



Department of Computer Science
UNIVERSITY OF COLORADO **BOULDER**



Mathematical Foundations

Introduction to Data Science Algorithms

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SLIDES ADAPTED FROM DAVE BLEI AND LAUREN HANNAH

Marginalization

If we know a joint distribution of multiple variables, what if we want to know the distribution of only one of the variables?

We can compute the distribution of $P(X)$ from $P(X, Y, Z)$ through *marginalization*:

$$\begin{aligned}\sum_y \sum_z P(X, Y = y, Z = z) &= \sum_y \sum_z P(X) P(Y = y, Z = z | X) \\ &= P(X) \sum_y \sum_z P(Y = y, Z = z | X) \\ &= P(X)\end{aligned}$$

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We'll explain this notation more next week for now the formula is the most important part.

Marginalization (from Leyton-Brown)

Joint distribution

temperature (T) and weather (W)

	T=Hot	T=Mild	T=Cold
W=Sunny	.10	.20	.10
W=Cloudy	.05	.35	.20

Marginalization allows us to compute distributions over smaller sets of variables:

- $P(X, Y) = \sum_z P(X, Y, Z = z)$
- Corresponds to summing out a table dimension
- New table still sums to 1

- Marginalize out weather
- Marginalize out temperature

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.15	.55	.30
- Marginalize out temperature

W=Sunny	.40
W=Cloudy	

Marginalization (from Leyton-Brown)

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.15	.55	.30
- Marginalize out temperature

W=Sunny	.40
W=Cloudy	.60