**(a)** This problem can be solved using network flow. We construct a graph with a node $v_i$ for each cannister, a node $w_j$ for each truck, and an edge $(v_i, w_j)$ of capacity 1 whenever cannister $i$ can go in truck $j$. We then connect a super-source $s$ to each of the cannister nodes by an edge of capacity 1, and we connect each of the truck nodes to a super-sink $t$ by an edge of capacity $k$.

We claim that there is a feasible way to place all cannisters in trucks if and only if there is an $s$-$t$ flow of value $n$. If there is a feasible placement, then we send one unit of flow from $s$ to $t$ along each of the paths $s, v_i, w_j, t$, where cannister $i$ is placed in truck $j$. This does not violate the capacity conditions, in particular on the edges $(w_j, t)$, due to the capacity constraints. Conversely, if there is a flow of value $n$, then there is one with integer values. We place cannister $i$ in truck $j$ if the edge $(v_i, w_j)$ carries one unit of flow, and we observe that the capacity condition ensures that no truck is overloaded.

The running is the time required to solve a max-flow problem on a graph with $O(m+n)$ nodes and $O(mn)$ edges.

**(b)** When there are conflicts between pairs of cannisters, rather than between cannisters and trucks, the problem becomes NP-complete.

We show how to reduce *3-Coloring* to this problem. Given a graph $G$ on $n$ nodes, we define a cannister $i$ for each node $v_i$. We have three trucks, each of capacity $k = n$, and we say that two cannisters cannot go in the same truck whenever there is an edge between the corresponding nodes in $G$.

Now, if there is a 3-coloring of $G$, then we can place all the cannisters corresponding to nodes assigned the same coloring in a single truck. Conversely, if there is a way to place all the cannisters in the three trucks, then we can use the truck assignments as colors; this gives a 3-coloring of the graph.

[1] ex460.602.46