# Online Learning

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LECTURE 13

## Content Questions

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## Administrivia Questions

- Grading Feature Engineering HW
- Learnability HW Due Friday
- No class Monday

## Perceptron Algorithm

$\vec{w}_1 \leftarrow \vec{0}$;
**for** $t \leftarrow 1 \dots T$ **do**
    Receive $x_t$;
    $\hat{y}_t \leftarrow \text{sgn}(\vec{w}_t \cdot \vec{x}_t)$;
    Receive $y_t$;
    **if** $\hat{y}_t \neq y_t$ **then**
        $\vec{w}_{t+1} \leftarrow \vec{w}_t + y_t \vec{x}_t$;
    **else**
        $\vec{w}_{t+1} \leftarrow w_t$;
**return** $w_{T+1}$

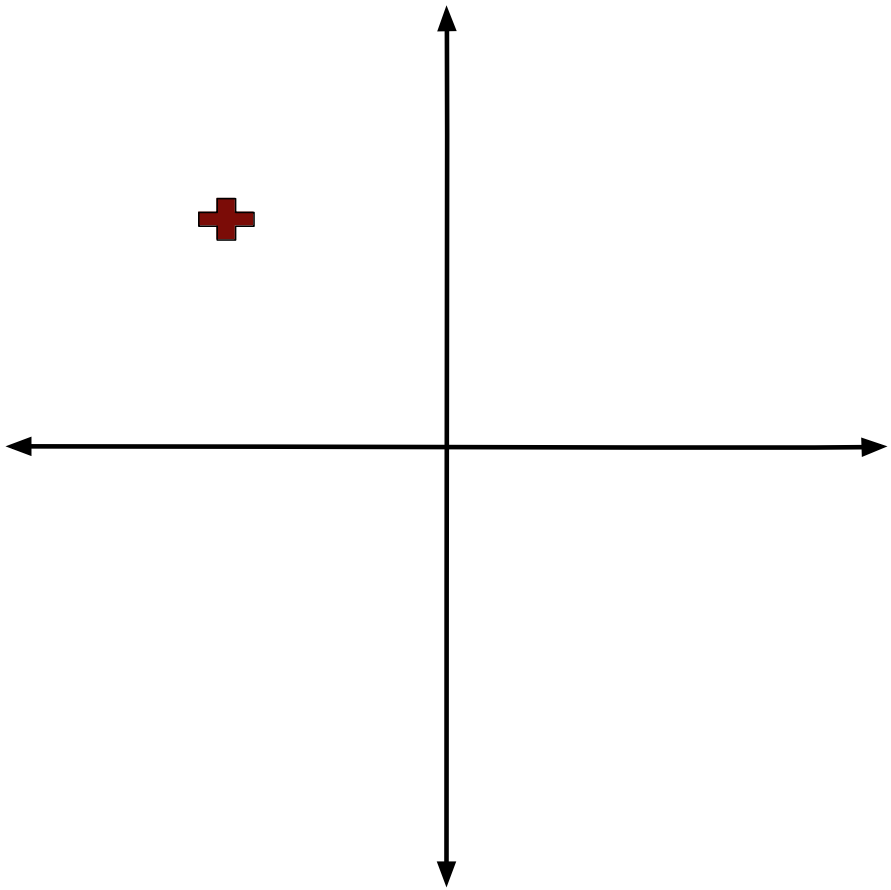
**Algorithm 1:** Perceptron Algorithm (Rosenblatt, 1958)

## 2D Example

Initially, weight vector is zero:

$$\vec{w}_1 = \langle 0, 0 \rangle \tag{1}$$

## Observation 1



$$x_1 = \langle -2, 2 \rangle \tag{2}$$

$$\hat{y}_1 = 0 \tag{3}$$

$$y_1 = +1 \tag{4}$$

## Update 1

$$\vec{w}_{t+1} \leftarrow \vec{w}_t + y_t \vec{x}_t \tag{5}$$

$$\vec{w}_2 \leftarrow \tag{6}$$

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$$\vec{w}_{t+1} \leftarrow \vec{w}_t + y_t \vec{x}_t \tag{5}$$

