

In a Steiner tree T on $X \cup Z \subseteq V$, $|X| = k$, we will refer to X as the *terminals* and Z as the *extra nodes*. We first claim that each extra node has degree at least 3 in T ; for if not, then the triangle inequality implies we can replace its two incident edges by an edge joining its two neighbors. Since the sum of the degrees in a t -node tree is $2t - 2$, every tree has at least as many leaves as it has nodes of degree greater than 2. Hence $|Z| \leq k$. It follows that if we compute the minimum spanning tree on all sets of the form $X \cup Z$ with $|Z| \leq k$, the cheapest among these will be the minimum Steiner tree. There are at most $\binom{n}{2k} = n^{O(k)}$ such sets to try, so the overall running time will be $n^{O(k)}$.