Topic Models

Machine Learning: Jordan Boyd-Graber
University of Colorado Boulder

LECTURE 18

Why topic models?

- Suppose you have a huge number of documents
- Want to know what's going on
- Can't read them all (e.g. every New York Times article from the 90's)
- Topic models offer a way to get a corpus-level view of major themes

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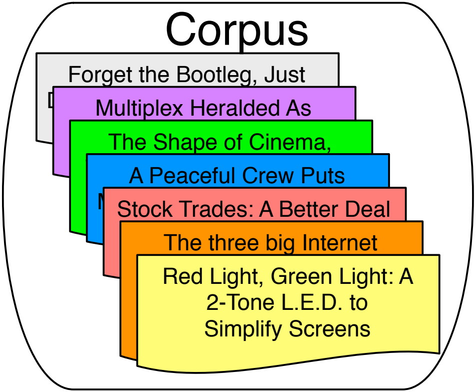
- Topic models offer a way to get a corpus-level view of major themes
- Unsupervised

Roadmap

- What are topic models
- How to know if you have good topic model
- How to go from raw data to topics

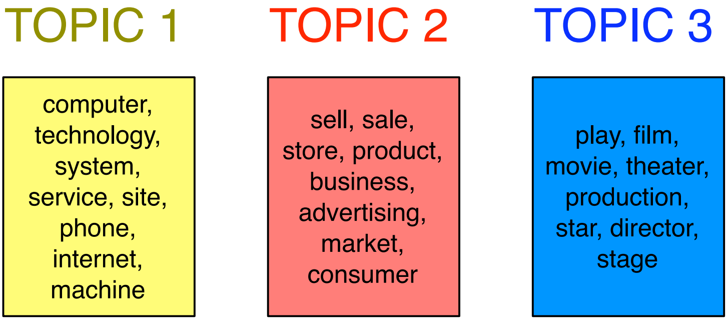
Conceptual Approach

From an **input corpus** and number of topics K → words to topics



Conceptual Approach

From an input corpus and number of topics $K \rightarrow$ **words to topics**



TOPIC 1

computer, technology, system, service, site, phone, internet, machine

TOPIC 2

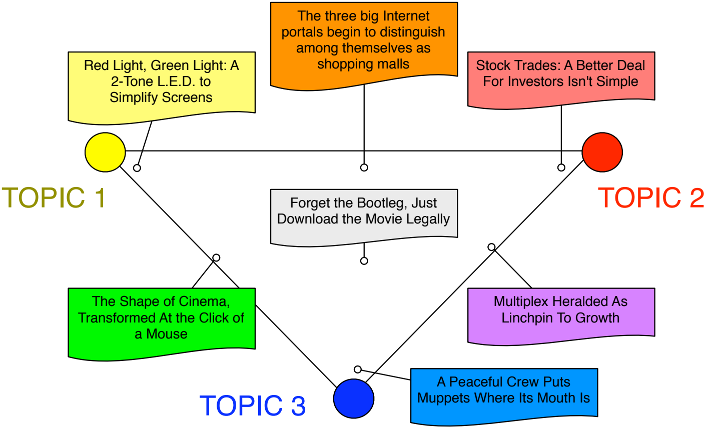
sell, sale, store, product, business, advertising, market, consumer

TOPIC 3

play, film, movie, theater, production, star, director, stage

Conceptual Approach

- For each document, what topics are expressed by that document?


The three big Internet portals begin to distinguish among themselves as shopping malls
Red Light, Green Light: A 2-Tone L.E.D. to Simplify Screens
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Topics from Science

human	evolution	disease	computer
genome	evolutionary	host	models
dna	species	bacteria	information
genetic	organisms	diseases	data
genes	life	resistance	computers
sequence	origin	bacterial	system
gene	biology	new	network
molecular	groups	strains	systems
sequencing	phylogenetic	control	model
map	living	infectious	parallel
information	diversity	malaria	methods
genetics	group	parasite	networks
mapping	new	parasites	software
project	two	united	new
sequences	common	tuberculosis	simulations

Why should you care?

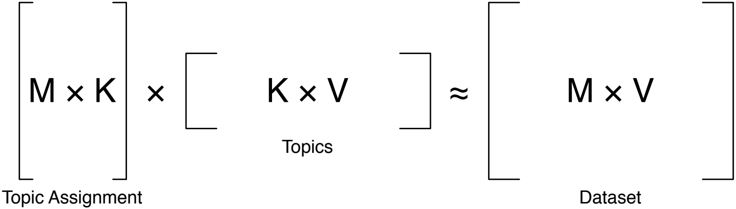
- Neat way to explore / understand corpus collections
    - E-discovery
    - Social media
    - Scientific data
- NLP Applications
    - Word Sense Disambiguation
    - Discourse Segmentation
    - Machine Translation
- Psychology: word meaning, polysemy
- Inference is (relatively) simple

Matrix Factorization Approach

$$\underset{\text{Topic Assignment}}{\begin{bmatrix} M \times K \end{bmatrix}} \times \underset{\text{Topics}}{\begin{bmatrix} K \times V \end{bmatrix}} \approx \underset{\text{Dataset}}{\begin{bmatrix} M \times V \end{bmatrix}}$$

K	Number of topics
M	Number of documents
V	Size of vocabulary

Matrix Factorization Approach



$M \times K$ × $K \times V$ ≈ $M \times V$

Topic Assignment

Topics

Dataset

K	Number of topics
M	Number of documents
V	Size of vocabulary

- If you use singular value decomposition (SVD), this technique is called latent semantic analysis.
- Popular in information retrieval.

Alternative: Generative Model

- How your data came to be
- Sequence of Probabilistic Steps
- Posterior Inference

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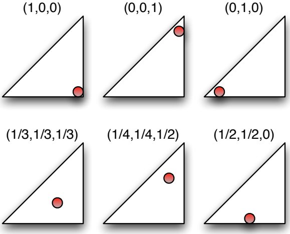
- Posterior Inference
- Blei, Ng, Jordan. Latent **Dirichlet** Allocation. JMLR, 2003.

