Department of Computer Science
UNIVERSITY OF COLORADO **BOULDER**

Chinese Restaurants and Backoff

Natural Language Processing: Jordan Boyd-Graber
University of Colorado Boulder

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Roadmap

After this class, you'll be able to:

- Understand probability distributions through the metaphor of the Chinese Restaurant Process
- Be able to calculate Kneser-Ney smoothing
- Understand the role of contexts in language models

Intuition

- Some words are “sticky”
- “San Francisco” is very common (high ungram)
- But Francisco only appears after one word

Intuition

- Some words are "sticky"
- "San Francisco" is very common (high ungram)
- But Francisco only appears after one word
- Our goal: to tell a statistical story of bay area restaurants to account for this phenomenon

Outline

How does a CRP encode a probability distribution?

How do many CRPs encode backoff?

Language Model Probabilities

Let's remember what a language model is

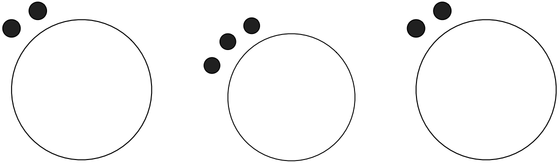
- It is a distribution over the *next word* in a sentence
- Given the previous $n-1$ words

Let's remember what a language model is

- It is a distribution over the *next word* in a sentence
- Given the previous $n - 1$ words
- The challenge: backoff and sparsity

The Chinese Restaurant as a Distribution

To generate a word, you first sit down at a table. You sit down at a table proportional to the number of people sitting at the table.



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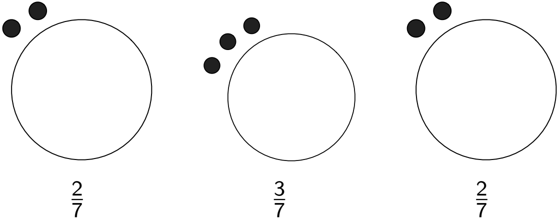

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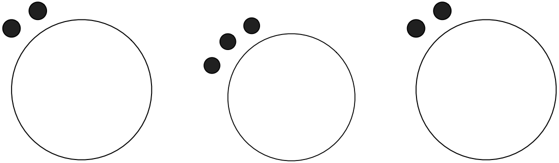

$\frac{2}{7}$ $\frac{3}{7}$ $\frac{2}{7}$

The Chinese Restaurant as a Distribution

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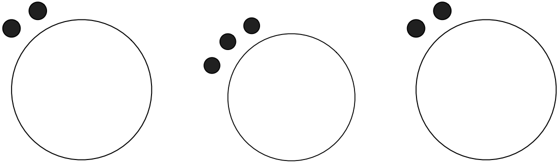
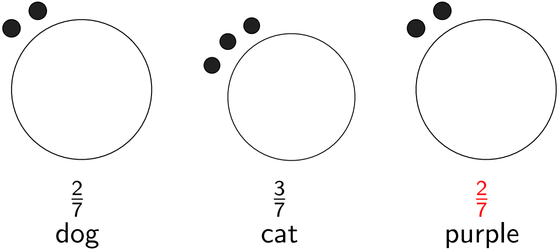


$\frac{2}{7}$	$\frac{3}{7}$	$\frac{2}{7}$
dog	cat	purple

The Chinese Restaurant as a Distribution

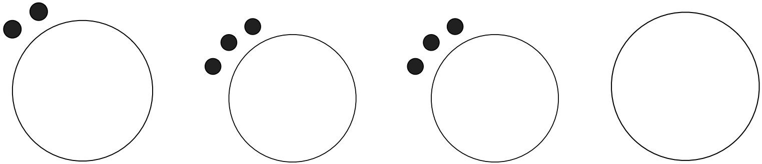
To generate a word, you first sit down at a table. You sit down at a table proportional to the number of people sitting at the table.