$$\vec{w}_2 \leftarrow \langle 0, 0 \rangle + \langle -2, 2 \rangle \tag{6}$$

$$\tag{7}$$

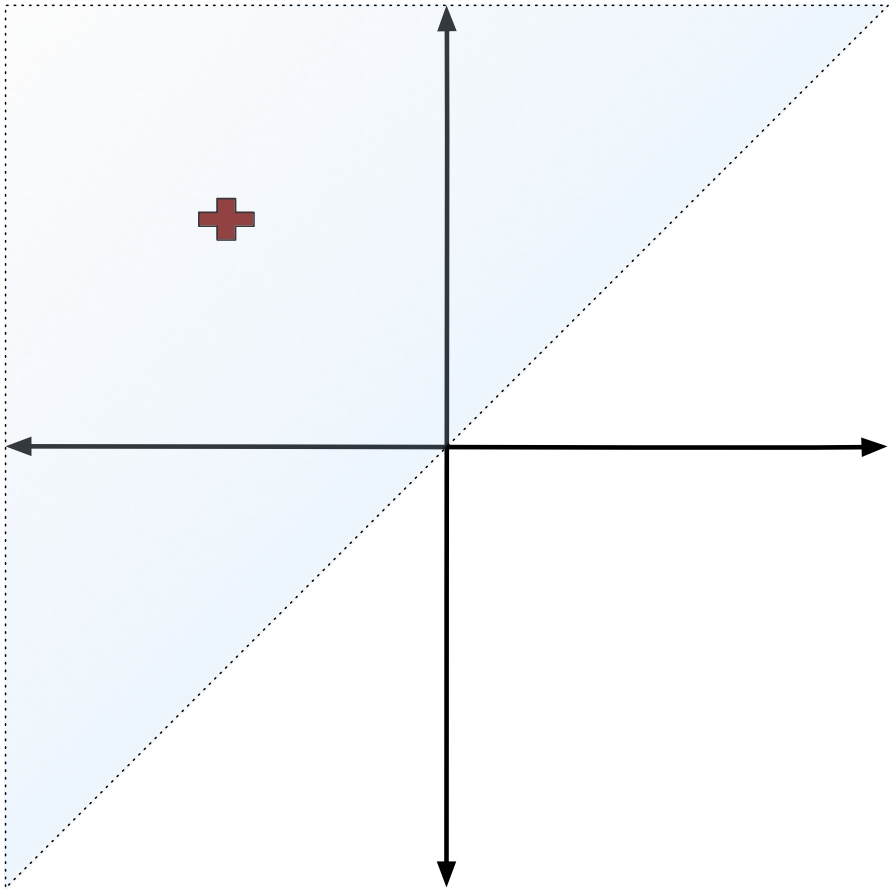
## Update 1

$$\vec{w}_{t+1} \leftarrow \vec{w}_t + y_t \vec{x}_t \tag{5}$$

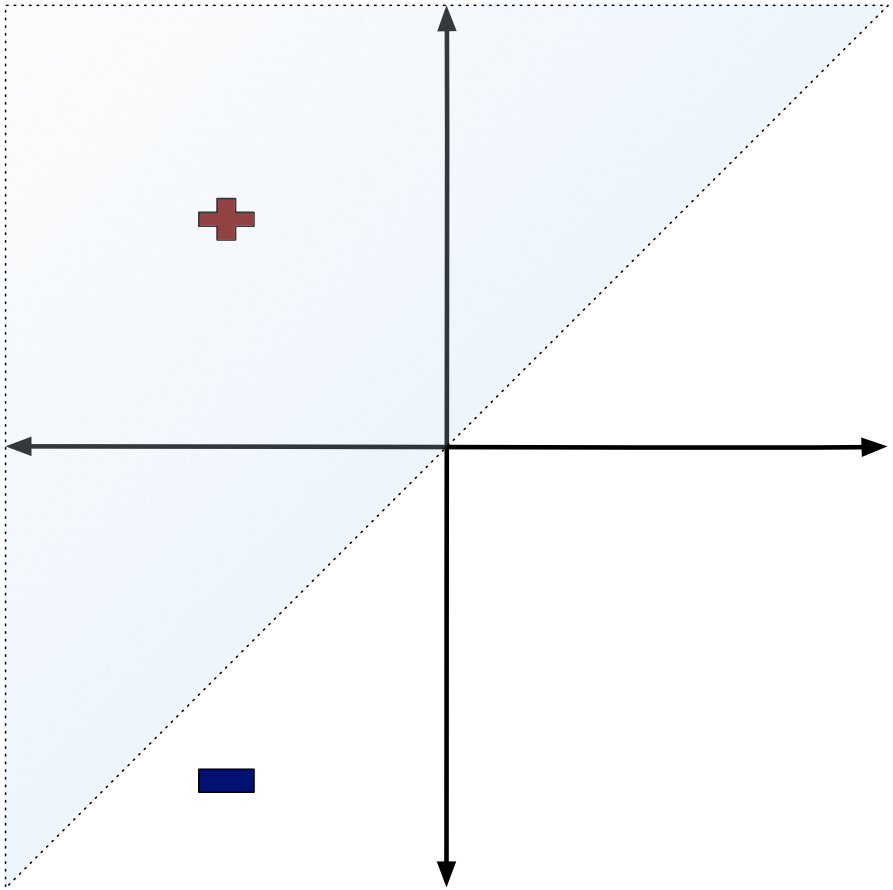
$$\vec{w}_2 \leftarrow \langle 0, 0 \rangle + \langle -2, 2 \rangle \tag{6}$$

$$\vec{w}_2 = \langle -2, 2 \rangle \tag{7}$$

# Observation 2



## Observation 2



$$x_2 = \langle -2, -3 \rangle \tag{8}$$

$$\hat{y}_2 = +4 + -6 = -2 \tag{9}$$

$$y_2 = -1 \tag{10}$$

## Update 2

$$\vec{w}_{t+1} \leftarrow \vec{w}_t \tag{11}$$

$$\vec{w}_2 \leftarrow \tag{12}$$

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$$\vec{w}_{t+1} \leftarrow \vec{w}_t \tag{11}$$