Multinomial Distribution

- Distribution over discrete outcomes
- Represented by non-negative vector that sums to one
- Picture representation



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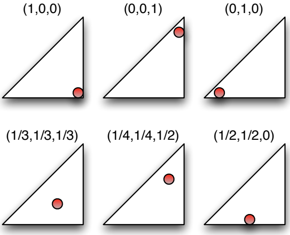

- Come from a Dirichlet distribution

Dirichlet Distribution

$$P(\boldsymbol{p} \mid \alpha \boldsymbol{m}) = \frac{\Gamma(\sum_k \alpha m_k)}{\prod_k \Gamma(\alpha m_k)} \prod_k p_k^{\alpha m_k - 1}$$

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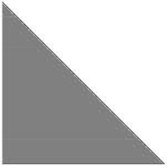
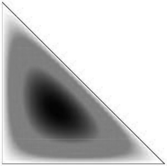
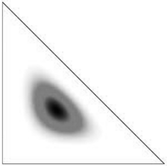


$\alpha = 3, \boldsymbol{m} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$



$\alpha = 6, \boldsymbol{m} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$



$\alpha = 30, \boldsymbol{m} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$

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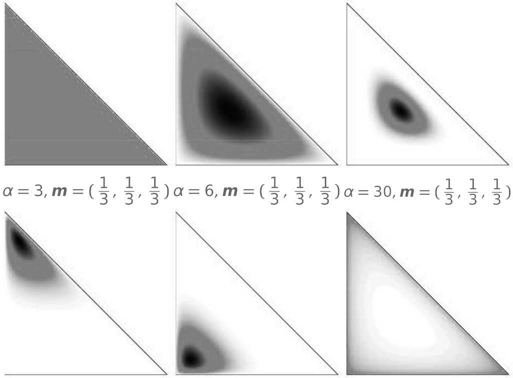

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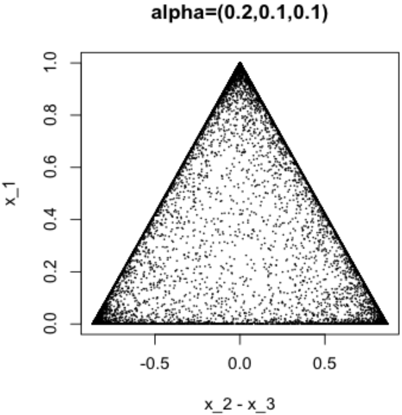
Dirichlet Distribution



Dirichlet Distribution

- If $\phi \sim \mathrm{Dir}(()\alpha)$, $\mathbf{w} \sim \mathrm{Mult}(()\phi)$, and $n_k = |\{w_i : w_i = k\}|$ then

$$\begin{aligned} p(\phi|\alpha, \mathbf{w}) &\propto p(\mathbf{w}|\phi)p(\phi|\alpha) & (1) \\ &\propto \prod_k \phi^{n_k} \prod_k \phi^{\alpha_k - 1} & (2) \\ &\propto \prod_k \phi^{\alpha_k + n_k - 1} & (3) \end{aligned}$$

- Conjugacy: this **posterior** has the same form as the **prior**

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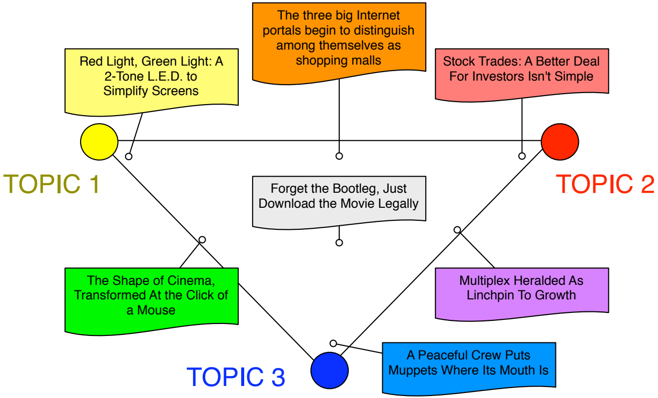
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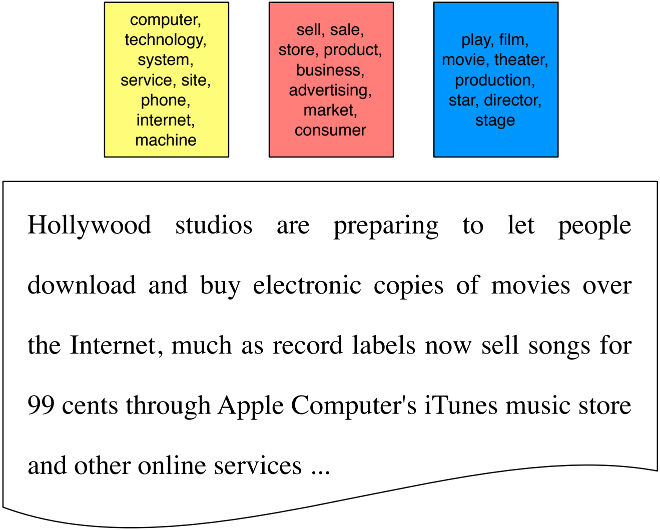

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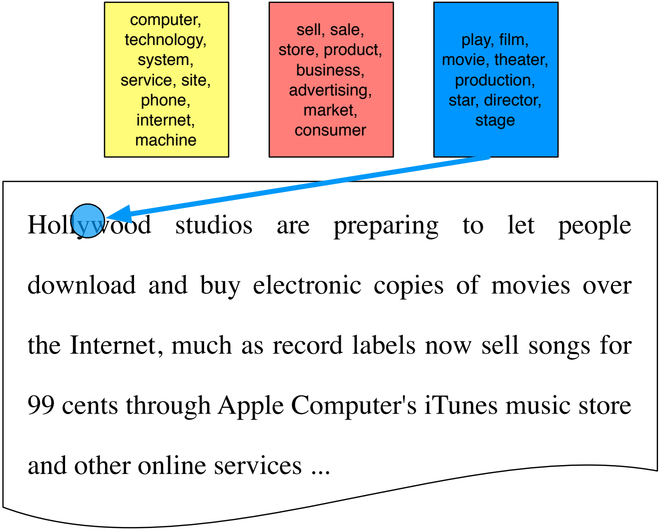

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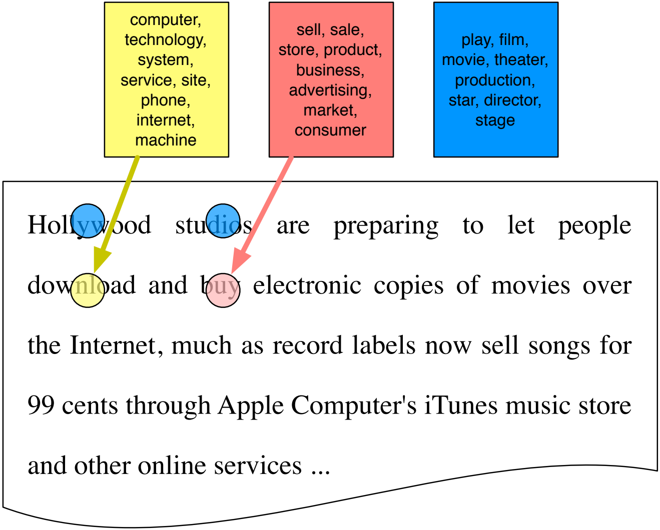

