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Study on short baseline location of underwater vehicle based on constrained least squares algorithm

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***Abstract*—Due to the effects of sea waves and the complexity of the underwater environment, the array tends to swing or drift, which can affect its stability. Qualitative factors are the primary determinants influencing the accuracy of short baseline positioning systems. In light of the factors affecting positioning accuracy, this study proposes a fast maneuvering array layout and a self-calibration technology based on a short baseline. To address the positioning challenges arising from random disturbances in array position, we introduce a global constrained least squares algorithm (CGLS) designed to minimize the disturbance vector and mitigate the impact of errors from disturbances by focusing on the overall error of the system. Simulation results demonstrate that the overall positioning accuracy of the CGLS algorithm outperforms that of both the CHAN algorithm and the LS algorithm. Additionally, as disturbance errors increase, the error associated with the CGLS algorithm remains largely stable. The positioning accuracy of the CGLS algorithm is significantly enhanced with an increased number of observed data points and corresponding available redundant data.**

Keywords—Short baseline; Array calibration; Multipath channel; Least square method

I. INTRODUCTION

The ocean is rich in natural resources, which are vital for the sustainable development of national economies. Additionally, it plays a crucial role in military affairs, as military powers utilize marine environments to showcase advanced weapons and equipment [1]. However, seawater, as a propagation medium, differs significantly from air. Commonly used media such as light waves and radio waves experience substantial attenuation in water, resulting in a very limited propagation distance [2]. Therefore, underwater acoustic technology based on sound detection has emerged as one of the most effective means for acquiring and transmitting underwater information. Various countries have conducted research and development on sonar-based underwater acoustic technology, driven by both resource exploitation and military needs. Underwater navigation technology utilizing acoustics is a key approach for achieving long-distance underwater positioning [3]. In the civilian sector, this technology is essential for the exploration and exploitation of mineral, oil, and natural gas resources. From a scientific perspective, it aids in understanding the genesis, structure, and distribution of sediments on the continental shelf. In the military realm, it is crucial for preparing underwater battlefields and for the positioning and navigation of submarines [4].

Underwater acoustic positioning technology primarily encompasses two methods: active positioning and passive positioning. Among these, active sonar-based positioning has been extensively researched and applied due to its high accuracy. Conventional underwater acoustic positioning systems can be categorized into three main types based on the length of the transponder array or the size of the receiving array: Long Baseline Positioning System (LBL), Short Baseline Positioning System (SBL), and Ultra-Short Baseline Positioning System (USBL) [5]. The Short Baseline Positioning System (SBL) derives its name from its compact array size, typically consisting of more than three elements. The baseline length usually ranges from several meters to tens of meters and is commonly situated on the seabed or mounted alongside a vessel [6]. This system calculates target orientation and distance information by measuring the time difference in the propagation of acoustic signals between the elements and the target, subsequently determining the target coordinates. In comparison to the Long Baseline system, the Short Baseline system offers advantages such as a simpler configuration, ease of mobility, and straightforward operation [7]. However, it has a limited tracking range; increasing the number of arrays can help to extend this range. Its positioning accuracy surpasses that of the Ultra-Short Baseline system, and it does not require calibration for installation errors [8]. Consequently, the SBL is well-suited for navigation and positioning of underwater robots in close proximity to a mother ship for precise operations.

The performance of the Short Baseline Positioning System is influenced by various factors, including array layout, calibration technology, refinement of positioning solutions, sound propagation correction methods, anti-multipath techniques, delay estimation technology, and signal detection methods [9]. Traditional sonar arrays typically consist of three-element subarrays for passive ranging sonar, as well as various configurations such as cross arrays or cube arrays for active positioning sonar, bulbous bow sonar using ball or cylindrical arrays, and side linear arrays [10]. For any positioning system, the accuracy relies heavily on the precision of the array configuration. If the configuration is insufficiently accurate, formation errors must be corrected. To obtain the parameters of the hydrophone array, there are two approaches: formation estimation and direct measurement of array position. The former involves modeling the array's layout and converting the formation estimation into a parameter estimation problem. The latter entails directly measuring the array's position to obtain the distance information between the array elements.

