Application of Sequential Convex Optimization in the Spacecraft Aero-assisted Orbital Transfer Problem

Liyan Ben
Shanghai Satellite Network Research Institute Co., Ltd
Shanghai, China
benliyan@126.com

Wang Zhou
Shanghai Satellite Network Research Institute Co., Ltd
Shanghai, China
zhouwang@163.com

***Abstract*—An algorithm based on sequential convex optimization (SCP) is introduced to facilitate the fast and efficient solution of a spacecraft aero-assisted orbital transfer problem. Firstly, an optimal model for this problem is formulated. Given the discontinuity of aerodynamic force, uniform discretization is applied, and the terminal time is used as a control variable. To convexify the optimal model, the spacecraft's equations of motion are linearized, and path constraints are convexified. Based on SCP, the detailed procedure for the solution of the spacecraft aero-assisted orbital transfer problem is outlined. Numerical simulations demonstrate that the efficiency and robustness of the sequential convex optimization are superior to that of the pseudo-spectral method.**

Keywords-component; spacecraft; aero-assisted; orbital transfer; convex optimization

I. INTRODUCTION

Aeroassisted orbital transfer represents a significant method for optimizing fuel efficiency in spacecraft operations. Due to its complexity, the spacecraft aero-assisted orbital transfer problem has garnered considerable attention from researchers in this field[1].

The spacecraft aero-assisted orbital transfer problem can be addressed through the application of the indirect method, which reformulates the original issue by discretizing control and state variables into a constrained parameter optimization framework. This approach can then be tackled using various optimization algorithms. In comparison to other optimization techniques, convex optimization offers numerous advantages in solving such problems.

In the field of trajectory planning through convex optimization, scholars have achieved rich research results. In the study of Acikmese[2], the soft-landing problem on Mars is studied. A non-convex constraint is converted into a relaxed convex constraint, thereby demonstrating that the original non-convex problem can be reformulated as a convex optimization problem. Liu et al. [3-5] utilized convex optimization techniques to tackle entry trajectory planning challenges. Wang et al.[6] utilized penalty function methods to manage boundary constraints within subproblems of convex optimization, thereby mitigating potential convergence issues arising from infeasible subproblems. Ma et al.[7] employed an improvement-based Picarchebyshev iterative technique aimed at reducing both the dimensionality of unknowns and the number of equality constraints.

Building on the aforementioned research, this paper introduces a new method for addressing the spacecraft aero-assisted orbital transfer problem through sequence convex optimization. Firstly, an optimization model of the spacecraft aero-assisted orbital transfer problem is established. The original issue is reformulated into a convex optimization framework through the application of discretization and convexity techniques, which can be effectively addressed using the sequential convex optimization algorithm. Numerical simulations indicate that the sequence convex optimization method can greatly improve the computational efficiency for the spacecraft aero-assisted orbital transfer problem, and the results meet all the constraints.

II. OPTIMIZED MODEL

The spacecraft's equations of motion when traversing through the atmosphere are as follows:

$$\begin{cases} \dfrac{dr}{dt} = V\sin\gamma \\ \dfrac{dV}{dt} = -\dfrac{\rho_0 S C_L^*}{4mE^*}\exp(-\beta h)V^2\left(1+\lambda^2\right) - \dfrac{\mu}{r^2}\sin\gamma \\ \dfrac{d\gamma}{dt} = \dfrac{\rho_0 S C_L^*}{2m}\exp(-\beta h)V\lambda + \left(\dfrac{V}{r} - \dfrac{\mu}{r^2 V}\right)\cos\gamma \end{cases} \tag{1}$$

Here, r , V , γ , m , S , ρ_0 , β and h represent geocentric distance, velocity, track-angle, mass, aerodynamic reference area, sea level atmospheric density, exponential factor and geocentric height. The drag coefficient and lift coefficients are in accordance with a parabolic relationship $C_L = C_{D0} + KC_D^2$, where C_{D0} represents zero-lift drag coefficient, and K represents induced drag factor. Assuming that drag coefficient C_D^* equals $2C_D^0$, lift coefficient C_L^* equals $\sqrt{C_D^0 / K}$, thus generalized lift coefficient λ equals C_L / C_L^* .

