

# Study on Ultrasonic Signal Attenuation in Concrete Considering Scatterer Configurations

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**Abstract**—This study investigates the effect of randomly distributed scatterers in concrete on ultrasonic signal attenuation, focusing on the distribution probabilities of scatterers and their interference mechanisms on wave propagation. By modeling concrete as a two-phase medium composed of a homogeneous isotropic matrix and randomly distributed aggregates, a mathematical framework for multiple scattering problems is established. The scalar Helmholtz equation is employed to describe wave propagation characteristics. Additionally, the probability distribution function of scatterers and partial averaging methods are explored to analyze wave behavior under specific configurations. Simulation experiments are conducted to examine the impact of uniform, normal, exponential, and Poisson distributions of aggregates on ultrasonic scattering. The simulations generate 36 aggregate positions following different distributions, and the ultrasonic propagation characteristics are compared. This study evaluates the influence of scatterer distribution on detection accuracy, providing theoretical insights and data support for improving ultrasonic testing methods in concrete structures.

**Keywords**—Concrete structures, Randomly distributed scatterers, Helmholtz equation, Probability distribution

## I. INTRODUCTION

Ultrasonic testing has established itself as a pivotal non-destructive evaluation (NDE) technique for analyzing the internal structure and properties of materials, owing to its rapid, accurate, and non-invasive nature<sup>[1]</sup>. Over the past few decades, extensive research has demonstrated the utility of ultrasonic methods for detecting defects, such as cracks and voids, evaluating homogeneity, and monitoring the integrity of structures across a wide range of industries, including construction, aerospace, and manufacturing<sup>[2-3]</sup>. In the context of civil engineering, concrete—a composite material consisting of a cement matrix and aggregates of different sizes—benefits significantly from ultrasonic testing for ensuring safety and longevity of infrastructure<sup>[4]</sup>.

Despite these advantages, interpreting ultrasonic signals in concrete remains challenging due to the material's inherent heterogeneity. Concrete is composed of aggregates dispersed within a cementitious matrix, often with additional voids and microcracks. These aggregates act as scatterers, causing reflection, refraction, and absorption of the ultrasonic waves, leading to signal attenuation and distortion<sup>[5]</sup>. Numerous studies have explored ultrasonic wave propagation in heterogeneous media to understand these complex interactions<sup>[6-7]</sup>. Early theoretical efforts often employed simplified models, assuming either a homogeneous matrix or regularly spaced inclusions to facilitate analytical tractability<sup>[8]</sup>. While these models provided foundational insights into scattering mechanisms, they do not fully capture the stochastic nature of real concrete.

More recent research has attempted to incorporate random distributions of scatterers through numerical simulations and advanced experimental methods. For instance, Monte Carlo simulations have been used to statistically analyze wave attenuation in media with randomly distributed inclusions<sup>[9]</sup>. Multiple scattering theories, such as Waterman Truett or Foldy's approximations, have also been adapted to model the complex interactions of waves in heterogeneous materials<sup>[10]</sup>. However, many of these approaches rely on assumptions regarding the spatial arrangement, size distribution, or shape of aggregates that may not hold true for actual concrete. Moreover, most studies focus on a single type of probabilistic distribution typically uniform or Poisson without systematically comparing how different distributions affect ultrasonic wave propagation and signal attenuation. These limitations underscore the necessity for a more generalized approach.

In practical terms, understanding the effect of varying scatterer configurations is critical for enhancing the reliability and accuracy of ultrasonic testing. Real-world concrete structures exhibit a wide spectrum of aggregate placements, influenced by factors like mix design, compaction methods, and curing processes. Each of these factors can result in distinct

scattering phenomena, potentially leading to misinterpretations of ultrasonic signals if the underlying distribution of aggregates is not accounted for. Consequently, a gap exists between the theoretical modeling of ultrasonic wave scattering in idealized scenarios and the complexities observed in field applications. Addressing this gap would significantly improve non-destructive testing protocols and structural health monitoring strategies for concrete.

