

The problem is in NP since, given a set of k counselors, we can check that they cover all the sports.

Suppose we had such an algorithm \mathcal{A} ; here is how we would solve an instance of *Vertex Cover*. Given a graph $G = (V, E)$ and an integer k , we would define a sport S_e for each edge e , and a counselor C_v for each vertex v . C_v is qualified in sport S_e if and only if e has an endpoint equal to v .

Now, if there are k counselors that, together, are qualified in all sports, the corresponding vertices in G have the property that each edge has an end in at least one of them; so they define a vertex cover of size k . Conversely, if there is a vertex cover of size k , then this set of counselors has the property that each sport is contained in the list of qualifications of at least one of them.

Thus, G has a vertex cover of size at most k if and only if the instance of *Efficient Recruiting* that we create can be solved with at most k counselors. Moreover, the instance of *Efficient Recruiting* has size polynomial in the size of G . Thus, if we could determine the answer to the *Efficient Recruiting* instance in polynomial time, we could also solve the instance of *Vertex Cover* in polynomial time.

¹ex195.705.667