W will prove LDC is NPC by reduction from k *Coloring*, that is, given a graph $G = (V, E)$ and an integer k, we want to know whether we can color V with k colors, s.t. no two adjacent nodes share the same color.

Construct an instance of LDC as follows: for each node $v_i \in V$, we have an object p_i, let

$$d(p_i, p_j) = \begin{cases} 0 & i = j \\ 1 & i \neq j \wedge (v_i, v_j) \notin E \\ 2 & i \neq j \wedge (v_i, v_j) \in E \end{cases}$$

and let $B = 1$. The goal is to partition $\{p_1, p_2, \ldots, p_n\}$ into k subsets.

Now we are going to prove that k *Coloring* is achievable if and only we can find a valid partition in LDC. If we have a valid coloring scheme, then we can partition those objects into k subsets, each of which is corresponding to a subset of nodes which have the same color. From the specification of k *Coloring* problem, we know that there is no edge connecting two nodes with the same color, and therefore their corresponding objects have distance no greater than 1, and hence we have a valid partition in LDC. If we have a valid partition in LDC, then each subset is corresponding to a different color, and we can color those nodes that have their counterparts in the same subset with the same color. By our construction of LDC instance, we know that any two objects in the same subset can't have distance of 2, which means that their corresponding nodes in k *Coloring* problem are not connected. So the coloring will be legal.

[1]ex463.411.47