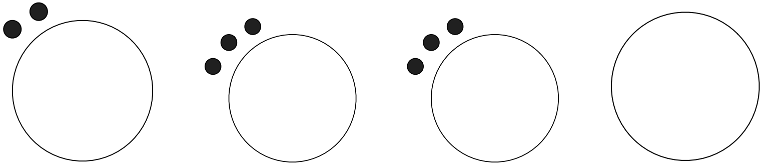
But this is just Maximum Likelihood

Why are we talking about Chinese Restaurants?

Always one more table . . .

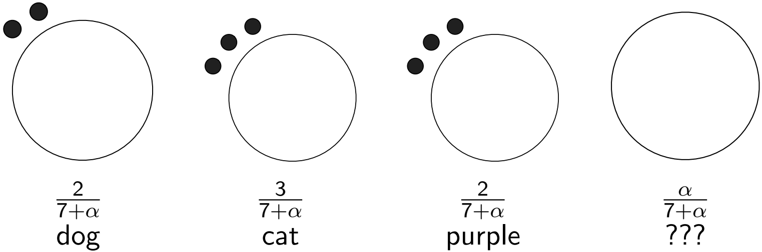


Always one more table . . .

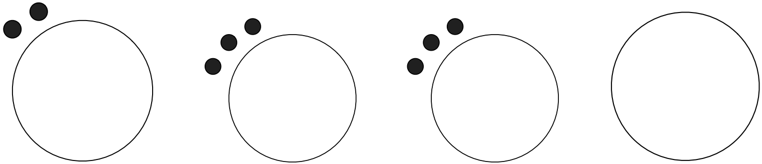


$\frac{2}{7+\alpha}$ $\frac{3}{7+\alpha}$ $\frac{2}{7+\alpha}$ $\frac{\alpha}{7+\alpha}$

Always one more table . . .



Always one more table . . .



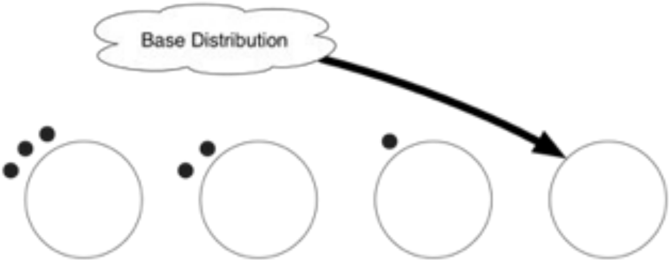
$\frac{2}{7+\alpha}$ dog

$\frac{3}{7+\alpha}$ cat

$\frac{2}{7+\alpha}$ purple

$\frac{\alpha}{7+\alpha}$???

What to do with a new table?



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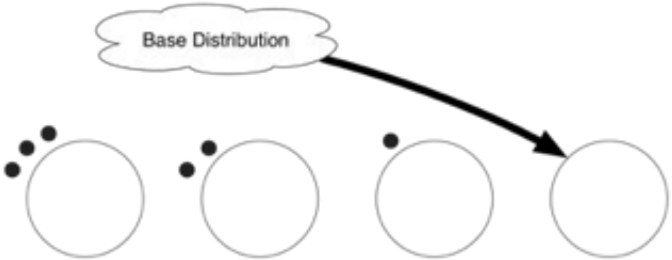
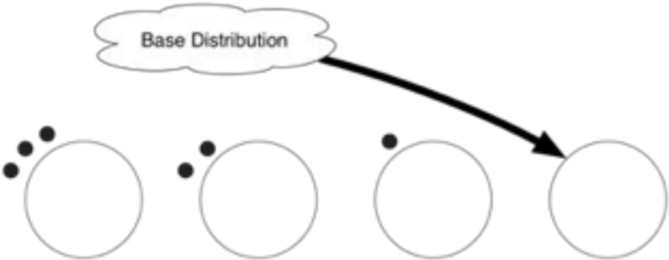


What can be a base distribution?

- Uniform (Dirichlet smoothing)

What to do with a new table?



What can be a base distribution?

