

Let's suppose that s has n characters total. To make things easier to think about, let's consider the repetition x' of x consisting of exactly n characters, and the repetition y' of y consisting of exactly n characters. Our problem can be phrased as: is s an interleaving of x' and y' ? The advantage of working with these elongated strings is that we don't need to "wrap around" and consider multiple periods of x' and y' — each is already as long as s .

Let $s[j]$ denote the j^{th} character of s , and let $s[1 : j]$ denote the first j characters of s . We define the analogous notation for x' and y' . We know that if s is an interleaving of x' and y' , then its last character comes from either x' or y' . Removing this character (wherever it is), we get a smaller recursive problem on $s[1 : n - 1]$ and prefixes of x' and y' .

Thus, we consider sub-problems defined by prefixes of x' and y' . Let $M[i, j] = \text{yes}$ if $s[1 : i + j]$ is an interleaving of $x'[1 : i]$ and $y'[1 : j]$. If there is such an interleaving, then the final character is either $x'[i]$ or $y'[j]$, and so we have the following basic recurrence:

$$M[i, j] = \text{yes} \text{ if and only if } M[i-1, j] = \text{yes} \text{ and } s[i+j] = x'[i], \text{ or } M[i, j-1] = \text{yes} \text{ and } s[i+j] = y'[j].$$

We can build these up via the following loop.

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M[0,0] = yes
For k = 1, 2, ..., n
  For all pairs (i, j) so that i + j = k
    If M[i-1, j] = yes and s[i+j] = x'[i] then
      M[i, j] = yes
    Else if M[i, j-1] = yes and s[i+j] = y'[j] then
      M[i, j] = yes
    Else
      M[i, j] = no
  Endfor
Endfor
Return "yes" if and only there is some pair (i, j) with i + j = n
so that M[i, j] = yes.

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There are $O(n^2)$ values $M[i, j]$ to build up, and each takes constant time to fill in from the results on previous sub-problems; thus the total running time is $O(n^2)$.

¹ex357.417.692