



$$V_s = i_L (R + R_L)$$ **KVL**

$$i_L = \frac{V_s}{R + R_L}$$ **Solve for i_L**

$$i_L = i_s \frac{R}{R + R_L}$$ **Solve for i_L by Current Division**

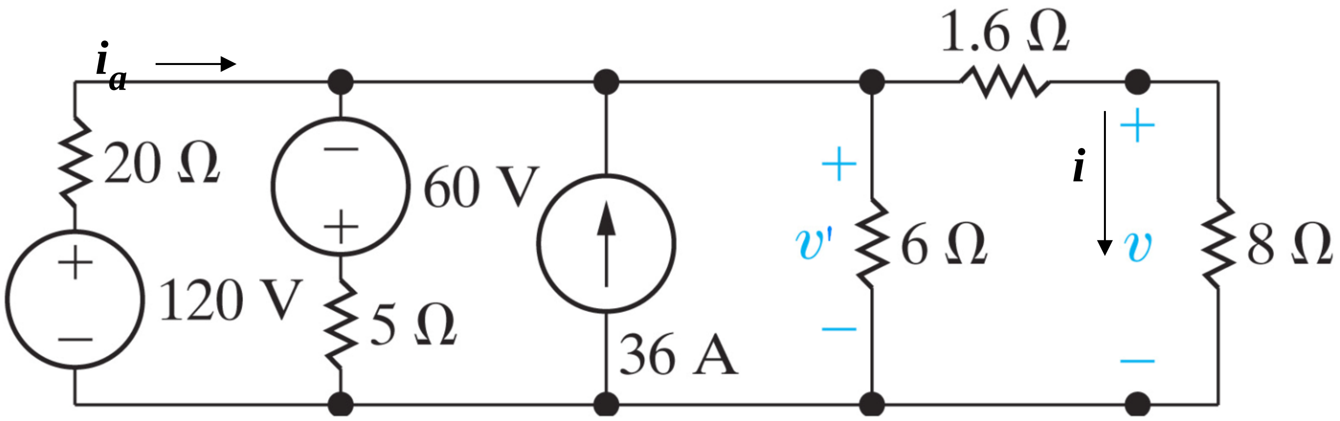
A ≡ B **For Equivalence** $i_L = i_L$

$$\frac{V_s}{\cancel{R + R_L}} = i_s \frac{R}{\cancel{R + R_L}}$$

$$\Rightarrow \quad V_s = i_s R \qquad \Rightarrow \quad i_s = \frac{V_s}{R}$$

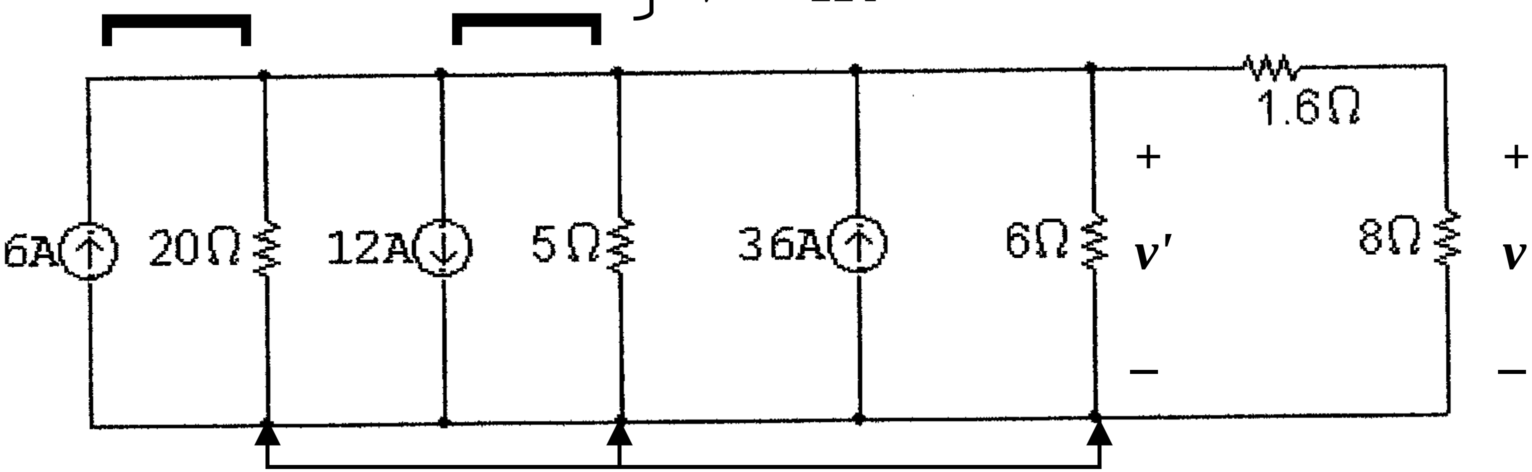
Drill Exercise

Finding *V* across the 8Ω Resistor



Find *V*

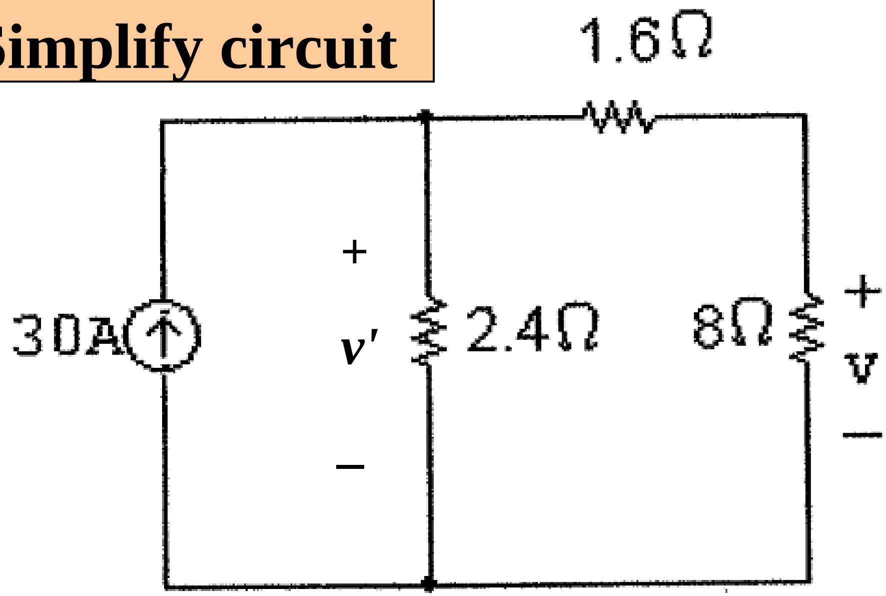
Source Transformations } $V = IR$



$20 \| 5 = 4$ $\Rightarrow$ $4 \| 6 = 2.4$ $\quad i_{source\ total} = 6A - 12A + 36 = 30(A)$

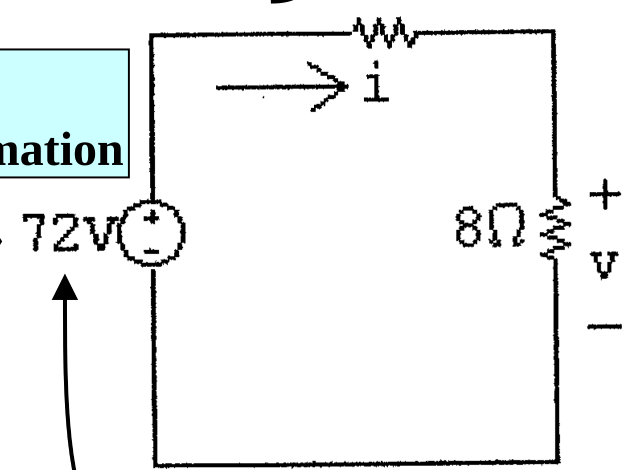
Drill Exercise (Contd.)

Simplify circuit



Source Transformation

Add resistors 4Ω = 2.4 + 1.6



$V = IR$

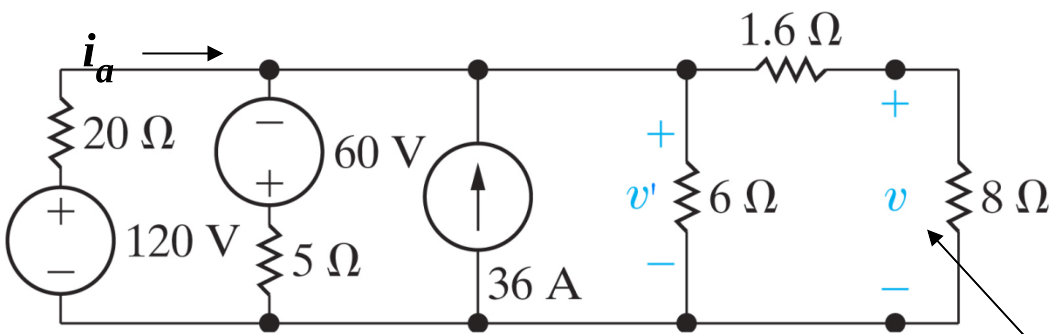
(Source Transformation) $V = 30(2.4) = \boxed{72(V)}$

Find V by Voltage Division

$$V = 72\frac{8}{8+4}$$

$$\boxed{V = 48\ (V)}$$

Find the Power of the 120 V Source



Find i_a

Go back to the original circuit. Work from right to left.

$$v = 48(V)$$

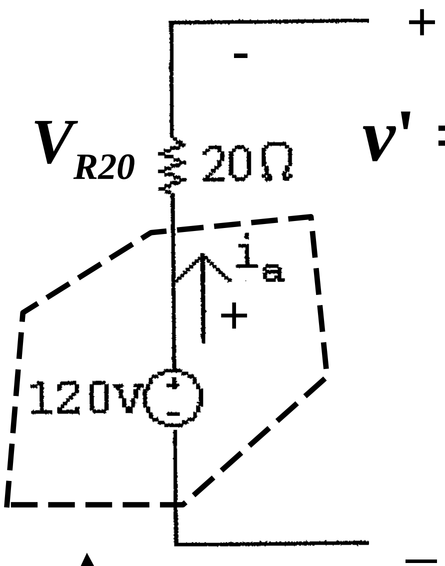
$$i_{8\Omega \text{ resistor}} = \frac{v}{8} = \frac{48}{8} = 6(A)$$

Find the Voltage across 6Ω Resistor

$$v' = i(1.6) + v = 6(1.6) + 48 = 57.6 \ (V) \Big\} \text{KVL}$$

v' is Dropped Across Each Essential Branch To The Left

Find the Power of the 120 V Source (Contd.)



$$v' = 57.6(V)$$

$$\left. v' = 120 - V_{R20} \right\} \text{KVL}$$

$$V_{R20} = 120 - 57.6 = 62.4\,(V)$$

$$\left. i_a = \frac{V_{R20}}{20} = \frac{62.4}{20} = 3.12(A) \right\} \text{Ohm's Law}$$

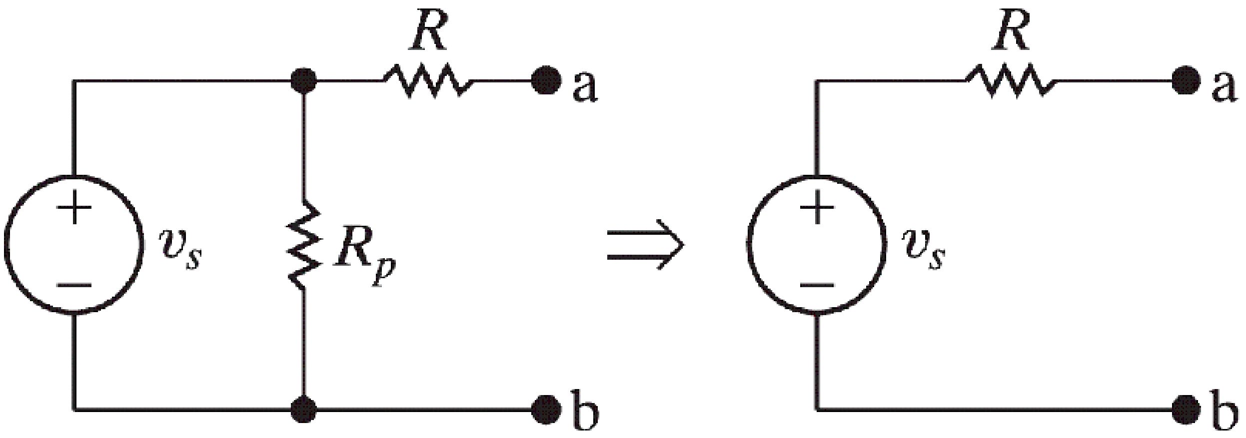
Passive sign convention

$$\left. p_{120} = -120 i_a = -120(3.12) \right\} \text{Power extracted}$$

$$\boxed{p_{120} = 374.4\,(W)}$$

Delivered

Resistor in Parallel With a Voltage Source

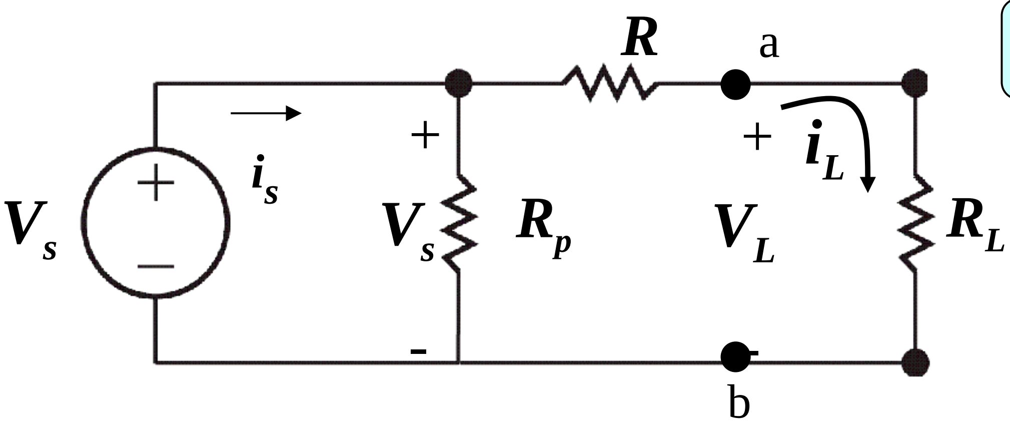


i.e., ***Simply Remove R_p. R_p has NO effect on the terminal V and i***

WHY?

