Mathematical Foundations

Introduction to Data Science Algorithms

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SLIDES ADAPTED FROM DAVE BLEI AND LAUREN HANNAH

Expectation

An *expectation* of a random variable is a weighted average:

$$\mathrm{E}[f(X)] = \sum_x f(x)\,p(x) \qquad \text{(discrete)}$$

$$= \int_{-\infty}^{\infty} f(x)\,p(x)\,dx \qquad \text{(continuous)}$$

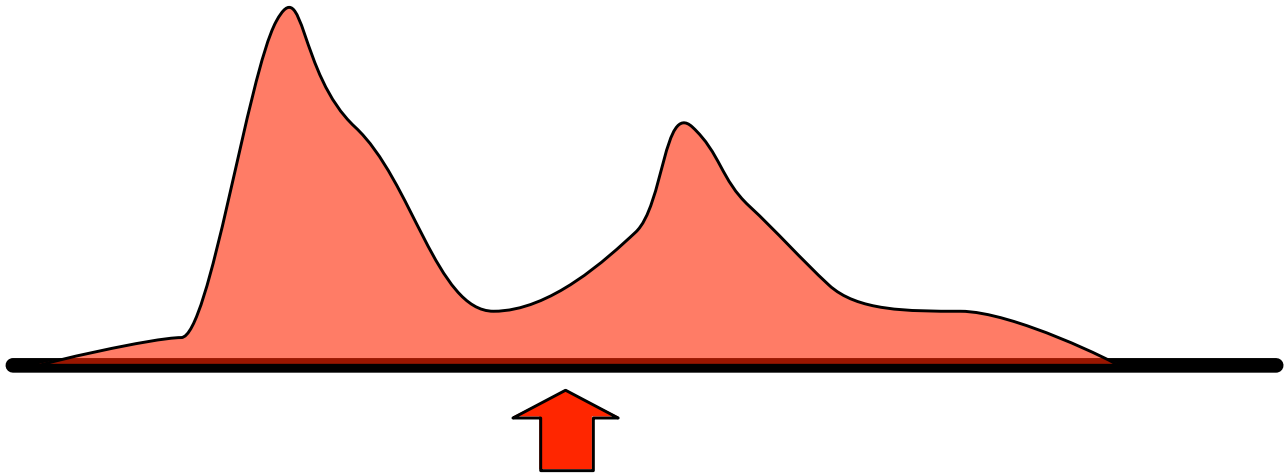
Expectation

Expectations of constants or known values:

- $\mathrm{E}[a] = a$
- $\mathrm{E}[Y \mid Y = y] = y$

Expectation Intuition

- Average outcome (might not be an event: 2.4 children)
- Center of mass



Expectation of die / dice

What is the expectation of the roll of die?

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One die

$$1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6} =$$

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What is the expectation of the sum of two dice?

Two die

$$2 \cdot \frac{1}{36} + 3 \cdot \frac{2}{36} + 4 \cdot \frac{3}{36} + 5 \cdot \frac{4}{36} + 6 \cdot \frac{5}{36} + 7 \cdot \frac{6}{36} + 8 \cdot \frac{5}{36} + 9 \cdot \frac{4}{36} + 10 \cdot \frac{3}{36} + 11 \cdot \frac{2}{36} + 12 \cdot \frac{1}{36} =$$

Expectation of die / dice

What is the expectation of the roll of die?

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What is the expectation of the sum of two dice?

Two die

$$2 \cdot \frac{1}{36} + 3 \cdot \frac{2}{36} + 4 \cdot \frac{3}{36} + 5 \cdot \frac{4}{36} + 6 \cdot \frac{5}{36} + 7 \cdot \frac{6}{36} + 8 \cdot \frac{5}{36} + 9 \cdot \frac{4}{36} + 10 \cdot \frac{3}{36} + 11 \cdot \frac{2}{36} + 12 \cdot \frac{1}{36} = 7$$