#### Classification II: Decision Trees and SVMs

Digging into Data: Jordan Boyd-Graber

February 25, 2013



Slides adapted from Tom Mitchell, Eric Xing, and Lauren Hannah

### Roadmap

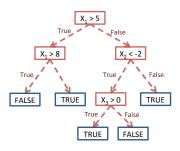
- Classification: machines labeling data for us
- Last time: naïve Bayes and logistic regression
- This time:
  - Decision Trees
    - ★ Simple, nonlinear, interpretable
  - SVMs
    - ★ (another) example of linear classifier
    - ★ State-of-the-art classification
  - Examples in Rattle (Logistic, SVM, Trees)
  - Discussion: Which classifier should I use for my problem?

### **Outline**

- Decision Trees
- Learning Decision Trees
- Overfitting
- 4 Vector space classification
- **5** Linear Classifiers
- Which hyperplane is best for me?
- Support Vector Machines
- Trying Out Classifiers in Rattle
- Recap

Suppose that we want to construct a set of rules to represent the data

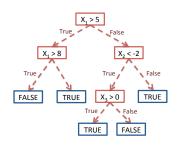
- can represent data as a series of if-then statements
- here, "if" splits inputs into two categories
- "then" assigns value
- when "if" statements are nested, structure is called a tree

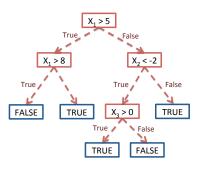


Ex: data  $(X_1, X_2, X_3, Y)$  with  $X_1, X_2, X_3$  are real, Y Boolean

First, see if  $X_1 > 5$ :

- if TRUE, see if  $X_1 > 8$ 
  - ▶ if TRUE, return FALSE
  - ▶ if FALSE, return TRUE
- if FALSE, see if  $X_2 < -2$ 
  - if TRUE, see if  $X_3 > 0$ 
    - ★ if TRUE, return TRUE
    - ★ if FALSE, return FALSE
  - ▶ if FALSE, return TRUE





Example 1: 
$$(X_1, X_2, X_3) = (1, 1, 1)$$

Example 2:  $(X_1, X_2, X_3) = (10, -3, 0)$ 

#### Terminology:

- branches: one side of a split
- leaves: terminal nodes that return values

#### Why trees?

- trees can be used for regression or classification
  - regression: returned value is a real number
  - classification: returned value is a class
- unlike linear regression, SVMs, naive Bayes, etc, trees fit local models
  - in large spaces, global models may be hard to fit
  - results may be hard to interpret
- fast, interpretable predictions

### **Example: Predicting Electoral Results**

#### 2008 Democratic primary:

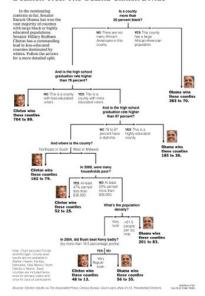
- Hillary Clinton
- Barack Obama

Given historical data, how will a count vote?

- can extrapolate to state level data
- might give regions to focus on increasing voter turnout
- would like to know how variables interact

### **Example: Predicting Electoral Results**

#### Decision Tree: The Obama-Clinton Divide



#### **Decision Trees**

#### Decision tree representation:

- Each internal node tests an attribute
- Each branch corresponds to attribute value
- Each leaf node assigns a classification

#### How would we represent:

- AND, OR, XOR
- $(A \text{ AND } B) \text{ OR } (C \text{ AND } \neg D \text{ AND } E)$
- M of N

#### When to Consider Decision Trees

- Instances describable by attribute-value pairs
- Target function is discrete valued
- Disjunctive hypothesis may be required
- Possibly noisy training data

#### Examples:

- Equipment or medical diagnosis
- Credit risk analysis
- Modeling calendar scheduling preferences

### **Outline**

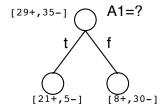
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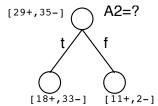
# **Top-Down Induction of Decision Trees**

