



Department of Computer Science  
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# Mathematical Foundations

## Introduction to Data Science Algorithms

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SLIDES ADAPTED FROM DAVE BLEI AND LAUREN HANNAH

## Expectation

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An *expectation* of a random variable is a weighted average:

$$\begin{aligned} \mathbb{E}[f(X)] &= \sum_{x=1}^{\infty} f(x) p(x) && \text{(discrete)} \\ &= \int_{-\infty}^{\infty} f(x) p(x) dx && \text{(continuous)} \end{aligned}$$

## Expectation

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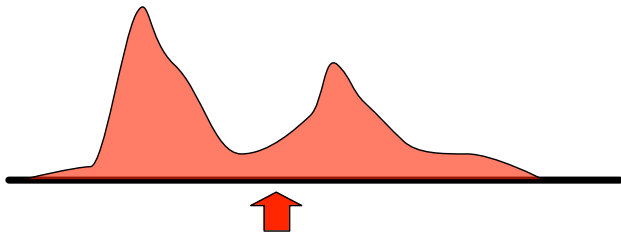
Expectations of constants or known values:

- $E[a] = a$
- $E[Y | Y = y] = y$

## Expectation Intuition

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- Average or outcome (might not be an event: 2.4 children)
- Center of mass



- “Fair Price” of a wager

## Expectation of die / dice

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What is the expectation of the roll of die?

## Expectation of die / dice

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What is the expectation of the roll of die?

**One die**

$$1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6} =$$

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What is the expectation of the sum of two dice?



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What is the expectation of the sum of two dice?

### Two die

$$2 \cdot \frac{1}{36} + 3 \cdot \frac{2}{36} + 4 \cdot \frac{3}{36} + 5 \cdot \frac{4}{36} + 6 \cdot \frac{5}{36} + 7 \cdot \frac{6}{36} + 8 \cdot \frac{5}{36} + 9 \cdot \frac{4}{36} + 10 \cdot \frac{3}{36} + 11 \cdot \frac{2}{36} + 12 \cdot \frac{1}{36} =$$

## Expectation of die / dice

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What is the expectation of the roll of die?

### One die

$$1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6} = 3.5$$

What is the expectation of the sum of two dice?

### Two die

$$2 \cdot \frac{1}{36} + 3 \cdot \frac{2}{36} + 4 \cdot \frac{3}{36} + 5 \cdot \frac{4}{36} + 6 \cdot \frac{5}{36} + 7 \cdot \frac{6}{36} + 8 \cdot \frac{5}{36} + 9 \cdot \frac{4}{36} + 10 \cdot \frac{3}{36} + 11 \cdot \frac{2}{36} + 12 \cdot \frac{1}{36} = 7$$