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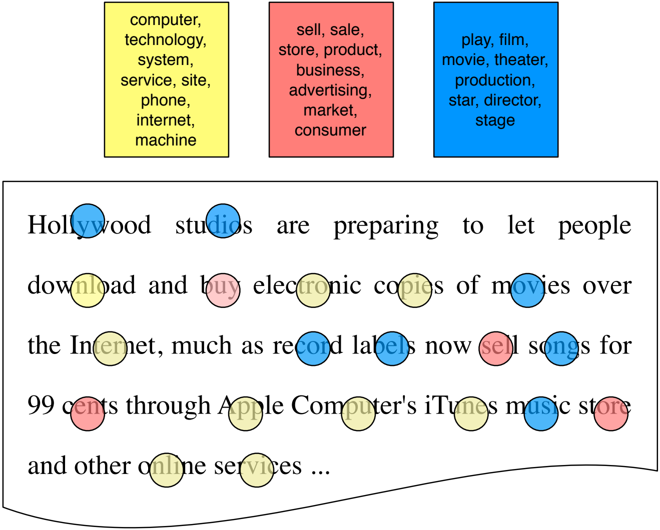

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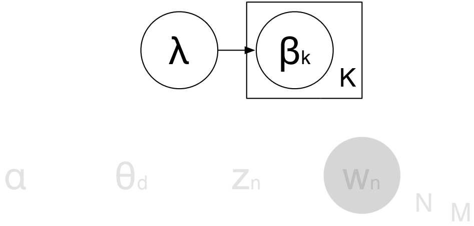

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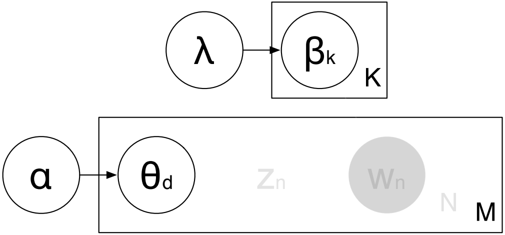

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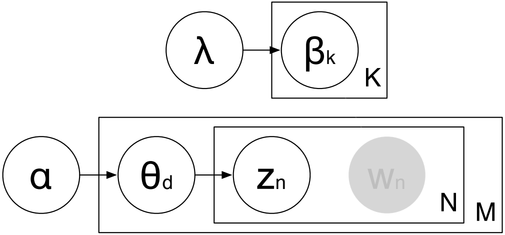

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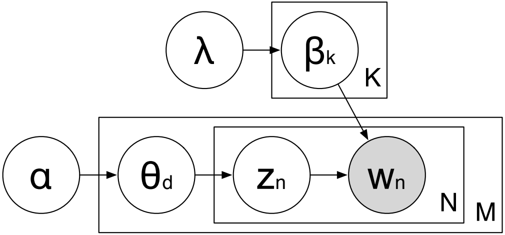

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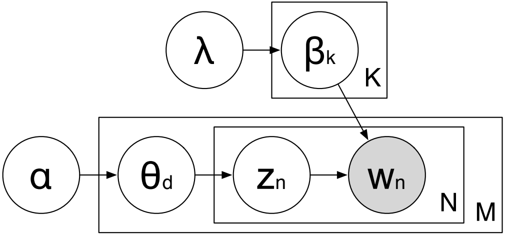
Generative Model Approach



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Topic Models: What's Important

- Topic models
    - Topics to word types—multinomial distribution
    - Documents to topics—multinomial distribution
- Focus in this talk: statistical methods
    - Model: story of how your data came to be
    - Latent variables: missing pieces of your story
    - Statistical inference: filling in those missing pieces
- We use latent Dirichlet allocation (LDA), a fully Bayesian version of pLSI, probabilistic version of LSA

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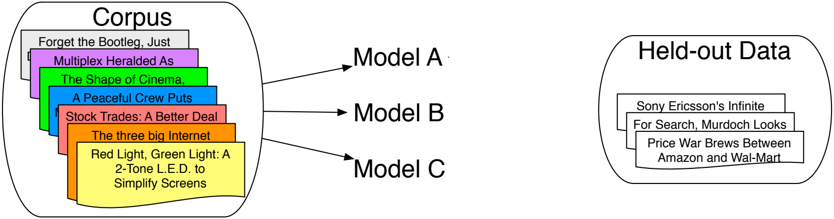
Evaluation



$$
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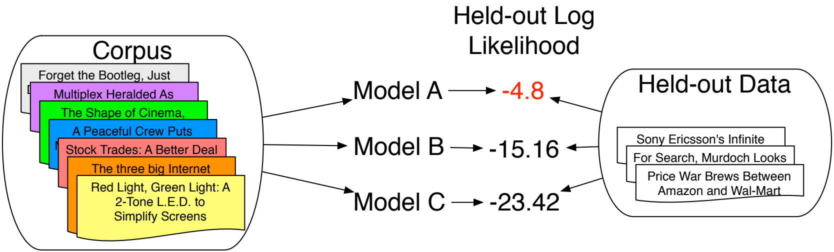
$$

How you compute it is important too (Wallach et al. 2009)

Evaluation



Measures predictive power, not what the topics are

$$P(\boldsymbol{w} \,|\, \boldsymbol{w}', \boldsymbol{z}', \alpha \boldsymbol{m}, \beta \boldsymbol{u}) = \sum_{\boldsymbol{z}} P(\boldsymbol{w}, \boldsymbol{z} \,|\, \boldsymbol{w}', \boldsymbol{z}', \alpha \boldsymbol{m}, \beta \boldsymbol{u})$$

How you compute it is important too (Wallach et al. 2009)

Word Intrusion

TOPIC 1

computer, technology, system, service, site, phone, internet, machine

TOPIC 2

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Word Intrusion

1. Take the highest probability words from a topic

Original Topic

dog, cat, horse, pig, cow

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2. Take a high-probability word from another topic and add it

Topic with Intruder

dog, cat, apple, horse, pig, cow

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dog, cat, horse, pig, cow

2. Take a high-probability word from another topic and add it

Topic with Intruder

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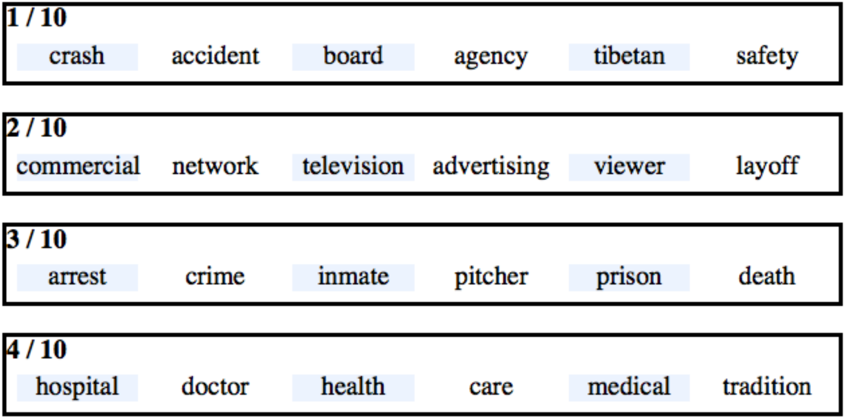
3. We ask users to find the word that doesn't belong

