

(a) This algorithm is too short-sighted; it might take a high-stress job too early and an even better one later.

|        | Week 1 | Week 2 | Week 3 |
|--------|--------|--------|--------|
| $\ell$ | 2      | 2      | 2      |
| $h$    | 1      | 5      | 10     |

The algorithm in (a) would take a high-stress job in week 2, when the unique optimal solution would take a low-stress job in week 1, nothing in week 2, and then a high-stress job in week 3.

(b) Let  $OPT(i)$  denote the maximum value revenue achievable in the input instance restricted to weeks 1 through  $i$ . The optimal solution for the input instance restricted to weeks 1 through  $i$  will select *some* job in week  $i$ , since it's not worth skipping all jobs — there are no future high-stress jobs to prepare for. If it selects a low-stress job, it can behave optimally up to week  $i - 1$ , followed by this job, while if it selects a high-stress job, it can behave optimally up to week  $i - 2$ , followed by this job. Thus we have justified the following recurrence.

$$OPT(i) = \max(\ell_i + OPT(i - 1), h_i + OPT(i - 2)).$$

We can compute all  $OPT$  values by invoking this recurrence for  $i = 1, 2, \dots, n$ , with the initialization  $OPT(1) = \max(\ell_1, h_1)$ . This takes constant time for each value of  $i$ , for a total time of  $O(n)$ . As usual, the actual sequence of jobs can be reconstructed by tracing back through the set of  $OPT$  values.

An alternate, but essentially equivalent, solution is as follows. We define the following sub-problems. Let  $L(i)$  be the maximum revenue achievable in weeks 1 through  $i$ , given that you select a low-stress job in week  $i$ , and let  $H(i)$  be the maximum revenue achievable in weeks 1 through  $i$ , given that you select a high-stress job in week  $i$ .

Again, the optimal solution for the input instance restricted to weeks 1 through  $i$  will select some job in week  $i$ . Now, if it selects a low-stress job in week  $i$ , it can select anything it wants in week  $i - 1$ ; and if it selects a high-stress job in week  $i$ , it has to sit out week  $i - 1$  but can select anything it wants in week  $i - 2$ . Thus we have

$$L(i) = \ell_i + \max(L(i - 1), H(i - 1)),$$

$$H(i) = h_i + \max(L(i - 2), H(i - 2)).$$

The  $L$  and  $H$  values can be built up by invoking these recurrences for  $i = 1, 2, \dots, n$ , with the initializations  $L(1) = \ell_1$  and  $H_1 = h_1$ .

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<sup>1</sup>ex695.414.330