#### Main loop:

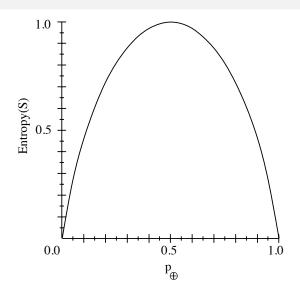
- A ← the "best" decision attribute for next node
- Assign A as decision attribute for node
- 3 For each value of A, create new descendant of node
- Sort training examples to leaf nodes
- If training examples perfectly classified, Then STOP, Else iterate over new leaf nodes

#### Which attribute is best?





# **Entropy**



S is a sample of training examples

### **Entropy**

Entropy(S) =expected number of bits needed to encode class ( $\oplus$  or  $\ominus$ ) of randomly drawn member of S (under the optimal, shortest-length code)

Why?

Information theory: optimal length code assigns  $-\log_2 p$  bits to message having probability p.

So, expected number of bits to encode  $\oplus$  or  $\ominus$  of random member of S:

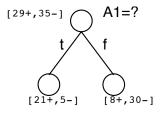
$$p_{\oplus}(-\log_2 p_{\oplus}) + p_{\ominus}(-\log_2 p_{\ominus})$$

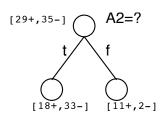
$$Entropy(S) \equiv -p_{\oplus} \log_2 p_{\oplus} - p_{\ominus} \log_2 p_{\ominus}$$

#### **Information Gain**

Gain(S, A) = expected reduction in entropy due to sorting on A

$$Gain(S, A) \equiv Entropy(S) - \sum_{v \in Values(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$



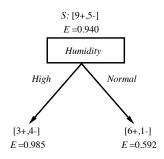


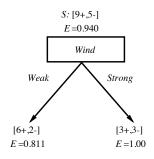
# **Training Examples**

Day	Outlook	Temperature	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

### **Selecting the Next Attribute**

#### Which attribute is the best classifier?





## **ID3 Algorithm**

- Start at root, look for best attribute
- Repeat for subtrees at each attribute outcome
- Stop when information gain is below a threshold
- Bias: prefers shorter trees (Occam's Razor)
  - → a short hyp that fits data unlikely to be coincidence
  - → a long hyp that fits data might be coincidence
  - Prevents overfitting

### **Outline**

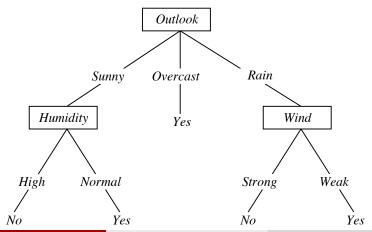
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## **Overfitting in Decision Trees**

Consider adding noisy training example #15:

Sunny, Hot, Normal, Strong, PlayTennis = No

What effect on earlier tree?



### **Overfitting**

#### Consider error of hypothesis h over

- training data: error<sub>train</sub>(h)
- entire distribution  $\mathcal{D}$  of data:  $error_{\mathcal{D}}(h)$

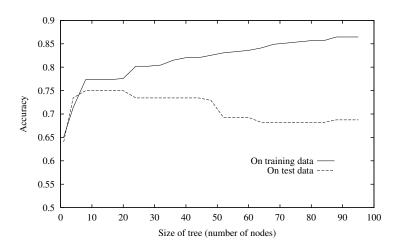
Hypothesis  $h \in H$  overfits training data if there is an alternative hypothesis  $h' \in H$  such that

$$error_{train}(h) < error_{train}(h')$$

and

$$error_{\mathscr{D}}(h) > error_{\mathscr{D}}(h')$$

# **Overfitting in Decision Tree Learning**



## **Avoiding Overfitting**

#### How can we avoid overfitting?

- stop growing when data split not statistically significant
- grow full tree, then post-prune

#### How to select "best" tree:

- Measure performance over training data
- Measure performance over separate validation data set
- MDL: minimize size(tree) + size(misclassifications(tree))