Either circuit ⟹ same V_L and i_L for a load R_L

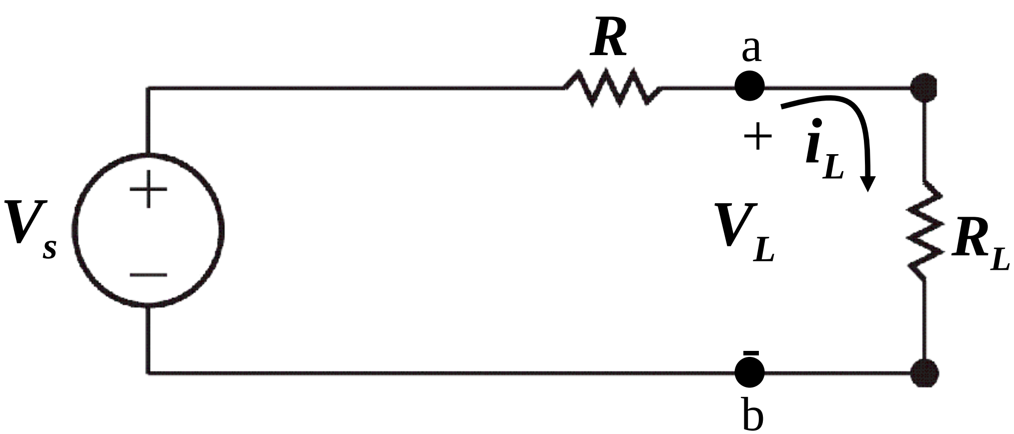
Resistor in Parallel With a Voltage Source (Contd.)



V_s Dropped Across R_p

$$V_L = V_S \frac{R_L}{R_L + R} = i_L R_L$$ **Voltage Division**

$$i_L = \frac{V_S}{R_L + R}$$ **Solve for i_L**



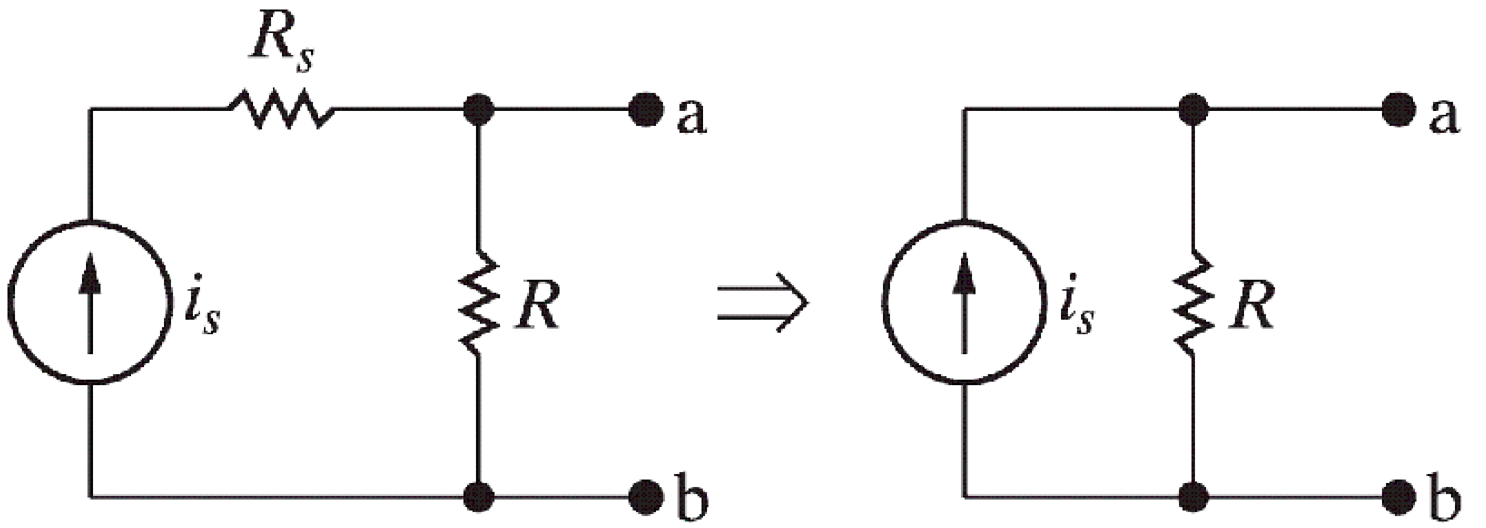
$$V_L = V_S \frac{R_L}{R_L + R} = i_L R_L$$ **Voltage Division**

$$i_L = \frac{V_S}{R_L + R}$$ **Solve for i_L**

Results are the same!

R_p Doesn't Matter for Equivalent Circuits

Resistor in Series With a Current Source

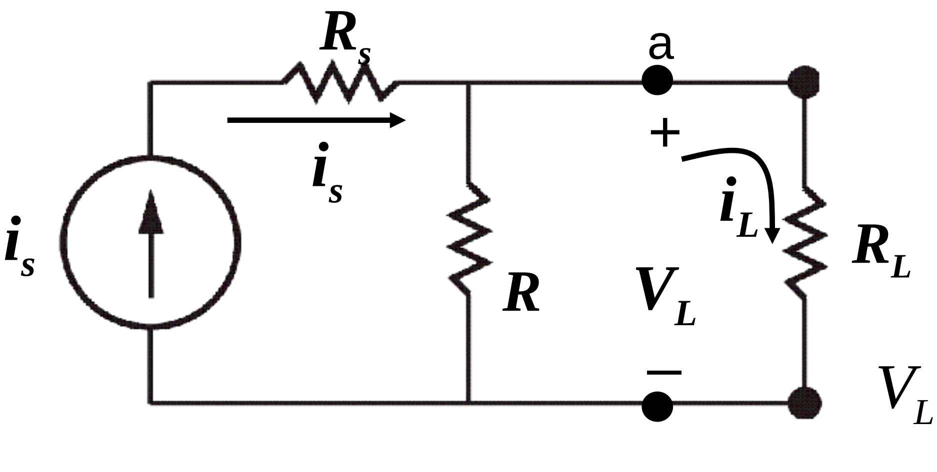


i.e., *Simply Remove* R_S. R_S *has NO effect on the terminal V and i*

WHY?

Either circuit ⟹ same V_L and i_L for a load R_L

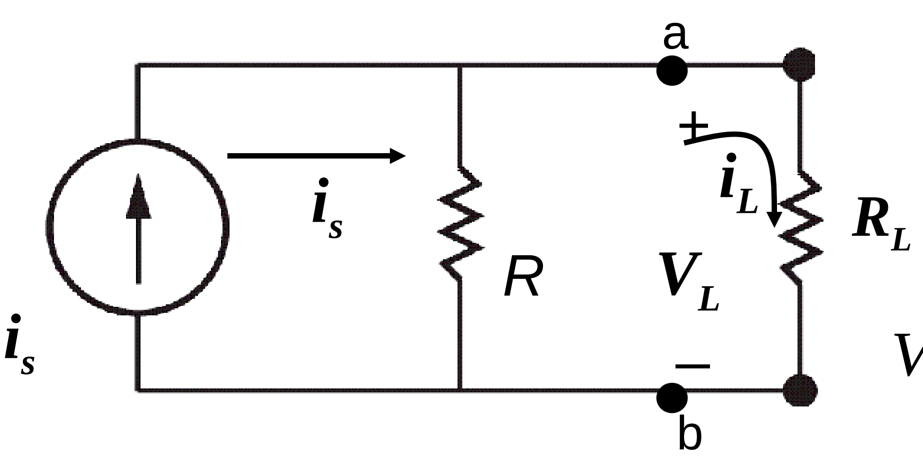
Resistor in Series With a Current Source (Contd.)



i_s flows into R_s and $R||R_L$

$$i_L = i_S \frac{R}{R_L + R} = \frac{V_L}{R_L}$$ **Current Division**

$$V_L = i_S \frac{R \cdot R_L}{R_L + R} = i_S (R \,||\, R_L)$$ **Ohm's Law**



$$i_L = i_S \frac{R}{R_L + R} = \frac{V_L}{R_L}$$ **Current Division**

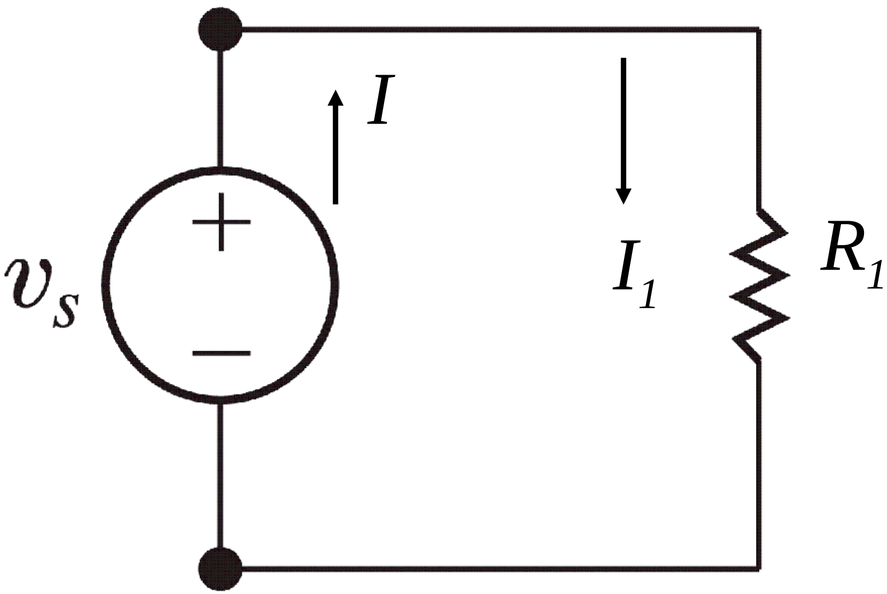
$$V_L = i_S \frac{R \cdot R_L}{R_L + R} = i_S (R \,||\, R_L)$$ **Ohm's Law**

Results are the same!

R_S Doesn't Matter for Equivalent Circuits.

