

The problem *Decisive Subset (DS)* is in NP because we can check in polynomial time that a given subset of committee members is of size at most k , and that its voting outcome is the same as that of the whole committee.

Now we show that *Vertex Cover* \leq_P *DS*. Given a graph $G = (V, E)$ and bound k , we create an issue I_e for each edge e , and a committee member m_v for each node v . If $e = (u, v)$, then we have members u and v vote “yes” on issue I_e , and all other committee members abstain. Note that the voting outcome by the whole committee on all issues is “yes.” We now ask whether there is a decisive subset of size at most k .

If there is a decisive subset S of size at most k , then it must lead to an affirmative decision for each issue. In particular, this means that for each edge e , S must include at least one of the members m_u or m_v , and so the corresponding set of nodes will constitute a vertex cover in G of size at most k .

If there is a vertex cover C of size at most k , then for each edge $e = (u, v)$, at least one of u or v will be in C . For the set S of members corresponding to C , the voting outcome will thus be “yes” on each issue I_e , so S is decisive.

¹ex7.111.521