Against this backdrop, the research motivation for this study is twofold. First, we seek to advance the understanding of how different probabilistic scatterer configurations influence ultrasonic wave propagation in concrete. By systematically examining distributions such as uniform, normal, exponential, and Poisson, we aim to elucidate how each affects key parameters including attenuation rate, scattering patterns, and wavefront distortions. Second, we strive to bridge the gap between theoretical modeling and real-world applications by providing insights that can enhance the interpretative accuracy of ultrasonic signals. In doing so, this work contributes to the development of more robust NDE methodologies and informs guidelines for optimizing concrete mix designs to improve test reliability.

To achieve these objectives, concrete is modeled as a two-phase medium: a homogeneous isotropic matrix representing the cement paste, and randomly distributed aggregates representing scatterers. The scalar Helmholtz equation serves as the fundamental framework for describing ultrasonic wave propagation. Statistical methods are employed to quantify the influence of scatterer configurations on wave scattering and attenuation under varying probabilistic distributions. Numerical simulations are used to validate and compare the effects of different scatterer distributions, thereby offering a comprehensive perspective on the stochastic nature of ultrasonic wave behavior in concrete.

In the sections that follow, we present the theoretical background, modeling techniques, simulation setup, and results. Finally, we discuss the implications of our findings for improving ultrasonic testing of concrete, particularly in the design of experiments and in the interpretation of wave propagation data in heterogeneous materials.

## II. PROBABILITY DISTRIBUTION OF AGGREGATE SCATTERER CONFIGURATIONS

Consider a function  $p(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$  that describes the probability of finding  $N$  scatterers in space, where the position of each scatterer is represented by a position vector  $\mathbf{r}_i$ . This function is a function of the  $N$  position vectors and represents the probability density of finding scatterers in a specific configuration. Specifically,  $p(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$  can be viewed as a multivariable probability density function that considers the spatial positions of each scatterer as well as their interactions. This probability density function is not only related to the position of individual scatterers but also closely linked to the distribution pattern, density, and spatial constraints between them. By analyzing this function, we can study the distribution laws of scatterers in space and provide a more accurate mathematical model for simulating ultrasonic wave propagation. Furthermore,  $p(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$  can be used to assess the variations in wave propagation characteristics under different

scatterer configurations, which in turn influence the attenuation properties of the ultrasonic waves.

$d\tau_i$  represents the volume element where the  $i$ -th scatterer is located. Therefore,  $p(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)d\tau_1 d\tau_2 \dots d\tau_N$  represents the probability of finding a scatterer in  $N$  volume elements. The integral of the probability distribution function over the entire space must equal 1, that is:

$$\int p(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)d\tau_1 d\tau_2 \dots d\tau_N = 1 \quad (1)$$

This ensures that the sum of probabilities for all possible configurations equals 1. The scatterers are indistinguishable (completely identical), meaning that swapping any two position vectors  $\mathbf{r}_i$  and  $\mathbf{r}_j$  in the probability distribution function does not change the value of the function. The scattering regions of the scatterers cannot overlap, so when any two position vectors  $\mathbf{r}_i$  and  $\mathbf{r}_j$  are very close, the probability distribution function  $p(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$  must be zero.

In multiple scattering problems, the specific configuration of scatterers is usually not of interest; rather, the average effect of all possible configurations is of concern. Therefore, it is necessary to average over all possible configurations, which involves integrating the probability distribution function  $p(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$  to obtain the average physical quantity. It is defined as:

$$\langle f(\mathbf{r}) \rangle \equiv \int \dots \int d\tau_1 \dots d\tau_N \times p(\mathbf{r}_1, \dots, \mathbf{r}_N) f(\mathbf{r} | \mathbf{r}_1, \dots, \mathbf{r}_N) \quad (2)$$

Here,  $f(\mathbf{r} | \mathbf{r}_1, \dots, \mathbf{r}_N)$  represents the function value under a given configuration.

