

Let the sequence  $S$  consist of  $s_1, \dots, s_n$  and the sequence  $S'$  consist of  $s'_1, \dots, s'_m$ . We give a greedy algorithm that finds the first event in  $S$  that is the same as  $s'_1$ , matches these two events, then finds the first event after this that is the same as  $s'_2$ , and so on. We will use  $k_1, k_2, \dots$  to denote the match have we found so far,  $i$  to denote the current position in  $S$ , and  $j$  the current position in  $S'$ .

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Initially  $i = j = 1$ 
While  $i \leq n$  and  $j \leq m$ 
    If  $s_i$  is the same as  $s'_j$ , then
        let  $k_j = i$ 
        let  $i = i + 1$  and  $j = j + 1$ 
    otherwise let  $i = i + 1$ 
EndWhile
If  $j = m + 1$  return the subsequence found:  $k_1, \dots, k_m$ 
Else return that " $S'$  is not a subsequence of  $S$ "

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The running time is  $O(n)$ : one iteration through the while loop takes  $O(1)$  time, and each iteration increments  $i$ , so there can be at most  $n$  iterations.

It is also clear that the algorithm finds a correct match if it finds anything. It is harder to show that if the algorithm fails to find a match, then no match exists. Assume that  $S'$  is the same as the subsequence  $s_{l_1}, \dots, s_{l_m}$  of  $S$ . We prove by induction that the algorithm will succeed in finding a match and will have  $k_j \leq l_j$  for all  $j = 1, \dots, m$ . This is analogous to the proof in class that the greedy algorithm finds the optimal solution for the interval scheduling problem: we prove that the greedy algorithm is always ahead.

- For each  $j = 1, \dots, m$  the algorithm finds a match  $k_j$  and has  $k_j \leq l_j$ .

*Proof.* The proof is by induction on  $j$ . First consider  $j = 1$ . The algorithm lets  $k_1$  be the first event that is the same as  $s'_1$ , so we must have that  $k_1 \leq l_1$ .

Now consider a case when  $j > 1$ . Assume that  $j - 1 < m$  and assume by the induction hypothesis that the algorithm found the match  $k_{j-1}$  and has  $k_{j-1} \leq l_{j-1}$ . The algorithm lets  $k_j$  be the first event after  $k_{j-1}$  that is the same as  $s'_j$  if such an event exists. We know that  $l_j$  is such an event and  $l_j > l_{j-1} \geq k_{j-1}$ . So  $s_{l_j} = s'_j$ , and  $l_j > k_{j-1}$ . The algorithm finds the first such index, so we get that  $k_j \leq l_j$ . ■

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<sup>1</sup>ex876.936.4