Hypothesis

If the topics are interpretable, users will consistently choose true intruder

Word Intrusion



1 / 10

crash accident board agency tibetan safety

2 / 10

commercial network television advertising viewer layoff

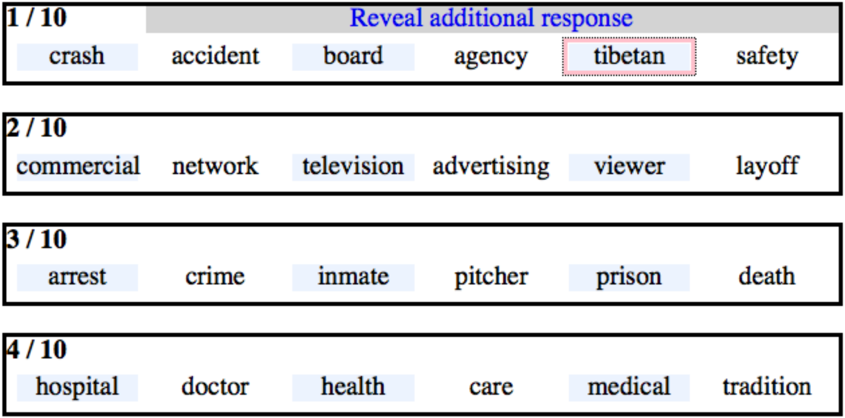
3 / 10

arrest crime inmate pitcher prison death

4 / 10

hospital doctor health care medical tradition

Word Intrusion



1 / 10 Reveal additional response

crash accident board agency tibetan safety

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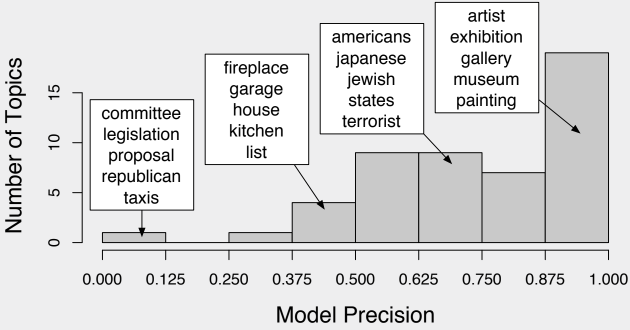
- Order of words was shuffled
- Which intruder was selected varied
- Model precision: percentage of users who clicked on intruder

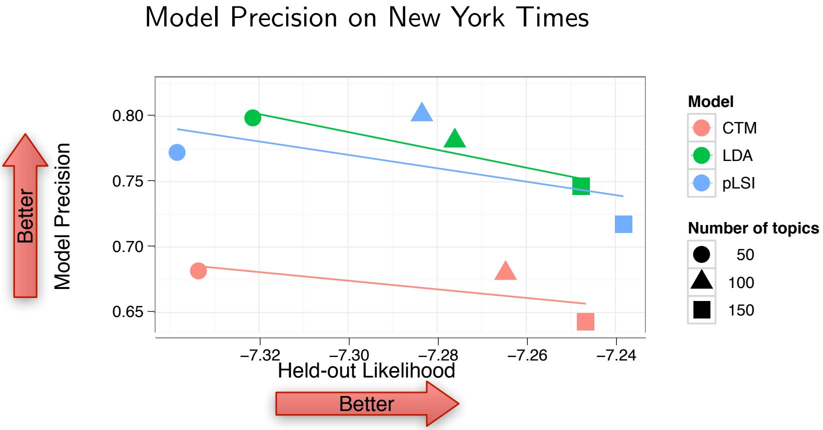
Word Intrusion: Which Topics are Interpretable?

New York Times, 50 LDA Topics



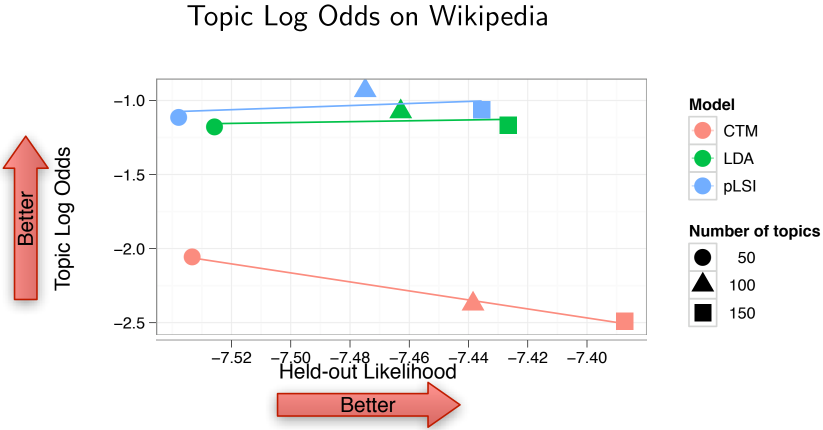
Model Precision: percentage of correct intruders found

Interpretability and Likelihood



within a model, higher likelihood $\neq$ higher interpretability

Interpretability and Likelihood



across models, higher likelihood $\neq$ higher interpretability

Evaluation Takeaway

- Measure what you care about
- If you care about prediction, likelihood is good
- If you care about a particular task, measure that

Inference

- We are interested in posterior distribution

$$p(Z|X,\Theta) \tag{4}$$

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- Here, latent variables are topic assignments z and topics θ. X is the words (divided into documents), and Θ are hyperparameters to Dirichlet distributions: α for topic proportion, λ for topics.

$$p(\mathbf{z},\boldsymbol{\beta},\boldsymbol{\theta}|\mathbf{w},\alpha,\lambda) \tag{5}$$

Inference

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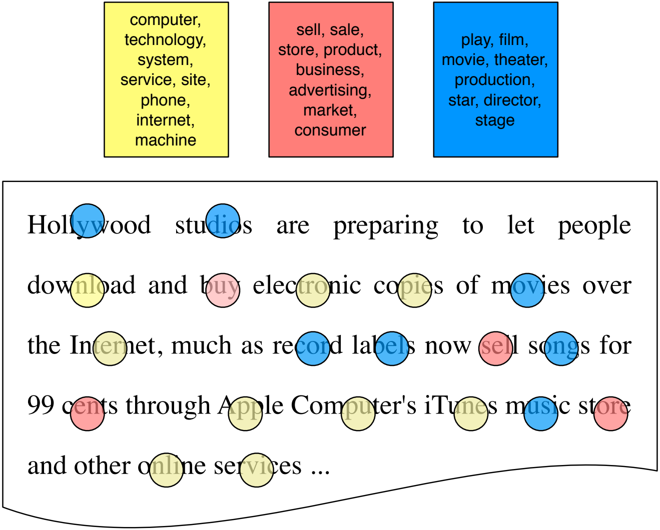
$$p(\mathbf{z},\boldsymbol{\beta},\boldsymbol{\theta}|\mathbf{w},\alpha,\lambda) \tag{5}$$

$$p(\mathbf{w},\mathbf{z},\boldsymbol{\theta},\boldsymbol{\beta}|\alpha,\lambda) = \prod_k p(\beta_k|\lambda)\prod_d p(\theta_d|\alpha)\prod_n p(z_{d,n}|\theta_d)p(w_{d,n}|\beta_{z_{d,n}})$$

