Department of Computer Science
UNIVERSITY OF COLORADO BOULDER

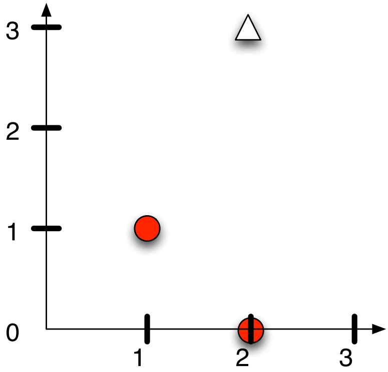
SVM

Introduction to Data Science Algorithms

Jordan Boyd-Graber and Michael Paul

SLIDES ADAPTED FROM HINRICH SCHÜTZE

Find the maximum margin hyperplane



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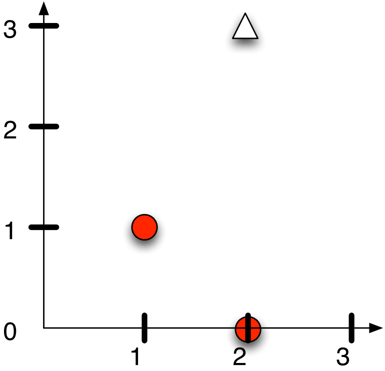

Which are the support vectors?

Walkthrough example: building an SVM over the data shown

Working geometrically:

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- Set up system of equations

$$w_1 + w_2 + b = -1 \qquad (1)$$

$$\frac{3}{2}w_1 + 2w_2 + b = 0 \qquad (2)$$

$$2w_1 + 3w_2 + b = +1 \qquad (3)$$

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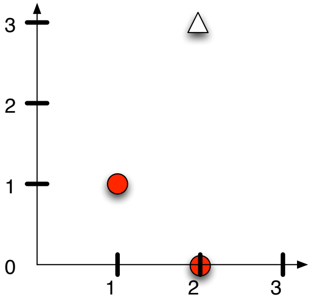
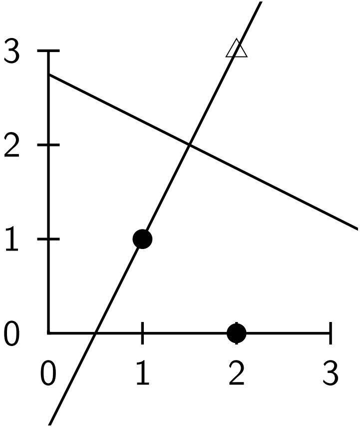


The SVM decision boundary is:

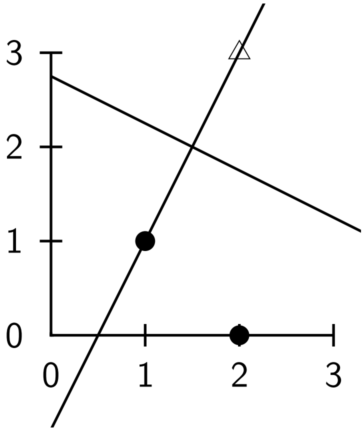
$$0 = \frac{2}{5}x + \frac{4}{5}y - \frac{11}{5}$$

Cannonical Form



$$w_1 x_1 + w_2 x_2 + b$$

Cannonical Form



$.4x_1 + .8x_2 - 2.2$

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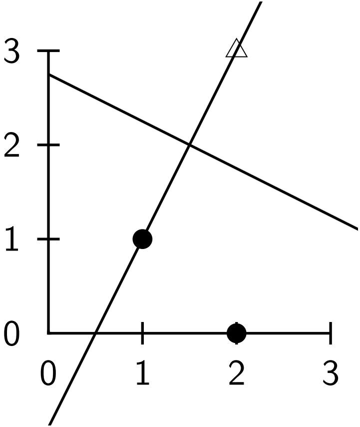

$.4x_1 + .8x_2 - 2.2$

- $.4 \cdot 1 + .8 \cdot 1 - 2.2 = -1$
- $.4 \cdot \frac{3}{2} + .8 \cdot 2 = 0$
- $.4 \cdot 2 + .8 \cdot 3 - 2.2 = +1$

What's the margin?