To address the impact of array disturbances on positioning accuracy, this study introduces a non-calibration positioning algorithm that employs a constrained global least squares approach to mitigate array disturbance errors. In the short

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baseline positioning process, traditional least squares methods typically focus solely on ranging errors. However, in fact, position deviations often arise from disturbances within the array itself, and in many cases, the two error components are not statistically independent after decomposition. This study takes a holistic approach to the overall system error, specifically targeting the positioning challenges posed by array position disturbances. By minimizing the disturbance vector of the array, we implement a more robust constrained global least squares positioning method.

II. CONSTRAINED GLOBAL LEAST SQUARES ALGORITHM

The least squares algorithm (LS) functions as a maximum likelihood estimation technique that does not rely on any prior knowledge. Its primary objective is to minimize the sum of the squares of the mean square error to achieve unbiased estimation. By minimizing the mean square error, we ensure that the difference between the estimator and the true value is statistically as small as possible. In the conventional short baseline least squares positioning algorithm, only the errors associated with matrix B, specifically the ranging error, are taken into account. However, in practical positioning scenarios, it is essential to also consider the positional deviations caused by disturbances in the matrix itself, implying there may be errors in matrix A. Furthermore, in many instances, the error components of matrices A and B are not statistically independent. In such situations, the constrained total least squares method offers a more robust approach to positioning.

The nonlinear equations ignore the quadratic coefficients and the linear observation equation is expressed as:

$$V = AX + B \tag{1}$$

$$A = 2\begin{bmatrix} x_1 - x_n & y_1 - y_n & z_1 - z_n \\ \dots & \dots & \dots \\ x_{n-1} - x_n & y_{n-1} - y_n & z_{n-1} - z_n \end{bmatrix} \tag{2}$$

$$B = \begin{bmatrix} {x_1}^2 - x_n^2 + y_1^2 - y_n^2 + z_1^2 - z_n^2 + R_n^2 - R_1^2 \\ \dots \\ {x_{n-1}}^2 - x_n^2 + y_{n-1}^2 - y_n^2 + z_{n-1}^2 - z_n^2 + R_n^2 - R_{n-1}^2 \end{bmatrix} \tag{3}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{4}$$

The constructed estimate $\hat{X}$ the performance indicator:

$$J(\hat{X}) = (X - A\hat{X})^T (X - A\hat{X}) \tag{5}$$

Assuming the measurement number be $M \geq 1$, then the least squares estimate of equation (5) can be written as:

$$\hat{X} = (A^T A)^{-1} A^T B \tag{6}$$

$$\begin{cases} A = \begin{bmatrix} A(1) \\ A(2) \\ \dots \\ A(n) \end{bmatrix} \\ B = \begin{bmatrix} B(1) \\ B(2) \\ \dots \\ B(n) \end{bmatrix} \end{cases} \tag{7}$$

Where $A = A^0 + \Delta A, B = B^0 + \Delta B$. When $XA = 0$, A^0 and B^0 indicates true values without error. The least squares solution is the estimate that corresponds to when the error variance is the smallest. Thus, when $\Delta A \neq 0$, the cost function constructed by the least square method is:

$$\begin{cases} \min\limits_X \| \Delta C \|_F^2 \\ s.t.(C + \Delta C)\begin{bmatrix} X \\ -1 \end{bmatrix} = 0 \end{cases} \tag{8}$$

Where $C = [A \vdots B], \Delta C = [\Delta A \vdots B]$ indicates total error of the system; $\| \bullet \|_F^2$ represents Frobenius norm. When the total error of the system is minimum, equation(8) under the minimal constraint conditions can be regarded as the $\hat{X}$ value when $(C + \Delta C)\begin{bmatrix} X \\ -1 \end{bmatrix} = 0$. Thus, equation (8) is the final global least squares with constraints (CGLS).

Compared with the conventional least square method, the CGLS model uniformly constructs the cost function for the analysis of the whole system error, and takes the minimization of the whole system error as the objective function for the overall parameter estimation. Therefore, its estimation performance is definitely better than the least square algorithm which only considers the ranging error ΔB. Expanding ΔC :

$$C(n) = \begin{bmatrix} v_{x1}(n) & v_{y1}(n) & v_{z1}(n) & x_1 v_{x1}(n) + y_1 v_{y1}(n) + z_1 v_{z1}(n) - d_1 v_{d1}(n) \\ v_{x2}(n) & v_{y2}(n) & v_{z2}(n) & x_2 v_{x2}(n) + y_2 v_{y2}(n) + z_2 v_{z2}(n) - d_2 v_{d2}(n) \\ \dots & \dots & \dots & \dots \\ v_{xL}(n) & v_{yL}(n) & v_{zL}(n) & x_L v_{xL}(n) + y_L v_{yL}(n) + z_L v_{zL}(n) - d_L v_{dL}(n) \end{bmatrix} \tag{9}$$

