The problem is in $\mathcal{NP}$, since we can exhibit a set of k edges, and it can be checked in polynomial time that all nodes in X can reach one another on paths using these edges.

Now we show *Vertex Cover* $\leq_P$ *Graphical Steiner Tree.* Given a graph G on n nodes and m edges, and a number k, we construct a new graph H as follows. We insert a new node w_e in the middle of each edge $e = (u, v)$ (connected only to u and v); let W denote this set of new nodes. We also include one additional node r connected to all the original nodes V of the graph G. We define $X = \{r\} \cup W$, and we ask whether X can be connected using $k' = k + m$ edges.

If G has a vertex cover S of size k, this can be done: we include edges from r to each node in S, and from each node in W to one of its neighbors in S.

Conversely, suppose there is a Steiner tree for X using a set F of at most k' edges. We may assume that F includes at most one edge incident to each node w_e: if it included both edges, we could modify it so that one of these edges was retained, and the other edge, say (u, w_e), was replaced by the edge (r, u). The resulting set would still form a Steiner tree.

Given this structure for F, we see that m edges are used to connect to nodes in W, and that leaves k edges from r to nodes of V. Let S be the ends in V of these edges from r. We claim that S is a vertex cover. Indeed, every node w_e has a path to r, and by the structure we have imposed on F, this path must consist of two steps: from w_e to one of its ends u, and then directly to r. Thus, every edge e is incident to at least one node in S, so S is a vertex cover.

[1]ex569.422.522