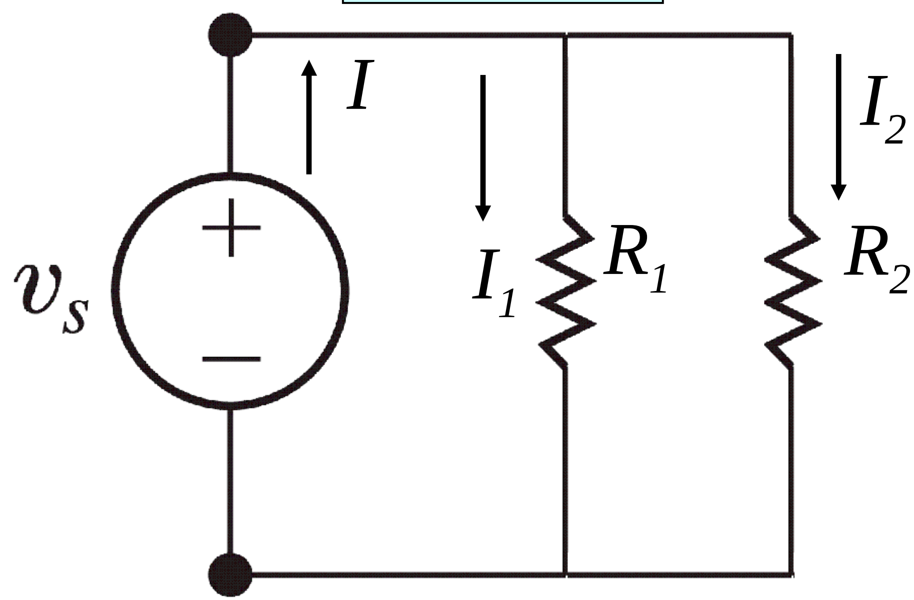
Supplemental Note on Parallel Resistors

One Resistor



$$I_1 = \frac{v_s}{R_1} \equiv I$$

Two Resistors

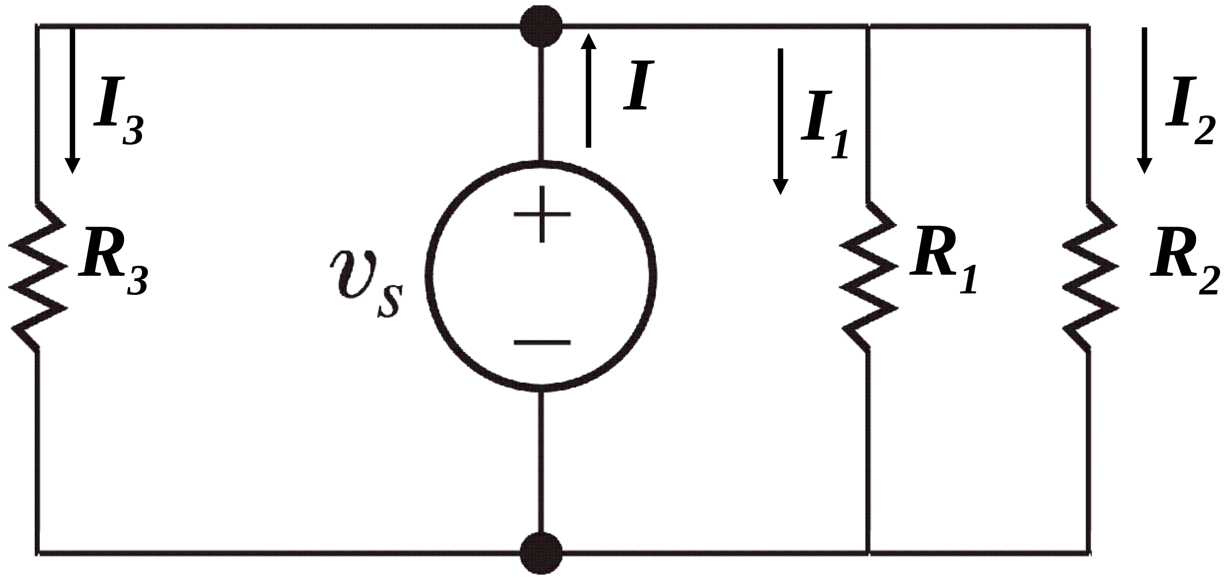


$$I_1 = \frac{v_s}{R_1} \qquad I_2 = \frac{v_s}{R_2}$$

$$\left. I = I_1 + I_2 \right\}$$ **KCL**

Supplemental Note on Parallel Resistors
(Contd.)

Three Resistors



$$I_1 = \frac{v_S}{R_1} \quad I_2 = \frac{v_S}{R_2} \quad I_3 = \frac{v_S}{R_3}$$

$$I = I_1 + I_2 + I_3$$

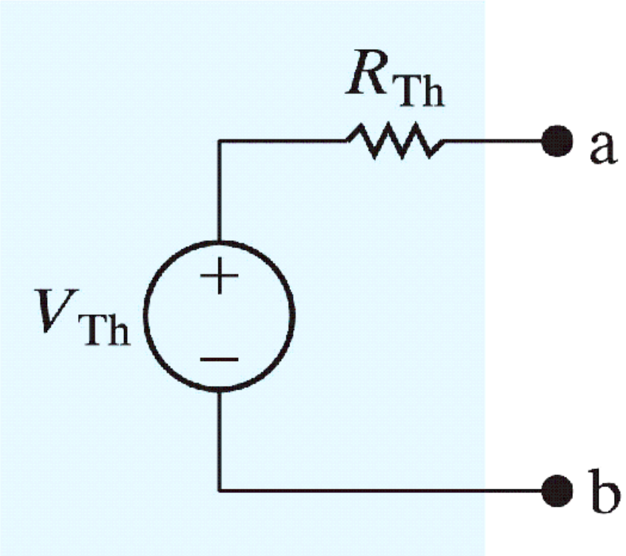
I increases with each added *R*.

More Resistors Cause More Current To Be Drawn From The Voltage Source

Thevenin Equivalents

Sometimes we want to simplify a circuit at a particular pair of nodes in a circuit.

Example: Power supply attached to a load



$V_{Th} \equiv$ *Thevenin Voltage*

$R_{Th} \equiv$ *Thevenin Resistance*

- **Thevenin Equivalent of some arbitrary circuit.**
- **Load R_L will "See" the same V and I in either the original or equivalent circuit.**
- **For ANY value of R_L**

Thevenin Equivalents (Contd.)

- A Thevenin Equivalent is really a generalization of the Source Transformation.

- In fact, you could obtain a Thevenin Equivalent using Source Transformations, but it might take a long time. (Dependent sources might make it tough as well.)

- How do we determine $\boldsymbol{V_{Th}}$ and $\boldsymbol{R_{Th}}$?

Look at the Two Extreme Cases

Thevenin Equivalents (Contd.)

Find the V_{Th}

$If\ R_L \equiv \infty$, **Terminals a & b are open**

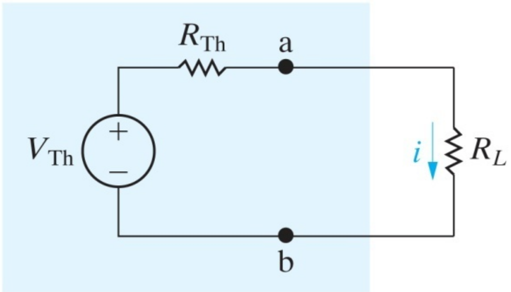


Figure: 04-59

$V_{oc} \equiv V_{Th}$ **in Thevenin Equivalent.**

No Current Through R_{Th}

Must be the same for the original circuit.

$\therefore$ **Calculate V_{oc} in original circuit – This is the V_{Th}**

Thevenin Equivalents (Contd.)

Find the R_{Th}

If $R_L = 0$, Terminals a and b are Shorted.

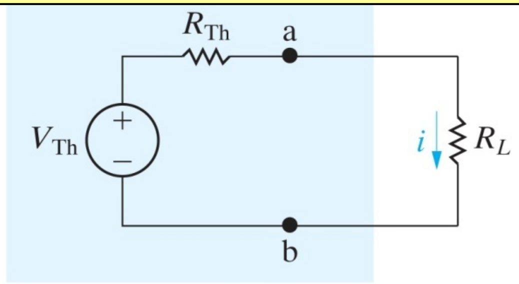


Figure: 04-59

$$i_{sc} = \frac{V_{Th}}{R_{Th}}$$

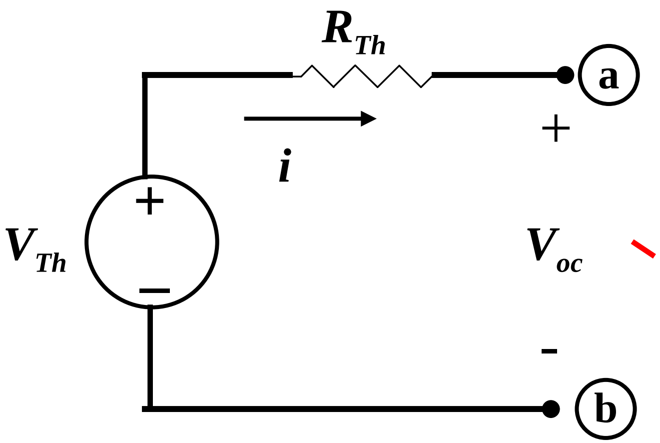
Thevenin Equivalent with a shorted to b.

Must be the same for original circuit, with a shorted to b

$$R_{Th} = \frac{V_{Th}}{i_{sc}}$$

$$R_{Th} \equiv \frac{\text{Open circuit voltage}}{\text{Short circuit current}}$$

Thevenin Illustration

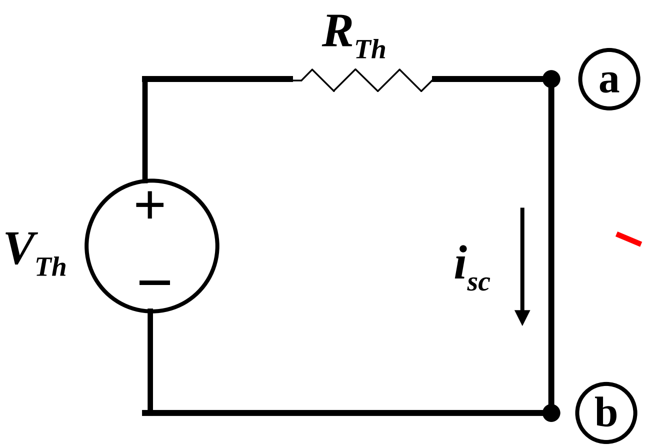


$$V_{oc} \equiv V_{Th}$$

since $i \equiv 0$

No closed paths

$$R_L \equiv \infty \qquad \text{[Open]}$$



$$V_{Th} = i_{sc} R_{Th}$$

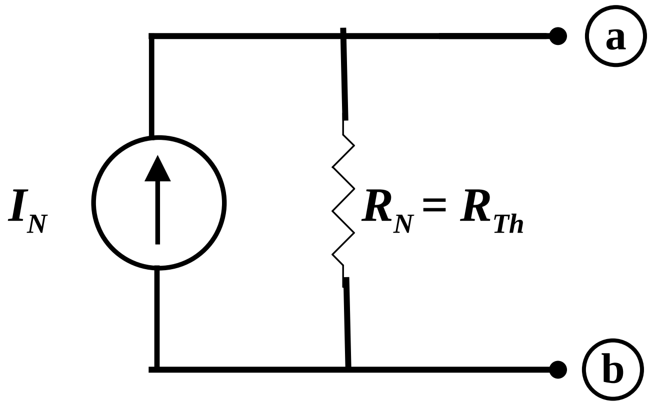
Ohm's Law

$$R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{V_{oc}}{i_{sc}}$$

$$R_L \equiv 0 \qquad \text{[Short]}$$

Norton Equivalent

Dual of Thevenin



Current Source in Parallel with a Resistor

$$I_N = \frac{V_{Th}}{R_{Th}}$$

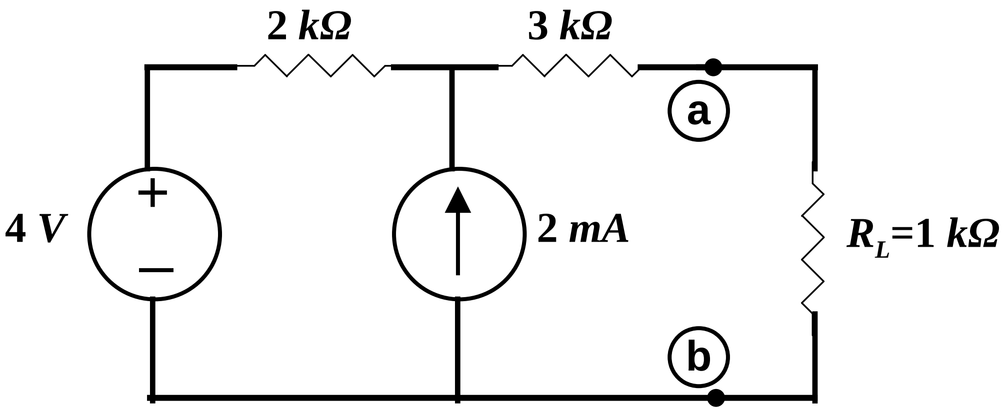
Source Transformation

$$R_N \equiv R_{Th}$$

Source Transformation

Example

Find Thevenin and Norton Equivalent at Terminals (a) and (b)

