The problem is in $\mathcal{NP}$ because we can exhibit a set of bidders, and in polynomial time is can be checked that no two bought bid on the same item, and that the total value of their bids is at least $B$.

We now show that *Independent Set* $\leq_P$ *Diverse Subset*. Given a graph $G$ and a number $k$, we construct a bidder for each node of $G$, and an item for each edge of $G$. Each bidder $v$ places a bid on each item $e$ for which $e$ is incident to $v$ in $G$. We set the value of each bid to 1. Finally, we ask whether the auctioneer can accept a set of bids of total value $B = k$.

We claim that this holds if and only if $G$ has an independent set of size $k$. If there is a set of acceptable bids of total value $k$, then the corresponding set of nodes has the property that no two are incident to the same edge — so it is an independent set of size $k$. Conversely, if there is an independent set of size $k$, then the corresponding set of bidders has the property that their bids are disjoint, and their total value is $k$.

[1] ex617.432.555