The claim is true; here is a proof. Let G be a graph with the given properties, and suppose by way of contradiction that it is not connected. Let S be the nodes in its smallest connected component. Since there are at least two connected components, we have $|S| \leq n/2$. Now, consider any node $u \in S$. Its neighbors must all lie in S, so its degree can be at most $|S| - 1 \leq n/2 - 1 < n/2$. This contradicts our assumption that every node has degree at least $n/2$.

[1] ex722.926.396