

(a) Consider the sequence 1, 4, 2, 3. The greedy algorithm produces the rising trend 1, 4, while the optimal solution is 1, 2, 3.

(b) Let  $OPT(j)$  be the length of the longest increasing subsequence on the set  $P[j], P[j+1], \dots, P[n]$ , including the element  $P[j]$ . Note that we can initialize  $OPT(n) = 1$ , and  $OPT(1)$  is the length of the longest rising trend, as desired.

Now, consider a solution achieving  $OPT(j)$ . Its first element is  $P[j]$ , and its next element is  $P[k]$  for some  $k > j$  for which  $P[k] > P[j]$ . From  $k$  onward, it is simply the longest increasing subsequence that starts at  $P[k]$ ; in other words, this part of the sequence has length  $OPT(k)$ , so including  $P[j]$ , the full sequence has length  $1 + OPT(k)$ . We have thus justified the following recurrence.

$$OPT(j) = 1 + \max_{k>j:P[k]>P[j]} OPT(k).$$

The values of  $OPT$  can be built up in order of decreasing  $j$ , in time  $O(n-j)$  for iteration  $j$ , leading to a total running time of  $O(n^2)$ . The value we want is  $OPT(1)$ , and the subsequence itself can be found by tracing back through the array of  $OPT$  values.

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<sup>1</sup>ex219.570.316