Gibbs Sampling

- A form of Markov Chain Monte Carlo
- Chain is a sequence of random variable states
- Given a state $\{z_1, \dots z_N\}$ given certain technical conditions, drawing $z_k \sim p(z_1, \dots z_{k-1}, z_{k+1}, \dots z_N | X, \Theta)$ for all k (repeatedly) results in a Markov Chain whose stationary distribution *is* the posterior.
- For notational convenience, call $\mathbf{z}$ with $z_{d,n}$ removed $\mathbf{z}_{-d,n}$

Inference



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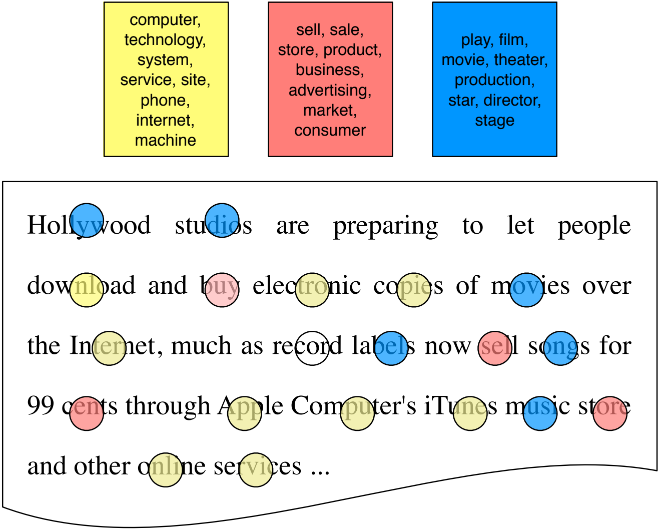

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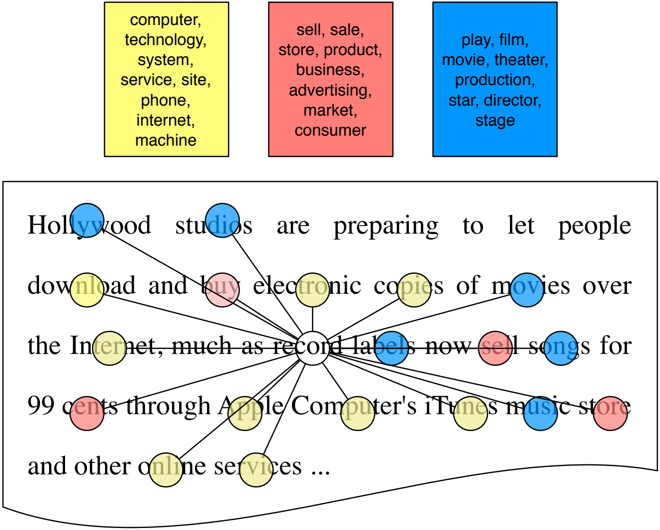

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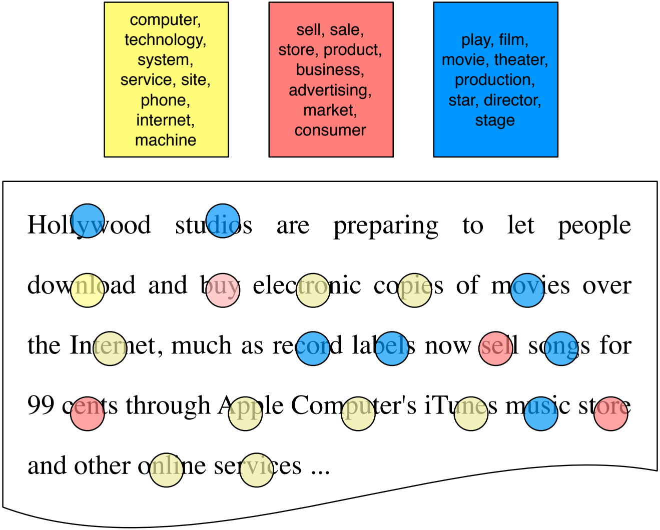

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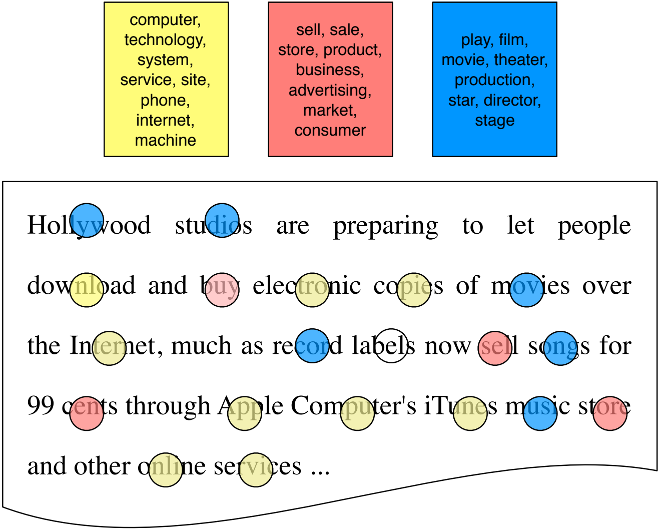
Inference



computer, technology, system, service, site, phone, internet, machine

sell, sale, store, product, business, advertising, market, consumer

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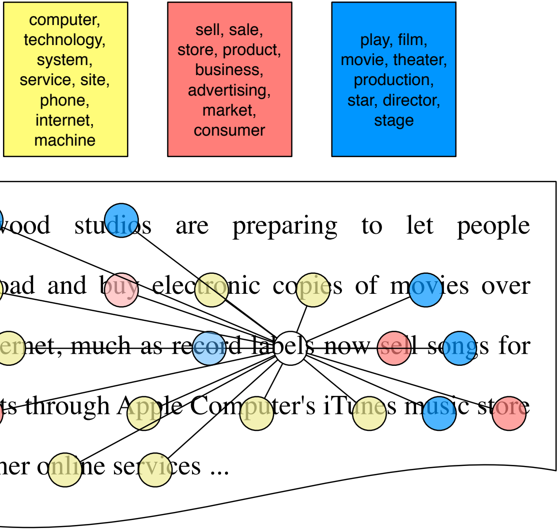

