

Consider a graph  $G$  with nodes  $s$  and  $t$ , and  $n - 2$  other nodes  $v_1, \dots, v_{n-2}$ . There are two parallel edges from  $s$  to each  $v_i$ , and one edge from  $v_i$  to  $t$ . The minimum  $s$ - $t$  cut is to separate  $t$  by itself.

If we run the version of the contraction algorithm described in the problem, it will independently contract each of the length-2 paths from  $s$  to  $t$  in some order. In order for it to find the minimum  $s$ - $t$  cut, it must contract each  $v_i$  into  $s$ , not into  $t$ . There is a  $2/3$  chance of this happening for each  $i$ , so the probability that the minimum  $s$ - $t$  cut is found is  $(2/3)^{n-2}$ , an exponentially small quantity.

(Note that this example poses no problem for the global minimum cut, which consists of any of the nodes  $v_i$  on its own.)

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<sup>1</sup>ex242.186.32