Web Science
Week 5 - Project Report

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Exercise 1.1

Let n represent the number of countries that do not fight against climate change by spending 5 billion Euros. When we investigate the decision of spending 5 billion Euros from perspective of a country that is already paying 5 billion Euros, we see the following situation:

- Currently, the country is spending 5 billion Euros on fighting climate change and it's also paying for the shared cost of not fighting climate change, n billion Euros. The total cost is $n + 5$ billion Euros.
- If this country decides to stop spending 5 billion Euros to fight climate change, then there will be $n + 1$ countries not spending 5 billion Euro. So the country will be paying $n + 1$ billion Euros.

We see that the cost of not paying for the fight against climate change ($n + 1$) will always be less than the cost of fighting it ($n + 5$). Assuming that countries act as selfish agents, countries will prefer not to pay 5 billion Euros even though cooperation is better for everyone.

Exercise 1.3

Let H_k represent the hotel that values each click on its advertisement k cents.

a)

To maximize its profit, the search engine can allocate the slots based on auctions. Whoever bids the most can get the highest slot. Only issue with auction is that H_{15} will compete with H_{12} and once the price becomes higher than $12c$, H_{12} will withdraw from the auction for slot 1 and the bid of H_{15} will be slightly higher than $12c$. This is not optimum from the perspective of the search engine since H_{15} is actually willing to pay even more for the clicks but pays less.

Hotels should submit their bids such that their gain from the deal is maximized. To do this, they should assume that other hotels are rational actors maximizing their gains and act accordingly. This can easily result in situations where a hotel that values clicks less ends up with a higher slot than another hotel that values clicks more.

This situation is inefficient from the perspective of the search engine. Search engine will be charging a lower amount compared to what the hotels are willing to pay.

b)

Lower bound for the bids of the hotels H_{15}, H_{12} and H_9 is 7c. This is because we have 4 hotels and only 3 slots. If any of these hotels bids less than $7c$, then H_7 will be able to replace them by bidding $7c$.

Now consider the case where the slots are given to H_{15}, H_9, H_{12} with the same order. In this case, benefit for H_{12} is $(12c - 7c) * 3 = 15c$ if it bids $7c$. Instead, H_{12} can bid a little higher than $9c$ and replace H_9 in the second slot. This way, benefit of H_{12} will become $(12c - 9c) * 6 = 18c$. We see that H_{12} will prefer replacing H_9 in the second slot.

For the first slot, competition between H_{12} and H_{15} is not that simple. H_{15} is in an interesting position. H_{15} either bid a little higher than $12c$ and get the first slot, or bid a little higher than $9c$ and bid for the second slot. When we calculate the gain fo H_{15} in these two situations, we see that gain of H_{15} from these two cases are $30c$ and $36c$. H_{15} can bid lower than it's gain from each click and profit more.

c)
If slot 2 receives 9 clicks, then the difference between slot 1 and slot 2 will be less. This will result in less competition between H_{12} and H_{15} for slots 1 and 2. Profits of H_{12} and H_{15} will increase while the profit of the search engine decreases.

Exercise 2.6

Let the mixed Nash Equilibrium be represented by (p, q), p being the strategy probability of the kicker and q being the strategy probability of the goalkeeper.

To find the mixed Nash Equilibrium, we calculate the expected pay-off of the kicker from kicking left and right:
- Kicker payoff from kicking left: $0.4 * q_{left} + 0.9 * (1 - q_{left}) = 0.9 - 0.5 * q_{left}$
- Kicker payoff from kicking right: $0.8 * q_{left} + 0.5 * (1 - q_{left}) = 0.5 + 0.3 * q_{left}$

To make it indifferent for kicker to kick left or right, goalkeeper should pick q_{left} such that the payoff of the kicker from kicking left and right are the same.

$0.9 - 0.5 * q_{left} = 0.5 + 0.3 * q_{left}$

$q_{left} = 0.5$

This means that strategy probability of goalkeeper is $q = (0.5,\ 0.5)$.

