Normal Probability Distribution and Process Capability Index Analysis Determine the Reference Value for Continuity Requirements of Aircraft External Surface

Ling Ding*

Shanghai Aircraft Design and Research Institute, Shanghai, China
*Corresponding author's email: dingling@comac.cc

Abstract. To ensure the aerodynamic performance of the aircraft, the continuity requirements of the external surface are determined to restrain the deviation between the manufacturing external surface and the design external surface of the aircraft. When formulating the continuity requirements of the external surface, we need to consider whether the manufacturing can be realized. This paper proposes the normal probability distribution analysis and process capability index analysis of the existing manufacturing detection data to provide the manufacturing reference value for the continuity requirements of the external surface. At present, this method has been verified and applied in practical work.

Keywords: aircraft external surface, normal probability distribution, process ability index, reference value

1 Introduction

The aircraft's external surface has the design external surface and the manufacturing external surface. The designed external surface is the appearance generated in the design stage; the manufacturing external surface is the actual external surface of the aircraft after production, and there is an inevitable deviation from the design external surface due to manufacturing errors and other terms [1]. To ensure the aerodynamic performance of the aircraft, the deviation between the manufacturing external surface and the design external surface is controlled by the continuity requirements, and the allowable deviation between the manufacturing external surface and the design external surface is specified.

Determining the appropriate continuity requirements for the aircraft's external surface is a challenging task. It requires striking a balance between the aerodynamic performance requirements and the manufacturing feasibility. The traditional practice is to set continuity requirements with the main goal of aerodynamic smoothness, and the continuity requirements are often set too high, which may result in increased manufacturing costs or even make it impossible to achieve. On the other hand, if the continuity requirements are set too low, it may compromise the aircraft's performance and competitiveness in the market.

To address this challenge, it is crucial to analyze the existing aircraft manufacturing test data using statistical methods. By examining the actual manufacturing data, we can gain insights into the current manufacturing capabilities and levels. This analysis provides valuable support and reference for determining the appropriate continuity requirements for the external surface.

The purpose of this paper is to propose a method for determining the reference value for the continuity requirements of the aircraft's external surface using normal probability distribution and process capability index analysis. By analyzing the existing manufacturing detection data, we aim to establish a data-driven approach to setting realistic and achievable continuity requirements.

In the following sections, we will delve into the details of the proposed method. We will discuss the continuity requirements for the aircraft's external surface, explain the concepts of normal probability distribution and process capability index analysis, and outline the steps involved in determining the reference value. Additionally, we will present an application case to demonstrate the effectiveness of the proposed method in a practical scenario.

The findings of this study have significant implications for the aircraft manufacturing industry. By establishing a robust method for determining the continuity requirements of the external surface, we can ensure that the manufacturing process is optimized to meet the desired aerodynamic performance while maintaining cost-effectiveness and feasibility. This approach can lead to improved aircraft quality, reduced manufacturing errors, and enhanced competitiveness in the market.

2 Continuity Requirements for Aircraft External Surface

The external surface of the aircraft is composed of skin, outer surface equipment, access covers, and other structures. Continuity requirements for an aircraft's external surface refer to the allowed deviation between the manufacturing external surface and the designed external surface. The items include: steps (shown in Figure 1), gap (shown in Figure 2), etc.

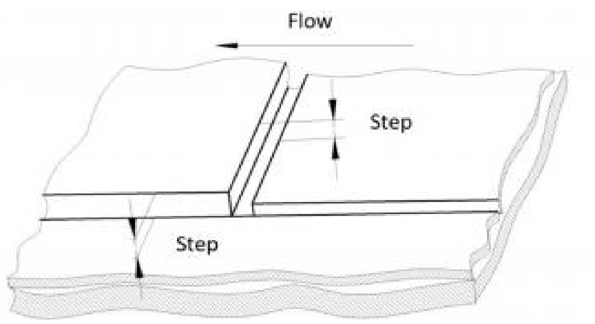

Fig. 1. Step.

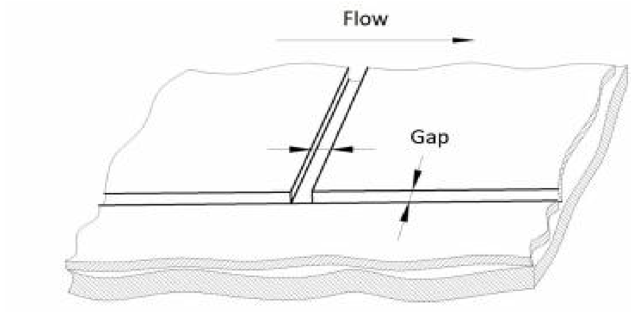

Fig. 2. Gap.