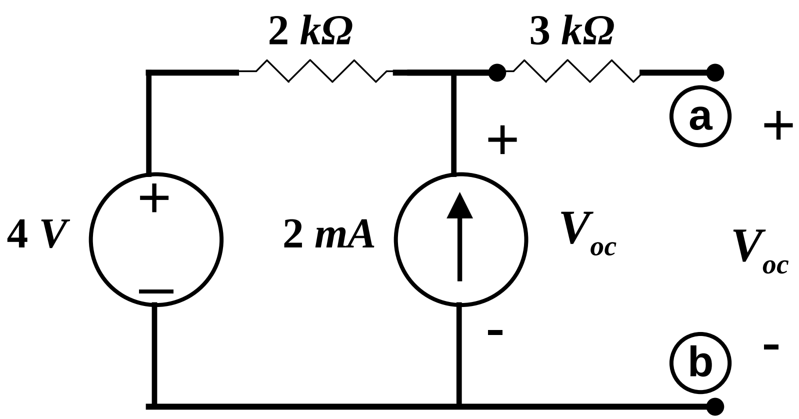


Do it three ways …

Example

Find Thevenin and Norton equivalent

Method 1: Calculate $V_{oc} = V_{Th}$ and $i_{sc} = V_{Th}/R_{Th}$



Open Circuit

⇩

Write Nodal Equation

⇩

$$\frac{V_{oc} - 4}{2(K\Omega)} - 2(mA) = 0$$

Node Equation

Current in 3 kΩ Resistor is Zero

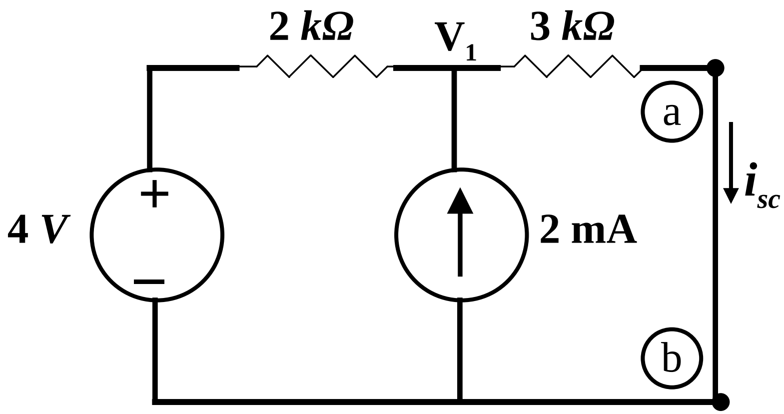
Solve for V_{oc} → $V_{oc} - 4 = 2(mA)(2(K\Omega)) = 4(V)$

Simplify → $V_{oc} = 4 + 4 = 8(V)$

$$\boxed{V_{Th} = 8(V)}$$

Example (Contd.)

Find Thevenin and Norton equivalent



Short Circuit → **Write Nodal Equation**

$$\frac{V_1 - 4}{2K} - 2(mA) + \frac{V_1}{3K} = 0$$

Node Equation

$$3V_1 - 12 - 12 + 2V_1 = 0$$

Simplify

$$\boxed{V_1 = 4.8(V)}$$

$$i_{SC} = \frac{V_1}{3K} = \frac{4.8}{3K}$$

Ohm's Law

$$\boxed{i_{SC} = 1.6(mA)}$$

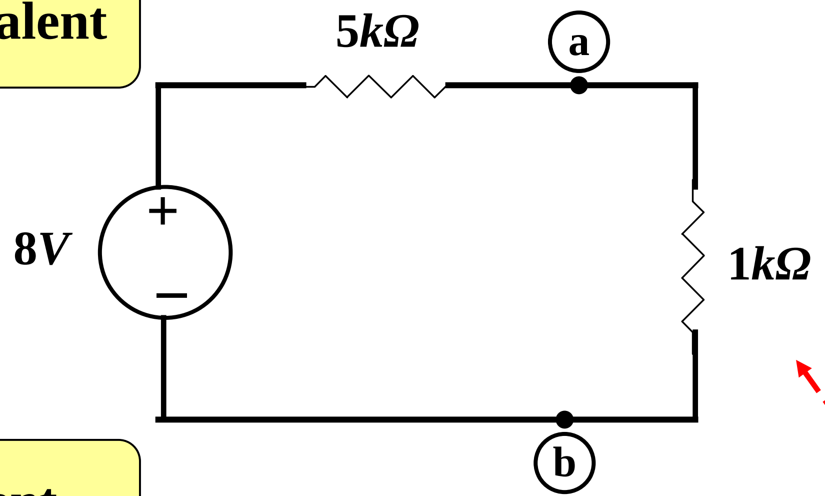
$$R_{Th} = \frac{V_{Th}}{i_{SC}} = \frac{8}{1.6(mA)} = \boxed{5(k\Omega)}$$

Find R_{Th}

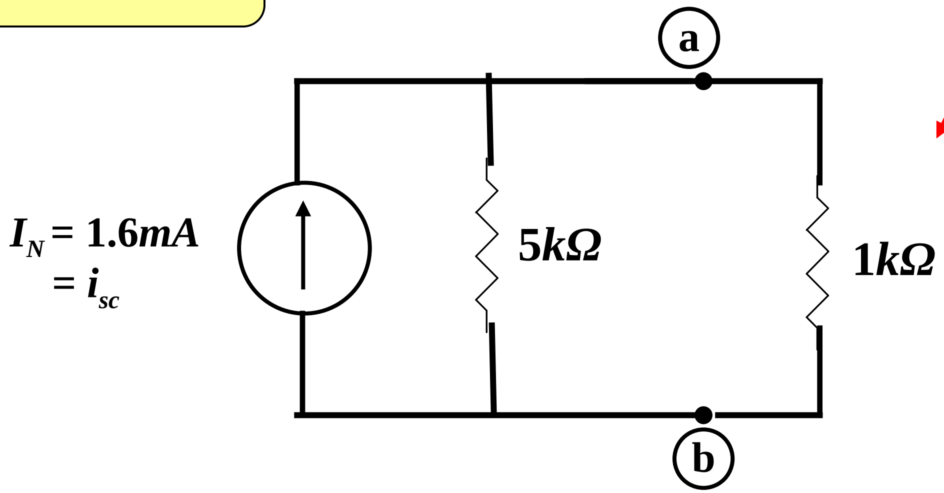
Example

Find Thevenin and Norton equivalent

Thevenin Equivalent



Norton Equivalent

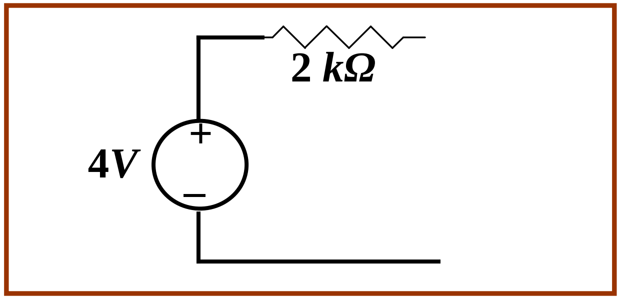


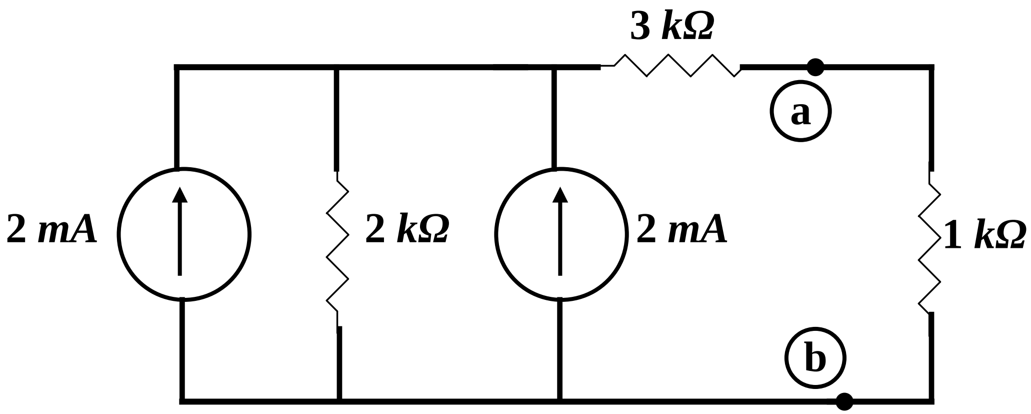
The 1kΩ Resistor is the "Load" and NOT part of the Thevenin or Norton Equivalent

Example

Find Thevenin and Norton equivalent

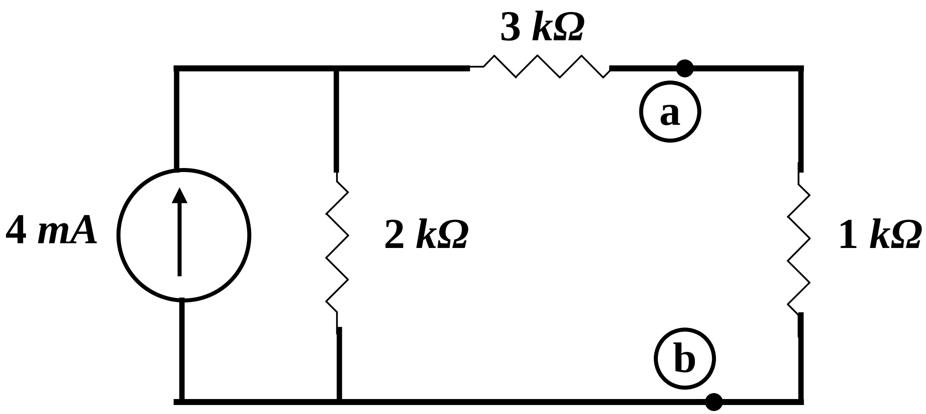
Method 2: **Use Source Transformations**





Already changed 4 (V) source to current source

4 V / 2K = 2 (mA)

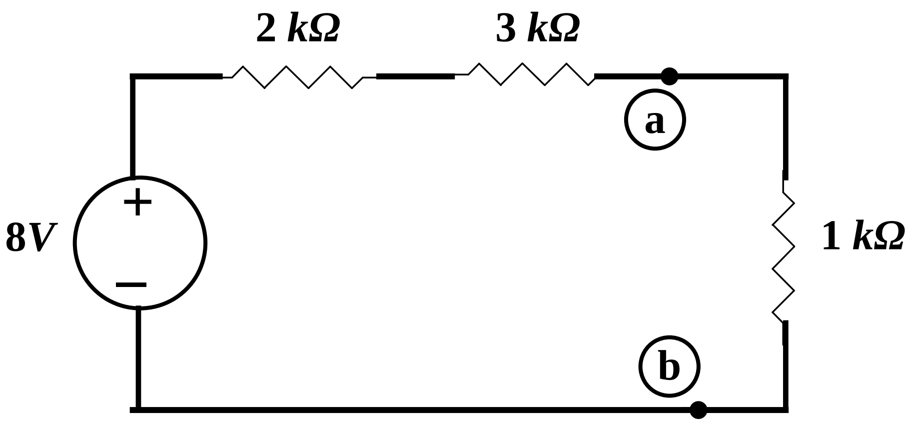


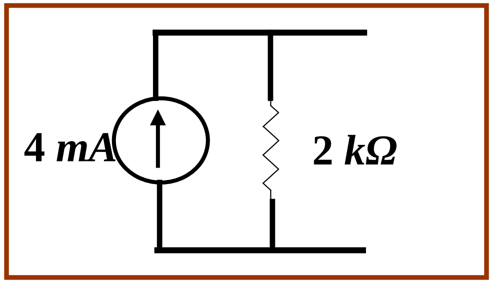
Sum Current Sources

(Contd..)

Example

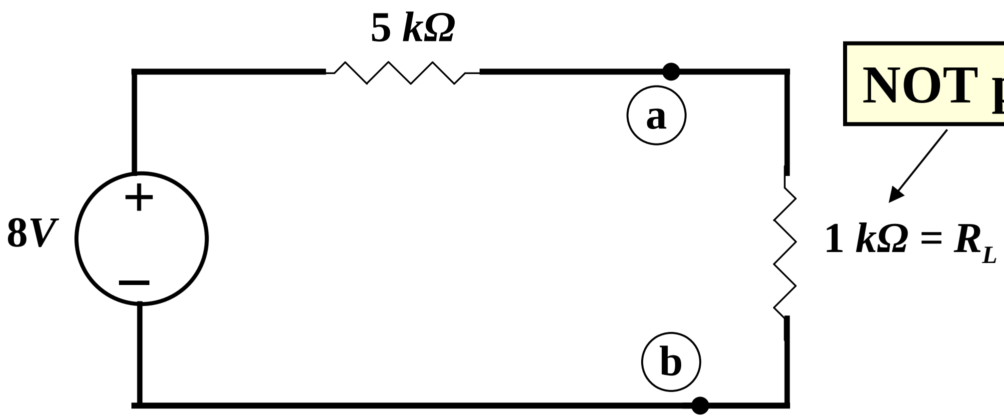
Find Thevenin and Norton equivalent





Change 4(mA) source to voltage source

$$8(V) = (4(mA))(2(K\Omega))$$



NOT part of R_{Th}

$V_{Th} = 8\ (V)$

$R_{Th} = 5\ (k\Omega) = R_N$

$I_N = 8/5K = 1.6\ (mA)$

Example

Find Thevenin and Norton equivalent

Method 3: [a] Independent Source Deactivation.