$\Delta C = [\Delta C(1) \vdots \Delta C(1) \dots \Delta C(M)]^T$, array disturbance noise can be expressed as $v_x(n), v_y(n), v_z(n), v_d(n)$, they follow $N(0, \sigma^2)$ distribution and independent with each other. It is can be seen that the error components of ΔC are correlated rather than independent. Due to the existence of correlation, the result will not be the optimal estimate, so it is necessary to eliminate this correlation, and use independent random vectors to express the overall error of the system ΔC is:

$$\Delta C = [F_1 \varepsilon, f_2 \varepsilon, F_3 \varepsilon, F_4 \varepsilon] \tag{10}$$

where ε is disturbance vector. Equation (10) is equivalent to:

$$\begin{cases} \min\limits_X \| \varepsilon \| \\ s.t.(C + \Delta C)\begin{bmatrix} X \\ -1 \end{bmatrix} + [F_1 \varepsilon, F_2 \varepsilon, F_3 \varepsilon, F_4 \varepsilon] = 0 \end{cases} \tag{11}$$

Here, it is assumed that the noise is zero mean Gaussian white noise, and if there is color noise, it can be pre-whitened first, and the autocorrelation matrix of the measurement error vector ω is used as $R_\omega = E[\omega\omega^H]$, which can be written as Cholesky decomposition: $R_\omega = PP^H$. Let $v = P^{-1}\omega$, v follows the white noise follows Gaussian distribution, the overall error of the system is $\Delta C = [F_1 P\varepsilon, F_2 P\varepsilon, F_3 P\varepsilon, F_4 P\varepsilon]$. Equation (11) can further be simplified as:

$$\min_X\{f(X)\} = \min_X\{\begin{bmatrix} X \\ -1 \end{bmatrix}^T, C^T(H_X G^{-1} H_X^T)^{-1} C \begin{bmatrix} X \\ -1 \end{bmatrix}\} \quad (12)$$

Where $H_X = [\sum_{i=1}^{3} X_i F_i - F_4]$, $G = [\sum_{i=1}^{4} F_i^T F_i]$ .Then Newton iteration method is used to solve equation (12):

$$X(m+1) = m(m) - u(m)H^{-1}(m)T(m) \quad (13)$$

Where m is the number of iteration, $u(m) = u/m$, $T(m)$ is gradient of $f(X)$ at $X(m)$, $H(m)$ is Hess matrix of $f(X)$ at $X(m)$.

$$T(m) = 2(U^T C I_{4,3} - U^T D_1)^T \quad (14)$$

Where $U = (H_X G^{-1} H_X{}^T)^{-1} C \begin{bmatrix} X \\ -1 \end{bmatrix}$, $D = [H_X G^{-1} F_1^T U, H_X G^{-1} F_2^T U, H_X G^{-1} F_3^T U]$. The diagonal element of $I_{4,3}$ is 1 and the remaining elements are 0. Similarly, according to the derivation of Hess matrix, the expression of $H(m)$ can be solved.

The localization performance of constrained global least squares algorithm in short baseline is simulated and analyzed. Considering only the two-dimensional positioning model, it is assumed that in the positioning process, the number of short baseline arrays is 4, whose position coordinates are (-5.0, 10.0)m, (-13.0, -5.0)m, (16.0, 0)m, (6.0, -14.0)m, and the position of the measured target is (-1.0, -2.0)m. The performance of constrained global least squares positioning method is analyzed through the following three sets of simulation experiments:

III. Results

The performance of the constrained global least squares algorithm (CGLS), CHAN algorithm, and least squares (LS) algorithm proposed in this paper was compared through Monte Carlo simulation experiments, with K set to 1000 and the number of positioning iterations M specified for each experiment. We analyzed and compared the performance of the CGLS, CHAN, and LS algorithms under identical array disturbances. First, we define the relative error of distance as follows:

$$\sigma_{xi} = \sigma_{yi} = \sigma \quad (15)$$

$$RMSE = \frac{\frac{1}{K}\{\sum_{k=1}^{K}(\hat{x}_{ms}(k) - x_T^0)^2 + (\hat{y}_{ms}(k) - y_T^0)^2\}^{0.5}}{\sqrt{(x^0{}_T)^2 + (y_T^0)^2}} \quad (16)$$

