

First note that it is enough to maximize one's *total* grade over the  $n$  courses, since this differs from the average grade by the fixed factor of  $n$ . Let the  $(i, h)$ -*subproblem* be the problem in which one wants to maximize one's grade on the first  $i$  courses, using at most  $h$  hours.

Let  $A[i, h]$  be the maximum total grade that can be achieved for this subproblem. Then  $A[0, h] = 0$  for all  $h$ , and  $A[i, 0] = \sum_{j=1}^i f_j(0)$ . Now, in the optimal solution to the  $(i, h)$ -subproblem, one spends  $k$  hours on course  $i$  for some value of  $k \in [0, h]$ ; thus

$$A[i, h] = \max_{0 \leq k \leq h} f_i(k) + A[i-1, h-k].$$

We also record the value of  $k$  that produces this maximum. Finally, we output  $A[n, H]$ , and can trace-back through the entries using the recorded values to produce the optimal distribution of time. The total time to fill in each entry  $A[i, h]$  is  $O(H)$ , and there are  $nH$  entries, for a total time of  $O(nH^2)$ .

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<sup>1</sup>ex680.762.178