### **Outline**

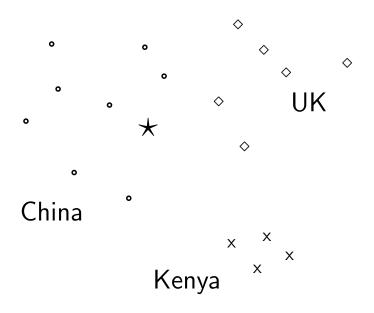
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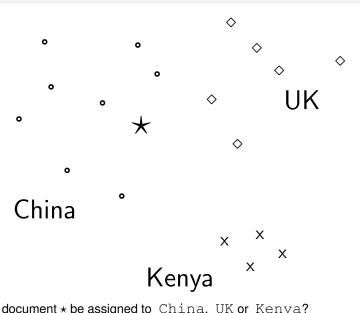
### **Recall vector space representation**

- Each document is a vector, one component for each term.
- Terms are axes.
- High dimensionality: 10,000s of dimensions and more
- Normalize vectors (documents) to unit length
- How can we do classification in this space?

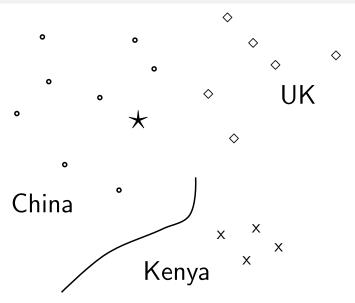
### **Vector space classification**

- As before, the training set is a set of documents, each labeled with its class.
- In vector space classification, this set corresponds to a labeled set of points or vectors in the vector space.
- Premise 1: Documents in the same class form a contiguous region.
- Premise 2: Documents from different classes don't overlap.
- We define lines, surfaces, hypersurfaces to divide regions.

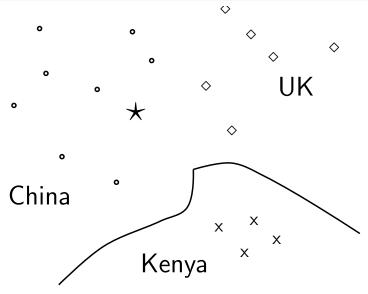




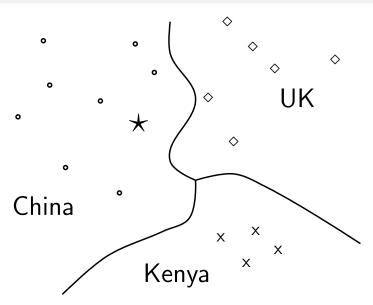
Should the document \* be assigned to China, UK or Kenya?



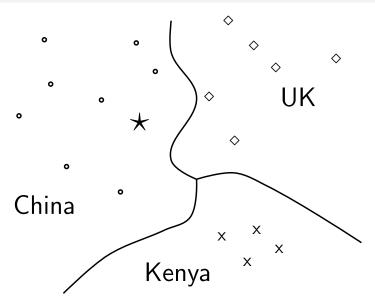
Find separators between the classes



Find separators between the classes



Based on these separators: \* should be assigned to China



How do we find separators that do a good job at classifying new documents like ★?

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#### Linear classifiers

- Definition:
  - A linear classifier computes a linear combination or weighted sum  $\sum_i w_i x_i$  of the feature values.
  - ► Classification decision:  $\sum_i w_i x_i > \theta$ ?
  - ... where  $\theta$  (the threshold) is a parameter.
- (First, we only consider binary classifiers.)
- Geometrically, this corresponds to a line (2D), a plane (3D) or a hyperplane (higher dimensionalities).
- We call this the separator or decision boundary.
- We find the separator based on training set.
- Methods for finding separator: Perceptron, Rocchio, Naive Bayes, linear SVM
- Assumption: The classes are linearly separable.