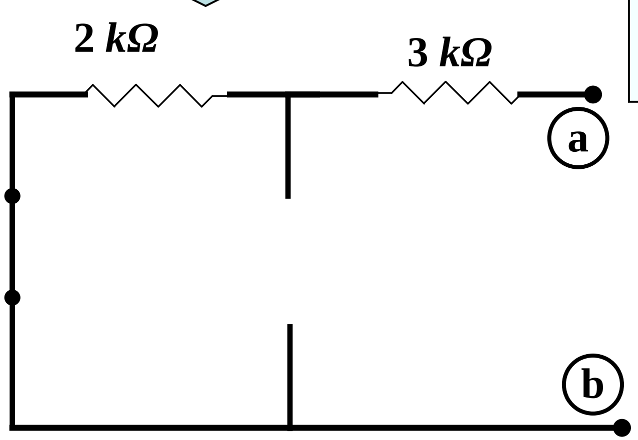
[b] Linear Superposition

Short Voltage Sources

Open Current Sources

Deactivate the Independent Sources

Original circuit becomes …



$R_{Th} \equiv R$ "Looking" into terminals a and b

***R* 'Looking Into' a & b**

What R_L "sees"

$$R_{Th} = 2K + 3K = 5(K\Omega)$$

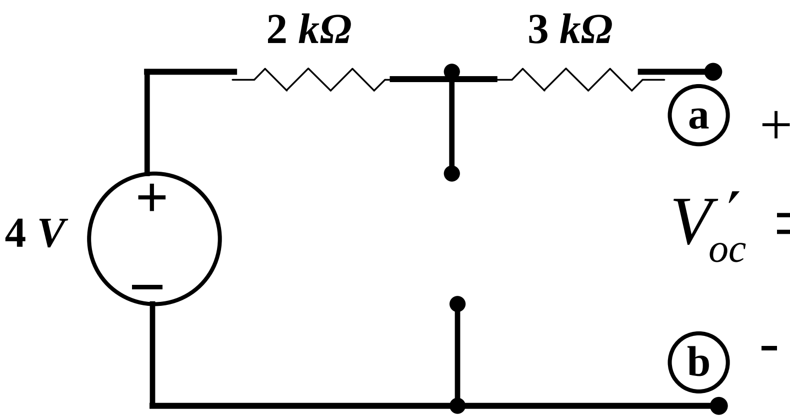
Example

Find Thevenin and Norton equivalent

Superposition

In a **linear** system, with independent sources:

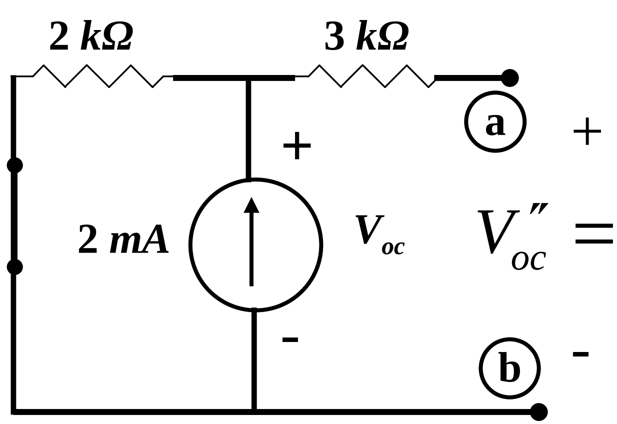
Total Response ≡ Sum of Individual Responses



Remove (open) current source

$$V_{oc}' = 4(V)$$

No current flows in resistors



Remove (short) voltage source

$$V_{oc}'' = 2mA(2K\Omega) = 4(V)$$

No current in 3*K* resistor

(Contd..)

Example

Find Thevenin and Norton equivalent

$$V_{oc} = V_{Th} = V_{oc}'' + V_{oc}' = 4(V) + 4(V) = 8(V)$$

$$V_{Th} = 8(V)$$

Recall $R_{Th} = 5(K\Omega)$

Same as method 1 and 2

Get Norton from Source Transformation

Most Common Way to Obtain Thevenin Equivalent

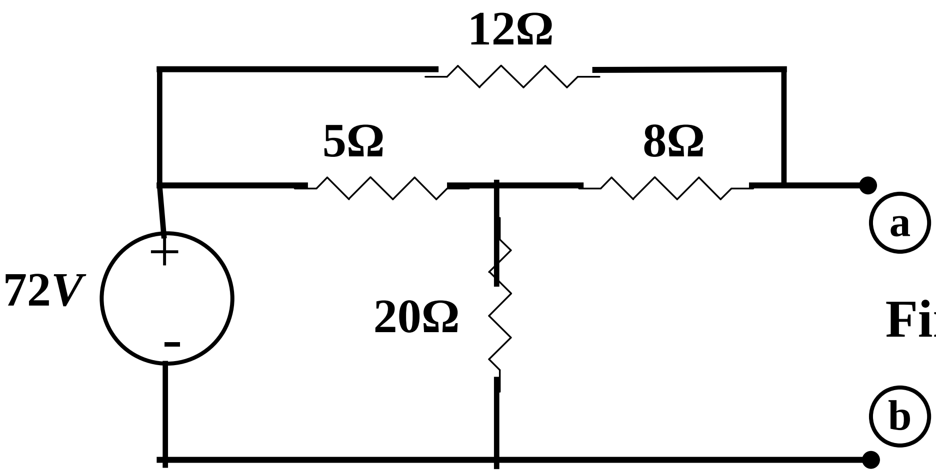
We could use Method 3 to obtain R_{Th} by Deactivating Sources,

and then

Obtain $V_{oc} = V_{Th}$ as in Method 1 to obtain V_{Th}

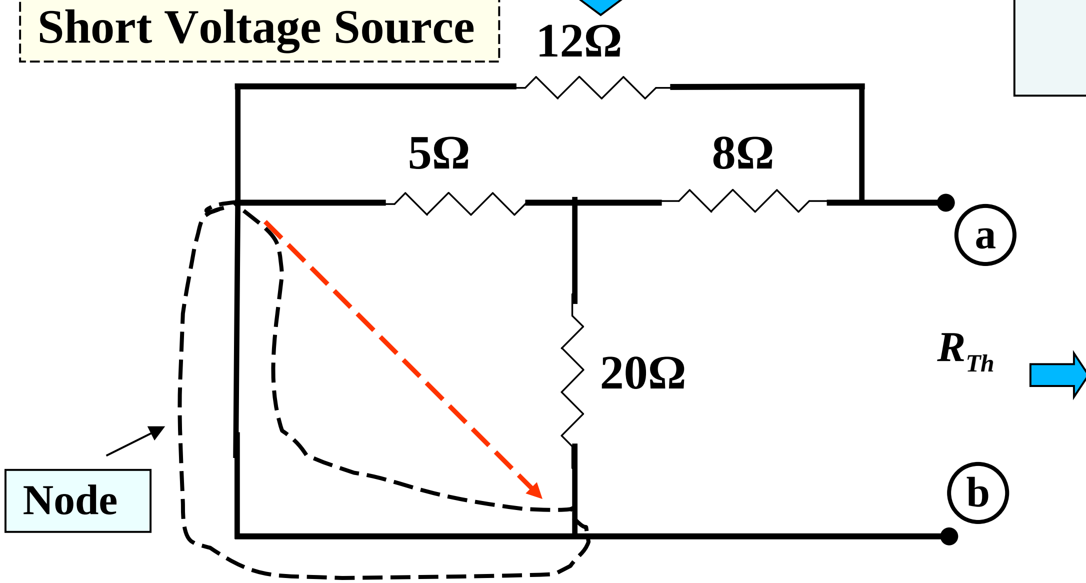
This is often the best method

Drill Exercise

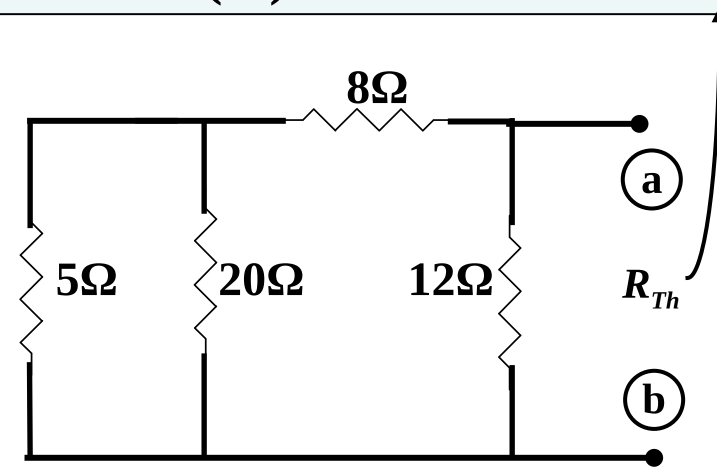


Find Thevenin Equivalent at a – b

Thevenin Resistance:
Short Voltage Source



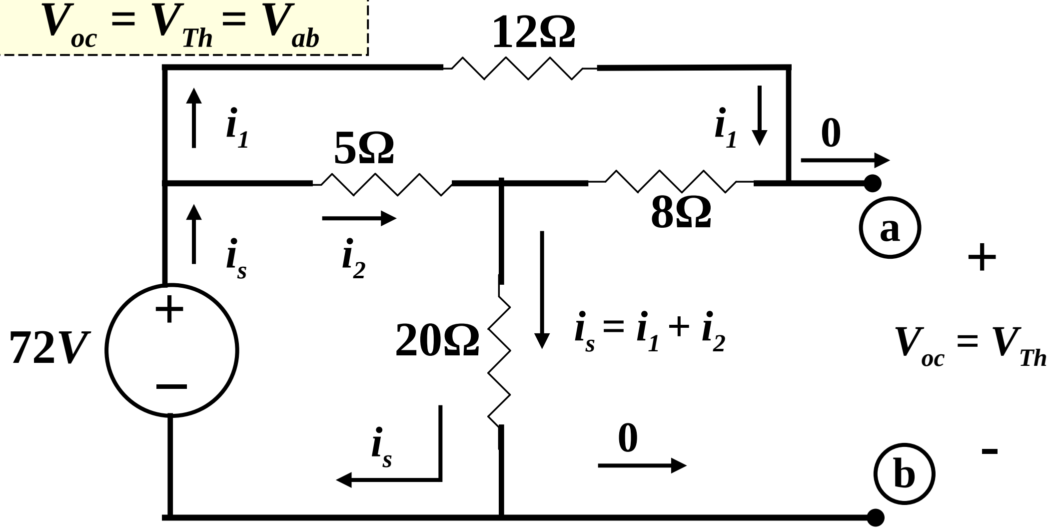
$$R_{Th} = [\ (5\ ||\ 20) + 8\ ]\ ||\ 12$$
$$= 6\ (\Omega)$$



Drill Exercise (Contd.)

Find Thevenin Equivalent at a – b

$V_{oc} = V_{Th} = V_{ab}$



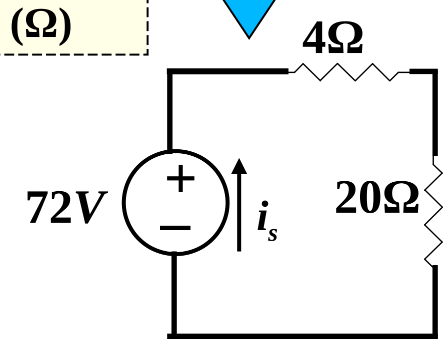
With the terminals open, no current flows to a or b

Thus, 12Ω is in Series with 8Ω

$R_{eq} = (12 + 8) \,||\, 5$

$= 20 \,||\, 5$

$= 4\ (\Omega)$



$$i_S = \frac{72(V)}{4+20} = 3(A)$$

Ohm's Law

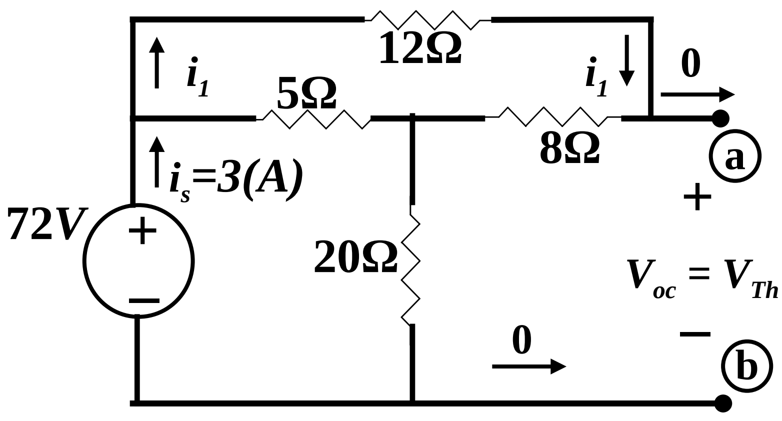
Current Supplied by Voltage Source

Lost Definition of V_{oc},

But OK For Now

Drill Exercise (Contd.)