The performance index of the optimal transfer time problem is described as:

$$\min \quad t_f \tag{2}$$

Initial and terminal conditions:

$$\begin{cases} r(0) = r_0, r(t_f) = r_f \\ V(0) = V_0, V(t_f) = V_f \\ \gamma(0) = \gamma_0, \gamma(t_f) = \gamma_f \end{cases} \tag{3}$$

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The path constraints are as follows:

$$\begin{cases}\rho V^2/2 \le q_{\max} \\ k_Q\sqrt{\rho}V^3 \le Q_{\max}\end{cases} \tag{4}$$

$q_{\max}$ denotes the maximum threshold of dynamic pressure, and $Q_{\max}$ denotes the maximum threshold of heat flow. The control constraints are as follows:

$$|\lambda| \le \lambda_{\max} \tag{5}$$

$\lambda_{\max}$ represents the maximum generalized lift coefficient. The optimization problem has been established, which is called P0 or the original problem:

$$\begin{aligned}&\min \ \ t_f \\ &\text{s.t.} \mathit{equation}(3)-(5)\end{aligned} \tag{6}$$

III. CONVEXITY AND DISCRETIZATION OF OPTIMIZED MODEL

First, the time domain of the original problem is mapped to the specified interval[0,1]. Assuming that τ equals t/t_f , the equation of motion (1) is described as

$$\frac{d\boldsymbol{x}}{d\tau} = t_f \boldsymbol{f}\left(\boldsymbol{x}, \lambda\right) \tag{7}$$

Following the process of time normalization, the terminal time is introduced into the motion equation, which can be solved as an optimization variable. Due to the presence of a nonlinear term at the right hand of Equation (7), it is essential to linearize the nonlinear term in accordance with the reference trajectory $\left\{\bar{\boldsymbol{x}}, \bar{\lambda}, \bar{t}_f\right\}$:

$$\begin{aligned}\frac{d\boldsymbol{x}}{d\tau} &= \boldsymbol{A}\left(\bar{\boldsymbol{x}}, \bar{\lambda}, \bar{t}_f\right)\boldsymbol{x} + \boldsymbol{B}\left(\bar{\boldsymbol{x}}, \bar{\lambda}, \bar{t}_f\right)\lambda \\ &+ \boldsymbol{f}\left(\bar{\boldsymbol{x}}, \bar{\lambda}, \bar{t}_f\right)t_f + \boldsymbol{c}\left(\bar{\boldsymbol{x}}, \bar{\lambda}, \bar{t}_f\right)\end{aligned} \tag{8}$$

$\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{c}$ are dependent on the reference trajectory and can be treated as a constant matrix in each iteration. This approach facilitates the linearization of the motion equation. Furthermore, the discretization process employs equidistant sampling under the assumption that control variables change only at discrete time intervals. Consequently, the discrete motion equation can be described as:

$$\boldsymbol{x}_{i+1} = \boldsymbol{G}_i\boldsymbol{x}_i + \boldsymbol{H}_{i\lambda}\lambda_i + \boldsymbol{H}_{it}t_f + \boldsymbol{C}_i \tag{9}$$

where: $i = 0,...N$, N denotes the number of discrete points, $\Delta\tau$ denotes the discrete step, and its value $\Delta\tau = 1/N$, thus $\tau_i = i\Delta\tau$. The expression of $\boldsymbol{G}_i$, $\boldsymbol{H}_{i\lambda}$, $\boldsymbol{H}_{it}$ and $\boldsymbol{C}_i$ are as follows:

$$\begin{cases}\boldsymbol{G}_i = e^{\boldsymbol{A}_i\Delta\tau} \\ \boldsymbol{H}_{i\lambda} = \int_0^{\Delta\tau} e^{\boldsymbol{A}_i\Delta\tau}dt\boldsymbol{B}_i \\ \boldsymbol{H}_{it} = \int_0^{\Delta\tau} e^{\boldsymbol{A}_i\Delta\tau}dt\boldsymbol{f}_i \\ \boldsymbol{C}_i = \Delta\tau\boldsymbol{c}_i\end{cases} \tag{10}$$

The path and control constraints of discrete points are linearized as follows:

$$\begin{cases}c_{11}V_i + c_{12}r_i \le q_{\max} + c_{11}\bar{V}_i + c_{12}\bar{r}_i - q\left(\bar{r}_i, \bar{V}_i\right) \\ c_{21}V_i + c_{22}r_i \le q_{\max} + c_{21}\bar{V}_i + c_{22}\bar{r}_i - Q\left(\bar{r}_i, \bar{V}_i\right) \\ \lambda_i \le \lambda_{\max} \\ -\lambda_i \ge -\lambda_{\max} \\ -t_f \le 0\end{cases} \tag{11}$$

where

$$\begin{aligned}c_{11} &= \bar{\rho}_i\bar{V}_i, \\ c_{12} &= -\beta\bar{\rho}_i\bar{V}_i^2/2, \\ c_{21} &= 3k_Q\sqrt{\bar{\rho}}\bar{V}_i^2, \\ c_{22} &= -k_Q\sqrt{\bar{\rho}}\bar{V}_i^3/2\end{aligned}$$

Here, the optimization problem P1 is formulated with Equation (2) as the performance index and Equations (3) and (11) as the constraint. The solution to P0 can be derived by addressing P1.