$$\vec{w}_2 \leftarrow \langle -2, 2 \rangle \tag{12}$$

$$\tag{13}$$

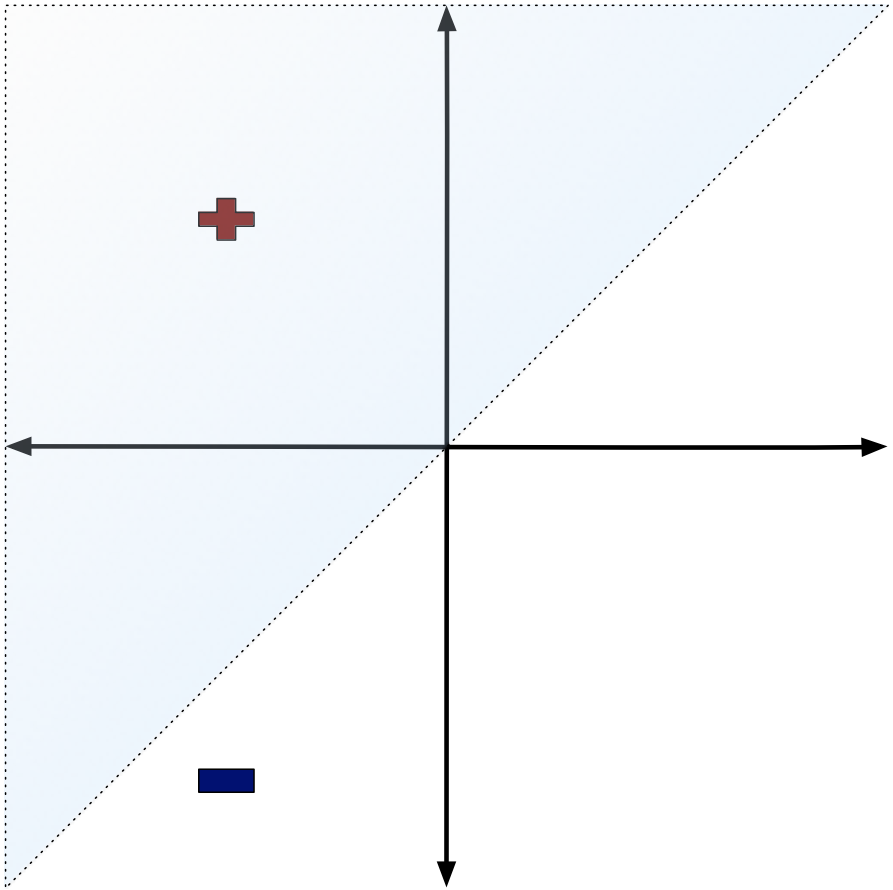
## Update 2

$$\vec{w}_{t+1} \leftarrow \vec{w}_t \tag{11}$$

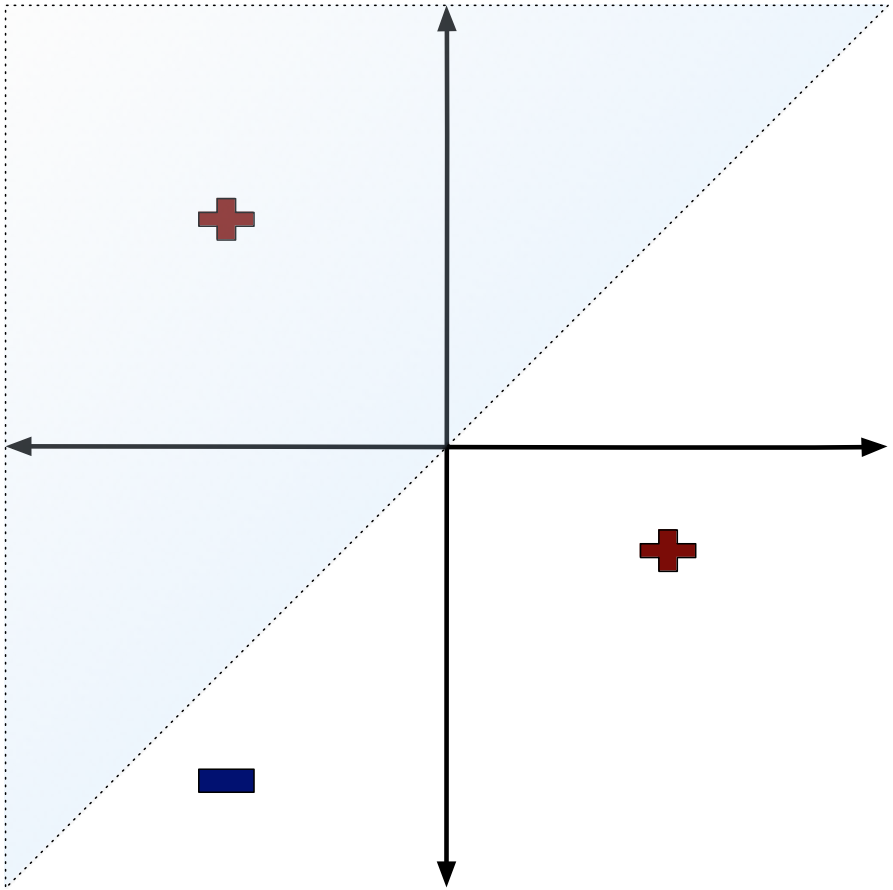
$$\vec{w}_2 \leftarrow \langle -2, 2 \rangle \tag{12}$$

