The problem is in $\mathcal{NP}$ because we can exhibit a choice of one element from each option set, and it can be checked there are no incompatibilities between these.

We now show that *3-SAT*$\leq_P$ *Fully Compatible Configuration*. The reduction will be very similar to the reduction that showed *3-SAT*$\leq_P$ *Independent Set*. Given an instance of *3-SAT*, we create an option set for each clause, and each of these sets has three options, corresponding to the terms in the clause. We now declare an option in A_i to be incompatible with an option in A_j if the terms corresponding to these two options correspond to a variable and its negation.

If there is a satisfying assignment for the *3-SAT* instance, then we can choose a term from each clause in such a way that we never choose both a variable and its negation. Thus, the corresponding selection of options will have no incompatibilities. Conversely, if there is a way to select options without incompatibilities, then there is a corresponding selection of terms from clauses so that we never choose a variable and its negation. Thus, we can set all variables as indicated by the selected terms (setting variables arbitrary when they appear in no selected term) so as to satisfy the *3-SAT* instance.

[1]ex755.846.885