- Uniform (Dirichlet smoothing)
- Specific contexts $\rightarrow$ less-specific contexts (backoff)

Outline

How does a CRP encode a probability distribution?

How do many CRPs encode backoff?

Language Model Probabilities

A hierarchy of Chinese Restaurants

Seating Assignments

Dataset:

```
<s> a a a b a c </s>
```

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

<s> Restaurant

a Restaurant

b Restaurant

c Restaurant

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

<s> Restaurant

$\boxed{*}^{1}$

a Restaurant

b Restaurant

c Restaurant

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

$\boxed{*}^{1}$

<s> Restaurant

$\boxed{*}^{1}$

a Restaurant

b Restaurant

c Restaurant

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

a [1]

<s> Restaurant

a [1]

a Restaurant

b Restaurant

c Restaurant

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

$\boxed{\texttt{a}}^{1}$

<s> Restaurant

$\boxed{\texttt{a}}^{1}$

a Restaurant

$\boxed{*}^{1}$

b Restaurant

c Restaurant

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

a [1]

<s> Restaurant

a [1]

a Restaurant

* [1]

b Restaurant

c Restaurant

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

a ^{2}

<s> Restaurant

a ^{1}

a Restaurant

a ^{1}

b Restaurant

c Restaurant

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

a ^{2}

<s> Restaurant

a ^{1}

a Restaurant

a ^{1}

b Restaurant

c Restaurant

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

a 2

<s> Restaurant

a 1

a Restaurant

a 2

b Restaurant

c Restaurant

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

a [2]

<s> Restaurant

a [1]

a Restaurant

a [2] * [1]

b Restaurant

c Restaurant

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

a 2 * 1

<s> Restaurant

a 1

a Restaurant

a 2 * 1

b Restaurant

c Restaurant

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

a [2] b [1]

<s> Restaurant

a [1]

a Restaurant

a [2] * [1]

b Restaurant

c Restaurant

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

a^2 b^1

<s> Restaurant

a^1

a Restaurant

a^2 b^1

b Restaurant

c Restaurant

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

$\boxed{a}^{2}$ $\boxed{b}^{1}$

<s> Restaurant

$\boxed{a}^{1}$

a Restaurant

$\boxed{a}^{2}$ $\boxed{b}^{1}$

b Restaurant

c Restaurant

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

a[2] b[1]

<s> Restaurant

a[1]

b Restaurant

*[1]

a Restaurant

a[2] b[1]

c Restaurant

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

a^{2} b^{1}

<s> Restaurant

a^{1}

a Restaurant

a^{2} b^{1}

b Restaurant

*^{1}

c Restaurant

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

$\boxed{a}^{3}$ $\boxed{b}^{1}$

<s> Restaurant

$\boxed{a}^{1}$

a Restaurant

$\boxed{a}^{2}$ $\boxed{b}^{1}$

b Restaurant

$\boxed{a}^{1}$

c Restaurant

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

a 3 b 1

<s> Restaurant

a 1

a Restaurant

a 2 b 1

b Restaurant

a 1

c Restaurant

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

a ^{3} b ^{1}

<s> Restaurant

a ^{1}

b Restaurant

a ^{1}

a Restaurant

a ^{2} b ^{1} * ^{1}

c Restaurant

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

a 3 b 1 * 1

<s> Restaurant

a 1

a Restaurant

a 2 b 1 * 1

b Restaurant

a 1

c Restaurant

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

a 3 | b 1 | c 1

<s> Restaurant

a 1

b Restaurant

a 1

a Restaurant

a 2 | b 1 | c 1

c Restaurant

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

a^{3} b^{1} c^{1}

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Seating Assignments

Dataset:

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Unigram Restaurant

a 3 | b 1 | c 1

<s> Restaurant

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a Restaurant

a 2 | b 1 | c 1

b Restaurant

a 1

c Restaurant

* 1

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

a^3 b^1 c^1 *^{1}

<s> Restaurant

a^1

a Restaurant

a^2 b^1 c^1

b Restaurant

a^1

c Restaurant

*^{1}

Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

a [3] b [1] c [1] </s> [1]

<s> Restaurant

a [1]

a Restaurant

a [2] b [1] c [1]

b Restaurant

a [1]

c Restaurant

</s> [1]

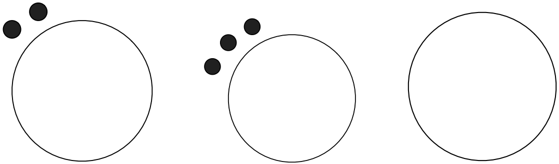
Outline

How does a CRP encode a probability distribution?

