Let Y denote the number of steps in which your net profit is positive. Then $Y = Y_1 + Y_2 + \cdots + Y_n$, where $Y_k = 1$ if your net profit is positive at step k, and 0 otherwise.

Now, consider a particular step k. $Y_k = 1$ if and only if you have had more than $k/2$ steps in which your profit increased. Since the expected number of steps in which your profit increased is $k/3$, we can apply the Chernoff bound (4.1) with $\mu = k/3$ and $1 + \delta = 3/2$ to conclude that EY_k is bounded by

$$\left[\frac{e^{1/2}}{(3/2)^{(3/2)}} \right]^{(k/3)} < (.97)^k.$$

Thus,

$$EY = \sum_{k=1}^{n} EY_k < \sum_{k=1}^{n} (.97)^k < \frac{1}{1 - (.97)} < 34,$$

which is a constant independent of n.

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[1] ex251.139.906