Partial averaging reflects the average behavior of function  $f$  when one scatterer is fixed. This is particularly useful when studying the interactions between scatterers, as we can fix one scatterer and then examine the influence of other scatterers on it. Partial averaging refers to the average value of a function  $f(\mathbf{r} | \mathbf{r}_1, \dots, \mathbf{r}_N)$  across all configurations, with one scatterer fixed at  $\mathbf{r}_1$ . It is defined as:

$$f(\mathbf{r} | \mathbf{r}_1) \equiv \int \dots \int d\tau_2 \dots d\tau_N \times p(\mathbf{r}_2, \dots, \mathbf{r}_N | \mathbf{r}_1) f(\mathbf{r} | \mathbf{r}_1, \dots, \mathbf{r}_N) \quad (3)$$

Here,  $p(\mathbf{r}_2, \dots, \mathbf{r}_N | \mathbf{r}_1)$  is the conditional probability distribution function of the other scatterers, given that the first scatterer is fixed at  $\mathbf{r}_1$ .

## III. UNIFORM ISOTROPIC MEDIUM

For concrete mixtures, when considering only the concrete slurry and aggregates, it can be treated as a two-phase medium. The concrete slurry is a homogeneous mixture composed of cement, sand, and water, while the aggregates are particles such as gravel and sand. Due to the isotropic nature of the slurry, it is considered a uniform isotropic medium, meaning that the physical properties of the medium (such as density, elastic modulus, etc.) are constant throughout space and do not change with position. This assumption simplifies the analysis of wave propagation in the medium, allowing us to describe the wave behavior in the medium with a simplified mathematical form.

The scalar Helmholtz equation is the fundamental equation describing wave propagation in a uniform medium, given by:

$$\nabla^2 \psi + k^2 \psi = 0 \quad (4)$$

where  $\psi$  is the Laplacian operator,  $\nabla^2$  is the wave function, and  $k$  is the wave number. This equation describes the propagation characteristics of waves in a uniform isotropic medium, indicating that wave propagation is a steady and constant process, independent of spatial position.

To simplify the wave propagation problem in concrete, the time dependence of the wave is assumed to be in a simple harmonic form, i.e.,  $\exp(-i\omega t)$ , where  $\omega$  is the angular frequency. The time-dependent scalar potential is  $\psi(r)\exp(-i\omega t)$ , where  $\psi(r)$  is the spatially dependent wave function.

The scalar potential is a scalar function whose gradient can represent a irrotational vector field. An irrotational vector field is a type of vector field whose curl is zero. In wave problems, an irrotational vector field can be expressed through the scalar potential:

$$\mathbf{E} = \text{Re}[\nabla\psi\exp(-i\omega t)] \quad (5)$$

#### IV. THE SCATTERING EXCITATION FIELD OF MULTIPLE SCATTERERS

In a concrete medium, it is common to embed NN identical or similar scattering regions (such as coarse and fine aggregates), located at  $r_1, r_2, \dots, r_N$ . When an external incident wave (for instance, an ultrasonic wave)  $\psi^{\text{inc}}(\mathbf{r})$  propagates through these aggregates, each aggregate generates a scattered field  $\psi^S(\mathbf{r} | \mathbf{r}_j; \mathbf{r}_1, \dots, \mathbf{r}_N)$ . However, unlike the single-scatterer case, these scatterers do not exist in isolation: the “excitation field” acting on any given scatterer is influenced not only by the incident wave itself but also by the scattered waves produced by all the other scatterers. This process is referred to as multiple scattering, and the core challenge in solving a multiple-scattering problem lies in accurately representing and iteratively updating the excitation and scattering fields of each scatterer.

