



Department of Computer Science
UNIVERSITY OF COLORADO **BOULDER**



Probability Distributions: Continuous

Introduction to Data Science Algorithms

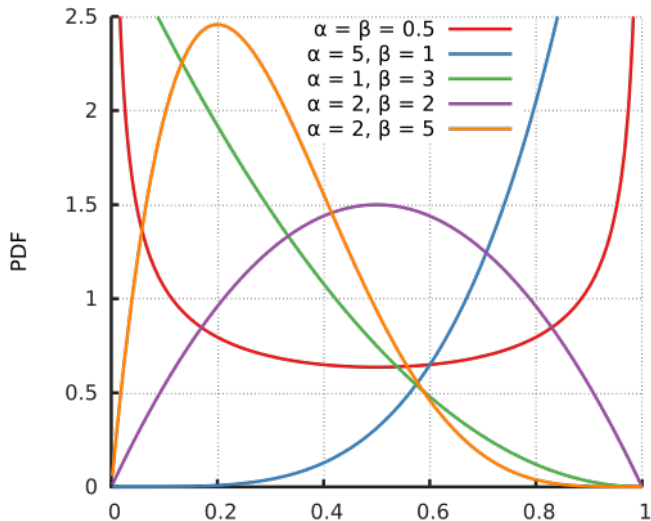
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Beta distribution

- The Beta distribution is over real values on the interval $[0, 1]$.
- Useful distribution for modeling percentages and proportions
 - Batting averages in baseball
 - Percentage of people with a disease in a country
- The density is proportional to: $x^{\alpha-1}(1-x)^{\beta-1}$
- Related to the Bernoulli distribution: $x^{\theta}(1-x)^{1-\theta}$

Beta distribution



Beta distribution

- The PDF for Beta is:

$$f(x) = \frac{\Gamma(\alpha + \beta)}{\underbrace{\Gamma(\alpha) + \Gamma(\beta)}} x^{\alpha-1} (1-x)^{\beta-1}$$

Inverse Beta function,
related to the binomial coefficient

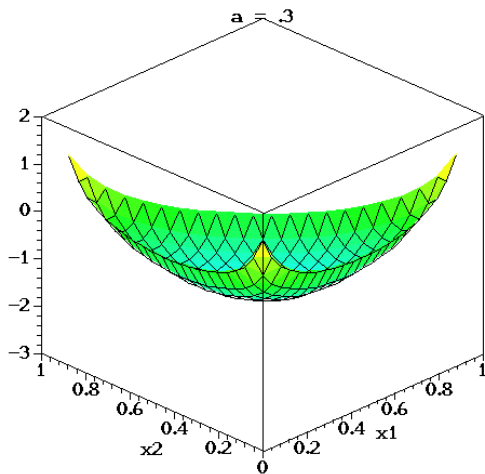
- Γ is the gamma function, $\Gamma(x) = (x-1)!$
 - Just like the factorial function, but works for real values in addition to integers
- Mean: $\mathbb{E}[X] = \frac{\alpha}{\alpha + \beta}$
- The parameters α and β must be > 0 .

Dirichlet distribution

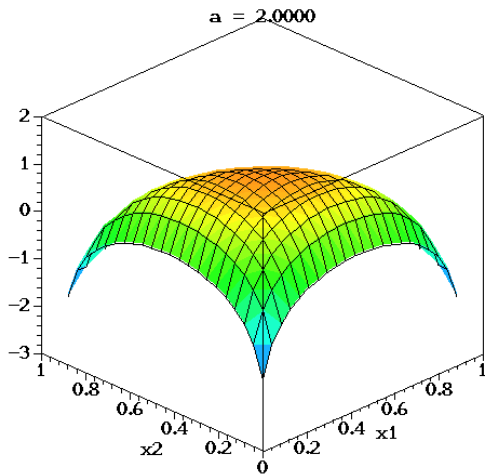
- The Dirichlet distribution is a generalization of the Beta distribution for multiple random variables
- The Dirichlet distribution is over *vectors* whose values are all in the interval $[0, 1]$ and the sum of values in the vector is 1.
 - In other words, the vectors in the sample space of the Dirichlet have the same properties as probability distributions.
 - The Dirichlet distribution can be thought of as a “distribution over distributions”.
- The PDF for a K -dimensional Dirichlet distribution has a vector of parameters denoted α , given by:

$$f(\mathbf{x}) = \frac{\prod_{k=1}^K \Gamma(\alpha_k)}{\Gamma(\sum_{k=1}^K \alpha_k)} \prod_{k=1}^K x_k^{\alpha_k - 1}$$

Dirichlet distribution



Dirichlet distribution



Dirichlet distribution

- The Dirichlet PDF looks similar to the multinomial distribution.
 - The Dirichlet density is proportional to: $\prod_k x_k^{\alpha_k-1}$
 - The multinomial mass is proportional to: $\prod_k x_k^{\theta_k}$
- Remember this analogy:
 - **Beta : binomial :: Dirichlet : multinomial**