

The mean for  $X_2$  is  $n$ , so in order to have  $X_1 - X_2 > c\sqrt{n}$ , we need  $X_2 < E[X_2] - \frac{c}{2}\sqrt{n} = (1 - \delta)E[X_2]$  for  $\delta = \frac{c}{2\sqrt{n}}$ . Plugging this into the Chernoff lower bound, the probability this happens is

$$e_{-\frac{1}{2}\delta^2 E[X_2]} = e^{-c^2/4}.$$

This can be made smaller than a constant  $\varepsilon$  by choosing the undetermined constant  $c$  large enough.