



Probability Distributions: Discrete

Introduction to Data Science Algorithms

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Binomial distribution

- Bernoulli: distribution over two values (success or failure) from a single event
- **binomial**: number of successes from *multiple* Bernoulli events
- Examples:
 - The number of times “heads” comes up after flipping a coin 10 times
 - The number of defective TVs in a line of 10,000 TVs
- Important: each Bernoulli event is assumed to be independent
- Notation: let X be a random variable that describes the number of successes out of N trials.
 - The possible values of X are integers from 0 to N : $\{0, 1, 2, \dots, N\}$

Binomial distribution

- Suppose we flip a coin 3 times. There are 8 possible outcomes:

$$P(HHH) = P(H)P(H)P(H) = 0.125$$

$$P(HHT) = P(H)P(H)P(T) = 0.125$$

$$P(HTH) = P(H)P(T)P(H) = 0.125$$

$$P(HTT) = P(H)P(T)P(T) = 0.125$$

$$P(THH) = P(T)P(H)P(H) = 0.125$$

$$P(THT) = P(T)P(H)P(T) = 0.125$$

$$P(TTH) = P(T)P(T)P(H) = 0.125$$

$$P(TTT) = P(T)P(T)P(T) = 0.125$$

- What is the probability of landing heads x times during these 3 flips?

Binomial distribution

- What is the probability of landing heads x times during these 3 flips?
- 0 times:
 - $P(TTT) = 0.125$
- 1 time:
 - $P(HTT) + P(THT) + P(TTH) = 0.375$
- 2 times:
 - $P(HHT) + P(HTH) + P(THH) = 0.375$
- 3 times:
 - $P(HHH) = 0.125$

Binomial distribution

- The probability mass function for the binomial distribution is:

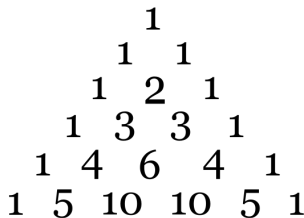
$$f(x) = \underbrace{\binom{N}{x}}_{\text{"N choose x"}} \theta^x (1 - \theta)^{N-x}$$

- Like the Bernoulli, the binomial parameter θ is the probability of success from one event.
- Binomial has second parameter N : number of trials.
- The PMF important: difficult to figure out the entire distribution by hand.

Aside: Binomial coefficients

- The expression $\binom{n}{k}$ is called a *binomial coefficient*.
 - Also called a *combination* in combinatorics.
- $\binom{n}{k}$ is the number of ways to choose k elements from a set of n elements.
- For example, the number of ways to choose 2 heads from 3 coin flips:
HHT, HTH, THH
 $\binom{3}{2} = 3$
- Formula:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$



Pascal's triangle depicts the values of $\binom{n}{k}$.

Bernoulli vs Binomial

- A Bernoulli distribution is a special case of the binomial distribution when $N = 1$.
- For this reason, sometimes the term binomial is used to refer to a Bernoulli random variable.

Example

- Probability that a coin lands heads *at least* once during 3 flips?

Example

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$$P(X \geq 1)$$

Example

- Probability that a coin lands heads *at least* once during 3 flips?

$$\begin{aligned}P(X \geq 1) &= P(X = 1) + P(X = 2) + P(X = 3) \\ &= 0.375 + 0.375 + 0.125 = 0.875\end{aligned}$$