A useful way to analyze large products in cases like this is to take logarithms, which causes them to become sums. Thus, let us build a graph G with a node for each stock, and a directed edge (i, j) for each pair of stocks. We put a cost of $-\log r_{ij}$ on edge (i, j).

Now, a trading cycle C in G is an opportunity cycle if and only if

$$\prod_{(i,j)\in C} r_{ij} > 1,$$

in other words, taking logarithms of both sides, if and only if

$$\sum_{(i,j)\in C} \log r_{ij} > 0,$$

or

$$\sum_{(i,j)\in C} -\log r_{ij} < 0.$$

Thus, a trading cycle C in G is an opportunity cycle if and only if it is a negative cycle. Hence we can use our polynomial-time algorithm for negative-cycle detection to determine whether an opportunity cycle exists.

[1] ex181.273.949