How do many CRPs encode backoff?

Language Model Probabilities

The rich get richer



$\frac{2}{5+\theta}$ $\frac{3}{5+\theta}$ $\frac{\theta}{5+\theta}$

Computing the Probability of an Observation

$$p(w = x|\vec{s}, \theta, u) = \underbrace{\frac{c_{u,x}}{\theta + c_{u,\cdot}}}_{\text{existing table}} + \underbrace{\frac{\theta}{\theta + c_{u,\cdot}} p(w = x|\vec{s}, \theta, \pi(u))}_{\text{new table}} \tag{1}$$

- Word type x
- Seating assignments $\vec{s}$
- Concentration θ
- Context u
- Number seated at table serving x in restaurant u, $c_{u,x}$
- Number seated at all tables in restaurant u, $c_{u,\cdot}$
- The backoff context $\pi(u)$

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- Number seated at all tables in restaurant u, $c_{u,\cdot}$
- The backoff context $\pi(u)$

Example: $p(w = \mathtt{b}|\vec{s}, \theta = 1.0, u = \mathtt{a})$

Unigram Restaurant

a^{3} b^{1} c^{1} </s>^{1}

<s> Restaurant

a^{1}

a Restaurant

a^{2} b^{1} c^{1}

b Restaurant

a^{1}

c Restaurant

</s>^{1}

$$p(w = \mathtt{b}| \dots) = \frac{c_{\mathtt{a},\mathtt{b}}}{\theta + c_{u,\cdot}} + \frac{\theta}{\theta + c_{u,\cdot}} p(w = x|\vec{s}, \theta, \pi(u)) \tag{2}$$

Example: $p(w = \mathtt{b} | \vec{s}, \theta = 1.0, u = \mathtt{a})$

Unigram Restaurant

a [3] b [1] c [1] </s> [1]

<s> Restaurant

a [1]

a Restaurant

a [2] b [1] c [1]

b Restaurant

a [1]

c Restaurant

</s> [1]

$$p(w = \mathtt{b} | \dots) = \frac{c_{\mathtt{a},\mathtt{b}}}{\theta + c_{u,\cdot}} + \frac{\theta}{\theta + c_{u,\cdot}} p(w = x | \vec{s}, \theta, \pi(u)) \tag{2}$$

Example: $p(w = \texttt{b} | \vec{s}, \theta = 1.0, u = \texttt{a})$

Unigram Restaurant

a[3] b[1] c[1] </s>[1]

<s> Restaurant

a[1]

a Restaurant

a[2] b[1] c[1]

b Restaurant

a[1]

c Restaurant

</s>[1]

$$p(w = \texttt{b} | \dots) = \frac{1}{\theta + c_{u,\cdot}} + \frac{\theta}{\theta + c_{u,\cdot}} p(w = x | \vec{s}, \theta, \pi(u)) \tag{2}$$

Example: $p(w = \mathtt{b} | \vec{s}, \theta = 1.0, u = \mathtt{a})$

Unigram Restaurant

$\boxed{\mathtt{a}}^{3}$ $\boxed{\mathtt{b}}^{1}$ $\boxed{\mathtt{c}}^{1}$ $\boxed{\mathtt{</s>}}^{1}$

