Added Mass Solution Methods for Water Entry Problem

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Abstract. The solution of added mass for water inflow can be summarized as the potential flow solution problem of the corresponding model. Using boundary element method and commercial software to solve UDS (User Defined Scalars) to solve the Laplace equation and obtain the potential flow solution of the corresponding problem. Firstly, the added mass of the plate water entry problem is calculated and compared with the theoretical solution to verify the reliability of the two methods. Furthermore, the added mass of wedges with different deadrise angles was calculated, and the results showed that the added mass values calculated by the two methods were close, while the added mass of wedges increased with the decrease of deadrise angles, gradually approaching the added mass value of flat plates entering the water.

Keywords: water entry, boundary element method, UDS, added mass

1 Introduction

The water entry problem is a complex hydrodynamics problem, which is often applied to the calculation of landing loads on navigation vehicles and slamming loads generated by ship motion in waves. Around the 1930s, Von Karman [1] and Wanger [2] were recognized as the first two scholars who carried out the research on slamming. Von Karman simplified the wedge water entry problem into the problem of solving added mass, and considered that the slamming force on the wedge during water entry was simplified into an inertial force, which was equal to the product of the added mass and the acceleration of the object. The momentum theorem was used to complete the solution of the water entry problem, and the added mass of the wedge was simplified into the calculation of the added mass of the plate with the same wetting width. In addition to Wagner, Zhao and Faltinsen [3] and Carcaterra [4] have also revised Von Karman's added mass formula, but the revised results are different. According to the theory of stripe, Wang and Fang [5] calculated the half width and added mass of the cross sections of the boat considering the rise of the liquid level during the water entry, and calculates impact force of the boat by the theory of momentum and kinetic energy respectively. Boundary element method, also known as boundary integral equation method, is a high-precision method developed after finite element method. It transforms the governing equation into boundary integral equation and discretely solves the boundary of the computational domain. Compared with other regional discrete solving methods for partial differential equations, the number of unknowns and computation amount are significantly reduced due to the reduction of solving dimension. At the same time, because it uses the basic solution of the differential equation as the kernel function of the boundary integral equation, it has the characteristics of combining analytic and numerical values, and usually has a high calculation precision. Dong et al. [6-7] used the fully nonlinear boundary element method to solve the water entry problem of a two-dimensional wing. The User Defined Scalars (UDS) has a wide range of application scenarios in fluid dynamics simulation. With UDS, users can simulate complex fluid dynamics problems more flexibly. UDS allows users to define custom scalar transfer equations in Fluent. These equations can include the transient term, the flux term, the diffusivity term, and the source term. UDS allows users to simulate physical processes that are not included in Fluent's standard model. In this paper, the boundary element method and UDS method are used to solve the added mass problem. After comparison and verification with the theoretical solution of the added mass of the binary plate, the variation rule of the added mass of the wedges with different deadrise angles is studied. Zheng [8] proposed a method of calculating the added mass based on commercial software ANSYS fluent, which reduces the grid requirements and greatly improves the calculation speed while ensuring the calculation accuracy.

2 Theoretical Foundation

2.1 Test Model

The added mass problem of water entry can be solved by the corresponding potential flow problem. That is, the Laplace equation and the corresponding boundary conditions are satisfied. In order to simplify the calculation, only the two-dimensional problem is considered, and the plate enters the water at a unit velocity. The corresponding governing equation and boundary conditions are as follows [9]:

Governing equation:

$$\Delta\phi = 0 \tag{1}$$

Surface boundary conditions:

$$\frac{\partial\phi}{\partial n} = -n_3 \tag{2}$$

Infinite water bottom boundary conditions:

$$\nabla\phi = \vec{0} \tag{3}$$

Linear free surface boundary conditions:

$$\frac{\partial\phi}{\partial z}+\frac{\omega^2}{g}\phi=0 \tag{4}$$

Where ϕ is the velocity potential, n_3 represents the vertical component of the normal direction of the surface, g is the gravitational acceleration, and ω is the circular frequency. For the water entry bang problem, it can be considered a high frequency response problem, that is, ω tends to infinity and the vertical derivative of the velocity potential is a finite definite value, so the linear free surface boundary condition holds only at ϕ=0 at the free liquid surface. After solving the above potential flow problem, the added mass M can be expressed as follows:

$$M=\rho\iint_s \phi\frac{\partial\phi}{\partial n}ds \tag{5}$$

The above equation represents the boundary integral on the surface of the computational domain. For the problem of water entry, since the free surface boundary condition is ϕ=0 and the boundary condition on the bottom is$\nabla\cdot\phi$=0, and the velocity potential at infinity is also close to 0, the integral of the above equation can be directly integrated on the boundary of the surface.

