

Suppose that to obtain  $n$  words, we need  $L$  lines (most of which will get repeated many times, as described above). We write the script as follows

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line 1 = <text of line 1 here>
line 2 = <text of line 2 here>
...
line  $L$  = <text of line  $L$  here>
For  $i = 1, 2, \dots, L$ 
  For  $j = 1, 2, \dots, i$ 
    Sing lines  $j$  through 1
  Endfor
Endfor

```

Now, the nested **For** loops have length bounded by a constant  $c_1$ , so the real space in the script is consumed by the text of the lines. Each of these lines in the script has length at most  $c_2$  (where  $c_2$  is the maximum line length  $c$  plus the space to write the variable assignment). So in total, the space required by the script is  $S = c_1 + c_2 L$ .

Recall that  $n$  denotes the number of words this produces when sung.  $n$  is at least  $1 + 2 + \dots + L = \frac{1}{2}L(L + 1)$ ; hence,  $\frac{1}{2}(L + 1)^2 \leq n$ , and so  $L \leq 1 + \sqrt{2n}$ . Plugging this into our bound on the length of the script, we have  $f(n) = S \leq c_1 + c_2 \sqrt{2n} = O(\sqrt{n})$ .

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<sup>1</sup>ex434.486.949