

(a) Change  $x_4$  to 2 in the given example. Then this algorithm would activate the EMP at times 2 and 4, for a total of 4 destroyed; but activating at times 3 and 4 as before still gets 5.

(b) Let  $OPT(j)$  be the maximum number of robots that can be destroyed for the instance of the problem just on  $x_1, \dots, x_j$ . Clearly if the input ends at  $x_j$ , there is no reason not to activate the EMP then (you're not saving it for anything), so the choice is just when to last activate it before step  $j$ . Thus  $OPT(j)$  is the best of these choices over all  $i$ :

$$OPT(j) = \max_{0 \leq i < j} [OPT(i) + \min(x_j, f(j-i))],$$

where  $OPT(0) = 0$ . The full algorithm is just

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Set  $OPT(0) = 0$ 
For  $i = 1, 2, \dots, n$ 
  Compute  $OPT(j)$  using the recurrence
Endfor
Return  $OPT(n)$ .

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The running time is  $O(n)$  per iteration, for a total of  $O(n^2)$ .

An alternate solution would define  $OPT'(j, k)$  to be the best solution for steps  $j$  through  $n$ , given that the EMP in step  $j$  has already been charging for  $k$  steps. The optimal way to solve this sub-problem would be to either activate the EMP in step  $j$  or not, and  $OPT'(j, k)$  is just the better of these two choices:

$$OPT'(j, k) = \max(\min(x_j, f(k)) + OPT'(j+1, 1), OPT'(j+1, k+1)).$$

We initialize  $OPT'(n, k) = \min(x_n, f(k))$  for all  $k$ , and the full algorithm is

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Set  $OPT'(n, k) = \min(x_n, f(k))$  for all  $k$ .
For  $j = n-1, n-2, \dots, 1$ 
  For  $k = 1, 2, \dots, j$ 
    Compute  $OPT'(j, k)$  using the recurrence
  Endfor
Endfor
Return  $OPT'(1, 1)$ .

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The running time is  $O(1)$  per entry of  $OPT'$ , for a total of  $O(n^2)$ .