Next, we make the same calculation for the payoff of goalkeeper to find p:
- Goalkeeper payoff from diving left: $0.6 * p_{left} + 0.2 * (1 - p_{left}) = 0.2 + 0.4 * p_{left}$
- Goalkeeper payoff from diving right: $0.1 * p_{left} + 0.5 * (1 - p_{left}) = 0.5 - 0.4 * p_{left}$

Then we find p_{left}:

$0.2 + 0.4 * p_{left} = 0.5 - 0.4 * p_{left}$

$p_{left} = 0.375$

This means that strategy probability of kicker is $p = (0.375,\ 0.625)$.

Exercise 3.5

(a)
We first calculate the total time spent T as a function of m. We do this by multiplying the cars taking each road by the time those roads take:

$$T(m) = m * \frac{am}{N} + (N - m) * 1 = \frac{am^2}{N} - m + N$$

To find the optimum value of $T(m)$, we found the derivative of $T(m)$ compared to m:

$$\frac{\Delta T(m)}{\Delta m} = \frac{\Delta}{\Delta m}\left(\frac{am^2}{N} - m + N\right) = \frac{2am}{N} - 1$$

Using the derivative, we can find the lowest value $T(m)$ can have by finding the point where the derivative of $T(m)$ crosses the x axis:

$\frac{2am}{N} - 1 = 0 \Rightarrow m = \frac{N}{2a}$

To find the social optimum, we now use $m = \frac{N}{2a}$ value in $T(m)$:

$T(m) = T(\frac{N}{2a}) = a(\frac{N}{2a})^2/N - \frac{N}{2a} + N = (1 - \frac{1}{4a})N$

We can not apply this for every m however. This is because when $a \leq 0.5$, the optimum $m = \frac{N}{2a} \geq N$. This doesn't make sense as m can not be higher than N. So we add a condition to our rule. $T(m) = (1 - \frac{1}{4a})N$ when $a > 0.5$ and $T(N) = \frac{aN^2}{N} = aN$ when $a \leq 0.5$

(b)

In the state of (i), all cars take the upper road. We have $m = N$. This means that the travel time on the upper road is a. When $a \leq 1$, no car will decide to change to the lower road since it takes longer than the upper road.

In the state of (ii), $a * m_{NE} = N$. Travel time on the upper road is 1. So travel time on both of the roads is 1.

- If a car on the upper road decides to take the lower road, then the travel time on the upper road will reduce to a value lower than 1. The car will prefer staying on the upper road.
- If a car on the lower road decides to take the upper road, then the travel time on the upper road will increase to a value higher than 1. The car will prefer staying on the lower road.

(c)

(iii) can not be an equilibrium because when all the cars take the lower road, $m = 0$ and travel time on the road above becomes 0. This means that cars taking the lower road will prefer switching to the road above, ending the potential equilibrium state.

(d)

The ratio of the social cost of the worst Nash equilibrium to the social cost of the social optimum is called the price of anarchy.

	Worst Nash Equilibrium	Social Optimum	Price of Anarchy
(I): $a \leq 0.5$	aN	aN	1
(II): $0.5 \leq a \leq 1$	aN	$(1 - \frac{1}{4a})N$	$\frac{4a^2}{4a-1}$
(III): $1 < a$	N	$(1 - \frac{1}{4a})N$	$\frac{4a}{4a-1}$

Price of Anarchy function with parameter a as parameter ($PaO(a)$) can be seen in the graph down below. This graph was generated with Desmos. Highest value of the price of anarchy is when $a = 1$, $PaO(1) = 1.333$.

1 $y = 1\{0 < x < 0.5\}$

2 $y = \frac{4x^2}{4x-1}\{0.5 < x < 1\}$

3 $y = \frac{4x}{4x-1}\{1 < x\}$

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