To describe the scattering behavior of the  $j$ -th scatterer under its excitation field, one typically introduces a linear scattering operator  $T$  for a single scatterer. This operator captures how the scatterer’s constitutive properties (e.g., density, elastic modulus, or acoustic impedance) and geometrical features (e.g., size, shape) affect the scattered wave. When the excitation field  $\psi^E(\mathbf{r} | \mathbf{r}_j; \mathbf{r}_1, \dots, \mathbf{r}_N)$  acts on  $T$ , it produces a new scattered wave  $\psi^S(\mathbf{r} | \mathbf{r}_j; \mathbf{r}_1, \dots, \mathbf{r}_N)$ . Notably, this excitation field is generally a superposition of the external incident wave plus the scattered waves generated by all the other scatterers. To fully represent this superposition, the excitation field at the  $j$ -th scatterer can be expressed as:

$$\psi^E(\mathbf{r} | \mathbf{r}_j; \mathbf{r}_1, \dots, \mathbf{r}_N) = \psi^{\text{inc}}(\mathbf{r}) + \sum_{k \neq j}^N T(\mathbf{r}_k) \psi^E(\mathbf{r} | \mathbf{r}_k; \mathbf{r}_1, \dots, \mathbf{r}_N) \quad (6)$$

#### V. SIMULATION EXPERIMENTS AND RESULT ANALYSIS

To investigate how different probability distributions of aggregates affect ultrasonic scattering and propagation, a  $50 \text{ mm} \times 50 \text{ mm} \times 50 \text{ mm}$  region was used to generate 36

aggregates under four typical probability density distributions: uniform, normal, exponential, and Poisson. Each aggregate was modeled as a circular area with a radius of  $2 \text{ mm}$ , approximating the typical size of aggregates in concrete. By establishing simulation models for aggregates positioned according to these distributions, we obtained time-domain signals (wave amplitude versus time) of ultrasonic waves propagating through the medium. This approach enabled comparisons of how the positional arrangements of aggregates influence ultrasonic testing results.

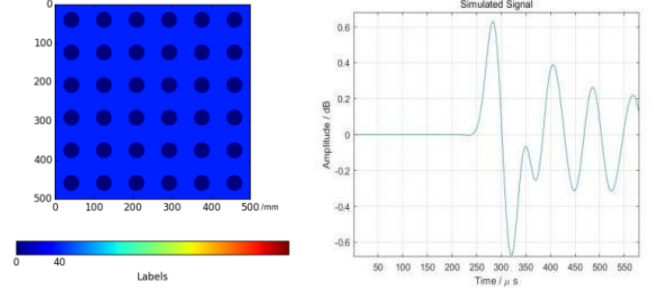


Figure 1. Uniform Distribution

Under a uniform distribution, aggregates are spaced fairly regularly, making the scattering effects relatively “predictable.” As shown in Figure 1, the time-domain signal exhibits a smooth pulse shape, where the main scattering peaks attenuate gradually after a few oscillations. Compared with the other distributions, the scattering response here is more stable and repeatable, and both positive and negative peaks tend to remain in a moderate range, indicating relatively consistent energy dissipation and scattering characteristics within the medium.

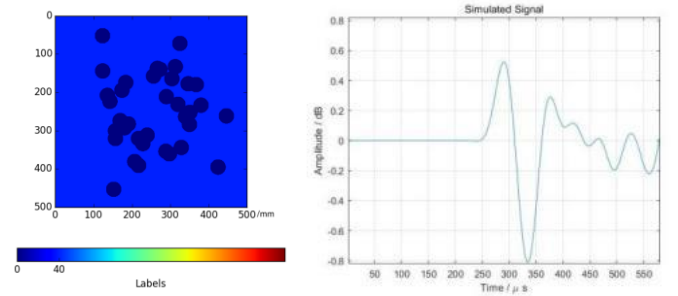


Figure 2. Normal Distribution

In the normal distribution, aggregates display a degree of clustering around a mean location; however, due to randomness, they also disperse across a certain area. Compared with a uniform arrangement, this local clustering in certain regions leads to stronger scattering, while the sparser regions produce weaker scattering. Consequently, the waveform in Figure 2 shows larger positive and negative peaks and multiple pronounced oscillations, reflecting more complex multiple scattering interactions. Overall, the normal distribution results in a more asymmetric waveform with more substantial amplitudes.