<s> Restaurant

$\boxed{\mathtt{a}}^{1}$

a Restaurant

$\boxed{\mathtt{a}}^{2}$ $\boxed{\mathtt{b}}^{1}$ $\boxed{\mathtt{c}}^{1}$

b Restaurant

$\boxed{\mathtt{a}}^{1}$

c Restaurant

$\boxed{\mathtt{</s>}}^{1}$

$$p(w = \mathtt{b} | \dots) = \frac{1}{1.0 + c_{u,\cdot}} + \frac{1.0}{1.0 + c_{u,\cdot}} p(w = x | \vec{s}, \theta, \pi(u)) \tag{2}$$

Example: $p(w = \mathtt{b} | \vec{s}, \theta = 1.0, u = \mathtt{a})$

Unigram Restaurant

a[3] b[1] c[1] </s>[1]

<s> Restaurant

a[1]

a Restaurant

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b Restaurant

a[1]

c Restaurant

</s>[1]

$$p(w = \mathtt{b} | \dots) = \frac{1}{1.0 + 4} + \frac{1.0}{1.0 + 4} p(w = x | \vec{s}, \theta, \pi(u)) \qquad (2)$$

Example: $p(w = \mathtt{b}|\vec{s}, \theta = 1.0, u = \mathtt{a})$

Unigram Restaurant

a[3] b[1] c[1] </s>[1]

<s> Restaurant

a[1]

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</s>[1]

$$p(w = \mathtt{b}| \dots) = \frac{1}{1.0 + 4} + \frac{1.0}{1.0 + 4} p(w = x|\vec{s}, \theta, \pi(u)) \quad (2)$$

Example: $p(w = \texttt{b} | \vec{s}, \theta = 1.0, u = \texttt{a})$

Unigram Restaurant

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<s> Restaurant

$\boxed{\texttt{a}}^{1}$

a Restaurant

$\boxed{\texttt{a}}^{2}\ \boxed{\texttt{b}}^{1}\ \boxed{\texttt{c}}^{1}$

b Restaurant

$\boxed{\texttt{a}}^{1}$

c Restaurant

$\boxed{\texttt{</s>}}^{1}$

$$p(w = \texttt{b} | \dots) = \frac{1}{1.0 + 4} + \frac{1.0}{1.0 + 4} p(w = x | \vec{s}, \theta, \pi(\emptyset)) \tag{2}$$

Example: $p(w = \texttt{b} | \vec{s}, \theta = 1.0, u = \texttt{a})$

Unigram Restaurant

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a^{1}

c Restaurant

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$$p(w = \texttt{b} | \dots) = \frac{1}{1.0 + 4} + \frac{1.0}{1.0 + 4} p(w = x | \vec{s}, \theta, \pi(\emptyset)) \quad (2)$$

Example: $p(w = \mathtt{b} | \vec{s}, \theta = 1.0, u = \mathtt{a})$

Unigram Restaurant

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<s> Restaurant

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b Restaurant

a[1]

c Restaurant

</s>[1]

$$p(w = \mathtt{b} | \dots) = \frac{1}{5} + \frac{1}{5}\left(\frac{c_{\emptyset,\mathtt{b}}}{c_{\emptyset,\cdot} + \theta} + \frac{\theta}{c_{\emptyset,\cdot} + \theta} \frac{1}{V}\right) \tag{2}$$

Example: $p(w = \texttt{b} | \vec{s}, \theta = 1.0, u = \texttt{a})$

Unigram Restaurant

$\texttt{a}^{3}$ $\texttt{b}^{1}$ $\texttt{c}^{1}$ $\texttt{</s>}^{1}$

<s> Restaurant

$\texttt{a}^{1}$

a Restaurant

$\texttt{a}^{2}$ $\texttt{b}^{1}$ $\texttt{c}^{1}$

b Restaurant

$\texttt{a}^{1}$

c Restaurant

$\texttt{</s>}^{1}$

$$p(w = \texttt{b} | \dots) = \frac{1}{5} + \frac{1}{5}\left(\frac{c_{\emptyset,\texttt{b}}}{c_{\emptyset,\cdot} + \theta} + \frac{\theta}{c_{\emptyset,\cdot} + \theta}\frac{1}{5}\right) \tag{2}$$

