(a) Consider a clause C_i with n variables. The probability that the clause is not satisfied is $\frac{1}{2^m}$ and so the probability that it is satisfied is 1 less this quantity. The worst case is when C_i has just one variable, i.e. $n = 1$, in which case the probability of the clause being satisfied is $\frac{1}{2}$. Since there are k clauses, the expected number of clauses being satisfied is at least $\frac{k}{2}$. Consider the two clauses x_1 and $\overline{x_1}$. Clearly only one of these can be satisfied.

(b) For variables that occur in single variable clauses, let the probability of setting the variable so as to satisfy the clause be $p \geq \frac{1}{2}$. For all other variables, let the probabilities be $\frac{1}{2}$ as before. Now for a clause C_i with n variables, $n \geq 2$, the probability of satisfying it is at worst $(1 - \frac{1}{2^n}) \geq (1 - p^2)$ since $p \geq \frac{1}{2}$. Now to solve for p, we want to satisfy all clauses, so solve $p = 1 - p^2$ to get $p \approx 0.62$. And hence the expected number of satisfied clauses is $0.62n$.

(c) Let the total number of clauses be k. For each pair of single variable conflicting clauses, i.e. x_i and $\overline{x_i}$, remove one of them from the set of clauses. Assume we have removed m clauses. Then the maximum number of clauses we could satisfy is $k - m$. Now apply the algorithm described in the previous part of the problem to the $k - 2m$ clauses that had no conflict to begin with. The expected number of clauses we satisfy this way is $0.62 * (k - 2m)$. In addition to this we can also satisfy m of the $2m$ conflicting clauses and so we satisfy $0.62 * (k - 2m) + m \geq 0.62 * (k - m)$ clauses which is our desired target. Note that this algorithm is polynomial in the number of variables and clauses since we look at each clause once.

[1] ex633.413.669