Galactic Shortest Path is in NP: given a path P in a graph, we can add up the lengths and risks of its edges, and compare them to the given bounds L and R.

Galactic Shortest Path involves adding numbers, so we naturally consider reducing from the *Subset Sum* problem. Specifically, we'll prove that *Subset Sum* $\leq_P$ *Galactic Shortest Path*.

Thus, consider an instance of *Subset Sum*, specified by numbers $w_1, \ldots, w_n$ and a bound W; we want to know if there is a subset S of these numbers that add up to exactly W. *Galactic Shortest Path* looks somewhat different on the surface, since we have *two kinds* of numbers (lengths and risks), and we are only given *upper bounds* on their sums. However, we can use the fact that we also have an underlying graph structure. In particular, by defining a simple type of graph, we can encode the idea of choosing a subset of numbers.

We define the following instance of *Galactic Shortest Path*. The graph G has a nodes $v_0, v_1, \ldots, v_n$. There are two edges from v_{i-1} to v_i, for each $1 \leq i \leq n$; we'll name them e_i and e'_i. (If one wants to work with a graph containing no parallel edges, we can add extra nodes that subdivide these edges into two; but the construction turns out the same in any case.)

Now, any path from v_0 to v_n in this graph G goes through edge one from each pair $\{e_i, e'_i\}$. This is very useful, since it corresponds to making n independent binary choices — much like the binary choices one has in *Subset Sum*. In particular, choosing e_i will represent putting w_i into our set S, and e'_i will represent leaving it out.

Here's a final observation. Let $W_0 = \sum_{i=1}^{n} w_i$ — the sum of all the numbers. Then a subset S adds up to W if and only if its complement adds up to $W_0 - W$.

We give e_i a length of w_i and a risk of 0; we give e'_i a length of 0 and a risk of w_i. We set the bound $L = W$, and $R = W_0 - W$. We now claim: there is a solution to the *Subset Sum* instance if and only if there is a valid path in G. For if there is a set S adding up to W, then in G we use the edges e_i for $i \in S$, and e'_j for $j \notin S$. This path has length W and risk $W_0 - W$, so it meets the given bounds. Conversely, if there is a path P meeting the given bounds, then consider the set $S = \{w_i : e_i \in P\}$. S adds up to at most W and its complement adds up to at most $W_0 - W$. But since the two sets together add up to exactly W_0, it must be that S adds up to exactly W and its complement to exactly $W_0 - W$. Thus, S is valid solution to the *Subset Sum* instance.

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