(a) We process the customers in an arbitrary order. At any given point in time, let V_j denote the total value of all customers who have been shown ad j. As we see each new customer, we show him or her the ad for which V_j is as small as possible.

Let s' denote the spread of this algorithm. We first claim that $s' \geq \overline{v}/2m$. To prove this, suppose that ad j is the one achieving the spread (i.e., $V_j = s'$), and let i be any other ad. Let c be the last customer to be shown ad i. Before c was shown ad i, the value of V_i was at most V_j (by the definition of our greedy algorithm), and so $V_i \leq V_j + v_c \leq V_j + (\overline{v}/2m)$ by our assumption about the maximum customer value. Thus, if $s' = V_j < \overline{v}/2m$, then

$$\overline{v} = \sum_j V_j < V_j + (m-1)(V_j + (\overline{v}/2m)) < mV_j + (\overline{v}/2m) < (\overline{v}/2m) + (\overline{v}/2m) = \overline{v},$$

a contradiction.

Next we claim that the optimum spread s satisfies $s \leq \overline{v}/m$. Indeed, the total customer value is $\overline{v}$, and there are m ads, so one must be allocated at most a customer value of $\overline{v}/m$.

Combining these two claims, we get $s \leq \overline{v}/m \leq 2s'$.

(b) Suppose the input begins with $N+m$ customers of value 1, for some very large N, and then $m/2$ customers of value 2. (Suppose m is even and N is divisible by m.) Then our greedy algorithm will produce a spread of $1 + N/m$, while the optimal spread is $2 + N/m$, obtained by grouping the final m customers of value 1 onto $m/2$ ads, and showing the remaining $m/2$ ads to the customers of value 2.

[1] ex43.640.595