(a) This is false. Let G have vertices $\{v_1, v_2, v_3, v_4\}$, with edges between each pair of vertices, and with the weight on the edge from v_i to v_j equal to $i + j$. Then every tree has a bottleneck edge of weight at least 5, so the tree consisting of a path through vertices v_3, v_2, v_1, v_4 is a minimum bottleneck tree. It is a not a minimum spanning tree, however, since its total weight is greater than that of the tree with edges from v_1 to every other vertex.

(b) This is true. Suppose that T is a minimum spanning tree of G, and T' is a spanning tree with a lighter bottleneck edge. Thus, T contains an edge e that is heavier than every edge in T'. So if we add e to T', it forms a cycle C on which it is the heaviest edge (since all other edges in C belong to T'). By the Cut Property, then, e does not belong to any minimum spanning tree, contradicting the fact that it is in T and T is a minimum spanning tree.

[1] ex582.808.674