Web Science
Week 6 - Project Report

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Reader Questions

4.4
(a) In order to maximize your payoff, how should you choose b?

To make a bid that will maximize payoff, we first write a function to estimate expected payoff. To do this we consider the following information provided by the question:

- There is only one competitor
- The other competitor has a uniform probability distribution for their bid. They will make a random choice.

First, we define a utility function $u(b_{second})$ for our payoff for the cases when we win the bid and our competitor bids b_{second}. We subtract the bid of the second bidder from 10 since we value the item 10 Euros:

$$= u(b_{second}) = 10 - b_{second}$$

Next, we define $p(b_{first})$ to represent the expected payoff from our bid b_{first}:

$$= p_1(b_{first}) = \sum_{b_{second}=1}^{b_{first}} \frac{u(b_{second})}{10} = \sum_{b_{second}=1}^{b_{first}} \frac{10-b_{second}}{10}$$

We observe that this is an increasing function in the range of 1 to 10. This can also be seen in the graph at the end of our solution to exercise 4.4. Hence, we should bid 10 Euros to maximize our expected payoff.

(b) What is your expected payoff?

Expected payoff when we choose $b_{first} = 10$ is $p_1(10) = 4.5$.

Now assume that the auction is first-price auction.

(c) Describe your expected payoff as a function of your bid

When we change the auction to a first-price auction, the expected payoff function changes. Instead of paying the amount bidded by the second player when we win, we are going to pay the amount we bid. We change the expected payoff method in the following way:

$$= p_2(b_{first}) = \sum_{b_{second}=1}^{b_{first}} \frac{10-b_{first}}{10}$$

(d) How should you bid in order to maximize your expected payoff? What is your expected payoff in this case?

We should bid $b_{first} = 5$ Euros in this case to maximize our expected payoff. Expected payoff is $p_2(5) = 2.5$
.

Now assume that we have an all-pay auction. This means that every bidder has to pay his bid, but only the highest bidder gets the painting. (Again, in case that you have bid highest and there is a tie, you get the painting.)

(e) Describe your expected payoff as a function of your bid b. How should you bid in order to maximize your expected payoff?

To solve this problem, we define two functions:

- $gain(bid) = \frac{x}{10} * (10 - x)$
- $loss(bid) = \frac{(10-x)}{10} * (- x)$

Then the expected payoff is $gain(bid) + loss(bid) = 0$ for all values of bid.In this case, we should bid 10 Euros to increase the chance that we will win the auction. We can also bid 0 since in this case we never lose money.

(f) How should you bid if you absolutely do not want to make any loss?

We must bid **10 Euros (or 0)** to not if we don't want to make any loss. When we bid 10 Euros, we will always win the bid and win the item. In all other cases, there is a chance that the competitor bids higher than us and win the item, leaving us with a loss.

Now we turn to second-price auctions again. But this time, there is a third bidder. Also her bid is some random amount a 'between 1 Euro and 10 Euros (again with all amounts being equally likely).

(g) Would you change your bid compared to (a)?

we shouldn't change our bid compared to (a). What's different to (a) in this case is that the likelihood of the second bid being 9 Euros is now higher than the second bid being 1 Euro. These probabilities were equal in (a). Even though the distribution of the second highest bid has changed, we should still bid **10 Euros** to maximize expected payoff. Another way to look at this is that the probability that the second highest bid is equal to B_i has dubbeld ? But this does not affect us since we always get the item if there is a tie.

(h) What is the expected price that you have to pay in this case? What is your expected payoff in this case?

To calculate the expected price, we first define $p(b_{second})$ to represent the probability of the b_{second}:

$$= p(b_{second}) = \frac{2(b_{second}-1)}{100} + \frac{1}{100}$$

In the equation of $p(b_{second})$, second term $\frac{1}{100}$ represents the probability of both competitors bidding b_{second} and the other term $\frac{2(b_{second}-1)}{100}$ represents the probability of one competitor bidding b_{second} and the other bidding something lower.

Then we define expected payoff function $p_3(b_{first})$:

$$= p_3(b_{first}) = \sum_{b_{second}=1}^{b_{first}} (p(b_{second}) * u(b_{second})) = \sum_{b_{second}=1}^{b_{first}} ((\frac{2(b_{second}-1)}{100} + \frac{1}{100}) * (10 - b_{second}))$$

Expected payoff when $b_{first} = 10$ is **2.85 Euros**, meaning that the expected price is **7.15 Euros**

(i) Explain the different outcomes of (b) and (h).

The difference is caused by the change in the probability distribution of the second bid b_{second} when we added a third competitor. This caused the expected payoff to decrease.

Now we turn to first-price auctions with a third bidder as described above. (You are only allowed to bid full Euro amounts, but not, say, 5 . 85 Euros.)

(j) What is your expected payoff as a function of your bid b?

To change the auction to a first price auction, we make a small modification to the last expected payoff function $p_3(b_{first})$. We simply change the parameter of the utility function to b_{first}:

$$= p_4(b_{first}) = \sum_{b_{second}=1}^{b_{first}} (p(b_{second}) * u(b_{first})) = \sum_{b_{second}=1}^{b_{first}} ((\frac{2(b_{second}-1)}{100} + \frac{1}{100}) * (10 - b_{first}))$$

(k) How should you bid in order to maximize your expected payoff? What is your expected payoff in this case?