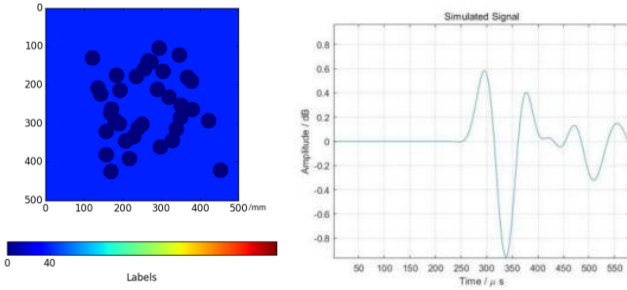


Figure 3. Exponential Distribution

An exponential distribution often yields high-density clustering in specific regions and sparser placements elsewhere. Ultrasonic waves thus undergo stronger scattering and attenuation in these dense areas, while propagation is relatively unhindered in more open spaces. As illustrated in Figure 3, the time domain signal exhibits a pronounced negative peak followed by a series of diminishing oscillations, suggesting significant energy attenuation and scattering in those concentrated areas of aggregates. Similar to the normal distribution, the randomness in exponential distributions causes less uniform waveform fluctuations and varying time intervals between scattering peaks.

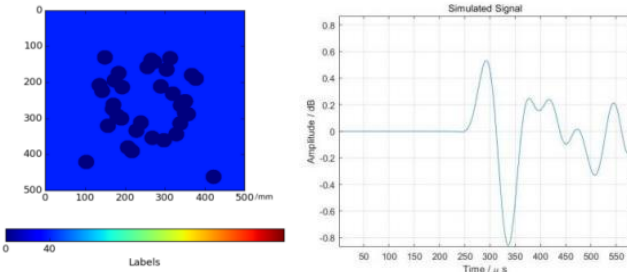


Figure 4. Poisson Distribution

A Poisson distribution is frequently used to model random scatter over a large scale, with aggregates more evenly “randomized” throughout the medium—lacking a clear central cluster. The waveform shown in Figure 4 has overall scattering amplitudes that lie between those observed in the other three distributions but still features multiple positive and negative oscillation peaks. Compared with the uniform distribution, the short-range clustering or voids arising from Poisson randomness lead to more variable signal fluctuations over time. In general, Poisson-distributed aggregates produce scattering characteristics that more closely resemble real-world random conditions, resulting in more discrete waveform changes at different time points.

From the time domain signals, it is evident that normal and exponential distributions often yield more pronounced scattering peaks and deeper negative minima, indicating that localized aggregate clustering or uneven distribution intensifies multiple scattering and energy loss. By contrast, the uniform distribution produces smoother waveforms with relatively regular oscillations and moderate peak magnitudes, while Poisson distributions, with their stronger randomness, result in greater variation in waveform shapes and correspondingly higher uncertainty. When aggregates are heavily clustered—regardless

of distribution—this adversely affects the penetration depth and resolution of ultrasonic tests, whereas random or uniform distributions shape the waveform differently and can influence measurement repeatability. In practice, concrete aggregates rarely exhibit a strictly uniform or singular distribution; more often, they approximate some form of random pattern (such as a Poisson distribution) or an even more complex mixture. Hence, understanding how various distributions alter the scattering environment is vital for targeted signal calibration and analysis in ultrasonic testing, ultimately enhancing the accuracy of internal defect detection and evaluation.

These simulation experiments, which compare ultrasonic propagation and scattering behavior under four representative aggregate distributions, highlight that both the regularity (or randomness) and clustering of aggregates play decisive roles in determining waveform amplitude, attenuation rates, and oscillatory patterns. Such findings offer valuable insights for more precise modeling and signal processing in non-destructive evaluation of concrete structures, paving the way for improved reliability in ultrasonic testing and more robust interpretations of measured signals

## VI. CONCLUSION

This study provides a comprehensive analysis of how different scatterer distributions in concrete affect ultrasonic signal attenuation. By establishing a mathematical framework for multiple scattering problems and utilizing various probability distributions, the research highlights the significant role of scatterer configurations in wave propagation. The simulations offer valuable insights into the influence of aggregate positioning on ultrasonic testing accuracy. Future work can further refine the model by considering other material properties and exploring more advanced ultrasonic detection techniques, which may lead to enhanced methods for evaluating concrete structures.

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