$$\vec{w}_2 = \langle -2, 2 \rangle \tag{13}$$

# Observation 3



## Observation 3



$$x_3 = \langle 2, -1 \rangle \tag{14}$$

$$\hat{y}_3 = -4 + -2 = -6 \tag{15}$$

$$y_3 = +1 \tag{16}$$

## Update 3

$$\vec{w}_{t+1} \leftarrow \vec{w}_t + y_t \vec{x}_t \tag{17}$$

$$\vec{w}_3 \leftarrow \tag{18}$$

## Update 3

$$\vec{w}_{t+1} \leftarrow \vec{w}_t + y_t \vec{x}_t \tag{17}$$

$$\vec{w}_3 \leftarrow \langle -2, 2 \rangle + \langle 2, -1 \rangle \tag{18}$$

$$\tag{19}$$

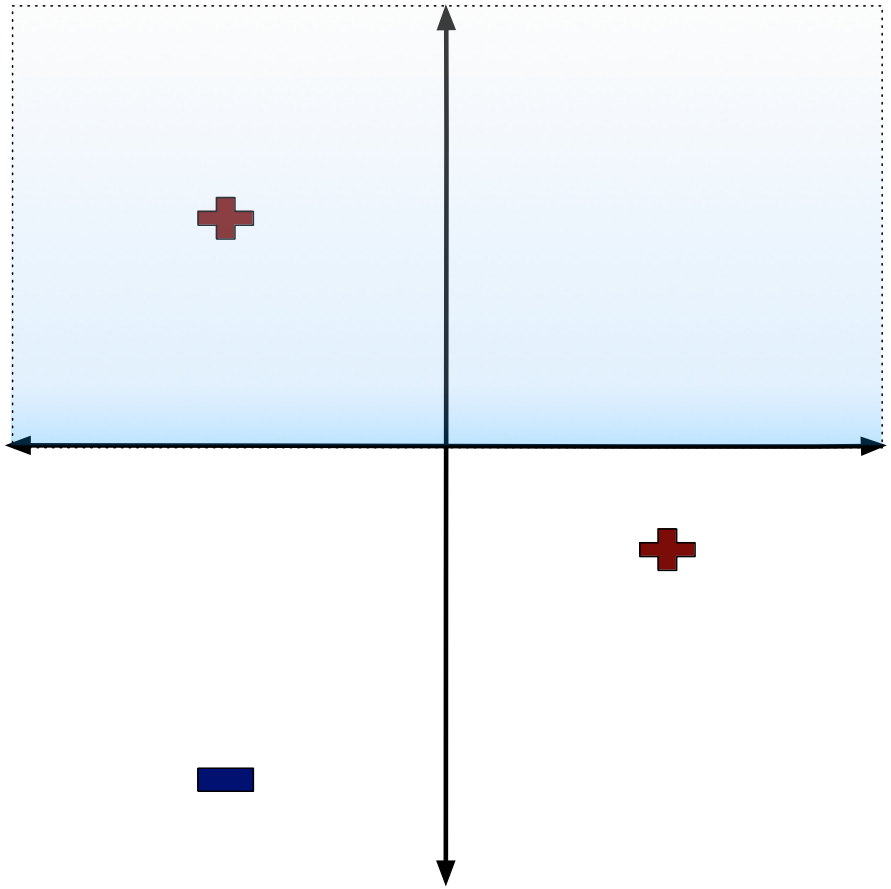
## Update 3

$$\vec{w}_{t+1} \leftarrow \vec{w}_t + y_t \vec{x}_t \tag{17}$$

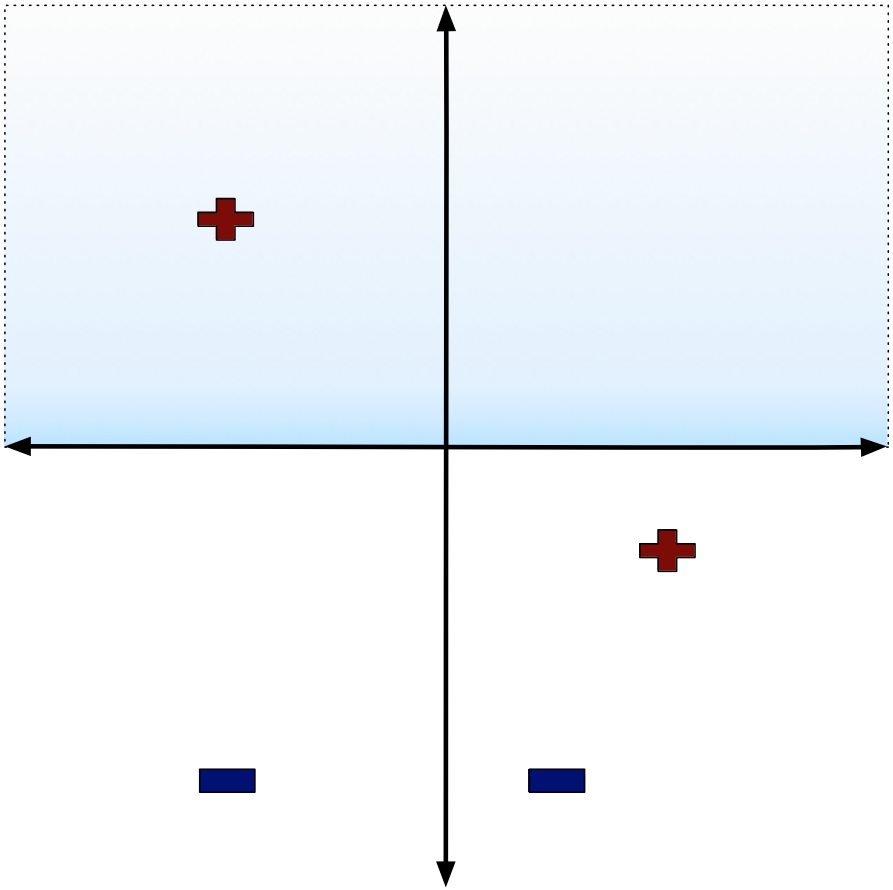
$$\vec{w}_3 \leftarrow \langle -2, 2 \rangle + \langle 2, -1 \rangle \tag{18}$$