IV. SEQUENCE CONVEX OPTIMIZATION ALGORITHM

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To ensure the efficiency of linearization and the convergence of sequence iteration, trust region constraints are introduced[8-9], which are outlined as follows:

$$\left|\boldsymbol{x}^{k+1} - \boldsymbol{x}^k\right| \le \varepsilon^x, \left|\lambda^{k+1} - \lambda^k\right| \le \varepsilon^\lambda, \left|t_f^{\ k+1} - t_f^{\ k}\right| \le \varepsilon^t \tag{12}$$

where: ε^x , ε^λ and ε^t are the radius of the trust region. The detailed procedure is outlined as follows:

Step 1: Selecting a set of $\bar{\lambda}$ and integrating Equation (8) under these variables, a set of initial trajectories $\left\{\bar{\boldsymbol{x}}^k, \bar{\lambda}^k, \bar{t}_f^k\right\}$ are solved.

Step2: The convex optimization model of problem P1 is formulated based on the trajectory $\left\{\bar{\boldsymbol{x}}^{k}, \bar{\lambda}^{k}, \bar{t}_{f}^{k}\right\}$ .The optimization model is solved, and the optimal control variables are obtained.

Step3: Integrating Equation (8) under the optimal control variables, the reference trajectory $\left\{\bar{\boldsymbol{x}}^{k+1}, \bar{\lambda}^{k+1}, \bar{t}_{f}^{k+1}\right\}$ is solved.

Step4: It progresses to determining whether the condition $\left|\bar{\boldsymbol{x}}^{k+1}-\bar{\boldsymbol{x}}^{k}\right| \leq e$ is met, where e is the error limit of iterative convergence. If the above condition is met, the iteration stops, and the solution of P0 is $\left\{\bar{\boldsymbol{x}}^{k+1}, \bar{\lambda}^{k+1}, \bar{t}_{f}^{k+1}\right\}$. Otherwise, let $\left\{\bar{\boldsymbol{x}}^{k}, \bar{\lambda}^{k}, \bar{t}_{f}^{k}\right\}$ be $\left\{\bar{\boldsymbol{x}}^{k+1}, \bar{\lambda}^{k+1}, \bar{t}_{f}^{k+1}\right\}$, and go to step 2.

V. EXAMPLE

A verification of the efficiency of the sequence convex optimization method is presented, along with a comparative analysis of its performance against that of the pseudo-spectral method. The initial and terminal conditions are derived[10], and specific values of each parameter employed in numerical simulations are presented in Table I. The convex optimization subproblem was addressed utilizing CVX. Trajectory planning employing the pseudo-spectral method was executed through the GPOPS-II software package.

The optimal results obtained through the application of the sequence convex optimization and the pseudo-spectral method are illustrated in Figures 1-3 and Table II. The performance indexes exhibit minimal discrepancy, with analogous variation trends observed for altitude, velocity, and flight angle. However, due to the discrepancies in the local optimal solutions yielded by the two methods, there are slight discrepancies in the control laws obtained by the two methods, which ultimately result in differences in peak dynamic pressure and peak heat flux.

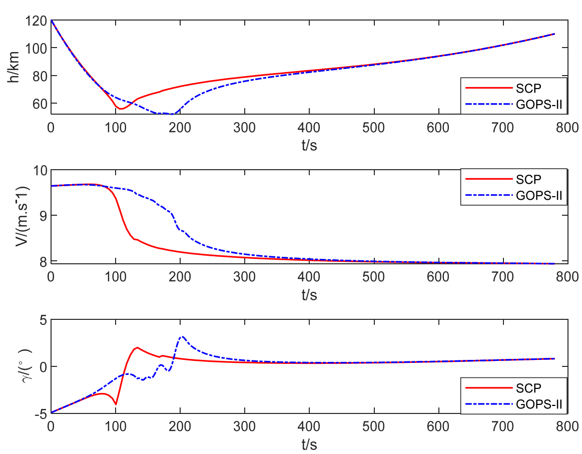

Figure 1. State variables vs time.