The continuity requirements for external surface is the control requirement for aircraft external surface quality, which must be achieved through manufacturing. If the continuity requirements of the external surface are too much higher, the manufacturing cost will be too high or impossible; if the continuity requirements of the external surface are much lower than the actual manufacturing level, the aircraft performance will be affected, and the competitiveness of the aircraft.

In the process of determining the continuity requirements for external surfaces, it is particularly important to analyze the manufacturing data of existing aircraft and understand the implementation level in the actual implementation. This requires recording and analyzing the relevant dimensional data in the manufacturing process, reflecting the actual manufacturing capacity and level, and providing support and reference for determining the continuity requirements of the external surface.

3 Normal Probability Distribution and Process Capability Index Analysis

The normal distribution has an extremely wide practical background, and the probability distribution of many random variables in production and scientific experiments can be described by the normal distribution [2][3]. Aircraft manufacturing and assembly are the statistical control state process of the system specification. Large amounts of detection data (step etc.) also conform to the law of normal distribution.

It is a continuous probability distribution that is symmetric and bell-shaped (shown in Figure 3), with the probability density function defined by the mean (μ) and variance (σ^2) of the distribution. The normal distribution is often denoted as x~N (μ, σ^2), where x is the random variable. The probability density function of the normal distribution is given by the following equation [4]:

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}, -\infty < x < +\infty \tag{1}$$

where μ presents the mean of the normal distribution, which is the center of the curve and describes the central tendency of the distribution. The closer the value of x is to μ, the higher the probability density is. Conversely, the further away x is from μ, the lower the probability density becomes.

The parameter σ^2 presents the variance of the normal distribution, while σ is the standard deviation of the normal distribution, which determines the shape of the curve and describes the dispersion of the normal distribution. The larger σ is, the more dispersed the data distribution is; the smaller σ is, the more concentrated the data distribution is.

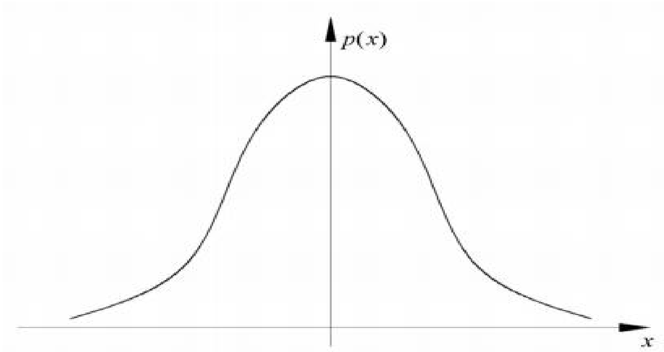

Fig. 3. A normally distributed probability density function curve.

One of the key characteristics of the normal probability distribution is its symmetry around the mean. The curve is divided into three typical intervals: [μ-σ, μ+σ], [μ-2σ, μ+2σ], and [μ-3σ, μ+3σ]. The probability of a random variable x falling within these intervals is 68.26%, 95.44%, and 99.73%, respectively (shown in Figure 4). This property makes the normal distribution a powerful tool for analyzing and interpreting data in various contexts.

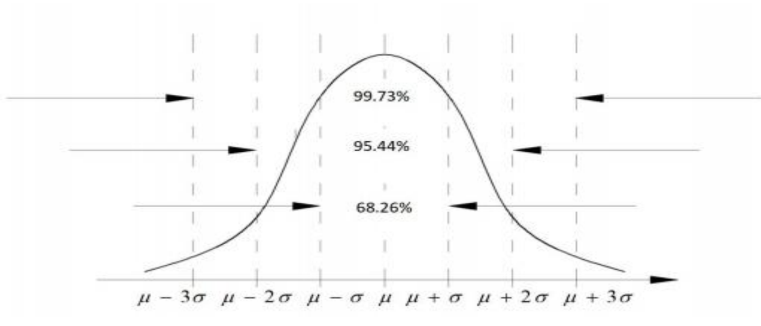

Fig. 4. Characteristics of the normal probability distribution.

The process capability index is another important concept that is closely related to the normal probability distribution. It is a measure of the relationship between the tolerance requirements and the manufacturing capability. The potential process capability index, denoted as C_p, is calculated by using Equation (2) [5].

$$C_p = \frac{USL - LSL}{6\sigma} \tag{2}$$

where USL and LSL represent the upper and lower specification limits of the tolerance, respectively. The process capability index provides valuable insights into the manufacturing process and its ability to meet the specified requirements.

When $C_p < 1$, it indicates that the tolerance requirements exceed the manufacturing capability. In other words, the process is not capable of consistently producing parts within the specified tolerance limits. This situation requires immediate attention and corrective actions to improve the process and bring it within the acceptable range.

When $1 \leq C_p < 1.33$, it suggests that the tolerance requirements match the manufacturing capability. The process is considered marginally capable, and there is room for improvement to ensure a more robust and reliable manufacturing process.