2.2 Boundary Element Method

Boundary element method, also known as boundary integral equation method, is a high-precision method developed after finite element method. It transforms the governing equation into boundary integral equation and discretely solves the boundary of the computational domain. Compared with other regional discrete solving methods for partial differential equations, the number of unknowns and computation amount are significantly reduced due to the reduction of solving dimension. At the same time, because it uses the basic solution of the differential equation as the kernel function of the boundary integral equation, it has the characteristics of combining analytic and numerical values, and usually has a high calculation precision [10].

By using Green's second formula (6), the calculated solution is transformed into the boundary integral equation. In the equation, ϕ is the velocity potential function to be solved, and $G(P,Q)$ becomes Green's function, which satisfies the basic solution of the governing equation: $\Delta G(P,Q)=\delta(P-Q)$, P and Q are the source point and the field point respectively. δ is the Dirac function. It can be simplified as formula (7).

$$\int_{\Omega}\phi\nabla^2 G(P,Q)-G(P,Q)\nabla^2\phi d\Omega=\int_{\partial\Omega}\phi\frac{\partial G(P,Q)}{\partial n}-G(P,Q)\frac{\partial\phi}{\partial n}ds \tag{6}$$

$$\phi(P)=\int_{\partial\Omega}\phi\frac{\partial G(P,Q)}{\partial n}-G(P,Q)\frac{\partial\phi}{\partial n}ds \tag{7}$$

There are two main methods to solve the above integral equations: one is the direct boundary element method, which constructs discrete equations according to formula (7), and the solved unknown is the velocity potential; The other is called the indirect boundary element method, that is, the basic solution of the governing equation is multiplied by a constant distributed on the boundary of the calculation domain, the solution is called the source strength, and then the velocity potential of each point in the calculation domain is solved. Both methods have higher calculation accuracy. The indirect boundary element method is used in this paper.

Lnr/2π is the basic solution of two-dimensional Laplace equation, which only satisfies the bottom-bottom boundary conditions of infinite water depth, and the source points should be arranged on the boundary conditions of the free liquid surface and the boundary of the object surface. In this paper, a constant element is used, that is, the two-dimensional boundary is discretized into multiple line segments, and the point source strength on each line segment is a constant. Singularity exists when the spot point coincides with the source point, which needs special treatment. One way is to stagger the source point and the field point to avoid singularity. Another way is to treat singularity in a special way. This article takes the latter approach.

Fig. 1. Singularity treatment of elements.

Singularity exists when calculating the contribution of the element itself to the velocity potential and normal derivative of the central field point of the element. As shown in figure 1, when calculating the partial derivative, the area around the field point is approximated to an infinitely small semicircle segment, where the point sources distributed in the straight segment have no contribution to the field point velocity potential partial derivative, so it can be considered that the partial derivative calculation can be approximated to the contribution of the point sources distributed on the semicircle, that is $\frac{\pi \mathrm{r} S}{2\pi} \cdot \frac{\partial \ln r}{\partial r} = \frac{S}{2}$,where S is the intensity of the point sources distributed on the unit. For the calculation of the velocity potential, only when the field point coincides with the source point, there is singularity, and Cauchy principal value integral can be used.

2.3 UDS Method

General fluid business software solves the Reynolds transport equation in the form of equation (8). In the formula, ϕ is the transport volume and $\vec{u}$ is the transport velocity in the flow field. The time derivative term and flux term in the Reynolds diffusion equation at steady state and zero velocity flow field are 0, leaving only the diffusion term on the right of the equation. If the diffusion coefficient Γ is set at 1, the Reynolds transport equation directly degenerates into Laplace equation [11]. The Laplace equation can be solved by solving the UDS equation if certain boundary conditions are specified.

$$\frac{\partial \phi}{\partial t} + \vec{u} \cdot \nabla \phi = \nabla \cdot (\Gamma \nabla \phi) \tag{8}$$

The general boundary conditions can be divided into three categories: the first type of boundary conditions, namely Dirichlet boundary conditions, specify the value at the boundary, namely $\phi_f=\phi_{specfied}$; The second type of boundary condition, the Neuman boundary condition, specifies the normal derivative on the boundary, determining $\phi_{n\_f}=\phi_{n\_specfied}$. The third type of boundary condition, Robin boundary condition, gives the linear combination of the physical quantity and its normal derivative on the boundary, namely $\alpha\phi_f+\beta\phi_{n\_f}=\gamma$, where α、β、γ are the given known quantities. The first two types of boundary conditions are given in general commercial software, and the third type of boundary conditions can be indirectly transformed into the form of the first or second type of boundary conditions and solved iteratively.