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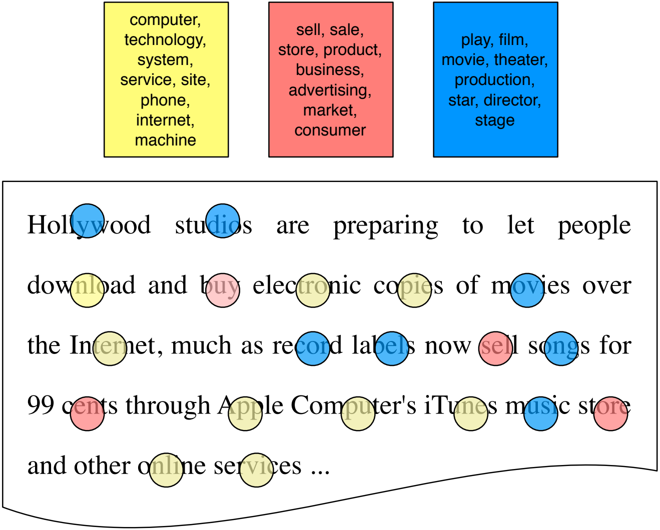

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Gibbs Sampling

- For LDA, we will sample the topic assignments
- Thus, we want:

$$p(z_{d,n} = k | \mathbf{z}_{-d,n}, \mathbf{w}, \alpha, \lambda) = \frac{p(z_{d,n} = k, \mathbf{z}_{-d,n} | \mathbf{w}, \alpha, \lambda)}{p(\mathbf{z}_{-d,n} | \mathbf{w}, \alpha, \lambda)}$$

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- The topics and per-document topic proportions are integrated out / marginalized
- Let $n_{d,i}$ be the number of words taking topic i in document d. Let $v_{k,w}$ be the number of times word w is used in topic k.

$$= \frac{\int_{\theta_d} \left(\prod_{i \neq k} \theta_d^{\alpha_i + n_{d,i} - 1} \right) \theta_d^{\alpha_k + n_{d,i}} d\theta_d \int_{\beta_k} \left(\prod_{i \neq w_{d,n}} \beta_{k,i}^{\lambda_i + v_{k,i} - 1} \right) \beta_{k,w_{d,n}}^{\lambda_i + v_{k,i}} d\beta_k}{\int_{\theta_d} \left(\prod_i \theta_d^{\alpha_i + n_{d,i} - 1} \right) d\theta_d \int_{\beta_k} \left(\prod_i \beta_{k,i}^{\lambda_i + v_{k,i} - 1} \right) d\beta_k}$$

Gibbs Sampling

- Integral is normalizer of Dirichlet distribution

$$\int_{\beta_k} \left(\prod_i \beta_{k,i}^{\lambda_i + v_{k,i} - 1} \right) d\beta_k = \frac{\prod_i^V \Gamma\left(\beta_i + v_{k,i}\right)}{\Gamma\left(\sum_i^V \beta_i + v_{k,i}\right)}$$

Gibbs Sampling

- Integral is normalizer of Dirichlet distribution

$$\int_{\beta_k} \left(\prod_i \beta_{k,i}^{\lambda_i + v_{k,i} - 1} \right) d\beta_k = \frac{\prod_i^V \Gamma\left(\beta_i + v_{k,i}\right)}{\Gamma\left(\sum_i^V \beta_i + v_{k,i}\right)}$$

- So we can simplify

$$\frac{\int_{\theta_d} \left(\prod_{i \neq k} \theta_d^{\alpha_i + n_{d,i} - 1} \right) \theta_d^{\alpha_k + n_{d,i}} d\theta_d \int_{\beta_k} \left(\prod_{i \neq w_{d,n}} \beta_{k,i}^{\lambda_i + v_{k,i} - 1} \right) \beta_{k,w_{d,n}}^{\lambda_i + v_{k,i}} d\beta_k}{\int_{\theta_d} \left(\prod_i \theta_d^{\alpha_i + n_{d,i} - 1} \right) d\theta_d \int_{\beta_k} \left(\prod_i \beta_{k,i}^{\lambda_i + v_{k,i} - 1} \right) d\beta_k} =$$

$$\frac{\frac{\Gamma\left(\alpha_k + n_{d,k} + 1\right)}{\Gamma\left(\sum_i^K \alpha_i + n_{d,i} + 1\right)} \prod_{i \neq k}^K \Gamma\left(\alpha_k + n_{d,k}\right)}{\frac{\prod_i^K \Gamma\left(\alpha_i + n_{d,i}\right)}{\Gamma\left(\sum_i^K \alpha_i + n_{d,i}\right)}} \frac{\frac{\Gamma\left(\lambda_{w_{d,n}} + v_{k,w_{d,n}} + 1\right)}{\Gamma\left(\sum_i^V \lambda_i + v_{k,i} + 1\right)} \prod_{i \neq w_{d,n}}^V \Gamma\left(\lambda_k + v_{k,w_{d,n}}\right)}{\frac{\prod_i^V \Gamma\left(\lambda_i + v_{k,i}\right)}{\Gamma\left(\sum_i^V \lambda_i + v_{k,i}\right)}}$$

Gamma Function Identity

$$z = \frac{\Gamma(z+1)}{\Gamma(z)} \tag{6}$$

$$\frac{\frac{\Gamma\left(\alpha_k+n_{d,k}+1\right)}{\Gamma\left(\sum_i^K \alpha_i+n_{d,i}+1\right)} \prod_{i\neq k}^K \Gamma\left(\alpha_k+n_{d,k}\right)}{\frac{\prod_i^K \Gamma\left(\alpha_i+n_{d,i}\right)}{\Gamma\left(\sum_i^K \alpha_i+n_{d,i}\right)}} \frac{\frac{\Gamma\left(\lambda_{w_{d,n}}+v_{k,w_{d,n}}+1\right)}{\Gamma\left(\sum_i^V \lambda_i+v_{k,i}+1\right)} \prod_{i\neq w_{d,n}}^V \Gamma\left(\lambda_k+v_{k,w_{d,n}}\right)}{\frac{\prod_i^V \Gamma\left(\lambda_i+v_{k,i}\right)}{\Gamma\left(\sum_i^V \lambda_i+v_{k,i}\right)}}$$

$$= \frac{n_{d,k}+\alpha_k}{\sum_i^K n_{d,i}+\alpha_i} \frac{v_{k,w_{d,n}}+\lambda_{w_{d,n}}}{\sum_i v_{k,i}+\lambda_i}$$

Gibbs Sampling Equation

$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i} \tag{7}$$

- Number of times document d uses topic k
- Number of times topic k uses word type $w_{d,n}$
- Dirichlet parameter for document to topic distribution
- Dirichlet parameter for topic to word distribution
- How much this document likes topic k
- How much this topic likes word $w_{d,n}$

Gibbs Sampling Equation

$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i} \tag{7}$$

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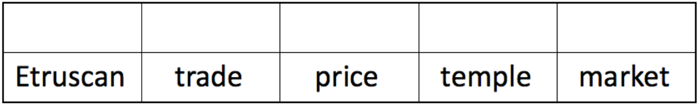
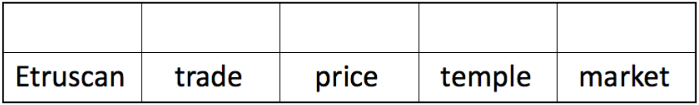
Sample Document



