We start off by showing that *HSoDT!* is in *NP*. Let the certificate t consist of a path P and a sequence of transactions to be performed along P. Then the certifier B should check if performing the given transactions along the given path P achieves the target bound.

We shall now show *HSoDT!* is NP-complete by showing *3-SAT* $\leq_P$ *HSoDT!*. Consider a *3-SAT* instance with n variables and k clauses. Construct a layered graph $G = (V, E)$ with $n + k$ layers. The first n layers correspond to the n variables and their negations and the last k layers correspond to the clauses. More specifically, layer i of the first n layers consists of two nodes (not adjacent), one that sells droid types corresponding to variable x_i and the other sells droid types corresponding to variable $\overline{x}_i$. The supply of x_i and $\overline{x}_i$ is the total number of times each of them occurs in the k clauses. Also, let their prices be zero. For layer i of the last k layers, construct three nodes (not adjacent) corresponding to the variables or their negations in clause i. If x is a variable or its negation in clause i, then the corresponding node in layer i of the last k layers has a demand for one unit of droid type x with unit cost. Now for each of the first $n + k - 1$ layers, construct directed edges from each of the nodes in layer i to each of the nodes in layer $i + 1$. Construct a starting node s with edges from s to each node in layer 1 and an ending node t with edges from each node in layer $n + k$ to t. Note that there are $2n$ droid types, $2 + 2n + 3k$ nodes including s and t. Now let the target bound be k. We claim that this bound can be reached on this instance of *HSoDT!* if and only if the given *3-SAT* instance has a solution.

Assume we have an *HSoDT!* solution. Note that for each of the layers, we have to pass through exactly one of the nodes. Layer i of the first n layers has two nodes, x_i and $\overline{x}_i$. If the solution passes through node x_i, then let variable x_i have a true assignment else let it have a false assignment. Since the target bound of k is reached, then one droid is sold at each of the last k layers which implies that each clause evaluates to true. Thus we have a *3-SAT* solution.

Now assume we have a *3-SAT* solution. Then we must have each clause evaluate to true, i.e. for each clause C_i, there must be some x_j or $\overline{x}_j$ in C_i such that the one in C_i evaluates to true. Now construct the path P such that for each of the first n layers we pass through node x_i if variable x_i has a true assignment else we pass through node $\overline{x}_i$. When passing through each node in the first n layers, take the available supply of droids. When passing through layer i of the last k layers, visit a node that causes clause i to evaluate to true and sell a unit of the corresponding droid. Since we sell a droid at each of the k layers, the target bound of k is achieved.

[1] ex182.967.464