3 Model Verification

The solution of the added mass in water of the plate is simplified into a two-dimensional model, as shown in figure 2. Its governing equation and boundary conditions are shown in Section 2, where the wetting length of the plate is 2C and the normal vertical component of the plate is 1, that is, the surface boundary condition is $\partial\phi/\partial n=-1$.

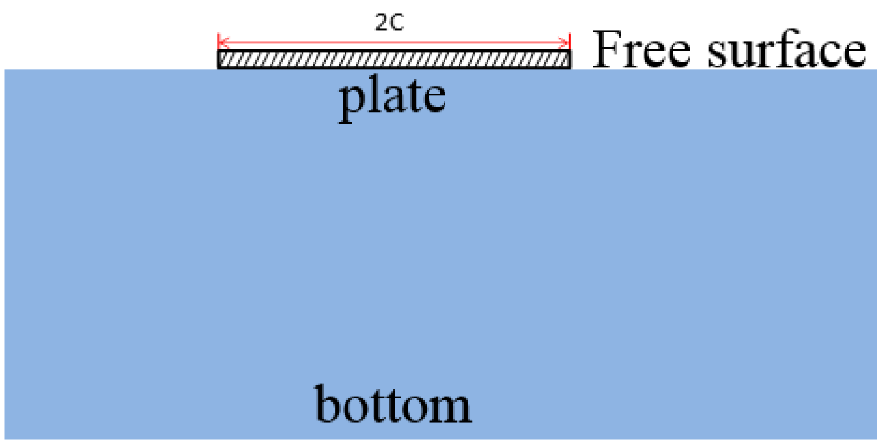

Fig. 2. Schematic diagram of two-dimensional plate model.

To solve the above problem, complex variable Z=x+iz is introduced, and the analytical solution of complex potential can be expressed as:

$$\Phi = \phi + i\varphi = iZ - i\frac{Z}{\sqrt{Z^2 - C^2}} \tag{9}$$

ϕ represents the velocity potential and φ represents the flow function. According to the above formula, the velocity potential distribution on the plate can be obtained as follows:

$$\phi_w = -\sqrt{C^2 - x^2} \tag{10}$$

The vertical velocity at the free liquid surface is expressed as:

$$\frac{\partial \phi}{\partial z} = \frac{|x|}{\sqrt{x^2 - C^2}} - 1 \tag{11}$$

For the boundary element method, the discretization is shown in figure 3. Only the free surface and the surface of the object surface are discretized into corresponding line units, and point sources of equal intensity are distributed on each line unit. The length of the free liquid surface should ensure that it has no obvious influence on the calculation results. 10 times the wetting length was taken for the calculation. According to the above discretization and solution methods, the flow field information such as velocity potential distribution on the surface and normal velocity distribution on the free liquid surface can be obtained.

However, the UDS method needs to discretely process the computational domain of the two-dimensional plane problem，as shown in figure 4. Similarly, only a finite length free liquid surface is selected, and the boundary of two sides and bottom is set as a symmetric surface, and the normal derivative of the velocity potential on the boundary is 0. Theoretically, this is inconsistent with the real situation. However, considering that when the calculation domain is large enough, the water entry disturbance of the plate has relatively little influence on both sides and bottom, and the symmetric boundary conditions of both sides and bottom have relatively little influence on the calculation results, The boundary conditions of the free liquid surface and the plate wall are set according to the previous discussion.

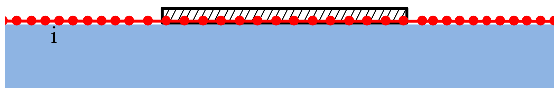

Fig. 3. Schematic diagram of boundary element dispersion.

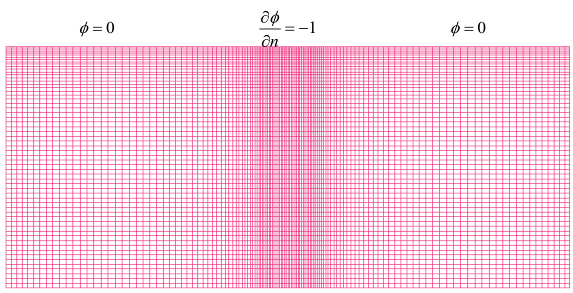

Fig. 4. Grid diagram of UDS.