#### A linear classifier in 1D



• A linear classifier in 1D is a point x described by the equation  $w_1d_1 = \theta$ 



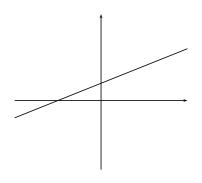
- A linear classifier in 1D is a point x described by the equation  $w_1d_1 = \theta$
- $x = \theta/w_1$



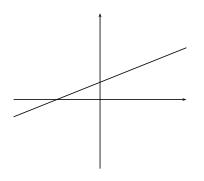
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- Points  $(d_1)$  with  $w_1 d_1 \ge \theta$  are in the class c.



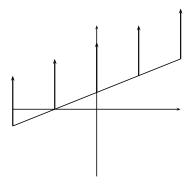
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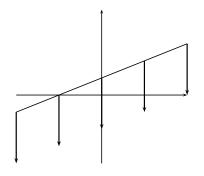
• A linear classifier in 2D is a line described by the equation  $w_1d_1 + w_2d_2 = \theta$ 



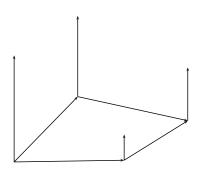
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- Example for a 2D linear classifier



- A linear classifier in 2D is a line described by the equation  $w_1 d_1 + w_2 d_2 = \theta$
- Example for a 2D linear classifier
- Points (d<sub>1</sub> d<sub>2</sub>) with  $w_1 d_1 + w_2 d_2 > \theta$  are in the class c.

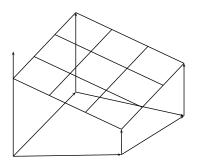


- A linear classifier in 2D is a line described by the equation  $w_1 d_1 + w_2 d_2 = \theta$
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- Points  $(d_1 \ d_2)$  with  $w_1 d_1 + w_2 d_2 \ge \theta$  are in the class c.
- Points  $(d_1 \ d_2)$  with  $w_1 d_1 + w_2 d_2 < \theta$  are in the complement class  $\overline{c}$ .



 A linear classifier in 3D is a plane described by the equation

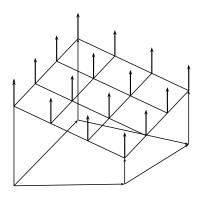
$$w_1 d_1 + w_2 d_2 + w_3 d_3 = \theta$$



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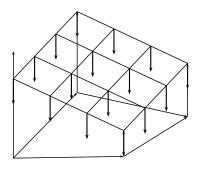
 Example for a 3D linear classifier



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- Example for a 3D linear classifier
- Points  $(d_1 \ d_2 \ d_3)$  with  $w_1 d_1 + w_2 d_2 + w_3 d_3 \ge \theta$  are in the class c.



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## Naive Bayes as a linear classifier

Multinomial Naive Bayes is a linear classifier (in log space) defined by:

$$\sum_{i=1}^{M} w_i d_i = \theta$$

where  $w_i = \log[\hat{P}(t_i|c)/\hat{P}(t_i|\bar{c})]$ ,  $d_i =$  number of occurrences of  $t_i$  in d, and  $\theta = -\log[\hat{P}(c)/\hat{P}(\bar{c})]$ . Here, the index i,  $1 \le i \le M$ , refers to terms of the vocabulary

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#### **Takeway**

Naïve Bayes, logistic regression and SVM (which we'll get to in a second) are all linear methods. They choose their hyperplanes based on different objectives: joint likelihood (NB), conditional likelihood (LR), and the margin (SVM).

# Reminder: Example of a linear classifier

$t_i$	$w_i$	$d_{1i}$	$d_{2i}$	t <sub>i</sub>	$w_i$	$d_{1i}$	$d_{2i}$
prime	0.70	0	1	dlrs	-0.71	1	1
rate	0.67	1	0	world	-0.35	1	0
interest	0.63	0	0	sees	-0.33	0	0
rates	0.60	0	0	year	-0.25	0	0
discount	0.46	1	0	group	-0.24	0	0
bundesbank	0.43	0	0	dlr	-0.24	0	0

- This is for the class interest in Reuters-21578.
- For simplicity: assume a simple 0/1 vector representation
- d<sub>1</sub>: "rate discount dlrs world"
- d<sub>2</sub>: "prime dlrs"
- $\theta = 0$
- Exercise: Which class is  $d_1$  assigned to? Which class is  $d_2$  assigned to?

