



Department of Computer Science  
UNIVERSITY OF COLORADO **BOULDER**



## Classification: VC Dimension

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University of Colorado Boulder

LECTURE 7

## Questions

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## VC Dimension

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To show VC dimension of a set of points

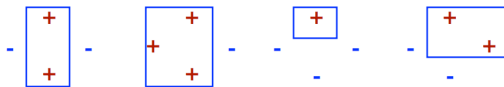
- Show that **a** set of  $d$  *can* be shattered
- Show that **no** set of  $d + 1$  can be shattered

## Axis Aligned Rectangles

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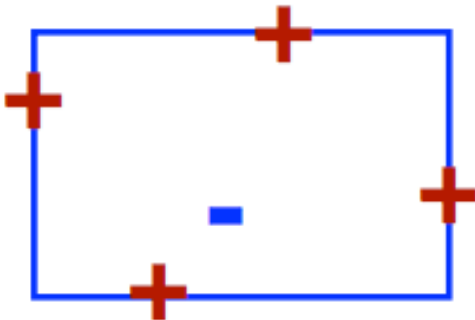
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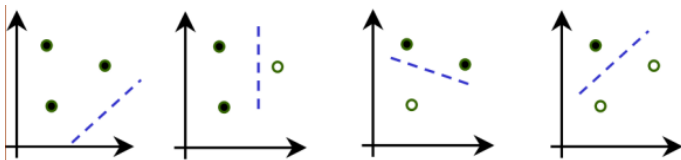
# Hyperplanes

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## Hyperplanes

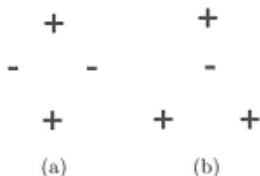
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**Figure 3.2** Unrealizable dichotomies for four points using hyperplanes in  $\mathbb{R}^2$ . (a) All four points lie on the convex hull. (b) Three points lie on the convex hull while the remaining point is interior.

## Hyperplanes

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**Figure 3.2** Unrealizable dichotomies for four points using hyperplanes in  $\mathbb{R}^2$ . (a) All four points lie on the convex hull. (b) Three points lie on the convex hull while the remaining point is interior.

In general, the VC dimension of  $d$ -dimensional hyperplanes is  $d + 1$

## Finite Subsets

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- If a set has  $d$  points, there are  $2^d$  ways to do that
- Each configuration requires a different hypothesis
- Solving for the number of hypotheses gives  $\lg |H|$



## Next time

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- Getting more practical
- SVMs
- Excellent theoretical properties