Find Thevenin Equivalent at a – b

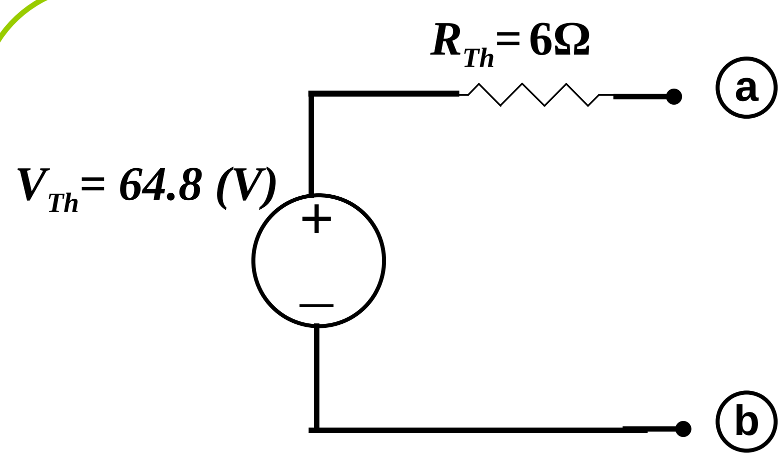


$$i_1 = i_s \frac{5}{5+(12+8)} = 3\frac{5}{5+20}$$

Find i_1 by Current Division

$$\boxed{i_1 = 0.6(A)}$$

$$V_{Th} = 72 - V_{12\Omega}$$
$$= 72 - 12 i_1$$
$$= 72 - 12(0.6)$$
$$= 72 - 7.2$$
$$\boxed{V_{Th} = 64.8(V)}$$



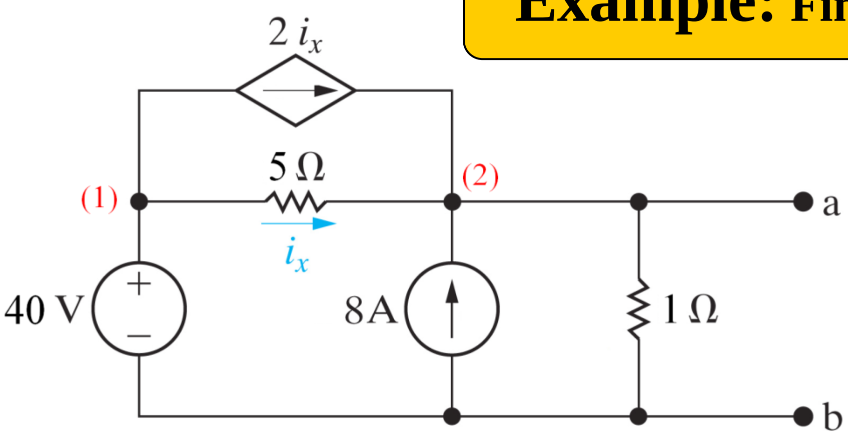
Thevenin Equivalent

THEVENIN WITH DEPENDENT SOURCES

To compute R_{Th}:

1. Short Independent Voltage Sources
2. Open Independent Current Sources
3. Apply Test Voltage or Current Source (a)-(b) } **Dummy Variable to deal with Dependent Sources**
4. $R_{Th} = V_T / i_T$ ⇨ {Also, you could use $R_{Th} = V_{OC} / i_{SC}$

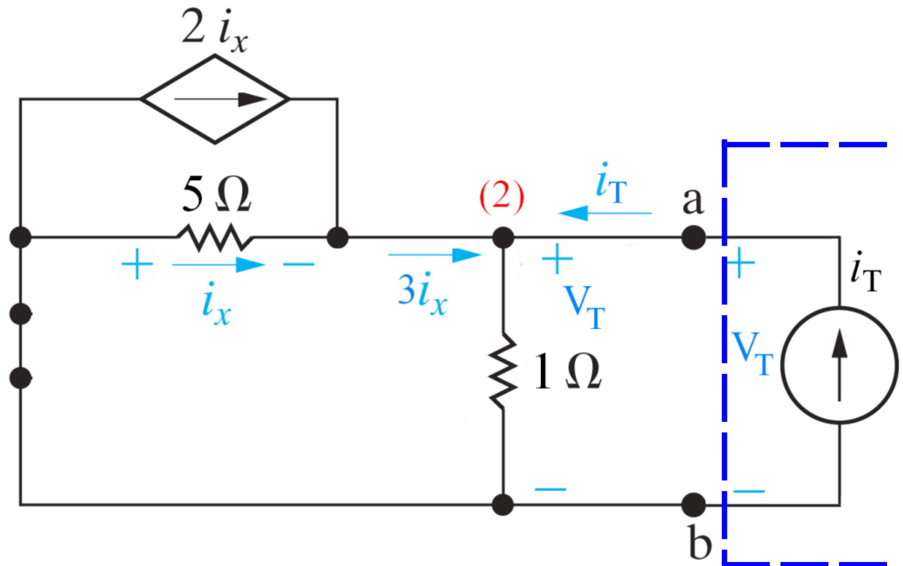
Example: Find R_{Th} and V_{Th}



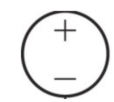
- **Short 40V source**
- **Open 8A source**

Example (Contd.)

Independent sources have been deactivated



Apply a test source

Doesn't really matter whether  or 

Find R_{Th}

1 $$\underbrace{3i_x + i_T = \frac{V_T}{1}}_{\text{KCL at node (2)}}$$

2 $$\underbrace{V_T = -5i_x}_{\text{KVL}} \Rightarrow i_x = -\frac{V_T}{5}$$

(Contd..)

Example: Find R_{Th} (Contd.)

Don't know V_T or i_T, but we can solve for the ratio

$$\text{③} \quad 3\underbrace{\left(-\frac{V_T}{5}\right)}^{i_x} + i_T = V_T$$

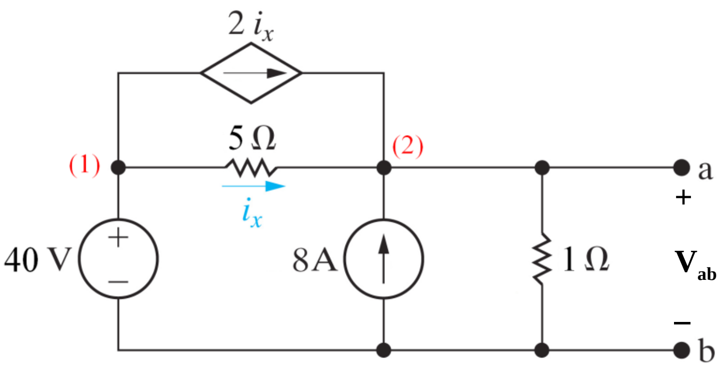
Substitute ② into ①

$$\text{③} \quad i_T = V_T\left[1+\frac{3}{5}\right] = \frac{8}{5}V_T$$

Simplify

$$\therefore R_{Th} \equiv \frac{V_T}{i_T} = \frac{5}{8} \quad \Rightarrow \quad \boxed{R_{Th} = 0.625\,(\Omega)}$$

Example: Find V_{Th} (Contd.)



$V_{Th} \equiv V_{oc} = V_{a\text{-}b}$: *Go back to the Original Circuit*

2 Node Voltages: $V_1 = 40\ V$

$V_2 = V_{ab} = V_{oc} = V_{Th}$

$$\frac{V_{Th} - 40}{5} - 8 - 2i_x + \frac{V_{Th}}{1} = 0$$

Node equation at 2

$$i_x + 2i_x + 8 = \frac{V_{Th}}{1} \Rightarrow \boxed{i_x = \frac{V_{Th} - 8}{3}}$$

KCL @ 2

Solve
2 equations
for
2 unknowns

RESULT: $V_{Th} = 20\ (V)$

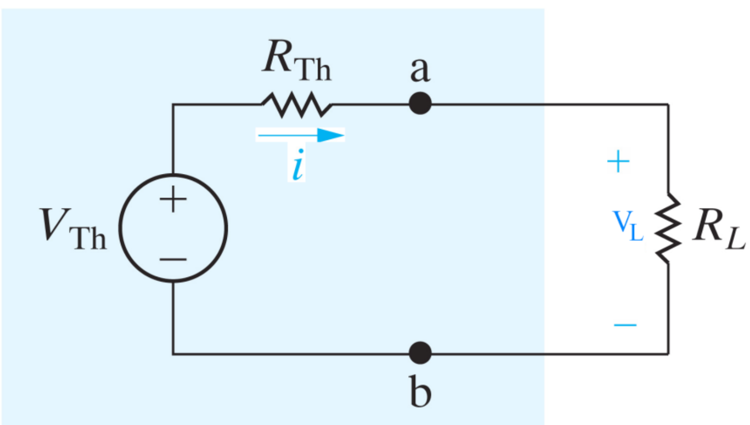
MAXIMUM POWER TRANSFER THEOREM

<u>Maximum Power</u> is delivered by a circuit to a load when:

$$\boxed{R_{Th}\,(\text{Circuit}) \equiv R_L\,(\text{load})}$$

Important for efficient transmission of power.
Related to impedance matching.

Proof: **- Any circuit can be reduced to a Thevenin Equivalent**
- Resistive Load modeled by R_L



What value of R_L gives the Maximum Power Transfer?

For a given circuit, we assume that V_{Th} and R_{Th} are fixed

MAXIMUM POWER TRANSFER THEOREM (Contd.)

1 $$p_L = i \cdot V_L = i^2 R_L = \underbrace{V_L^2 / R_L}$$ **Power absorbed at the Load**

2 $$V_L = V_{Th} \frac{R_L}{R_L + R_{Th}}$$

Voltage Division

3 $$p_L = \frac{V_{Th}^2 \frac{R_L^2}{(R_L + R_{Th})^2}}{R_L} \equiv \frac{V_{Th}^2}{(R_L + R_{Th})^2} \cdot R_L$$

Substitute 1 into 2

To find R_L ➡ Maximize p_L

$$\frac{\partial p_L}{\partial R_L} = 0 = V_{Th}^2 \left[\frac{(1)(R_L + R_{Th})^2 - R_L 2 (R_L + R_{Th})(1)}{(R_L + R_{Th})^4} \right]$$

Take partial derivative and set to zero

(Contd…)

MAXIMUM POWER TRANSFER THEOREM (Contd.)