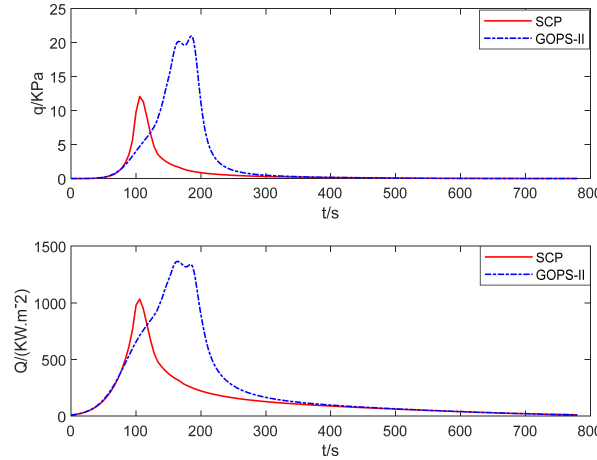

Figure 2. Dynamic pressure and heating rate vs time.

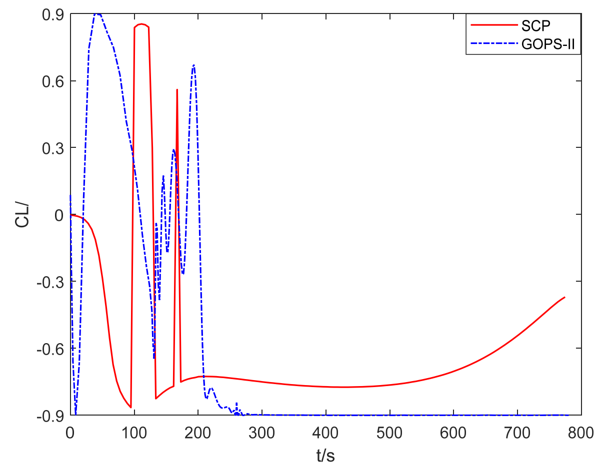

Figure 3. Optimal control vs time.

TABLE I. PARAMETERS FOR SIMULATION.

Parameter	Value	Parameter	Value
C_{D0}	0.1	$r_0\,(km)$	6498
K	1.11	$V_0\,(m.s^{-1})$	9641
$m/S(kg.m^{-2})$	300	$\gamma_0\,(^\circ)$	-4.92
$\beta(km^{-1})$	6.7^{-1}	$r_f\,(km)$	6498
$\gamma_f\,(^\circ)$	0.5	$V_f\,(m.s^{-1})$	7864
$\rho_0(kg.m^{-3})$	1.225	$q_{\max}\,(Kpa)$	50
$Q_{\max}\,(KW.m^{-2})$	1500	$\lambda_{\max}$	0.9
$k_Q\left(Js^2.m^{-3.5}.kg^{-0.5}\right)$	7.9×10^{-5}		

Numerical simulations indicate that the solution time for the sequence convex optimization method is shorter. By employing this approach, the spacecraft aero-assisted orbital transfer problem can be efficiently addressed using an established convex optimization algorithm. Although the pseudo-spectral method can reduce the number of optimization variables by using a global interpolation polynomial, the control law is relatively volatile. To ensure calculation accuracy, it is essential to add the number of collocation points, which reduces the efficiency of the pseudo-spectral method. Furthermore, the control law obtained by the pseudo spectral method appears to exhibit virtual numerical oscillation. Monte Carlo simulations are conducted to verify the robustness of the sequence convex optimization algorithm. The solutions meet all the constraints, achieving the goals of optimal transfer time.

TABLE II. COMPARISON OF THE OPTIMAL RESULTS.

	SCP	**GOPS-II**
Flight time (s)	780.1	780.2
Solution time (s)	15.8	24.7
Peak dynamic pressure (Kpa)	12.1	20.9
Peak heat flow ($KW.m^{-2}$)	1033.6	1361.3

VI. CONCLUSIONS

An algorithm for the spacecraft aero-assisted orbital transfer problem through the sequence convex optimization method is presented in this paper. An optimal model of this problem is established, and the original issue is reformulated into a convex optimization framework through the application of discretization and convexity techniques. The detailed procedure for the solution of the spacecraft aero-assisted orbital transfer problem is outlined. Numerical simulations demonstrate that the efficiency and robustness of the algorithm are superior to that of the pseudo-spectral method. The proposed algorithm can be extended to more complex spacecraft aero-assisted orbital transfer problems.

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