

The statement is false and the following is a counterexample to it. Let us be given a number $b > 1$ (we will without loss of generality assume that it is an integer, otherwise we will round it up). We consider the following graph. It has $2(b + 1) + 2$ vertices: source s sink t , and vertices u_1, u_2, \dots, u_{b+1} that have an edge coming from the source and vertices v_1, v_2, \dots, v_{b+1} that have an edge going into the sink. There is also an edge from u_i to v_i and from v_i to u_{i+1} . All the edge capacities are 1.

Now assume that the first augmenting path was the path $s \rightarrow u_1 \rightarrow v_1 \rightarrow u_2 \rightarrow v_2 \rightarrow \dots u_{b+1} \rightarrow v_{b+1} \rightarrow t$. Then since all the backward edges are deleted from the residual graph according to the super-fast algorithm, the residual graph would contain no path from s to t , and therefore our final flow would equal 1. But there is a flow of value $b + 1$ by using the horizontal edges (that is $u_i \rightarrow v_i$). Therefore we failed to reach within b of the optimum.

Notice that for different b 's we would be considering different graphs, but we are allowed to do this, since the problem asks whether there exists a *universal* b that is independent of the choice of the flow graph G .

¹ex70.281.132