

By *transaction* (i, j) , we mean the single transaction that consists of buying on day i and selling on day j . Let $P[i, j]$ denote the monetary return from transaction (i, j) . Let $Q[i, j]$ denote the maximum profit obtainable by executing a single transaction somewhere in the interval of days between i and j . Note that the transaction achieving the maximum in $Q[i, j]$ is either the transaction (i, j) , or else it fits into one of the intervals $[i, j - 1]$ or $[i + 1, j]$. Thus we have

$$Q[i, j] = \max(P[i, j], Q[i, j - 1], Q[i + 1, j]).$$

Using this formula, we can build up all values of $Q[i, j]$ in time $O(n^2)$. (By going in order of increasing $i + j$, spending constant time per entry.)

Now, let us say that an *m-exact strategy* is one with *exactly* m non-overlapping buy-sell transactions. Let $M[m, d]$ denote the maximum profit obtainable by an m -exact strategy on days $1, \dots, d$, for $0 \leq m \leq k$ and $0 \leq d \leq n$. We will use $-\infty$ to denote the profit obtainable if there isn't room in days $1, \dots, d$ to execute m transactions. (E.g. if $d < 2m$.) We can initialize $M[m, 0] = -\infty$ and $M[0, d] = -\infty$ for each m and each d .

In the optimal m -exact strategy on days $1, \dots, d$, the final transaction occupies an interval that begins at i and ends at j , for some $1 \leq i < j \leq d$; and up to day $i - 1$ we then have an $(m - 1)$ -exact strategy. Thus we have

$$M[m, d] = \max_{1 \leq i < j \leq d} Q[i, j] + M[m - 1, i - 1].$$

We can fill in these entries in order of increasing $m + d$. The time spent per entry is $O(n)$, since we've already computed all $Q[i, j]$. Since there are $O(kn)$ entries, the total time is therefore $O(kn^2)$. We can determine the strategy associated with each entry by maintaining a pointer to the entry that produced the maximum, and tracing back through the dynamic programming table using these pointers.

Finally, the optimal k -shot strategy is, by definition, an m -exact strategy for some $m \leq k$; thus, the optimal profit from a k -shot strategy is

$$\max_{0 \leq m \leq k} M[m, n].$$

¹ex541.91.349