

We prove this by induction on the number of nodes in T . Let $n_0(T)$ denote the number of leaves of a binary tree T , and let $n_2(T)$ denote the number of nodes with two children.

The basis of the induction is a tree with a single node. This node is the only leaf, and there are no nodes with two children.

Now, let T be an arbitrary binary tree on more than one node, and let v be a leaf. Since T has more than one node, v is not the root, so it has a parent u . Let T' be the tree obtained by deleting v .

If u had no other child in T , then it becomes a leaf in T' , so we have $n_0(T') = n_0(T)$ and $n_2(T') = n_2(T)$. Applying the induction hypothesis to T' completes the induction step in this case. On the other hand, if u had another child in T , then it does not become a leaf after the deletion; but it used to have two children and now it doesn't. Thus we have $n_0(T') = n_0(T) - 1$ and $n_2(T') = n_2(T) - 1$. Again, applying the induction hypothesis to T' completes the induction step in this case.