- Distance to closest point

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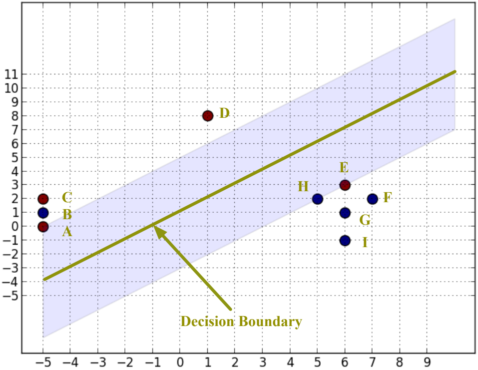
- Weight vector

$$\frac{1}{||w||}=\frac{1}{\sqrt{\left(\frac{2}{5}\right)^2+\left(\frac{4}{5}\right)^2}}=\frac{1}{\sqrt{\frac{20}{25}}}=\frac{5}{\sqrt{5}\sqrt{4}}=\frac{\sqrt{5}}{2} \tag{5}$$

Slack Example

Decision function:

$$w = \begin{bmatrix} -\frac{1}{4} \\ \frac{1}{4} \end{bmatrix}; b = -\frac{1}{4}$$



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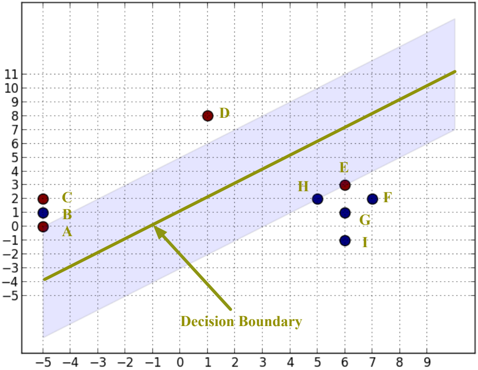

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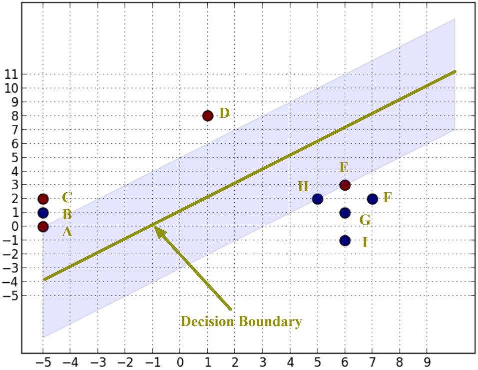

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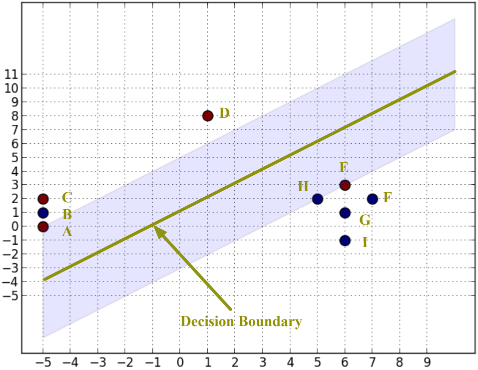

- What are the support vectors?
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- Compute ξ_B, ξ_E

Computing slack

$$y_i(\vec{w}_i \cdot x_i + b) \geq 1 - \xi_i \tag{6}$$

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Point B

$$y_B(\vec{w}_B \cdot x_B + b) = \tag{7}$$

$$-1(-0.25 \cdot -5 + 0.25 \cdot 1 - 0.25) = -1.25 \tag{8}$$

Thus, $\xi_B = 2.25$

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Point E

$$y_E(\vec{w}_E \cdot x_E + b) = \tag{9}$$

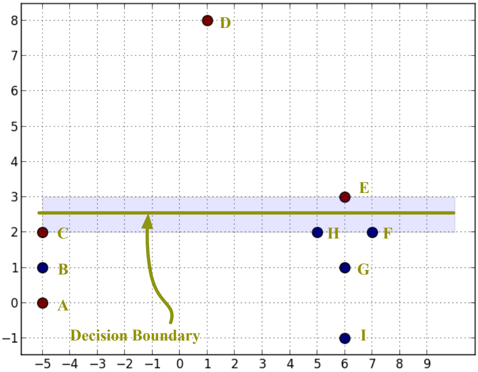
$$1(-0.25 \cdot 6 + 0.25 \cdot 3 + -0.25) = -1 \tag{10}$$