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- We assign document  $\vec{d}_1$  "rate discount dirs world" to interest since  $\vec{w}^T \vec{d}_1 = 0.67 \cdot 1 + 0.46 \cdot 1 + (-0.71) \cdot 1 + (-0.35) \cdot 1 = 0.07 > 0 = \theta$ .

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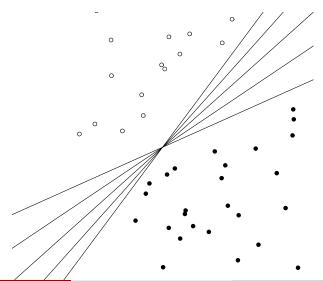
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- We assign  $\vec{o}_2$  "prime dlrs" to the complement class (not in interest) since  $\vec{w}^T \vec{o}_2 = -0.01 \le \theta$ .

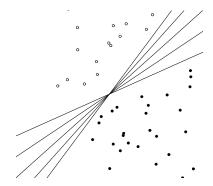
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# Which hyperplane?



# Which hyperplane?



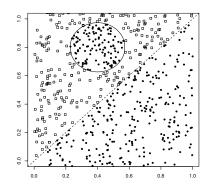
## Which hyperplane?

- For linearly separable training sets: there are infinitely many separating hyperplanes.
- They all separate the training set perfectly . . .
- ...but they behave differently on test data.
- Error rates on new data are low for some, high for others.
- How do we find a low-error separator?
- Naive Bayes: ok; linear SVM: good

### **Linear classifiers: Discussion**

- Many common text classifiers are linear classifiers: Naive Bayes, logistic regression, linear support vector machines etc.
- Each method has a different way of selecting the separating hyperplane
  - Huge differences in performance on test documents
- Can we get better performance with more powerful nonlinear classifiers?
- Not in general: A given amount of training data may suffice for estimating a linear boundary, but not for estimating a more complex nonlinear boundary.

## A nonlinear problem



- Linear classifier like Naive Bayes does badly on this task.
- kNN will do well (assuming enough training data)

## **Outline**

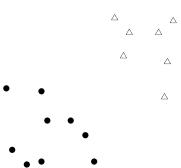
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- Machine-learning research in the last two decades has improved classifier effectiveness.
- New generation of state-of-the-art classifiers: support vector machines (SVMs), boosted decision trees, regularized logistic regression, neural networks, and random forests
- Applications to IR problems, particularly text classification

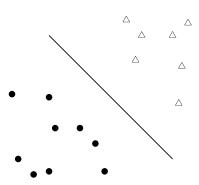
#### SVMs: A kind of large-margin classifier

Vector space based machine-learning method aiming to find a decision boundary between two classes that is maximally far from any point in the training data (possibly discounting some points as outliers or noise)

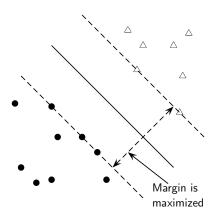
2-class training data



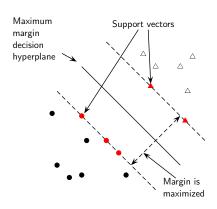
- 2-class training data
- ◆ decision boundary → linear separator



- 2-class training data
- ◆ decision boundary → linear separator
- criterion: being maximally far away from any data point
   → determines classifier margin

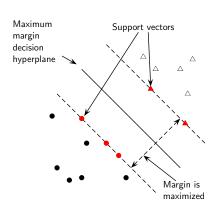


- 2-class training data
- ◆ decision boundary → linear separator
- criterion: being maximally far away from any data point
   → determines classifier margin
- linear separator position defined by support vectors



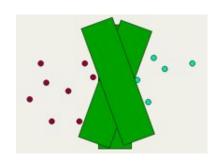
# Why maximize the margin?

