Probability Distributions: Continuous

Introduction to Data Science Algorithms

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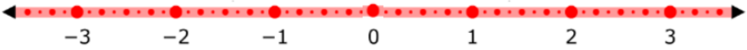
Continuous random variables

- Today we will look at *continuous* random variables:
  - Real numbers: $\mathbb{R}$; $(-\infty, \infty)$
  - Positive real numbers: $\mathbb{R}^+$; $(0, \infty)$
  - Real numbers between -1 and 1 (inclusive): $[-1, 1]$
- The *sample space* of continuous random variables is uncountably infinite.



Continuous distributions

- Last time: a discrete distribution assigns a probability to every possible outcome in the sample space
- How do we define a continuous distribution?
- Suppose our sample space is all real numbers, $\mathbb{R}$.
  - What is the probability of $P(X = 20.1626338)$?
  - What is the probability of $P(X = -1.5)$?

Continuous distributions

- Last time: a discrete distribution assigns a probability to every possible outcome in the sample space
- How do we define a continuous distribution?
- Suppose our sample space is all real numbers, $\mathbb{R}$.
  - What is the probability of $P(X = 20.1626338)$?
  - What is the probability of $P(X = -1.5)$?
- **The probability of any continuous event is always 0.**
  - Huh?
  - There are infinitely many possible values a continuous variable could take. There is zero chance of picking any one exact value.
  - We need a slightly different definition of probability for continuous variables.

Probability density

- A *probability density function* (PDF, or simply *density*) is the continuous version of probability mass functions for discrete distributions.
- The density at a point x is denoted $f(x)$.
- Density behaves like probability:
  - $f(x) \geq 0$, for all x
  - $\int_x f(x) = 1$
- Even though $P(X = 1.5) = 0$, density allows us to ask other questions:
  - Intervals: $P(1.4999 < X < 1.5001)$
  - Relative likelihood: is 1.5 more likely than 0.8?

Probability of intervals

- While the probability for a specific value is 0 under a continuous distribution, we can still measure the probability that a value falls within an interval.
  - $P(X \geq a) = \int_{x=a}^{\infty} f(x)$
  - $P(X \leq a) = \int_{x=-\infty}^{a} f(x)$
  - $P(a \leq X \leq b) = \int_{x=a}^{b} f(x)$
- This is analogous to the disjunction rule for discrete distributions.
  - For example if X is a die roll, then $P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6)$
  - An integral is similar to a sum

Likelihood

- The *likelihood function* refers to the PMF (discrete) or PDF (continuous).
- For discrete distributions, the likelihood of x is $P(X = x)$.
- For continuous distributions, the likelihood of x is the density $f(x)$.
- We will often refer to likelihood rather than probability/mass/density so that the term applies to either scenario.