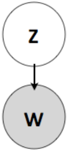
Etruscan	trade	price	temple	market

Sample Document



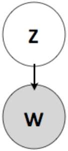
Etruscan	trade	price	temple	market

Randomly Assign Topics

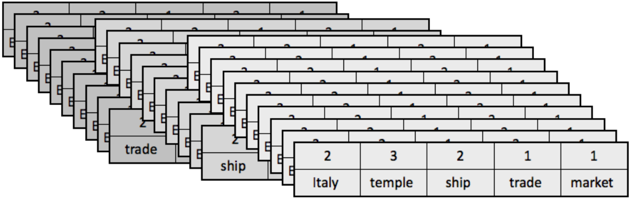


3	2	1	3	1
Etruscan	trade	price	temple	market

Randomly Assign Topics



3	2	1	3	1
Etruscan	trade	price	temple	market



2	3	2	1	1
Italy	temple	ship	trade	market

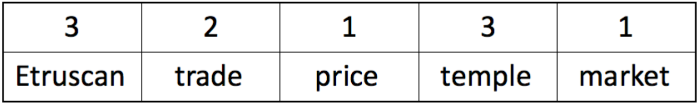
Total Topic Counts

3	2	1	3	1
Etruscan	trade	price	temple	market

Total counts from **all** docs

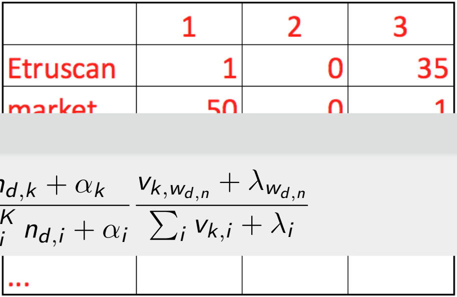
	1	2	3
Etruscan	1	0	35
market	50	0	1
price	42	1	0
temple	0	0	20
trade	10	8	1
...			

Total Topic Counts



3	2	1	3	1
Etruscan	trade	price	temple	market

Total

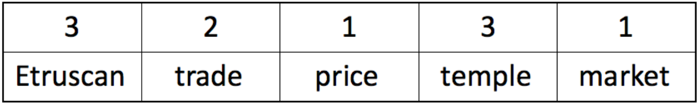


	1	2	3
Etruscan	1	0	35
market	50	0	1
...			

Sampling Equation

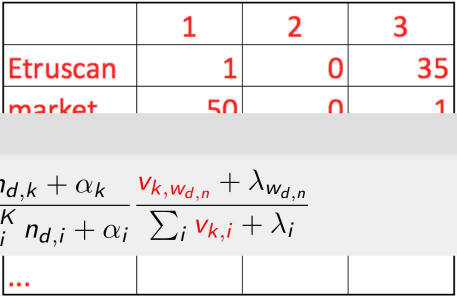
$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i}$$

Total Topic Counts



3	2	1	3	1
Etruscan	trade	price	temple	market

Total



	1	2	3
Etruscan	1	0	35
market	50	0	1
...			

Sampling Equation

$$\frac{n_{d,k}+\alpha_k}{\sum_i^K n_{d,i}+\alpha_i}\;\frac{v_{k,w_{d,n}}+\lambda_{w_{d,n}}}{\sum_i v_{k,i}+\lambda_i}$$

We want to sample this word . . .

3	**2**	1	3	1
Etruscan	trade	price	temple	market

	1	2	3
Etruscan	1	0	35
market	50	0	1
price	42	1	0
temple	0	0	20
trade	10	8	1
...			

We want to sample this word . . .

3	**2**	1	3	1
Etruscan	trade	price	temple	market

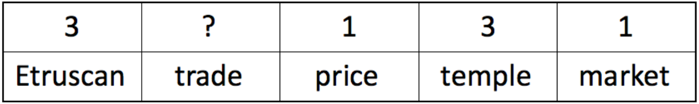
	1	2	3
Etruscan	1	0	35
market	50	0	1
price	42	1	0
temple	0	0	20
trade	10	**8**	1
...			

Decrement its count

3	?	1	3	1
Etruscan	trade	price	temple	market

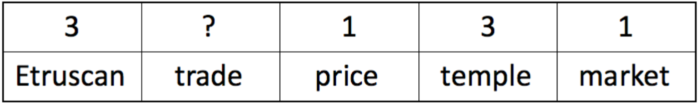
	1	2	3
Etruscan	1	0	35
market	50	0	1
price	42	1	0
temple	0	0	20
trade	10	**7**	1
...			

What is the conditional distribution for this topic?



3	?	1	3	1
Etruscan	trade	price	temple	market

Part 1: How much does this document like each topic?



3	?	1	3	1
Etruscan	trade	price	temple	market

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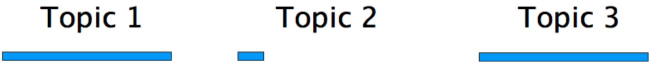

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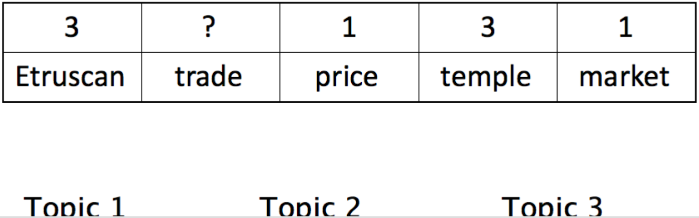


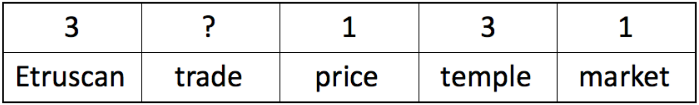
3	?	1	3	1
Etruscan	trade	price	temple	market

Topic 1 Topic 2 Topic 3

Sampling Equation

$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i}$$

Part 1: How much does this document like each topic?



3	?	1	3	1
Etruscan	trade	price	temple	market

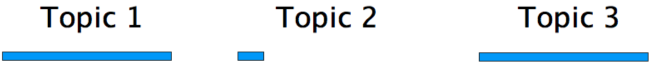
Topic 1 Topic 2 Topic 3

Sampling Equation

$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i}$$

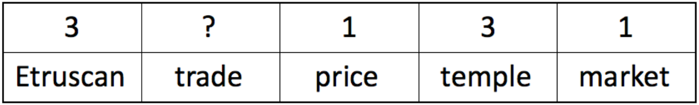
Part 2: How much does each topic like the word?

3	?	1	3	1
Etruscan	trade	price	temple	market

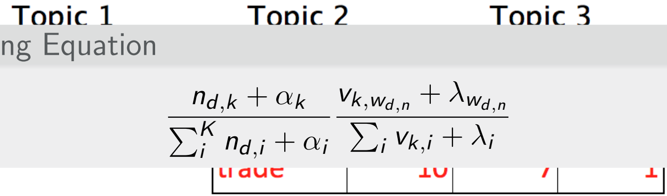


	1	2	3
trade	10	7	1

Part 2: How much does each topic like the word?