$$\vec{w}_3 = \langle 0, 1 \rangle \tag{19}$$

# Observation 4



## Observation 4



$$x_4 = \langle 1, -4 \rangle \tag{20}$$

$$\hat{y}_4 = -4 \tag{21}$$

$$y_4 = -1 \tag{22}$$

## Update 4

$$\vec{w}_4 \leftarrow \tag{23}$$

## Update 4

$$\vec{w}_4 \leftarrow \vec{w}_3 \tag{23}$$

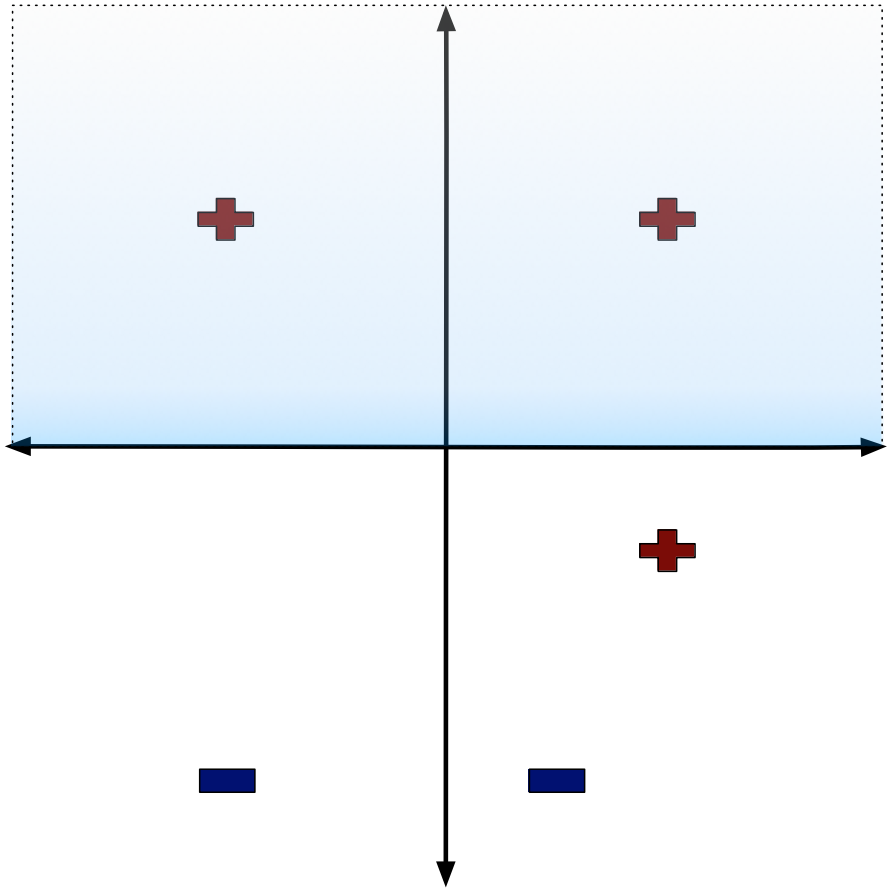
$$\tag{24}$$

## Update 4

$$\vec{w}_4 \leftarrow \vec{w}_3 \tag{23}$$

$$\vec{w}_4 = \langle 0, 1 \rangle \tag{24}$$

## Observation 5



$$x_5 = \langle 2, 2 \rangle \tag{25}$$

$$\hat{y}_5 = 2 \tag{26}$$

$$y_5 = +1 \tag{27}$$

## Update 5

$$\vec{w}_5 \leftarrow \tag{28}$$

