Exercises for K-Means

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Digging into Data

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1 Clustering Example

2 K-means Algorithm

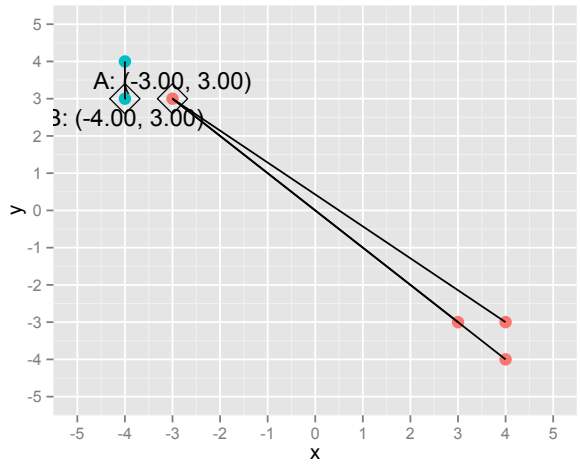
Input: A set of data points $X \equiv \{x_1, x_2, \ldots x_N\}$, a number of clusters K, and initial means $M = \{\mu_1, \ldots \mu_K\}$.
Output: Updated means $\{\mu_1, \ldots \mu_K\}$ and cluster assignments $\{z_1, \ldots z_N\}$. Such that $f(X, Z, M) = \sum_i ||x_i - \mu_{z_i}||$ is minimized.

While $f(X, Z, M)$ has not converged:

1. Assign each x_i to the mean c such that $d(\mu_c, x_i)$ is smallest ($z_i = c$).
2. Recompute each of the means: $\mu_j = \frac{1}{V_j} \sum_i \mathbb{1}[z_i = j] x_i$

3 K-means Warmup

3.1 Two Cluster 1



Updated Means:

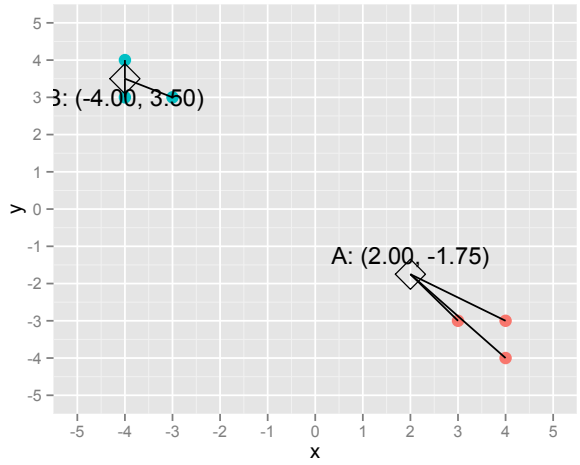
$$\mu_A = \frac{1}{4}\left((-3,3) + (3,-3) + (4,-3) + (4,-4)\right)$$

$$=$$

$$\mu_B = \frac{(-4,3) + (-4,4)}{2}$$

$$=$$

3.2 Two Cluster 2



Updated Means:

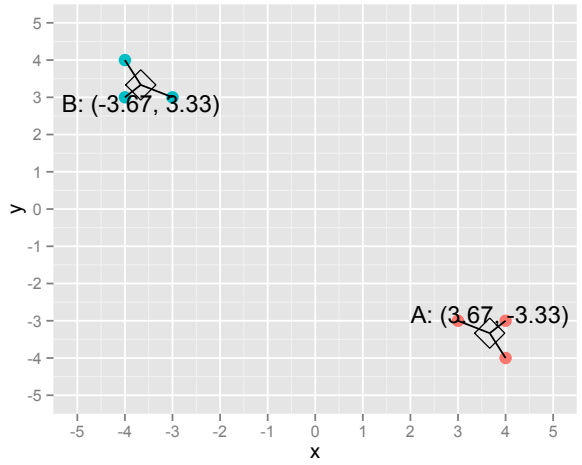
$$\mu_A = \frac{(3,-3) + (4,-3) + (4,-4)}{3}$$

$$=$$

$$\mu_B = \frac{(-4,3) + (-4,4) + (-3,3)}{3}$$

$$=$$

3.3 Two Cluster 3



Updated Means:

$$\mu_A = \frac{(3,-3)+(4,-3)+(4,-4)}{3}$$

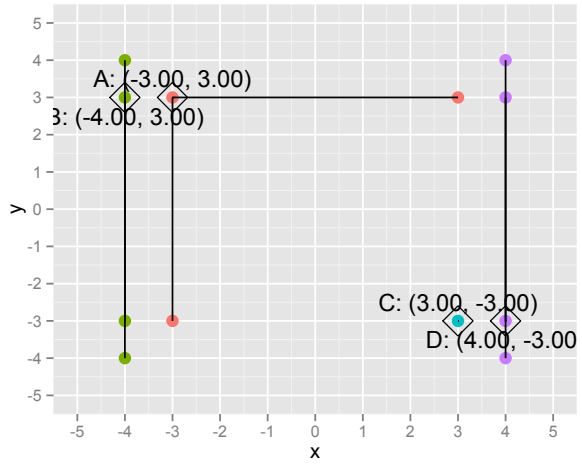
$$=$$

$$\mu_B = \frac{(-4,3)+(-4,4)+(-3,3)}{3}$$

$$=$$

4 Four Clusters

4.1 Four Clusters 1



Updated Means:

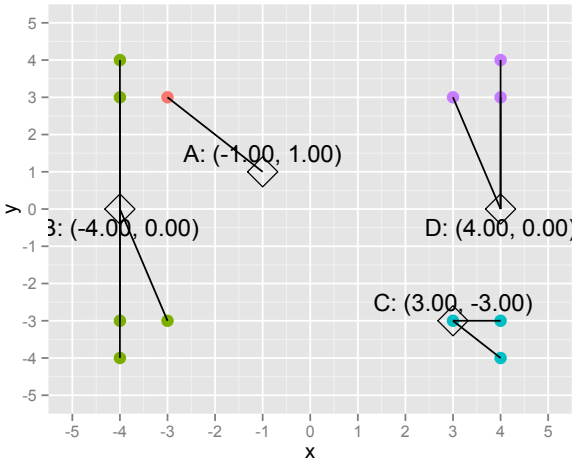
$$\mu_A =$$

$$\mu_B =$$

$$\mu_C =$$

$$\mu_D =$$

4.2 Four Clusters 2



Updated Means:

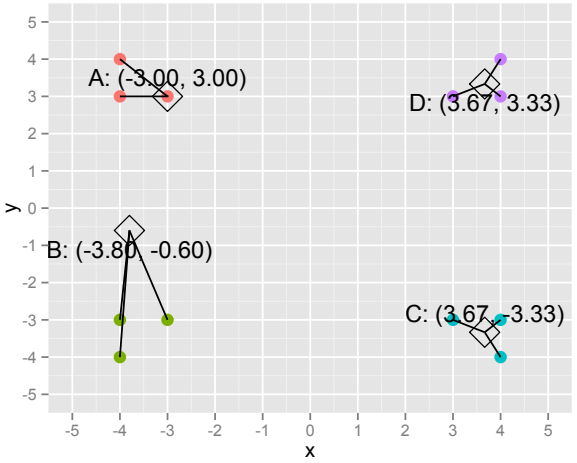
$\mu_A =$

$\mu_B =$

$\mu_C =$

$\mu_D =$

4.3 Four Clusters 3



Updated Means:

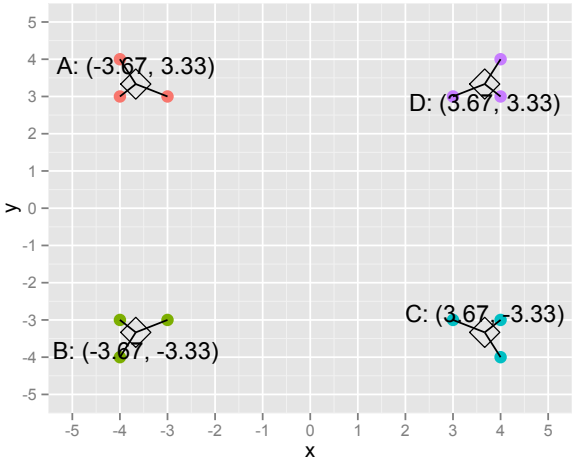
$\mu_A =$

$\mu_B =$

$\mu_C =$

$\mu_D =$

4.4 Four Clusters 4



Updated Means:

$\mu_A =$

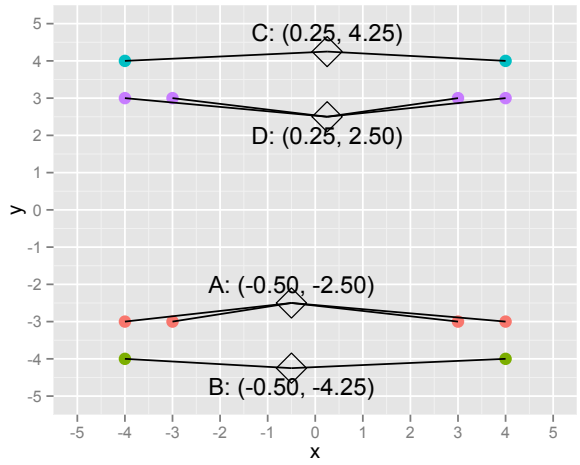
$\mu_B =$

$\mu_C =$

$\mu_D =$

5 Strange Initialization

5.1 Strange Initialization 1



Updated Means:

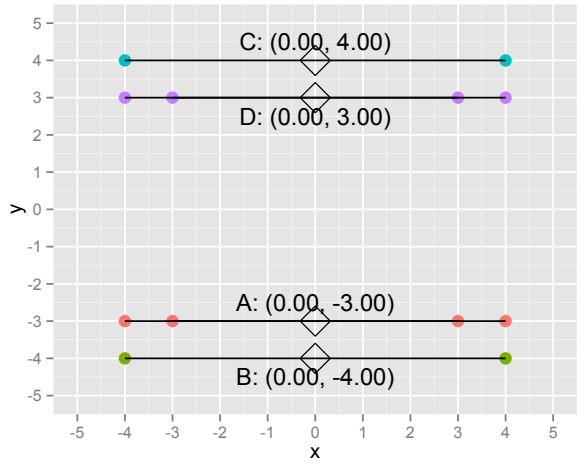
$\mu_A =$

$\mu_B =$

$\mu_C =$

$\mu_D =$

5.2 Strange Initialization 2



Updated Means:

$\mu_A =$

$\mu_B =$

$\mu_C =$

$\mu_D =$