$$(R_L + R_{Th})^2 = 2R_L(R_L + R_{Th})$$ **Simplify**

$$R_L^2 + \cancel{2R_L R_{Th}} + R_{Th}^2 = 2R_L^2 + \cancel{2R_L R_{Th}}$$ **Simplify**

$$R_L^2 = R_{Th}^2$$

$$\boxed{R_L = R_{Th} \quad \textit{For Maximum Power Transfer}}$$

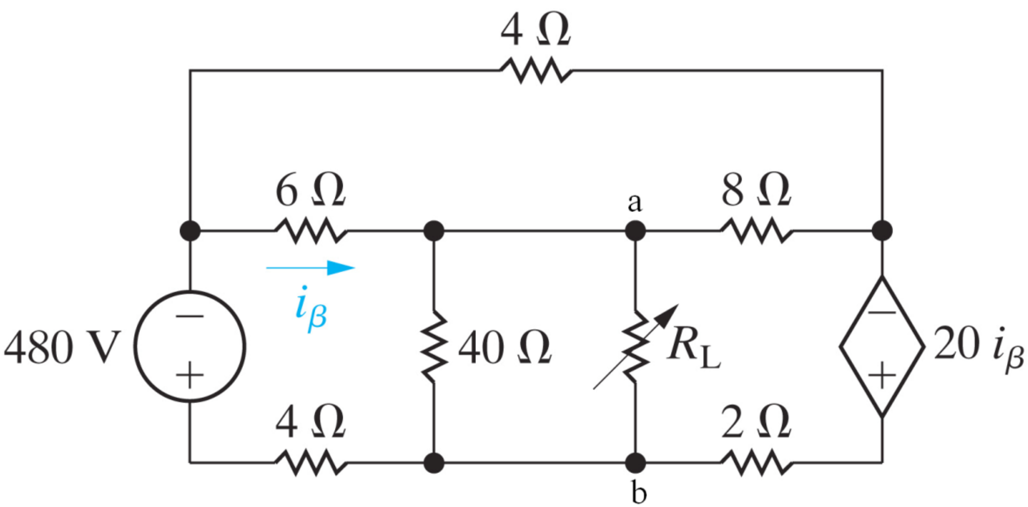
3

$$p_{L\,max} = \frac{V_{Th}^2}{(R_L + R_L)^2}.R_L$$

Since R_L = R_{Th}

$$\boxed{p_{L\,max} = \frac{V_{Th}^2}{4R_L}}$$ **Maximum Power**

Example

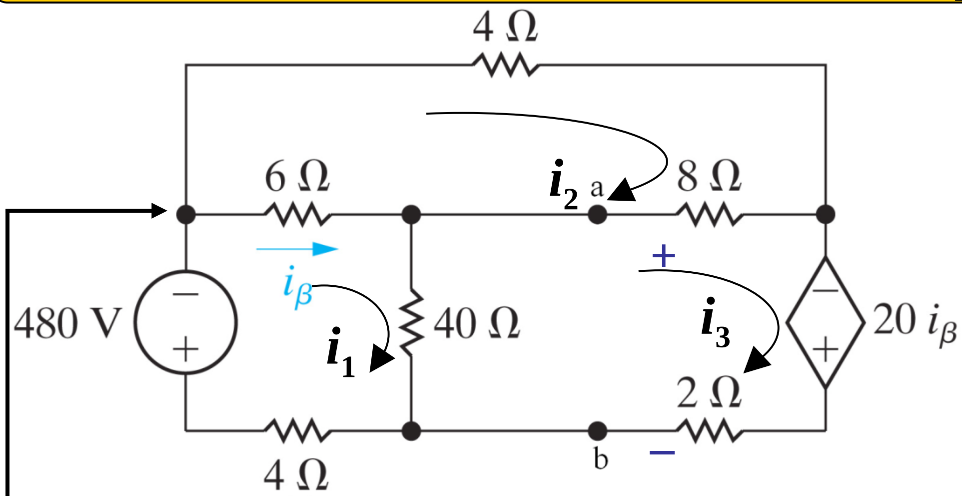


Find R_L for Maximum Power Transfer

- **Need to find Thevenin Equivalent between nodes a and b**
- **So remove R_L and find:**

$$V_{ab} \equiv V_{oc} \equiv V_{Th}$$

Example (Contd.)



Use Mesh Analysis

Find V_{Th} at (a)-(b)

1 $\boxed{-480 = 6(i_1 - i_2) + 40(i_1 - i_3) + 4i_1}$ **Mesh 1**

1 $-480 = 50i_1 - 6i_2 - 40i_3$ **Simplify**

2 $\boxed{0 = -6i_1 + 18i_2 - 8i_3}$ **Mesh 2**

3 $8(i_3 - i_2) - 20\underbrace{(i_1 - i_2)}_{i_\beta} + 2i_3 + 40(i_3 - i_1) = 0$ **Mesh 3**

$i_\beta = i_1 - i_2$

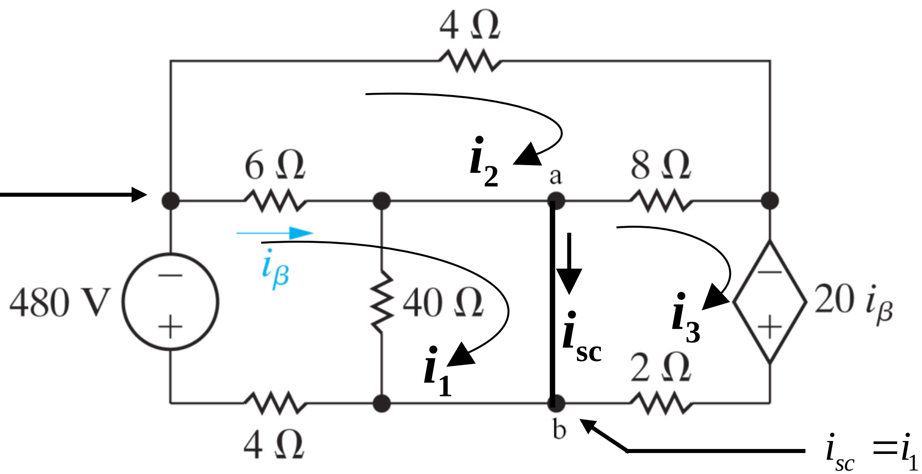
3 $\boxed{0 = -60i_1 + 12i_2 + 50i_3}$ **Simplify**

$i_1 = -99.60\ (A)$ $i_3 = -100.80\ (A)$ **Solve 3 equations for 3 unknowns**

$V_{ab} \equiv V_{40} = 40(i_1 - i_3) = 48\ (V)$ **Ohm's Law**

$V_{Th} = 48(V)$

Example (Contd.)



$i_{sc} = i_1 - i_3$

Find i_{sc}

i.e., Short terminals a and b
40 KΩ Resistor shorted out

Use Mesh Analysis

1 $\quad -480 = 6(i_1 - i_2) + 4i_1$ **Mesh 1**

1 $\quad \boxed{-480 = 10i_1 - 6i_2}$ **Simplify**

2 $\quad \boxed{0 = -6i_1 + 18i_2 - 8i_3}$ **Mesh 2**

3 $\quad 8(i_3 - i_2) - 20\underbrace{(i_1 - i_2)}_{i_\beta} + 2i_3 = 0$ **Mesh 3**

$i_\beta = i_1 - i_2$

3 $\quad \boxed{0 = -20i_1 + 12i_2 + 10i_3}$ **Simplify**

$i_1 = -92(A) \qquad i_3 = -96(A)$

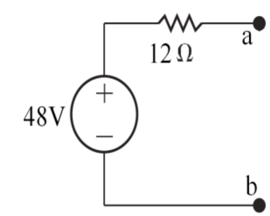
Solve
3 equations for
3 unknowns

$$\boxed{i_{sc} = i_1 - i_3 = 4(A)}$$

$$\Rightarrow \quad R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{48}{4} = 12(\Omega)$$

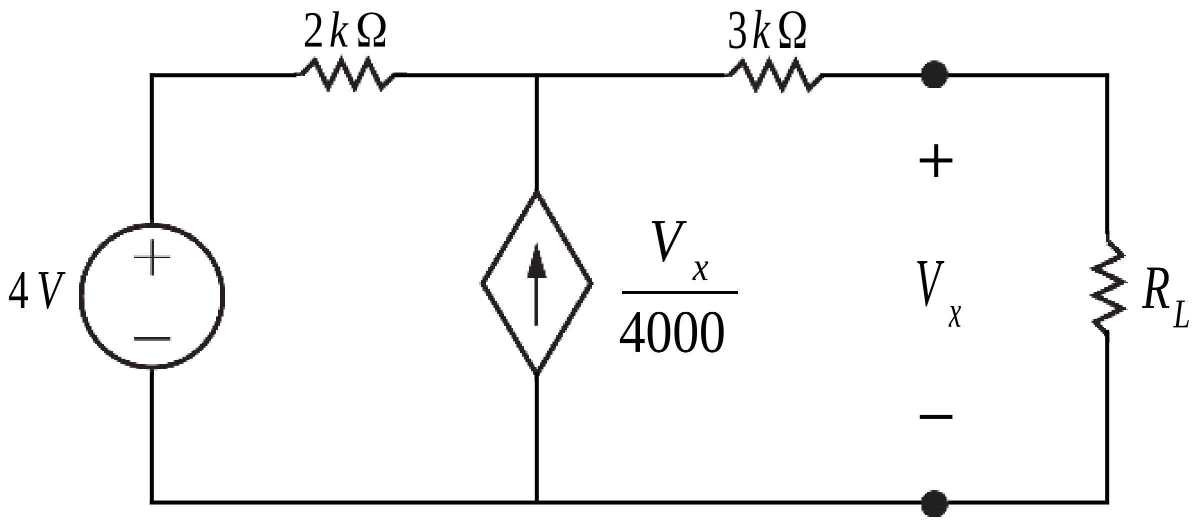
$$\boxed{R_L = R_{Th} = 12(\Omega)}$$

Maximum Power Transferred



Thevenin Equivalent

Thevenin Equivalent Example and Maximum Power Transfer



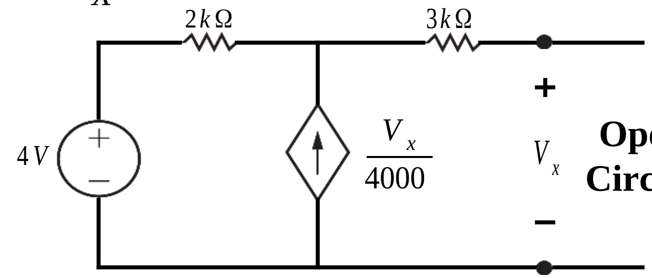
a) Find R_L for Max Power Transfer

b) Find Max Power Transferred to R_L

Example (Contd.)

a) Find Thevenin Equivalent seen by R_L

$$V_{Th} = V_{oc} = V_X$$



Open Circuit

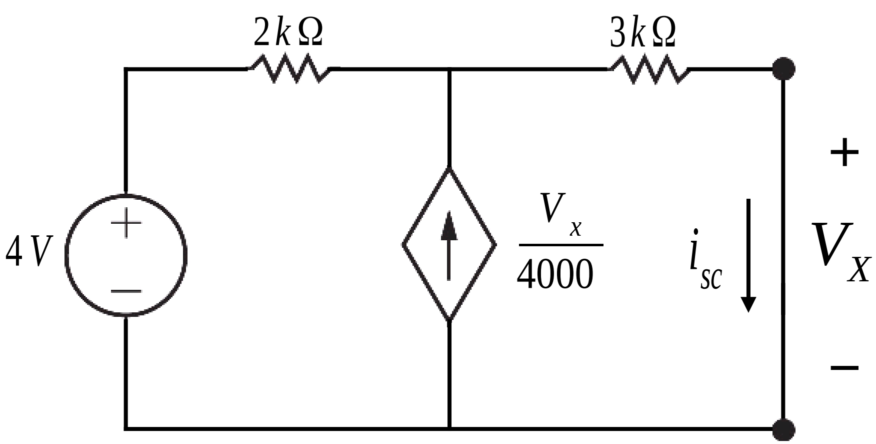
With terminals open, no current in 3kΩ resistor

All the current , V_X/ 4000, goes into the 2kΩ resistor

KVL $\left\{ 4 = -2000\left[\frac{V_X}{4000}\right] + V_X \right.$

Simplify $\left\{ 4 = -\frac{1}{2}V_X + V_X = \frac{1}{2}V_X \right.$ $\Rightarrow$ $\boxed{V_X = V_{oc} = V_{Th} = 8(V)}$

Find R_{Th} Find i_{sc} Note V_x = 0



$4(V) = (2K + 3K)i_{sc}$ **Ohm's Law**

$i_{sc} = \frac{4}{5K}$ **Simplify**

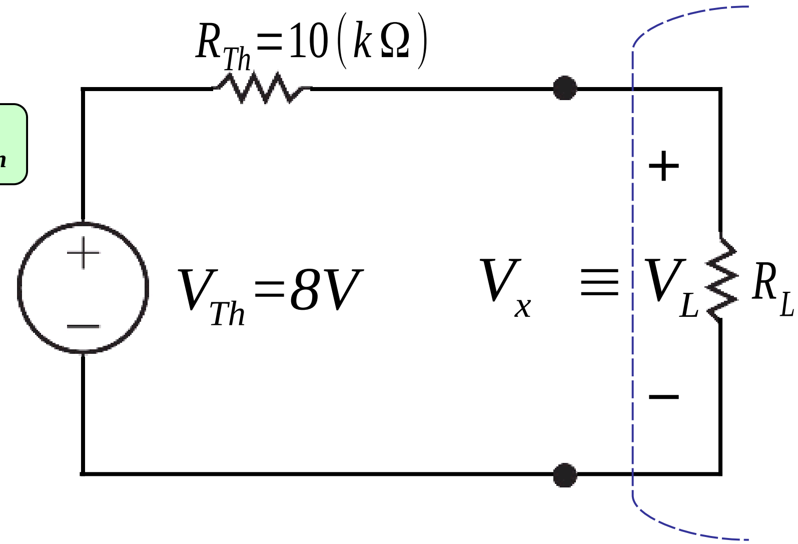
$\boxed{i_{sc} = 0.8(mA)}$

Example (Contd.)

$$R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{8}{0.8 \times 10^{-3}}$$

