We run BFS starting from node s. Let d be the layer in which node t is encountered; by assumption, we have $d > n/2$. We claim first that one of the layers $L_1, L_2, \ldots, L_{d-1}$ consists of a single node. Indeed, if each of these layers had size at least two, then this would account for at least $2(n/2) = n$ nodes; but G has only n nodes, and neither s nor t appears in these layers.

Thus, there is some layer L_i consisting of just the node v. We claim next that deleting v destroys all s-t paths. To see this, consider the set of nodes $X = \{s\} \cup L_1 \cup L_2 \cup \cdots \cup L_{i-1}$. Node s belongs to X but node t does not; and any edge out of X must lie in layer L_i, by the properties of BFS. Since any path from s to t must leave X at some point, it must contain a node in L_i; but v is the only node in L_i.

[1] ex758.356.752