When $1.33 \leq C_p < 1.67$, the process is deemed capable, and the manufacturing capability is sufficient to meet the tolerance requirements consistently. This range indicates a well-controlled and stable manufacturing process.

4 The Reference Value Step Determined by Normal Probability Distribution Analysis

Determining the reference value for the smooth requirements of the aircraft's external surface is a critical task that requires a systematic and data-driven approach. By analyzing the existing manufacturing detection data using normal probability distribution analysis, we can establish realistic and achievable smooth requirements that align with the manufacturing capabilities. The following steps outline the process of determining the reference value for smooth requirements:

Step 1: Random sampling of sufficient manufacturing test data

The first step in the analysis is to collect a representative sample of manufacturing test data. It is essential to ensure that the sample size is sufficient to capture the variability and characteristics of the manufacturing process. Typically, a sample size greater than 100 is recommended to obtain reliable results. The data should be randomly sampled from the existing manufacturing detection records to avoid any bias or selective sampling.

Step 2: The mean value and standard deviation of the sample data

Once the sample data is collected, the next step is to calculate the key statistical parameters, the mean value, and the standard deviation. These parameters provide a quantitative description of the manufacturing process and are essential for subsequent analysis.

Step 3: The normal distribution curve of the manufacturing detection data

Using the calculated mean value and standard deviation, the normal distribution curve of the manufacturing detection data can be plotted. This curve visually represents the probability distribution of the data and helps identify the reference values corresponding to different qualification rates.

Step 4: The reference values corresponding to different qualification rates

By analyzing the normal distribution curve, engineers can determine the reference values corresponding to different qualification rates. The qualification rate refers to the percentage of manufactured parts that meet the specified smooth requirements. Commonly used qualification rates include 68.26%, 95.44%, and 99.73%, which correspond to the intervals of [μ-σ, μ+σ], [μ-2σ, μ+2σ], and [μ-3σ, μ+3σ], respectively. These reference values provide a range of options for setting smooth requirements based on the desired level of quality and manufacturing capability.

Step 5: The potential process capability index

To assess whether the initial smooth requirements match the manufacturing capability, the potential process capability index (C_p) is calculated. A C_p value greater than 1 indicates that the manufacturing process is capable of meeting the specified requirements consistently. If the C_p value is less than 1, it suggests that the initial smooth requirements may be too stringent and exceed the manufacturing capability.

Step 6: The initial smooth requirements based on the reference values

If the potential process capability index indicates that the initial smooth requirements are not achievable or exceed the manufacturing capability, adjustments can be made based on the reference values obtained from the normal probability distribution analysis. The reference values corresponding to different qualification rates provide a range of options for setting the smooth requirements. Engineers can select the appropriate reference value that balances the desired surface quality with the practical limitations of the manufacturing process.

By following these steps, engineers can determine the reference value for the smooth requirements of the aircraft's external surface using normal probability distribution analysis. This systematic approach ensures that the smooth requirements are realistic, achievable, and aligned with the manufacturing capabilities. It allows for the optimization of the manufacturing process, reducing the risk of non-conformance and improving the overall quality of the aircraft components.

5 Application Case

We take an aircraft to determine the different requirements of external surface equipment and skin as an example. The initial value of the step requirement is [-2.08, -0.8] mm.

The measured data of the existing aircraft were randomly sampled to obtain the data of 1, 284 samples shown in Table 1.

Table 1. Sample data.

NO.	Steps for surface-installed equipment and skin of an aircraft (unit: mm)											
1	-0.832	-1.88	-1.928	-1.856	-2.312	-1.84	-2.512	-1.936	-2.224	-2.056	-2.24	-1.768
2	-1.936	-2.424	-1.984	-1.4	-1.52	-1.928	-2.01	-1.584	-2.12	-2.248	-1.736	-2.056
3	-1.608	-2.384	-2.48	-2.264	-2.36	-2.096	-2.176	-2.3	-2.248	-1.896	-2.176	-2.272
4	-2.184	-2.424	-2.048	-1.656	-1.968	-1.488	-0.952	-0.984	-1.104	-1.616	-1.768	-2.264
5	-2.16	-1.92	-2.776	-2.176	-2.264	-1.856	-2.24	-2.096	-1.928	-2.216	-1.584	-1.44
6	-2.064	-2.384	-1.56	-1.712	-1.736	-1.488	-2.24	-2.112	-2.544	-1.968	-1.776	-1.672
7	-2.176	-1.56	-2.152	-1.912	-1.824	-1.584	-1.84	-1.808	-1.744	-1.84	-2.016	-2.104
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The mean value of the sample data was -1.8 mm with a standard deviation of 0.289 mm. Normal distribution plots of the sample data are shown in Figure 5.

