For simplicity, we will say u *is smaller than* v, or $u \prec v$, if $x_u < x_v$. We will extend this to sets: if S is a set of nodes, we say $u \prec S$ if u has a smaller value than any node in S.

The algorithm is the following. We begin at the root r of the tree, and see if r is smaller than its two children. If so, the root is a local minimum. Otherwise, we move to any smaller child and iterate.

The algorithm terminates when either (1) we reach a node v that is smaller than both its children, or (2) we reach a leaf w. In the former case, we return v; in the latter case, we return w.

The algorithm performs $O(d) = O(\log n)$ probes of the tree; we must now argue that the returned value is a local minimum. If the root r is returned, then it is a local minimum as explained above. If we terminate in case (1), v is a local minimum because v is smaller than its parent (since it was chosen in the previous iteration) and its two children (since we terminated). If we terminate in case (2), w is a local minimum because w is smaller than its parent (again since it was chosen in the previous iteration).

[1] ex739.448.876