To maximize profit, we should bid 7 Euros. Expected payoff is $p_4(7) = 1.47$.

(l) Explain the different outcomes of (j) and (k) on the one hand and (c) and (d) on the other hand.

The difference is caused by the same reason explained in (i). Probability distribution of the highest second bid shifts towards higher values. Expected payoff when we bid low and win decreases because possibility of winning decreases when we bid low. Likelihood of winning is now comparatively higher for bids higher than 5 Euros.

5.3

		fruits		
		apple	banana	cherry
buyers	Arjen	10	4	1
	Bram	4	2	1
	Claudios	6	5	3

Figure 5.6. The first matching market for Exercise 5.3.

		fruits			
		apple	banana	cherry	date
buyers	Alina	10	4	6	8
	Britt	4	12	9	8
	Claire	12	11	9	8
	Dael	13	15	9	11

Figure 5.7. The second matching market for Exercise 5.3.

Compute market-clearing prices and the resulting assignments of fruits to buyers for the matching markets shown in Figure 5.6 and Figure 5.7.
Verify that your market-clearing prices form indeed a matching of maximum total valuation.

You can see the market clearing prices and the resulting assignments in Figure 5.6 down below:

Item	Price	Buyer
Apple	3	Arjen
Banana	2	Claudios
Cherry	0	Bram

You can see the market clearing prices and the resulting assignments in Figure 5.7 down below:

Item	Price	Buyer
Apple	3	Claire
Banana	3	Aline
Cherry	0	Britt
Date	1	Dael

Verify

6.4

Consider the example of a matching market shown in Figure 6.5 and do the following:

		items			
		A	B	C	D
buyers	W	12	8	7	2
	X	8	8	8	4
	Y	9	6	3	0
	Z	9	2	0	0

Figure 6.5. The matching market for Exercise 6.4.

(a) Run the procedure of Section 5.2 to compute market-clearing prices. (Instead of raising prices for just 1 for the neighborhood of a constricted set, you can immediately raise the price by the minimal amount for which the preferred-seller graph changes.)

Item	Price	Buyer
A	9	Z
B	5	Y
C	4	W
D	0	X

(b) Run the VCG procedure to compute VCG prices. (You do not have to compute all $V^{-?}_{-?}$ values, but only the four relevant ones.)

	A	**B**	**C**	**D**
W	23-14=9	23-17=6	**23-19=4**	23-23=0
X	22-13=9	22-16=6	22-18=4	**22-22=0**
Y	25-16=9	**25-20=5**	25-21=4	25-25=0
Z	**26-17=9**	26-20=6	26-22=4	26-26=0

Kidney Questions

1a) What could be a reason to use the Tie-breaking rule that prioritizes chains starting with a blood type O friend?

The reason may be that there is correlation between the blood type of the patient and the donor friend.

1b) What could be a reason to use the Tie-breaking rule that prioritizes chains starting with a blood type O patient?

This makes sense because patients with blood type O are less likely to find a compatible donor compared to other blood types. By giving them priority, we reduce the risk of not being able to match patients with blood type 0.

2) As mentioned in section 2, operations in cycles have to take place simultaneously. Suppose due to a small number of operating rooms, we would like to do the operations from chains sequentially. What order of operations would you recommend? start of chain to w, or w to start of chain? Exhibit advantages and disadvantages for both orders.

Consider the case when the chain is $a -> b -> c -> d -> e > w$ and we can only do two operations simultaneously. This means that there are 4 kidneys to be transferred and it will take two simultaneous operations.

	advantages	**disadvantages**
start of chain to w	After the first simultaneous operation, there is a chance that the friend of the patient a decides not to give his kidney anymore. Even if this is the case; next operation between c,d and e can take place since the friend of a is not involved..	
w to start of chain		When d receives the kidney from e and c receives the kidney from d, there is a possibility that friend of c will not carry on with the next operation to give his kidney to b.

Based on the advantages and disadvantages we identified, we believe that ordering the operations to go from the start of the chain to w is makes more sense,

3a) Suppose we do not care about misreporting and the values in the example from section5 are the true values. What matching would maximize the total expected utility?

Matching that would maximize total expected utility is can be seen in the patient-donor matchings below:

- a,b
- b,c
- c,d
- d,e
- e,f
- f,a

3b) Is there a set of pairs that have incentive to withdraw from the matching in 3a) and trade among themselves?.

Pairs c,d and e would prefer to trade amongst themselves to improve their total utility.

4) Suppose some patient prefers to wait for a better kidney instead of being assigned a kidney with an expected time until rejection of 12 years or less. How would you model such a patient?

We can model such a patient *P* by adding a rule to the creation of the *expected time until rejection matrix*. Normally, when there is a kidney that will not be rejected by *P*, we would add it to the matrix without considering the expected time until rejection. To model patient *P*, we can add a new rule to include the value in the matrix if and only if the expected time until rejection is higher than 12..