## Update 5

$$\vec{w}_5 \leftarrow \vec{w}_4 \tag{28}$$

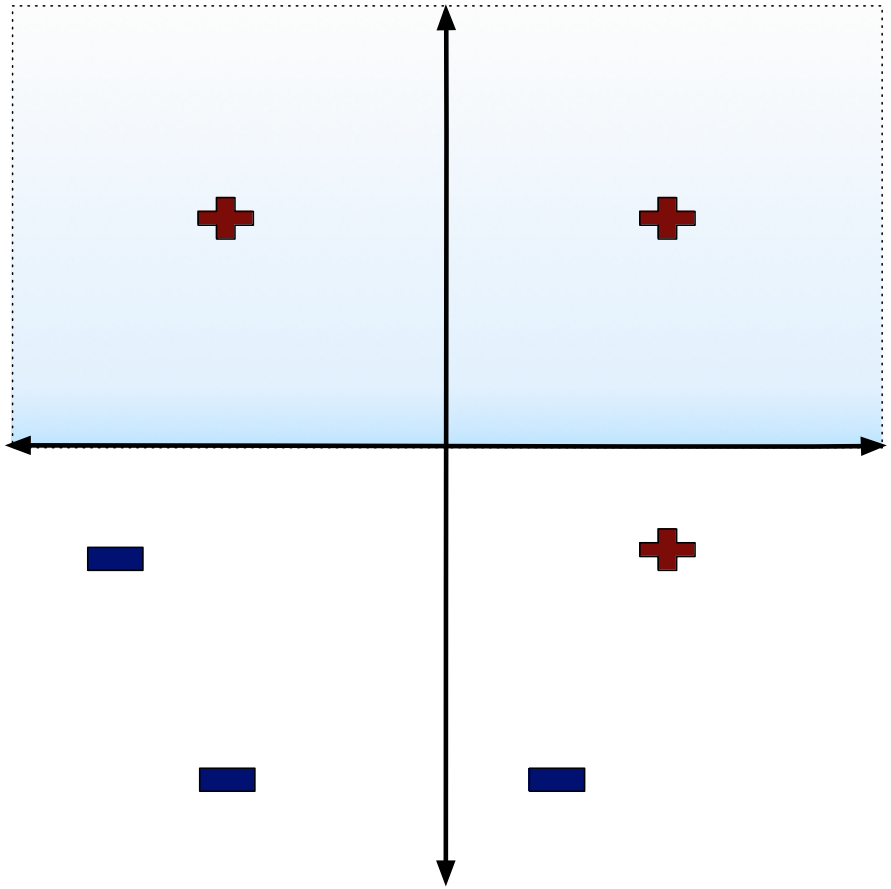
$$\tag{29}$$

## Update 5

$$\vec{w}_5 \leftarrow \vec{w}_4 \tag{28}$$

$$\vec{w}_5 = \langle 0, 1 \rangle \tag{29}$$

## Observation 6



$$x_6 = \langle 2, 2 \rangle \tag{30}$$

$$\hat{y}_6 = 2 \tag{31}$$

$$y_6 = +1 \tag{32}$$

## Update 6

$$\vec{w}_6 \leftarrow \tag{33}$$

## Update 6

$$\vec{w}_6 \leftarrow \vec{w}_5 \tag{33}$$

(34)

## Update 6

$$\vec{w}_6 \leftarrow \vec{w}_5 \tag{33}$$

$$\vec{w}_6 = \langle 0, 1 \rangle \tag{34}$$

## Beyond Binary Classification

- Multiclass
- Ranking
- Regression