



# Probability Distributions: Continuous

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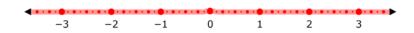
#### Administrivia

- HW1 Grading Underway
- Shockingly similar submissions
  - As an exercise, at some point we'll compute the probability
- All code must come from your fingers
- HW2 released
- Python review



#### Continuous random variables

- Today we will look at continuous random variables:
  - Real numbers:  $\mathbb{R}$ ;  $(-\infty, \infty)$
  - Positive real numbers:  $\mathbb{R}^+$ ;  $(0, \infty)$
  - Real numbers between -1 and 1 (inclusive): [-1,1]
- The sample space of continuous random variables is uncountably infinite.



#### Continuous distributions

- Last time: a discrete distribution assigns a probability to every possible outcome in the sample space
- How do we define a continuous distribution?
- Suppose our sample space is all real numbers, R.
  - What is the probability of P(X = 20.1626338)?
  - What is the probability of P(X = -1.5)?

### Continuous distributions

- Last time: a discrete distribution assigns a probability to every possible outcome in the sample space
- How do we define a continuous distribution?
- Suppose our sample space is all real numbers, ℝ.
  - What is the probability of P(X = 20.1626338)?
  - What is the probability of P(X = -1.5)?
- The probability of any continuous event is always 0.
  - Huh?
  - There are infinitely many possible values a continuous variable could take. There is zero chance of picking any one exact value.
  - We need a slightly different definition of probability for continuous variables.

## **Probability density**

- A probability density function (PDF, or simply density) is the continuous version of probability mass functions for discrete distributions.
- The density at a point x is denoted f(x).
- · Density behaves like probability:
  - $f(x) \ge 0$ , for all x
  - $\circ \int_X f(x) = 1$
- Even though P(X = 1.5) = 0, density allows us to ask other questions:
  - Intervals: P(1.4999 < X < 1.5001)
  - Relative likelihood: is 1.5 more likely than 0.8?

- While the probability for a specific value is 0 under a continuous distribution, we can still measure the probability that a value falls within an interval.
  - $P(X \ge a) = \int_{x=a}^{\infty} f(x)$
  - $P(X \le a) = \int_{x=-\infty}^{a} f(x)$
  - $P(a \le X \le b) = \int_{x=a}^{b} f(x)$
- This is analogous to the disjunction rule for discrete distributions.
  - For example if X is a die roll, then P(Y > 3) = P(Y = 3) + P(Y = 4) + P(Y = 5) + P(Y = 6)
    - $P(X \ge 3) = P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6)$
  - An integral is similar to a sum

#### Likelihood

- The likelihood function refers to the PMF (discrete) or PDF (continuous).
- For discrete distributions, the likelihood of x is P(X = x).
- For continuous distributions, the likelihood of x is the density f(x).
- We will often refer to likelihood rather than probability/mass/density so that the term applies to either scenario.