Thus, $\xi_E = 2$

Slack Example

Decision function:

$$w = \begin{bmatrix} 0 \\ 2 \end{bmatrix}; b = -5$$



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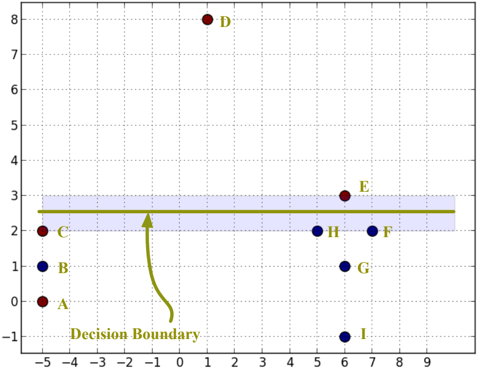

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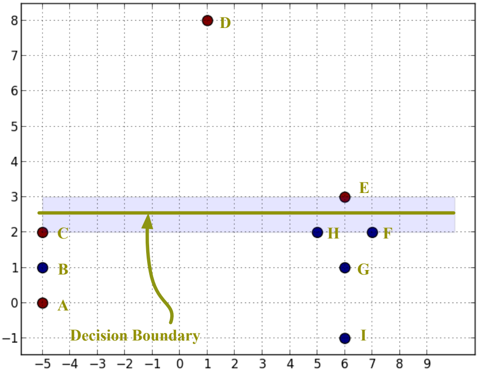

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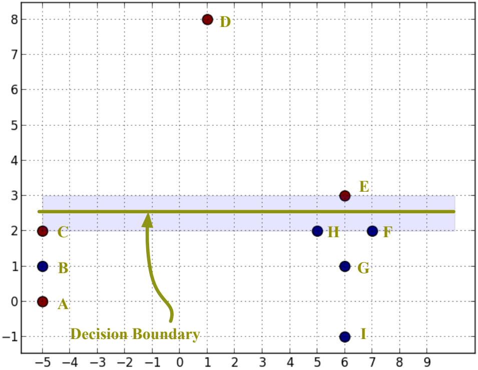

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Point A

$$y_A(\vec{w}_A \cdot x_A + b) = \tag{12}$$

$$1(0 \cdot -5 + 2 \cdot 0 + -5) = -5 \tag{13}$$

Thus, $\xi_A = 6$

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Thus, $\xi_A = 6$

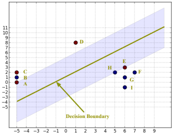
Point C

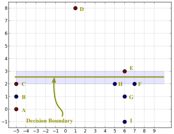
$$y_C(\vec{w}_C \cdot x_C + b) = \tag{14}$$

$$1(0 \cdot -5 + 2 \cdot 2 + -5) = -1 \tag{15}$$

Thus, $\xi_C = 2$

Which one is better?





- Which decision boundary (wide / narrow) has the better objective?

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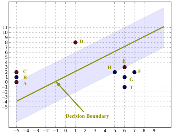

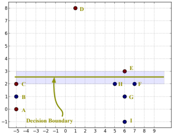

- Which decision boundary (wide / narrow) has the better objective?

$$\min_{w} \frac{1}{2}||w||^2 + C\sum_{i} \xi_i \tag{16}$$

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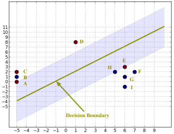

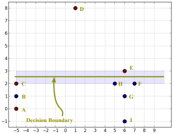

$$\frac{1}{2}||w||^2 = \frac{1}{2}\left(\frac{-1}{4}^2 + \frac{1}{4}^2\right) = 0.0625 \tag{16}$$

$$\sum_i \xi_i = 4.25 \tag{17}$$

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$$\min_w \frac{1}{2}||w||^2 + C\sum_i \xi_i \tag{18}$$

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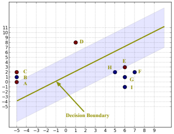

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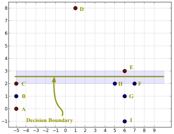

$$\frac{1}{2}||w||^2 = \frac{1}{2}\left(2^2\right) = 2 \tag{18}$$

$$\sum_i \xi_i = 8 \tag{19}$$

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$$\min_w \frac{1}{2}||w||^2 + C\sum_i \xi_i \tag{20}$$

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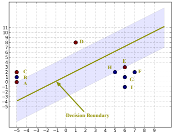

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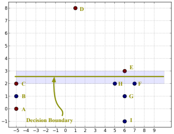

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- In this case it doesn't matter. Common C values: 1.0, $\frac{1}{m}$

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- Don't trust default values
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- Next time: how to find w