As this is a maximization problem, we need an upper bound of c^*, and there is an easy one:

$$c^* \leq m$$

where $m = |E|$.

The algorithm is: coloring every node independently with one of the three colors, each with probability $\frac{1}{3}$.

Let random variable

$$X_e = \begin{cases} 1 & \text{edge } e \text{ is satisfied} \\ 0 & \text{otherwise} \end{cases}$$

Then for any given edge e, there are 9 ways to color its two ends, each of which appears with the same probability, and 3 of them are not satisfying.

$$Exp[X_e] = Pr[e \text{ is satisfied}] = \frac{6}{9} = \frac{2}{3}$$

Let Y be the random variable denoting the number of satisfied edges, then by linearity of expectations,

$$Exp[Y] = Exp\left[\sum_{e \in E} X_e\right] = \sum_{e \in E} Exp[X_e] = \frac{2}{3}m \geq \frac{2}{3}c^*$$

[1] ex568.721.313