Example: $p(w = \texttt{b} | \vec{s}, \theta = 1.0, u = \texttt{a})$

Unigram Restaurant

$\texttt{a}^{3}$ $\texttt{b}^{1}$ $\texttt{c}^{1}$ $\texttt{</s>}^{1}$

<s> Restaurant

$\texttt{a}^{1}$

a Restaurant

$\texttt{a}^{2}$ $\texttt{b}^{1}$ $\texttt{c}^{1}$

b Restaurant

$\texttt{a}^{1}$

c Restaurant

$\texttt{</s>}^{1}$

$$p(w = \texttt{b} | \dots) = \frac{1}{5} + \frac{1}{5} \left(\frac{c_{\emptyset,\texttt{b}}}{c_{\emptyset,\cdot} + 1.0} + \frac{1.0}{c_{\emptyset,\cdot} + 1.0} \frac{1}{5} \right) \tag{2}$$

Example: $p(w = \mathtt{b} | \vec{s}, \theta = 1.0, u = \mathtt{a})$

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$\boxed{\mathtt{a}}^{3}$ $\boxed{\mathtt{b}}^{1}$ $\boxed{\mathtt{c}}^{1}$ $\boxed{\mathtt{</s>}}^{1}$

<s> Restaurant

$\boxed{\mathtt{a}}^{1}$

a Restaurant

$\boxed{\mathtt{a}}^{2}$ $\boxed{\mathtt{b}}^{1}$ $\boxed{\mathtt{c}}^{1}$

b Restaurant

$\boxed{\mathtt{a}}^{1}$

c Restaurant

$\boxed{\mathtt{</s>}}^{1}$

$$p(w = \mathtt{b} | \dots) = \frac{1}{5} + \frac{1}{5}\left(\frac{1}{c_{\emptyset,\cdot} + 1.0} + \frac{1.0}{c_{\emptyset,\cdot} + 1.0}\frac{1}{5}\right) \tag{2}$$

Example: $p(w = \mathtt{b} | \vec{s}, \theta = 1.0, u = \mathtt{a})$

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a[3] b[1] c[1] </s>[1]

<s> Restaurant

a[1]

a Restaurant

a[2] b[1] c[1]

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a[1]

c Restaurant

</s>[1]

$$p(w = \mathtt{b} | \ldots) = \frac{1}{5} + \frac{1}{5}\left(\frac{1}{6+1.0} + \frac{1.0}{6+1.0}\frac{1}{5}\right) \tag{2}$$

Example: $p(w = \texttt{b} | \vec{s}, \theta = 1.0, u = \texttt{a})$

Unigram Restaurant

a [3] b [1] c [1] </s> [1]

<s> Restaurant

a [1]

a Restaurant

a [2] b [1] c [1]

b Restaurant

a [1]

c Restaurant

</s> [1]

$$p(w = \texttt{b} | \dots) = \frac{1}{5} + \frac{1}{5}\left(\frac{1}{7} + \frac{1}{7}\frac{1}{5}\right) = 0.24 \quad (2)$$

Discounting

- Empirically, it helps favor the backoff if you have more tables
- Otherwise, it gets too close to maximum likelihood
- Idea is called *discounting*
- Steal a little bit of probability mass δ from every table and give it to the new table (backoff)

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Interpolated Kneser-Ney!

More advanced models

- Interpolated Kneser-Ney assumes **one table with a dish (word)** per restaurant
- Can get slightly better performance by assuming you can have duplicated tables: **Pitman-Yor** language model
- Requires Gibbs Sampling of the seating assignments (GS, later, but not for language models)

Exercise

- Start with restaurant we had before
- Assume you see `<s> b b a c </s>`; add those counts to tables
- Compute probability of `b` following `a` $(\theta = 1.0, \delta = 0.5)$
- Compute the probability of `a` following `b`
- Compute probability of `</s>` following `<s>`