During the calculation, the half-width of the plate was set to 0.5m. The velocity potential distribution of the plate wall and the vertical velocity distribution of the free liquid surface obtained by the two methods and the corresponding theoretical solutions are shown in figure 5 and figure 6. The theoretical results show that the vertical velocity of the free surface is singular at both ends of the plate. The results show that the velocity potential distribution of the plate and the vertical velocity distribution of the free liquid surface obtained by the two methods are in good agreement with the theoretical solution, but the deviation is obvious at the end of the plate. Then the added mass of the plate is calculated, and the calculation results are shown in Table 1. The theoretical value of added mass is 391.91 kg, the calculated value of added mass is 396.9 kg by BEM and 391.52 kg by UDS. The added mass deviation of the two methods is 1.27% and 0.10% respectively. The results show that both methods are suitable for solving added mass problems without considering the water surface uplift effect, and the calculation accuracy is high.

Table 1. Comparison of solution results for added mass.

Solution method	Added mass/kg		deviation
	Calculated value	Theoretical value	
BEM	396.90	391.91	1.27%
UDS	391.52	391.91	0.10%

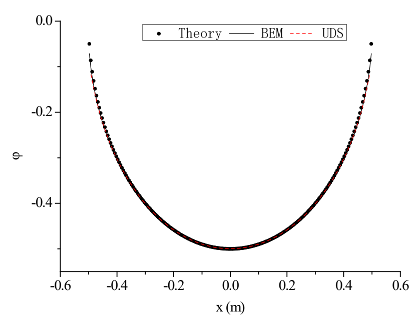

Fig. 5. Comparison of the velocity potential distribution of the plate.

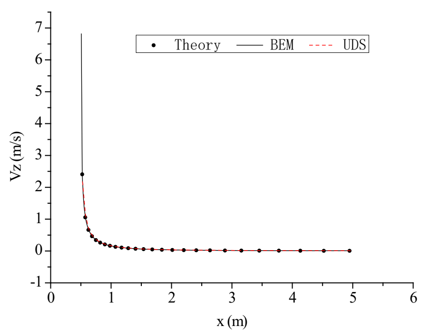

Fig. 6. Comparison of vertical velocity distribution at the free surface.

4 Calculation of Added Mass of Wedges

Nine wedge shapes with different deadrise angles of 0, 5, 10, 15, 20, 30, 40, 50 and 60° were selected for calculation, and the wetted half-width was set at 0.5m. The boundary element method and the UDS boundary element method are used respectively to solve the added mass, and the water surface uplift and air cushion effect are not considered in the calculation process.

The velocity potential distribution on the wetting surface of the wedge is shown in figure 7. The velocity potential distributions obtained by the two methods are close, and the differences are also concentrated in the two end points where the wedge body intersects with the free surface of still water. The velocity potential distribution of the wedge with different deadrise angles is mainly concentrated at the lower edge of the wedge, showing a local peak value, and the peak value distribution becomes more obvious with the increase of the deadrise angle.

Figure 8 shows the vertical velocity distributions on the free surface of the wedges with different deadrise angles. Theoretically, the velocity distributions are symmetrical. As shown in the figure, the calculation results of the two methods are quite different, which are also mainly concentrated on the two ends of the wedge. The UDS method fails to calculate the velocity distribution of the two ends of the wedge well, while the BEM method fails to capture the velocity of the large gradient at the two ends of the wedge. It is worth explaining that the UDS method mainly adopts the finite volume method, and the gradient calculation on the boundary of the calculation domain mainly adopts the interpolation method, which may be one of the reasons for the large deviation in the calculation of the vertical velocity of the free water surface in UDS. At the same time, the figure shows that the vertical velocity near the two ends of the wedge decreases with the increase of the angular velocity.

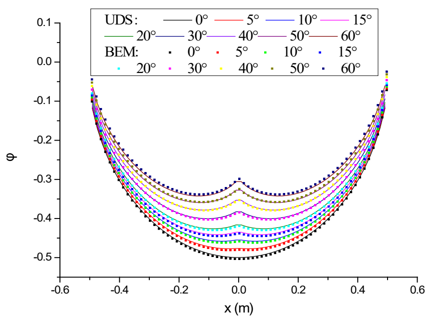

Fig. 7. Velocity Potential Distribution of Wedge with Different Deadrise Angles.

