In this problem, you basically have a set of n points (the account events) and a set of intervals (the "error bars" around the suspicious transactions, i.e. $[t_i - e_i, t_i + e_i]$), and you want to know if there is a perfect matching between points and intervals so that each point lies in its corresponding interval). Without loss of generality, let us assume $x_1 \le x_2 \le \ldots \le x_n$.

A greedy style algorithm goes like this:

```
for i = 1, 2, ..., n
  if there are unmatched intervals containing x_i
    Match x_i with the one that ends earliest
  else
    Declare that there is no perfect matching
```

It is obvious that if the algorithm succeeds, it really finds a perfect matching. We want to prove that if there is a perfect matching, the algorithm will find it. We prove this by an exchange argument, which we will express in the form of a proof by contradiction.

Suppose by way of contradiction that there is a perfect matching, but that the above greedy algorithm does not construct one. Choose a perfect matching M, in which the first i points $x_1, x_2, \ldots, x_i$ match to intervals in the same way described in the algorithm, and i is the largest number with this property. Now suppose x_{i+1} matches to an interval centered at t_l in M, but the algorithm matches x_{i+1} to another interval centered at t_j. According to the algorithm, we know that $t_j + e_j \le t_l + e_l$. Suppose t_j is matched to x_k ($x_k \ge x_{i+1}$) in M. Then we have

$$t_l - e_l \le x_{i+1} \le x_k \le t_j + e_j \le t_l + e_l,$$

so in M we can instead match x_k to t_l and match x_{i+1} to t_j to have a new perfect matching M', which agrees with the algorithm. M' agrees with the output of the greedy algorithm on the first $i+1$ points, contradicting our choice of i.

To bound the running time, note that if we simply enumerate all unmatched intervals in each iteration of the `for` loop, it will take $O(n)$ time to find the unmatched one that ends earliest. There are n iterations, so the algorithm takes $O(n^2)$ time.

[1] ex18.628.375