Path Selection is in NP, since we can be shown a set of k paths from among P_1, \ldots, P_c and check in polynomial time that no two of them share any nodes.

Now, we claim that 3-Dimensional Matching $\leq_P Path$ Selection. For consider an instance of 3-Dimensional Matching with sets X, Y, and Z, each of size n, and ordered triples T_1, \ldots, T_m from $X \times Y \times Z$. We construct a directed graph G = (V, E) on the node set $X \cup Y \cup Z$. For each triple $T_i = (x_i, y_j, z_k)$, we add edges (x_i, y_j) and (y_j, z_k) to G. Finally, for each $i = 1, 2, \ldots, m$, we define a path P_i that passes through the nodes $\{x_i, y_j, z_k\}$, where again $T_i = (x_i, y_j, z_k)$. Note that by our definition of the edges, each P_i is a valid path in G. Also, the reduction takes polynomial time.

Now we claim that there are n paths among P_1, \ldots, P_m sharing no nodes if and only if there exist n disjoint triples among T_1, \ldots, T_m . For if there do exist n paths sharing no nodes, then the corresponding triples must each contain a different element from X, a different element from Y, and a different element from Z— they form a perfect three-dimensional matching. Conversely, if there exist n disjoint triples, then the corresponding paths will have no nodes in common.

Since *Path Selection* is in NP, and we can reduce an NP-complete problem to it, it must be NP-complete.

(Other direct reductions are from Set Packing and from Independent Set.)

 $^{^{1}}$ ex529.979.546