

Suppose that  $G$  has an edge  $e = \{a, b\}$  that does not belong to  $T$ . Since  $T$  is a depth-first search tree, one of the two ends must be an ancestor of the other — say  $a$  is an ancestor of  $b$ . Since  $T$  is a breadth-first search tree, the distance of the two nodes from  $u$  in  $T$  can differ by at most one.

But if  $a$  is an ancestor of  $b$ , and the distance from  $u$  to  $b$  in  $T$  is at most one greater than the distance from  $u$  to  $a$ , then  $a$  must in fact be the direct parent of  $b$  in  $T$ . From this it follows that  $\{a, b\}$  is an edge of  $T$ , contradicting our initial assumption that  $\{a, b\}$  did not belong to  $T$ .