Find R_{Th}

$$\boxed{R_{Th} = 10(k\Omega)}$$



Thevenin Equivalent Circuit

$$\boxed{R_L = R_{Th} = 10(k\Omega)}$$

<u>**For Max Power**</u>

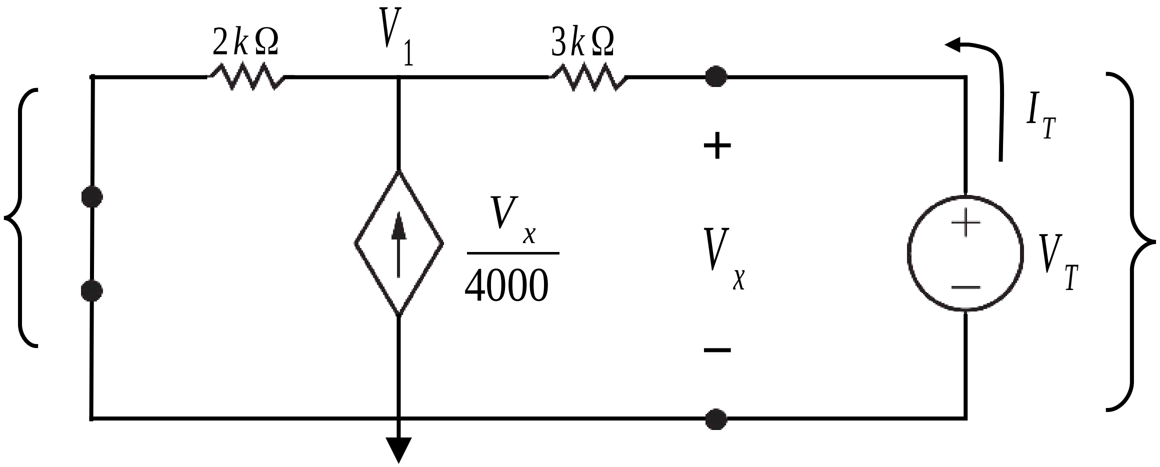
$$V_L = \frac{1}{2} \cdot 8$$

Voltage Division

$$Maximum\ Power\ p_{RL} = \frac{V_L^2}{R_L} = \frac{16}{10(k\Omega)} \Rightarrow \boxed{p_{RL} = 1.6(mW)}$$

Another method for Finding R_{Th}: Test Source

Deactivate Independent sources



Insert Test Source

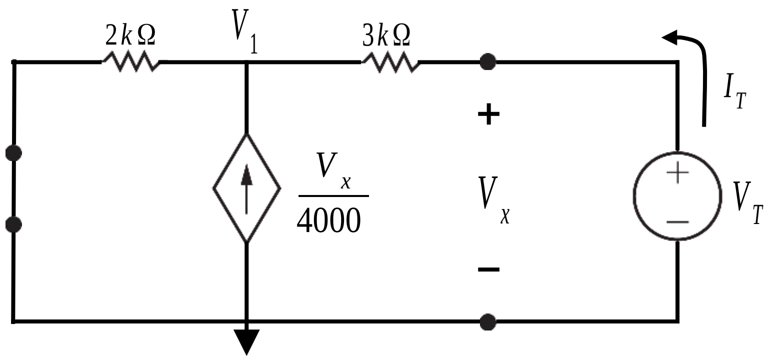
Nodal Equation

$$① \quad \frac{V_1}{2K} - \frac{V_X}{4K} + \frac{V_1 - V_T}{3K} = 0$$ Node Equation

$$② \quad V_X = V_T$$ From Circuit

$$③ \quad \frac{V_1}{2K} - \frac{V_T}{4K} + \frac{V_1 - V_T}{3K} = 0$$ Substitute ② into ①

Another method for Finding R_{Th}: Test Source (Contd.)



③ $6V_1 - 3V_T + 4V_1 - 4V_T = 0$ } **Multiply by 12K**

③ $\boxed{10V_1 - 7V_T = 0}$ } **Simplify**

Relate V_1 to I_T and V_T

④ $V_1 = -3KI_T + V_T$ } **KVL**

$-30KI_T + 10V_T - 7V_T = 0$ } **Substitute ③ into ④**

$-30KI_T + 3V_T = 0$ } **Simplify**

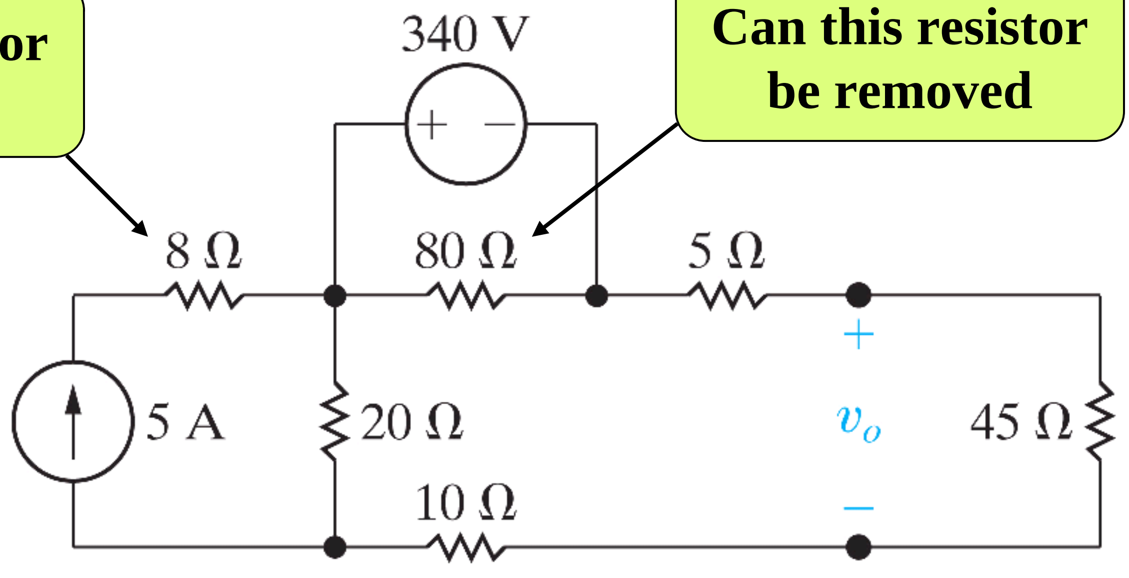
$V_T = \frac{30K}{3} I_T$ } **Simplify**

$$\frac{V_T}{I_T} = \frac{30K}{3} = R_{Th} = 10(k\Omega)$$

Example: Source Transformation

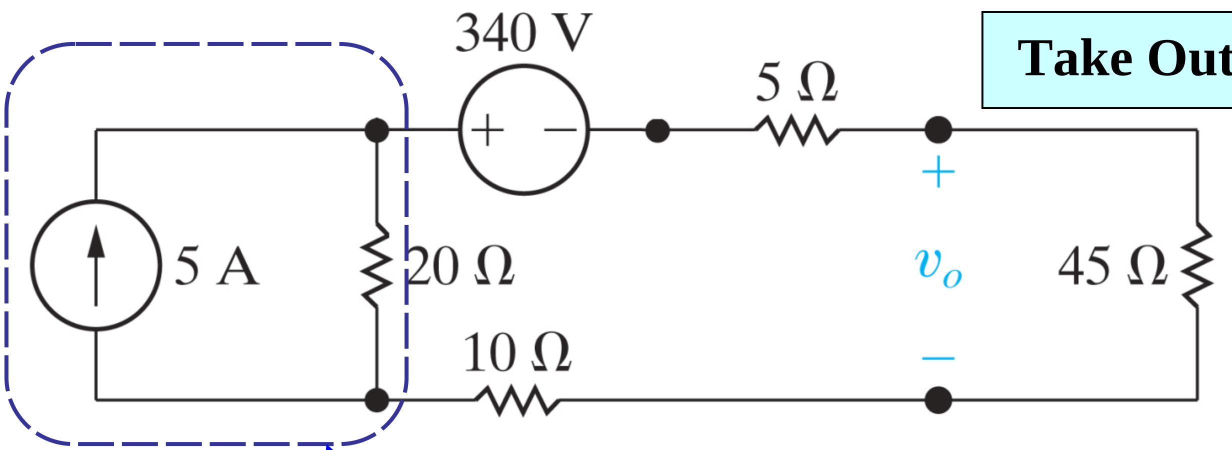
Can this resistor be removed

Can this resistor be removed



Find V_0 Using Source Transformations

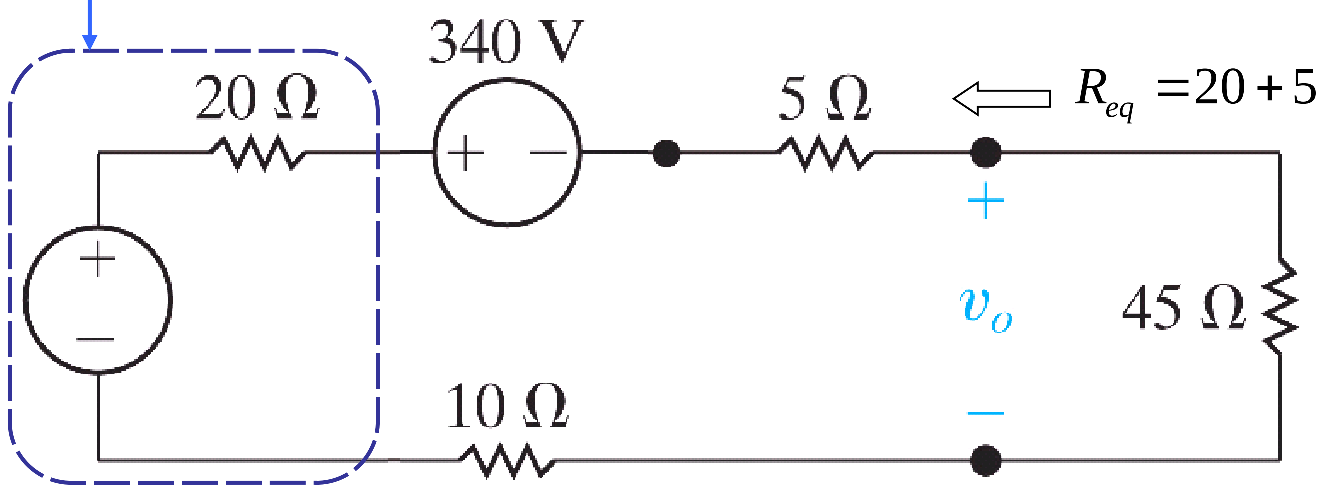
Example: Source Transformation (Contd.)



Take Out 8Ω and 80Ω Resistors

Current Source in series with 8Ω resistor
Voltage in parallel with 80Ω resistor

Source Transformation

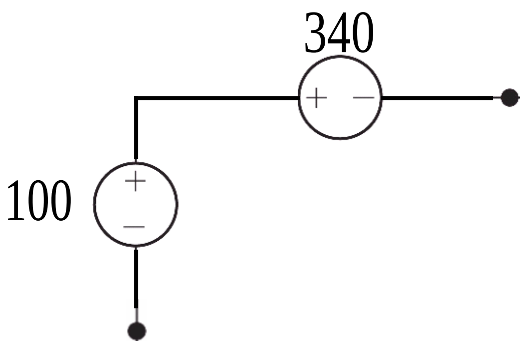


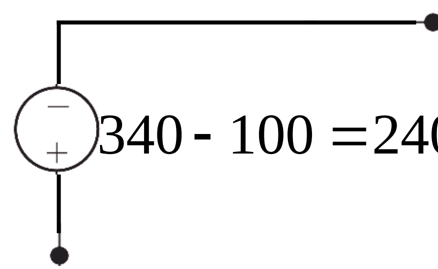
$5A(20\Omega) = 100V$

$R_{eq} = 20 + 5 + 10 = 35\Omega$

Example: Source Transformation (Contd.)

$$R_{eq}=20\,\Omega+5\,\Omega+10\,\Omega=35\,(\Omega)$$

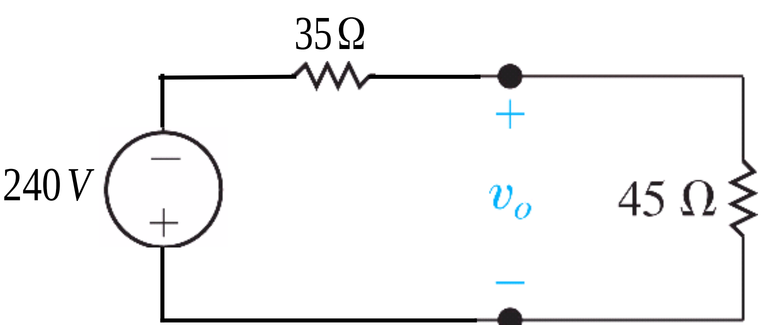




$$340-100=240(V)$$

Combine Voltage Sources

Circuit Becomes



$$v_0 = -\;240 \cdot \frac{45}{45+35}$$

<u>Voltage Division</u>

$$v_0 = -\,135(V)$$