3	?	1	3	1
Etruscan	trade	price	temple	market

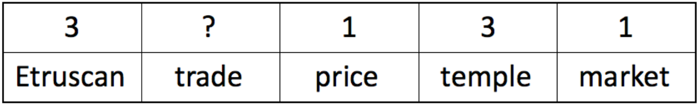


	Topic 1	Topic 2	Topic 3
trade	10	7	1

Sampling Equation

$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i}$$

Part 2: How much does each topic like the word?



3	?	1	3	1
Etruscan	trade	price	temple	market

Topic 1 Topic 2 Topic 3

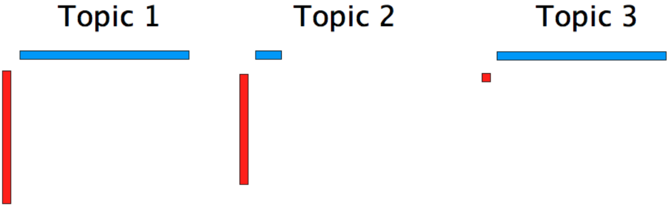
Sampling Equation

$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i}$$

trade	10	7	1

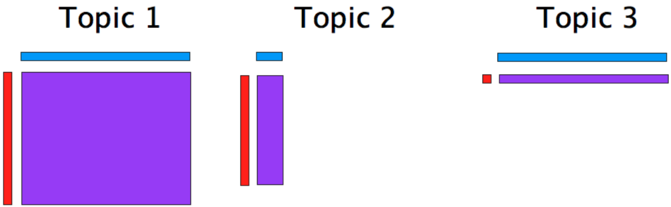
Geometric interpretation

3	?	1	3	1
Etruscan	trade	price	temple	market



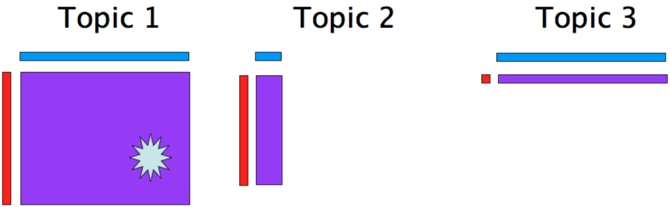
Geometric interpretation

3	?	1	3	1
Etruscan	trade	price	temple	market

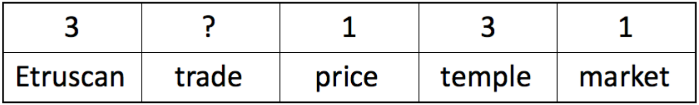


Geometric interpretation

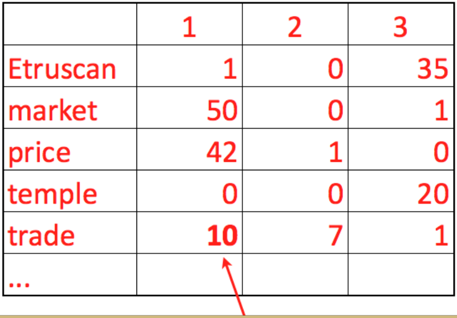
3	?	1	3	1
Etruscan	trade	price	temple	market



Update counts



3	?	1	3	1
Etruscan	trade	price	temple	market



	1	2	3
Etruscan	1	0	35
market	50	0	1
price	42	1	0
temple	0	0	20
trade	**10**	7	1
...			

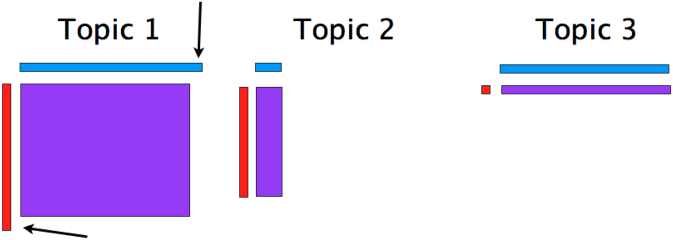
Update counts

3	**1**	1	3	1
Etruscan	trade	price	temple	market

	1	2	3
Etruscan	1	0	35
market	50	0	1
price	42	1	0
temple	0	0	20
trade	**11**	7	1
...			

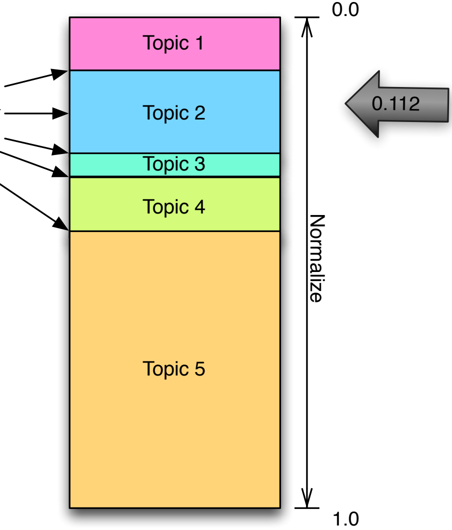
Update counts

3	1	1	3	1
Etruscan	trade	price	temple	market



Details: how to sample from a distribution

$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i}$$



Algorithm

```
1. For each iteration $i$:
    1.1 For each document $d$ and word $n$ currently assigned to $z_{old}$:
        1.1.1 Decrement $n_{d,z_{old}}$ and $v_{z_{old},w_{d,n}}$
        1.1.2 Sample $z_{new} = k$ with probability proportional to $\frac{n_{d,k}+\alpha_k}{\sum_i^K n_{d,i}+\alpha_i} \frac{v_{k,w_{d,n}}+\lambda_{w_{d,n}}}{\sum_i v_{k,i}+\lambda_i}$
        1.1.3 Increment $n_{d,z_{new}}$ and $v_{z_{new},w_{d,n}}$
```

Implementation

Algorithm

```
1. For each iteration $i$:
   1.1 For each document $d$ and word $n$ currently assigned to $z_{old}$:
       1.1.1 Decrement $n_{d,z_{old}}$ and $v_{z_{old},w_{d,n}}$
       1.1.2 Sample $z_{new} = k$ with probability proportional to $\frac{n_{d,k}+\alpha_k}{\sum_i^K n_{d,i}+\alpha_i} \frac{v_{k,w_{d,n}}+\lambda_{w_{d,n}}}{\sum_i v_{k,i}+\lambda_i}$
       1.1.3 Increment $n_{d,z_{new}}$ and $v_{z_{new},w_{d,n}}$
```

Desiderata

- Hyperparameters: Sample them too (slice sampling)
- Initialization: Random
- Sampling: Until likelihood converges
- Lag / burn-in: Difference of opinion on this
- Number of chains: Should do more than one

Available implementations

- Mallet (http://mallet.cs.umass.edu)
- LDAC (http://www.cs.princeton.edu/ blei/lda-c)
- Topicmod (http://code.google.com/p/topicmod)

Wrapup

- Topic Models: Tools to uncover themes in large document collections
- Another example of Gibbs Sampling
- In class: Gibbs sampling example