Consider the minimum spanning tree T of G under the edge weights $\{a_e\}$, and suppose T were not a minimum-altitude connected subgraph. Then there would be some pair of nodes u and v, and two u-v paths $P \neq P^*$ (represented as sets of edges), so that P is the u-v path in T but P^* has smaller height. In other words, there is an edge $e' = (u', v')$ on P that has the maximum altitude over all edges in $P \cup P^*$. Now, if we consider the edges in $(P \cup P^*) - \{e'\}$, they contain a (possibly self-intersecting) u'-v' path; we can construct such a path by walking along P from u' to u, then along P^* from u to v, and then along P from v to v'. Thus $(P \cup P^*) - \{e'\}$ contains a simple path Q. But then $Q \cup \{e\}$ is a cycle on which e' is the heaviest edge, contradicting the Cycle Property. Thus, T must be a minimum-altitude connected subgraph.

Now consider a connected subgraph $H = (V, E')$ that does not contain all the edges of T; let $e = (u, v)$ be an edge of T that is not part of E'. Deleting e from T partitions T into two connected components; and these two components represent a partition of V into sets A and B. The edge e is the minimum-altitude edge with one end in A and the other in B. Since any path in H from u to v must cross at some point from A to B, and it cannot use e, it must have height greater than a_e. It follows that H cannot be a minimum-altitude connected subgraph.

[1] ex976.901.589