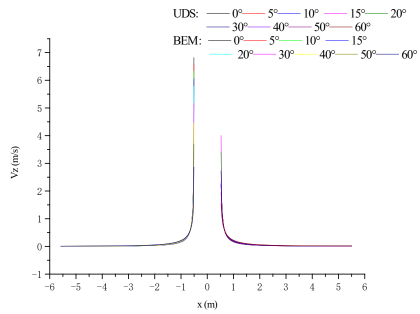

Fig. 8. Vertical velocity distribution of free surface of Wedge with Different Deadrise Angles.

Then, the variation of the added mass of the 2D wedge model with different incline angles can be obtained by using formula 5. As shown in fig 9, the added mass obtained by the two methods is relatively close, and the added mass of the wedge with the same wetted width also decreases with the increase of the deadrise angle.

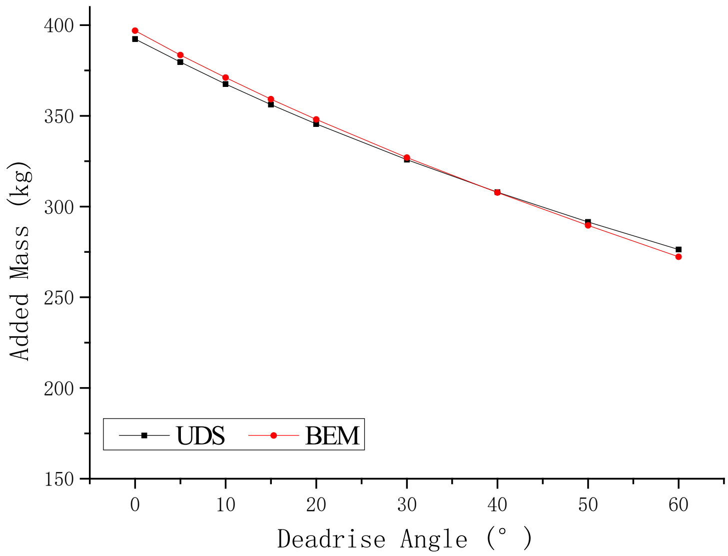

Fig. 9. The variation of the added mass of 2D wedge with the deadrise angle.

5 Conclusion

The core advantage of the Boundary Element Method (BEM) lies in its direct handling of boundary conditions. It relates the boundary information to the internal field through the Green's function, which is particularly efficient for problems with strong analyticity, such as potential flow problems and acoustic problems. However, this also brings a key challenge: it relies on the exact Green's function corresponding to the equation, which means that not all types of equations, especially hyperbolic equations, are suitable for using BEM, because the Green's function of such equations is usually difficult to obtain.

The advantage of the User-Defined Scalar Equation Method (UDS) is its strong flexibility. Users can define equations according to their own understanding of the problem and specific requirements without relying on the Green's function. This is very useful when dealing with some special, non-standard physical phenomena or engineering problems. It can also conveniently combine the actual physical process to build a model and can reflect the physical essence of the problem more intuitively. However, this also leads to the fact that the user-defined equations may lack the support of universal theories, and users need to have profound professional knowledge and practical experience to ensure the rationality and correctness of the equations. Moreover, in terms of solution efficiency, it may not be as good as some mature general methods. Especially for complex large-scale problems, it may take more time to debug and optimize the user-defined equations.

In summary, both BEM and UDS have their respective applicable scenarios. Each method has its own unique advantages and limitations. Which one to choose depends on the nature of the specific problem and the needs of engineers.

In this paper, the added mass of two-dimensional wedge model is solved by using boundary element and UDS, and the reliability and accuracy of the calculation model are verified by comparing with the theoretical solution of two-dimensional plate added mass. The results show that both methods are suitable for solving added mass problems without considering the water surface uplift effect, and the calculation accuracy is high. Furthermore, the added mass of wedges with different deadrise angles was analyzed. The calculation results showed that under the same wetting half width, the added mass of wedges decreased with the increase of deadrise angle. When the inclination angle approached 0 degrees, the added mass of wedges approached the value of the added mass of flat plates.

Because the calculation model refers to the neglected water surface uplift effect, the added mass solved will be smaller than the real situation. The next step is to use the instantaneous calculation model to solve the water entry process of the wedge according to the momentum theorem. At the same time, the position of the free surface is explicitly traced according to the vertical velocity of the free surface to consider the free surface uplift effect. At the same time, Bernoulli formula is used to calculate the distribution of wet surface pressure during the process of wedge entering water, and the calculation model is perfected. The reliability and accuracy of the method are further verified by wedge experiments.

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