Figure 1 illustrates the variation curve of the root RMSE for the three algorithms in relation to the disturbance error σ of the array position. Since this is a positioning algorithm specifically designed to address array disturbances, we maintain the error variance of the delay difference estimation as a constant value. This approach effectively highlights the changes in positioning performance relative to the array disturbance error σ . In this study, we analyze the relationship between the positioning mean square error of the three algorithms when the delay difference estimation error $\sigma = 0.5m$ is held constant.

As shown in Figure 1, the overall positioning accuracy of the CGLS algorithm proposed in this paper is superior to that of both the CHAN algorithm and the LS algorithm. Furthermore, as the disturbance error σ increases, the error associated with the CGLS algorithm remains relatively stable, while the error of the LS algorithm deteriorates and the error of the CHAN algorithm escalates rapidly. This analysis demonstrates that the CGLS algorithm exhibits stability in the presence of array disturbances and consistently outperforms the other two algorithms in terms of positioning accuracy.

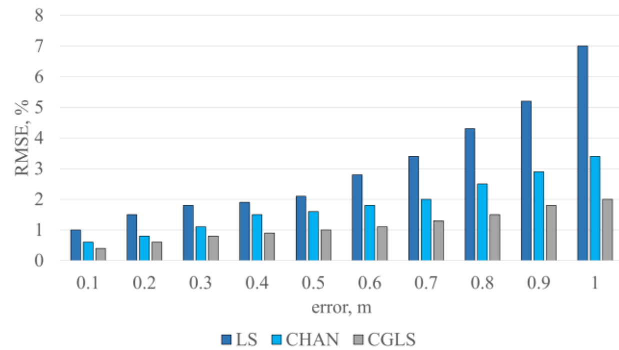

Fig.1 Effect of array deviation on the performance of three algorithms (LS, CHAN, and CGLS)

The CGLS algorithm is evaluated under various observation times (different values of m). The simulation conditions are established with array disturbance errors ranging from 0.1 to 1 meter, while the delay difference estimation error is maintained at a constant value. When the σ value is set to 0.5 m, the array coordinate position aligns with that of Experiment 1. The observation times σ are set to 2, 4, 8, and 12, respectively. Figure 2 presents the positioning error results for each scenario. As illustrated in Figure 2, the positioning accuracy of the CGLS algorithm markedly improves with an increase in the number of observations σ . This enhancement is primarily due to the increased number of observations, which leads to a corresponding rise in the available redundant data, ultimately resulting in improved positioning accuracy.

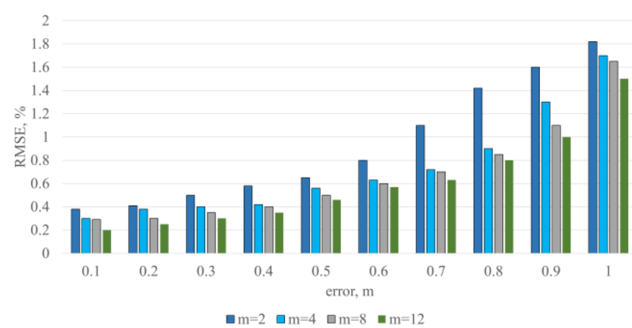

Fig.2 Error performance under different conditions of m value

As illustrated in Figure 2, when the disturbance error is 1 meter and two sets of data are utilized (m=2), the relative positioning error of the CGLS positioning algorithm remains within 2%. However, when utilizing 8 sets of data, the relative positioning error decreases to below 1%, thereby fully meeting the positioning system's requirements. This simulation experiment leads us to conclude that acquiring a certain amount of redundant data during the positioning process is essential for achieving high positioning accuracy with the CGLS algorithm. Therefore, when conditions allow, increasing the number of observations can significantly enhance positioning accuracy.