- Points near decision surface → uncertain classification decisions (50% either way).
- A classifier with a large margin makes no low certainty classification decisions.
- Gives classification safety margin w.r.t slight errors in measurement or documents variation

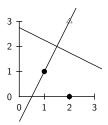


# Why maximize the margin?

- SVM classifier: large margin around decision boundary
- compare to decision
  hyperplane: place fat separator
  between classes
  - unique solution
- decreased memory capacity
- increased ability to correctly generalize to test data

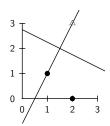


Working geometrically:



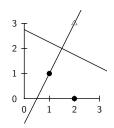
#### Working geometrically:

 The maximum margin weight vector will be parallel to the shortest line connecting points of the two classes, that is, the line between (1,1) and (2,3), giving a weight vector of (1,2).



### Working geometrically:

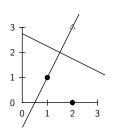
- The maximum margin weight vector will be parallel to the shortest line connecting points of the two classes, that is, the line between (1,1) and (2,3), giving a weight vector of (1,2).
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### Working geometrically:

- The maximum margin weight vector will be parallel to the shortest line connecting points of the two classes, that is, the line between (1,1) and (2,3), giving a weight vector of (1,2).
- The optimal decision surface is orthogonal to that line and intersects it at the halfway point. Therefore, it passes through (1.5,2).
- The SVM decision boundary is:

$$0 = \frac{1}{2}x + y - \frac{11}{4} \iff 0 = \frac{2}{5}x + \frac{4}{5}y - \frac{11}{5}$$



#### **SVM** extensions

- Slack variables: not perfect line
- Kernels: different geometries
- Loss functions: Different penalties for getting the answer wrong

## **Outline**

- Decision Trees
- Learning Decision Trees
- Overfitting
- 4 Vector space classification
- **5** Linear Classifiers
- Which hyperplane is best for me?
- Support Vector Machines
- Trying Out Classifiers in Rattle
- Recap

# Selecting a model

- Go to "model" tab and select one of the models
- Make sure the model makes sense
- For logistic regression, select "linear" and "logistic"



## Selecting a model

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- Make sure the model makes sense
- For logistic regression, select "linear" and "logistic"



 For SVM, you also need to select a kernel (try linear first, then "Gaussian" which will be much slower)



- Output varies by model
  - SVM is least informative (hard to summarize)
  - Note you can click draw to see decision trees

# **Decision Trees Have Many Options...**



- Prior: The prior observation probabilities (in case your training data are skewed)
- Min Split: How many observations can be in an expanded leaf (pre-test)
- Min Bucket: How many observations can be in any resulting leaf (post-test)
- Max Depth: How many levels the tree has
- Complexity: How many "if" statements the tree has

Defaults are reasonable; tweak if you are having complexity issues.

#### How'd we do?

Fit the model by clicking on the "execute" button



- Click on the evaluate tab, have your boxes checked for the models you want to compare
- For the weather dataset, SVM does best (.14)
- We'll learn about the other metrics next week!

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#### Text classification

- Many commercial applications
- There are many applications of text classification for corporate Intranets, government departments, and Internet publishers.
- Often greater performance gains from exploiting domain-specific text features than from changing from one machine learning method to another.
- Understanding the data is one of the keys to successful categorization, yet this is an area in which many categorization tool vendors are weak.

- None?
- Very little?
- A fair amount?
- A huge amount

- None? Hand write rules or use active learning
- Very little?
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- None? Hand write rules or use active learning
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- None? Hand write rules or use active learning
- Very little? Naïve Bayes
- A fair amount? SVM
- A huge amount Doesn't matter, use whatever works

## Recap

- Is there a learning method that is optimal for all text classification problems?
- No, because there is a tradeoff between bias and variance.
- Factors to take into account:
  - How much training data is available?
  - How simple/complex is the problem? (linear vs. nonlinear decision boundary)
  - How noisy is the problem?
  - How stable is the problem over time?
    - For an unstable problem, it's better to use a simple and robust classifier.
    - ★ You'll be investigating the role of features and classifiers in HW2!