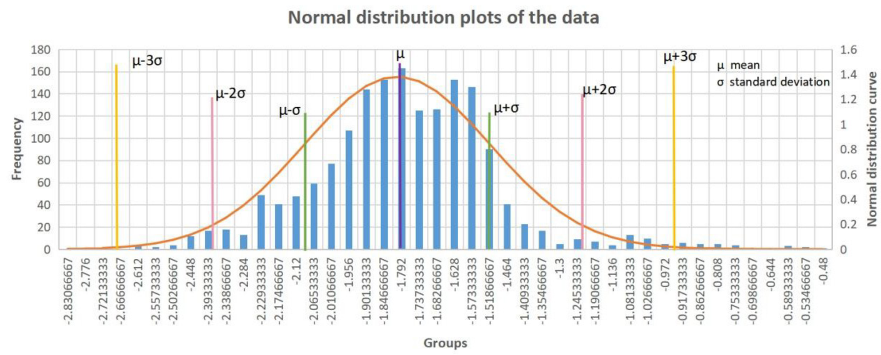

Fig. 5. Normal distribution map of sample data.

As can be determined from the figure, the step data intervals of different pass rates are shown in Table 2.

Table 2. Reference step interval.

Steps interval	Lower value (mm)	High value (mm)	Percent of pass
[μ-σ, μ+σ]	-2.6716	-0.9382	99.73%
[μ-2σ, μ+2σ]	-2.3827	- 1.2271	95.44%
[μ-3σ, μ+3σ]	-2.0938	- 1.5160	68.26%

In addition, the step requires an initial value of [-2.08, -0.8] mm, and the calculated C_p is equal to 0.74. C_p<1, beyond the manufacturing capacity. Correction of the step requirement is required. The tolerance requirement and the manufacturing capacity match $1 \le C_p < 1.33$ (at least C_p=1), when the reference interval of the step requirement [-2.67, -0.94] mm, and the qualified rate is expected to reach 99.73%. The step requirement of this place is adjusted from [-2.08, -0.8] mm to [-2.67, -0.94] mm.

The step requirements adjusted after the normal probability distribution and process capability index analysis of the existing manufacturing detection data are used to determine the steps between the equipment and skin of an aircraft. Step data of 100 points are recorded, as shown in Table 3.

Table 3. Validation data.

NO.	Validation data							
1	-1.944	-2.008	-1.856	-1.872	-2.36	-1.976	-2.144	-2.112
2	-2.024	-1.968	-1.848	-1.952	-2.136	-1.584	-2.072	-1.336
3	-2.088	-0.888	-1.936	-2.008	-2.296	-1.888	-2.128	-1.752
4	-2.152	-1.96	-1.928	-1.48	-1.576	-1.608	-1.696	-1.544
5	-2.16	-1.816	-2.208	-1.184	-1.648	-1.584	-1.464	-1.52
6	-2.288	-1.8	-2.2	-1.992	-2.24	-2.256	-2.344	-2.112
7	-2.224	-1.92	-1.944	-1.816	-2.072	-1.848	-2.04	-1.84
8	-1.424	-1.872	-2.36	-1.88	-1.848	-1.768	-1.896	-1.688
9	-2.176	-1.688	-2.104	-1.528	-2.056	-1.672	-2.112	-1.928
10	-1.92	-0.584	-1.808	-1.448	-2.096	-0.672	-2.112	-1.744
11	-2.328	-1.976	-2.184	-1.856	-1.976	-1.96	-2.072	-1.616
12	-2.184	-1.856	-2.136	-1.464	-2.024	-1.728	-2.208	-1.864
13	-2.01	-1.864	-2.104	-1.736	—	—	—	—

According to Table 3, according to the requirements of [-2.67, -0.94] mm, the qualified rate of test data is 97%, and the difference from the expected qualified rate of 99.73% when the difference requirement is within 3%.

Moreover, the step requirement is adjusted from [-2.08, -0.8] mm to [-2.67, -0.94] mm, the lower difference is 0.59 mm, the upper difference is 0.14 mm, and the order difference is 0.59 mm, resulting in the influence of aerodynamic resistance increment is less than 0.00001 count, which can be ignored.

The application case demonstrates the effectiveness of using normal probability distribution and process capability index analysis to determine the reference value for the step requirement between the equipment and skin of an aircraft. By analyzing the existing manufacturing detection data, realistic and achievable step requirements can be established, ensuring a balance between the desired surface quality and manufacturing feasibility.

6 Conclusion

This paper introduced the methods and steps of the normal probability distribution and process capability index analysis of the existing manufacturing detection data and then applied them in the work practice. Taking the reference value of the step between the equipment and skin as an example, the method is demonstrated and the reference values are compared with the actual results. The method of normal probability distribution and processability index analysis to determine the reference value of the continuity requirements of the external surface can reflect the actual manufacturing level and have reference significance for determining the continuity requirements for the aircraft's external surface.

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