To demonstrate the positioning accuracy of the constrained least squares algorithm, we employ this algorithm to analyze the positioning of uniformly moving targets, comparing it with the least squares algorithm and the CHAN algorithm. In the simulation, we assume a delay difference estimation error of 0.5 meters and exclude the effects of random disturbances in the array position. The target moves from a distance towards the center of the array, with a maximum distance of 1000 meters, while the array's position conditions remain unchanged. Figure 3 illustrates the relative errors of the CGLS algorithm, CHAN algorithm, and LS algorithm at various distances. The horizontal axis indicates the distance from the initial point, and the vertical axis represents the mean square error.

As shown in Figure 3, during the initial stage of positioning, the relative distance error of the three algorithms is substantial due to the target's distance from the observation station. As the target gradually moves closer to the geometric center of the array, the positioning errors decrease for all three algorithms. Notably, the CGLS algorithm demonstrates better relative positioning accuracy compared to the other two, although the differences are not substantial. Additionally, the experiment indicates that the constrained least squares algorithm shows only modest improvements in performance when array position disturbances are absent.

As the level of noise increases, the error margin of the LS and CHAN algorithms also increases rapidly. Consequently, the LS and CHAN algorithms are unstable in complex underwater environments, while the CGLS algorithm demonstrates greater resilience. This method has been shown to be highly effective in suppressing the effects of noise. Even under conditions of increasing noise, the method has been demonstrated to maintain stability and avoid sudden deterioration, thus demonstrating its significant practical value.

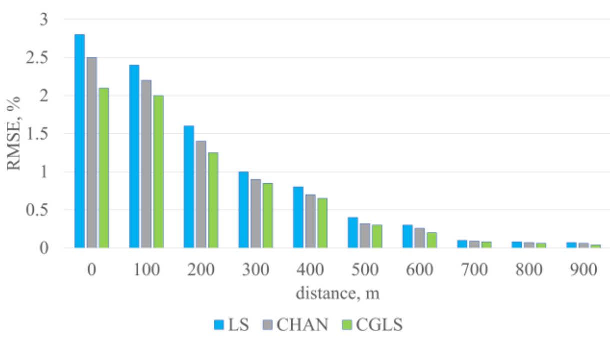

Fig.3 The effect of positioning distance on the performance of three algorithms (LS, CHAN, CGLS).

The motorized flexible array is used as the receiving base array, the base array position can be changed according to the installation requirements, and is not rigidly connected to the hull of the ship, the base elements are connected in a soft way (not rigidly fixed), the advantages of this array are to avoid the noise of the hull caused by the rigid connection, and can be placed on the hull of the ship according to the needs of any position. The base element position of the flexible array is uncertain and randomly perturbed, to obtain accurate localization information, it is necessary to obtain the coordinate position of the base element of the flexible array in real time.

The towed short baseline array is deployed by detaching it from the towing platform and positioning it away from the ship’s noise. The acoustic modules of the towed short baseline array consist of flexible arrays that necessitate specialized equipment for lifting, lowering, and depth adjustment. Alternatively, critical angle towing can be employed, allowing for changes in the array's depth by adjusting both the cable length and the ship's speed. Additionally, the base arrays are interconnected flexibly, which introduces uncertainty regarding the positions of the array elements; thus, real-time calibration of the attitude information for the base arrays is essential.

IV. Conclusion

To investigate the impact of array perturbation on positioning accuracy, this study proposes a non-calibration positioning algorithm that employs the constrained global least squares (CGLS) method to mitigate array perturbation errors. In short baseline positioning, the traditional least squares approach primarily focuses on ranging errors. However, in practice, disturbances inherent to the array can introduce additional positional deviations, and in many instances, the two types of errors are not independent. This paper addresses the positioning challenges posed by array position disturbances by adopting a more robust CGLS method to minimize the array disturbance vector. The results demonstrate that the CGLS algorithm offers enhanced stability and localization accuracy compared to traditional algorithms in the presence of array disturbances. It is important to note that the proposed algorithm does not take into account the effects of Doppler, which can have significant negative implications for signal processing in positioning and navigation systems.

The algorithm presented in this paper does not currently take into account the effects of the Doppler phenomenon, which can significantly impact signal processing in positioning and navigation systems. Future research should focus on analyzing the robustness of the algorithm in the presence of Doppler effects. Additionally, as the field of underwater positioning evolves, the trends are moving towards combined and integrated positioning. Therefore, exploring how to effectively merge short, ultra-short, and long baseline systems, as well as integrating them with other navigation equipment and hydroacoustic communication devices, will be crucial areas for further investigation.

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